Old Dominion University

# Measurement of the Photon Beam Asymmetry in $\boldsymbol{\gamma} P \rightarrow K^{+} \boldsymbol{\Sigma}^{0}$ at $E_{\boldsymbol{\gamma}}$ $=8.5 \mathrm{GeV}$ with GlueX 

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# MEASUREMENT OF THE PHOTON BEAM ASYMMETRY IN $\gamma P \rightarrow K^{+} \Sigma^{0}$ AT $E_{\gamma}=$ 8.5 GEV WITH GLUEX 

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M.S. May 2017, Old Dominion University

A Dissertation Submitted to the Faculty of Old Dominion University in Partial Fulfillment of the Requirements for the Degree of<br>DOCTOR OF PHILOSOPHY<br>PHYSICS<br>OLD DOMINION UNIVERSITY<br>May 2020

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## ABSTRACT

# MEASUREMENT OF THE PHOTON BEAM ASYMMETRY IN $\gamma P \rightarrow K^{+} \Sigma^{0}$ AT $E_{\gamma}=8.5$ GEV WITH GLUEX 

Nilanga Indrajie Wickramaarachchi

Old Dominion University, 2020
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In this work the photon beam asymmetry $\Sigma$ for the reaction $\gamma p \rightarrow K^{+} \Sigma^{0}(1193)$ is measured using the GlueX experiment in Hall D at Jefferson Lab. The analysis used data that were collected using a linearly polarized photon beam in the energy range (8.2-8.8) GeV incident on a liquid hydrogen target. The beam asymmetries are measured as a function of the Mandelstam variable $t$ and as a single value for the low $u$ region. These are the first exclusive measurements of the photon beam asymmetry $\Sigma$ in this reaction at energies above the baryon resonance region. For the $t$-channel, results are close to unity and show that the reaction is dominated by the natural-parity exchange of the $K^{*}$ (892) Regge trajectory as predicted by theoretical models. A value of $\Sigma=0.410 \pm 0.090$ is obtained for the $u$-channel and suggests the exchange of both $\Sigma(J=1 / 2)$ and $Y^{*}(J=3 / 2)$ trajectory contributions to the production of $K^{+} \Sigma^{0}$.

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## ACKNOWLEDGMENTS

I would first like to express my gratitude to my supervisor Dr. Moskov Amaryan for giving me the opportunity to work on this analysis. I am thankful for his support and guidance throughout my graduate research. I would also like to thank Dr. Anatoly Radyushkin, Dr. Stephen Bueltmann, Dr. Alexander Gurevich and Dr. Alvin Holder for kindly agreeing to serve on my dissertation committee. I'm also grateful to Dr. Charles Hyde for the support he gave during this work as the Chair of the Physics Department at Old Dominion University.

I would like to specially thank everyone of the Hall D at Jefferson Lab including the group leader Eugene Chudakov for their extreme dedication during the experiment. I am grateful to the GlueX collaboration for all the comments made regarding the analysis and specially like to thank Mark Dalton, Alexander Austregesilo and Naomi Jarvis for their review which made many improvements to the analysis. I am also grateful to Dr. Justin Stevens and Simon Taylor for their suggestions and support throughout my research work. I would also like to thank Marouen Baalouch for giving me advice and guidance to be familiar with the analysis work when I was new to the research area.

I am very grateful to my parents, Wickramaarachchi Siriwardane and Leelawathie Meegahakotuwa for their support and love which helped me to focus on my research work. I am also thankful to my brother, Thilina Wickramaarachchi for his support during my time as a graduate student.

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## CHAPTER 1

## INTRODUCTION

The state $K^{+} \Sigma^{0}$ belongs to the so-called $K Y$ final states which have a kaon (a pseudoscalar meson) and a hyperon Y with one strange quark. Photoproduction is an interesting reaction to generate these states since there are no strange quarks initially in the system therefore a strange ( $s$ ) anti-strange ( $\bar{s}$ ) quark pair should be formed to conserve the strangeness. Polarization observables extracted using these reactions would be helpful to understand the production mechanisms dominating such processes. The work of this thesis is done using data from the GlueX experiment for photoproduction from a proton target: $\gamma p \rightarrow K^{+} \Sigma^{0}$.

The $\Sigma^{0}$ has a mean lifetime of $(7.4 \pm 0.7) \times 10^{-20}$ s and instantaneously decays to $\Lambda^{0} \gamma$. The branching fraction for this decay mode is $100 \%$.

High statistics measurements of differential cross sections and polarization observables for the reaction $\gamma p \rightarrow K^{+} \Sigma^{0}$ have been done by collaborations like CLAS [1, 2], SAPHIR [3], GRAAL [4] and LEPS [5] at lower energies. A list of these measurements can be found in the SAID database [6]. The highest beam energy measurements for differential cross section and polarized-photon asymmetry for $\gamma p \rightarrow K^{+}\left(\Lambda, \Sigma^{0}\right)$ have been made by SLAC at 16 GeV [7] and the individual asymmetries for $K^{+} \Lambda$ and $K^{+} \Sigma^{0}$ have been only determined based on cross section measurements.

Figure 1 shows Feynman diagrams for the different channels that can contribute to the reaction $\gamma p \rightarrow K^{+} \Sigma^{0}$. The $s, t$ and $u$-Mandelstam variables for the reaction are defined as:

$$
\begin{align*}
& s=\left(p_{\gamma}+p_{T}\right)^{2}  \tag{1}\\
& t=\left(p_{\gamma}-p_{K}\right)^{2}  \tag{2}\\
& u=\left(p_{T}-p_{K}\right)^{2} \tag{3}
\end{align*}
$$

where $p_{\gamma}, p_{T}$ and $p_{K}$ are four-momenta of the incoming photon beam, the target proton and the produced $K^{+}$meson respectively. For the lower photon beam energies, s-channel contribution is expected to dominate the production mechanism where the proton gets excited in the intermediate state. At higher energies above the resonance region $(\geq 4 \mathrm{GeV})$ the $t$ - and $u$-channel exchanges contribute to the production with the exchange of $K^{+}$or $K^{*+}$ for the
$t$-channel and baryons like $\Lambda, \Sigma$ and $Y^{*}$ for the $u$-channel respectively. In the case where the incoming photon beam is polarized, the study of beam asymmetry allows us to determine which type of exchange is happening during the reaction.


FIG. 1: Different channels contributing to the reaction $\gamma p \rightarrow K^{+} \Sigma^{0}$.

### 1.1 STRUCTURE OF THE THESIS

The structure of the thesis is organized as follows. Chapter 2 gives an overview of the theory. Starting with the standard model and then the quark model, a brief description is given about Regge theory and helicity amplitudes.

An overview of the detector components used in the GlueX experiment is presented in Chapter 3.

The data analysis primarily focusing on the event selection and the MC simulation is described in Chapter 4.

Chapters 5 and 6 explain how the beam asymmetry has been extracted and outlines studies to estimate different sources of systematic uncertainties.

Chapter 7 is devoted to comparison of experimental data with the model predictions and explains physics conclusions obtained from this comparison.

## CHAPTER 2

## THEORY

### 2.1 THE STANDARD MODEL

In the standard model, the fundamental particles of matter are divided into two categories. They are fermions that have half-integer spin and bosons with integer spins. Fermions obey Fermi-Dirac statistics while bosons obey Bose-Einstein statistics. The interactions between particles are considered to come through four fundamental forces: the strong, the electromagnetic, the weak and the gravitational force. Although the gravity is the much easier force to be observed in the world, in the high-energy physics, it is considered to be negligible.

Fermions are further divided into two types: quarks and leptons which have spin $1 / 2$. Quarks carry fractional electric charge and six types of them are observed: up (u), down $(d)$, charm $(c)$, strange $(s)$, top $(t)$ and bottom $(b)$. These are known as quarks of different flavors. Similarly there are six types of leptons: electron $\left(e^{-}\right)$, muon $\left(\mu^{-}\right)$, tau $\left(\tau^{-}\right)$and their corresponding neutrinos $\left(\nu_{e}, \nu_{\mu}\right.$ and $\left.\nu_{\tau}\right)$. Neutrinos don't have electric charge. Furthermore each quark and lepton has its anti-particle which has opposite electric charge and the same mass. Figure 2 shows the fundamental particles of the standard model.

Quarks can not be observed as individual free states. They combine to form hadrons which can be either mesons or baryons depending on how the quarks and (or) anti-quarks are combined. A quark anti-quark pair forms a meson which in turn has spin 0 or 1 making them bosons. A baryon is a combination of three valence quarks ( $q q q$ ). The proton (uud) and neutron ( $d d u$ ) are examples of baryons. Baryons have half-integer spins and therefore are fermions. One of the important feature about quarks is that only they can carry the color charge, the Quantum Chromo Dynamics (QCD) equivalent of the electric charge. Because of this, only quarks are involved in the strong interactions. Hadrons are formed as color-neutral states. For this criteria to be satisfied anti-quarks are assumed to have anti-colors.

Leptons can interact via electromagnetic or weak forces depending on whether they are charged or not. Neutrinos are mainly found in radioactive decays like $\beta$ decay of atomic nuclei or hadrons.


FIG. 2: Fundamental particles in the Standard Model. Quarks and leptons are arranged as columns corresponding to their generations.

Quarks and leptons are further classified as three generations. The $u$ - and $d$-quarks and the electron and electron-neutrino are considered as the first generation. The $c$ - and $s$-quarks and $\mu$ and $\nu_{\mu}$ form the second generation while the $t$ - and $b$-quarks and $\tau$ and $\nu_{\tau}$ form the third generation. When going from lower to higher generation, particle masses increase by orders of magnitude. Because of their lower masses, hadrons made out from $u, d$ and $s$ quarks are the ones that are extensively studied in experiments.

The four forces are considered to occur through exchange of particles called gauge-bosons between the interacting fermions. These mediators of forces have spin 1 so making them types of bosons. Each force can be described by an associated Quantum Field Theory (QFT). Quantum Electrodynamics (QED) explains the electromagnetic force between two charged particles via the exchange of a virtual photon. In the QCD, force between two quarks is described to occur through the exchange of a gluon. Both photon and gluon are massless. Contrary to these gauge bosons, the weak force is exchanged by massive W and

Z bosons. Weak charged-current interaction is mediated by the $\mathrm{W}^{+}$and $\mathrm{W}^{-}$bosons while the weak neutral-current interaction is mediated by the neutral Z boson. Weak interactions are unique in a sense that they can change the flavor of a quark from one to another. For example, a neutron can decay to a proton by the $\beta^{-}$decay and a down quark in the neutron changes to an up quark with the emission of a $\mathrm{W}^{-}$boson. The electroweak theory unifies electromagnetic and weak interactions.

Additionally there is a scalar boson (boson with spin 0 ) called Higgs boson which is considered to give masses to the particles that interacts with the so-called Higgs field. The Higgs boson was discovered by the ATLAS and CMS experiments at the Large Hadron Collider (LHC) in 2012. It has a mass of approximately 125 GeV .

### 2.2 THE QUARK MODEL

The quark model is based on the work by Murray Gell-Mann [8, 9], Yuval Ne'eman [10] and George Zweig [11]. It's a classification scheme where hadrons are made out of quarks and (or) antiquarks.

In quantum mechanics, for each symmetry of the Hamiltonian, we can find a conserved observable quantity. Mathematically, this deals with a unitary operator $\hat{U}$ which commutes with the Hamiltonian, i.e. $[\hat{H}, \hat{U}]=0$. This unitary operator can be associated with a Hermitian generator $\hat{G}$ which also commutes with the Hamiltonian and has real eigenvalues. This can be used to model hadrons with a symmetry group consideration.

It was proposed by Heisenberg that the nuclear force is approximately charge independent and neutron and proton can be considered as two states of the nucleon. The concept of isospin was introduced and the proton and neutron were assumed to have a total isospin $I=1 / 2$ and third component of isospin $I_{3}=1 / 2$ and $I_{3}=-1 / 2$ respectively. Similar to the spin-states of an electron this can be written as,

$$
\begin{equation*}
p=\binom{1}{0} \quad n=\binom{0}{1} \tag{4}
\end{equation*}
$$

Assuming up, down quark masses to be same, the strong interaction can be incorporated with a $u, d$ flavor symmetry. Following the way proton and neutron was written in isospin
space, then the $u$ and $d$ quarks are written as states in flavor space,

$$
\begin{equation*}
u=\binom{1}{0} \quad d=\binom{0}{1} \tag{5}
\end{equation*}
$$

In group theory terms, this allows to find the generators of the unitary operator which makes the QCD interaction to be invariant under transformation in $u, d$ flavor space as,

$$
\begin{equation*}
\hat{U}=\exp \left(i \alpha_{i} \hat{G}_{i}\right) \tag{6}
\end{equation*}
$$

where $\alpha_{i}$ is an infinitesimal transformation. Three of the generators form a special unitary $\mathrm{SU}(2)$ group and the Pauli spin-matrices given below are chosen that gives conserved observables.

$$
\sigma_{1}=\left(\begin{array}{ll}
0 & 1  \tag{7}\\
1 & 0
\end{array}\right) \quad \sigma_{2}=\left(\begin{array}{cc}
0 & -i \\
i & 0
\end{array}\right) \quad \sigma_{3}=\left(\begin{array}{cc}
1 & 0 \\
0 & -1
\end{array}\right)
$$

The isospin components can be written using Pauli matrices as,

$$
\begin{equation*}
\hat{T}_{i}=\frac{1}{2} \hat{\sigma}_{i} \tag{8}
\end{equation*}
$$

The total isospin is defined as $\hat{T}^{2}=\hat{T}_{1}^{2}+\hat{T}_{2}^{2}+\hat{T}_{3}^{2}$ and commutes with each of the isospin component. This allows to write $u$ and $d$ quarks as states $\phi\left(I, I_{3}\right)$ since the eigenvalue equation is given by,

$$
\begin{equation*}
\hat{T}^{2} \phi\left(I, I_{3}\right)=I(I+1) \phi\left(I, I_{3}\right) \tag{9}
\end{equation*}
$$

The up and down quarks are written in terms of isospin as,

$$
\begin{equation*}
u=\binom{1}{0}=\phi\left(\frac{1}{2},+\frac{1}{2}\right) \quad \text { and } d=\binom{0}{1}=\phi\left(\frac{1}{2},-\frac{1}{2}\right) \tag{10}
\end{equation*}
$$

Then, combinations of two quark states can be formed by $u$ and $d$ quarks considering the rules of adding $I$ and $I_{3}$. Adding another $u$ or $d$ quark allows to make 8 possible combinations uuu, uud, $u d u, u d d, d u u, d u d, d d u$ and $d d d$. The flavor wavefunctions of these states can be found using isospin arguments and isospin ladder operators for $u$ and $d$ quarks. This
gives a $I=\frac{3}{2}$ quadruplet and two $I=\frac{1}{2}$ doublets. This can be written using $\mathrm{SU}(2)$ group representation as,

$$
\begin{equation*}
2 \otimes 2 \otimes 2=4 \oplus 2 \oplus 2 \tag{11}
\end{equation*}
$$

with the three quark combinations are represented as isospin doublets. Similarly, spin wavefunctions can be constructed for three quarks and will have the same form since isospin and spin are analogous. The total wavefunction of three quark bound state has extra terms for color and spatial wavefunctions, but it can be found that the product of flavor and spin wavefunctions should be symmetric for the case with zero orbital angular momentum, i.e. $L=0$. This comes from the fact that the total wavefunction should be antisymmetric under interchange of any two quarks.

The up and down antiquarks ( $\bar{u}$ and $\bar{d}$ ) are written as a doublet in the following way so that they transform the same way as the quarks. This gives two $\mathrm{SU}(2)$ doublets for quarks and antiquarks as,

$$
\begin{equation*}
q=\binom{u}{d} \quad \bar{q}=\binom{-\bar{d}}{\bar{u}} \tag{12}
\end{equation*}
$$

Using this isospin representation, four states of mesons can be formed by up and down quarks and antiquarks. Written as states of $\phi\left(I, I_{3}\right)$ they are, [12]

$$
\begin{align*}
\phi(1,-1) & =d \bar{u} \\
\phi(1,0) & =\frac{1}{\sqrt{2}}(u \bar{u}-d \bar{d}) \\
\phi(1,+1) & =-u \bar{d} \\
\phi(0,0) & =\frac{1}{\sqrt{2}}(u \bar{u}+d \bar{d}) \tag{13}
\end{align*}
$$

So, this gives an isospin- 1 triplet and an isospin- 0 singlet and represented as,

$$
\begin{equation*}
2 \otimes \overline{2}=3 \oplus 1 \tag{14}
\end{equation*}
$$

Extending this procedure to include the $s$ quark, an $\mathrm{SU}(3)$ representation can be developed to associate a $u d s$ flavor symmetry of the Hamiltonian. This is only approximate, as the strange quark mass is much higher than masses of $u, d$ quarks but this difference is small compared to binding energies of hadrons. The $u, d$ and $s$ quarks are reprented in $\mathrm{SU}(3)$
reprentation as,

$$
u=\left(\begin{array}{l}
1  \tag{15}\\
0 \\
0
\end{array}\right) \quad d=\left(\begin{array}{l}
0 \\
1 \\
0
\end{array}\right) \quad s=\left(\begin{array}{l}
0 \\
0 \\
1
\end{array}\right)
$$

There are eight generators in $\mathrm{SU}(3)$ symmetry called Gell-Mann matrices defined as,

$$
\begin{equation*}
\hat{T}_{i}=\frac{1}{2} \lambda_{i} \tag{16}
\end{equation*}
$$

Only two of them ( $\hat{T}_{3}$ and $\hat{T}_{8}$ ) commute. Therefore, in $\mathrm{SU}(3)$, hadrons are descibed in terms of the third component of isospin $\left(I_{3}\right)$ and hypercharge $(Y)$ with operators,

$$
\begin{equation*}
\hat{T}_{3}=\frac{1}{2} \lambda_{3}, \hat{Y}=\frac{1}{\sqrt{3}} \lambda_{8} \tag{17}
\end{equation*}
$$

Similar to the case of $\mathrm{SU}(2)$, the addition of $I_{3}$ and $Y$ components of quarks leads to formation of hadrons. The corresponding ladder operators are used to determine the flavor wavefunctions. A quark ( $u, d$ or $s$ ) and an antiquark is combined to give nine possible combinations which are decomposed into an octet and a singlet as,

$$
\begin{equation*}
3 \otimes \overline{3}=8 \oplus 1 \tag{18}
\end{equation*}
$$

The singlet state is a linear combination of $u \bar{u}, d \bar{d}$ and $s \bar{s}$ and its flavor wavefunction is written as,

$$
\begin{equation*}
\psi_{s}=\frac{1}{\sqrt{3}}(u \bar{u}+d \bar{d}+s \bar{s}) \tag{19}
\end{equation*}
$$

A hadron state is usually represented by its quantum numbers as $J^{P C}$. Here, $J=L+S$ is the total angular momentum found by adding orbital and spin angular momenta quantum numbers. $P$ and $C$ are called parity and charge conjugation and they can be found by $P=(-1)^{L+1}$ and $C=(-1)^{L+S}$ respectively. The $L=0$ meson states are the simplest case and are called light mesons. Since the $q \bar{q}$ pair can give spin $S=0$ or $S=1$, this gives a nonet with $J=0$ pseudoscalar mesons and another nonet with $J=1$ vector mesons. They have parities of -1 . Figures 3 and 4 show the corresponding mesons and Table 1 represents the $J^{P C}$ assignments for mesons with $L<2$.


FIG. 3: Nonet of pseudoscalar mesons. $S$ and $Q$ indicate the strangeness and the electric charge of the mesons.

The combination of three quarks gives total of 27 flavor states in $\mathrm{SU}(3)$ representation. These baryons are decomposed to a symmetric decuplet, two mixed symmetry octets and an antisymmetric singlet state as,

$$
\begin{equation*}
3 \otimes 3 \otimes 3=10 \oplus 8 \oplus 8 \oplus 1 \tag{20}
\end{equation*}
$$

Figures 5 and 6 show the observed octet and decuplet of $L=0$ light baryons.


FIG. 4: Nonet of vector mesons. $S$ and $Q$ indicate the strangeness and the electric charge of the mesons.

| L | S | J | P | C | $\mathrm{J}^{P C}$ | Type | Examples |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | - | + | $0^{-+}$ | Pseudoscalar | $\pi, \eta, \eta^{\prime}, K$ |
| 0 | 1 | 1 | - | - | $1^{--}$ | Vector | $\rho, \omega, \phi, K^{*}$ |
| 1 | 0 | 1 | + | - | $1^{+-}$ | Pseudovector | $b_{1}, h_{1}, h_{1}^{\prime}, K_{1}$ |
| 1 | 1 | 0 | + | + | $0^{++}$ | Scalar | $a_{0}, f_{0}, f_{0}^{\prime}, K_{0}^{*}$ |
| 1 | 1 | 1 | + | + | $1^{++}$ | Axial Vector | $a_{1}, f_{1}, f_{1}^{\prime}, K_{1}$ |
| 1 | 1 | 2 | + | + | $2^{++}$ | Tensor | $a_{2}, f_{2}, f_{2}^{\prime}, K_{2}^{*}$ |

TABLE 1: $J^{P C}$ assignments for mesons.


FIG. 5: Observed octet of baryons. $S$ and $Q$ indicate the strangeness and the electric charge of the baryons.


FIG. 6: Observed decuplet of baryons. $S$ and $Q$ indicate the strangeness and the electric charge of the baryons.

### 2.3 THE REGGE MODEL

In the Regge model the scattering amplitude of a two particle collision is written as a function of a complex angular momentum variable. It was proposed as an alternative approach to the quantum mechanical treatment of potential scattering. At high energies the poles of the scattering amplitude correspond to the exchange of the Regge trajectories. A Regge trajectory consists of a family of resonances with identical internal quantum numbers but different total angular momenta $J$. Members of such a family are related approximately by the linear relation $J_{i}=\alpha\left(m_{i}^{2}\right)$ between their total angular momenta and squared masses [13]. Regge trajectories for $K(494)$ and $K^{*}(892)$ families and $\Lambda, \Sigma$ and $\Sigma^{\star}$ families are shown in Figs. 7 and 8 respectively.


FIG. 7: Chew-Frautschi plots for the $K(494)$ and $K^{*}(892)$ trajectories [14].

The amplitude for $\gamma p \rightarrow K Y$ reaction is written considering the exchange of entire Regge trajectories rather than a finite selection of individual particles. For the forward scattering angles, $K^{*}$ trajectories are exchanged in the $t$-channel while $Y^{*}$ trajectories are exchanged in the $u$-channel for backward scattering angles.


FIG. 8: Chew-Frautschi plots for the $\Lambda, \Sigma$ and $\Sigma^{\star}$ trajectories [15].

A linear meson trajectory has the following form,

$$
\begin{equation*}
\alpha_{X}(t)=\alpha_{X, 0}+\alpha_{X}^{\prime}\left(t-m_{X}^{2}\right) \tag{21}
\end{equation*}
$$

where $m_{X}$ and $\alpha_{X, 0}$ are the mass and spin of the lightest member in the trajectory. The slope of the trajectory is given by $\alpha_{X}^{\prime}$.

The amplitude can then be written by replacing the Feynman propagator with the Regge propagator:

$$
\begin{equation*}
\frac{1}{t-m_{X}^{2}} \rightarrow \mathcal{P}_{\text {Regge }}^{X}\left[s, \alpha_{X}(t)\right] . \tag{22}
\end{equation*}
$$

Then the Regge amplitude is presented as

$$
\begin{equation*}
\mathcal{M}_{\text {Regge }}^{X}(s, t)=\mathcal{P}_{\text {Regge }}^{X}\left[s, \alpha_{X}(t)\right] \times \beta_{X}(s, t) \tag{23}
\end{equation*}
$$

where $\beta_{X}(s, t)$ is the residue of the original Feynman amplitude calculated from the interaction Lagrangians at the $\gamma K X$ and $p X Y$ vertices in the reaction $p(\gamma, K) Y$ with $X$ exchange particle.

Following this method, the general Regge propagator can be formed as shown in the Ref. [14].

$$
P_{\text {Regge }}^{X}(s, t)=\frac{\left(\frac{s}{s_{0}}\right)^{\alpha_{X}(t)-\alpha_{X, 0}}}{\sin \left(\pi \alpha_{X}(t)\right)}\left\{\begin{array}{c}
1  \tag{24}\\
e^{-i \pi \alpha_{X}(t)}
\end{array}\right\} \frac{\pi \alpha_{X}^{\prime}}{\Gamma\left(1+\alpha_{X}(t)-\alpha_{X, 0}\right)}
$$

where $s_{0}$ is a scale factor fixed at $1 \mathrm{GeV}^{2}$.
For fixed $s,\left|P_{\text {Regge }}^{X}(s, t=0)\right|$ increases with decreasing $\left|\alpha_{X}(0)-\alpha_{X, 0}\right|=\alpha_{X}^{\prime} m_{X}^{2}$. Slopes $\alpha_{X}^{\prime}$ for all meson trajectories are nearly the same, therefore trajectories with lowest mass first member of the corresponding family are expected to dominate the exchange. Because of this, for the $\gamma p \rightarrow K Y$ channel, $K(494)$ and $K^{*}(892)$ exchanges dominate.

### 2.4 RPR MODEL

In this model, a contribution from the resonance region is added to the Regge part of the amplitude to generate the "Regge-Plus-Resonance" model. Figure 9 shows the RPR amplitude for $\gamma p \rightarrow K Y$ process in the forward-angle region. According to this the kaon trajectories are exchanged in the $t$-channel and individual baryon resonances are also contributing in the $s$-channel. For photon energies greater than about 4 GeV , the $s$-channel contributions vanish by construction and only the Regge part of the amplitude is considered.


FIG. 9: General forward-angle RPR amplitude for the $\gamma p \rightarrow K Y$ process [14].
Using Eqn. 24, Regge propagators for $K(494)$ and $K^{*}(892)$ exchange can be written. When the interaction Lagrangian for $\gamma p \rightarrow K^{+} \Sigma^{0}$ process is used, the high energy amplitude contains only three parameters related to the coupling constants

$$
\begin{equation*}
g_{K^{+} \Sigma^{0} p}, G_{K^{*+}(892)}^{v, t}=\frac{e g_{K^{*+}(892) \Sigma^{0} p}^{v, t}}{4 \pi} \kappa_{K^{+} K^{*+}(892)} \tag{25}
\end{equation*}
$$

Here, $g_{K^{*+}(892) \Sigma^{0} p}^{v, t}$ are the strong vector and tensor couplings to the $K^{*+}(892)$ vector meson trajectory. Additionally, the phase of the trajectory is chosen to be constant or rotating. These parameters are used in the RPR model to predict the values for polarization observables, in particular the $t$-dependence of beam asymmetry.

### 2.5 HELICITY AMPLITUDES

The helicity of a particle is defined as the projection of its spin $\vec{s}$ along the direction of linear momentum $\vec{p}$ :

$$
\begin{equation*}
H=\frac{\vec{s} \cdot \vec{p}}{|\vec{s} \cdot \vec{p}|} \tag{26}
\end{equation*}
$$

Depending on whether the spin and momentum vectors are aligned in the same direction or in opposite directions, the helicity can be positive or negative respectively.

The photoproduction amplitude can be written in terms of functions which are related to the helicities of incoming and outgoing particles. These functions are called "helicity amplitudes" and provide information about the exchange mechanisms during the process. For a photoproduction of a spin zero meson and a nucleon, these have the amplitude written in general form $f_{a b c d}$ where $a, b, c$ and $d$ are the helicities of the incoming photon, target proton, produced spin zero meson and the nucleon.

In 1964, P. Stichel developed a theorem relating the photoproduction amplitude for $\gamma+$ $N \rightarrow \pi+N$ channel and the parity of the exchange particle $\pi_{j}[16]$. In brief, the theory goes as follows.

Let $\vec{k}$ and $\vec{q}$ be the momenta of the incoming photon and the outgoing pion in the center of mass system for the process $\gamma+N \rightarrow \pi+N$. In the Coloumb gauge we have the transversality condition $\vec{\epsilon} \cdot \vec{k}=0$ where $\vec{\epsilon}$ is the polarization vector of the photon. Then the differential cross section maybe written as 17]:

$$
\begin{equation*}
\frac{d \sigma}{d \Omega}=\frac{q}{k}\left|X_{f} F \quad X_{i}\right|^{2} \tag{27}
\end{equation*}
$$

where the photoproduction amplitude $F$ is a $2 \times 2$ matrix in spin space and $X_{i}$ and $X_{f}$ are the Pauli spinors for the initial and final nucleon spin states. The general form of $F$ is written as:

$$
\begin{equation*}
F=i \sin \phi\left[C_{1}(\vec{\sigma} \cdot \hat{k})+C_{2}(\vec{\sigma} \cdot \hat{n} \times \hat{k})\right]+\cos \phi\left[C_{3}+i(\vec{\sigma} \cdot \hat{n}) C_{4}\right] \tag{28}
\end{equation*}
$$

where $\phi$ is an angle which describes the orientation of $\vec{\epsilon}$ in a plane orthogonal to $\hat{k}$ :

$$
\begin{equation*}
\vec{\epsilon}=-\cos \phi \hat{n}+\sin \phi(\hat{n} \times \hat{k}) \tag{29}
\end{equation*}
$$

with $\hat{n} \equiv \hat{k} \times \hat{q}$. The $\vec{\sigma}$ are the Pauli spin matrices and the $C_{i}$ are functions of Mandelstam variables $s$ and $t$.

When summed over the nucleon spin states, Eqn. 27 gives,

$$
\begin{equation*}
\frac{\overline{d \sigma}}{d \Omega}=\frac{q}{k}\left[\sin ^{2} \phi\left(\left|C_{1}\right|^{2}+\left|C_{2}\right|^{2}\right)+\cos ^{2} \phi\left(\left|C_{3}\right|^{2}+\left|C_{4}\right|^{2}\right)\right] \tag{30}
\end{equation*}
$$

The Stichel theorem says,
"Provided that to the amplitude $F(\phi)$ for $\gamma+N \rightarrow \pi+N$ only exchange of particles or particle-systems with total angular momenta $j$ and either parity $\pi_{j}=(-1)^{j+1}$ or $\pi_{j}=(-1)^{j}$ in the crossed channel $\gamma+\pi \rightarrow N+\bar{N}$ contributes, then the $\phi$-dependence of $F(\phi)$ at high energies $s$ and small momentum transfer $t$ becomes:
a) $F(\phi) \sim \sin \phi$ for $\pi_{j}=(-1)^{j+1}$ resp.
b) $F(\phi) \sim \cos \phi$ for $\pi_{j}=(-1)^{j} "$ 16].

The theorem has been proved by writing functions $C_{i}$ in terms of helicity amplitudes and considering which amplitudes are non-vanishing for the cases $\pi_{j}=(-1)^{j+1}$ (unnatural parity) and $\pi_{j}=(-1)^{j}$ (natural parity). The theorem can be extended to complex angular momenta in the case that $\pi_{j}$ denotes a Regge trajectory exchanged in the $t$-channel.

Stichel theorem can also be used for the reaction $\gamma p \rightarrow K^{+} \Sigma^{0}$ and in Ref. [18] a relation between the photon beam asymmetry $\Sigma$ and exchanged parity is given. There a set of $s$ channel helicity amplitudes with definite parity in the $t$-channel are defined to leading order in $s$ as:

$$
\begin{align*}
f_{1} & =f_{1+, 0+}, \\
f_{2} & =f_{1+, 0-}, \\
f_{3} & =f_{1-, 0+}, \\
f_{4} & =f_{1-, 0-}, \tag{31}
\end{align*}
$$

Then the following combinations can be formed:

$$
\begin{align*}
f_{1}^{ \pm} & =\frac{1}{2}\left(f_{1} \pm f_{4}\right), \\
f_{2}^{ \pm} & =\frac{1}{2}\left(f_{2} \mp f_{3}\right), \tag{32}
\end{align*}
$$

where the superscript $+(-)$ indicates natural (unnatural) parity exchange in the $t$-channel.
The polarized photon asymmetry is given by

$$
\begin{align*}
\Sigma & =\left[\frac{d \sigma_{\perp}}{d t}-\frac{d \sigma_{\|}}{d t}\right] /\left[\frac{d \sigma_{\perp}}{d t}+\frac{d \sigma_{\|}}{d t}\right] \\
& =\frac{\left(\left|f_{1}^{+}\right|^{2}+\left|f_{2}^{+}\right|^{2}-\left|f_{1}^{-}\right|^{2}-\left|f_{2}^{-}\right|^{2}\right)}{\left(\left|f_{1}^{+}\right|^{2}+\left|f_{2}^{+}\right|^{2}+\left|f_{1}^{-}\right|^{2}+\left|f_{2}^{-}\right|^{2}\right)}, \tag{33}
\end{align*}
$$

where $\frac{d \sigma_{\perp}}{d t}\left(\frac{d \sigma_{\|}}{d t}\right)$ is for the cross section with a photon beam polarized perpendicular (parallel) to the reaction plane.

Using a linearly polarized photon beam allows us to study about the type of parity exchanged in the $t$-channel. Since beam asymmetry $\Sigma$ is a measure of the difference between contributions from perpendicular and parallel polarized photons to the cross section, it can be used to distinguish the relative contribution of natural and unnatural parity exchange.

## CHAPTER 3

## GLUEX SPECTROMETER

The GlueX spectrometer is located in Hall D at the Thomas Jefferson National Accelerator Facility (JLab) in Newport News, Virginia. The Continuous Electron Beam Accelerator Facility (CEBAF) (shown in Fig. 10) in JLab provides a continuous wave 12 GeV electron beam to Hall D. A high frequency laser is used in the CEBAF to produce electrons. The laser is incident on a GaAs photocathode and produce a highly polarized electron beam. The electron bursts are distributed as evenly spaced beam bunches (pulses) and accelerated by superconducting radiofrequency ( RF ) cavities before reaching the experimental halls [19]. The Nb RF cavities produce standing electromagnetic waves which are matched with the phase and frequency of the electron bunch. The cavities are designed with an accelerating frequency of 1497 MHz . The electron beam produced should match this frequency but to operate four halls simultaneously four lower frequency lasers are used providing 4 distinct electron beams to the halls $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D . The electron beams are accelerated due to the gradient from the standing waves in the cavities and run through the accelerating sections in the CEBAF up to five passes. Each pass provides energy of 2.2 GeV to the electron beam. Halls A, B and C receives the electron beam after 1-5 passes through the accelerator. A 249.5 MHz electron beam is delivered to Hall D and arrives the hall after 5.5 passes through the accelerator. This gives the potential to have a 12 GeV electron beam delivered to the Hall D and in turn allows to be used for many physics interests.

The electron beam hits a diamond radiator and produces a linearly polarized photon beam. The photon beam then reaches a liquid hydrogen target and produces many particles and showers after the interaction. The GlueX detector is a nearly $4 \pi$ hermetic detector and allows to identify all the particles in the final state of the reaction. The GlueX spectrometer consists of many detector components as shown in Fig. 11 and each of the major components are briefly described in this chapter.


FIG. 10: CEBAF at the Jefferson Lab. Figure from Ref. [20].


FIG. 11: The GlueX spectrometer.

### 3.1 BEAMLINE

The GlueX beamline consists of components that are used to create a linearly polarized photon beam and to measure the energy, polarization and the flux of the photon beam. The main components of the beamline are shown in Fig. 12 .


FIG. 12: Schematic of the GlueX beamline components and the spectrometer. Figure from Ref. [21].

### 3.1.1 DIAMOND RADIATOR

The 12 GeV electron beam from CEBAF enters the Tagger Hall that houses the radiator and the photon tagging system. The electron beam is incident on a thin (58 $\mu \mathrm{m}$ ) diamond wafer and produces a photon beam. The photon beam is created using the coherent Bremsstrahlung technique [22, 23]. The electron beam transfers momentum to a struck nucleus in the diamond radiator and forms a continuum of momentum transfer and the momentum values are averaged to give the incoherent component of the Bremsstrahlung process. The coherent component is obtained by allowing the momentum transfer to match with the reciprocal lattice vectors of the diamond. A goniometer device is used to hold the diamond radiator in order to align it precisely to achieve the desired coherent photon beam spectrum. The coherent spectra occurs at certain energy ranges of the photon beam but the nominal GlueX configuration uses the main coherent peak set around 9 GeV .

### 3.1.2 PHOTON TAGGING SYSTEM

The GlueX photon tagging system primarily consists of dipole magnets and two scintillation detectors called Tagger Hodoscope (TAGH) and Tagger Microscope (TAGM). The energy of the photon $\left(E_{\gamma}\right)$ is "tagged" by measuring the energy of the scattered electron $\left(E_{e}\right)$ from the diamond radiator and using the precisely known incoming electron beam energy $\left(E_{0}\right)$. It can be written as $E_{\gamma}=E_{0}-E_{e}$. Figure 13 shows a schematic of the photon tagging system.


FIG. 13: Schematic of the photon tagging system. Red dotted lines indicate the scattered electron trajectories and $k / E_{0}$ is the ratio of the photon and incoming electron beam energy. Blue dotted lines indicate the three layers of hodoscope counters. Blue shaded region indicates the TAGM active volume. Figure from Ref. [21].

The scattered electrons are deflected by the tagger magnet which is a dipole magnet. When the electrons have only small amount of energy lost due to the Bremsstrahlung process, or are not interacted with the radiator, they are deflected by $13.4^{\circ}$ into the electron beam dump. The electrons that lose more than $25 \%$ of the initial energy are deflected by the magnet and detected in TAGH or TAGM which lie on the focal plane on the magnet. Depending on the energy the scattered electrons are deflected by smaller or larger angles and go through the scintillating fiber bundles of TAGH or TAGM and read out by photomultipliers (PMTs). Those having large energies are bent less and those with smaller energies bend more. The tagger microscope detects scattered electrons with energies between 3.0 and 3.6 GeV and as a result tags the coherent photons between 8.4 and 9.0 GeV . It has counters that provide width of 9.1 MeV to 11.2 MeV and has a $0.1 \%$ energy resolution and timing resolution of 200 ps . The tagger hodoscope detects scattered electrons with energies from 0.22 GeV to
9.0 GeV and tags the photons from 3.0 GeV to 11.7 GeV . It has coarse counters with 8.5 MeV to 30 MeV energy width.

### 3.1.3 COLLIMATORS

The active collimator is located before the primary collimator. It consists of a large tungsten plate divided into two radial rings and four quadrants. The active collimator is used to monitor the photon beam position. The deviations of the beam from the nominal position are detected by studying the induced currents on the tungsten plates from particle showers caused by the photons.

The primary collimator is used to increase the fraction of linearly polarized photons delivered to the target. It is located 75 m downstream of the radiator and is shielded by lead. It is used with a 3.4 mm or 5.0 mm aperture. The coherent part of the photon spectrum produced from the diamond radiator has the maximum polarization and has a lower angular spread compared to the incoherent part. The coherent spectrum is usually spread within 25 $\mu \mathrm{rad}$ from the beam direction. Additionally there is a secondary collimator and a sweeping magnet after the primary collimator. These are used to prevent the secondary photons and charged particles produced in the primary collimator being delivered any further.

### 3.1.4 TRIPLET POLARIMETER

The triplet polarimeter (TPOL) is used to determine the degree of polarization of the photon beam after the collimation. The TPOL works based on the "triplet photoproduction" process [24, 25] where the photon beam interacts with an atomic electron in a Be target foil in front of a Silicon Strip Detector (SSD). A high energy $e^{+} e^{-}$pair is produced and the atomic electron recoils with enough momentum to leave the atom. The recoil $e^{-}$is detected by the SSD which contains 32 azimuthal sectors and 24 concentric rings. By measuring the azimuthal angular distribution of the recoil $e^{-}$, the degree of polarization of the photon beam can be found. The cross section for triplet photoproduction is given by,

$$
\begin{equation*}
\sigma_{t}=\sigma_{0}[1-P \Sigma \cos (2 \phi)], \tag{34}
\end{equation*}
$$

where $\sigma_{0}$ is the unpolarized cross section, $P$ is the polarization of the photon beam, $\Sigma$ the beam asymmetry for the process and $\phi$ the azimuthal angle of the recoil $e^{-}$from the polarzation plane of the incident photon beam. $\Sigma$ is calculated using QED [26]. Recoil electrons passing through the Be converter can also produce $\delta$-rays so instead of triplet beam asymmetry $\Sigma$, the TPOL measures the analyzing power $\Sigma_{A}$ which is a combination
of triplet production and $\delta$-ray production. By fitting the yield of azimuthal distribution of recoil $e^{-}$with

$$
\begin{equation*}
f(\phi)=A[1-B \cos (2 \phi)]=\sigma_{0}\left[1-P \Sigma_{A} \cos (2 \phi)\right], \tag{35}
\end{equation*}
$$

$P$ can be found as $B / \Sigma_{A}$. The analyzing power for different photon beam energies can be found by simulation [26].

In Fig. 14 the beam polarization is shown for different diamond orientations as a function of the incoming photon beam energy. Table 2 shows the corresponding polarization values for each diamond orientation. They are obtained from polarimeter measurements [26].


FIG. 14: Photon beam polarization as a function of the beam energy for different diamond orientations, as measured by the triplet polarimeter.

| Diamond orientation | Polarization | Statistical uncertainty |
| :---: | :---: | :---: |
| $0^{\circ}$ | 0.3536 | 0.0100 |
| $45^{\circ}$ | 0.3484 | 0.0102 |
| $90^{\circ}$ | 0.3472 | 0.0098 |
| $-45^{\circ}$ | 0.3513 | 0.0102 |

TABLE 2: Photon beam polarization for different diamond orientations from TPOL measurements.

### 3.1.5 PAIR SPECTROMETER

The Pair Spectrometer (PS) is used to measure the energy and the flux of photons after the collimation and also provides identification of the electron beam bunch corresponding to events with a coincidence hit in PS and TAGH/TAGM. The incoming beam photon produces a $e^{+} e^{-}$pair after interacting with the Be foil in front of the TPOL. This lepton pair is bent by a dipole magnet and detected by two scintillator detectors: a high-resolution hodoscope (PS) and a course hodoscope (PSC) as shown in Fig. 15. The high-resolution hodoscope reads out the light signals using Silicon Photomultipliers (SiPMs). Light from the PSC counters is read out by PMTs. Each detector has two arms positioned symmetrically with respect to the photon beamline and covers a momentum range between $3.0 \mathrm{GeV} / c$ and $6.2 \mathrm{GeV} / c$ of the lepton pair. This allows to reconstruct photons between energy 6.0 GeV to 12.4 GeV . PSC trigger helps to reduce the background originating from interactions of leptons in the magnet. PS hodoscope has energy resolution of about 25 MeV . PSC counters have timing resolution of about 120 ps . The pair spectrometer can also be used for energy calibration of the TAGH and TAGM.


FIG. 15: Schematic of the pair spectrometer [27]. The blue and purple dashed lines indicate the trajectories of $e^{-}$and $e^{+}$after being bent by the dipole magnet. PS and PSC indicate the high-resolution hodoscope and the coarse hodoscope.

### 3.2 MAIN SPECTROMETER

The GlueX main spectometer is shown in Fig. 16. The photon beam is incident on a liquid hydrogen target housed in a superconducting solenoid magnet. It provides a 2 T magnetic field and contains the start counter, central and forward drift chambers and the barrel calorimeter. The start counter is used to measure primary interaction time. Drift chambers detect charged particles while the barrel calorimeter is for detection of neutral showers. The forward calorimeter (FCAL) provides detection for forward photons and the time-of-flight is used for particle identification.


FIG. 16: Schematic of the main GlueX spectrometer.

### 3.2.1 LIQUID HYDROGEN TARGET

Figure 17 shows a schematic of the GlueX target. It's a cryogenic target and consists of a kapton cell containing liquid hydrogen $\left(\mathrm{LH}_{2}\right)$. The target is unpolarized and 30 cm long and has a diameter around 2 cm . The target is kept at a temperature and pressure around 20 K and 19 psi.

### 3.2.2 START COUNTER

The Start Counter (ST) surrounds the 30 cm long liquid hydrogen target and provides $\sim 90 \%$ of $4 \pi$ solid angle coverage relative to the target center (Fig. 18.) The start counter


FIG. 17: Schematic of the GlueX liquid hydrogen target. Figure from Ref. [28].
consists of an array of 30 scintillator paddles. These are arranged in a cylindrical shape and have pointed ends bending towards the beamline at the downstream end. Light sensing is done by SiPM detectors which are not affected by the 2 T magnetic field produced by the GlueX solenoid magnet. The ST is primarily used for the identification of the photon beam bunch associated with the primary interaction with the target. The time of the interaction is compared with the RF beam bunch timing to select the main beam bunch. The ST operates at tagged photon rates of up to $10^{8} \gamma / \mathrm{s}$. The start counter has an average measured timing resolution of 550 ps (FWHM) which considerably surpasses its design resolution of 825 ps (FWHM) [29]. The ST also can be used in particle identification by comparing hit times of particles in other sub detectors with the measured ST time.


FIG. 18: Start counter surrounding the $\mathrm{LH}_{2}$ target. The beam direction is from left to right down the central axis of the ST. Figure from Ref. [29].

### 3.2.3 TIME OF FLIGHT DETECTOR

The Time-of-flight (TOF) detector is located about 5.5 m downstream from the target. It consists of an array of scintillator paddles aligned perpendicular to each other as shown in Fig. 19. Most of the paddles are 252 cm long and 6 cm wide with a thickness of 2.54 cm . There is a $12 \times 12 \mathrm{~cm}^{2}$ square hole in the center of the TOF that matches the beam hole in the forward calorimeter to allow photon beam to travel to the beam dump. The TOF covers polar angles from $0.6^{\circ}$ to $13^{\circ}$ and has a timing resolution of around 100 ps . It is used for the particle identification for forward going charged particles. There are PMTs to read out energy and time of those particles that have hits in the TOF. Flight time of a particle is taken as the difference between measured arrival time in the TOF and the time of the corresponding RF beam bunch. Figure 20 shows the relativistic velocity $\beta$ measured using the TOF vs momentum for different charged particles. As can be seen from the bands on the plot, it allows to separate proton from other lighter particles up to $3 \mathrm{GeV} / \mathrm{c}$ in the momentum. Pion-kaon separation is possible only up to around $2 \mathrm{GeV} / c$ as the kaon band tends to merge with the pion and positron bands for higher momenta.


FIG. 19: Design of the GlueX TOF wall. Figure from Ref. [30].


FIG. 20: Relativistic velocity $\beta$ measured using TOF vs momentum for positively charged particles. Positron, pion, kaon and proton bands are labelled as $e, \pi, K$ and $p$ respectively. The horizontal band around $\beta \approx 0.8$ is due to an accidentally-tagged photon beam bunch. Figure from Ref. [30].

### 3.2.4 CENTRAL DRIFT CHAMBER

The Central Drift Chamber (CDC) is a cylindrical straw-tube drift chamber surrounding the target and the start counter. The CDC is primarily used to track charge particles in the polar angle range $6^{\circ}$ to $168^{\circ}$ and has the optimum coverage for polar angles between $29^{\circ}$ and $132^{\circ}$. The CDC consists of 3522 straws that are 1.5 m long and 1.6 cm in diameter. Each tube has a gold-plate tungsten wire of $20 \mu \mathrm{~m}$ diameter as the anode. The straw tube walls are made of several layers of kapton with a innermost layer of aluminum forming the cathode. A 50:50 mixture of argon and $\mathrm{CO}_{2}$ gases flows through the straws. A charged particle travelling through a straw ionizes the gas and electrons drift towards the anode while ions drift towards the cathode. The CDC measures the drift time that is the time the ionization electron generated closest to the wire takes to reach the wire. This is measured using highresolution flash Analog-to-Digital Converters (fADCs). The drift time is converted into a
distance from the wire and using the information from adjacent straw tubes charged particle tracks can be reconstructed. The straw tubes are arranged as 12 layers of axial (along the beam direction) and 16 layers of stereo (with a $\pm 6^{\circ}$ offset from axial) orientations as shown in Fig. 21. The offset of stereo layers improves the reconstruction of a tracks z-position. The CDC provides position measurements with a resolution in $z$ of 2 mm and $150 \mu \mathrm{~m}$ resolution in the $r \phi$ plane.


FIG. 21: Drawing of the CDC straw positions. Black dots represent axial straws, red and blue dots represent stereo straws that are offset by $\pm 6^{\circ}$ respectively. Figure from Ref. [31].

The CDC also provides a method of particle identification. For this the fADC is used to measure $d E / d x$, the amount of energy lost by the charged particle per unit distance in the gas. Figure 22 shows a plot of CDC $d E / d x$ vs momentum for positively charged particles and it shows a clear separation of curved proton band from the horizontal band of other lighter particles for momenta below $1 \mathrm{GeV} / c$.


FIG. 22: Energy loss $d E / d x$ in the CDC as a function of proton momentum. The curved band is for the proton candidates while the horizontal band is for lighter charged particles such as $e^{+}, \pi^{+}$and $K^{+}$.

### 3.2.5 FORWARD DRIFT CHAMBER

The Forward Drift Chamber (FDC) system is located downstream of the CDC and within the solenoid magnet. It is used for the track reconstruction of forward going charged particles in the polar angle range from $1^{\circ}$ to $20^{\circ}$. The FDC consists of 24 planar drift chambers of 1 m diameter. These are grouped into 4 identical packages each containing 6 chambers. Each chamber contains of 1 "U" and 1 "V" plane of cathode wire strips and a plane of anode wires between the two cathode planes as shown in Fig. 23. The two cathode strip planes are oriented at $\pm 75^{\circ}$ with respect to the anode wires. Each chamber is rotated by $60^{\circ}$ relative to the previous one.

A gas mixture of $\mathrm{CO}_{2}$ and Ar with ratio of 40:60 is filled in a chamber and the ionization of the gas by traversing charged particles is used for the track reconstruction process. The two cathode strip planes provide reconstruction of the avalanche/hit position along the wire. The drift times from the anode wires are used to reconstruct the hit position perpendicular to the wire. Each chamber thus provides a three-dimensional space point of the track. By combining the information from the chambers the track can be reconstructed. The FDC can
reconstruct tracks with a position resolution better than the design resolution of $200 \mu \mathrm{~m}$.


FIG. 23: (a) Front view of an FDC package. Anode wires are shown as red vertical lines. The thick green lines represent the u-strips and the thick blue lines represent the v-strips. The cathode strip planes are oriented at $\pm 75^{\circ}$ with respect to the anode wires. (b) Side view of a six-chamber package. The red-dashed lines represent the anode wires and the solid-green lines represent the cathode planes. Figure from Ref. [32].

### 3.2.6 BARREL CALORIMETER

The barrel calorimeter (BCAL) is an electromagnetic calorimeter located within the solenoid magnet surrounding CDC and FDC. It's cylindrically shaped with a length of 390 cm and inner and outer radii of 65 cm and 90 cm respectively. The BCAL can detect photon showers with energies between 0.05 GeV and few GeV . It has angular coverage of $11^{\circ}-126^{\circ}$ in polar angle and $0^{\circ}-360^{\circ}$ in azimuthal angle. The BCAL is made as a lead-scintillating fiber matrix. The 0.5 mm thick grooved lead sheets produce electromagnetic showers while the 1.0 mm diameter scintillating fibers collect the light. The fibers run parallel to the cylindrical axis. The particle showers deposit energy in the fibers. The fibers re-emit light with an amount that is directly proportional to the energy deposited.

Figure 24 shows the geometry of the BCAL and readout segmentation. The BCAL contains 48 trapezoidal modules with a single module having 4 layers that are radially oriented. The containment of showers is a function of the angle of the photon incidence. It is 15.3 radiation lengths for particles entering normal to the BCAL face and 67 radiation lengths for $14^{\circ}$ incidence. At the each end of a module, the light guides collect the light from the
fibers and transport to silicon photomultipliers (SiPMs). The SiPMs have the advantage of being insensitive to the magnetic field.

Each of an upstream and downstream end of a BCAL module contains 40 SiPMs so there are 3840 SiPMs in total. The SiPMs within a layer are arranged such that there are 16 readout channels at a single end of a module resulting in a total of 1536 readout channels. The elecronic signal from a SiPM is delivered to a flash ADC (fADC). These readouts provide information about the energy, position and timing of the corresponding shower. In general, the showers can be produced by photons or charged tracks. The position information from the calorimeter hits are compared with the charged track information from drift chambers to find out if the showers are caused by the charged particles. The energy resolution of the BCAL is found to be $\sigma_{E} / E=5.2 \% \sqrt{E(\mathrm{GeV})} \oplus 3.6 \%$. The timing resolution of BCAL is $\sigma=150 \mathrm{ps}$ at 1 GeV .


FIG. 24: (a) Schematic view showing dimensions of the BCAL. (b) Top-half cutaway of a BCAL module showing its polar angle coverage and position with respect to the hydrogen target. (c) End view of the BCAL showing 48 azimuthal modules. (d) End view of a single module showing the 40 SiPMs and the layer-wise orientation producing the 16 readout channels. Figure from Ref. [33].

### 3.2.7 FORWARD CALORIMETER

The forward calorimeter (FCAL) is located 5.6 m downstream from the GlueX target. It can detect photon showers in the energy range 100 MeV to 5 GeV and has a polar angle coverage of $\theta<12^{\circ}$ [34]. The FCAL contains 2800 lead-glass blocks stacked in a circular array that has a radius of 1.2 m as shown in Fig. 25. The PMTs attached to lead-glass blocks detect the Cherenkov light emitted from the charged particles in the electromagnetic showers caused by the particles traveling through the blocks. The amount of light produced depends on the energy deposited by the initial particle. This can be caused by a charged particle or a neutral shower and is distinguished by matching calorimeter hits to charged tracks. Usually a shower will deposit energy in multiple blocks so they are grouped as clusters during the shower reconstruction process. Using the cluster information the energy, position and timing of the incident particle can be determined. The photon energy resolution of FCAL is found to be $\sigma_{E} / E=5.931 \% \sqrt{E(\mathrm{GeV})} \oplus 3.656 \%$ from the studies of reconstruction of the $\pi^{0} \rightarrow \gamma \gamma$ decay as shown in Fig. 26.

The FCAL also provides a method for separating electrons from charged hadrons. The $E / p$ ratio of energy deposited in the calorimeter and the momentum of the charged track matched to that shower can be used for this purpose. Figure 27 shows an example plot of $E / p$ versus the track polar angle and the electrons have values close to 1 because of their lighter mass compared to the charged hadrons.


FIG. 25: View of front face of the FCAL [35].


FIG. 26: Photon energy resolution in FCAL. Figure from Ref. [36.


FIG. 27: Ratio of energy measured by the FCAL and the momentum of the charged track versus polar angle of the track. Figure from Ref. [37].

## CHAPTER 4

## DATA ANALYSIS

### 4.1 SPRING 2017 DATASET

In this analysis the beam asymmetry was measured with the data collected from the GlueX experiment during spring of 2017. The CEBAF provided a 11.6 GeV electron beam into Hall D. The data used corresponds to the first run period with physics data production and is about $20 \%$ of the GlueX phase I data set. Figure 28 shows a brief overview of the run conditions relevant to this data which is available in Ref. [38].


FIG. 28: Summary of the beam current and the detector conditions for spring 2017 GlueX data as a function of run number.

The analysis was done with the data taken with JD70-100 $\quad 58 \mu \mathrm{~m}$ thickness diamond radiator. The data corresponds to a total luminosity of $20.8 \mathrm{pb}^{-1}$ in the photon beam energy range 8.2 to 8.8 GeV . The trigger signal was generated for events that deposited sufficient energy in FCAL and BCAL.

Two sets of polarization directions of beam were used in this analysis. One set contains $\phi=0^{\circ}$ as PARA and $\phi=90^{\circ}$ as PERP where $\phi$ denotes the polarization plane of the beam with respect to the lab floor. The other set is $\phi=-45^{\circ}$ as PARA and $\phi=45^{\circ}$ as PERP.

The complete list of runs that was approved for physics analysis purposes for each diamond orientation is given in Appendix A.

The ROOT [39] trees used in this work were generated with the REST version 03 of 2017 data. The REST files consists of special format of files that are produced from the raw data of the runs after calibrations and monitoring and by the use of GlueX reconstruction code. These files were then converted to tree format to use with ROOT for physics analysis and the channel specific files were created in the analysis launch version 26.

### 4.2 EVENT SELECTION

The selection of $\gamma p \rightarrow K^{+} \Sigma^{0}$ was performed using the standard analysis software framework [40], using the standard JANA plugin, ReactionFilter. It produced ROOT trees containing the particle combinations for the channel. Then, using the DSelector software [41], these TTrees were analyzed.

The following standard cuts were used by the analysis library.

- Number of extra charged tracks $\leq 3$
- Number of photons $\leq 15$
- Event timing: 4 beam bunches before and after primary bunch ( $-18.036 \mathrm{~ns}<t_{v e r t e x}-$ $\left.t_{R F}<18.036 \mathrm{~ns}\right)$
- Particle timing cuts (Table 3)
- Standard CDC dE/dx cuts 42]
- Cut around $\Lambda$ mass $M_{\pi^{-} p}=1.0-1.2 \mathrm{GeV} / c^{2}$

Additionally, a cut on BCAL energy $>50 \mathrm{MeV}$ was used to keep events having low energy final photons.

Samples of $K^{+} \Sigma^{0}$ and $K^{+} \Lambda$ Monte-Carlo (MC) events were generated using genr8 software and they were compared with real data after detector simulation and reconstruction to study the validity of cuts applied and the sources of background. Parameters used for MC simulation are given in Appendix B.

### 4.2.1 PLUGIN REQUIREMENTS

In the ReactionFilter plugin, candidate particles were selected to match the reaction under study. This is known as a "combo" or a "combination". It was required to have at
least one tagged beam photon, two reconstructed positively charged tracks ( $K^{+}$and proton), one negatively charged track $\left(\pi^{-}\right)$and a neutral shower $(\gamma)$ in calorimeters.

The reaction tag 1_14__1_11_18_B4_M18 was used for the ReactionFilter plugin. The output TTree was called "gkplamb__B4_M18_Tree" according to the final state particles it had. So, the reaction was reconstructed as two steps.

1. $\gamma p \rightarrow \gamma K^{+} \Lambda$
2. $\Lambda \rightarrow \pi^{-} p$

The $\pi^{-} p$ mass spectrum was not constrained just to the $\Lambda$ mass region (with the use of M18 flag) in order to study if there's any background under the $\Lambda$ peak.

A kinematic fit was applied satisfying 4-momentum conservation and constraining the event vertex. Then, two charged tracks for the particles coming from $\Lambda$ decay were required to come from a secondary vertex. At the plugin level, the kinematic fit was required to converge (Confidence Level >0).

The resulting reconstructed event was required to have a missing mass squared, $M M^{2}=$ $\left(p_{i}-p_{f}\right)^{2}$, be of magnitude less than $0.1\left(\mathrm{GeV} / c^{2}\right)^{2}$, where

$$
\begin{equation*}
p_{i}-p_{f}=\left(p_{\gamma}^{\text {beam }}+p_{\text {proton }}^{\text {target }}\right)-\left(p_{\gamma}+p_{K^{+}}+p_{\pi^{-}}+p_{\text {proton }}\right) \tag{36}
\end{equation*}
$$

where $p_{\gamma}^{\text {beam }}, p_{\text {proton }}^{\text {target }}, p_{\gamma}, p_{K^{+}}, p_{\pi^{-}}$and $p_{\text {proton }}$ are the 4 -momentum of the tagged beam photon, target proton at rest, measured final state photon, measured final state tracks of kaon, pion and proton respectively.

Two cuts were applied for particle identification (PID). The first cut applied is on $\mathrm{dE} / \mathrm{dx}$ in order to distinguish between the final state proton and other positively charged particles. Although the reaction doesn't have any $\pi^{+}$or $e^{+}$on it, a positively charged track in the combination can satisfy four hypotheses $\left(K^{+}, \pi^{+}, e^{+}\right.$and proton). The energy deposited in the drift chambers by charged tracks can be used to distinguish these. The cut applied is $\mathrm{dE} / \mathrm{dx}>e^{(-4.0 \times x+2.25)}+1.0$ [42] and is effective for particle momenta $\leq 1.0 \mathrm{GeV} / c$. Figure 29 shows the energy loss $\mathrm{dE} / \mathrm{dx}$ as a function of proton momentum after the cut.

The second cut used for particle identification is for particle timing. For each candidate particle, track flight time from the track vertex to each detector was calculated assuming the mass of the hypothesis. Then, the time difference $(\Delta t)$ between this vertex time and RF beam bunch was used to identify particles. These cuts are shown in Table 3 [42].


FIG. 29: Energy loss $\mathrm{dE} / \mathrm{dx}$ in the Central Drift Chamber (CDC) as a function of proton momentum. Red curve represents the loose cut on $\mathrm{dE} / \mathrm{dx}$ for proton identification.

| Particle Type | BCAL cut | TOF cut | FCAL cut |
| :---: | :---: | :---: | :---: |
| Proton | $\|\Delta t\|<1.0$ | $\|\Delta t\|<0.6$ | $\|\Delta t\|<2.0$ |
| $K^{+}$ | $\|\Delta t\|<0.75$ | $\|\Delta t\|<0.3$ | $\|\Delta t\|<2.5$ |
| $\pi^{-}$ | $\|\Delta t\|<1.0$ | $\|\Delta t\|<0.5$ | $\|\Delta t\|<2.0$ |
| $\gamma$ | $\|\Delta t\|<1.5$ | - | $\|\Delta t\|<2.5$ |

TABLE 3: Timing cuts ( $\Delta t$ ) for different particles in the BCAL, TOF and FCAL. Times are in ns.

### 4.2.2 COMBINATION SELECTION

The $\gamma p \rightarrow \gamma K^{+} \Lambda$ candidate combinations were selected at the DSelector stage using the following cuts:

- Beam energy: $8.2 \mathrm{GeV}<E_{\text {beam }}<8.8 \mathrm{GeV}$
- Measured missing mass squared: $\left|M M^{2}\right|<0.08\left(\mathrm{GeV} / c^{2}\right)^{2}$
- Kinematic fit confidence level $(\mathrm{CL})>10^{-4}$ (Corresponds to $\chi^{2} /$ NDF (number of degrees of freedom) $<5$ )
- $K^{+}$vertex z position: $51.2 \mathrm{~cm}<z<78.8 \mathrm{~cm}$
- $K^{+}$vertex radial position: $r<1.0 \mathrm{~cm}$
- FCAL shower quality $>0.5$

The quantities plotted were found using kinematically fit 4 -vectors except for the measured missing mass squared.

The coherent peak energy range of $8.2 \mathrm{GeV}<E_{\text {beam }}<8.8 \mathrm{GeV}$ was chosen for the events, because the beam polarization is maximum in that energy range and to match the TPOL analysis of polarization values.

Figure 30 shows the time difference $\Delta t=t_{v e r t e x}-t_{R F}$ between the event vertex time, as measured with the final state particles and projected back to the vertex, and the RF time. The electron beam provided from the accelerator comes as bunches that are in 4 ns intervals. The region between the blue lines indicates the signal region ( $-2.004 \mathrm{~ns}<\Delta t<2.004 \mathrm{~ns}$ ) consisting of a single RF bunch. The red lines indicate the sample of out of time RF bunches used as accidentals. Contribution from this accidentally tagged beam photons as a background must be estimated and subtracted. It will be discussed further in Sec. 4.2.3.

Figure 31 shows the measured missing mass squared distribution. It should be peaking around zero since the reaction under study is exclusive (all final particles are detected). A loose cut on the magnitude of the missing mass squared was applied to be less than 0.08 $\left(\mathrm{GeV} / c^{2}\right)^{2}$ (as shown by red dashed lines).


FIG. 30: Time difference between event vertex and RF time. The dashed blue lines indicate the in-time beam photons and the dashed red lines indicate the out-of-time beam photons that will be used for accidental subtraction.


FIG. 31: Missing mass squared distribution for the reaction $\gamma p \rightarrow \gamma K^{+} \pi^{-} p$

As can be seen in Appendix E, the loss of signal events is only $0.7 \%$ when tightening the cut to $0.08\left(\mathrm{GeV} / c^{2}\right)^{2}$ from the default analysis library cut at $0.1\left(\mathrm{GeV} / c^{2}\right)^{2}$. Furthermore, Fig. 32 shows the missing mass squared distribution in $\log$ scale with all the other cuts
applied and $M_{\Lambda \gamma}$ within the $\Sigma^{0}$ peak. It clearly shows that the reduction of signal events due to the cut at $0.08\left(\mathrm{GeV} / c^{2}\right)^{2}$ is negligible. The effect of loosening and tightening of the cut on beam asymmetry results will be shown in Sec. 5.5.1.


FIG. 32: Missing mass squared distribution in $\log$ scale for the reaction $\gamma p \rightarrow \gamma K^{+} \pi^{-} p$ with events selected in the $\Sigma^{0}$ peak.

The kinematic fit was introduced in Sec. 4.2.1. A cut of $\mathrm{CL}>10^{-4}$ was used in order to remove the background. A very loose cut was not applied on the confidence level as it results in similar statistical errors for yield asymmetry fits described in Sec. 5.3 but giving much higher value for $\chi^{2} / \mathrm{NDF}$ of the kinematic fit. (For an example CL $>10^{-10}$ corresponds to a $\chi^{2} / \mathrm{NDF}<10.6$ and an example plot of yield asymmetry fits for this cut is given in Appendix E.) Figure 33 shows the kinematic fit confidence level distribution and the applied nominal cut is indicated by red dashed line. This cut corresponds to a cut on $\chi^{2} / \mathrm{NDF}<5$ and is shown in Fig. 34.

Since the $K^{+}$track should be originating from the target region, cuts were applied on the $z$ and $r$ vertex positions of the reconstructed kaon's distance of closest approach to the beamline. A study was done to find the resolution in target z using $K^{+} \Sigma^{0} t$-channel MC and the relevant figures are shown in Appendix D.1. A $3 \sigma$ cut was made for the target windows from positions at $\mathrm{z}=50.3 \mathrm{~cm}$ and $\mathrm{z}=79.6 \mathrm{~cm}$. Based on this studies the $K^{+}$vertex z position is required to be between 51.2 and 78.8 cm (Fig. 35) in order to avoid events that can be
produced from the target windows. The vertex radius r was required to be less than 1.0 cm (Fig. 36) consistent with the radial size of the liquid hydrogen target.


FIG. 33: Kinematic fit confidence level distribution for data. X-axis is in the log scale.


FIG. 34: Kinematic fit $\chi^{2} /$ NDF distribution.


FIG. 35: Vertex position along the beamline for $K^{+}$.


FIG. 36: Radial vertex position for $K^{+}$.

Figure 37 shows the invariant mass of $\pi^{-} p$ after the cuts mentioned above. $\Lambda^{0}(1116)$ peak is clearly seen with some background remaining above the $\Lambda$ mass range. However, it'll be shown that $\Lambda$ 's from the $\Sigma^{0}$ peak don't have any background under it.


FIG. 37: The accidental subtracted histogram for invariant mass of $\pi^{-} p$ after the cuts applied.

Figure 38 shows the fit to accidental subtracted histogram of $M_{\pi^{-} p}$. Analogous Gaussian width $(\sigma)$ was found as 3 MeV using the full width half maximum (FWHM) of the Voigtian $(\sigma=$ FWHM $/(2 \sqrt{2 \ln 2}))$. Events within the range $\left|M_{\pi^{-} p}-M_{\Lambda}\right|<3 \sigma$ were selected and invariant mass of $\Lambda \gamma$ was constructed. Figure 39 shows this and according to this, a considerable amount of background is underneath the $\Sigma^{0}(1193)$ peak.


FIG. 38: Invariant mass of $\pi^{-} p$ distribution fitted with the Voigtian function for a signal (red dot dashed curve) plus first order Chebychev polynomial for the background (green dashed curve). The blue solid curve is a result of the total fit. The fit gives a $\chi^{2} / \mathrm{NDF}=314.50 / 66$ $=4.77$. Dashed vertical lines represent the region of $\Lambda$ events selected for the analysis.


FIG. 39: The accidental subtracted histogram for invariant mass of $\Lambda \gamma$.

There can be split-off showers that produced from charged hadrons interacting with calorimeters. These are not matched to charged tracks and can be mis-identified as photons.

So, the shower quality is used as a kinematic topology to study any correlation to this background.

The shower quality variable is a new method introduced by GlueX FCAL group [43]. It is included in the ROOT trees as NeutralHypo_-ShowerQuality. It gives a quality score between 0 and 1 to a neutral shower in FCAL using machine learning techniques. So, it can be used to remove uncorrelated showers.

Figure 40 shows the invariant mass of $\Lambda \gamma$ versus shower quality in FCAL. It is evident that the background with higher masses above the $\Sigma^{0}$ region and the lower mass peak is associated with lower quality values of the shower (close to 0 ). Furthermore it was found from simulation that $K^{+} \Lambda t$ - and $u$-channel events combined with a neutral shower form this background.


FIG. 40: Invariant mass of $\Lambda \gamma$ vs FCAL shower quality.

A cut of FCAL shower quality $>0.5$ was used to get rid of this background. Effect of this cut on invariant mass of $\Lambda \gamma$ spectrum can be seen in Fig. 41. Figures 42 and 43 show that the shower quality cut eliminates the background coming from $K^{+} \Lambda$ channel.


FIG. 41: Invariant mass of $\Lambda \gamma$ before (blue solid circles) and after shower quality $>0.5$ cut (red open circles).


FIG. 42: Comparison of $K^{+} \Lambda t$-channel events before (blue solid circles) and after (red open circles) shower quality $>0.5$ cut (MC).


FIG. 43: Comparison of $K^{+} \Lambda u$-channel events before (blue solid circles) and after (red open circles) shower quality $>0.5$ cut (MC).

Figure 44 shows a fit to the accidental subtracted histogram of invariant mass of $\Lambda \gamma$ after all the cuts including FCAL shower quality $>0.5$. The width $(\sigma)$ of $\Sigma^{0}$ peak is about 8 MeV . Events within the range $\left|M_{\Lambda \gamma}-M_{\Sigma}\right|<3 \sigma$ were selected for the beam asymmetry analysis. Background under the $\Sigma^{0}$ peak was found to be $\sim 2 \%$. The fractions of background under the $\Sigma^{0}$ peak for different $-t$ bins and the $-u$ bin used in the analysis are shown in Appendix D.3.


FIG. 44: Invariant mass of the $\Lambda \gamma$ distribution fitted with a Voigtian function for the signal (red dot dashed curve) plus first order Chebychev polynomial for the background (green dashed curve). The blue solid curve is a result of the total fit. The fit gives a $\chi^{2} / \mathrm{NDF}=$ $249.31 / 56=4.45$. Dashed vertical lines represent the region of $\Sigma^{0}$ events selected for the analysis.

Using the MC for $K^{+} \Sigma^{0}$ reaction, a study was done about the photon polar angle $\left(\theta_{\gamma}\right)$ distribution. Figures 45 and 46 show $\theta_{\gamma}$ distributions for $K^{+} \Sigma^{0} t$-channel and $u$-channel. Events were selected in the range $1.169 \mathrm{GeV} / c^{2}<M_{\Lambda \gamma}<1.217 \mathrm{GeV} / c^{2}$ to match with the mass region selected for data. From this, we see that almost all photons in the $t$-channel go to BCAL $\left(\theta_{\gamma}>11^{\circ}\right)$ while most photons in the $u$-channel go to FCAL (fraction of photons in the range $1^{\circ}-11^{\circ}$ from Fig. 46 is $67 \%$.)


FIG. 45: Photon polar angle distribution for $K^{+} \Sigma^{0} t$-channel MC.


FIG. 46: Photon polar angle distribution for $K^{+} \Sigma^{0} u$-channel MC.

In addition to this, the effect of the shower quality $>0.5$ cut on $\Sigma^{0}$ yield was also investigated using MC. This is shown in Fig. 47. It appears that there is almost no effect on the $t$-channel $\Sigma^{0}$ events. For the $u$-channel, the fraction of signal events lost is negligible, on the order of a few percent as one can see from Fig. 48.


FIG. 47: Comparison of $K^{+} \Sigma^{0} t$-channel yield before (blue solid circles) and after (red open circles) shower quality $>0.5$ cut (MC).


FIG. 48: Comparison of $K^{+} \Sigma^{0} u$-channel yield before (blue solid circles) and after (red open circles) shower quality $>0.5$ cut (MC).

Figure 49 shows the $-t$ distribution after all the cuts for events selected within range
of $1.169 \mathrm{GeV} / c^{2}<M_{\Lambda \gamma}<1.217 \mathrm{GeV} / c^{2}$. Figure 50 shows the $-u$ distribution with the same cuts. The two peaks in low and high $-t(-u)$ regions suggest both $t$ and $u$-channel contributions for the reaction. Here, Mandelstam variables $t$ and $u$ were calculated using four-momenta of particles in the following way.
$t=\left(p_{\text {beam }}-p_{K^{+}}\right)^{2}$ and $u=\left(p_{\text {target }}-p_{K^{+}}\right)^{2}$.

From Fig. 50, we see that there are few events with $-u<0$. This is physically possible if the mass is transferred along with momentum, in particular, if a proton is replaced by a lighter kaon. It was decided to keep these events for beam asymmetry analysis in the $u$-channel.


FIG. 49: $K^{+} \Sigma^{0}-t$ distribution after all the cuts.


FIG. 50: $K^{+} \Sigma^{0}-u$ distribution after all the cuts.

In order to understand the large gap between events in the low $-t$ and $-u$ regions, a MC sample was generated with a very flat $t$ slope of $0.01(\mathrm{GeV} / c)^{-2}$ and the acceptance was found in the entire range of $t$ between 0 and $16(\mathrm{GeV} / c)^{2}$. Figure 51 shows the thrown and accepted $-t$ distributions for this case and Fig. 52 shows the corresponding acceptance. The acceptances for low $|t|$ and low $|u|$ regions were also found using simulations with corresponding $t(u)$ slopes and are shown in Appendix D.2.

The acceptance was calculated as:

$$
\begin{equation*}
\text { Acceptance }=\frac{\text { Number of reconstructed events }}{\text { Number of generated events }} \tag{37}
\end{equation*}
$$

It can be seen that the acceptance is very low (by 3 orders of magnitude) in the mid $-t$ region between low $-t$ and $-u$ regions leading to the gap observed in the distribution in data.



FIG. 51: $K^{+} \Sigma^{0}$ thrown (left) and accepted (right) - $t$ distributions for a $t$-slope of 0.01 $(\mathrm{GeV} / c)^{-2}$.


FIG. 52: Acceptance for the events from MC with $t$-slope of $0.01(\mathrm{GeV} / c)^{-2}$ (red, dashed line), $-t$ distribution from data is shown by solid circles.

### 4.2.3 ACCIDENTAL SUBTRACTION

The contribution of accidentally tagged beam photons to the yield was introduced in Sec. 4.2.2. As shown in Fig. 30, a sample of accidentally tagged combos with $6.012 \mathrm{~ns}<|\Delta t|<$ 18.036 ns was used to estimate this contribution to the $K^{+} \phi$ distributions used in the beam asymmetry analysis. The accidental distributions were scaled by a factor of $1 / 6$ because they contain 6 accidental RF bunches. Then, they were subtracted from the distribution for main RF bunch.

### 4.2.4 NUMBER OF COMBINATIONS PER EVENT

In GlueX reconstruction there are more than one combination for an event. This is done to prevent the efficiency from getting too low. But after all the cuts in the analysis, number of combinations per event should be close to 1 (Exactly 1.0 is the ideal case) which means majority of combos are for true signal. So the following was done to estimate the number of combinations per event.

Figure 53 shows the number of events that was in the TTree after the full event selection. Figure 54 shows the number of combinations for "IsComboCut" was flagged as true (1) or not (0). The combinations which survive the event selection are those with flag $=0$. Combinations with different beam photons were excluded by selecting combinations only in the primary beam bunch.


FIG. 53: The number of events after the full event selection.


FIG. 54: Combinations that were flagged as cut or uncut.

Then, number of surviving combos per event is,

$$
\begin{equation*}
\frac{\text { combos not cut }}{\text { number of events }}=\frac{25930}{22511}=1.15 \tag{38}
\end{equation*}
$$

### 4.2.5 BACKGROUND UNDER THE SIGNAL PEAK

In Sec. 4.2.2, it was mentioned that there is some background under the $\Lambda$ peak in the $M_{\pi^{-} p}$ spectrum (Fig. 37). This should be accounted for while doing the beam asymmetry analysis. Figure 55 shows the invariant mass of $\pi^{-} p \gamma$ vs that of $\pi^{-} p$. Figure 56 shows the $M_{\pi^{-} p}$ spectrum for events within the range of $1.169 \mathrm{GeV} / c^{2}<M_{\pi^{-} p \gamma}<1.217 \mathrm{GeV} / c^{2}$. As one can see the $\Lambda$ peak has no background and only the $2 \%$ background under the $\Sigma^{0}$ peak was considered while estimating the beam asymmetry from background as described in Sec. 5.5.7,


FIG. 55: Invariant mass of $\pi^{-} p \gamma$ vs invariant mass of $\pi^{-} p$ (data).


FIG. 56: Invariant mass of $\pi^{-} p$ within the $\Sigma^{0}$ region (data).

## CHAPTER 5

## BEAM ASYMMETRY MEASUREMENT (t-CHANNEL)

### 5.1 BEAM ASYMMETRY METHOD

For the photoproduction of psuedoscalar mesons using a linearly polarized photon beam and an unpolarized target, the polarized cross section is given by,

$$
\begin{equation*}
\sigma_{\text {pol }}\left(\phi, \phi_{\text {lin }}\right)=\sigma_{\text {unpol }}\left[1-P_{\gamma} \Sigma \cos \left(2\left(\phi-\phi_{\text {lin }}\right)\right)\right] . \tag{39}
\end{equation*}
$$

Where $\sigma_{\text {unpol }}$ is the unpolarized cross section, $\phi$ is the angle between plane parallel to the lab floor and $K^{+}$production plane. It is equivalent to the azimuthal angle of $K^{+}$in the lab frame. The magnitude of the photon beam polarization is given by $P_{\gamma}$. Figure 57 shows the relevant angles in the lab frame.

The beam asymmetry $\Sigma$ is a measure of the difference between cross sections with two mutually perpendicular polarizations.

$$
\begin{equation*}
\Sigma=\frac{\sigma_{\perp}-\sigma_{\|}}{\sigma_{\perp}+\sigma_{\|}} \tag{40}
\end{equation*}
$$

Where $\sigma_{\perp}$ and $\sigma_{\|}$are cross sections for the two mutually perpendicular polarization directions which we call "PERP" and "PARA". PARA is normally taken to be when the polarization plane of the beam is parallel $\left(0^{\circ}\right)$ to the lab floor $(x-z$ plane) and PERP when it is perpendicular $\left(90^{\circ}\right)$. Another set with $\mathrm{PARA}=-45^{\circ}$ and $\mathrm{PERP}=45^{\circ}$ is chosen to allow an independent check of systematics for the two sets.

Cross sections for each orientation can be found using the following relations:

$$
\begin{align*}
& \sigma_{\|}(\phi)=\sigma_{\text {pol }}\left(\phi, \phi_{\text {lin }}=0\right)=\sigma_{\text {unpol }}\left(1-P_{\|} \Sigma \cos 2 \phi\right)  \tag{41}\\
& \sigma_{\perp}(\phi)=\sigma_{\text {pol }}\left(\phi, \phi_{\text {lin }}=90\right)=\sigma_{\text {unpol }}\left(1+P_{\perp} \Sigma \cos 2 \phi\right) . \tag{42}
\end{align*}
$$

Here $P_{\|}$and $P_{\perp}$ are the magnitude of the photon beam polarization in the PARA and PERP orientations, respectively.


FIG. 57: Illustration of the angles used for the beam asymmetry analysis in the lab frame. Figure from Ref. [44].

The event yield in each orientation is given by

$$
\begin{align*}
Y_{\|}(\phi) & \sim N_{\|}\left[\sigma_{\text {unpol }} A(\phi)\left(1-P_{\|} \Sigma \cos 2 \phi\right)\right],  \tag{43}\\
Y_{\perp}(\phi) & \sim N_{\perp}\left[\sigma_{\text {unpol }} A(\phi)\left(1+P_{\perp} \Sigma \cos 2 \phi\right)\right] \tag{44}
\end{align*}
$$

where:

- $A(\phi)$ is an arbitrary function for the $\phi$-dependent detector acceptance and efficiency,
- $N_{\perp(\|)}$ is the flux of photons in the PERP (PARA) orientation.

If there is no background, a polarization-dependent yield asymmetry $S(\phi)$ can be defined as

$$
\begin{equation*}
S(\phi)=\frac{Y(\phi)_{\perp}-F_{R} Y(\phi)_{\|}}{Y(\phi)_{\perp}+F_{R} Y(\phi)_{\|}}=\frac{\sigma(\phi)_{\perp} A(\phi)-\sigma(\phi)_{\|} A(\phi)}{\sigma(\phi)_{\perp} A(\phi)+\sigma(\phi)_{\|} A(\phi)} . \tag{45}
\end{equation*}
$$

The $\phi$-dependent acceptance, $A(\phi)$ cancels if it is the same for both PARA and PERP modes. $F_{R}=\frac{N_{\perp}}{N_{\|}}$is the ratio of the integrated photon flux for PERP and PARA for a given data set. Then, measured yield asymmetry can be fit by the following function [45]

$$
\begin{equation*}
f(\phi)=\frac{\left(P_{\perp}+P_{\|}\right) \Sigma \cos 2\left(\phi-\phi_{0}\right)}{2+\left(P_{\perp}-P_{\|}\right) \Sigma \cos 2\left(\phi-\phi_{0}\right)} \tag{46}
\end{equation*}
$$

A phase shift $\phi_{0}$ was added to allow for the misalignment of polarization of the beam and will be discussed in Sec. 5.3. Measurements of $P_{\perp}$ and $P_{\|}$are found using TPOL data as mentioned in Sec. 3.1.4. Then, from the only fit parameter, beam asymmetry $\Sigma$ can be obtained.

### 5.2 RELATIVE FLUX NORMALIZATION

Using the PS hits in coincidence with a hit in the TAGM or TAGH, the integrated photon flux was determined. As explained in Sec. 4.2.3, accidentally tagged photons were subtracted. Here $\Delta t$ is the time difference between the PS pair and the tagger hit. The ratio of tagged flux for PARA and PERP data sets in the beam energy range $8.2<E_{\gamma}<8.8 \mathrm{GeV}$ is shown in Table 4.

| Data Set | PARA Integrated Flux | PERP Integrated Flux | $F_{R}$ |
| :---: | :---: | :---: | :---: |
| $0 / 90$ | $4.18800 \times 10^{12}$ | $4.34682 \times 10^{12}$ | 1.03792 |
| $-45 / 45$ | $4.09501 \times 10^{12}$ | $4.07606 \times 10^{12}$ | 0.99537 |

TABLE 4: Flux normalization ratios $\left(F_{R}\right)$ for different polarization directions.

### 5.3 EXAMPLE FIT TO THE $\Sigma^{0}$ SAMPLE ( $t$-CHANNEL)

Using the relative flux ratio, the $\phi$-dependent yields were fit to extract the $\Sigma$ beam asymmetry. A set of example fits is shown in Figs. 58 and 59 for the $\Sigma^{0}$ event sample integrated over the entire $t$ region used for this analysis $\left(0.1(\mathrm{GeV} / c)^{2}<-t<1.4(\mathrm{GeV} / c)^{2}\right)$. The upper plots are fits to the PARA (left) and PERP (right) yields independently. There are two free parameters in these fits of the form

$$
\begin{equation*}
f(\phi)=C\left[1+P \Sigma \cos \left(2\left(\phi-\phi_{0}\right)\right)\right] \tag{47}
\end{equation*}
$$

with $p_{0}=C$ for normalization and $p_{1}=P \Sigma$.
The phase constant $\phi_{0}$ allows for the PARA (PERP) polarization plane orientation to not be exactly parallel to the $x-z(x-y)$ plane in the lab. Phase constants were obtained from a study of $\rho(770)$ decay asymmetries and are given in Table 5 [46. The value $\phi_{0}=3.13^{\circ}$ was used for fitting yield asymmetry in each individual bins of $-t$ for $0 / 90$ data set. The value $\phi_{0}=3.16^{\circ}$ was used for fitting the yield asymmetry in the $-45 / 45$ data set.

| Nominal $\phi$ | $\phi_{0}$ | statistical | systematic |
| :---: | :---: | :---: | :---: |
| $0^{\circ}$ | 1.77 | $\pm 0.04$ | $\pm 0.56$ |
| $90^{\circ}$ | 4.50 | $\pm 0.04$ | $\pm 0.50$ |
| Asymmetry | 3.13 | $\pm 0.03$ | $\pm 0.14$ |
| $45^{\circ}$ | 2.85 | $\pm 0.05$ | $\pm 0.53$ |
| $-45^{\circ}$ | 3.43 | $\pm 0.04$ | $\pm 0.59$ |
| Asymmetry | 3.16 | $\pm 0.03$ | $\pm 0.17$ |

TABLE 5: Deviation of polarization plane orientation with uncertainties.

The lower left plot in Fig. 58 is a fit to the normalized yield asymmetry $S(\phi)$ (Eqn. 45) according to the fit function in Eqn. 46 .

The fit results for the two beam orientation sets in the individual $-t$ bins are shown in Appendix C.1. The lower right plot in Fig. 58 is relevant for understanding of the impact of instrumental asymmetries and is discussed in Sec. 5.5.2.


FIG. 58: Polarization direction set to $0 / 90$. Fits to the $\phi$ distributions in the beam energy range $8.2<E_{\gamma}<8.8 \mathrm{GeV}$, integrated over the entire $t$ range. Upper row: fits to the PARA (left) and PERP (right) distributions independently. Lower row (left): fit to the asymmetry (Eqn. 45). Lower row (right): fit to the sum $Y(\phi)_{\perp}+F_{R} Y(\phi)_{\|}$, which is sensitive to instrumental asymmetries.


FIG. 59: Polarization direction set to $-45 / 45$. Fits to the $\phi$ distributions in the beam energy range $8.2<E_{\gamma}<8.8 \mathrm{GeV}$, integrated over the entire $t$ range. Upper row: fits to the PARA (left) and PERP (right) distributions independently. Lower row (left): fit to the asymmetry (Eqn. 45). Lower row (right): fit to the $\operatorname{sum} Y(\phi)_{\perp}+F_{R} Y(\phi)_{\|}$, which is sensitive to instrumental asymmetries.

### 5.4 SUMMARY OF FIT RESULTS

Figure 60 shows the beam asymmetry results obtained from fitting the yield asymmetry histograms for individual $-t$ bins.


FIG. 60: The beam asymmetry $\Sigma$ for $\gamma p \rightarrow K^{+} \Sigma^{0}$ in the $t$ channel. The black solid circles correspond to polarization direction set to $0 / 90$, red open circles correspond to polarization direction set to $-45 / 45$ ( $-t$ coordinates are shifted by $0.02(\mathrm{GeV} / c)^{2}$ to the right for clarity). Vertical error bars represent statistical uncertainties while horizontal error bars are drawn to indicate the RMS widths of $-t$ bins used.

### 5.5 SYSTEMATIC STUDIES

### 5.5.1 EVENT SELECTION

By varying the parameters of the event selection described in Sec 4.2.2, the systematic dependence of the asymmetry on the event selection criteria was studied. The list of nominal cuts and the looser and tighter cuts that were used for this purpose are given in Table 6. For each variation of the cut, the asymmetry was extracted in bins of $-t$.

Figures 61 and 62 show the values of the asymmetry as a function of $-t$, where each parameter is represented by an individual color and two marker shapes for tighter and looser cuts. The asymmetry for the default cut parameters is shown by the open black circles.

| Looser Cut | Nominal Cut | Tighter Cut |
| :--- | :--- | :--- |
| $50.9 \mathrm{~cm}<z<79.1 \mathrm{~cm}$ | $51.2 \mathrm{~cm}<z<78.8 \mathrm{~cm}$ | $51.5 \mathrm{~cm}<z<78.5 \mathrm{~cm}$ |
| $r<1.2 \mathrm{~cm}$ | $r<1.0 \mathrm{~cm}$ | $r<0.8 \mathrm{~cm}$ |
| $\left\|M M^{2}\right\|<0.1 \mathrm{GeV}^{2}$ | $\left\|M M^{2}\right\|<0.08 \mathrm{GeV}^{2}$ | $\left\|M M^{2}\right\|<0.06 \mathrm{GeV}^{2}$ |
| $\mathrm{CL}>10^{-5}$ | $\mathrm{CL}>10^{-4}$ | $\mathrm{CL}>10^{-3}$ |
| $\left\|M_{\pi^{-} p}-M_{\Lambda}\right\|<0.012 \mathrm{GeV}$ | $\left\|M_{\pi^{-} p}-M_{\Lambda}\right\|<0.009 \mathrm{GeV}$ | $\left\|M_{\pi^{-}-}-M_{\Lambda}\right\|<0.006 \mathrm{GeV}$ |
| $\left\|M_{\Lambda \gamma}-M_{\Sigma^{0}}\right\|<0.032 \mathrm{GeV}$ | $\left\|M_{\Lambda \gamma}-M_{\Sigma^{0}}\right\|<0.024 \mathrm{GeV}$ | $\left\|M_{\Lambda \gamma}-M_{\Sigma^{0}}\right\|<0.016 \mathrm{GeV}$ |
| shower quality $>0.3$ | shower quality $>0.5$ | shower quality $>0.7$ |

TABLE 6: Variation of the event selection cut parameters to study systematic impact on the asymmetry.

All the data points associated with cut systematics are within the $1 \sigma$ statistical error of the nominal data points. Colors for different cut variations are as shown in Fig. 63.


FIG. 61: $\Sigma$ asymmetry measured for each of the cut parameter variations, shown as different colored points in each $-t$ bin for the $0 / 90$ orientation. Full circles represent tighter cuts and triangles represent looser cuts. The open circles and vertical error bars are the asymmetry values and statistical errors using the nominal cut parameters.


FIG. 62: $\Sigma$ asymmetry measured for each of the cut parameter variations, shown as different colored points in each $-t$ bin for the $-45 / 45$ orientation. Full circles represent tighter cuts and triangles represent looser cuts. The open circles and vertical error bars are the asymmetry values and statistical errors using the nominal cut parameters.

| $\phi$ | Nominal cuts |
| :---: | :---: |
| - | $\mathrm{K}^{+}$vertex z (Tighter) |
| 」 | $\mathrm{K}^{+}$vertex z (Looser) |
| - | $\mathrm{K}^{+}$vertex r (Tighter) |
| A | $\mathrm{K}^{+}$vertex r (Looser) |
| - | Missing mass squared (Tighter) |
| - | Missing mass squared (Looser) |
| - | KinFit CL (Tighter) |
| * | KinFit CL (Looser) |
| - | $\pi$ - ${ }^{\text {p mass (Tighter) }}$ |
| - | $\pi \mathrm{p}$ mass (Looser) |
| - | $\Lambda \gamma$ mass (Tighter) |
| 」 | $\Lambda \gamma$ mass (Looser) |
| - | shower quality (Tighter) |
| , | shower quality (Looser) |

FIG. 63: Legend for cut systematics plots (Figs. 61 and 62).

Tables 7 and 8 give systematic uncertainties for each cut variation for $0 / 90$ and $-45 / 45$ orientation sets. Here the uncertainty was calculated as the difference between asymmetry obtained from the cut variation and the nominal cut. For each cut variable, errors associated with looser and tighter values were averaged. Then, systematic uncertainties due to cuts were calculated by adding uncertainties due to each cut variable in quadrature. The total uncertainty due to the cut systematics is obtained as follows.

$$
\begin{equation*}
\delta_{\text {CUTSYS }}=\sqrt{\sum_{n=1}^{N} \delta_{n}^{2}}, \tag{48}
\end{equation*}
$$

where $\delta_{n}$ is the average uncertainty of the cut variable ' $n$ ' and $N$ is the number of cut variables.

Tables 9 and 10 give the calculated systematic uncertainties for $0 / 90$ and $-45 / 45$ beam orientations respectively.

|  | $-\mathrm{t}(\mathrm{GeV} / c)^{2}$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| cut variation | $0.10-0.35$ | $0.35-0.50$ | $0.50-0.70$ | $0.70-1.40$ |
| $K^{+}$vertex z loose | 0.001 | 0.040 | 0.018 | 0.010 |
| $K^{+}$vertex z tight | 0.010 | 0.024 | 0.004 | 0.038 |
| $K^{+}$vertex r loose | 0 | 0 | 0.001 | 0.002 |
| $K^{+}$vertex r tight | 0 | 0 | 0 | 0.001 |
| Missing mass squared loose | 0.001 | 0.004 | 0.002 | 0.004 |
| Missing mass squared tight | 0.002 | 0.019 | 0.002 | 0.010 |
| KinFit CL loose | 0.028 | 0.028 | 0.009 | 0.003 |
| KinFit CL tight | 0.016 | 0.030 | 0.029 | 0.024 |
| $M_{\pi^{-p}}$ loose | 0.020 | 0.017 | 0.007 | 0.001 |
| $M_{\pi^{-} p}$ tight | 0.011 | 0.053 | 0.029 | 0.016 |
| $M_{\Lambda \gamma}$ loose | 0.006 | 0.031 | 0.011 | 0.002 |
| $M_{\Lambda \gamma}$ tight | 0.026 | 0 | 0.012 | 0.004 |
| FCAL shower quality loose | 0.008 | 0.003 | 0.004 | 0.006 |
| FCAL shower quality tight | 0 | 0.002 | 0.004 | 0.007 |

TABLE 7: Systematic uncertainty for each cut variation for individual $-t$ bins for $0 / 90$ orientation.

|  | $-\mathrm{t}(\mathrm{GeV} / c)^{2}$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| cut variation | $0.10-0.35$ | $0.35-0.50$ | $0.50-0.70$ | $0.70-1.40$ |
| $K^{+}$vertex z loose | 0.006 | 0.007 | 0.007 | 0.005 |
| $K^{+}$vertex z tight | 0.005 | 0.035 | 0.004 | 0.010 |
| $K^{+}$vertex r loose | 0 | 0 | 0 | 0 |
| $K^{+}$vertex r tight | 0 | 0.003 | 0 | 0 |
| Missing mass squared loose | 0.027 | 0.002 | 0.003 | 0.002 |
| Missing mass squared tight | 0.027 | 0.025 | 0.005 | 0.005 |
| KinFit CL loose | 0.002 | 0.014 | 0.007 | 0.026 |
| KinFit CL tight | 0.001 | 0.009 | 0.001 | 0.016 |
| $M_{\pi^{-} p}$ loose | 0.001 | 0 | 0.004 | 0.007 |
| $M_{\pi^{-} p}$ tight | 0.046 | 0.002 | 0.023 | 0.016 |
| $M_{\Lambda \gamma}$ loose | 0.037 | 0.015 | 0.017 | 0.002 |
| $M_{\Lambda \gamma}$ tight | 0.035 | 0.040 | 0.032 | 0.018 |
| FCAL shower quality loose | 0.021 | 0.005 | 0.022 | 0.019 |
| FCAL shower quality tight | 0.004 | 0.001 | 0.002 | 0.004 |

TABLE 8: Systematic uncertainty for each cut variation for individual $-t$ bins for $-45 / 45$ orientation.

| $-t$ bin $\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | Systematic uncertainty |
| :---: | :---: |
| $0.10<-t<0.35$ | 0.032 |
| $0.35<-t<0.50$ | 0.059 |
| $0.50<-t<0.70$ | 0.031 |
| $0.70<-t<1.40$ | 0.031 |

TABLE 9: Cut dependence systematic uncertainty on $\Sigma$ for individual $-t$ bins for $0 / 90$ orientation.

| $-t$ bin $\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | Systematic uncertainty |
| :---: | :---: |
| $0.10<-t<0.35$ | 0.053 |
| $0.35<-t<0.50$ | 0.039 |
| $0.50<-t<0.70$ | 0.031 |
| $0.70<-t<1.40$ | 0.030 |

TABLE 10: Cut dependence systematic uncertainty on $\Sigma$ for individual $-t$ bins for $-45 / 45$ orientation.

### 5.5.2 INSTRUMENTAL ASYMMETRY

Any instrumental asymmetries not canceled by the PERP - PARA subtraction described in Sec. 5.1 can be seen in the sum $Y_{\perp}(\phi)+F_{R} Y_{\|}(\phi)$. For every $-t$ bin, fits to the sum of yields were done using the functional form

$$
\begin{equation*}
f_{\text {inst }}(\phi)=C\left[1+A_{\text {inst }} \cos 2\left(\phi-\phi_{0}\right)\right] . \tag{49}
\end{equation*}
$$

There are two free parameters, $p_{0}=C$ for normalization and $p_{1}=A_{\text {inst }}$. The same value was used for the phase constant of the fit $\phi_{0}$ as for the fits to the yield asymmetry $S(\phi)$. Figure 64 shows the measured instrumental asymmetries as a function of $-t$ from the fits to the sum of yields shown in Figs. 65 and 66. From the fits, we see that instrumental asymmetry is close to zero for both $0 / 90$ and $-45 / 45$ modes. So, a systematic uncertainty will not be assigned due to the instrumental asymmetry. Additionally, a linear fit was done for the sum of yields to compare $\chi^{2}$ values with that for cosine fit. Linear fits are shown in Figs. 67 and 68 . Both type of fits seem to have similar $\chi^{2}$ values.


FIG. 64: Instrumental asymmetry for $0 / 90$ orientation (top) and $-45 / 45$ orientation (bottom) as a function of $-t$.


FIG. 65: Cosine fits to the sum $Y(\phi)_{\perp}+F_{R} Y(\phi)_{\|}$for $0 / 90$ orientation.


FIG. 66: Cosine fits to the sum $Y(\phi)_{\perp}+F_{R} Y(\phi)_{\|}$for $-45 / 45$ orientation.


FIG. 67: Linear fits to the sum $Y(\phi)_{\perp}+F_{R} Y(\phi)_{\|}$for $0 / 90$ orientation.


FIG. 68: Linear fits to the sum $Y(\phi)_{\perp}+F_{R} Y(\phi)_{\|}$for $-45 / 45$ orientation.

### 5.5.3 PHASE DEPENDENCE

While extracting beam asymmetry values, we chose to fix the phase parameter $\phi_{0}$ in the fits to the yield asymmetry for all $-t$ bins. The linear polarization axis cannot change with $-t$. But, any sensitivity to fixing of this phase is studied with fitting the yield asymmetry allowing phase constant to be a free parameter. Figures 69 and 70 show yield asymmetry plots with phase parameter being free. Figure 71 shows the measured asymmetry values along with the nominal results.


FIG. 69: Yield asymmetry plots for allowing the phase to be a free parameter in the fit for $0 / 90$ orientation. The $\phi_{0}$ values on which fits converge are given in the text boxes inside plots.

Using the standard deviation of histograms for differences between floating $\phi_{0}$ fit asymmetries and nominal asymmetries, a systematic error of $0.11 \%$ was assigned to $0 / 90$ orientation and an error of $0.70 \%$ was assigned to $-45 / 45$ orientation.


FIG. 70: Yield asymmetry plots for allowing the phase to be a free parameter in the fit for $-45 / 45$ orientation. The $\phi_{0}$ values on which fits converge are given in the text boxes inside plots.



FIG. 71: $\Sigma$ asymmetry measured while allowing the phase to be a free parameter in the fit (red closed point) in the $0 / 90$ orientation (top) and $-45 / 45$ orientation (bottom). The open black circles and error bars are the asymmetry values and statistical errors using the fixed phase.

### 5.5.4 FLUX NORMALIZATION DEPENDENCE

To estimate the sensitivity of the measured $\Sigma$ values to the flux normalization determined by PS yields, asymmetries were measured again by changing the nominal flux ratio by $\pm 5 \%$. The measured asymmetries with the varied flux normalization are shown in Fig. 72, The deviations of the red and blue points from the nominal asymmetry values were computed for all $-t$ bins and standard deviations of histograms were taken as the systematic uncertainty of the flux normalization. The average deviation for the $0 / 90$ orientation is 0.002 while the maximum deviation is 0.011 . The average deviation for the $-45 / 45$ orientation is 0.002 while the maximum deviation is 0.008 . A systematic error of $0.496 \%$ was assigned to the $0 / 90$ orientation and an error of $0.38 \%$ was assigned to the $-45 / 45$ orientation respectively.


FIG. 72: $\Sigma$ asymmetry measured after changing flux normalization by $\pm 5 \%$ (red closed points for $+5 \%$ and blue closed points for $-5 \%$ ) in the $0 / 90$ (top) and $-45 / 45$ (bottom) orientations. The open circles and error bars are the asymmetry values and statistical errors using the nominal flux normalization.

### 5.5.5 FINITE $\phi$ BIN SIZE DEPENDENCE

The effect of the bin width of $\phi$ distribution was studied analytically based on the method of Ref. [47]. The correction function to the binned data is defined as (using Table 1 of Ref. [47])

$$
\begin{equation*}
R=\frac{\frac{1}{2} \alpha \Delta}{\sin \frac{1}{2} \alpha \Delta} . \tag{50}
\end{equation*}
$$

where $\alpha=2$ and $\Delta=\frac{2 \pi}{20}$ is the bin width used in $\phi$ histogram. The correction function is a constant as shown in Fig. 73 and has a value of 1.0166 . The beam asymmetry values of each set $(0 / 90$ and $-45 / 45)$ were multiplied by this factor for the final results shown in $\operatorname{Sec} 7.1$.


FIG. 73: Correction function R defined in Eqn. 50 for a bin width of $\Delta=\frac{2 \pi}{20}$

### 5.5.6 MINIMUM PHOTON ENERGY DEPENDENCE

The reaction under study involves low energy radiated photons from $\Sigma^{0}$ decay. The BCAL energy threshold was lowered to 50 MeV while making the ROOT TTrees. Figure 74 shows the photon energy $\left(E_{\gamma}\right)$ distribution in the lab frame for events with $0.1(\mathrm{GeV} / c)^{2}<-t<$ $1.4(\mathrm{GeV} / c)^{2}$. It was estimated that $63 \%$ of the total events are below 100 MeV .


FIG. 74: $E_{\gamma}$ distribution in the lab frame for $0.1(\mathrm{GeV} / c)^{2}<-t<1.4(\mathrm{GeV} / c)^{2}$.

To estimate the sensitivity of the measured $\Sigma$ values to the minimum photon energy in the final state, asymmetries were measured again using the minimum photon energy 55 MeV and 60 MeV . The measured asymmetries with the varied photon energies are shown in Fig. 75. The deviations of the red and blue points from the nominal asymmetry value were computed for all $-t$ bins and the standard deviations of histograms were taken as the systematic uncertainty on minimum photon energy. Average deviation for 0/90 orientation is -0.004 while the maximum deviation is 0.048 . Average deviation for $-45 / 45$ orientation is 0.012 while the maximum deviation is 0.079 . A systematic error of $2.648 \%$ was assigned to $0 / 90$ orientation and an error of $2.866 \%$ was assigned to $-45 / 45$ orientation respectively.

The impact of this cut variation on the total number of signal events was estimated using the distribution shown in Fig. 74. It was found that $8.4 \%$ of signal is lost when applying a minimum photon energy cut at 60 MeV .


FIG. 75: $\Sigma$ asymmetry measured after changing minimum photon energy (red closed points for 55 MeV and blue closed points for 60 MeV ) in the $0 / 90$ (top) and $-45 / 45$ (bottom) orientations. The open circles and error bars are the asymmetry values and statistical errors using the nominal minimum photon energy ( 50 MeV ).

### 5.5.7 ASYMMETRY FROM THE BACKGROUND

In order to find the systematic uncertainty from the $2 \%$ background under the $\Sigma^{0}$ peak the beam asymmetry was found using a sideband region of the invariant mass of $\Lambda \gamma$. Events were selected in the region $1.23 \mathrm{GeV} / c^{2}<M_{\Lambda \gamma}<1.4 \mathrm{GeV} / c^{2}$ where the $\Lambda$ events were in the range $1.107 \mathrm{GeV} / c^{2}<M_{\pi^{-} p}<1.125 \mathrm{GeV} / c^{2}$.

The relation between measured asymmetry, $A_{m}$ and asymmetry of the signal, $A_{s}$ and the background, $A_{b}$ is as follows:

$$
\begin{equation*}
A_{m}=A_{s} \cdot f_{s}+A_{b} \cdot f_{b}, \tag{51}
\end{equation*}
$$

that leads to

$$
\begin{equation*}
A_{s}=\frac{A_{m}-A_{b} \cdot f_{b}}{f_{s}} \tag{52}
\end{equation*}
$$

where $f_{s}$ and $f_{b}$ are the fractions of signal (0.98) and background (0.02) events. Due to the low number of events in the sideband region, we made a sample of events by combining the two data sets $0 / 90$ and $-45 / 45$ and found the beam asymmetries from the background in the entire $t$ range used in the analysis. Figure 76 shows the fit to the yield asymmetry in the chosen $M_{\Lambda \gamma}$ sideband region.


FIG. 76: Fit to the yield asymmmetry in the region $1.23 \mathrm{GeV} / c^{2}<M_{\Lambda \gamma}<1.4 \mathrm{GeV} / c^{2}$ for $0.1(\mathrm{GeV} / c)^{2}<-t<1.4(\mathrm{GeV} / c)^{2}$.

From the fit parameter the beam asymmetry was extracted and after finite bin correction
we obtained a value $A_{b}=0.814 \pm 0.076$. For the average measured asymmetry we calculated the statistical weighted mean of the values given in Table 33 and it was found to be $A_{m}=$ 1.000. Then, using Eqn. 52, we obtained $A_{s}=1.004$ therefore a systematic uncertainty of 0.004 was included as the uncertainty associated with the background for the $t$-channel.

### 5.5.8 NON-UNIFORMITY OF ACCEPTANCE

For the fit function given by Eqn. 46 we were only considering the beam asymmetry $\Sigma$ as the polarization observable in the yields for a given orientation. In general for an experiment with a linearly polarized photon beam and an unpolarized target, when the polarization of the recoiling hyperon can be determined via a weak decay asymmetry, the differential cross section is given by,

$$
\begin{align*}
\frac{d \sigma}{d \Omega}=\left(\frac{d \sigma}{d \Omega}\right)_{0}\{ & 1-P^{\gamma} \Sigma \cos 2 \phi+\alpha \cos \theta_{x} P^{\gamma} O_{x} \sin 2 \phi \\
& +\alpha P \cos \theta_{y}-\alpha \cos \theta_{y} P^{\gamma} T \cos 2 \phi  \tag{53}\\
& \left.+\alpha \cos \theta_{z} P^{\gamma} O_{z} \sin 2 \phi\right\}
\end{align*}
$$

where $\left(\frac{d \sigma}{d \Omega}\right)_{0}$ is the unpolarized cross section, $P^{\gamma}$ is the polarization of the beam and $P, T, O_{x}$ and $O_{z}$ are polarization observables. The direction cosines $\cos \theta_{x, y, z}$ indicate the direction of the decay proton in the hyperon $\left(\Sigma^{0}\right)$ rest frame and $\alpha$ is the weak decay asymmetry [48.

Figure 77 shows the definition of axis used. The three axes $\hat{x}, \hat{y}$ and $\hat{z}$ are defined in the center of mass frame as,

$$
\begin{equation*}
\hat{z}=\frac{\vec{p}_{\gamma}}{\left|\vec{p}_{\gamma}\right|}, \hat{y}=\frac{\vec{p}_{\gamma} \times \vec{p}_{K}}{\left|\vec{p}_{\gamma} \times \vec{p}_{K}\right|}, \hat{x}=\hat{y} \times \hat{z} \tag{54}
\end{equation*}
$$

where $\overrightarrow{p_{\gamma}}$ and $\overrightarrow{p_{K}}$ are momenta of the incoming photon beam and the outgoing kaon. Then the angles $\theta_{x}, \theta_{y}$ and $\theta_{z}$ are measured from the axes $\hat{x}, \hat{y}$ and $\hat{z}$ after boosting to the $\Sigma^{0}$ rest frame and using the final proton momentum vector in the $\Sigma^{0}$ frame.

In the standard analysis method it is assumed that the acceptances in $\theta_{x}, \theta_{y}$ and $\theta_{z}$ are flat and we can try to derive a general formula for the yield asymmetry in the case where acceptances $A\left(\theta_{x}\right), A\left(\theta_{y}\right)$ and $A\left(\theta_{z}\right)$ are not uniform. Then we can make an estimation to see how much uncertainty will be due to this non-uniform acceptance.

Using Eqn. 53 the event yields for $\|$ and $\perp$ photon beam polarization orientations can


FIG. 77: The definitions of the axes used in the study. Figure is from Ref. [49].
be written as,

$$
\begin{align*}
Y_{\|}\left(\phi, \theta_{x}, \theta_{y}, \theta_{z}\right) \propto A\left(\phi, \theta_{x}, \theta_{y}, \theta_{z}\right)[ & -P_{\|} \Sigma \cos 2 \phi+\alpha \cos \theta_{x} P_{\|} O_{x} \sin 2 \phi \\
& +\alpha P \cos \theta_{y}-\alpha \cos \theta_{y} P_{\|} T \cos 2 \phi  \tag{55}\\
& \left.+\alpha \cos \theta_{z} P_{\|} O_{z} \sin 2 \phi\right] \\
Y_{\perp}\left(\phi, \theta_{x}, \theta_{y}, \theta_{z}\right) \propto A\left(\phi, \theta_{x}, \theta_{y}, \theta_{z}\right)[1 & +P_{\perp} \Sigma \cos 2 \phi-\alpha \cos \theta_{x} P_{\perp} O_{x} \sin 2 \phi \\
& +\alpha P \cos \theta_{y}+\alpha \cos \theta_{y} P_{\perp} T \cos 2 \phi  \tag{56}\\
& \left.-\alpha \cos \theta_{z} P_{\perp} O_{z} \sin 2 \phi\right]
\end{align*}
$$

if $P_{\|}=P_{\perp}=P_{\gamma}$, the yield asymmetry will be

$$
\begin{align*}
& P_{\gamma} \Sigma \cos 2 \phi \int A\left(\theta_{x}\right) d\left(\cos \theta_{x}\right) \int A\left(\theta_{y}\right) d\left(\cos \theta_{y}\right) \int A\left(\theta_{z}\right) d\left(\cos \theta_{z}\right) \\
&-\alpha P_{\gamma} O_{x} \sin 2 \phi \int A\left(\theta_{x}\right) \cos \theta_{x} d\left(\cos \theta_{x}\right) \int A\left(\theta_{y}\right) d\left(\cos \theta_{y}\right) \int A\left(\theta_{z}\right) d\left(\cos \theta_{z}\right) \\
&+\alpha P_{\gamma} T \cos 2 \phi \int A\left(\theta_{x}\right) d\left(\cos \theta_{x}\right) \int A\left(\theta_{y}\right) \cos \theta_{y} d\left(\cos \theta_{y}\right) \int A\left(\theta_{z}\right) d\left(\cos \theta_{z}\right) \\
& \frac{Y(\phi)_{\perp}-Y(\phi)_{\|}}{Y(\phi)_{\perp}+Y(\phi)_{\|}}=\frac{-\alpha P_{\gamma} O_{z} \sin 2 \phi \int A\left(\theta_{x}\right) d\left(\cos \theta_{x}\right) \int A\left(\theta_{y}\right) d\left(\cos \theta_{y}\right) \int A\left(\theta_{z}\right) \cos \theta_{z} d\left(\cos \theta_{z}\right)}{\int A\left(\theta_{x}\right) d\left(\cos \theta_{x}\right) \int A\left(\theta_{y}\right) d\left(\cos \theta_{y}\right) \int A\left(\theta_{z}\right) d\left(\cos \theta_{z}\right)} \\
&+\alpha P \int A\left(\theta_{x}\right) d\left(\cos \theta_{x}\right) \int A\left(\theta_{y}\right) \cos \theta_{y} d\left(\cos \theta_{y}\right) \int A\left(\theta_{z}\right) d\left(\cos \theta_{z}\right) \tag{57}
\end{align*}
$$

This can be written as,

$$
\frac{Y(\phi)_{\perp}-Y(\phi)_{\|}}{Y(\phi)_{\perp}+Y(\phi)_{\|}}=\frac{\begin{array}{l}
P_{\gamma} \Sigma \cos 2 \phi-\alpha P_{\gamma} O_{x} \sin 2 \phi \frac{\int A\left(\theta_{x}\right) \cos \theta_{x} d\left(\cos \theta_{x}\right)}{\int A\left(\theta_{x}\right) d\left(\cos \theta_{x}\right)} \\
+\alpha P_{\gamma} T \cos 2 \phi \frac{\int A\left(\theta_{y}\right) \cos \theta_{y} d\left(\cos \theta_{y}\right)}{\int A\left(\theta_{y}\right) d\left(\cos \theta_{y}\right)}-\alpha P_{\gamma} O_{z} \sin 2 \phi \frac{\int A\left(\theta_{z}\right) \cos \theta_{z} d\left(\cos \theta_{z}\right)}{\int A\left(\theta_{z}\right) d\left(\cos \theta_{z}\right)}  \tag{58}\\
1+\alpha P \frac{\int A\left(\theta_{y}\right) \cos \theta_{y} d\left(\cos \theta_{y}\right)}{\int A\left(\theta_{y}\right) d\left(\cos \theta_{y}\right)}
\end{array}}{\qquad}
$$

If we consider $R_{x}=\frac{\int A\left(\theta_{x}\right) \cos \theta_{x} d\left(\cos \theta_{x}\right)}{\int A\left(\theta_{x}\right) d\left(\cos \theta_{x}\right)}, R_{y}=\frac{\int A\left(\theta_{y}\right) \cos \theta_{y} d\left(\cos \theta_{y}\right)}{\int A\left(\theta_{y}\right) d\left(\cos \theta_{y}\right)}, R_{z}=\frac{\int A\left(\theta_{z}\right) \cos \theta_{z} d\left(\cos \theta_{z}\right)}{\int A\left(\theta_{z}\right) d\left(\cos \theta_{z}\right)}$ and calculate the integrals in $\cos \theta_{i}$ from -1 to 1 we can find the contribution coming from the asymmetry of events in the corresponding angle.

Tables $11-13$ show the calculated integrals and the corresponding ratios for each $-t$ bin used in the analysis. The values from integrals contained more significant figures and are used to find the ratio but the values given in tables are rounded. Acceptance plots in angles $\theta_{x}, \theta_{y}$ and $\theta_{z}$ are shown in Appendix F. The acceptance histograms were fitted with a 4th order polynomial and the acceptance functions were found for use in the integrals.

Additionally, 2 d plots were made for recoil angles $\theta_{x}, \theta_{y}$ and $\theta_{z}$ vs. $K^{+}$lab $\phi$ angle from $t$-channel MC and corresponding acceptances are shown in Appendix F. Acceptances are uniform in $\phi$.

| $-t$ bin $\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | $\int A\left(\theta_{x}\right) d\left(\cos \theta_{x}\right)$ | $\int A\left(\theta_{x}\right) \cos \theta_{x} d\left(\cos \theta_{x}\right)$ | $R_{x}$ |
| :---: | :---: | :---: | :---: |
| $0.10<-t<0.35$ | 0.026 | 0.010 | 0.385 |
| $0.35<-t<0.50$ | 0.073 | 0.031 | 0.419 |
| $0.50<-t<0.70$ | 0.084 | 0.030 | 0.362 |
| $0.70<-t<1.40$ | 0.082 | 0.022 | 0.266 |

TABLE 11: The integrals associated with the asymmetry of events in recoil angle $\theta_{x}$ for $t$-channel

| $-t \operatorname{bin}\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | $\int A\left(\theta_{y}\right) d\left(\cos \theta_{y}\right)$ | $\int A\left(\theta_{y}\right) \cos \theta_{y} d\left(\cos \theta_{y}\right)$ | $R_{y}$ |
| :---: | :---: | :---: | :---: |
| $0.10<-t<0.35$ | 0.026 | 0.004 | 0.148 |
| $0.35<-t<0.50$ | 0.073 | 0.009 | 0.127 |
| $0.50<-t<0.70$ | 0.084 | 0.011 | 0.126 |
| $0.70<-t<1.40$ | 0.082 | 0.011 | 0.138 |

TABLE 12: The integrals associated with the asymmetry of events in recoil angle $\theta_{y}$ for $t$-channel

| $-t \operatorname{bin}\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | $\int A\left(\theta_{z}\right) d\left(\cos \theta_{z}\right)$ | $\int A\left(\theta_{z}\right) \cos \theta_{z} d\left(\cos \theta_{z}\right)$ | $R_{z}$ |
| :---: | :---: | :---: | :---: |
| $0.10<-t<0.35$ | 0.026 | 0.00009 | 0.003 |
| $0.35<-t<0.50$ | 0.073 | -0.005 | -0.075 |
| $0.50<-t<0.70$ | 0.084 | -0.008 | -0.090 |
| $0.70<-t<1.40$ | 0.082 | -0.007 | -0.083 |

TABLE 13: The integrals associated with the asymmetry of events in recoil angle $\theta_{z}$ for $t$-channel

Writing Eqn. 58 in terms of $R_{x}, R_{y}$ and $R_{z}$ gives,

$$
\frac{Y(\phi)_{\perp}-Y(\phi)_{\|}}{Y(\phi)_{\perp}+Y(\phi)_{\|}}=\frac{\begin{array}{l}
P_{\gamma} \Sigma \cos 2 \phi-\alpha P_{\gamma} O_{x} \sin 2 \phi R_{x} \\
+\alpha P_{\gamma} T \cos 2 \phi R_{y}-\alpha P_{\gamma} O_{z} \sin 2 \phi R_{z}  \tag{59}\\
1+\alpha P R_{y}
\end{array}}{\frac{1}{}}
$$

This can be written in the following form,

$$
\begin{equation*}
\frac{Y(\phi)_{\perp}-Y(\phi)_{\|}}{Y(\phi)_{\perp}+Y(\phi)_{\|}}=\frac{P_{\gamma}[(\Sigma+B) \cos 2 \phi+C \sin 2 \phi]}{1+D} \tag{60}
\end{equation*}
$$

where $B=\alpha T R_{y}, C=-\left(\alpha O_{x} R_{x}+\alpha O_{z} R_{z}\right)$ and $D=\alpha P R_{y}$.

In order to find the constants $B, C$ and $D$ we used $\alpha=0.750[50]$ and the RPR-2007 model [14] predictions for $P, T, O_{x}$ and $O_{z}$. Figure 78 shows the predictions for those polarization observables and beam asymmetry $\Sigma$ for the $t$-channel. Values found for $B, C$ and $D$ are given in Table 14.


FIG. 78: RPR-2007 predictions for polarization observables for $\gamma p \rightarrow K^{+} \Sigma^{0}$ at $E_{\gamma}=8.5$ GeV for the $t$-channel.

| $-t\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | $B$ | $C$ | $D$ |
| :---: | :---: | :---: | :---: |
| 0.268 | 0.0305994 | 0.00828148 | 0.0308824 |
| 0.424 | 0.0229152 | 0.0368844 | 0.0233467 |
| 0.593 | 0.0183028 | 0.0376789 | 0.0188395 |
| 0.912 | 0.00917362 | 0.0173528 | 0.00963026 |

TABLE 14: Constants $B, C$ and $D$ for the mean values of $-t$ bins used in the analysis.

Then we can use the new fit function given in Eqn. 60 and find the beam asymmetry values (only fit parameter) to compare with the nominal fit results. Tables 15 and 16 show the values from the fits and the differences.

| $-t\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | nominal fit value | new fit value | difference |
| :---: | :---: | :---: | :---: |
| 0.268 | 1.100 | 1.104 | 0.004 |
| 0.424 | 1.001 | 1.002 | 0.001 |
| 0.593 | 1.057 | 1.058 | 0.001 |
| 0.912 | 0.9263 | 0.9267 | 0.0004 |

TABLE 15: Comparison of $\Sigma$ after the new fit with the nominal values for $0 / 90$ orientation set.

| $-t\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | nominal fit value | new fit value | difference |
| :---: | :---: | :---: | :---: |
| 0.268 | 0.8191 | 0.8141 | 0.0050 |
| 0.424 | 1.0144 | 1.0159 | 0.0015 |
| 0.593 | 0.9612 | 0.9611 | 0.0001 |
| 0.912 | 0.9530 | 0.9521 | 0.0009 |

TABLE 16: Comparison of $\Sigma$ after the new fit with the nominal values for $-45 / 45$ orientation set.

This suggests that the maximum systematic uncertainty due to non-uniform acceptance is $\sim 0.5 \%$ for the $t$-channel. However as can be seen in Fig. 78, $P$ and $T$ have almost same values from the model prediction. This can be due to some theoretical constraints on the polarization observables the model has used and we did another study to find the systematic uncertainty using more conservative values for $P$ and $T$.
The values used are,
(i) $T=0$ and $P=0.25$
(ii) $T=0.25$ and $P=0.1$.
$O_{x}$ and $O_{z}$ were assumed to be zero.

Case (i): $T=0, P=0.25$ and $O_{x}=O_{z}=0$

| $-t\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | $B$ | $C$ | $D$ |
| :---: | :---: | :---: | :---: |
| 0.268 | 0 | 0 | 0.02775 |
| 0.424 | 0 | 0 | 0.0238125 |
| 0.593 | 0 | 0 | 0.023625 |
| 0.912 | 0 | 0 | 0.025875 |

TABLE 17: Constants $B, C$ and $D$ for the mean values of $-t$ bins used in the analysis for $T=0$ and $P=0.25$.

| $-t\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | nominal fit value | new fit value | difference |
| :---: | :---: | :---: | :---: |
| 0.268 | 1.100 | 1.131 | 0.031 |
| 0.424 | 1.001 | 1.025 | 0.024 |
| 0.593 | 1.057 | 1.081 | 0.024 |
| 0.912 | 0.926 | 0.951 | 0.025 |

TABLE 18: Comparison of $\Sigma$ values after the new fit with the nominal values for $0 / 90$ orientation set for $T=0$ and $P=0.25$.

| $-t\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | nominal fit value | new fit value | difference |
| :---: | :---: | :---: | :---: |
| 0.268 | 0.819 | 0.842 | 0.023 |
| 0.424 | 1.014 | 1.038 | 0.024 |
| 0.593 | 0.961 | 0.984 | 0.023 |
| 0.912 | 0.953 | 0.977 | 0.024 |

TABLE 19: Comparison of $\Sigma$ values after the new fit with the nominal values for $-45 / 45$ orientation set for $T=0$ and $P=0.25$.

Case (ii): $T=0.25, P=0.1$ and $O_{x}=O_{z}=0$

| $-t\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | $B$ | $C$ | $D$ |
| :---: | :---: | :---: | :---: |
| 0.268 | 0.02775 | 0 | 0.0111 |
| 0.424 | 0.0238125 | 0 | 0.009525 |
| 0.593 | 0.023625 | 0 | 0.00945 |
| 0.912 | 0.025875 | 0 | 0.01035 |

TABLE 20: Constants $B, C$ and $D$ for the mean values of $-t$ bins used in the analysis for $T=0.25$ and $P=0.1$.

| $-t\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | nominal fit value | new fit value | difference |
| :---: | :---: | :---: | :---: |
| 0.268 | 1.100 | 1.085 | 0.015 |
| 0.424 | 1.001 | 0.987 | 0.014 |
| 0.593 | 1.057 | 1.043 | 0.014 |
| 0.912 | 0.926 | 0.910 | 0.016 |

TABLE 21: Comparison of $\Sigma$ values after the new fit with the nominal values for $0 / 90$ orientation set for $T=0.25$ and $P=0.1$.

| $-t\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | nominal fit value | new fit value | difference |
| :---: | :---: | :---: | :---: |
| 0.268 | 0.819 | 0.801 | 0.018 |
| 0.424 | 1.014 | 1.000 | 0.014 |
| 0.593 | 0.961 | 0.947 | 0.014 |
| 0.912 | 0.953 | 0.937 | 0.016 |

TABLE 22: Comparison of $\Sigma$ values after the new fit with the nominal values for $-45 / 45$ orientation set for $T=0.25$ and $P=0.1$.

From Tables 18 - 22 we can see that the systematic uncertainties range from $\sim 1 \%-3 \%$. We chose to include $3 \%$ as a more conservative systematic uncertainty due to the non-uniform acceptance for the $t$-channel.

### 5.5.9 BEAM POLARIZATION

The beam polarization values used for beam asymmetry analysis and their statistical uncertainties were given in Table 2. We see that for each orientation polarization P is close to 0.35 while the statistical uncertainty is around 0.01 . This gives $\frac{\delta P}{P} \approx 0.03$. This uncertainty will shift beam asymmetry measurements by an overall scale factor. Systematic uncertainties on TPOL measurements also contribute to this normalization uncertainty. The systematic uncertainty from TPOL is estimated to be $1.5 \%$ [26]. Since the final beam asymmetry result contains statistically weighted result from four orientations, the statistical uncertainty must be scaled by a factor of 4 . The total normalization uncertainty on the final beam asymmetry result is therefore, $\sqrt{\frac{(0.03)^{2}}{4}+(0.015)^{2}}=2.1 \%$. This will not be combined with other systematic uncertainties.

### 5.5.10 SYSTEMATICS SUMMARY

A summary of systematic uncertainties on the $\Sigma$ beam asymmetries obtained for the $t$-channel is presented in Table 23. Total systematic uncertainty for each orientation set was found by adding individual contributions given in Table 23 in quadrature. As mentioned in Sec. 5.5.9, the uncertainty from the beam polarization was not combined with this systematics.

| Study | $0 / 90$ Systematic Error | -45/45 Systematic Error |
| :---: | :---: | :---: |
| Event selection | $0.031-0.059$ | $0.030-0.053$ |
| Phase dependence | 0.001 | 0.007 |
| Flux normalization dependence | 0.005 | 0.004 |
| Minimum photon energy | 0.026 | 0.029 |
| Background | 0.004 | 0.004 |
| Non-uniform acceptance | 0.030 | 0.030 |
| Total | $0.051-0.071$ | $0.052-0.068$ |

TABLE 23: Summary of systematic uncertainties from the studies described in Sec. 5.5.

## CHAPTER 6

## BEAM ASYMMETRY MEASUREMENT (u-CHANNEL)

### 6.1 FITS TO THE $\Sigma^{0}$ SAMPLE

Figures 79 and 80 show the fits used for the $\Sigma^{0}$ event sample integrated over the entire $u$ region used for the analysis $\left(-u<2.0(\mathrm{GeV} / c)^{2}\right)$. The upper plots are fits to the PARA (left) and PERP (right) yields independently. There are two free parameters in these fits of the form $f(\phi)=C\left[1+P \Sigma \cos \left(2\left(\phi-\phi_{0}\right)\right)\right]$, with $p_{0}=C$ for normalization and $p_{1}=P \Sigma$. Fits to the sum of yields were done using the functional form given in Eqn. 49 .

Same phase constant values were used for $\phi_{0}$ which were given in Sec. 5.3.
From the yield asymmetry plots, beam asymmetry values were obtained for $-u<2.0$ $(\mathrm{GeV} / c)^{2}$ and they are,
$\Sigma=0.428 \pm 0.099$ for $0 / 90$ orientation,
$\Sigma=0.338 \pm 0.099$ for $-45 / 45$ orientation
where uncertainties are statistical errors from the fits.
Additionally, a linear fit was done for the sum of yields to compare $\chi^{2}$ values with that for cosine fit. Linear fits are shown in Figs. 81 and 82. Both types of fits seem to have similar $\chi^{2}$ values.


FIG. 79: Polarization direction set to $0 / 90$. Fits to the $\phi$ distributions in the beam energy range $8.2<E_{\gamma}<8.8 \mathrm{GeV}$, integrated over the entire $u$ range. Upper row: fits to the PARA (left) and PERP (right) distributions independently. Lower row (left): fit to the asymmetry (Eqn. 45). Lower row (right): fit to the sum $Y(\phi)_{\perp}+F_{R} Y(\phi)_{\|}$, which is sensitive to instrumental asymmetries.


FIG. 80: Polarization direction set to $-45 / 45$. Fits to the $\phi$ distributions in the beam energy range $8.2<E_{\gamma}<8.8 \mathrm{GeV}$, integrated over the entire $u$ range. Upper row: fits to the PARA (left) and PERP (right) distributions independently. Lower row (left): fit to the asymmetry (Eqn. 45). Lower row (right): fit to the sum $Y(\phi)_{\perp}+F_{R} Y(\phi)_{\|}$, which is sensitive to instrumental asymmetries.


FIG. 81: Linear fit to the sum $Y(\phi)_{\perp}+F_{R} Y(\phi)_{\|}$for $0 / 90$ orientation.


FIG. 82: Linear fit to the sum $Y(\phi)_{\perp}+F_{R} Y(\phi)_{\|}$for $-45 / 45$ orientation.

### 6.2 SYSTEMATIC STUDIES

### 6.2.1 EVENT SELECTION

By varying the parameters of the event selection described in Sec. 4.2.2, the systematic dependence of the asymmetry on the event selection criteria was studied. The list of nominal cuts and the looser and tighter cuts that were used for this are same as those in Table 6. For each variation of the cut, the asymmetry was extracted for the $-u$ bin.

Figure 83 shows the values of the asymmetry where each parameter is represented by an individual color, and the asymmetries for the nominal cut parameters are shown by the open black circles. All the filled data points are within the $1 \sigma$ statistical error of the nominal cut value points. Colors for different cut variations are the same as shown in Fig. 63 .


FIG. 83: $\Sigma$ asymmetry measured for each of the cut parameter variations, shown as different colored points for the $0 / 90$ (top) and $-45 / 45$ (bottom) orientations. The open circles and vertical error bars are the asymmetry values and statistical errors using the nominal cut parameters.

Table 24 gives the systematic uncertainties for each cut variation for $0 / 90$ and $-45 / 45$ orientation sets.

The total systematic uncertainty due to the cut variation is $5 \%$ for $0 / 90$ orientation and

| cut variation | $0 / 90$ uncertainty | $-45 / 45$ uncertainty |
| :---: | :---: | :---: |
| $K^{+}$vertex z loose | 0.015 | 0.007 |
| $K^{+}$vertex z tight | 0.003 | 0.016 |
| $K^{+}$vertex r loose | 0 | 0 |
| $K^{+}$vertex r tight | 0 | 0 |
| Missing mass squared loose | 0.004 | 0.025 |
| Missing mass squared tight | 0.013 | 0.008 |
| KinFit CL loose | 0.029 | 0.014 |
| KinFit CL tight | 0.049 | 0.006 |
| $M_{\pi^{-} p}$ loose | 0.018 | 0.004 |
| $M_{\pi^{-} p}$ tight | 0.006 | 0.011 |
| $M_{\Lambda \gamma}$ loose | 0.011 | 0.021 |
| $M_{\Lambda \gamma}$ tight | 0.013 | 0.018 |
| FCAL shower quality loose | 0.017 | 0.007 |
| FCAL shower quality tight | 0.030 | 0.043 |

TABLE 24: Systematic uncertainty for each cut variation for $0 / 90$ and $-45 / 45$ orientations.
$4 \%$ for $-45 / 45$ orientation.

### 6.2.2 PHASE DEPENDENCE

Any sensitivity that can be due to fixing of the phase parameter $\phi_{0}$ was studied with fitting the yield asymmetry allowing phase constant to be a free parameter. Figures 84 and 85 show yield asymmetry plots with phase parameter being free. Figure 86 shows the measured asymmetry values along with the nominal results.

Using the difference of beam asymmetry values from nominal, a systematic error of $2.2 \%$ was assigned to $0 / 90$ orientation and an error of $2.1 \%$ was assigned to $-45 / 45$ orientation.


FIG. 84: Yield asymmetry allowing the phase to be a free parameter in the fit for $0 / 90$ orientation. The $\phi_{0}$ value on which fit converge is given in the text box inside the plot.


FIG. 85: Yield asymmetry allowing the phase to be a free parameter in the fit for $-45 / 45$ orientation. The $\phi_{0}$ value on which fit converge is given in the text box inside the plot.


FIG. 86: $\Sigma$ asymmetry measured while allowing the phase to be a free parameter in the fit (red closed point) in the $0 / 90$ orientation (top) and $-45 / 45$ orientation (bottom). The open black circle and error bar are asymmetry value and statistical error using the fixed phase.

### 6.2.3 FLUX NORMALIZATION DEPENDENCE

To estimate the sensitivity of the measured $\Sigma$ values to the flux normalization determined by PS yields, asymmetries were measured again by changing the nominal flux ratio by $\pm 5 \%$. The measured asymmetries with the varied flux normalization are shown in Fig. 87. Using the difference between the measured asymmetries from the nominal, a systematic error of $0.6 \%$ was assigned to $0 / 90$ orientation and an error of $0.2 \%$ was assigned to $-45 / 45$ orientation respectively.


FIG. 87: $\Sigma$ asymmetry measured after changing flux normalization by $\pm 5 \%$ (red closed points for $+5 \%$ and blue closed points for $-5 \%$ ) in the $0 / 90$ (top) and $-45 / 45$ (bottom) orientations. The open circles and error bars are the asymmetry values and statistical errors using the nominal flux normalization.

### 6.2.4 FINITE $\phi$ BIN SIZE DEPENDENCE

The effect of the bin width of $\phi$ distribution was studied analytically using Eqn. 50 . $\Delta=\frac{2 \pi}{10}$ was used as the bin width. The correction function is a constant as shown in Fig. 88 and has a value of 1.069 . Values of $\Sigma$ shown in Sec. 6.1 were multiplied by this number and the corrected beam asymmetry values are given in Sec. 7.2 ,


FIG. 88: Correction function R defined in Eqn. 50 for a bin width of $\Delta=\frac{2 \pi}{10}$

### 6.2.5 MINIMUM PHOTON ENERGY DEPENDENCE

In Sec. 5.5.6, it was shown that there is a large fraction of photons from $\Sigma^{0}$ decay with energy below 100 MeV for the $t$-channel. Figure 89 shows the photon energy $\left(E_{\gamma}\right)$ distribution in the lab frame for events with $-u<2.0(\mathrm{GeV} / c)^{2}$. It was estimated that only $5 \%$ of the total events are below 100 MeV . So, the event sample in the low $|u|$ region is insensitive to the minimum shower energy around 50 MeV . Therefore a systematic uncertainty was not estimated for the $u$-channel due to minimum photon energy.


FIG. 89: $E_{\gamma}$ distribution in the lab frame for $-u<2.0(\mathrm{GeV} / c)^{2}$.

### 6.2.6 ASYMMETRY FROM THE BACKGROUND

Following the method used in Sec. 5.5.7for the $t$-channel, beam asymmetry was calculated for the background using the region $1.23 \mathrm{GeV} / c^{2}<M_{\Lambda \gamma}<1.4 \mathrm{GeV} / c^{2}$ for $-u<2.0$ $(\mathrm{GeV} / c)^{2}$. Figure 90 shows the fit to the yield asymmetry.


FIG. 90: Fit to the yield asymmmetry in the region $1.23 \mathrm{GeV} / c^{2}<M_{\Lambda \gamma}<1.4 \mathrm{GeV} / c^{2}$ for $-u<2.0(\mathrm{GeV} / c)^{2}$.

From the fit parameter the beam asymmetry was extracted and the finite bin correction gave a value $A_{b}=0.199 \pm 0.300$. The average measured asymmetry was found to be $A_{m}=$ 0.410. Then, using Eqn. 52, we obtained a value of $A_{s}=0.414$ therefore a systematic uncertainty of 0.004 was included as the uncertainty associated with the background for the $u$-channel.

### 6.2.7 NON-UNIFORMITY OF ACCEPTANCE

Similar to the method shown in Sec. 5.5.8 the non-uniformity of acceptance in recoil angles of the decay proton were estimated for the $u$-channel. Corresponding histograms are shown in Appendix F.

Additionally, 2 d plots were made for recoil angles $\theta_{x}, \theta_{y}$ and $\theta_{z}$ vs. $K^{+}$lab $\phi$ angle from $u$-channel MC and the corresponding acceptances are shown in Appendix F. Acceptances are uniform in $\phi$.

| $-u \operatorname{bin}\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | $\int A\left(\theta_{x}\right) d\left(\cos \theta_{x}\right)$ | $\int A\left(\theta_{x}\right) \cos \theta_{x} d\left(\cos \theta_{x}\right)$ | $R_{x}$ |
| :---: | :---: | :---: | :---: |
| $-u<2.0$ | 0.2159 | -0.0008 | -0.0039 |

TABLE 25: The integrals associated with the asymmetry of events in recoil angle $\theta_{x}$ for $u$-channel.

| $-u$ bin $\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | $\int A\left(\theta_{y}\right) d\left(\cos \theta_{y}\right)$ | $\int A\left(\theta_{y}\right) \cos \theta_{y} d\left(\cos \theta_{y}\right)$ | $R_{y}$ |
| :---: | :---: | :---: | :---: |
| $-u<2.0$ | 0.2158 | 0.0058 | 0.0268 |

TABLE 26: The integrals associated with the asymmetry of events in recoil angle $\theta_{y}$ for $u$-channel.

| $-u \operatorname{bin}\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | $\int A\left(\theta_{z}\right) d\left(\cos \theta_{z}\right)$ | $\int A\left(\theta_{z}\right) \cos \theta_{z} d\left(\cos \theta_{z}\right)$ | $R_{z}$ |
| :---: | :---: | :---: | :---: |
| $-u<2.0$ | 0.2159 | -0.0073 | -0.0340 |

TABLE 27: The integrals associated with the asymmetry of events in recoil angle $\theta_{z}$ for $u$-channel.

In order to find the uncertainty using the ratios $R_{i}$, we made predictions for other polarization observables for a beam asymmetry value of $\Sigma=0.4$. Based on the Ref. [51], the following constraint equations were used while calculating the observables.

$$
\begin{align*}
\left|\left[\Sigma P-\left(C_{x} O_{z}-C_{z} O_{x}\right)\right]-T\right| & <0.005  \tag{61}\\
\left|\left[O_{x}^{2}+O_{z}^{2}+C_{x}^{2}+C_{z}^{2}+\Sigma^{2}-T^{2}+P^{2}\right]-1\right| & <0.0001  \tag{62}\\
O_{x}^{2}+O_{z}^{2}+\Sigma^{2} & <1  \tag{63}\\
|T-P| & \leq 1-\Sigma  \tag{64}\\
|T+P| & \leq 1+\Sigma \tag{65}
\end{align*}
$$

Figure 91 show the relations between $B, C$ and $D$ and Fig. 92 shows $P$ vs. $T$ for 1000 possible values for $P, T, O_{x}, O_{z}, C_{x}$ and $C_{z}$ satisfying the above constraint equations. From the plot of $D$ vs. $B$ values for two extremes were used in the fit function given by Eqn. 60 while keeping $C=0$.
The values used are,
(i) $B=0.008$ and $D=0.018$
(ii) $B=-0.018$ and $D=-0.008$


FIG. 91: Plots of the ratios $C$ vs. $B$ (left), $D$ vs. $C$ (middle) and $D$ vs. $B$ (right) found for $\Sigma=0.4$.


FIG. 92: $P$ vs. $T$ for $\Sigma=0.4$.
Case (i): $B=0.008$ and $D=0.018$

| orientation set | nominal fit value | new fit value | difference |
| :---: | :---: | :---: | :---: |
| $0 / 90$ | 0.4282 | 0.4284 | 0.0002 |
| $-45 / 45$ | 0.3378 | 0.3358 | 0.0020 |

TABLE 28: Comparison of $\Sigma$ values after the new fit with the nominal values for $B=0.008$ and $D=0.018$.

Case (ii): $B=-0.018$ and $D=-0.008$

| orientation set | nominal fit value | new fit value | difference |
| :---: | :---: | :---: | :---: |
| $0 / 90$ | 0.4282 | 0.4432 | 0.0150 |
| $-45 / 45$ | 0.3378 | 0.3530 | 0.0152 |

TABLE 29: Comparison of $\Sigma$ values after the new fit with the nominal values for $B=-0.018$ and $D=-0.008$.

From Tables 28 and 29 we can see that the maximum systematic uncertainty due to the non-uniformity of acceptance is $\sim 1.5 \%$ for the $u$-channel.

### 6.2.8 SYSTEMATICS SUMMARY

A summary of systematic uncertainties on the $\Sigma$ beam asymmetries obtained for the $u$-channel is presented in Table 30. Total systematic uncertainty for each orientation set was found by adding individual contributions given in Table 30 in quadrature.

| Study | $0 / 90$ Systematic Error | $-45 / 45$ Systematic Error |
| :---: | :---: | :---: |
| Event selection | 0.050 | 0.040 |
| Phase dependence | 0.022 | 0.021 |
| Flux normalization dependence | 0.006 | 0.002 |
| Background | 0.004 | 0.004 |
| Non-uniform acceptance | 0.015 | 0.015 |
| Total | 0.057 | 0.048 |

TABLE 30: Summary of systematic uncertainties from the studies described in Sec. 6.2.

## CHAPTER 7

## RESULTS

## $7.1 t$-CHANNEL

Figure 93 shows the results for $\Sigma$ beam asymmetry for $\gamma p \rightarrow K^{+} \Sigma^{0}$ as a function of $-t$ for $0 / 90$ and $-45 / 45$ orientation sets. Total uncertainties for each orientation set were found from adding statistical and systematic uncertainties in quadrature. Results are compared with previous SLAC 16 GeV results and RPR-2007 model prediction at 8.5 GeV .


FIG. 93: The beam asymmetry $\Sigma$ for $\gamma p \rightarrow K^{+} \Sigma^{0}$ in the $t$ channel. The black solid circles correspond to the polarization direction set to $0 / 90$, magenta open circles correspond to the polarization direction set to $-45 / 45\left(-t\right.$ coordinates are shifted by $0.02(\mathrm{GeV} / c)^{2}$ to the right for clarity), vertical error bars represent total uncertainties while horizontal error bars are to indicate the RMS widths of the $-t$ bins used. Red squares are previous SLAC [7] results at $E_{\gamma}=16 \mathrm{GeV}$ and the blue dotted curve represents predicted values from RPR-2007 model [14] at $E_{\gamma}=8.5 \mathrm{GeV}$.

Figure 94 shows the final result found by combining results of $0 / 90$ and $-45 / 45$. We used a conservative approach and the larger systematic uncertainty from the two data sets were used as the average systematic uncertainty for each $-t$ bin. Combined beam asymmetry values and statistical uncertainties were found by using statistical errors of the two data sets as weights as given below following the method in Ref. [52].

The weighted average asymmetry is given by,

$$
\begin{equation*}
\hat{\mu}=\frac{\sum x_{i} / \sigma_{i}^{2}}{\sum 1 / \sigma_{i}^{2}} \tag{66}
\end{equation*}
$$

where $x_{i}$ are measured beam asymmetry values for $0 / 90$ and $-45 / 45$ orientation sets for a given $-t$ bin and $\sigma_{i}$ are their statistical uncertainties.

The average error on the weighted beam asymmetry was found as,

$$
\begin{equation*}
\sigma(\hat{\mu})=\sqrt{\frac{1}{\sum 1 / \sigma_{i}^{2}}} \tag{67}
\end{equation*}
$$

Then, total uncertainties of the combined results were found by adding the combined statistical and systematic errors in quadrature. In addition to the prediction from RPR2007 model we added a curve (red, dashed) for the model prediction from Guidal et al. [53]. Both models use Regge theory and give similar prediction for beam asymmetry as a function of $-t$ for energies beyond resonance region.

Tables 31 - 33 summarize the $\Sigma$ beam asymmetry results for individual orientation sets and the average result for the $t$-channel.


FIG. 94: Beam asymmetry $\Sigma$ for $\gamma p \rightarrow K^{+} \Sigma^{0}$ in the $t$ channel. The black solid circles correspond to combined results from $0 / 90$ and $-45 / 45$ data sets in this analysis, vertical error bars represent total uncertainties while horizontal error bars are to indicate the RMS widths of the $-t$ bins used. Gray triangles are previous SLAC results [7] at $E_{\gamma}=16 \mathrm{GeV}$ and the curves show predicted values from RPR-2007 [13, 14] (blue, solid) and Guidal et al. [53] (red, dashed) at $E_{\gamma}=8.5 \mathrm{GeV}$ respectively.

| $-t$ bin $\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | $\Sigma$ | Stat. uncert. | Syst. uncert. | Total uncert. |
| :---: | :---: | :---: | :---: | :---: |
| $0.10<-t<0.35$ | 1.118 | 0.100 | 0.051 | 0.112 |
| $0.35<-t<0.50$ | 1.018 | 0.097 | 0.071 | 0.120 |
| $0.50<-t<0.70$ | 1.075 | 0.092 | 0.051 | 0.105 |
| $0.70<-t<1.40$ | 0.941 | 0.092 | 0.051 | 0.105 |

TABLE 31: Summary of $\Sigma$ beam asymmetry for the $0 / 90$ orientation.

| $-t$ bin $\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | $\Sigma$ | Stat. uncert. | Syst. uncert. | Total uncert. |
| :---: | :---: | :---: | :---: | :---: |
| $0.10<-t<0.35$ | 0.833 | 0.114 | 0.068 | 0.133 |
| $0.35<-t<0.50$ | 1.031 | 0.102 | 0.058 | 0.117 |
| $0.50<-t<0.70$ | 0.977 | 0.100 | 0.053 | 0.113 |
| $0.70<-t<1.40$ | 0.969 | 0.098 | 0.052 | 0.111 |

TABLE 32: Summary of $\Sigma$ beam asymmetry for the $-45 / 45$ orientation

| $-t$ bin $\left[(\mathrm{GeV} / \mathrm{c})^{2}\right]$ | $\Sigma$ | Stat. uncert. | Syst. uncert. | Total uncert. |
| :---: | :---: | :---: | :---: | :---: |
| $0.10<-t<0.35$ | 0.994 | 0.075 | 0.068 | 0.101 |
| $0.35<-t<0.50$ | 1.024 | 0.070 | 0.071 | 0.100 |
| $0.50<-t<0.70$ | 1.030 | 0.068 | 0.053 | 0.086 |
| $0.70<-t<1.40$ | 0.954 | 0.067 | 0.052 | 0.085 |

TABLE 33: Summary of average $\Sigma$ beam asymmetry for the $t$-channel

## 7.2 -CHANNEL

Tables 3436 show the results for $\Sigma$ beam asymmetry for $K^{+} \Sigma^{0}$ in the $u$-channel. Combined asymmetry values and uncertainties were found the same way as explained in Sec. 7.1 .

| $-u$ bin $\left((\mathrm{GeV} / \mathrm{c})^{2}\right)$ | $\Sigma$ | Stat. uncert. | Syst. uncert. | Total uncert. |
| :---: | :---: | :---: | :---: | :---: |
| $-u<2.0$ | 0.458 | 0.099 | 0.057 | 0.114 |

TABLE 34: Summary of $\Sigma$ beam asymmetry for the $0 / 90$ orientation.

| $-u \operatorname{bin}\left((\mathrm{GeV} / \mathrm{c})^{2}\right)$ | $\Sigma$ | Stat. uncert. | Syst. uncert. | Total uncert. |
| :---: | :---: | :---: | :---: | :---: |
| $-u<2.0$ | 0.361 | 0.099 | 0.048 | 0.110 |

TABLE 35: Summary of $\Sigma$ beam asymmetry for the $-45 / 45$ orientation

| $-u \operatorname{bin}\left((\mathrm{GeV} / \mathrm{c})^{2}\right)$ | $\Sigma$ | Stat. uncert. | Syst. uncert. | Total uncert. |
| :---: | :---: | :---: | :---: | :---: |
| $-u<2.0$ | 0.410 | 0.070 | 0.057 | 0.090 |

TABLE 36: Summary of average $\Sigma$ beam asymmetry for the $u$-channel

### 7.3 CONCLUSION

The measurement for $t$-channel is the first measurement done exclusively for $K^{+} \Sigma^{0}$ beam asymmetry above the resonance region. As shown by Fig. 94 , the beam asymmetry values for $t$-channel are close to 1 . According to Eqn. 33 this implies that only the perpendicular polarized photon beam contributes to the production of $K^{+} \Sigma^{0}$ in the $t$-channel. Also, it tells that the natural parity exchange dominates presumably with the $K^{*}(892)$ meson as discussed in Sec. 2.3. Results are consistent with the predictions from RPR-2007 [13, 14] and Guidal et al. [53] at $E_{\gamma}=8.5 \mathrm{GeV}$. Our results have much precise statistical precision than the previous SLAC results at $E_{\gamma}=16 \mathrm{GeV}$. The SLAC measurement was done using the cross section ratios for $K^{+} \Lambda$ and $K^{+} \Sigma^{0}$ reactions to get the individual beam asymmetries for the two reactions. This is because they only detected the $K^{+}$in the final state so their measurement of asymmetry included the sum of $K^{+} \Lambda$ and $K^{+} \Sigma^{0}$.

The $u$-channel beam asymmetry for $K^{+} \Sigma^{0}$ has never been measured before. As shown in Table 36 an average value of approximately 0.4 is obtained for the low $-u$ region $(-u<$ $\left.2.0(\mathrm{GeV} / c)^{2}\right)$ measured. It indicates that in this kinematic domain, $u$-channel hyperon exchanges of both $\Sigma(J=1 / 2)$ and $Y^{*}(J=3 / 2)$ trajectories contribute to the production of the $K^{+} \Sigma^{0}$ final state. There is no model prediction for beam asymmetry in the low $-u$ region yet. The measurement in this work will provide valuable information for theorists to understand the exchange mechanisms contributing to kaon and in general pseudoscalar meson photoproduction.

## BIBLIOGRAPHY

[1] R. Bradford et al. (CLAS Collaboration), "Differential cross sections for $\gamma+p \rightarrow$ $K^{+}+Y$ for $\Lambda$ and $\Sigma^{0}$ hyperons", Phys. Rev. C 73, 035202 (2006).
[2] B. Dey et al. (CLAS Collaboration), "Differential cross sections and recoil polarizations for the reaction $\gamma p \rightarrow K^{+} \Sigma^{0} "$, Phys. Rev. C 82, 025202 (2010).
[3] M. Tran et al. (SAPHIR Collaboration), "Measurement of $\gamma p \rightarrow K^{+} \Lambda$ and $\gamma p \rightarrow$ $K^{+} \Sigma^{0}$ at photon energies up to $2 \mathrm{GeV}^{\prime \prime}$, Phys. Lett. B 445, 20 (1998).
[4] A. Lleres et al. (GRAAL Collaboration), "Polarization observable measurements for $\gamma p \rightarrow K^{+} \Lambda$ and $\gamma p \rightarrow K^{+} \Sigma^{0}$ for energies up to 1.5 GeV ", Eur. Phys. J. A. 31, 79 (2007).
[5] M. Sumihama et al. (LEPS Collaboration), "The $\vec{\gamma} p \rightarrow K^{+} \Lambda$ and $\vec{\gamma} p \rightarrow K^{+} \Sigma^{0}$ reactions at forward angles with photon energies from 1.5 to $2.4 \mathrm{GeV}^{\prime \prime}$, Phys. Rev. C 73, 035214 (2006).
[6] W. Briscoe, M. Doering, H. Haberzettl, I. Strakovsky and R. Workman, http: //gwdac.phys.gwu.edu/analysis/prk_analysis.html.
[7] D. Quinn, J. Rutherford, M. Shupe, D. Sherden, R. Siemann and C. Sinclair, "Study of charged-pseudoscalar-meson photoproduction from hydrogen and deuterium with 16-GeV linearly polarized photons", Phys. Rev. D 20, 1553 (1979).
[8] M. Gell-Mann, "The EightfoldWay: A Theory of Strong Interaction Symmetry", California Institute of Technology, USA (1961).
[9] M. Gell-Mann, "The symmetry group of vector and axial vector currents", Physics 1, 63 (1964).
[10] M. Gell-Mann and Y. Ne'eman, The Eightfold Way, W. A. Benjamin (1964).
[11] G. Zweig, "An $\mathrm{SU}_{3}$ Model for Strong Interaction Symmetry and its Breaking", CERN, report number CERN-TH-412 (1964).
[12] M. Thomson, Modern Particle Physics, Cambridge University Press (2013).
[13] T. Corthals, J. Ryckebusch and T. Van Cauteren, "Forward-angle $K^{+} \Lambda$ photoproduction in a Regge-plus-resonance approach", Phys. Rev. C73, 045207 (2006).
[14] T. Corthals, T. Van Cauteren, J. Ryckebusch and D.G. Ireland, "Regge-plusresonance treatment of the $p\left(\gamma, K^{+}\right) \Sigma^{0}$ and $p\left(\gamma, K^{0}\right) \Sigma^{+}$reactions at forward kaon angles", Phys. Rev. C75, 045204 (2007).
[15] M. Guidal, PhD thesis, DAPNIA/SPhN-96-03T, CE Saclay (1996).
[16] P. Stichel, "Single pion production by linearly polarized photons at high energies", Z. Phys. 180, 170 (1964).
[17] G.F. Chew, M.L. Goldberger, F.E. Low and Y. Nambu, "Relativistic dispersion relation approach to photomeson production", Phys. Rev. 106, 1345 (1957).
[18] G. R. Goldstein, J.F. Owens III and J. Rutherfoord, "Polarization predictions for $K^{+}$photoproduction", Nucl. Phys. B53, 197, (1973).
[19] C. Reece, "Continuous Wave Superconducting Radio Frequency Electron Linac for Nuclear Physics Research", Phys. Rev. Accel. Beams. 19, 124801 (2016).
[20] https://www.jlab.org/physics/GeV.
[21] E.S. Smith et al., "The GlueX beam line and detector", https://halldweb.jlab. org/doc-private/DocDB/ShowDocument?docid=4294
[22] U. Timm, "Coherent bremsstrahlung of electrons in crystals", Fortschritt der Physik 17, 765-808 (1969).
[23] K. Livingston, "The stonehenge technique. A method for aligning coherent bremsstrahlung radiators", Nucl. Instrum. Meth. A 603 (3), 205-213 (2009).
[24] K. Mork, "Pair production by photons on electrons", Phys. Rev. 160, 1065-1071 (1967).
[25] L. Maximon, H. Gima, "Pair production in the field of atomic electrons", Phys. Rev. A 23, 172-185 (1981).
[26] M. Dugger et al., "Design and construction of a high-energy photon polarimeter", Nucl. Instrum. Meth. A867, 115-127 (2017).
[27] F. Barbosa et al.," Pair spectrometer hodoscope for HALL D at Jefferson Lab", Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, 795, 376-380 (2015).
[28] C. Keith, "Targets for a neutral kaon beam", Jefferson Lab, Newport News, VA (2016).
[29] E. Pooser et al., "The GlueX Start Counter Detector", Nucl. Instrum. Meth. A927, 330 (2019).
[30] E. Smith, "Time-of-Flight Measurements with the Barrel Calorimeter and the Forward Scintillators", From the GlueX Document Database, Document \#1001 (2008).
[31] Y. van Haarlem et al., "The GlueX Central Drift Chamber: Design and Performance", Nucl. Instrum. Meth. A622, 142 (2010).
[32] D. S. Carman and S. Taylor, "Hall D Forward Drift Chamber Technical Design Report", Jefferson Lab, Newport News, VA (2008).
[33] T. D. Beattie et al., "Construction and performance of the barrel electromagnetic calorimeter for the GlueX experiment", Nucl. Instrum. Meth. A896, 24 (2018).
[34] K. Moriya et al., "A Measurement of the energy and timing resolution of the GlueX Forward Calorimeter using an electron beam", Nucl. Instrum. Meth. A726, 60 (2013).
[35] GlueX Collaboration, Gluex photos at https://gluexweb.jlab.org/photos
[36] Cristiano Fanelli, "Photon energy resolution", https://halldweb.jlab.org/ doc-private/DocDB/ShowDocument?docid=3500
[37] M. Patsyuk, "GlueX Overview: Status and Some Future Plans", https:// halldweb.jlab.org/doc-private/DocDB/ShowDocument?docid=3109
[38] J. Stevens, "Spring 2017 Dataset Summary", https://halldweb.jlab.org/ wiki-private/index.php/Spring_2017_Dataset_Summary (2017).
[39] https://root.cern.ch
[40] P. Mattione, "GlueX Analysis Software", https://halldweb.jlab.org/wiki/ index.php/GlueX_Analysis_Software (2013).
[41] P. Mattione, "DSelector", https://halldweb.jlab.org/wiki/index.php/ DSelector (2016).
[42] P. Mattione, "Spring 2017 Analysis Launch Cuts", https://halldweb.jlab.org/ wiki/index.php/Spring_2017_Analysis_Launch_Cuts (2017).
[43] R. Barsotti and M.R. Shepherd, "Using machine learning to separate hadronic and electromagnetic interactions in the GlueX forward calorimeter", submitted to JINST, arXiv:2002.09530 [physics.data-an].
[44] W. McGinley, "Development of the barrel calorimeter reconstruction and measurement of the beam asymmetry $(\Sigma)$ for photoproduced $\pi^{0}$ and $\eta$ mesons with the GlueX experiment", Ph.D. Thesis, Carnegie Mellon University (2019).
[45] H. Al Ghoul et al. (GlueX Collaboration), "Measurement of the beam asymmetry $\Sigma$ for $\pi^{0}$ and $\eta$ photoproduction on the proton at $E_{\gamma}=9 \mathrm{GeV} "$, Phys. Rev. C 95, 042201(R) (2017).
[46] A. Austregesilo, "Orientations of the photon beam polarization from $\rho(770)$ decay asymmetries",https://halldweb.jlab.org/doc-private/DocDB/ShowDocument? docid=3977 (2019).
[47] E. Smith, "Finite bin corrections in 1-dimension", https://misportal.jlab. org/ul/Physics/Hall-B/clas/viewFile.cfm/2006-007.pdf?documentId=264 (2006).
[48] C.A. Paterson et al. (CLAS Collaboration), "Photoproduction of $\Lambda$ and $\Sigma^{0}$ hyperons using linearly polarized photons", Phys. Rev. C 93, 065201 (2016).
[49] R. Bradford, "Measurement of differential cross sections and $C_{x}$ and $C_{z}$ for $\gamma p \rightarrow$ $K^{+} \Lambda$ and $\gamma p \rightarrow K^{+} \Sigma^{0}$ using CLAS at Jefferson Lab", Ph.D. thesis, Carnegie Mellon University (2005).
[50] M. Ablikim et al. (BESIII Collaboration), "Polarization and entanglement in baryonantibaryon pair production in electronpositron annihilation", Nature Phys. 15, 631 (2019).
[51] I.S. Barker, A. Donnachie and J.K. Storrow, "Complete Experiments in Pseudoscalar Photoproduction", Nucl. Phys. B95, 347 (1975).
[52] W. Leo, https://ned.ipac.caltech.edu/level5/Leo/Stats4_5.html
[53] M. Guidal, M. Laget and J. -M. Vanderhaeghen, "Pion and kaon photoproduction at high energies: forward and intermediate angles", Nucl. Phys. A627, 645 (1997).

## APPENDIX A

## LIST OF GLUEX SPRING 2017 RUNS

## A. 1 0/90 PARA (69 RUNS)

3103131000309993099830964309633095430953309243092330893308903084430843 3083330830308293081330812308023080130780307793074330742307363073130694 3068630677306723065830654306493063730627306223062130611305983059730587 3058030579305683049430486304743046630461304513044130429304283042430422 30408303803035530345303433032930327303233032230321302833028230279

## A. 2 0/90 PERP (78 RUNS)

3104631029309963099530962309613095230951309473092030903309023089130889 3085930858308423084130840308273082630824308113081030809308003079730796 3077830770307413074030735307303070130693306843067630668306573065330648 3063630626306253062030610305963059530586305783057730567304993049330490 3048530473304653046030450304473044630442304323043130410304093038330381 3036130346303443033030326303243028130280

## A. 3 -45/45 PARA (74 RUNS)

3103231002310013096630965309563095530927309263089630895308483084730835 3083430816308153080430803307843078330745307443073730733307323069630695 3068830687306793067830673306593065530650306383063030629306233061230607 3060230600305903058930582305813057030496304953048730480304773046830467 3046330462304533045230448304333042030411304043040330385303843034930347 30331302993028530284

## A. 4 -45/45 PERP(69 RUNS)

3103431004310033099430993309583095730929309283089930898308563085530838 3083630821308183080730805307873078530754307493073830734306973069030680 3067530674306663066030656306513064330642306413063930633306323062430616

3061430608305923059130571304973048830484304823048130470304693046430454 30449304343042130402304013038730386303503034830332303003029830286

## APPENDIX B

## PARAMETERS FOR MC

- software version set: version_recon-2017_01-ver03_8.xml
- halld_recon software version: recon-2017_01-ver03_hdr
- halld_sim software version: 4.2.0
- generator used: genr8
- random trigger background input: Random:recon-2017_01-ver03
- run range: 30274-31057
- no of events generated: 10 million
- Geant version: 3 and 4 (version 3 was used for all the figures in this thesis. Version 4 was used for comparison of acceptance.)
- photon beam energy: $8.2 \mathrm{GeV}-8.8 \mathrm{GeV}$
- $t$-slopes used are, (extracted from data)
$-K^{+} \Sigma^{0} t$-channel $=3.654$
- $K^{+} \Sigma^{0} u$-channel $=3.21$
$-K^{+} \Lambda t$-channel $=3.792$
$-K^{+} \Lambda u$-channel $=3.064$
- After reconstruction, ReactionFilter plugin was applied with the version set version_4.3.0.xml in order to get FCAL shower quality values into the tree.


## APPENDIX C

## FIT RESULTS

## C. 1 FIT RESULTS FOR $t$-CHANNEL

Figures 95102 show the fit results for each individual $-t$ bin, with panels for the PARA (upper left), PERP (upper right), asymmetry (lower left) and sum (lower right) as described for Fig. 58 .


FIG. 95: Polarization direction set to $0 / 90$. Fits to the $\phi$ distributions for $0.10 \mathrm{GeV}^{2}<-t<$ $0.35 \mathrm{GeV}^{2}$. The orientation of the plots is the same as Fig. 58 .


FIG. 96: Polarization direction set to $0 / 90$. Fits to the $\phi$ distributions for $0.35 \mathrm{GeV}^{2}<-t<$ $0.50 \mathrm{GeV}^{2}$. The orientation of the plots is the same as Fig. 58 .


FIG. 97: Polarization direction set to $0 / 90$. Fits to the $\phi$ distributions for $0.50 \mathrm{GeV}^{2}<-t<$ $0.70 \mathrm{GeV}^{2}$. The orientation of the plots is the same as Fig. 58 .


FIG. 98: Polarization direction set to $0 / 90$. Fits to the $\phi$ distributions for $0.70 \mathrm{GeV}^{2}<-t<$ $1.40 \mathrm{GeV}^{2}$. The orientation of the plots is the same as Fig. 58 .


FIG. 99: Polarization direction set to $-45 / 45$. Fits to the $\phi$ distributions for $0.10 \mathrm{GeV}^{2}<$ $-t<0.35 \mathrm{GeV}^{2}$. The orientation of the plots is the same as Fig. 58 .


FIG. 100: Polarization direction set to $-45 / 45$. Fits to the $\phi$ distributions for $0.35 \mathrm{GeV}^{2}<$ $-t<0.50 \mathrm{GeV}^{2}$. The orientation of the plots is the same as Fig. 58 .


FIG. 101: Polarization direction set to $-45 / 45$. Fits to the $\phi$ distributions for $0.50 \mathrm{GeV}^{2}<$ $-t<0.70 \mathrm{GeV}^{2}$. The orientation of the plots is the same as Fig. 58 .


FIG. 102: Polarization direction set to $-45 / 45$. Fits to the $\phi$ distributions for $0.70 \mathrm{GeV}^{2}<$ $-t<1.40 \mathrm{GeV}^{2}$. The orientation of the plots is the same as Fig. 58 .

## C. 2 LINEAR FITS TO $u$-CHANNEL YIELD ASYMMETRY



FIG. 103: Polarization direction set to 0/90. Linear fit to the yield asymmetry.


FIG. 104: Polarization direction set to $-45 / 45$. Linear fit to the yield asymmetry.

## C. 3 FITS TO YIELD ASYMMETRY FROM COMBINED DATA SET

In order to cross check the average beam asymmetry results obtained from the 0/90 and $-45 / 45$ sets we made an event sample combining the events from all 4 orientations. The following fits were done with Eqn. 46 for the $-t$ bins and the $-u$ bin in this event sample. Average values of polarization and flux ratio were used and they are,
$P_{\|}=0.3525$
$P_{\perp}=0.3478$
$F_{R}=1.01689$


FIG. 105: Fits to yield asymmetry from combined data set from $0 / 90$ and $-45 / 45$ for $t$ channel. Beam asymmetry values and statistical uncertainties are consistent with the average asymmetry results given in Table 33 .

Combined result for $-\mathrm{u}<2.0 \mathrm{GeV}^{2}$


FIG. 106: Fit to the yield asymmetry from combined data set from $0 / 90$ and $-45 / 45$ for $u$ channel. Beam asymmetry value and statistical uncertainty are consistent with the average asymmetry result given in Table 36 .

## APPENDIX D

## ADDITIONAL PLOTS



FIG. 107: Distribution of photon energy in the $\Sigma^{0}$ rest frame.

$$
\Sigma^{0} \rightarrow \Lambda \gamma
$$

So, in the $\Sigma^{0}$ rest frame, $\left(p_{\Sigma}-p_{\gamma}\right)^{2}=p_{\Lambda}^{2}$
$\Longrightarrow M_{\Sigma}^{2}-2 E_{\gamma} M_{\Sigma}=M_{\Lambda}^{2}$
$\Longrightarrow E_{\gamma}=\frac{M_{\Sigma}^{2}-M_{\Lambda}^{2}}{2 M_{\Sigma}}=74.5 \mathrm{MeV}$

The mean value of distribution in Fig. 107 is consistent with the calculated value.

## D. $1 K^{+}$Z-VERTEX STUDY



FIG. 108: $K^{+} \Delta z$ vs. $z$ event by event, where $\Delta z=z_{\text {recon }}-z_{\text {gen }}$ is the difference between the thrown and reconstructed $z$ positions.


FIG. 109: A Gaussian fitted to the projection from $z=48-52 \mathrm{~cm}$ region. The resolution is $\sigma=0.308 \pm 0.003 \mathrm{~cm}$.


FIG. 110: A Gaussian fitted to the projection from $z=78-82 \mathrm{~cm}$ region. The resolution is $\sigma=0.283 \pm 0.003 \mathrm{~cm}$.

## D. 2 ACCEPTANCE FROM MC

Distributions for accepted events were made with the thrown P4 vectors. Geant4 shows a higher acceptance than Geant3 for both $K^{+} \Sigma^{0}$ and $K^{+} \Lambda$ reactions, in the entire $-t$ range (Fig. 112 and Fig. 116) for the $t$-channel. For $u$-channel, the higher acceptance in Geant4 is seen in the region $-u<0.5(\mathrm{GeV} / c)^{2}$ (Fig. 114 and Fig. 118).

## D.2.1 $K^{+} \Sigma^{0} t$-CHANNEL




FIG. 111: $K^{+} \Sigma^{0} t$-channel thrown (left) and accepted (right) $-t$ distributions.


FIG. 112: Acceptance for $K^{+} \Sigma^{0} t$-channel.

## D.2.2 $K^{+} \Sigma^{0} u$-CHANNEL




FIG. 113: $K^{+} \Sigma^{0} u$-channel thrown (left) and accepted (right) $-u$ distributions.


FIG. 114: Acceptance for $K^{+} \Sigma^{0} u$-channel.

## D.2.3 $K^{+} \Lambda t$-CHANNEL

For $K^{+} \Lambda \gamma$ accepted $-t$ distribution plots of $K^{+} \Lambda$ reaction, cuts on $\mathrm{M}_{\Lambda \gamma}$ and shower quality were not used as this is the background reaction for $K^{+} \Sigma^{0}$.



FIG. 115: $K^{+} \Lambda t$-channel thrown (left) and $K^{+} \Lambda \gamma$ accepted (right) - $t$ distributions.
$\mathrm{K}^{+} \Lambda \gamma$ t-channel acceptance


FIG. 116: Acceptance for $K^{+} \Lambda \gamma t$-channel.

## D.2.4 $K^{+} \Lambda u$-CHANNEL




FIG. 117: $K^{+} \Lambda u$-channel thrown (left) and $K^{+} \Lambda \gamma$ accepted (right) $-u$ distributions.


FIG. 118: Acceptance for $K^{+} \Lambda \gamma u$-channel.

## D. 3 BACKGROUND UNDER THE $\Sigma^{0}$ PEAK FOR DIFFERENT BINS OF $t$ AND $u$



FIG. 119: The fraction of background under $\Sigma^{0}$ peak $\left(\frac{B_{\Sigma}}{S_{\Sigma}+B_{\Sigma}}\right)$ for the $-t$ bins used in the analysis. $B_{\Sigma}$ and $S_{\Sigma}$ are background and signal yields within the range $1.169 \mathrm{GeV} / c^{2}$ $<M_{\Lambda \gamma}<1.217 \mathrm{GeV} / c^{2}$ respectively.


FIG. 120: The fraction of background under $\Sigma^{0}$ peak $\left(\frac{B_{\Sigma}}{S_{\Sigma}+B_{\Sigma}}\right)$ for the $-u$ bin used in the analysis.

## APPENDIX E

## CUT STUDY



FIG. 121: Invariant mass of $\Lambda \gamma$ for different values of kinematic fit confidence level cut. $S_{\Sigma}$ and $B_{\Sigma}$ are signal and background yields within the range $1.169 \mathrm{GeV} / c^{2}<M_{\Lambda \gamma}<1.217$ $\mathrm{GeV} / c^{2}$.


FIG. 122: Invariant mass of $\Lambda \gamma$ for different values of measured missing mass squared cut.


FIG. 123: Invariant mass of $\Lambda \gamma$ for different values of $K^{+}$vertex radial position cut.


FIG. 124: Invariant mass of $\Lambda \gamma$ for different values of $K^{+}$vertex $z$ position cut.


FIG. 125: Invariant mass of $\Lambda \gamma$ for different values of FCAL shower quality cut.


FIG. 126: Invariant mass of $\Lambda \gamma$ for different values of minimum photon energy cut.


FIG. 127: Fits for yield asymmetry for $-t$ bins for $-45 / 45$ orientation with kinematic fit confidence level $>10^{-10}$ cut.

## APPENDIX F

## ACCEPTANCE OF RECOIL ANGLES



FIG. 128: Acceptance for angular distributions of the proton for $0.10(\mathrm{GeV} / c)^{2}<-t<0.35$ $(\mathrm{GeV} / c)^{2}$. Histograms are for three angles $\theta_{x}$ (top), $\theta_{y}$ (middle) and $\theta_{z}$ (bottom).


FIG. 129: Acceptance for angular distributions of the proton for $0.35(\mathrm{GeV} / c)^{2}<-t<0.50$ $(\mathrm{GeV} / c)^{2}$.



FIG. 130: Acceptance for angular distributions of the proton for $0.50(\mathrm{GeV} / c)^{2}<-t<0.70$ $(\mathrm{GeV} / c)^{2}$.


FIG. 131: Acceptance for angular distributions of the proton for $0.70(\mathrm{GeV} / c)^{2}<-t<1.40$ $(\mathrm{GeV} / c)^{2}$.


FIG. 132: Acceptances for decay angular distributions of the proton vs. $\phi$ lab angle of $K^{+}$ from $t$-channel MC.


FIG. 133: Acceptance for angular distributions of the proton for $-u<2.0(\mathrm{GeV} / c)^{2}$.


FIG. 134: Acceptances for decay angular distributions of the proton vs. $\phi$ lab angle of $K^{+}$ from $u$-channel MC.

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