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Frolov V., Redkina K., Chen Xinjian, He Fupeng

VORTEX SHEETS IN FLOW AROUND AIRFOILS WITH SPOILERS

The main purpose of this paper is to build the mathematical model of the unsteady flow around the combination of airfoil and spoiler by the numerical-analytical method (NAM) [1]. Previously, the problem of flow airfoil with spoiler was considered in paper [2]. In this paper two types of airfoils are investigated: arc-aerofoil and aerofoil with ellipse nose part.

The geometry structure of airfoil with spoiler is shown in Figure 1. The nose part is an ellipse, the tail consists of two symmetrical arcs, and the spoiler is simplified as a straight line. For the E-xxxx airfoil [1] the boundary surface consists of many small elements. The elements in the tail are curved. The control point is located on the 3/4 of each element and the discrete vortex point is located on the 1/4 of each element.

The geometry structure of the arc-airfoil with spoiler is shown in Figure 2. The main body part is two symmetrical arcs, and the spoiler we simplify as a straight line. For the arc-airfoil, it is just need divided only spoiler, because the arc-airfoil have conform mapping function. The boundary of the arc-airfoil and the structure for each element are shown in Figure 2.

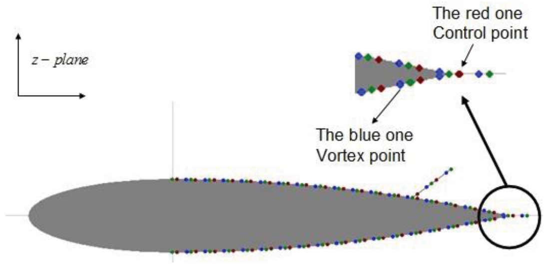


Figure1: Boundary elements of airfoil

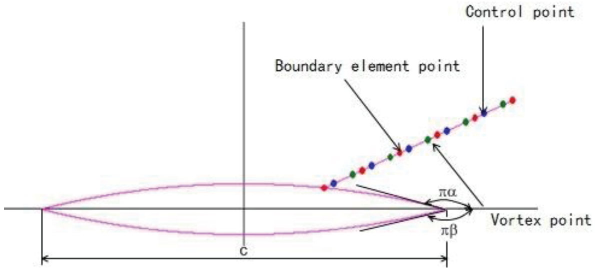


Figure2: Boundary elements of arc-airfoil

We assume that with two sharp edges on the end of the spoiler and the trail edge of the airfoil will break away free vortices.

Let the sharp edge on the spoiler will be the last element of the spoiler and the sharp edge of the airfoil is on the tail point. The free vortices will start to separate from the two vortex points on the two sharp edges.

The model is built in ζ -plane, so it is necessary to introduce conform mapping functions for two airfoils

$$\zeta = z \pm \sqrt{z^2 - c^2} \quad \text{- E-xxxx airfoil;}$$

$$\zeta = \left[\frac{(z+1)^k + (z-1)^k}{(z+1)^k - (z-1)^k} - i \operatorname{ctg} \gamma \right] \sin \gamma \quad \text{- arc-airfoil} \qquad (1)$$

Before adding Kutta condition, the complex potential [3] is

$$W(\zeta)=m_\infty(\overline{V_\infty}+V_\infty\frac{R^2}{\zeta})+\frac{1}{2\pi i}\sum_{j=1}^{N_0+1}\Gamma_j\ln\left[\frac{(\zeta-\zeta_{vj})\zeta}{\zeta-\dfrac{R^2}{\overline{\overline{\zeta_{vj}}}}}\right], \tag{2}$$

where $m_\infty=0,5$ for E-xxxx aerofoil and $m_\infty=k$ for arc-aerofoil.

The boundary conditions on the surface requires that the velocity component directed in each control point along normal to the solid surface must be zero, thus

$$[A][\Gamma]=[R] \tag{3}$$

If we use the complex potential (2) to solve the system (3) at the first time, the circulation matrix can be obtained. Two important circulations must be used, the first is the circulation of the vortex on the sharp edge of the spoiler $\Gamma_{S1}\left(\Gamma_{N_0}\right)$ and the second is the circulation of the vortex on the sharp edge of the airfoil $\Gamma_{T1}(\Gamma_{N_0+1})$. According to Kelvin's theorem, the circulation of the Kutta point will be $\Gamma_k=-(\Gamma_{S1}+\Gamma_{T1})$. The intensity of the separated vortex breaking away from the spoiler is equal to the intensity of the last vortex and a new vortex will appear at the location, so we have $\Gamma_{spoiler}=\Gamma_{N_0}=\Gamma_{S1}$. Before the first pair of moving vortices, the initial velocity is obtained under the guidance of the complex potential below, Kutta condition can make sure the velocity direction of the vortex on the airfoil tail is along the chord line

$$W(\zeta)=m_\infty\cdot(\overline{V_\infty}+V_\infty\frac{R^2}{\zeta})+\frac{1}{2\pi i}\sum_{j=1}^{N_0+1}\Gamma_j\ln\left[\frac{(\zeta-\zeta_{vj})\zeta}{\zeta-\dfrac{R^2}{\overline{\overline{\zeta_{vj}}}}}\right]+\frac{\Gamma_{N_0+2}}{2\pi i}\ln(\zeta). \tag{4}$$

The complex potential for many moving vortices will become dynamic

$$W(\zeta)=m_\infty\overline{V_\infty}+V_\infty\frac{R^2}{\zeta})+\frac{1}{2\pi i}\sum_{j=1}^{N_0+1}\Gamma_j\ln\left[\frac{(\zeta-\zeta_{vj})\zeta}{\zeta-\dfrac{R^2}{\overline{\overline{\zeta_{vj}}}}}\right]+\frac{\Gamma_{N_0+2}}{2\pi i}\ln(\zeta)+\frac{1}{2\pi i}\sum_{p=1}^{M_S}\Gamma_{Sp}\ln\left[\frac{(\zeta-\zeta_{Svp})\zeta}{\zeta-\dfrac{R^2}{\overline{\overline{\zeta_{Svp}}}}}\right]+\frac{1}{2\pi i}\sum_{q=1}^{M_T}\Gamma_{Tq}\ln\left[\frac{(\zeta-\zeta_{Tqp})\zeta}{\zeta-\dfrac{R^2}{\overline{\overline{\zeta_{Tqp}}}}}\right]. \tag{5}$$

M_S is the total number of dynamic vortices breaking out from spoiler. M_T is the total number of dynamic vortices coming from airfoil tail. At each time step, the system (3) should be solved with the new pair of free vortices. Using formula (5), the velocity field can obtain. The column vector $[R]$ will be refreshed at each time step and the difference is that the number of free vortices will be another. To calculate the new locations of the free vortices to solve a system of equations for the trajectories this vortices

$$\begin{cases} \dfrac{dx}{dt} = u \\ \dfrac{dy}{dt} = v \end{cases} \tag{6}$$

Velocity components in system (6) are calculated by the following formula [3]

$$u = \mathrm{Re}\left(\frac{dW}{d\zeta}\frac{d\zeta}{dz}\right); v = -\mathrm{Im}\left(\frac{dW}{d\zeta}\frac{d\zeta}{dz}\right). \tag{7}$$

At each time step size of the matrix $[A]$ will not change and will be equal $(N_0 + 2)$.

To verify this modeling method, it is necessary to build the model of unsteady flow around flat plate. In Figure 3 can see a comparison of the location of the vortex sheets at various time points. It is obvious that the core of vortex sheet is located in one line and the angle of this line approximately equals the angle of attack.

Comparison with Aubakirov's and etc. result [4] is shown in Figure 3 and Figure 4.

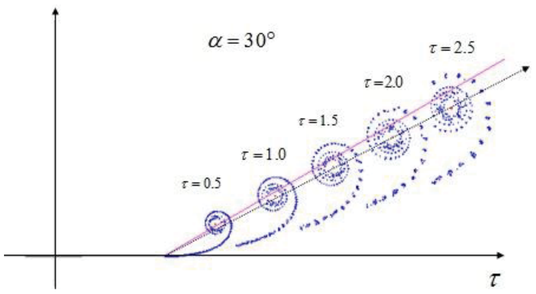


Figure 3: Unsteady flow around the flat plate

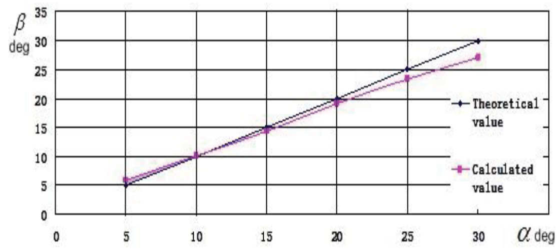


Figure 4: Angle of line and angle of attack

Another way to get the accuracy is as follows. The theoretical lift coefficient of flat plate is

$$C_L = 2\pi \sin(\alpha).$$

When $\alpha = 30°$, $C_L = \pi \approx 3{,}142$. The calculated lift coefficient is

$$C_L = -2\overline{\Gamma}, \overline{\Gamma} = \frac{\sum_{i=1}^{n} \Gamma_i}{V_\infty \cdot C}.$$

In each time step C_L can be calculated and a convergent result will be obtained after enough time as Figure 5 shows. Dimensionless time is $\tau = \dfrac{V_\infty \cdot t}{c}$.

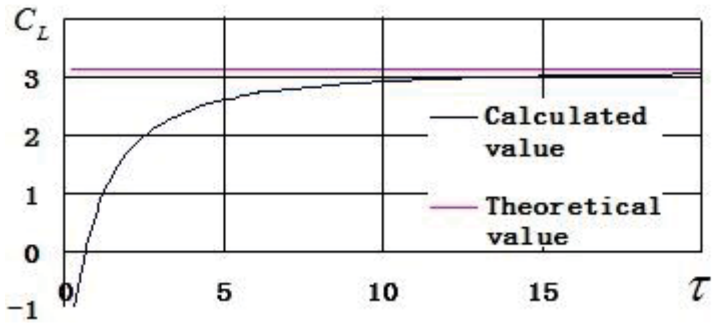
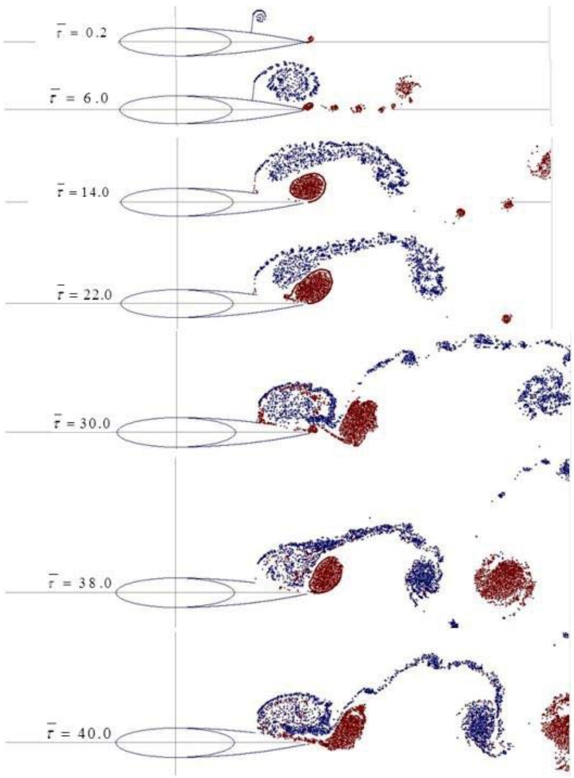
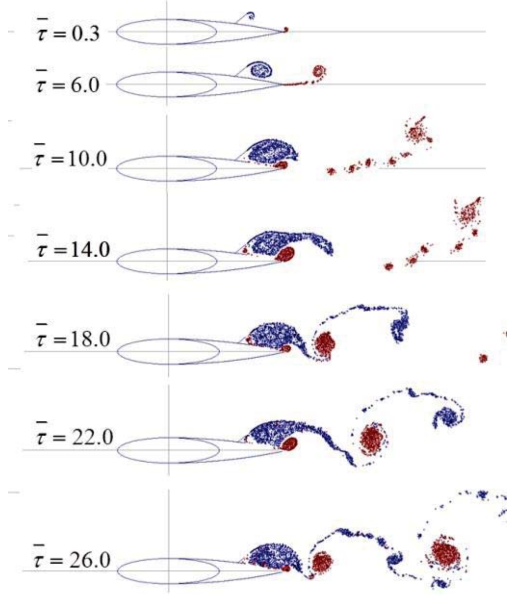


Figure 5: Variation of C_L with dimensionless time

The calculated C_L converges to 3.121 and the accuracy is 0.63%, which can be accepted in this method.

Figure 6, 7 shows the variation of the separation zone with non-dimensional time.



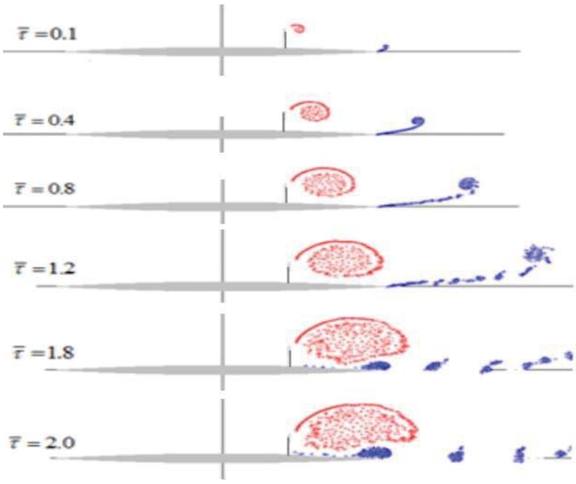


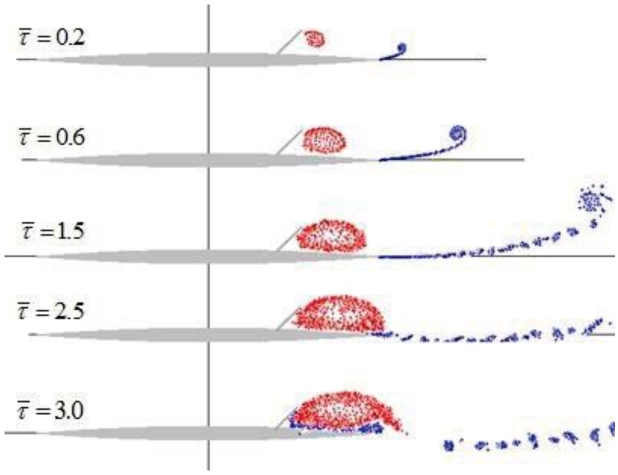
$\alpha = 10^\circ$; $\bar{x}_s = 0,7$; $\bar{c}_s = 0,1$; $\bar{t} = 15\%$; $\bar{x}_t = 0,3$

Figure 6: Case $\delta_s = 90^\circ$

Figure 7: Case $\delta_s = 45^\circ$

Figure 8 and Figure 9 shows the variation of the separation zone at non-dimensional time for arc-airfoil with different deflection angle for spoiler.





$\alpha=10^{\circ}$; $\bar{x}_s=0,7$; $\bar{c}_s=0,1$; $\bar{t}=4\%$; $\bar{x}_t=0,5$

Figure 8: $\delta_s=90^{\circ}$ Figure 9: $\delta_s=45^{\circ}$

Figures show if we change the deflection angle of spoiler, the size and shape of the separation zone will be different with time increasing. If we observe the dynamic process, we can get the result that when the deflection angle of the spoiler increases, the non-dimensional time will we need be less for the combination of the two clouds which break out from the edge of the spoiler and airfoil. Another result is also obtained that with time increasing, the clouds are going to be bigger and change little, at the same time, the cloud coming off from the edge of airfoil will go far away.

This paper has provided a method to build the mathematical model of the unsteady flow around the arc-airfoil and E-xxxx airfoil with spoiler under the assumption of ideal potential flow. We can see how the shape and size of the separation zone developing with time.

References

1 Frolov, V.A. Method for Building Potential Flow Around Symmetrical Aerofoil Formed Arcs of Ellipse and Circle [Text] /V.A. Frolov //Proceedings XII Russian Science-Technical Seminar Moving Control and Navigation of Vehicles /SSAU, Samara, 2003 – P. 260-265 (in Russian).

2 Cheng Xu. A vortex method for separated flow around an airfoil with detached spoiler [Text] //J. Computational Mechanics, 23, 1999, P.271-278.

3 Milne-Thomson, L. M. Theoretical Aerodynamics [Text] /L. M. Milne-Thomson – London: Constable and Company – 1973, 430 p.

4 Aubakirov, T.O. Nonlinear Theory of Wing and Its Applications [Text] /T.O. Aubakirov et al. – Almaty, "Gulum" – 1997, 448 p. (in Russian).