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Analyzing Math-to-Mastery through Brief Experimental Analysis

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Abstract

The current study evaluated the effectiveness of individualized math-to-mastery (MTM) interventions, selected though brief experimental analysis (BEA), at increasing math fluency skills for 3 elementary-aged females. As MTM has only been investigated as a multicomponent intervention, the present study utilized BEA to identify those specific components which led to math skills gains in the most efficient manner possible. BEA results indicated that for 2 of 3 participants only a partial MTM intervention was necessary to prompt fluency gains, while the entire intervention was the most effective for the third. During extended analysis all 3 participants displayed math skills gains above those seen during repeated baseline assessments. Results are discussed in terms of further refining MTM through BEA procedures so as to individually target math skill deficits by considering both intervention effectiveness and efficiency.

Analyzing Math-to-Mastery through Brief Experimental Analysis

Concerns regarding the math difficulties of U.S. students persist and span all education levels. Most first, second and third grade students do not meet grade-level fluency recommendations for basic addition and subtraction facts (Stickney, Sharp, & Kenyon, 2012) and only 23% of U.S. twelfth graders are judged proficient in math (National Mathematics Advisory Panel, 2008). Given the importance of math skills in both academic and real-world contexts the identification of effective, evidence-based techniques for students who are struggling and at-risk for further difficulty is important. In this manner, school personnel may be able to remediate current concerns with the goal of preventing future difficulty.

One intervention that may hold promise for targeting mathematics skill deficits is math-to-mastery (MTM). A multicomponent intervention, MTM includes problem previewing, repeated practice, corrective feedback, performance feedback, and self-monitoring of progress (Mong & Mong, 2010, 2012; Mong, Doggett, Mong, & Henington, 2012). Recently, MTM has shown to be effective in prompting gains in both calculation fluency and skill generalization from instructional to grade-level material. As described by Mong and Mong (2012), during MTM the interventionist manually and verbally demonstrates target problem completion for the student (i.e., problem previewing) who then practices completion of the same problems more than once (i.e., repeated practice) while the interventionist follows along so as to correct when necessary (i.e., corrective feedback). Following problem completion, the interventionist updates the student of their progress on the previous trial (i.e., performance feedback) and the student graphs this performance as a measure of ongoing progress (i.e., self-monitoring of progress).

Previous studies of MTM have investigated the intervention either in isolation (Mong et al., 2012) or compared directly with other math fluency interventions, including cover-copy-compare (CCC) and taped problems (Mong & Mong, 2010, 2012). When investigated individually, Mong et al. (2012) provide evidence that MTM prompted increased math fact fluency for three general education third graders performing below grade level. Regarding direct comparisons of intervention efficacy, Mong and Mong (2010) first indicated MTM was more effective than CCC for 2 of 3 general education second graders and then more effective than both CCC and taped problems for 2 of 3 general education third graders (2012). Although such investigations provide important data indicating the *overall* effectiveness of MTM, they also suggest additional empirical questions involving issues regarding the efficacy and efficiency of *individual* intervention components separate from the multicomponent intervention as a whole.

One technique useful in determining both intervention efficacy and efficiency is brief experimental analysis (BEA). Specifically, BEA is a process described as intervention "test-driving" (Witt, Daly, & Noell, 2000) through which entire interventions, or their individual components, are implemented in quick succession to determine their effectiveness without fully implementing any of them (VanAuken, Chafouleas, Bradley, & Martens, 2002). Typically, BEA procedures are conducted within a multielement design in which interventions to be tested may be ordered in various ways including ease of use (e.g., least to most intrusive/complex) so as to select that which is the most effective, yet feasible, for extended implementation (Wilber & Cushman, 2006). In this manner both intervention efficacy and efficiency may be evaluated in a short time period to prevent full implementation of an ineffective intervention (McComas et al., 2009) and identify core strategies foundational for intervention success.

Past research has largely evaluated BEA as an intervention selection tool in the area of reading with much less empirical attention to mathematics. Regarding math-specific examples, Carson and Eckert (2003) used BEA to compare performance feedback, goal setting, contingent reinforcement, and timed sprints on the computation skills of three elementary-aged students. The BEA-selected intervention was then compared with both a student-selected intervention and baseline with results indicating greater effectiveness for the BEA-identified intervention for all students. More recently, Codding et al. (2009) used BEA to select amongst incentive, performance feedback, goal setting, and CCC interventions on the computational fluency of four elementary school students. Conditions were sequenced according to intensity, with the intervention requiring the fewest demands (i.e., incentive) implemented first with the most demanding (i.e., CCC) conducted last. During extended analysis the intervention identified as most effective during BEA was compared with baseline with results for all participants indicating continued efficacy of the BEA-selected intervention.

Presently, only Mong and Mong (2012) have used BEA to select amongst math interventions inclusive of MTM. Here, MTM was compared with CCC (Grafman & Cates, 2010) and taped problems (McCallum, Skinner, & Hutchins, 2004); all math skill building interventions including common methodological components (e.g., problem previewing, repeated practice, and corrective feedback). Similar to other investigations, results indicated the predictive utility of BEA procedures, as for all participants the experimentally-identified intervention was also the most effective during extended analysis. Importantly, although MTM was the most effective intervention for 2 of 3 participants, it was also the most time intensive requiring the greatest interventionist involvement. That is, there was a disconnect between MTM efficacy and efficiency. As such, it is important to further investigate MTM so as to determine the efficacy of differing intervention components in order to most appropriately balance the additive effects of individual components with the most judicious use of resources.

Given the need to further identify and investigate empirically-based mathematics interventions, the current study was designed to build upon previous MTM research in two important ways. First, and more generally, although MTM has demonstrated past empirical effectiveness (Mong & Mong 2010, 2012; Mong et al., 2012), such research has only begun to document intervention efficacy. As such, the current investigation was conducted to build upon this promising, yet limited, research base in an attempt to further refine MTM. Second, and more specifically, as all previous MTM studies have employed the intervention in its entirety (i.e., as a time- and labor-intensive multi-step package), the current study was designed to investigate intervention efficiency in an attempt to identify the most essential MTM components. Through the use of BEA methodology, the current study evaluated the additive contributions of sequenced MTM components in an attempt to identify those procedures that best balanced intervention efficacy with intervention efficiency. Thus, the current investigation also expands the BEA literature base through a novel mathematics-related application.

Methods

Participants and Setting

Participants were two second-grade students and one fourth-grade student enrolled in general education classes from a Midwestern elementary school with approximately 320 students enrolled in grades pre-kindergarten through fourth grade selected through the following multi-step procedure post-IRB approval. First, potential participants were selected from a pool of 60 students in grades one through four based on teacher nomination or scoring at the lowest level of math benchmarking. Next, of these 60, 44 were excluded due to reasons including limited English proficiency, current special education placement, adequate class performance in math, or placement at a different school. Third, each of the remaining 16 students was then screened for inclusion using AIMSweb® Mathematics Computation (M-COMP) probes (Pearson, 2012). Of these 16, 10 were excluded because they scored above instructional level (i.e., 25th percentile) for grade level math computation when compared to AIMSweb® M-COMP national norms. Of the six students who scored below instructional level (i.e., 25th percentile) when compared to national norms, three were excluded due to student difficulties during the completion of the screening assessment. Finally, the remaining three students served as participants in the study.

Participants were (pseudonyms used) Abigail, an 8-year-old Caucasian girl enrolled in 2nd grade, Becca, a 7-year-old Caucasian girl enrolled in 2nd grade, and Cassandra, a 10-year-old Caucasian girl enrolled in 4th grade. Although none of the participants received special education services, Abigail and Cassandra were both referred for comprehensive evaluations of potential learning disabilities in the areas of math and reading during the course of the investigation. Individual assessment results indicated (a) Abigail performed between the 6th and 7th percentiles on second grade M-COMP, (b) Becca performed between the 17th and 20th percentiles on second grade M-COMP, and (c) Cassandra performed below the 1st percentile on fourth grade M-COMP. Following their selection as participants, each student's assessment results were also individually analyzed through an error analysis procedure so as to identify which math skills were most frequently missed and, therefore, should be targeted during BEA and extended analysis. For Abigail and Becca, error analysis indicated significant difficulty with (a) subtraction with 2 one-digit numbers, (b) addition with 2 one-digit numbers with sums of 11-18, and (c) addition with 2 two-digit numbers with no regrouping. For Cassandra, error analysis indicated problems with (a) multiplication with 2 one-digit numbers, (b) addition with 2 two-digit numbers with regrouping, and (c) subtraction with 2 two-digit numbers with regrouping.

The study's second author served as the interventionist for all sessions, which took place in various unoccupied school offices.

Materials

AIMSweb® M-COMP worksheets (Pearson, 2012) specific to each participant's grade level were used for all screening sessions. Separately, for all BEA and extended analysis sessions math worksheets including those skills identified during error analysis were constructed for each participant using the *Math Worksheet Generator* available at www.interventioncentral.org (Intervention Central n.d.). As this program allows the user to create math worksheets targeting specific skills, worksheets were individualized so as to include only problems of the type identified during error analysis. All experimenter-created worksheets contained 24 problems divided equally between each participant's three targeted skills as identified during error analysis (e.g., Abigail's worksheets had eight problems targeting subtraction with 2 one-digit numbers, eight targeting addition with 2 one-digit numbers with sums of 11-18, and eight targeting addition with 2 two-digit numbers with no regrouping). In addition, Microsoft Excel® was used to graph student performance during those MTM sessions that included a charting component, and was presented to students via a laptop or desktop computer present in the session room.

Dependent Measures

To judge both the efficacy and efficiency of the MTM intervention, two dependent measures were assessed. First, digits correct per minute (DCPM) served as a measure of intervention efficacy and was calculated by dividing the number of digits correct on experimental worksheets by the total number of seconds and multiplying by 60 (Shapiro, 2004). Second, rate of learning (ROL) served as a measure of intervention efficiency and was calculated during BEA sessions by dividing DCPM by the total time spent in each condition as measured in minutes and seconds. That is, as the current study was designed to investigate both MTM efficacy and efficiency, ROL was calculated during all BEA sessions so as to select for extended analysis the particular combination of MTM components that best balanced effectiveness (i.e., DCPM) with efficiency (i.e., ROL).

Procedures

Brief Experimental Analysis (BEA)

Brief Experimental Analysis (BEA) was conducted to evaluate the additive effectiveness of individual MTM components as outlined in Mong and Mong (2012). Specifically, problem previewing, repeated practice, corrective feedback, performance feedback, and self-monitoring of progress were individually added in sequence so that each BEA session represented a unique combination of MTM components. Consistent with common BEA procedures (Jones, Wickstrom, & Daly, 2008), conditions were sequenced in order from least intensive (i.e., inclusive of only one MTM components) to most intensive (i.e., in which all MTM components were used). During each BEA session the specific combination of MTM components was implemented so that both effectiveness (as measured through DCPM) and efficiency (as measured though ROL) could be assessed. For all sessions the interventionist completed the specified MTM components using individualized worksheets created from the *Math Worksheet Generator* (Intervention Central n.d.) while using a stopwatch to record the time required to complete each session. In addition to first implementing each MTM combination once, current BEA sessions also included a mini-reversal procedure during which a baseline session was re-implemented followed by the re-introduction of the specific MTM intervention(s) judged to be the best combination of effectiveness and efficiency. For all participants, BEA sessions took place over the course of two days in order to accommodate classroom schedules.

Baseline

During the BEA for each participant, baseline assessments were conducted both prior to the progressive introduction of MTM components and as a mini-reversal during which baseline was re-implemented in an attempt to confirm initial BEA findings. In addition, baseline assessments were also conducted throughout extended analysis for each participant as a means of examining ongoing intervention effectiveness. During both BEA and extended analysis baseline sessions participants received no intervention. Baseline assessments were conducted through the use of a 24 problem individualized worksheet for 1 min during which DCPM were counted.

Math-to-Mastery: 1

MTM1 included only the problem previewing component of the intervention. During MTM1, the interventionist modeled correct worksheet completion by manually and verbally completing all problems while participants followed along on a separate but identical worksheet. Following problem previewing, participants were given 1 min to complete the previewed worksheet on which DCPM were counted.

Math-to-Mastery: 2

In addition to problem previewing, MTM2 included repeated practice on the experimental worksheets. That is, each participant was given three separate 1 min trials to complete their worksheet following interventionist previewing. Although MTM methodology often includes up to 10 repetitions with the experimental worksheet (e.g., Mong & Mong, 2012), as the current study was also interested in intervention efficiency, a ceiling of three repetitions was used to maintain uniformity across participants and limit participant time outside of class. Such procedures are similar to other repeated practice interventions in which three trials were recognized as providing sufficient opportunities to respond (Therrien, 2004). Following three repeated trials participants were given 1 min to complete the outcome worksheet.

Math-to-Mastery: 3

During MTM3, corrective feedback procedures were added to the problem previewing and repeated practice components. Specifically, during completion of the three repeated practice trials the interventionist followed along so as to immediately identify calculation errors. When identified, the interventionist then immediately marked the incorrect digits and provided feedback regarding correct problem completion. Following three repeated trials, inclusive of corrective feedback, participants were given 1 min to complete the outcome worksheet so as to judge intervention efficacy.

Math-to-Mastery: 4

For MTM 4 performance feedback procedures were added to those already in place. That is, after each repeated practice trial the interventionist calculated and reported to the participants their DCPM. Participants were also provided verbal praise for performance and effort by the interventionist. Again, each session ended with participant completion of the 1 min outcome worksheet for DCPM calculation.

Math-to-Mastery: 5

MTM5 represented inclusion of all individual intervention components and was methodologically similar to past MTM investigations which have investigated the intervention as a whole (i.e., Mong & Mong, 2010, 2012; Mong et al., 2012). Specifically, in addition to problem previewing, repeated practice, corrective feedback, and performance feedback procedures as previously described, MTM5 sessions included a participant self-monitoring of progress component. Here, following performance feedback on each 1 min repeated practice trial, participants charted their performance using Microsoft® Excel on a laptop computer. Participants were then able to view their graphed performance for each of the three trials during that session. As with all other MTM sessions, participants then completed the 1 min outcome worksheet so as to judge intervention efficacy.

Extended Analysis

During extended analysis, the specific MTM intervention identified through BEA as the best combination of efficacy (DCPM) and efficiency (ROL) was implemented individually to assess the extended utility of specific MTM procedures. So as to provide a means of ongoing comparison, extended analysis also included recurring baseline assessment for each participant. Specific MTM intervention implementation followed the same procedures as during BEA with the intervention implemented first followed by a 1 min outcome assessment. Intervention sessions were conducted two to three times per week for approximately five weeks. Regarding baseline assessment, following each third intervention session participants were given a 1 min probe that included problems reflective of their specific targeted skills, but were untrained during MTM sessions.

<u>Design</u>

During BEA, a multielement design with a mini-reversal (Martens, Eckert, Bradley, & Ardoin, 1999), was used to select the specific MTM intervention for extended analysis. Each MTM intervention was presented once in an abridged data series with BEA sessions ordered from least intrusive to most intrusive. Outcome data (i.e., DCPM) were graphed and visually analyzed. After each intervention was administered once, a mini-reversal to baseline was conducted. Following reversal, the specific MTM intervention that balanced efficacy (i.e., increased DCPM) with efficiency (i.e., ROL) was re-administered so as to further investigate replication of initial BEA findings. For two participants, (i.e., Abigail and Cassandra) this replication involved the re-administration of two MTM interventions due to school-based disruptions during data collection (for Abigail) and very similar ROLs for two MTM interventions (for Cassandra).

During extended analysis an alternating treatments design was used to assess the ongoing effectiveness of the specific MTM intervention selected during BEA as compared to baseline assessment. That is, each extended analysis session included the administration of the individualized MTM intervention with each third session concluding with a baseline assessment on untrained problems. All data collection was completed over the course of 5 weeks.

Procedural Integrity, Interscorer Agreement, and Acceptability

To ensure procedural integrity, all BEA and extended analysis sessions were implemented according to checklist outlining required procedures specific to which combination of MTM components were administered. That is, each component of the MTM intervention was listed on a checklist and checked off immediately after implementation during all intervention sessions. Procedural integrity was monitored through both self-observation (i.e., by the interventionist) for all sessions and having a second observer monitor adherence to required MTM procedures during 36% of experimental sessions. Integrity was calculated by dividing the number of steps completed correctly by the total number of steps required of the MTM intervention during a given session and multiplied by 100 to obtain a percentage. For all sessions, procedural integrity equaled 100%.

Interscorer agreement for DCPM was also calculated on 36% of experimental worksheets used across all phases of the study. Interscorer agreement was calculated by dividing the total number of agreements by the total number of agreements plus disagreements and multiplying by 100 to obtain a percentage. Interscorer agreement averaged 97% across all worksheets.

In addition, at study conclusion, participants were asked to complete a brief experimenter-created measure of acceptability containing three Likert items in 5-point format. Each participant was asked to rate MTM according to their (a) opinion of the intervention, (b) willingness to use it again, and (c) perception of how time-consuming it was. Results indicated that all participants liked MTM and would use it again (item scores of 5). Finally, Becca did not find MTM to be time consuming, although Abigail and Cassandra believed it took a moderate amount of time.

Results

Abigail

Figure 1 displays Abigail's results across all BEA and MTM extended analysis sessions. Table 1 displays results of intervention efficacy (i.e., DCPM) and efficiency (i.e., ROL) as well as time in condition across all BEA sessions for each participant. As seen in the top panel of Figure 1, MTM3 (i.e., problem previewing, repeated practice, and corrective feedback) led to the highest DCPM (10) during the initial BEA sessions prior to the mini-reversal. In addition, as evident in Table 1, MTM3 also produced the highest ROL (1.20) across all initial BEA sessions. Following a return to baseline, both MTM3 and MTM5 were re-implemented so as to identify the specific MTM intervention for use during extended analysis. Although initial plans called for the re-implementation of the MTM intervention that best balanced efficacy with efficiency (i.e., MTM3 for Abigail), due to unforeseen interruption during the MTM5 session it was also re-implemented during mini-reversal. That is, while Abigail was completing her outcome probe during initial MTM5 implementation school announcements over the building's intercom system interrupted her progress. Results of the re-implemented MTM sessions indicated that although MTM5 led to increased DCPM when compared with MTM3, such increases did not outweigh the additional time required for such gains. Table 1 indicates that these gains in effectiveness did not offset the loss of efficiency required of MTM5. As such, MTM3 best balanced DCPM and ROL and was chosen for extended analysis implementation.

Extended analysis results indicated MTM3 was consistently more effective than baseline with a mean of 11.4 DCPM during MTM3 (range 7 – 15) sessions and 5 DCPM during baseline (range 4 – 6). As seen in the bottom panel of Figure 1, following implementation of MTM3 there was immediate separation and clear divergence from baseline with no instances of overlap where intervention results fell below those of baseline. Although Abigail's initial MTM3 performance displayed increased variability, her final four sessions evidenced an increasing trend with her highest DCPM outcome achieved during the final MTM3 session. As an additional measure of intervention effectiveness, following the discontinuation of extended analysis sessions Abigail was again assessed with grade level AIMSweb® M-COMP worksheets (Pearson, 2012). Results of this post-intervention assessment revealed that Abigail improved her AIMSweb percentile rank to between the 13th and 14th percentiles on second grade probes as compared to her initial screening results, which fell between the 6th and 7th percentiles. Collectively, such results indicate not only the relative effectiveness of MTM3 as compared to baseline, but also provide evidence of generalization effects to grade-level material routinely employed in the classroom.

Becca

Figure 2 displays Becca's results across all BEA and MTM extended analysis sessions. As seen in the top panel of Figure 2, MTM3 (i.e., problem previewing, repeated practice, and corrective feedback) evidenced the highest DCPM (15) during initial BEA sessions. Also, as seen in Table 1, MTM3 produced the highest ROL (1.85) during BEA prior to the mini-reversal. Following a return to baseline, MTM3 was re-implemented prior to its use in extended analysis. Although the ROL for MTM3 was slightly less during its second BEA presentation, it remained higher than all other MTM interventions in previous BEA sessions. As such, MTM3 was chosen for extended implementation.

Becca's extended analysis results indicated MTM3 was consistently more effective than baseline, with a mean of 11.5 DCPM during MTM3 (range 7 -18) sessions compared with a mean of 4 DCPM (range 2 – 6) during baseline assessments. As seen in the bottom panel of Figure 2, there was immediate separation and divergence of performance throughout extended analysis sessions. There were also no instances of overlap where MTM3 results fell below baseline performance. Although Becca's MTM3 results were clearly superior to baseline, they also displayed a high degree of variability with no clear trend for improved performance (although she achieved 2 of her 3 highest DCPM results during her final two intervention sessions). Results of Becca's post-intervention AIMSweb® M-COMP (Pearson, 2012) assessment provide additional evidence of MTM3 effectiveness. Specifically, she improved to between the 34^{th} – 35^{th} percentiles on second grade probes from her pre-intervention performance of between the 17^{th} – 20^{th} percentiles. Taken together, Becca's results indicate both the efficacy of MTM3 during extended analysis and as a procedure that may prompt generalization to grade-level material.

<u>Cassandra</u>

Figure 3 displays Cassandra's BEA and MTM extended analysis results. As seen in the top panel of Figure 3, MTM5 (i.e., problem previewing, repeated practice, corrective feedback, performance feedback, and self-monitoring of progress) led to the highest DCPM (12) during initial BEA sessions. In addition, as seen in Table 1, Cassandra's ROL (.97) was highest during MTM5, although results indicate only slightly improved performance when compared to her ROL in MTM3 (i.e., a ROL difference of only .01). Given such results, Cassandra's mini-reversal included the re-implementation of both MTM3 and MTM5 following a return to baseline. The results of these second BEA presentations further substantiated the initial findings, in which MTM5 resulted in both the highest DCPM and ROL, thereby best balancing efficacy and efficiency. As such, it was selected for extended implementation.

As seen in the bottom panel of Figure 3, initial extended analysis results indicate MTM5 was no better than baseline at prompting DCPM gains (i.e., four of Cassandra's MTM5 results were equal to or worse than baseline performance across the first six sessions). However, clear divergence between intervention and baseline performance did occur during the final six MTM5 sessions with only one such session falling to the level of baseline performance (i.e., intervention session 11). Overall, mean DCPM were 10.7 (range 6 – 16) for MTM5 and 7.5 (range 6 – 9) during baseline. In addition, although her MTM5 results indicated high variability, collectively Cassandra evidenced an increasing trend in DCPM across all intervention sessions. Cassandra's DCPM growth was also substantiated by improved performance on her outcome AIMSweb® M-COMP (Pearson, 2012) assessment. That is, post-intervention she scored between the 15^{th} – 16^{th} percentiles on fourth grade probes compared to her screening results, which were below the 1^{st} percentile on grade-level material. Similar to both Abigail and Becca's results, Cassandra's performance provides data-based evidence of both the comparative and generalization effects of MTM.

Discussion

The current study was designed to investigate math-to-mastery (MTM), a multicomponent math intervention, through the use of brief experimental analysis (BEA) methodology. Although past research has documented the effectiveness of MTM (i.e., Mong & Mong 2010, 2012; Mong et al., 2012), all have studied the intervention as a whole without identifying those components most responsible for math skill gains. Similarly, although BEA has been employed as a math intervention selection technique, such usage has been limited to a few empirical examples (e.g., Carson & Eckert, 2003; Codding et al., 2009). As such, this study sought to examine the utility of MTM not as a multicomponent intervention package, but as a potentially effective combination of individual procedural variables. Through the use of BEA the specific combination of MTM components that best balanced efficacy and efficiency was identified for three elementary school students. Following BEA, each unique MTM intervention was then compared to baseline during extended analysis in an attempt to maximize academic gains in the most efficient way possible.

Overall, current results provide initial evidence for the effectiveness of a partial MTM intervention, composed not of all procedural components, but of only those determined to individually affect mathematics outcomes. For 2 of 3 participants an abbreviated version of MTM, rather than the entire intervention package, was identified as the best combination of efficacy and efficiency. Specifically, for both Abigail and Becca MTM3, which included only the problem previewing, repeated practice and corrective feedback components resulted in the highest ROL during initial BEA sessions and those that followed a return to baseline. Although Abigail's post-reversal BEA results indicated higher DCPM for MTM5 than MTM3, her slight gains did not outweigh the additional intervention time required (almost 3 ½ mins). Therefore, as MTM3 allowed Abigail to learn at an increased rate it was selected for extended implementation. During extended analysis both Abigail and Becca evidenced increased math performance with MTM3, thereby indicating the utility of an intervention including only problem previewing, repeated practice, and corrective feedback components at targeting math skill deficits. Such results represent the most important contribution of the current work and suggest that for some students positive math gains may be prompted by a more concise, less involved intervention. In addition to empirical relevance of this finding, such results may have direct implications for school-based practice given the common preference for time-saving; as opposed to time-intensive interventions.

In addition to providing evidence of an abbreviated MTM intervention, current results also add to previous work regarding the effectiveness of the entire procedure (i.e., Mong & Mong 2010, 2012; Mong et al., 2012). That is, for Cassandra MTM5 produced both the highest DCPM and ROL across initial and post-reversal BEA sessions and was selected for extended implementation. During extended analysis MTM5 resulted in higher mean DCPM when compared to baseline, although there were several instances in which her intervention performance was equal to or less than baseline. Such results may be related to the more persistent nature of Cassandra's math-related struggles as a fourth grade student (compared to both Abigail and Becca as second graders) and because she began the MTM intervention the furthest behind her current grade placement. Specifically, as Cassandra's screening results indicated placement below the 1st percentile on grade-level mathematics (while Abigail and Becca performed higher on grade-level material) it may be that the entire MTM intervention was necessary to prompt improved math performance. Although the less involved MTM3 was effective for Abigail and Becca, their initial deficits were not as severe as Cassandra's and may have responded better to an abridged intervention. It may be that the additions of performance feedback and participant self-monitoring components of MTM5 were more necessary to prompt Cassandra's skill gains compared to both Abigail and Becca for which they were unnecessary.

The present study also further supports the predictive utility of BEA when applied to math-specific interventions. Although BEA procedures have been routinely employed as an intervention selection tool, past usage has largely focused on reading rather than mathematics (Codding et al., 2009). Specific to MTM usage, although Mong and Mong (2012) found MTM to be the most effective intervention for 2 of 3 participants (when selected though BEA), it was also the most time consuming requiring the most interventionist involvement. Here, the current study provides for continued documentation of BEA as a math-specific intervention selection vehicle, and also offers additional refinement of MTM though the potential benefits of time and resource savings. More specifically, for all three participants the MTM intervention chosen during BEA was more effective than baseline during extended analysis. For both Abigail and Becca this superior effectiveness was evident in both higher mean DCPM during intervention and no instances of overlap between MTM and baseline. For Cassandra, BEA was validated during extended analysis through higher intervention mean DCPM and more gradual separation between MTM and baseline performance, especially during the second half of extended analysis. In sum, though the application of BEA technology a previously more intensive intervention was shown to be effective for 2 of 3 participants in an abbreviated form.

The current study has a few notable limitations warranting discussion. First, although current MTM procedures included all methodological components found in previous work, they did not allow for any participant to reach "mastery" level performance. That is, current repeated practice methodology included a series of only three 1 min trials, as opposed to 10 repeated trails or the achievement of mastery performance as employed previously (Mong & Mong, 2012). This change was made to better facilitate uniform BEA assessments and minimize participant time away from class. Although no participant achieved mastery level performance, current results nonetheless indicate substantial math skills gains as evidenced by both extended analysis and AIMSweb® M-COMP (Pearson, 2012) generalization results which showed improvement on untrained, grade-level material for all participants. Future research should further examine the potential utility of fewer repeated practice trials. Second, given the current participants and math skills targeted, limits to generalizability are present. That is, as all participants were elementary-aged Caucasian females with addition and subtraction-related deficits the current results may not generalize beyond them. However, given that previous MTM studies have largely included male participants the current study may be viewed as expanding the gender-related applicability of the procedures. Third, as all sessions were conducted by the second author it is difficult to generalize results to other school personnel. As such, future research should investigate the utility of MTM procedures (both abridged and complete) as implemented by classroom teachers or others. Fourth, threats to interval validity are also impossible to rule out as some math-related skill gains may be due to multiple treatment interference and practice effects or ongoing in-class instruction as opposed to direct intervention procedures.

In conclusion, the current study provides both initial evidence regarding the utility of a partial MTM intervention and adds to the literature another example of the effectiveness of the entire intervention. Both findings are important and suggest BEA as an optimal intervention selection tool with which to find the best intervention fit for individual students. In the current study, the two participants with less significant math-skills deficits responded better to an abbreviated MTM intervention including only the problem previewing, repeated practice and corrective feedback components, while the entire intervention was found to be more effective for the participant with the most severe deficits. Future research should continue to investigate MTM in both forms so as to add to the literature base for this promising intervention. This may allow for broader application of an intervention with potentially wide school-based promise and appeal.

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Table 1

Dependent Measures of Efficacy and Efficiency during Brief Experimental Analysis (BEA) Sessions

		BEA Session								
Participant		BL	MTM1	MTM2	MTM3	MTM4	MTM5	BL	MTM3	MTM5
Abigail	DCPM	2	3	0	10	7	6	3	6	8
	TIC	1	4.20	8.04	8.31	8.05	9.38	1	6.52	10.13
	ROL	-	0.71	0	1.20	0.87	0.64	-	0.92	0.79
Becca	DCPM	2	2	4	15	10	13	9	12	
	TIC	1	4.45	7.49	8.09	9.30	9.45	1	8.35	
	ROL	-	0.45	0.53	1.85	1.08	1.38	-	1.44	
Cassandra	DCPM	1	3	5	9	10	12	9	9	13
	TIC	1	7.05	9.30	9.36	11.25	12.41	1	9.31	11.27
	ROL	-	0.43	0.54	0.96	0.89	0.97	-	0.97	1.15

Note. MTM = Math-to-Mastery, DCPM = Digits Correct per Minute, TIC = Time in Condition, ROL = Rate of Learning. Dashes indicate ROL not calculated during baseline sessions. Empty cells for Becca indicate only one MTM re-implementation during mini-reversal.

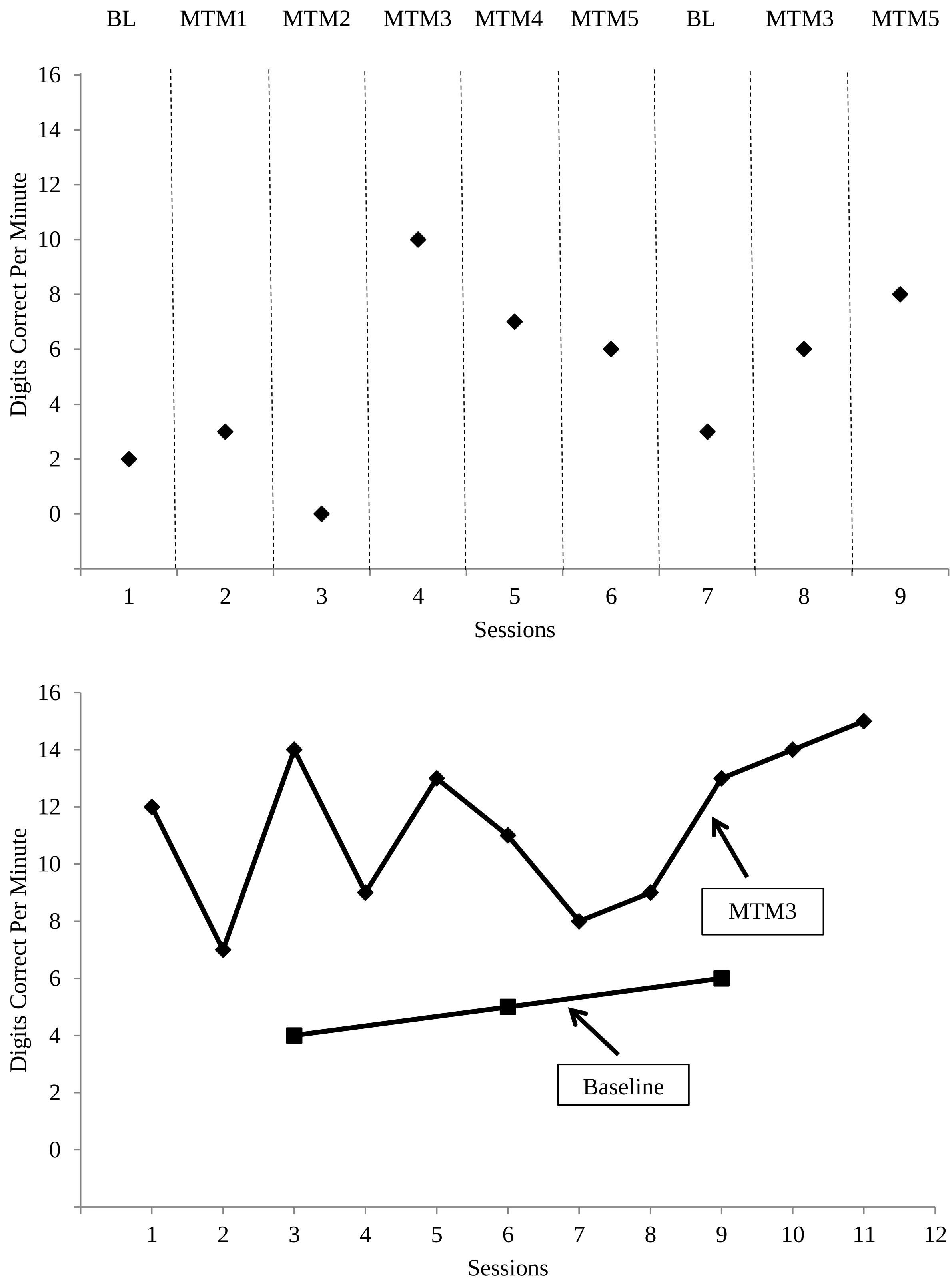
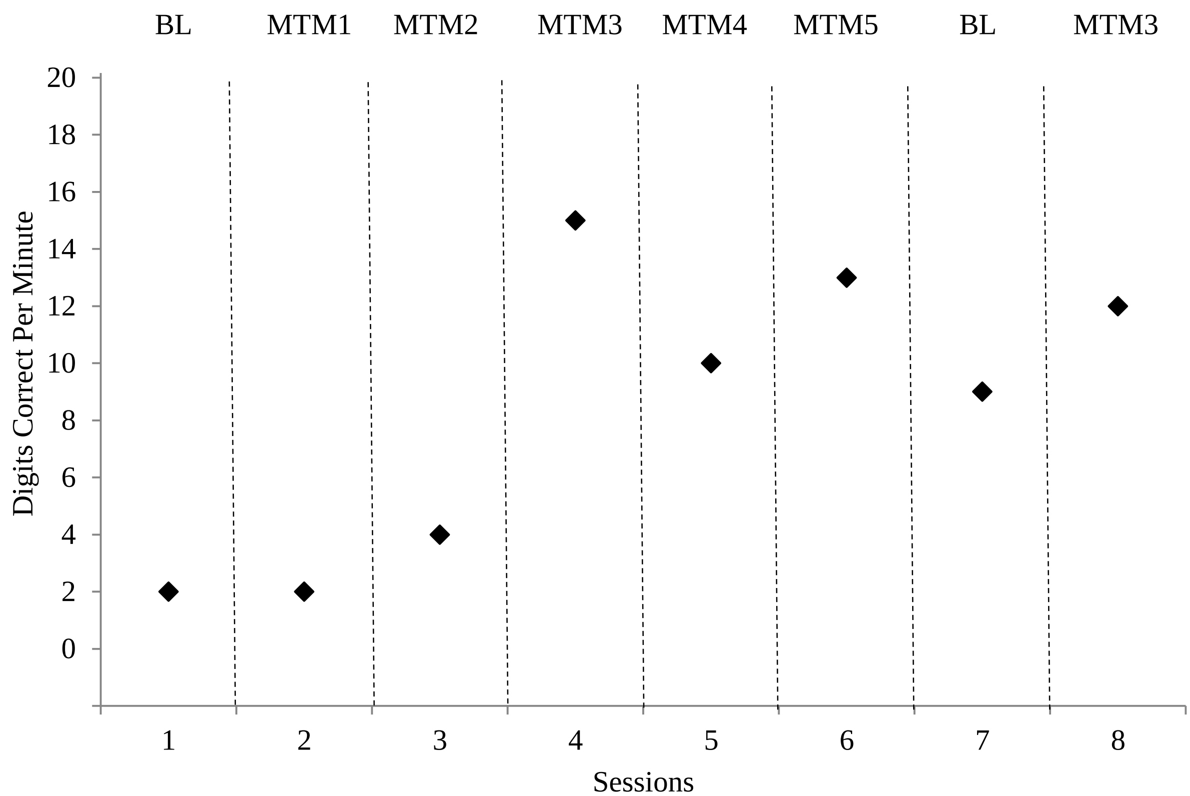


Figure 1. Brief experimental analysis (top panel) and extended analysis (bottom panel) results for Abigail.



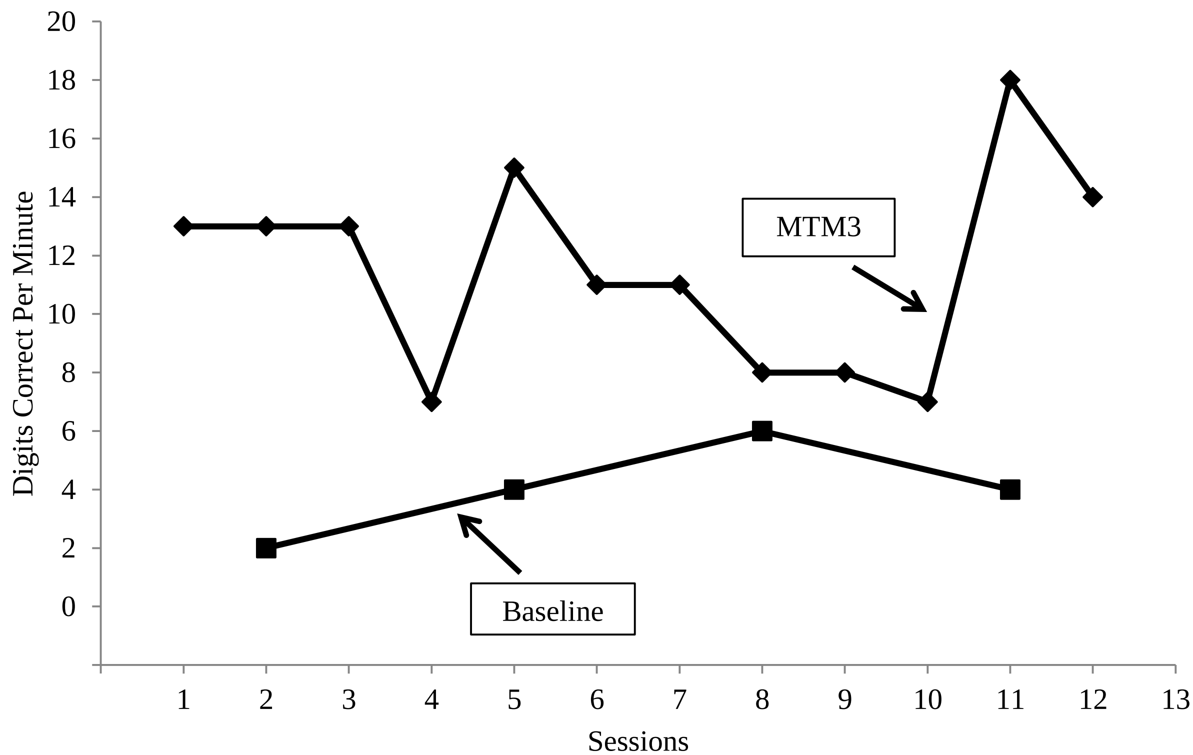


Figure 2. Brief experimental analysis (top panel) and extended analysis (bottom panel) results for Becca.

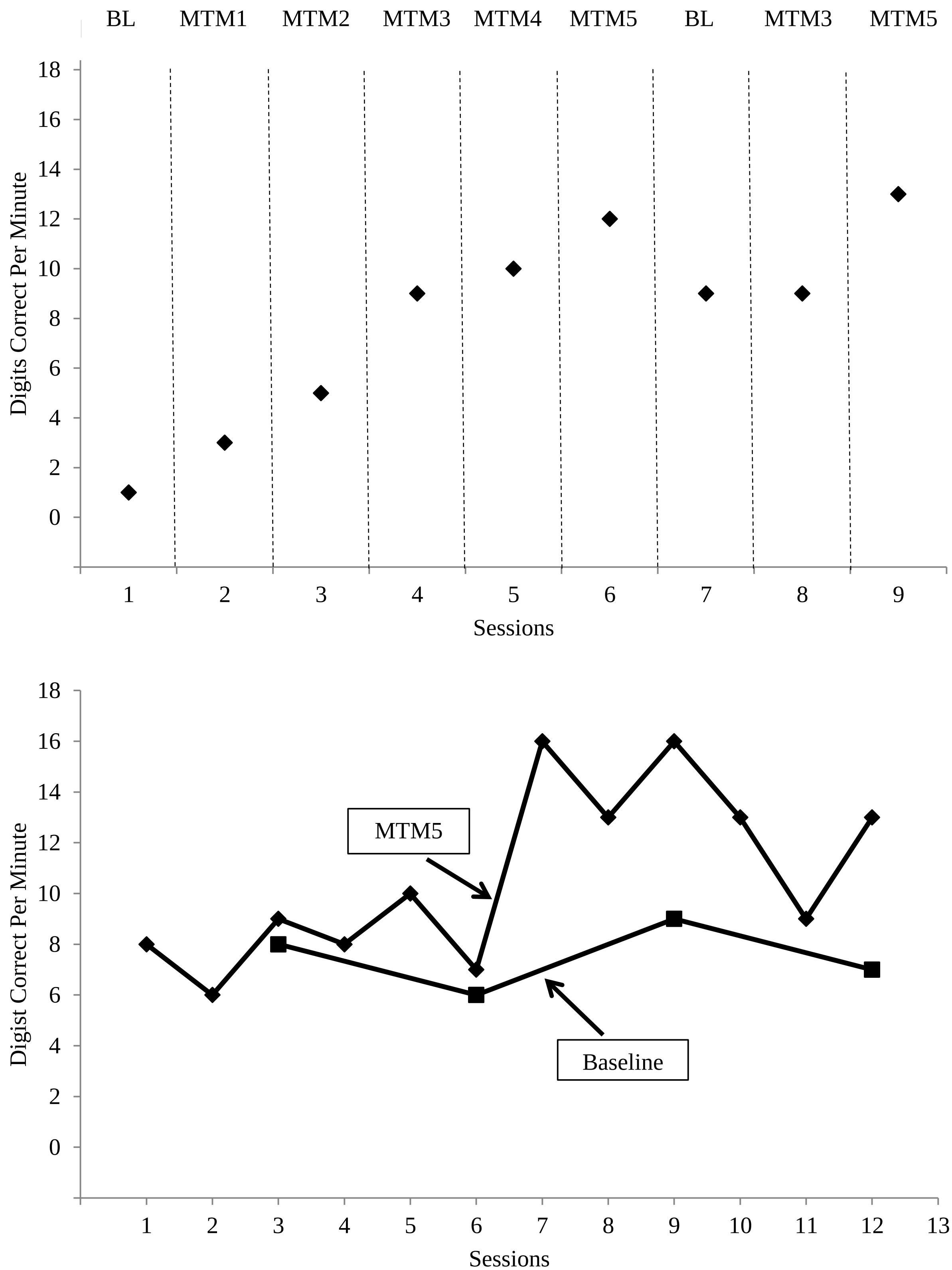


Figure 3. Brief experimental analysis (top panel) and extended analysis (bottom panel) results for Cassandra.