1-1-1995

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COWLES FOUNDATION DISCUSSION PAPER NO. 1090

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January 1995

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by

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November 1992, revised November 21, 1994

Abstract

In this paper we propose a modelling approach for labor supply and consumption decisions that is firmly grounded within a utility maximizing framework and allows for a role of such institutional constraints as limited access to borrowing and involuntary unemployment. We report estimations for a system of dynamic probit models with data from the Panel Study of Income Dynamics. These estimations test broad predictions of the theoretical model.

One of our models describes a household's propensity to be liquidity constrained in a given period. The second is a dynamic ordered probit model for a labor constraint indicator describing qualitative aspects of the conditions of employment, that is whether the household head is involuntarily overemployed, voluntarily employed, or involuntarily underemployed or unemployed. These models are estimated separately as well as jointly. Our results provide strong support for the basic theory of constrained behavior and the interaction between liquidity constraints and exogenous constraints on labor supply.

KEYWORDS: Intertemporal Optimization, Quantity Constraints, Liquidity Constraints, Unemployment, Dynamic Probit Models, Simulation Estimation

JEL Classification: D91, E24, C61, C33, C35

* Financial support by the National Science Foundation under grants SES-9000200 and SES-9211913 is gratefully acknowledged. We thank Steven Zeldes for kindly making his PSID data available to us, Fumio Hayashi for supplying us with a user-friendly program for reading the PSID data, and Kevin Lee, Ariel Pakes, Jonathan Thomas and Guglielmo Weber for useful discussions and suggestions. We also thank Steve Bergantino, Stelios Corres, Kamhon Kan, Anne Royalty, and Ann-Margret Westin for comments and expert research assistance. We benefitted from comments made during presentations at the 1992 National Science Foundation / National Center for Supercomputing Applications Conference on "Large Scale Computation in Economics" in the University of Illinois, the 1992 ASSET Congress in Toulouse, the University of Uppsala, the University of Guelph, Columbia University, the Congress of the Society for Economic Dynamics and Control, Nafplion, Greece, Yale University, VPI&SU, UCLA, LSE, the Department of Applied Economics, Cambridge University, and the Conference on "International Perspectives on the Macroeconomic and Microeconomic Implications of Financing Constraints," University of Bergamo and CEPR, October 1994. Ioannides thanks the Department of Economics and the Centre for Economic Performance at the London School of Economics for its hospitality during 1994.

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Unemployment and Liquidity Constraints

1 Introduction

The present paper uses panel data on households to address empirically the interaction between liquidity constraints and exogenous restrictions on labor supply decisions. Our techniques allow us to estimate with panel data general dynamic limited dependent variable models with a flexible dynamic structure. The presence of constraints is taken as an institutional datum. Whether and when they bind for particular individuals in a given population are the endogenous variables of interest.

We take as a starting point that capital market imperfections may prevent individuals from borrowing against their future income without collateral.[1] Intuitively, households are most likely to be liquidity constrained at times of events that are closely related to labor market conditions (e.g., unemployment) or other events, such as ill health, that have direct consequences for labor supply behavior. When labor supply is jointly considered with food consumption[2] some serious analytical difficulties emerge. These stem from the fact that observed hours of work (or employment) are not necessarily the outcome of free choice in the same way as food consumption is. Specifically, individuals may be involuntarily unemployed, underemployed, or overemployed. For such individuals, the equilibrium model of fluctuations in employment and hours worked is not appropriate. This paper addresses such qualitative aspects of employment jointly with liquidity constraints.

The paper proposes an estimation framework that utilizes an endogenous concept of liquidity constraints which, when exploited fully, will allow us to nest theoretically and econometrically previous research on consumption theory and labor supply behavior with liquidity constraints that has employed additively separable intertemporal preferences. The economic model of the paper belongs to the class of dynamic decision problems with mixed discrete and continuous decisions. Pakes (1994) and Rust (1994) provide excellent reviews of this class of problems. The econometric models of this paper address only discrete aspects of the mixed discrete continuous decisions. They permit us to incorporate quite naturally exogenous, institutional and/or economic, restrictions on labor supply and to investigate their interdependence with consumption behavior.

In a companion paper [Hajivassiliou and Ioannides (1992c)] we address mixed discrete continuous decisions. A generalized model along those lines encompasses the models of Altonji (1986), Ball (1990), Ham (1982), Ham (1986), and Zeldes (1989a). Those authors estimate single or simultaneous equations for food consumption and labor supply, but (with the notable exception of Ham's and Ball's work) neglect exogenous restrictions on labor supply and/or endogenous liquidity constraints. These works are discussed in greater detail in Section 4 below.

Our treatment of the endogeneity of regime switching and of the possible dependence between

[1] See Hall and Mishkin (1982); Flavin (1985); Altonji and Siow (1987); Zeldes (1989a); Ball (1990).

[2] Food and housing are the only major components of the consumption bundle for which data are consistently available in the Panel Study of Income Dynamics (PSID).

liquidity constraints and restrictions on labor supply behavior goes further than previous work. Typically, the past literature has only considered agents who were thought to be either liquidity-constrained or not constrained but remained so throughout the period of observation. For example, Ball restricts his sample to those who have never been constrained in the labor market. Casual empiricism suggests, and the data confirm, that switches in the state of households do occur when they are observed over long time periods. Households are most likely to be constrained early in their lifetimes, or at times of major purchases, changes in employment conditions, or other unforeseen events (death, catastrophic illnesses, etc.). The evolution over time of a household's socioeconomic circumstances makes it all the more important to allow for endogenous constraints with a dynamic structure.

Allowing for the coexistence of exogenous restrictions on labor supply and liquidity constraints is a novel feature of the present work. It is firmly rooted in the modern life cycle theory of labor supply, while at the same time it encompasses a dynamic generalization of the approach pioneered by Ashenfelter (1980). The latter studies unemployment as a "constraint on choice rather than a result of it," the latter being the hallmark of neoclassical theory of freely chosen labor supply. Our results provide strong support for the basic theory of constrained behavior and the interaction between liquidity constraints and constraints on labor supply that we propose in this paper.

Our econometric models may be estimated in their full generality only by simulation estimation methods, based on the pioneering work of McFadden (1989) and Pakes and Pollard (1989). In this paper we apply the method of maximum smoothly simulated likelihood (MSSL) developed in Börsch-Supan and Hajivassiliou (1993) that is applicable to LDV models, such as the ones here, in which the distribution of the latent variables belongs to the linear exponential class. The MSSL method also has the desirable property of continuity in the unknown parameters, which eases considerably certain computational problems.[3]

The paper is organized as follows. Section 2 presents the life cycle optimization model and derives a dynamic discrete choice model for liquidity constraints and quantity constraints on labor supply. Section 3 discusses the econometric specification of the model. Section 4 discusses the data, presents the empirical results, and reviews diagnostic tests performed on the estimated models. These results pertain to dynamic models for the discrete events of whether or not a household is liquidity constrained and whether or not household heads are subject to quantity restrictions in their labor supply behavior. Section 5 concludes.

[3] Other simulation methods sharing such desirable properties are discussed, *inter alia*, in Hajivassiliou (1989b), Hajivassiliou and McFadden (1992), and Hajivassiliou and Ruud (1993). For an extensive survey of simulation estimation methods for LDV models, see Hajivassiliou (1993).

2 Life Cycle Optimization with Liquidity and Other Quantity Constraints

We introduce basic notation for a standard life cycle optimization model. Time is discrete, lifetime horizon is of finite length T, and lifetime utility is additively separable per period and depends on the consumption of leisure and other goods. Let h_t denote hours worked per year, $\bar{L}_t$ the endowment of leisure, W_t the hourly wage rate, $\mathbf{G}_t$ the vector of consumption goods other than leisure, $\mathbf{P}_{G_t}$ the respective vector of prices, and $\mathbf{P}_t$ the full price vector, $\mathbf{P}_t = (W_t, \mathbf{P}_{G_t})$. Also, let: A_t denote the market value of total financial wealth at the beginning of period t (before expenditure decisions for that period are made); A_t^* the value of total financial wealth held after expenditures in period t have been incurred (alternatively, end-of-period t financial wealth); and b_t net asset decumulation in period t, defined as:

$$b_t = \mathbf{P}'_{G_t}\mathbf{G}_t - W_t h_t. \tag{1}$$

Note that b_t may be positive or negative, implying that the household may want to dissave or save, respectively.

The household's budget constraint for period t is:

$$A_t = b_t + A_t^*, \tag{2}$$

and the intertemporal asset accumulation constraint is:

$$A_{t+1} = (1 + r_{t+1})A_t^*, \tag{3}$$

where r_{t+1} is a one-period rate of return that may be uncertain as of time t.

We introduce uncertainty by letting $\{\mathcal{N}_t \ , t = 0, 1, ...\}$ be a stochastic process with well-defined transition probabilities. These transition probabilities may be used to define conditional expectations, where $\mathcal{N}^t = \{\mathcal{N}_0, \mathcal{N}_1, ..., \mathcal{N}_t\}$. We shall interpret $\mathcal{N}_t$ as the new information the household receives at time t and $\mathcal{N}^t$ as the information state as of time t, which comprises the set of exogenous state variables. The framework allows for taste uncertainty as well as price uncertainty, and sequential resolution of uncertainty may be associated with either of those items.

Let $u[h_t, \mathbf{G}_t|\mathcal{N}_t]$ be the utility from supplying h_t labor and consuming a vector of consumption goods $\mathbf{G}_t$. This function is assumed to be increasing in $\mathbf{G}_t$, decreasing in h_t, and concave with respect to all of its arguments. Dependence on the information state allows for taste shocks. The problem the typical household faces in period t is to maximize the expectation of a time-additive lifetime utility index:

$$u[h_t, \mathbf{G}_t|\mathcal{N}_t] + E_t\left\{\sum_{k=t+1}^{T} \frac{1}{(1+\rho)^{k-t}} u_k[h(k), \mathbf{G}(k) \mid \mathcal{N}_k] \mid \mathcal{N}^t\right\}, \tag{4}$$

with respect to $\{h_t, \mathbf{G}_t; ...\}$, subject to (1–3), and to a predetermined value of A_t. This statement of the problem follows MaCurdy (1983) and constitutes a multidimensional version of the problem addressed by Altonji (1986), Browning, Deaton and Irish (1985), and MaCurdy (1983). This problem may be equivalently stated in terms of the choice, in each period, of net asset decumulation, $b(k)$, and end-of-period financial wealth, $A^*(k)$, $k = t, \ldots, T$.

2.1 Liquidity Constraints and Constraints on Labor Supply

The problem of maximizing (4) subject to (1–3), the workhorse of modern life cycle theory, is well understood. Here we add constraints on borrowing and on labor supply.

2.1.1 Liquidity Constraints

We introduce a liquidity constraint that prevents a household from holding negative financial wealth at the end of period t; that is:

$$A_t - b_t \geq 0, \qquad t = 1, ..., T. \tag{5}$$

We regard this version of the liquidity constraint as the canonical case. Clarida (1987), Ritter (1987), Alessie *et al.* (1989), and Zeldes (1989a) employ similar assumptions about the borrowing constraint.[4] The constraint $A_T - b_T \geq 0$ is, of course, a standard feature of the lifecycle allocation problem in the absence of a bequest motive. [5]

2.1.2 Unemployment, Underemployment and Overemployment Constraints

We extend formally the problem of maximizing (4), subject to (1–3), so as to allow for qualitative aspects of employment. We accomplish this by introducing exogenous restrictions on labor supply. Such an extension may be interpreted as a dynamic generalization of Ashenfelter (1980).[6] In

[4]Chamberlain and Wilson (1984) have articulated a more general formulation for a lower bound on financial wealth, which may be a function of information available about the household up until period t. That is, a "consistent" borrowing constraint requires that non-human wealth may never be so low as to make it impossible for the household to satisfy its borrowing constraint next period even if nothing is spent in the current period. They then show that as long as the value of the endowment of leisure is non-negative one may re-define income, that is the value of the endowment of leisure, to be the increase in available purchasing power the household receives in each period (which is equal to the excess of income in period t over the change in the minimum allowable wealth from one period to the next). Then the consistent borrowing constraint is transformed to (5).

[5]A mathematically equivalent statement of the problem follows Deaton (1989) and Zeldes (1989a) and defines A_t as beginning of period t resources inclusive of the value of the endowment of leisure in that period:

$$A_{t+1} = (1 + r_{t+1})A_t^* + W_{t+1}\bar{L}_{t+1}. \tag{3'}$$

In that case, utility from consumption in period t is defined as a function of full expenditure in period t, b_t^*:

$$b_t^* = A_t - A_t^* \tag{2'}$$

and the period t expenditure constraint becomes:

$$b_t^* = \mathbf{P}'_{G_t}\mathbf{G}_t + W_t(\bar{L}_t - h_t). \tag{1'}$$

A disadvantage of this formulation is that an assumption must be made about the endowment of leisure, $\bar{L}_t$, which is not observable. The liquidity constraint would not be affected by such a redefinition of b_t.

[6]Ashenfelter's static model was designed to aggregate readily so that it be used with aggregate annual data. See Neary and Stiglitz (1983) for a two-period macro model with Keynesian-type quantity constraints. That paper's

section 2.3 we incorporate such restrictions formally into the dynamic programming formulation.

This extension is motivated by availability, within the PSID data, of answers to a number of questions that we interpret as pertaining to voluntary versus involuntary aspects of employment. Section 4.3.1 below provides details on how we recode that information in order to measure unemployment, underemployment, or overemployment.

Let us consider, in particular, that the decision maker expects that his labor supply must satisfy a sequence of constraints

$$h_t \leq h_{RU_t}, \qquad t = 0, 1, \ldots, T; \tag{6}$$

$$h_{RO_t} \leq h_t, \qquad t = 0, 1, \ldots, T; \tag{7}$$

with probability one.[7] Quantity constraint (6) may be used to represent *involuntary* unemployment. Quantity constraint (7) may be used to represent, symmetrically, *involuntary* overemployment.[8]

When compared to liquidity constraints (5), quantity constraints (6–7) may have an even better claim to possessing a strong "Keynesian" flavor. We think of h_{RU_t} and h_{RO_t} as representing demand for an individual's labor in his local labor market. Likely determinants are various cyclical factors and, in addition, such local factors as the local unemployment rate, the difference between the number of applicants and vacancies in an individual's labor market, the unemployment rate in an individual's (one-digit) occupation, and regional dummies. However, as Ham (1986) notes, the effect on a worker of demand shocks to an industry or a region may depend on his characteristics and various human capital variables, which, following others, we include in the model as determinants of labor supply behavior.[9] True to the neoclassical model of freely chosen labor supply, if the labor supply model is correctly specified "market-level information should be irrelevant to individual hours decisions, controlling for individual-specific wages" [e.g., see Card (1994)].

2.2 Life Cycle Optimization with Liquidity Constraints Only

We consider first the problem of maximizing lifetime utility subject to liquidity constraints only. We thus ignore for the time being constraints (6) and (7). Let $v(\cdot)$ denote the indirect utility function corresponding to $u(\cdot)$, and $V(\cdot)$ the value function corresponding to (4), as a function of A_t and of the information state $\mathcal{N}^t$. Once b_t is known, labor supply and commodity demands in period t are obtained from $v[b_t; W_t, \mathbf{P}_{G_t}|\mathcal{N}_t]$ via Roy's identity.

notable result on the bootstrap properties of rational expectations equilibria in such a setting suggests it would be interesting to incorporate quantity constraints on choice in a dynamic setting.

[7]For reasons of brevity we suppress the fact that $h_{RU}(\cdot)$ and $h_{RO}(\cdot)$ are conditional on the information state $\mathcal{N}^t$.

[8]Following Heckman and MaCurdy (1980), we may also consider *voluntary* unemployment (or non-participation in the labor force) as an additional discrete state that occurs as a corner solution at 0, when an individual's reservation wage rate is greater than the market wage. This may be handled as a special case of (7), where $h_{RO} = 0$. In order for this event to be identified as separate from overemployment (at 0), additional qualitative information must be brought to bear. We do not pursue this further in the present paper.

[9]Card (1994) notes that Keynesian-style labor market constraints are not indispensable for rationalizing Ham's findings on the importance of demand factors. He suggests the alternative explanation that individuals decide on their labor supply at a higher frequency time unit than the year (for which data are available) and that there may be significant fixed costs on either the worker's side or the employer's side of the labor market.

According to a standard dynamic programming formulation [Bertsekas (1987)] of problem (4), the value function is defined as follows:

$$V[A_t;\mathcal{N}^t] =$$

$$\max_{A_t \geq b_t} : v(b_t;W_t,\mathbf{P}_{G_t}|\mathcal{N}_t) + \frac{1}{1+\rho}E_t\left\{V[(1+r_{t+1})(A_t-b_t)\,;\mathcal{N}^{t+1}]\right\}. \tag{8}$$

We assume, like others before us [e.g., Altonji (1986)] that parameter values ensure that hours worked do not exceed the endowment of leisure, $h_t \leq \bar{L}_t$.

We now continue with problem (8) in the absence of labor supply quantity constraints. We introduce Lagrange multipliers $\lambda(t|\mathcal{N}^t)$ and $\mu(t|\mathcal{N}^t)$ to adjoin constraints (2) and (5) respectively. We shall suppress the dependence on $\mathcal{N}^t$ whenever no confusion arises. Necessary conditions for the maximization in the RHS of (8) are:

$$\frac{\partial}{\partial b}v(b_t;W_t,\mathbf{P}_{G_t}) = \lambda_t; \tag{9}$$

$$\lambda_t = \mu_t + \frac{1}{1+\rho}E_t\{(1+r_{t+1})\lambda_{t+1}\}; \tag{10}$$

$$\mu_t[A_t - b_t] = 0, \tag{11}$$

where we have made use of the envelope property:

$$\frac{\partial}{\partial b}v(b_t;W_t,\mathbf{P}_{G_t}) = \frac{\partial}{\partial A}V(A_t;\mathcal{N}^t).$$

Conditions (10–11) are simplified further as follows:

$$\text{If } \ A_t > b_t, \ \ \mu_t = 0, \ \ \lambda_t = \frac{1}{1+\rho}E_t\{(1+r_{t+1})\lambda_{t+1}\}. \tag{12}$$

or

$$\text{If } \ A_t = b_t, \ \ \mu_t > 0, \ \ \lambda_t = \mu_t + \frac{1}{1+\rho}E_t\{(1+r_{t+1})\lambda_{t+1}\}. \tag{13}$$

The intertemporal optimization conditions (12–13) may be written concisely in terms of a single functional equation in the standard fashion of dynamic programming [Bertsekas (1987)] in terms of the partial derivative[10] of the indirect utility function with respect to b:

$$\frac{\partial v(b_t;\cdot)}{\partial b_t} = \max\left\{\frac{\partial v(A_t;\cdot)}{\partial b_t}, \frac{1}{1+\rho}E_t\left\{(1+r_{t+1})\frac{\partial v(b_{t+1};\cdot)}{\partial b_{t+1}}\right\}\right\}. \tag{14}$$

An important conclusion follows from (14): marginal utility is a supermartingale (with a drift), a result that is crucial for proving existence of the optimal policy. We take that up in Subsection 2.4 below and elsewhere [Hajivassiliou and Ioannides (1992a)].

[10]By using the envelope property, equation (14) may be alternatively stated in terms of $\frac{\partial V}{\partial A}$, the derivative of the value function with respect to beginning-of-period financial wealth A_t.

We now follow Hajivassiliou and Ioannides (1992a) and state the general solution to this problem, which is fully characterized by the existence of a threshold value of A_t, $\tilde{A}(\cdot)$, such that the optimal net asset decumulation has the form:

$$b_t = A_t, \qquad A_t < \tilde{A}(W_t, \mathbf{P}_{G_t}; \mathcal{N}^t); \tag{15}$$

$$b_t = B(A_t; W_t, \mathbf{P}_{G_t}; \mathcal{N}^t), \qquad A_t \geq \tilde{A}(W_t, \mathbf{P}_{G_t}; \mathcal{N}^t). \tag{16}$$

The monotonicity property of the Lagrange multiplier, proven in Proposition 4 and Section 3.4 of Hajivassiliou and Ioannides (1992a), and equation (9) imply that b_t is an increasing function of beginning-of-period assets, conditional on the information state $\mathcal{N}^t$. Note that (15) implies $A_t^* = 0$ and the household is liquidity constrained, and (16) implies that $A_t^* > 0$ and the household is unconstrained. In a non-stationary setting as ours, the threshold value of financial wealth $\tilde{A}_t$ in general depends explicitly on period t prices and on all of the attributes of the information state $\mathcal{N}^t$. In particular, it is possible that the household may want to accumulate, by setting $b < 0$, when beginning-of-period financial assets are too low. For example, in our setting, it is not necessarily the case that a household is constrained if beginning-of-period assets are zero. We demonstrate in *ibid.* the existence of a threshold value of assets for a special case of our model. That analysis suggests that in a steady state setting where individual effects are the only source of randomness, the threshold value of assets depends upon prices. In particular, it depends negatively upon the real wage.[11]

In the econometric analysis reported in the present paper we will account for liquidity constraints by means of a single endogenous variable representing the discrete event of whether or not an individual is liquidity constrained. That is, we define the binary variable $S_t \equiv S(A_t; W_t, \mathbf{P}_{G_t}; \mathcal{N}^t)$ as:

$$S_t = \mathbf{1}[\tilde{A}(W_t, \mathbf{P}_{G_t}; \mathcal{N}^t) - A_t \geq 0], \tag{17}$$

where the indicator function $\mathbf{1}[\mathcal{C}]$ takes the value of 1 if condition $\mathcal{C}$ is true and 0, otherwise. S_t may be referred to as a discrete decision variable for the decision problem defined in (8) above. In section 3 below we use this definition to derive estimable econometric specifications.

[11] Research on this topic has been hindered by the fact that no closed-form solutions to problem (8) have been obtained, even without the liquidity constraint (5), except for two well known special cases, namely linear-quadratic utility and the case of fully-diversifiable wealth for the HARA class of utility functions [Merton (1971)]. It is for this reason that interest has recently turned to using the dynamic programming algorithm to solve numerically for the optimal policy functions with simple specifications of the utility function. For the unconstrained problem see Lam (1989) and Zeldes (1989b), and for the constrained one Deaton (1991).

Deaton (1991) demonstrates the feasibility of the computational approach in the presence of liquidity constraints with consumption as the single decision, and under several alternative assumptions about the stochastic process describing (exogenous) labor income. Taylor and Uhlig (1990) provide an exhaustive review of alternative solution methods that were tried out by the members of the Nonlinear Rational Expectations Modelling Group and are reported in that same issue of the *Journal of Business and Economic Statistics*.

2.2.1 Example

The following example demonstrates certain estimable consequences of the simplest possible generalization proposed by this paper, namely of introducing leisure in addition to consumption as decision variables in each period. We assume both consumption good and leisure to be normal. We simplify by setting $P_{G_t} = 1$, $\bar{L}_t = 1$, and by letting $W_{h_t} = W_t$, the real wage, be the sole source of uncertainty. The real wage is assumed to be independently and identically distributed over time. The indirect utility function representing preferences per period does not depend on $\mathcal{N}_t$. There exists a single riskless asset with a constant rate of return r, satisfying $r \geq -1$.

The difference from the classic treatment of the liquidity–constrained problem with a single decision variable according to Deaton (1991) is the presence of two state variables (A_t, W_t): W_t is an exogenous state variable and A_t is, as before, an endogenous one.

To the direct utility function $u(h, C) \equiv \frac{1}{1-\alpha}C^{1-\alpha} + \beta\frac{1}{1-\alpha}(1-h)^{1-\alpha}$, $\alpha > 0$, there corresponds an indirect utility function $\overset{*}{v}(b; W) \equiv \frac{1}{1-\alpha}b^{1-\alpha}\left[1 + \beta^{\frac{1}{\alpha}}W^{-\frac{1-\alpha}{\alpha}}\right]^{\alpha}$. It follows that relative to the single decision variable case as examined by Deaton, *op. cit.*, the presence of leisure in the utility function implies *ceteris paribus* that the optimal decision is a function of assets and the real wage. Under the assumption that an optimal asset decumulation function, according to (14)–(15) of the form $b = b(A; W)$, exists for the infinite horizon version of the problem, we may characterize the critical curve in (A, W) space as follows. According to (14), it is defined implicitly by the value of A, $A_c(W)$, for which the two terms in the RHS are equal to one another, $v_b(b(A_c; W); W) = \frac{1+r}{1+\rho}E_{\tilde{W}}\left\{v_b((1+r)[A_c - b(A_c; W)] + \tilde{W}; \tilde{W})\right\}$. Since, $b(A_c(W); W) = A_c(W)$, the critical curve is given by:

$$A_c^{-\alpha}\left[1 + \beta^{\frac{1}{\alpha}}W^{-\frac{1-\alpha}{\alpha}}\right]^{\alpha} = \frac{1+r}{1+\rho}E_{\tilde{W}}\left\{b(\tilde{W}; \tilde{W})^{-\alpha}\left[1 + \beta^{\frac{1}{\alpha}}\tilde{W}^{-\frac{1-\alpha}{\alpha}}\right]^{\alpha}\right\}. \tag{18}$$

The RHS of (18) is a constant at the optimal solution, and thus $A_c(W)$ is an decreasing (increasing) function of W, if $\alpha < (>)1$. Furthermore, labor supply is backward bending (upward sloping), if $\alpha < (>)1$. Of course, this result depends critically on one particular assumption, absence of serial dependence in W.

2.3 Life Cycle Optimization with Liquidity Constraints and Quantity Constraints on Labor Supply

The optimization problem described by (8) may be adapted to allow components of the consumption bundle to be treated as predetermined[12] or exogenously given, as appropriate. That is, if we know whether an individual is *involuntarily* unemployed (or, alternatively, overemployed) in a particular period, then observed labor supply may not be identified with the outcome of unrestricted choice and instead must be treated as being subject to exogenous restrictions. Such information is indeed

[12]The quantities of durables and of housing, in particular, are also subject to optimization but may not be changed every period (due to transactions costs). In contrast to food consumption and labor supply, it is appropriate to treat them as endogenous but predetermined.

available in the PSID, and this feature of the data is exploited by the present study.

In the presence of exogenous restrictions on labor supply, the utility per period enjoyed by the individual in general is no longer equal to the unrestricted indirect utility function $v(\cdot)$. The utility per period in the RHS of (8) is a function of the corresponding period's decision and is given by a suitably *restricted* indirect utility function, $v_R(\cdot)$.[13] The value function $V[\cdot]$ is redefined accordingly. If labor supply is constrained in a particular period, it would be equal from (6) or (7) to either h_{RU_t} or h_{RO_t}. Then $v(\cdot)$ in (8) should be replaced by $v_R(b_{R_t}; h_{R_t}; W_t, \mathbf{P}_{G_t}|\mathcal{N}_t)$, where b_{R_t}, from (1), is suitably redefined to reflect the fact that labor supply is given exogenously:

$$b_{R_t} = \mathbf{P}'_{G_t}\mathbf{G}_t - W_t h_{R_t}; \tag{19}$$

Whether $h_{R_t} = \{h_{RU_t} \text{ or } h_{RO_t}\}$ corresponds to the constraints on labor supply in the cases of underemployment and overemployment, respectively.

Which of the three possibilities holds specifies, of course, a decision variable, to be referred to as an employment state indicator. Its value depends upon all endogenous (A_t) and exogenous ($\mathbf{P}_t$ and $\mathcal{N}^t$) state variables:

$$E_t \equiv E(A_t; \mathbf{P}_t, \mathcal{N}^t). \tag{20}$$

E_t is defined as having a value equal to 0, if the individual is unconstrained in his labor supply behavior, equal to 1, if he is constrained by (6), in which case we say that the individual is underemployed or unemployed, and equal to -1, if he is constrained by (7), in which case we say that the individual is overemployed. We return to a further characterization of this definition after we have discussed existence and uniqueness.

2.4 Existence, Uniqueness and Characterization of the Optimal Solution

We now turn to a formal characterization of the problem of maximizing lifetime utility (4) with respect to $\{h_t, \mathbf{G}_t; ...\}$, subject to constraints (1)–(5), to quantity constraints on labor supply, (6)–(7), and to a predetermined value of A_t, as a dynamic programming problem with Markovian structure.[14] This problem belongs to a general class of dynamic Markov decision problems with mixed discrete-continuous decisions [Pakes (1994)].[15]

Let $C(\varsigma_t)$ denote the set of feasible decisions as a function of the state ς_t, and let Θ denote the state space, $\varsigma_t \in \Theta$. The set of state variables ς_t at time t consists of the endogenous state variable A_t, beginning-of-period assets, and of the exogenous state variables, which consist of the price vector

[13] This is the counterpart for a consumer of the restricted profit function. See McFadden (1978).

[14] Existence and uniqueness for the problem with a single decision variable in the presence of liquidity constraints have been shown by Chamberlain and Wilson (1984), who follow a primal approach in discrete time, He and Pagès (1990) and Pagès (1989), who follow a dual approach in continuous time, and Deaton (1991), who develops fully the stationary case and provides numerical solutions. Hajivassiliou and Ioannides (1992a) also employ a dual approach, allow for a vector of goods, and relate the formulation to Frisch demand theory. Quantity constraints have not been handled before in a dynamic setting.

[15] Pakes (1994), the only paper that considers formally such cases, shows that the discrete aspect of such problems does not interfere, in general, with our ability to derive Euler-type equations.

$\mathbf{P}_t = (W_t, \mathbf{P}_{G_t})$, and of the information state as of time t, $\mathcal{N}^t$. The decision variables D_t consist of the discrete-valued indicators (S_t, E_t) and of the continuous variable b_t, the corresponding net asset decumulation in period t, that is, $D_t = \{S_t, E_t, b_t\}$. $S_t \in \{0, 1\}$, where a value of 1, respectively 0, denotes that the liquidity constraint is, respectively is not, binding. $E_t \in \{-1, 0, 1\}$, where a value of 0 indicates that the individual is unconstrained in his labor supply behavior (that is, both constraints (6) and (7) are not binding), a value of -1 indicates that constraint (7) is binding, and a value of 1 indicates that constraint (6) is binding.

We note that the utility payoff in period t depends upon the state as well as the decision, $v[\varsigma; (s, e, b)]$. For example, if $S_t = 1$, $E_t = -1$, and $b_{RO_t} = b$, then the utility payoff is given by the restricted indirect utility function $v_R(b; h_{RO_t}; W_t, \mathbf{P}_{G_t}|\mathcal{N}_t)$. Given the state vector ς_t and once S_t, E_t and b_t have been determined, commodity demands follow by applying Roy's identity with the corresponding indirect utility function. When employment constraints bind, labor supply functions are given by the respective constraints.

We define the value function corresponding to the problem as follows:

$$V(\varsigma_t) = \sup : E\left\{\sum_{k=t}^{T} \frac{1}{(1+\rho)^{k-t}} v[\varsigma_k, D_k]|\varsigma_t\right\}, \tag{21}$$

where the supremum is taken with respect to all feasible policies $\{D_t, D_{t+1}, \ldots, D_T\}$.

2.4.1 Characterization of the Employment State Indicator

We denote the solution for the unconstrained (*notional*) labor supply from problem (21), conditionally upon S_t, by $h_t = H(A_t; W_t, \mathbf{P}_{G_t}; \mathcal{N}^t|S_t)$. As this is a function of assets, following MaCurdy (1983) we may refer to it as the *pseudo* labor supply function. The employment state indicator may now be defined in terms of the pseudo labor supply function as follows:

$$E(\mathbf{P}_t; \mathcal{N}^t|S_t) = -1, \qquad \text{if } h_t = h_{RO_t} > H(A_t; W_t, \mathbf{P}_{G_t}; \mathcal{N}^t|S_t); \tag{22}$$

$$E(\mathbf{P}_t; \mathcal{N}^t|S_t) = 0, \qquad \text{if } h_{RO_t} \leq H(A_t; W_t, \mathbf{P}_{G_t}; \mathcal{N}^t|S_t) \leq h_{RU_t}; \tag{23}$$

$$E(\mathbf{P}_t; \mathcal{N}^t|S_t) = 1, \qquad \text{if } h_t = h_{RU_t} < H(A_t; W_t, \mathbf{P}_{G_t}; \mathcal{N}^t|S_t); \tag{24}$$

Below in subsection 4.3.1 we link this definition with all categories available in the data, so as to take advantage of their full detail. An important feature of our model, namely that E_t may be modelled as an *ordered* discrete-choice model, readily follows from its definition.

It is helpful to try and visualize the determination of the employment state indicator in a static-equivalent setting. We note that once the period t net asset decumulation b_t has been determined, we may refer to a standard consumption-leisure choice diagram, such as in Figure 1. Given prices and net asset decumulation, the position of the "budget line" is determined. Furthermore, given parameters and values for the observables and unobservables, a particular individual who is in the labor force, may be in one of three categories. An individual may be of type V, in which case employment is determined according to point V_T, and the individual is voluntarily employed. We note that in this case $h_{RO} \leq h \leq h_{RU}$. Alternatively, an individual may be of type U, i.e., one who wishes to work according to point U_T. He may not, however, work as much as he wishes because of the underemployment constraint h_{RU}. In such a case employment is determined according to point U_R, and the individual is involuntarily underemployed (or unemployed) working h_{RU} hours. Finally, an individual may be of type O, i.e., one who wishes to work according to point O_T. Such an individual may not, however, be able to work as little as he wishes because of the overemployment constraint h_{RO}. In such a case employment is determined according to point O_R, and the individual is involuntarily overemployed, working h_{RO} hours.[16]

An appropriate analytical representation of this choice problem requires that it always be the case that $h_{RU_t} > h_{RO_t}$. The economic intuition of this assumption is straightforward. The maximum amount an individual is allowed to work must not be less than the minimum.

In view of the discussion of the determinants of h_{RU_t} and h_{RO_t} in Section 2.1.2 above, we would expect that an upturn in the business cycle would increase the magnitudes of both of the constraining quantities. This would cause the overemployment constraint to become tighter and the underemployment one to be relaxed. Both those outcomes accord with economic intuition.

2.4.2 Formulation as a Markovian Decision Problem

Even though in the present paper we are interested only in estimation of a discrete decision problem, the original problem does not seem to be reducible in terms of discrete decisions only, and a statement of the full problem is called for. We turn to this here. Let $\pi[\varsigma_{t+1}|\varsigma_t, D_t]$ denote the law of

[16]If it may be assumed that the notional labor supply function is locally monotonic with no backward bending portion, the definition of E_t may be alternatively stated in terms of wage comparisons. In fact, such a definition may be more appropriate, given that h_{RU}, and h_{RO} are actually not observed. The discrete events are then defined as follows. We invert $h_{RU} = H(A_t; W_t, \mathbf{P}_{G_t}; \mathcal{N}^t|S_t)$ in terms of the wage rate to obtain a threshold value W_{RU} for the wage rate as a function of A_t, $\mathbf{P}_{G_t}$, and the information state. The underemployment region is defined as the set of points for which $W > W_{RU}$. Similarly we can define a threshold value W_{RO}, corresponding to h_{RO}. The overemployment region is defined as the set of points for which $W < W_{RO}$. The functions W_{RU} and W_{RO} are the dynamic counterparts of the *virtual* wage, a concept which was introduced by Deaton and Muellbauer (1981) in a static commodity demands-labor supply context. Graphically, they are defined by the marginal rates of substitution at points U_R and O_R, respectively. [See Figure 1.] The discussion in subsection 2.3 suggests that the virtual wage function must be defined conditionally upon S.

The employment indicator E_t may then be characterized conditionally upon S_t as follows:
Overemployment:

$$E_t = -1, \quad \text{if } W_t < W_{RO}(A_t, \mathbf{P}_{G_t}; \mathcal{N}^t|S_t); \tag{22'}$$

Voluntary Employment:

$$E_t = 0, \quad \text{if } W_{RO}(A_t, \mathbf{P}_{G_t}, \mathcal{N}^t|S_t) \leq W_t \leq W_{RU}(A_t, \mathbf{P}_{Gt}, \mathcal{N}^t|S_t); \tag{23'}$$

Under/Unemployment:

$$E_t = 1, \quad \text{if } W_{RU}(A_t, \mathbf{P}_t; \mathcal{N}^t|S_t) < W_t. \tag{24'}$$

The assumption that the labor supply function does not contain a backward bending portion may, of course, be too strong.

motion of the system, a Markov transition density for next period's state ς_{t+1}, as a function of the current state, ς_t, and of the decision, D_t. Under appropriate regularity conditions[17] there exists a *value function* $V: \ \Theta \mapsto \Theta$ which is the unique solution to the Bellman equation:

$$V(\varsigma) = \max_{D \in C(\varsigma)} : v(\varsigma, D) + \frac{1}{1+\rho} \int_{\Theta} V(\varsigma') \pi(d\varsigma' | \varsigma, D). \tag{25}$$

The decision rule $\mathcal{D}$, the optimal policy of (25), is defined by:

$$\mathcal{D}(\varsigma_t) = \arg \max_{D \in C(\varsigma_t)} : v(\varsigma_t, D) + \frac{1}{1+\rho} \int_{\Theta} V(\varsigma_{t+1}) \pi(d\varsigma_{t+1} | \varsigma_t, D). \tag{26}$$

It follows from the statement of the problem according to (21) that the optimal solution for (S_t, E_t, b_t) is a mapping $\mathbf{R}_+ \times \mathbf{R}_+^{|\mathbf{P}|} \times \mathcal{N} \mapsto (\{0,1\} \times \{-1,0,1\}) \times \mathbf{R}$.

With this Markovian decision problem notation and by following Rust (1994), we note that the *primitive objects* of the model are (ρ, v, π), the *reduced-form* is the decision rule $\mathcal{D}$, and the *structure* is the mapping $(\rho, v, \pi) \Rightarrow \mathcal{D}$ defined by (25) and (26) above. As Rust (1994), pp.11–14, notes, it should not come as a surprise that the general dynamic programming problem is unidentified, i.e., the above mapping is many-to-one.

Given that the value function exists, we may rewrite it in order to make it easier to explore its empirical implications. In doing so and for the purpose of structuring the estimation problem, we augment the original state space and include an additive utility component, which is unobserved by the econometrician, and may in general depend upon both the state and the decision, $\varepsilon_t = \varepsilon(\varsigma_t, D_t)$. We assume that ε_t satisfies Rust's conditional independence assumption [Rust (1988), p. 1010], namely that the law of motion specifies that the additive utility component depends upon the evolution of the state variables only through its contemporaneous dependence with the rest of the state variables. That is, $\pi[\cdot|\cdot]$ satisfies:

$$\pi[\varsigma_{t+1}, \varepsilon_{t+1} | \varsigma_t, D_t] = \pi_\varepsilon[\varepsilon_{t+1} | \varsigma_{t+1}, \beta_\varepsilon] \cdot \pi_\varsigma[\varsigma_{t+1} | \varsigma_t, D_t, \beta_\varsigma], \tag{27}$$

where β_ε and β_ς are vectors of parameters. We also assume that the unobserved additive utility component ε_t is independent of the endogenous components of the state vector and when no confusion arises we suppress the dependence upon the information state and write: $\varepsilon_t = \varepsilon(S_t, E_t)$.

Under these assumptions, we may proceed to write the value function $V[\varsigma_t, \varepsilon_t]$ in terms of the conditional valuation functions, where we solve out for b_t and thus condition only on the discrete

[17] See Bertsekas (1987), Bertsekas and Shreve (1978), and Rust (1994). The weakest regularity conditions are those of Bhattacharya and Majumdar (1989) who require :
(A1) Θ be a nonempty Borel subset of a complete separable metric space;
(A2) $C(x)$ be a compact metric space for $\forall x \in \Theta$;
(A3) $v(x,d)$ be uppersemicontinuous $\forall d \in C(x)$, $\forall x \in \Theta$, and $U(x) \equiv \sum_{k=0}^{\infty} (1+\rho)^{-t} v_k(x) < \infty$, $\ \forall x \in \Theta$, where $v_0(x) \equiv \sup_{d \in C(x)} |v(x,d)|$, $\ v_{k+1}(x) \equiv \sup_{d \in C(x)} \int v_k(z) \pi(dz|x,d)$, $k = 0, 1, \ldots$.
(A4) $\pi(dz|x,d)$ is a weakly continuous function of (x,d).

If $v(x,d)$ is unbounded then (A3) and (A4) must be modified, [*ibid.*] p.375. (A3) must require that $v(x,d)$ be continuous. (A4) must require that the mapping $(x,d) \mapsto \int \phi(z) \pi(dz|x,d)$ be continuous on $\Theta \times D$ for all Borel measurable ϕ satisfying $|\phi(x)| \leq U(x) + 1$, $\forall x \in \Theta$.

As Rust (1994), p.11, notes, the assumption of a finite choice is not necessary for this result. See also Bertsekas and Shreve (1978) for measurability requirements, which are practically satisfied in most relevant settings.

decision variables. It is convenient to decompose the state vector as follows: $\varsigma_t = (A_t, \varsigma_{-t})$. The conditional valuation functions, V^{se}, are defined as the optimal values of expected lifetime utility conditional on a particular set of discrete decisions, $(S_t = s, E_t = e)$. The conditional valuation functions are given by the following recursions:

$$V^{se}[A_t, \varsigma_{-t}] = \max_{\{b_t, b_t \in C(A_t, \varsigma_{-t})\}} : v[\varsigma_t; s, e, b_t]$$

$$+ \frac{1}{1+\rho} E_{\varepsilon,\varsigma_-} \left\{ \max_{(s',e')} : \varepsilon(s', e') + V^{s'e'}[(1 + r_{t+1})(A_t - b_t), \varsigma_-] \right\}, \qquad (28)$$

It is in this fashion that the full problem, defined in (21), is transformed into an equivalent discrete-choice problem, where the continuous decision, b_t, is construed conditionally upon the discrete one. The determination of b_t is thus subsumed in the objective function of the discrete choice problem.

The value function $V[\varsigma, \varepsilon]$ may now be written in terms of the conditional valuation functions:

$$V[\varsigma_t, \varepsilon_t] = \max_{\{se \in \{\{0,1\} \times \{1,0,-1\}\}} : V^{se}[A_t, \varsigma_{-t}] + \varepsilon(s, e). \qquad (29)$$

It is a crucial consequence of this definition for the value function that leads, quite naturally, to a discrete choice problem, once assumptions about the stochastic structure of the additive utility components, $\pi_\varepsilon(\varepsilon_t|\varsigma_t, \beta_\varepsilon)$ have been made. The usefulness of the conditional valuation function readily follows [Hotz and Miller (1993); Rust (1987; 1988)]. The dynamic discrete choice problem is equivalent to a static one, given $\pi_\varepsilon(\cdot|\cdot)$, where the conditional valuation functions play the role of values of a static utility associated with discrete alternatives.

The above statement of the problem is used below to motivate the structural estimation model. It admits as special cases the problems examined by several previous researchers, including in particular Ball (1990) and Zeldes (1989a). A number of additional remarks are in order. First, if an individual in a particular period is unconstrained with respect to either liquidity or employment, anticipation of constraints' possibly binding some time in the future are reflected in current decisions through the conditional value functions $V^{se}[\cdot]$. This is true regardless of whether or not an individual is liquidity constrained. Intuitively, to the extent that constraints (5), (6) and (7) ever bind, they would affect total lifetime resources. Second, in spite of considerable research efforts during the last few years, structural estimation of a general mixed discrete continuous model like ours has run up against insurmountable, at present, computational difficulties.[18] It is for this reason that we pursue estimation of reduced form aspects of problem (21). Third, the functional form of the optimal solution for b_t as a function of state variables does depend upon whether or not the individual is constrained with respect to either liquidity or employment or both. This dependence is, in turn, transmitted to commodity demands and to labor supply, a fact that we exploit in specifying our estimation models in Section 3 below. We now turn to clarifying these statements through a graphical analysis.

[18] There are no breakthroughs in the mixed discrete-continuous decision problems comparable to Rust (1987, 1988) and to Hotz and Miller (1993). The reviews by Pakes (1994) and Rust (1994) provide excellent details on this issue.

2.4.3 A Graphical Analysis for the Steady State

We can obtain additional intuition by returning to the original notation of the problem, ignoring the stochastic structure ε, considering a steady-state version of the problem defined in (21), and simplifying the preference structure to assume a single consumption good. We further assume, as in subsection 2.2.1 above, that the consumption good is the numeraire and the only stochastic shock is in the form of a stochastic (real) wage rate, which is independently and identically distributed over time. We use the actual definition of the problem to derive a more precise economic interpretation of the solution and to refine the definition of the employment state indicator. For expositional simplicity, we only consider the case $s = 0$. Adapting (21) to this case yields

$$
\begin{aligned}
V^{0e}[A;W] &= \max_{\{e\in\{-1,0,1\}\}} \{V^{01}[A;W], V^{00}[A;W], V^{0-1}[A;W],\} \\
&= \max\Bigg\{ \max_{\{b:A>b\}}{}^{*}: v(b;W) + \frac{1}{1+\rho}E_{w'}\{\max_{s'e'} : V^{s'e'}[(1+r))(A-b);w']\} \\
&\quad \max_{\{G_{RU}:A>G_{RU}-Wh_{RU}\}} : v_R(G_{RU}-Wh_{RU};h_{RU};W) \\
&\quad +\frac{1}{1+\rho}E_{w'}\{\max_{s'e'} : V^{s'e'}[(1+r)(A-G_{RU}+Wh_{RU});w']\}, \\
&\quad \max_{\{G_{RO}:A>G_{RO}-Wh_{RO}\}} : v_R(G_{RO}-Wh_{RO};h_{RO};W) \\
&\quad + \frac{1}{1+\rho}E_{w'}\{\max_{s'e'} : V^{s'e'}[(1+r)(A-G_{RO}+Wh_{RO});w']\}\Bigg\}.
\end{aligned} \tag{30}
$$

The solution for labor supply when $E = 0$ is obtained from Roy's identity and would take the form $h = H(A, W|S)$, where under the assumption that leisure is a normal good $H(\cdot|S)$ is decreasing in A and increasing in W. For a given individual in a given period, constraints h_{RU} and h_{RO} imply a locus of points in (A, W) space, such that $H(A, W|S) = h_{RU}$, and $H(A, W|S) = h_{RO}$, respectively. These loci will, of course, depend on whether $S = 1$ or $S = 0$. In particular, we would expect that labor supply of unconstrained people would be more responsive to a given change in the real wage, so that these loci would be kinked, with the kink occurring at the threshold point.[19] We have drawn typical such loci in Figure 2.

We would expect, in general, that the employment state loci would partition the space into three regions corresponding to involuntary overemployment, voluntary employment, and involuntary underemployment or unemployment. These regions are drawn in Figure 2 under the assumption that the labor supply function is upward sloping. They may be interpreted as follows. For a given value

[19] The slopes, corresponding respectively to $S = 0$ and $S = 1$, are given by:

$$\frac{dW}{dA}|_{S=0} = -b_A \frac{hv_{bb} + v_{bW}}{v_{bW}b_W + v_{WW} + h[v_{bbb}b_W + v_{bW}]},$$

$$\frac{dW}{dA}|_{S=1} = -\frac{hv_{bb} + v_{bW}}{v_{WW} + hv_{bW}}.$$

The existence of a kink, which in effect follows from the Le Chatelier Principle, is implied by the above expressions and may be tested empirically.

of assets, there exists a threshold value of the real wage above which the individual would wish to work more but is constrained from doing so, and a lower threshold value of the real wage below which the person would wish to work less but is constrained from doing so.

Consideration of liquidity constraints, on the other hand, implies a critical value of assets that, according to (18), is decreasing in the real wage. Given assets, an individual is less likely to be constrained the larger is the real wage. By superimposing the two partitions of the (A, W) space in Figure 2, we obtain six regions, which fully characterize the discrete decisions of the problem. The estimation problem involves estimation of a stochastic process defined over such a discrete sample space.

3 Econometric Models

We shall contend in this paper with the objective of estimating the parameters of the discrete components of the decision rule $\mathcal{D}$ as a function of observable characteristics of the decision maker and his environment, while allowing for unobservable persistent heterogeneity. Even though the decision rule $\mathcal{D}$, defined in (26), is well defined as a reduced-form, it would be interesting to investigate it in a quasi-structural form setting too, where S_t is defined conditionally on E_t and, symmetrically, E_t conditionally on S_t.

Such a representation better conveys the economic intuition of the problem, though, of course, $\text{Prob}[S, E] = \text{Prob}[E]\text{Prob}[S|E] = \text{Prob}[S]\text{Prob}[E|S]$. For example, the individual is liquidity unconstrained and voluntarily employed in the region of the state space where the first of the components within the curly brackets of the max operator in the RHS of (21) dominates all others and $b_t = A_t$. He is involuntarily overemployed and liquidity unconstrained in the region of the state space where the third of the components within the curly brackets of the max operator in the RHS of (21) dominates all others and $b_{RO_t} < A_t$. Thus, some of the "structural" flavor of this mutual dependence of S_t and E_t according to (26) is obtained by means of structural form estimation of (17) and (20)–(22), a task that we take up further in Section 3.2 below.

3.1 Structural Forms vs. Reduced Forms

We now consider functional forms for the conditional valuation functions according to (28). We assume for simplicity linear functional forms for individual i at time t, given by

$$V_{it}^{se} = \Psi_{it}^{se}\beta_{se} + \epsilon_{it}^{se}, \tag{31}$$

where Ψ_{it}^{se} are vectors of independent variables, β_{se} is a corresponding vector of parameters, and ϵ_{it}^{se} are random variables that correspond to the unobserved components of utility $\varepsilon(S_t, E_t)$ defined after equation 27 in subsection 2.4 above. The terms Ψ_{it}^{se} are meant to represent different (in general nonlinear) functional forms associated with the same set of underlying independent variables.

Once we have assumed a stochastic structure for the ϵ_{it}^{se}'s, we may use (31) to estimate the model. This specification yields a six-nomial model, as becomes evident immediately below. As an example, the probability that an individual is observed voluntarily employed and unconstrained in period t is given by

$$Prob[S=0,E=0|\Psi_{it}^{se}] = Prob\left[\varepsilon_{it}^{00}-\varepsilon_{it}^{0-1}\geq \Psi_{it}^{0-1}\beta_{0-1}-\Psi_{it}^{00}\beta_{00},\right.$$
$$\varepsilon_{it}^{00}-\varepsilon_{it}^{01}\geq \Psi_{it}^{01}\beta_{01}-\Psi_{it}^{00}\beta_{00},\varepsilon_{it}^{00}-\varepsilon_{it}^{10}\geq \Psi_{it}^{10}\beta_{10}-\Psi_{it}^{00}\beta_{00},$$
$$\left.\varepsilon_{it}^{00}-\varepsilon_{it}^{11}\geq \Psi_{it}^{11}\beta_{11}-\Psi_{it}^{00}\beta_{00},\varepsilon_{it}^{00}-\varepsilon_{it}^{1-1}\geq \Psi_{it}^{1-1}\beta_{1-1}-\Psi_{it}^{00}\beta_{00}\right]. \tag{32}$$

This probability may be written in terms of the probability distribution functions of the ε_{it}^{se}'s. Since the ϵ_{it}^{se}'s are unobserved components of the state vector, it is appropriate to treat them as unobservable random shocks. Given the state of the art in dynamic discrete choice models, the most general assumption we can make is to treat them as random effects with a time- invariant component and an $AR(1)$ component. Hence, we assume the ε_{it}^{se}'s are of the form

$$\varepsilon_{it}^{se}=\eta_i^{se}+\zeta_{it}^{se}, \tag{33}$$

where the η_i^{se}'s are time-invariant individual effects, which we treat as random effects, and the ζ_{it}^{se}'s obey the AR(1) structure:

$$\zeta_{it}^{se}=\rho_{AR}^{se}\zeta_{it-1}^{se}+\xi_{it}^{se}, \tag{34}$$

where the ξ_{it}^{se}'s are random variables independently and identically distributed over time with means equal to zero, and a 6×6 variance-covariance matrix. In general, because the limited dependent variables in this model are purely discrete, to achieve identification one needs to normalize the conditional valuation functions of one of the six outcomes to zero. Hence, the parameters that can be estimated are as follows: five of the six parameter vectors β_{se} in (31), fourteen $(=5\times 6/2-1)$ elements of the contemporaneous variance-covariance matrix of the ξ_{it}^{se}'s in (34), fifteen $(=5\times 6/2)$ elements of the contemporaneous variance-covariance matrix of the η_i^{se} random effects in (33), and five of the autoregressive coefficients ρ_{AR}^{se} in (34).

To summarize, consideration of all possible liquidity and labor supply constraints leads to switching regressions, with switching occurring in two dimensions. The first is on account of liquidity constraints, the second is on account of quantity constraints on labor supply. The introduction of exogenous constraints on labor supply augments the number of the possible regimes in a given period from two, in the case of liquidity constraints alone, to six. An individual may be either liquidity unconstrained or constrained, denoted respectively by whether S_t from (17) is equal to 0 or 1. With respect to employment, an individual may be either involuntarily overemployed, voluntarily employed, or involuntarily unemployed or underemployed, according to whether E_t from (22)-(24) is equal to -1, or 0, or 1, respectively. Thus, the number of possible outcomes corresponds to the six possibilities defined by $\{\{0,1\}\times\{1,0,-1\}\}$.[20] This may be handled as a system of simultaneous discrete response models, corresponding to the discrete events $[S,E]$. In practice, of course, not all of regimes will be equally important, an issue that will be settled by the data.

[20] If the status of being out of the labor force (voluntarily unemployed) is included and underemployment is distinguished from unemployment we have ten states.

The economic interpretation of the model does suggest a more specific (and thus potentially testable) stochastic structure, namely one involving two discrete endogenous variables that jointly generate six regimes with a set of implied restrictions, namely that the employment state indicator is naturally ordered. Of those endogenous variables, the liquidity constraint indicator, S_t, introduced in (17), may be handled by means of dynamic binary-valued discrete choice model (such as a probit model). The definition of the employment state indicator E_t, on the other hand, according to (20), suggests that it be modelled as an ordered discrete-choice variable. For the pair $[S, E]$ to constitute a complete description of the discrete outcomes associated with (21), E_t must be defined conditionally upon S_t.

The first model, the estimation of which is presented in Section 4.2 below, assumes that a binary regime indicator for S_t is perfectly observable[21] for every household in every period. The second model, the estimation of which is presented in Section 4.3, is a dynamic ordered probit model that assumes a perfectly observed labor constraint indicator E_t is available. This model describes qualitative aspects of the conditions of employment, (i.e., whether the household head is involuntarily overemployed, voluntarily employed, or involuntarily underemployed or unemployed), and introduces the feature that consumption behavior may be affected by an individual's being constrained in the labor market. The third model, discussed in Section 4.4, is a simultaneous system that combines the above dynamic probit and ordered probit models.

It is interesting to highlight the fact that the ordered probit model may be nested in the classical sense into the general unrestricted six-nomial model introduced above. It is simpler to show this if we concentrate on the labor employment indicator E_t and dropping the time subscript. We then have that :

[21] Our assumption that the binary regime indicator for liquidity constraints S_t and the ordered employment indicator E_t are perfectly observable, while serving well to illustrate our basic approach, is problematic in its most general setting. A particular threshold amount of financial assets $\tilde{A}_{i_t}$, which depends on individual characteristics as well as market variables but is not observed directly, was identified by our approach as determining switching of regimes for S_t. It is holding assets A_{it} exceeding the threshold level that signifies that the household is not subject to a borrowing constraint in a particular period t. Of course, this would not be a problem if respondents were asked specifically about whether they felt they had been constrained, which is in fact the case with the 1983 wave of the Survey of Consumer Finances. This event is not observed, however, in our data.

Consider generalizing our econometric model to allow for an imperfect indicator, J_{it}, specifying whether or not liquidity constraints are binding, based on $I_{ai_{t+1}}$, the observed value of asset income for household i at the beginning of period $t+1$. Since typically assets vary in their liquidity characteristics, which are unobservable, the procedure we (and many others before us) have used to impute asset stocks is at best imperfect. It is, therefore, important to account for implied imperfections in the regime indicators and thus allow for misclassification. [*c.f.*, Lee and Porter (1984); Hajivassiliou (1989a).]

One approach would be to allow for random *coding errors* in the equations defining the regime indicators. This model, in contrast to the Lee and Porter (1984) formulation, allows the probability of misclassification to vary endogenously and to be determined by economic fundamentals. Such coding errors, however, do not affect the consistency up to scale of the discrete estimation procedures we adopt here, which assume perfect regime indicators.

An alternative approach would be to model directly the stochastic relation between the imperfect (J_t) and perfect (S_t) indicators, through a distribution function $F(S_t|J_t)$. In this paper we proceed to assume that the regime classification information is either perfect (i.e., $S_t = J_t$) or that possible imperfections in it do not affect the consistency up to scale of the estimators for the discrete models we consider here. (For example, the coding error model for the regime classification information considered in Hajivassiliou (1989a) exhibits such a feature.) We take up a detailed analysis of the possible misclassification issue elsewhere (Hajivassiliou and Ioannides (1992b)).

$$Prob[E=0] = Prob[\varepsilon_0 - \varepsilon_{-1} \geq \Psi_{-1}\beta_{-1} - \Psi_0\beta_0,\ \ \varepsilon_0 - \varepsilon_1 \geq \Psi_1\beta_1 - \Psi_0\beta_0].$$

By defining $\varepsilon_1' \equiv \varepsilon_1 - \varepsilon_0$ and $\varepsilon_{-1}' \equiv \varepsilon_{-1} - \varepsilon_0$, the above probability may be written in terms of the bivariate distribution function: $Prob[E = 0] = Prob[\varepsilon_{-1}' \leq \Psi_{-1,1}\beta_{-1,1},\ \ \varepsilon_1' \leq \Psi_{1,1}\beta_{1,1}]$. It now follows that the expression may be rewritten equivalently in terms of an ordered probit model in terms of a single underlying random variable, ε_{-1}', if and only if $\varepsilon_{-1}' \equiv -\varepsilon_1'$ (which implies $\varepsilon_1 \equiv \varepsilon_{-1}$), and provided that, in addition, the following conditions are satisfied: first, the variable components of $\Psi_{-1,1}\beta_{-1,1}$ and $\Psi_{1,1}\beta_{1,1}$ have coefficients which are opposite to one another (i.e., their variable components sum up to 0); and second, their intercepts differ. These testable restrictions are investigated in subsection 4.5 below.

In the subsections that follow we discuss a number of econometric issues that arise in estimating our simultaneous system with both types of restrictions, liquidity constraints and quantity restrictions on labor supply.

3.2 Estimation Models

The discrete response system (17) and (22)–(24) is modelled by a generalized selection model consisting of a simultaneous binary probit and ordered probit model. Since households must adapt their behavior to the presence of constraints on asset holdings and on labor supply, the path of the regime indicators $[S_t, E_t]$ is endogenous. Neither Zeldes (1989a) nor Ritter (1987), both of whom work with food consumption only, deal with switching. Similarly, Altonji (1986) and Ball (1990), who work with food consumption and labor supply data, and Ham (1986), who uses only labor supply data, do not deal with switching.[22] Given specific assumptions about the distribution of the unobservables, this endogeneity can be analyzed by simulated maximum likelihood estimation methods. In this paper we make a descriptive first cut and proceed with estimating separate reduced forms for S_t and E_t, which we use in turn to estimate a particular set of structural forms.[23] We note, nonetheless, that our reduced-form approach is, as we discussed in section 2.4 above, firmly rooted in the theory of Markovian decision problems.

It is instructive to highlight the interaction between the liquidity and labor supply constraint indicators, S_t and E_t, by considering structural forms for the pair of two endogenous variables $[S_t, E_t]$ as a system. Consider first structural forms for $[S_t, E_t]$ symmetrically defined with dummy endogenous variables as follows:

$$S_{it} = \mathrm{BP}(\delta E_{it} + X_{it}'\beta + \varepsilon_{it}^{bp}); \tag{35}$$

[22]Zeldes assumes that regimes are perfectly observable and uses only data for the unconstrained group in the estimations. If, as expected, regimes are endogenously determined, his procedure will give unreliable inferences. Ritter assumes that regimes do not change and are completely unobservable. Altonji excludes constrained individuals. Ball's approach differs from Zeldes' only in his using jointly food consumption and labor supply data. Ham's use of dummy endogenous variables to account for the impact of constraints is less general than ours.

[23]We leave a full set of structural models for future work.

$$E_{it} = \text{OP}(\kappa S_{it} + Z_{it}'\gamma + \varepsilon_{it}^{op}); \quad (36)$$

where BP and OP denote binary probit and ordered probit respectively. These are, of course, the simplest possible cases one could consider for the system (17) and (22)–(24), where the impact of the other endogenous variable is in shifting the intercept.[24]

In our setting, coherency conditions [Schmidt (1981)] reduce to conditions that the model be recursive, that is the coefficients δ and κ in (35–36) satisfy $\delta \cdot \kappa = 0$. Structural forms (35)–(36) imply the reduced forms:

$$S_{it} = \text{BP}(\mathcal{X}_{it}'\beta^{bp} + \mathcal{Z}_{it}'\gamma^{bp} + \vartheta_{it}^{bp}); \quad (37)$$

$$E_{it} = \text{OP}(\mathcal{X}_{it}'\beta^{op} + \mathcal{Z}_{it}'\gamma^{op} + \vartheta_{it}^{op}), \quad (38)$$

where $\mathcal{X}$ and $\mathcal{Z}$ consist, respectively, of X and Z as in (35)–(36) and, in addition, higher order terms which express the non-linearities implied in the reduced form equations. The stochastic structure $(\vartheta_{it}^{bp}, \vartheta_{it}^{op})$ in (37)–(38) is what we estimate in Section 4 below. Note that the correlation between the errors ϑ_{it}^{bp} and ϑ_{it}^{op} in (37) and (38) is of particular interest here, because the presence of unemployment may accentuate the propensity of an individual to be liquidity constrained.

Another set of structural forms can be obtained when instead of having E and S in the RHS of (35) and (36) we have the latent (threshold) functions $S^* \equiv \tilde{A} - A$, and $E^* \equiv W - W_R$, respectively, where W_R denotes our counterpart of the virtual wage defined earlier. These new structural forms are:

$$S_{it} = \text{BP}(\phi E_{it}^* + X_{it}'\beta + u_{it}^{bp}); \quad (39)$$

$$E_{it} = \text{OP}(\psi S_{it}^* + Z_{it}'\gamma + u_{it}^{op}). \quad (40)$$

Structural forms (39) and (40) imply reduced forms, in terms of X and Z, which are identical to (37) and (38), except that just linear terms and not higher-order ones appear on the RHS. That is:

$$S_{it} = \text{BP}(X_{it}'\beta^{bp} + Z_{it}'\gamma^{bp} + v_{it}^{bp}); \quad (41)$$

$$E_{it} = \text{OP}(Z_{it}'\gamma^{op} + X_{it}'\beta^{op} + v_{it}^{op}). \quad (42)$$

Thus we end up with a pair of reduced forms, (37–38) and (41–42), where the second is statistically nested into the first. Our estimations, reported in section 4 below, are along the lines of the reduced-form model (41)–(42), which follows from the structural-form model (39)–(40) above. The

[24] More general specifications are possible to consider and estimate. For example, we may allow for a different functional form for E conditional on S as follows:

$$E_{it} = S_{it} \cdot \text{OP}(Z_{1it}'\gamma_1 + \varepsilon_{it}^{op_1}) + (1 - S_{it}) \cdot \text{OP}(Z_{0it}'\gamma_0 + \varepsilon_{it}^{op_0}); \quad (36')$$

two reduced-form models may be compared and formally tested in terms of fit using the χ^2 cell diagnostic tests of Andrews (1988).

3.3 The Likelihood Function of the Joint Discrete Response Model

We carry out estimations with the discrete events S_t and E_t as the only dependent variables. Let us now introduce concise vector notation for the T periods of data on a household:

$$\boldsymbol{X} = (X_1', ..., X_t', ..., X_T')';$$

$$\boldsymbol{Z} = (Z_1', ..., Z_t', ..., Z_T')';$$

$$\boldsymbol{S} = (S_1, ..., S_t, ..., S_T)'.$$

$$\boldsymbol{E} = (E_1, ..., E_t, ..., E_T)'.$$

The T-vectors **S**, and **E** denote, respectively, the path of the liquidity constraint indicator and of the employment regime indicator; X and Z denote a $T \times K$ matrices of exogenous variables (prices, characteristics, etc.).

Since regime switching is endogenous, we need to compute the probability that any particular path of $[\mathbf{S}, \mathbf{E}]$ would be observed. In view of the intertemporal correlation we allow in our model, this probability is a high-dimensional integral, necessitating the use of recently developed simulation estimation methods. [See Hajivassiliou and McFadden (1992), Börsch-Supan and Hajivassiliou (1993), and Hajivassiliou (1993), *inter alia.*] These methods offer considerable advantages over the method of sequential marginal moments of Avery *et al.* (1983) in terms of asymptotic efficiency and of the possibility of imposing specific structures of serial correlation patterns.

4 Empirical Results

We present here our results from estimating econometric models of liquidity constraints and qualitative aspects of labor supply behavior as joint decisions. These models are structured according to (35–36) and (41–42). The remainder of the section is organized as follows. First, we discuss the data. Then we present estimation results for a dynamic probit equation according to (35) and (41), and for a dynamic ordered probit equation according to (36) and (42).

4.1 Data

Our panel data comes from the first twenty waves of the Panel Study of Income Dynamics, corresponding to years 1968-1987. In processing the data, we followed Zeldes (1989) and Ball (1990) as closely as possible and applied selection criteria similar to theirs.[25] Zeldes and Ball stopped with Wave 14, which includes data from the 1981 wave of interviews and was the latest wave available at the time their research was completed. We include data up to Wave 20 (which reports on the

[25]We benefitted from their kind advice, too.

1987 wave of interviews). We too excluded from our extract the non-random subsample of the PSID, known as the Office of Economic Opportunity sample. Wherever variables are constructed, such as the end-of-period stock of financial assets which we detail below, we followed exactly the calculations performed by Zeldes and Ball.

We heeded the advice of the PSID data team at the University of Michigan and worked with the family-individual tapes. Our data are organized according to the following principle. From all panel members interviewed in 1987 we selected those who were heads of households at the time of the interview, or had been household heads at least once prior to 1987. We then follow them back up until 1970 and select "household spells", i.e., periods that lasted for at least four consecutive years and during which these individuals were household heads.[26] The restriction that a household spell be at least four years long was dictated by our desire to study higher than first-order dynamics in our switching regressions models. Finally, because of unavailability of crucial data, we go back only as far as 1970, thus deleting two years of panel data. We end up with 2410 household spells (thus defined) with male heads, and with a mean length of household spell being equal to 13.45 years. The distribution of spell lengths is fairly uniform, with about one-fifth of the sample comprised of spells of length equal at least to 20.[27]

The PSID contains data on housing wealth, but not on non-housing net worth. We follow bold assumptions made by several others [Zeldes (1989); Ball (1990); Feinstein and McFadden (1988)] and by one of us in previous work [Ioannides (1988)] and circumvent the lack of direct data on assets by calculating nonhousing wealth by using the flow of asset income and an assumed rate of return on wealth. Specifically, the first $250 of interest and dividend income is assumed to be held in savings accounts at commercial banks earning the passbook rate, and all additional such income is assumed to be saved in 3-month Treasury bills or equivalent. These rates are used to "scale up" interest and dividend asset income to provide an approximation for the amount of wealth held in savings accounts. Because of the obvious difficulties in adopting this procedure for any asset income other than interest and dividends, observations with substantial "other asset income" are excluded. Real non-housing wealth is equal to the nominal amount deflated by the personal consumption expenditure deflator. Housing equity is equal to house value minus outstanding mortgage principal, both of which are reported in the PSID. More details may be found in Zeldes (1989), p.341.[28]

We have explored two samples, all heads and male heads. The sample of all heads contains 46,031 observations on 3,206 separate household spells. The sample of male heads contains 32,408

[26]This design is in accordance with Zeldes' definition, even though his model did not require that he keep track of the panel structure of the data on households, after first differencing the relevant variables. This is an important difference between our data and the data as used by Zeldes and Ball. Our need for the full panel structure of the data causes us to end up with a smaller data set because of missing values. It is also a reason why their data (and, in particular, Zeldes' data, to which he kindly gave us access) do not suffice for the full set of econometric experiments we are interested in.

[27]The frequency distribution is as follows:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	≥ 20
10	13	23	30	130	131	93	132	121	116	103	121	138	124	124	118	125	127	103	528

The existence of observations for spells of less than 4 years is due to missing values.

[28]The potential of this method is enhanced by the availability of data on holdings of certain key assets in the 1984 wave of the PSID. Thus estimates for asset stocks obtained in this fashion can be anchored on actual data.

observations on 2,410 separate household spells. We have chosen in the end to work with the sample of male heads because it is more homogeneous. We report summary statistics for key variables in Table 1 below. In addition to the usual ones, the statistics include interquartile range (IQ), a more robust measure of dispersion. Tables 2–7 report additional aspects of the data, which we refer to in further detail below. Even within such a homogeneous sample (that inevitably involves smaller variability than the sample of all heads in a number of key dimensions), all key dynamic aspects of the data that pertain to regime switching display a fair amount of hitherto unexplored richness. The regression results, reported in Tables 8–9 below, were obtained with the sample of male heads.[29]

4.2 A Dynamic Binary Probit Model for Liquidity Constraints

An overview of the pattern of transitions and the underlying dynamics of regime switching observed in the data may be obtained by looking at cross-tabulations for the transitions from being constrained to unconstrained and vice versa, given in Table 3. A household is classified as liquidity-constrained in a particular time period if the ratio of total wealth (the sum of housing wealth and calculated nonhousing wealth) to the average of disposable income in the current and previous years is less than 2/12.[30]

Under this definition, in the sample of male heads approximately 74.4% of the observations are associated with unconstrained households and the remainder are constrained. As reported in Table 3, of the households with male heads approximately 53.3% remain unconstrained in two successive periods, 21.1% move from constrained to unconstrained, and 16.9% move from unconstrained to constrained.

Table 2 shows that about 84.6% of household observations in the sample of male heads exhibit a

[29]The above number of 32,408 observations on 2,410 household spells with male heads, used in the estimations that are reported in Tables 8 and 9, includes observations with missing values, where continuous variables were filled in by individual means and discrete ones by most likely individual values.

[30]This is the only one of the "splits" used by Zeldes to indicate presence of binding liquidity constraints [*ibid.*, pp.338-344] that utilizes information available during the entire panel. Another one, Zeldes' "most stringent" split, is defined as follows. A household is classified as constrained if at least one of the following is true: (a) a household says it has no current savings (a question that is available for only waves 1–5, 8 and 13) of the PSID; (b) its ratio of nonhousing wealth to income is equal to zero; (c) a household does not have at least two months' worth of average income in the form of liquid wealth (a question that is available for only a limited number of waves). A household is unconstrained if only both of the following are true: (a) a household reports they have current savings (if that question is asked in that wave), and (b) the ratio of nonhousing wealth to income is greater than or equal to .5. All "middle" observations are excluded. If the ratio of nonhousing wealth to income is missing, the observation is excluded from both groups.. A third one involves comparison of two months' worth of income to liquid wealth. Yet the key difference among the various alternative measures for S_t centers, we believe, on the use of those direct questions asked of respondents only in a limited number of waves of the PSID, which are not available for a substantial fraction of our sample. This is particularly serious for us, because we are interested in the full panel structure of the data. Missing values for certain variables throw out a greater fraction of observations in our case than in Zeldes' case.

In any case, our extensive experiments with the data suggest that even though the proportions of constrained and unconstrained households as measured by the various indicators may differ, the pattern of switching in and out of being constrained is quite similar across the two splits other than the most stringent one. We have obtained similar results for the dynamic probit model for liquidity constraints with Zeldes' most stringent split. See Hajivassiliou and Ioannides (1990).

switch to a different liquidity constraint regime at least once during the period of observation, and 13.5% switch at least 10 times. Furthermore, more than 98% of the sample changes employment at least once, and 33.7% (or 34.9%, if the more detailed definition is employed) exhibit 10 or more such transitions. These numbers justify our argument that the dynamics of regime switching need to be investigated properly when working with long panel data sets.

In view of the discussion above, we proceed with (35) and (41), the structural and reduced forms for S_t. The reduced form includes as RHS variables the vectors of independent variables for S_t and E_t, X_{it} and Z_{it}, respectively:

$$\begin{aligned} S_{it} = 1 \quad \text{if} \quad X_{it}'\beta^{bp} + Z_{it}'\gamma^{bp} + \eta_i^{bp} + \zeta_{it}^{bp} \geq 0; \\ S_{it} = 0 \quad \text{if} \quad X_{it}'\beta^{bp} + Z_{it}'\gamma^{bp} + \eta_i^{bp} + \zeta_{it}^{bp} < 0, \end{aligned} \tag{43}$$

where bp is a mnemonic for binary probit, η_i^{bp} is a time-invariant characteristic of household i assumed to be normally distributed over the sample and ζ_{it}^{bp} is an AR(1) random process, with autocorrelation coefficient ρ_{AR}^{bp} and i.i.d. error ξ_{it}^{bp},

$$\zeta_{it}^{bp} = \rho_{AR}^{bp}\zeta_{it-1}^{bp} + \xi_{it}^{bp}.$$

We assume that the individual effects η_i^{bp} and ζ_{it}^{bp} are uncorrelated with the explanatory variables X_{it}. We also assume that the errors have a variance-covariance structure conditional on all explanatory variables, including lagged dependent variables. This is often done, *cf.* Heckman (1981), to express coexistence of state dependence and heterogeneity. Under this assumption we need not instrument for the lagged dependent variables, because we employ the correct full information maximum likelihood procedure, which is made possible by our simulation estimator. Estimation results with this equation are reported in Table 8. The dependent variable S_t is measured by a dummy variable identical to Zeldes' "total wealth split" of the data into constrained ($S = 1$) and unconstrained ($S = 0$) households.[31]

Column 1 of Table 8 reports estimation of the marginal dynamic probit model for liquidity constraints according to the specification (43). Column 2 of Table 8 reports the results for the structural form discussed in Subsection 4.4 below. A time-invariant individual effect is allowed for in that regression in the form of a random effect, and in addition, an AR(1) shock. The presence of the random effect is statistically very significant. The coefficients of most explanatory variables are also very significant and generally have the expected sign. These estimates are obtained by using the MSSL method of Börsch-Supan and Hajivassiliou (1993) with the Recursive Conditioning Simulator. The importance of the panel structure is confirmed by comparing with estimations of a homogeneous probit model with an identical set of explanatory variables restricted to have an i.i.d. error structure (i.e., $\sigma_\eta^{bp} = 0$ and $\rho_{AR}^{bp} = 0$). We have carried out such estimations but do not report them here for reasons of brevity. Such a comparison also reveals that a number of key coefficients, such as that of the real rate of interest, have the wrong sign when the panel structure

[31]As already said, 32408 observations from 2410 spells (implying, on average, 14 years of data) are used in these regressions.

is ignored. The lagged values of all endogenous variables are always very significant and imply substantial state dependence.

The results highlight the importance of the dynamic structure: the autoregressive coefficient is estimated to be 0.327 with an asymptotic t-statistic of 2.00. The variance of the random effect η_i^{bp} is also statistically significant, with a t-statistic of 3.10, and so are the estimated coefficients of the two lags of the endogenous variables included in the regression, `liqconl1` and `liqconl2`, are very significant. Their t-statistics are 60.18 and 18.04, respectively. Both those effects and, in addition, the sign of the autocorrelation coefficient suggest a high degree of persistence in the likelihood of being liquidity constrained.

The dummies for overemployment and for under- or unemployment in the previous two periods, (`dumovrl1, dumovrl2`) and (`dumunl1, dumunl2`) respectively, are not significant in this regression. The other regressors form two groups, the X variables and the Z variables. Education, food needs (a PSID variable measuring household composition, a weighted sum of the current ages of family members adjusted for total family size), age, race, religion and the real rate of interest are considered in the X group and shown as those variables, in addition to the dummies discussed above, in Column 2 of Table 8 which are included in the regressions. Several of these variables have also been used by Zeldes (1989). The Z group consists of such labor demand variables as county unemployment, local labor market conditions, unemployment rate in the household head's occupation and of labor supply variables as job tenure, access to unemployment insurance, imputed wage, number of children below the age of five, union membership and being being disabled. Marital status, race, religion and geographical dummies, are included in both groups. A cubic structure for age is very significant, implying a highly nonlinear negative effect of age upon the probability of being constrained. A higher real rate of interest is associated with a higher probability of being constrained. A household head's being black has a positive and very significant effect on that probability, and being married and highly educated have very significant negative effects. All these results accord with intuition.

4.3 A Dynamic Ordered Probit Model for Labor Constraints

A rich pattern of dynamics characterizes transitions over time over different states of qualitative aspects of employment. As Table 4 indicates, about 63.2% of households with male heads are voluntarily employed in a given period and more than half of them (38.06% overall) remain voluntarily employed in the subsequent period. An additional 4.9% percent are classified as overemployed, and the remainder are underemployed (17.0%), unemployed (2.2%) and out of the labor force (12.7%). Furthermore, Table 2 reveals that only 2% of households in the male sample remain in the same employment regime over the entire period of observations, whereas 33.7% (or 34.9, for the finer definition involving five cells) exhibit at least 10 transitions.[32]

[32] The 4-cell and 5-cell divisions with respect to the employment state are defined as follows: *4 cells* $\equiv$ {Overemployed, Voluntarily employed, Underemployed or unemployed, Out of the labor force}. {5 cells} $\equiv$ {Overemployed, Voluntarily employed, Underemployed, Unemployed, Out of the labor force}.

Cross-tabulations between labor supply status and liquidity constraint regime, reported in Table 6 suggest substantial correlation between the respective indicators. According to Table 6, only 24.3% of voluntarily employed and 23.6% of overemployed individuals are liquidity constrained, whereas 41.9% of underemployed and 58.2% of unemployed individuals are so constrained.

4.3.1 Construction of Labor Constraint Indicators

What follows is a description of the labor constraint indicator LABCON_t we used to construct the employment indicator, E_t, we introduced in (20) above and used in the econometric models reported in Table 9.[33] Further details on the actual questions asked of survey respondents are presented in the Appendix in the form of flow charts. The classifications are:

$\text{LABCON}_t = -1$: *overemployment.* (4.9%)

This is the case if, in year $t-1$, the person was an employed member of the labor force who answered yes to the question "Now thinking about your job(s) over the past year, was there more work available on your job [or "any of your jobs" if more than one] so that you could have worked more if you wanted to?",[34] and answered no to the question "Could you have worked less if you had wanted to?"[35]

$\text{LABCON}_t = 0$: *voluntary employment.* (63.2%)

This is the case if, in year $t-1$, the person was an employed member of the labor force who was classified as neither overemployed, according to the above definition, nor underemployed, as defined below.

$\text{LABCON}_t = 1$: *underemployment/unemployment.* (17.0%)

This is the case if in year $t-1$ the person was an an employed member of the labor force who who answered no to the question whether more work was available, and answers yes to the question "Would you have liked to work more if you could have found more work?"[36] or who was temporarily laid off, on a maternity or sick leave, or unemployed and looking for work. The latter possibilities were ascertained on the basis of the question "Are you working now, looking for work, retired, keeping house, a student or what?"[37] An important attribute of this variable is that it pertains to the employment status of the respondent as of the actual time of the interview.

[33] In view of our data, the endogenous variable E_t is inherently ordinal. The actual numbers used to code the employment indicator E are not, of course, of any consequence.

[34] This is variable V14230 in wave 20 of the PSID.

[35] This question was asked of those who answered yes to the previous question, and was coded as variable V14232 in wave 20 of the PSID. It was also asked of those who answered no, and was coded as v14235 in that same wave. Unfortunately, the latter variable does not appear to be available for years prior to 1979.

[36] This is variable V14234 in wave 20 of the PSID. The questions which lead to variables V14230, V14232, and V14234 became more precise over the years but retained their basic meaning. Our definition is consistent with Ham's [Ham (1982); Ham (1986)]. As he says, there is some ambiguity in how individuals may respond to these questions; e.g., there is no indication in the data as to whether or not a worker would require a premium to work overtime. Nevertheless, we think the phenomenon of involuntary overemployment is real enough and makes sufficiently good sense as an element of labor contracts to warrant attention within our framework.

[37] For years 1975 and earlier, the coding of the variable used to determine employment status, that is whether a person is employed, unemployed or out of the labor force for a variety of reasons is coarser unfortunately, so that it includes temporarily laid off workers among the employed.

$\text{LABCON}_t = 2$: *unemployment.* (2.2%)

This is the case, if in year $t-1$ the person was an unemployed member of the labor force.

$\text{LABCON}_t = 99$: *out of the labor force.* (12.7%)

This is the case, if in year $t-1$ the person was not a member of the labor force.

The coding of the labor constraint indicator was chosen to reflect the inherent ordering implied by the theory. Our econometric techniques allow us to test, quite naturally, whether such an ordering is supported by the data.

Whenever inconsistent answers to the above questions are reported, we proceed in the following way. If an individual reports that he/she is neither voluntarily employed, nor underemployed nor unemployed, then we classify the respondent as involuntarily unemployed if the person was out of the labor force last year, and as voluntarily employed, if the respondent was a member of the labor force. If, on the other hand, a person reports belonging to more than one of the above categories, and was out of the labor force in that same year, then such a person is recoded as involuntarily unemployed. Alternatively, if he/she was in the labor force and classified as involuntarily unemployed, then he/she was recoded as not voluntarily employed and not involuntarily overemployed; finally, if he/she was classified as involuntarily overemployed, then he/she was recoded as not voluntarily employed and not involuntarily unemployed. Such a set of variables have never before been used in their full generality to analyze employment status and, in particular, the possibly involuntary nature of reported unemployment or underemployment.

4.3.2 Overview of Past Work with Quantity Constraints

We highlight the significance of our approach by discussing previous works that have utilized such qualitative information. Zeldes (1989) makes no use of employment constraints information and restricts his attention to whether a household is liquidity constrained. Ball (1990) uses data from the PSID for 1968-1981 and classifies a worker as constrained in a given year if he either experiences a spell of unemployment or cannot work as many hours as he wants. The latter criterion is met, according to Ball, if the person answers no to the question "Was there more work available on your job [or "any of your jobs" if more than one] so that you could have worked more if you had wanted to?" and the person answers yes to the question "Would you have liked to work more if you could have found more work?" However, a person is considered constrained in all years if he was classified as constrained *in any year.* Thus, the sample split employed by Ball is time-invariant and results in 9290, or 70%, of his 13,265 annual observations being classified as constrained. We, on the other hand, exploit the substantial time variation associated with these qualitative employment status categories. This is one of the reasons for which this paper may be considered as a generalization of Ball's. The other is that we allow for the possibility of overemployment constraints.

Biddle (1988) uses data from the PSID for years 1976-1980. He uses a scheme similar to Ball's to classify workers who are constrained in their labor supply behavior. Workers are classified as constrained, if they are against either an upper bound on hours of work, (i.e., if they answer no to the question of whether more work was available and yes to the question of whether they would like to work more), or if they are against a lower bound, (i.e., if they answer no to the question of whether they could work less). Biddle works with a full sample of 1249 observations on first-differences, of which only 205, or 16%, are classified as unconstrained.

Ham (1982) explores the qualitative aspects of labor supply in much detail with a single cross-section of 835 workers from the PSID for 1971. He considers workers as unconstrained if they are neither underemployed nor unemployed and distinguishes three categories of constrained workers: unemployed but not underemployed, underemployed but not unemployed, and underemployed and unemployed. Ham ignores the possibly constraining effect of overemployment, by arguing that it is numerically relatively unimportant. He defines a worker as underemployed if a worker is constrained in terms of hours of work per week. That is, if the worker answers no to the question of whether more work was available and yes to the question of whether he wanted to work more then such a worker is classified as underemployed. A worker is classified as unemployed if the worker is constrained in terms of weeks per year. It is thus possible for a worker to be both underemployed and unemployed. We, however, do not wish to draw such a distinction, especially in view of the fact that the unit of time for hours of work is a year. Ham uses univariate and bivariate probit models for underemployment and unemployment as distinct selection rules to correct for sample selection bias affecting labor supply behavior. He finds support for the notion that unemployment and underemployment reflect constraints on behavior.

In examining the data, we have also replicated Ham's criteria and confirmed the consistency of his and our selection. The difference of his from the above selection by means of the labor constraint indicator E_t is that his selection is not *ordered* and does not distinguish overemployment. With respect to the latter, Ham is right in saying that it is numerically not very important.

The present paper with its emphasis on possibly time-varying discrete events in panel data is more closely related to Ham (1986), who uses PSID data for 473 individuals from 1971-1979. Ham's experiments with dummy variables for underemployment and unemployment, defining them identically to Ham (1982) as time-varying right hand side variables, is an improvement over Ball's notion of time-invariant constraints. Ham (1986) uses dummy variables to express the states of being underemployed or unemployed are recognized as endogenous variables and are instrumented by means of a set of exogenous variables chosen to proxy the labor market conditions facing a worker. However, those events are inherently discrete, and Ham's econometric procedures do not handle them as such. The present paper in effect makes up for these shortcomings.

4.3.3 Estimation

Table 9 reports estimation results for an ordered probit model for an employment indicator E_t as the dependent variable according to equations (36) and (42) for the sample of male heads. This variable corresponds to the definition (20) for members of the labor force only.[38] We present first,

[38] E_{it} is related to the labor constraint indicator LABCON $_{it}$, introduced in subsection 4.3.1, as follows:

$$E_{it} = \begin{cases} LABCON_{it} & \text{if } LABCON_{it} = \{-1, 0, \text{ or } 1\} \\ 1 & \text{if } LABCON_{it} = 2. \end{cases} \tag{44}$$

in column 1, estimation results for the reduced form of a marginal ordered probit model. Then we present, in column 2, the results for a structural form of the same model, but discuss those further below in Subsection 4.4.

We use data for members of the labor force only and do not distinguish econometrically the cases of underemployment and unemployment.[39] These suffice to provide an ordering of outcomes according to the theory presented above. Under the assumption of a random effects plus AR(1) structure for the unboservables, the ordered probit model with panel data is given by

$$\begin{array}{lll} E_{it}=-1 & Z_{it}\gamma^{op}+X'_{it}\beta^{op}+\eta_i^{op}+\zeta_{it}^{op}<\theta^-, & \text{overemployment} \\ E_{it}=0 & \theta^-\leq Z_{it}\gamma^{op}+X'_{it}\beta^{op}+\eta_i^{op}+\zeta_{it}^{op}\leq\theta^+, & \text{voluntary employment} \\ E_{it}=1 & \theta^+<Z_{it}\gamma^{op}+X'_{it}\beta^{op}+\eta_i^{op}+\zeta_{it}^{op}, & \text{under/unemployment.} \end{array} \tag{45}$$

where op is a mnemonic for ordered probit, η_i^{op} is a time-invariant characteristic of household i that is assumed to be normally distributed over the sample and ζ_{it}^{op} is an AR(1) random process, with autocorrelation coefficient ρ_{AR}^{op} and i.i.d. error ξ_{it}^{op},

$$\zeta_{it}^{op}=\rho_{AR}^{op}\zeta_{it-1}^{op}+\xi_{it}^{op}.$$

We assume that the individual effects η_i^{op} and ζ_{it}^{op} are uncorrelated with the explanatory variables. The ordered probit model estimates an intercept, denoted by **one** in Table 9, a vector of unknown coefficients, the upper threshold, denoted above by θ^+, and the variance, σ_ξ^{op}, of the i.i.d. component, ζ_{it}^{op}, of the ordered probit equation (45). A lower threshold, θ^-, is normalized at 0.

Column 1 of Table 9 reports results according to a reduced form equation (42), as specified in (45) above. These regressions also use 32,408 observations for 2,410 spells with male household heads. The panel structure is very significant, with the variance of the random effect η_i^{OP} being numerically large and having a t-statistic of 3.82. The autocorrelation coefficient for the AR(1) structure is 0.218 but its t-statistic of 1.89 is not significant. Two lagged values of the dummy variable indicating that a household head is involuntarily unemployed are both very significant, with $t-$statistics of 36.86 and 19.37 respectively, and have numerically reasonably large coefficients. Thus, being involuntarily unemployed makes one more likely to be involuntarily unemployed again in the future. Dummies for being overemployed have the opposite effect and are also very significant. Only the first lag of the liquidity constraint variable is significant, suggesting that being liquidity constrained in the past increases the likelihood of being involuntarily unemployed in the present. Also very significant is the threshold θ^+ (**thetapos** in Table 9) associated with involuntary under- or unemployment relative to voluntary employment. These findings strengthen an earlier but somewhat tentative result by Clark and Summers (1982) on the importance of persistence elements in explaining cyclical behavior in labor supply. These results imply a rich dynamic structure for the labor constraint indicator.

The remaining explanatory variables included in the regression coincide with those used by Ham (1982). Of the regional dummies the one indicating residence in Western U.S. is highly

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An observation is dropped altogether, if the household head is out of the labor force ($LABCON_{it}=99$).

[39] The model can be extended to account for the labor force participation decision.

significant. We also use a wage index[40] which is an imputed hourly wage instrumented so as to purge it of its endogeneity. A cubic effect for age is significant and implies that age reduces the probability of being underemployed. Similar and even more significant is the effect of job tenure on the likelihood of being underemployed. Race and religion are significant. Having a disabling health condition, being male, a union member, and having many children all have very strong positive effects that are statistically very significant. Collecting unemployment insurance has a very large and very significant effect. Being married and being educated both have very significant and negative effects. A set of variables representing demand effects are all very significant. Higher values of the unemployment rate in the county of residence and in the occupation of the household head imply higher values for the likelihood of underemployment or unemployment. With a few exceptions, these results accord with intuition. They do imply a persistent and possibly "trapping" effect caused by past unemployment and underemployment.

4.4 Structural Form Models for Liquidity Constraints and Labor Supply Constraints

As we argued above, the potential joint dependence of being liquidity constrained upon the qualitative state of employment and of the qualitative state of employment upon being liquidity constrained follow readily from our theoretical model. The presence of unemployment contemporaneously or in the past, may accentuate, in and of its own, the propensity of a worker to be liquidity constrained. This is strongly suggested by the cross-tabulation of the liquidity constraint S_t and employment E_t indicators presented in Table 5. A distinguishing feature of our paper is its unified treatment of both liquidity constraint and labor constraint approaches to modelling household behavior.

We report in Columns 2 of Tables 8 and 9 structural forms for S_t and E_t according to (35) and (36), respectively. These estimations are conducted as follows. The own lagged values are included as predetermined variables. For the contemporaneous endogenous variables, that is E_t in (35) and S_t in (36), we put in the fitted values from the corresponding reduced form, (45) and (43), respectively. That is, the estimation reported in Column 2 of Table 8 utilizes the results in Column 1 of Table 9, and vice versa. Fitted values are determined according to the criterion of maximum probability: for the probit model a fitted value is 1, if the fitted probability is greater than .5, and 0, otherwise; for the ordered probit model it is -1, 0, or 1, depending upon a comparison of the predicted values with the estimated thresholds. The panel structure is very significant for both models, with the variance of the random effect being numerically large and very significant. The autocorrelation coefficient of the i.i.d. component of the random effect is very significant for S_t though just below significance for E_t.[41]

[40] The result for the wage may be refined by means of a reported wage variable, which we may define by following Altonji (1986) for only those household heads who report an hourly wage. Such households make up only 36.47% of the full sample.

[41] More efficient estimates may be obtained by allowing for contemporaneous correlation between the errors of the two simultaneous equations.

We see that being unemployed has a strong positive effect on the likelihood of being liquidity constrained. Being overemployed is not significant. Most of the determinants of being liquidity constrained remain significant in the structural form too, and so do the own lagged dependent variables, whose coefficients are in fact even more precisely estimated. Being black is associated with higher likelihood of being constrained, while being other nonwhite, e.g., Asian, a lower one.

Turning now to the likelihood of being underemployed or unemployed, we see that being liquidity constrained has a very significant positive effect, and so do the lagged values of these variables. Most of the determinants of the likelihood of being underemployed retain their significance. In particular, the sign of the imputed wage variable suggests a strong cyclical effect. The higher the real wage, the less likely it is that a person is involuntarily underemployed. This estimate reflects the joint effect of an increase in the real wage upon notional labor supply and upon the (exogenous) ration. Unless we have the case of a strong backward-bending labor supply, the estimate suggests a strong cyclical effect of the real wage upon the underemployment/unemployment ration. Unemployment rate in the county of residence (**cunemp**) and in the occupation of the head of household (**occunemp**), and tightness of local labor market conditions (**labmkt**[42]) are all very significant and with signs in accord with intuition. Being nonwhite is associated with higher likelihood of being underemployed or unemployed.

4.5 Diagnostics

We report at the top of Tables 8 and 9 average predicted probabilities and the percentage of observations that are correctly predicted by the model according to the criterion of maximum probability. We note that our estimation results suggest surprisingly good fits. Specifically, the percentages of correctly predicted values are 87.59% and 88.05%, for the two models, respectively. The average predicted probability of being liquidity-constrained is .2712 and .2715, for the structural-form and reduced-form models respectively. This is very close to .2724, the mean value for the sample.

The percentages of correctly predicted values of the employment indicator are 75.52% and 75.54%, for the structural-form and the reduced-form models, respectively. The average predicted probabilities of being overemployed, voluntarily employed and underemployed/unemployed are: .0572, .7218 and .2210, for the structural- form model; .0571, .7219 and .2210, for the reduced-form model. The respective mean values for the sample are .0552, .7233, and .2215. Perhaps the fit of the labor constraint model is more impressive if one considers the fact that it pertains to three outcomes.

Additional information on how well our models fit the data is provided by Figures 3, 4a and 4b, where we have plotted predicted probabilities implied by our models over time, for mean values of explanatory variables. Specifically, Figure 3 reports the time variation in the predicted probability of being liquidity constrained for the structural form and the reduced form models. We note that

[42]This categorical variable measures tightness of the local labor market for unskilled workers, with values ranging from 1, for good conditions, to 5, for bad conditions.

the predictions of the reduced form model exhibit considerably greater variation than that of the structural form model and the two are quite far apart. Also, we note that the contemporaneous effect of the labor constraint indicator upon the predicted probability of being liquidity constrained is very small. Turning now to the labor constraint model, we note from the plots in Figures 4a and 4b that being liquidity constrained in the past reduces the predicted probability of being voluntarily employed and increases the predicted probability of being underemployed or unemployed. The predicted probabilities of the structural form and reduced form models for the labor constraint do not differ much, unlike their counterparts for the liquidity constraint model. Figures 3, 4a, and 4b suggest that all predicted probabilities exhibit considerable variation over time around the mean values for the entire sample, but the plots do not suggest any obvious biases. On the contrary, the plots do agree broadly with business cycle timing during 1970-1987.

As should be evident from equations (41–42), the joint 6-regime discrete response model we estimate has the specific binary/ordered structure we described above. A powerful test of this specification, which readily follows from the theoretical model, is to estimate the model as an unrestricted, i.e., *unordered* 6-nomial probit, and test the over-identifying restrictions. Such an estimation is feasible using the simulated maximum likelihood method we employ in this paper. The model we derive from our theory is clearly nested in the classical sense in such a standard 6-nomial probit model, which makes this testing approach have good asymptotic power properties.[43]

We discussed in Section 3.1 above that the unordered 6-nomial probit model involves a staggering increase in the number of parameters to be estimated relative to the ordered bivariate model. E.g., the slope parameters of the valuation functions amount to 180, since 5 β's are estimated for each explanatory variable. In order to conduct such a test by means of state-of-the-art technology we have to restrict ourselves to a subset of the data. We estimated an unrestricted 3-nomial probit model for the labor constraint indicator and an unrestricted 6-nomial probit model for the full model and compared them with the respective restricted ones. We refrain from reporting all of our estimation results because of the number of parameters involved. We are happy to note that key aspects of the overidentifying restrictions are not rejected. In particular, we may refer to the discussion on p.18 above and note that the estimated correlation coefficient between the i.i.d. terms of the AR(1) components of the errors for the unrestricted 3-nomial model is nearly -1, exactly as predicted by the ordering theory. Similarly, the most highly significant of the components of the parameter vectors of the indicator functions are quite near the theoretical prediction that they sum up to zero. We take these results as powerful evidence in favor of our theoretical model.

5 Conclusion

We propose in this paper a theory of labor supply and consumption decisions that is firmly grounded within a utility maximizing framework. Our theory goes further than previous research, in that

[43]The correct distribution theory required for these tests is complicated by the fact that the null hypothesis involves restrictions on the boundary of the parameter space.

it allows for a role of such institutional constraints as limited access to borrowing and involuntary unemployment. The novelty lies in the fact that we let the data pick the most likely institutional environment that is compatible with the observed consequences of rational choice.

We report estimations for two dynamic probit models. The first model describes a household's propensity to be liquidity constrained. The second one is a dynamic ordered probit model for a labor constraint indicator, which describes qualitative aspects of the conditions of employment, that is whether the household head is involuntarily overemployed, voluntarily employed, or involuntarily underemployed or unemployed. These models are estimated as reduced forms as well as structural forms. We believe that the labor constraint model has not been considered before in the literature. The estimation results we discuss and report are novel. The structural forms we estimate capture important interactions and dynamic components of the underlying decisions. Our diagnostics suggest that our estimation models exhibit remarkably good fits.

In terms of its structure and empirical objectives, the paper may be considered as an integration of two separate strands of the literature, both with respect to the equilibrium/disequilibrium dichotomy, on one hand, and with respect to the interaction between labor supply and consumption decisions on the other. These strands are represented in the literature respectively by Ashenfelter (1980) and Ham (1986), and by Altonji (1986), Ball (1990), and Zeldes (1989a).

Individuals may face restrictions on the amount of work they can supply to their employers just as much as they may resent restrictions on borrowing against their future incomes. They still adapt their lifetime plans to all restrictions and in the light of the best information they have about the presence of such constraints in the future. The assumption that is usually made, namely that all fluctuations in employment status and hours worked over time is voluntary, is an undue restriction that may therefore lead to inconsistent estimation and misinterpretation of the data. These problems can be overcome when information is utilized, just as in this paper, about the voluntary/involuntary nature of changes in employment over time. From among the numerous unexplored areas of research that our approach has opened up we note the possibility of estimating and testing the extent of the dependence of the structural form for each of the endogenous variables conditionally on the regime characterizing the other.

Table 1: Descriptive Statistics
Number of Household Spells: 2410

Variable		Nobs	Mean	StdDev	Med	Mode	Min	Max	IQ
cunemp	county unempl. rate	32870	6.5878	2.862655	6	5	1	34	3
disab	hd disabled?	35860	0.127133	0.333127	0	0	0	1	0
dumolf	out of lab. force?	36963	0.123231	0.328707	0	0	0	1	0
dumovr	overemployed?	36963	0.048427	0.214669	0	0	0	1	0
dumund	underemployed?	36963	0.172362	0.377699	0	0	0	1	0
dumune	unemployed?	36963	0.021806	0.14605	0	0	0	1	0
dumvol	vol. employed?	36963	0.634175	0.481667	1	1	0	1	1
edycat	education hd	34631	4.943808	1.814456	5	4	0	8	2
female	female?	36963	0	0	0	0	0	0	0
fneed	food needs	36963	1054.895	417.5646	1016	669	337	9999	555
fs	family size	35917	3.116992	1.447204	3	2	1	14	2
gfneed	growth food needs	35913	-0.01343	0.229644	0	0	-2.70445	2.704454	0.024992
hage	hd age	34828	41.5437	15.06523	38	29	17	92	23
htenure	tenure hd (months)	32654	82.11398	96.36219	39	0	0	960	156
impwage	imputed hourly wage	34808	7.73394	9.748598	6.25	0	-9	625	6.895742
indunemp	industry unempl. %	28257	7.336966	3.094807	6.599998	5.5	2.699999	20	2.999996
labinc	total labor income	34808	16401.72	17050.39	13000	0	0	700000	15250
liveinnc	live in north-centr?	36961	0.315955	0.464901	0	0	0	1	1
liveinne	live in north-east?	36961	0.198588	0.398942	0	0	0	1	0
liveinso	live in south?	36961	0.305538	0.460641	0	0	0	1	1
msm	hd married?	34828	0.877397	0.327985	1	1	0	1	0
numch017	num.child. 0-17 yrs	34828	1.031469	1.221776	1	0	0	8	2
numch05	num.child. 0-5 yrs	27951	0.373153	0.681522	0	0	0	4	1
occunemp	occup. unempl. rate	28737	5.882471	3.640575	4.699997	3	1.4	17.09999	4.800001
pestatus	parent econ. status	35918	2.697979	1.485812	3	3	1	5	2
raceb	race of hd: black?	36954	0.052308	0.222651	0	0	0	1	0
racew	race of hd: white?	36954	0.897224	0.303671	1	1	0	1	0
rdispy	real disposable inc	35793	10588.7	8535.672	9535.164	0	-223144	530110.5	6680.785
religceo	hd cath./eastorthdx?	32846	0.135481	0.342242	0	0	0	1	0
relignon	hd no religion/DK?	36963	0.480075	0.49961	0	0	0	1	1
religpro	hd 'protestant'?	32846	0.423461	0.494115	0	0	0	1	1
rheq	real house equity	35917	13754.83	18222.91	9167.43	0	-259444	271546.3	20996.63
rri	real int rate aft.tx	33088	0.024216	0.024142	0.023172	0.002456	-0.03573	0.094695	0.036805
rtainc	real total asset inc	34767	861.1424	4227.124	7.004997	0	-4085.33	466999.5	404.4456
sage	spouse age	34820	34.65103	18.59086	33	0	0	87	23
unemphrs	hours unempld (t-1)	36963	61.95111	232.3767	0	0	0	2080	0
unempld	unempld in (t-1)?	35917	0.114653	0.318607	0	0	0	1	0
valhouse	nominal val house	36963	37675.56	47694.8	26000	0	0	900000	55000
wagerate	hourly wage rate	8500	8.506098	5.649202	7.504999	5	0	99.97998	4.809998
zdumc2	liquidity constrained? *	34563	0.272401	0.445202	1	1	0	1	1

* zdumc2 = 1 if total wealth over income in current and previous periods is less the 1/6. See Zeldes (1989), p.341 for details.

Table 2: Dynamic Transition Counts — Male Heads

Number of Transitions	ΔS_t Frequency	ΔS_t Cumulative	ΔE_t Frequency 4 cells	ΔE_t Frequency 5 cells	ΔE_t Cumulative 4 cells	ΔE_t Cumulative 5 cells
0	15.4	15.4	2.1	2.0	2.1	2.0
1	9.7	25.1	5.0	4.6	7.2	6.7
2	9.4	34.5	6.5	6.2	13.6	12.9
3	8.2	42.7	7.2	7.1	20.9	20.0
4	9.3	52.0	8.3	8.1	29.1	28.1
5	7.9	59.1	8.7	8.8	37.9	37.0
6	7.1	67.0	7.5	7.2	45.4	44.1
7	6.9	74.0	7.4	7.7	52.8	51.8
8	5.1	79.1	7.2	7.4	60.1	59.1
9	4.1	83.2	6.2	6.0	66.3	65.1
10	3.3	86.5	5.4	5.6	71.7	70.7
11	3.2	89.6	4.8	4.8	76.5	75.6
12	2.7	92.4	4.7	4.7	81.3	80.3
13	2.0	94.4	4.5	4.5	85.8	84.8
14	2.0	96.4	4.2	4.8	90.0	89.6
15	1.0	97.4	3.4	3.5	93.3	93.1
16	1.0	98.4	2.0	2.1	95.4	95.3
17	0.8	99.2	1.9	2.0	97.3	97.2
18	0.8	100.0	1.4	1.4	98.6	98.6
19	0.0	100.0	1.4	1.4	100.0	100.0

Table 3: One-Period Transitions in Liquidity Indicator S_t – Male Heads

	$S_t = 1$ liq. constrained	$S_t = 0$ not liq. constrained	Row Per Cent
$S(t-1) = 1$ liq. constrained	8.7	21.1	29.8
$S(t-1) = 0$ not liq. constrained	16.9	53.3	70.2
Column Per Cent	25.6	74.4	100.00

Table 4: One-Period Transitions in $LABCON_t$ (Five Cells) – Male Heads

	-1 over/ed	0	1 under/ed	2 unemployed	99 out-of-the-labor-force	Row Per Cent
-1 overemployed	0.23	3.08	0.74	0.07	0.66	4.78
0	3.01	38.06	9.51	1.09	7.30	58.96
1 underemployed	0.76	9.70	3.32	0.45	1.87	16.10
2 unemployed	0.11	1.33	0.49	0.11	0.34	2.38
99 out-of-the-labor-force	0.84	11.00	2.96	0.46	2.53	17.78
Column Per Cent	4.94	63.16	17.01	2.18	12.71	100.00

Table 5: One-Period Transitions in E_t (Four Cells) – Male Heads

	-1 over/ed	0	1 under/- or un/ed	99 out-of-the-labor-force	Row Per Cent
-1 overemployed	0.23	3.08	0.81	0.66	4.78
0	3.01	38.06	10.60	7.30	58.96
1 under/unemployed	0.87	11.03	4.36	2.21	18.48
99 out-of-the-labor-force	0.84	11.00	3.42	2.53	17.78
Column Per Cent	4.94	63.16	19.19	12.71	100.00

Table 6: Frequency Counts of S_t vs. $LABCON_t$ (Five Cells) – Male Heads

	-1 over/ed	0	1 under/ed	2 unemployed	99 out-of-the-labor-force	Row Per Cent
$S_t = 1$ liq. constrained	1.15	15.17	7.37	1.33	2.22	27.24
$S_t = 0$ not liq. constrained	3.73	47.34	10.20	0.95	10.54	72.76
Column Per Cent	4.86	62.51	17.57	2.28	12.76	100.00

Table 7: Frequency Counts of S_t vs. E_t (Four Cells) – Male Heads

	$E_t = -1$ over/ed	$E_t = 0$	$E_t = 1$ under/- or un/ed	$E_t = 99$ out-of-the-labor-force	Row Per Cent
$S_t = 0$ not liq. constrained	1.15	15.17	8.69	2.22	27.24
$S_t = 1$ liq. constrained	3.73	47.34	11.15	10.54	72.76
Column Per Cent	4.88	62.51	19.85	12.76	100.00

Table 8: Liquidity Constraint Equation, Male Heads, In-the-Labor-Force
Dependent Variable: zdumc2 (S)

Reduced Form llf= -9,868.866			Structural Form llf= -9,892.162		
	Correct Pred.=	28,536 (88.05%)		Correct Pred.=	28,385 (87.59%)
	Aver.Prob.LC=	0.2715		Aver.Prob.LC=	0.2712
Variable	**Estimate**	**T-statistic**	**Variable**	**Estimate**	**T-statistic**
$\sigma_{\eta^{bp}}$	0.953	3.103	$\sigma_{\eta^{bp}}$	0.823	3.216
ρ^{bp}_{AR}	0.327	2.001	ρ^{bp}_{AR}	0.374	2.106
one	3.090	-91.81	one	3.066	-82.91
cunemp	-0.1076E-01	-2.511	cunemp	—	—
disab	-0.1018E-01	-0.2693	disab	—	—
dumovr	—	—	dumovr	0.8764E-02	0.1921
dumovrl1	0.1629E-01	0.3492	dumovrl1	—	—
dumovrl2	-0.4775E-01	-1.020	dumovrl2	—	—
dumun	—	—	dumun	0.1953	8.377
dumunl1	0.4592E-01	1.812	dumunl1	—	—
dumunl2	0.2314E-01	0.9172	dumunl2	—	—
edycat	-0.1611E-01	-2.441	edycat	-0.3190E-01	-5.338
era7679	0.3191E-01	1.027	era7679	0.1058	3.519
era8083	-0.1166	-3.721	era8083	-0.1162	-3.776
fneed	-0.2414E-05	-0.7435E-01	fneed	0.6601E-05	0.2086
gfneed	-0.4542	-11.57	gfneed	-0.4518	-11.47
hagecb	-0.3870E-04	-7.321	hagecb	-0.3759E-04	-6.891
hagesq	0.5599E-02	8.434	hagesq	0.5468E-02	7.952
hage	-0.2787	-10.50	hage	-0.2793	-10.11
htenure	-0.1097E-02	-4.525	htenure	—	—
htenursq	0.1419E-05	1.715	htenursq	—	—
hunemins	0.3320E-04	1.964	hunemins	—	—
impwage	-0.7170E-02	-5.854	impwage	—	—
labmkt	0.1355E-01	1.214	labmkt	—	—
liqconl1	1.496	60.18	liqconl1	1.534	62.58
liqconl2	0.4478	18.04	liqconl2	0.4703	19.17
liveinnc	-0.8753E-01	-2.902	liveinnc	-0.7823E-01	-2.688
liveinot	0.4264	3.770	liveinot	0.5612	3.917
liveinso	0.4478E-01	1.449	liveinso	0.6624E-01	1.561
liveinve	0.4609E-01	1.349	liveinve	0.3261E-01	1.287
msm	0.5551	15.84	msm	0.5470	16.15
numch05	0.4822E-02	0.2923	numch05	—	—
occunemp	0.1406E-01	4.083	occunemp	—	—
raceb	0.3326	7.532	raceb	0.3739	8.589
raceo	-0.2760	-5.516	raceo	-0.3123	-6.423
religceo	0.1027	3.730	religceo	0.7786E-01	2.327
religjsh	0.1891	2.650	religjsh	0.1307	1.916
religpro	0.1235	4.988	religpro	0.1388	5.500
rri	4.878	7.550	rri	6.239	10.07
unionmem	-0.3105E-01	-1.179	unionmem	—	—

Table 9: Labor Constraint Equation, Male Heads, In-the-Labor-Force
Dependent Variable: LabCon3 (E)

Reduced Form			Structural Form		
llf= -20,157.19			llf= -20,157.55		
Correct Pred.=		24,480 (75.54%)	Correct Pred.=		24,474 (75.52%)
Avg.Prob.(Ovr/Vol)=		0.0571 / 0.7219	Avg.Prob.(Ovr/Vol)=		0.0572 / 0.7218
Variable	**Estimate**	**T-statistic**	**Variable**	**Estimate**	**T-statistic**
$\sigma_{\eta^{op}}$	0.428	3.819	$\sigma_{\eta^{op}}$	0.517	4.182
ρ_{AR}^{op}	0.218	1.885	ρ_{AR}^{op}	0.288	1.791
$\sigma_{\xi^{op}}$	1.117	9.916	$\sigma_{\xi^{op}}$	1.261	6.828
one	2.842	143.5	one	2.619	137.5
θ^+	2.713	32.9959128	θ^+	2.709	29.0342471
cunemp	0.7041E-02	2.300	cunemp	0.7144E-02	2.329
disab	0.6182E-01	2.344	disab	0.5804E-01	2.197
dumovrl1	-0.6723	-21.06	dumovrl1	-0.6751	-21.15
dumovrl2	-0.3241	-10.10	dumovrl2	-0.3256	-10.15
dumunl1	0.7031	36.86	dumunl1	0.7031	36.91
dumunl2	0.3659	19.37	dumunl2	0.3650	19.38
edycat	-0.2616E-01	-5.745	edycat	-0.2902E-01	-6.418
era7679	0.1772E-01	0.7694	era7679	0.1869E-01	0.8102
era8083	0.2742E-01	1.177	era8083	0.3164E-01	1.348
fneed	0.1290E-03	5.792	fneed	—	—
gfneed	-0.1066	-3.284	gfneed	—	—
hagecb	-0.1180E-04	-4.085	hagecb	-0.1049E-04	-3.662
hagesq	0.1771E-02	4.538	hagesq	0.1456E-02	3.782
hage	-0.9095E-01	-5.419	hage	-0.7077E-01	-4.299
htenure	-0.1586E-02	-7.816	htenure	-0.1466E-02	-7.229
htenursq	0.2948E-05	4.963	htenursq	0.2670E-05	4.490
hunemins	0.2633E-03	17.42	hunemins	0.2619E-03	17.34
impwage	-0.1681E-02	-1.920	impwage	-0.1582E-02	-1.792
labmkt	0.2198E-01	2.761	labmkt	0.2209E-01	2.776
liqcon	—	—	liqcon	0.1332	7.111
liqconl1	0.5783E-01	2.567	liqconl1	—	—
liqconl2	0.7958E-02	0.3632	liqconl2	—	—
liveinnc	-0.4114E-01	-1.953	liveinnc	-0.3992E-01	-1.892
liveinot	0.3776E-01	0.4347	liveinot	0.1273E-01	0.1471
liveinso	-0.8723E-02	-0.3938	liveinso	-0.1398E-01	-0.6283
liveinwe	-0.1361	-5.560	liveinwe	-0.1403	-5.719
msm	0.4536E-01	1.684	msm	-0.2476E-01	-1.014
numch05	0.2141E-01	1.671	numch05	0.2694E-01	2.107
occunemp	0.1926E-01	7.611	occunemp	0.1902E-01	7.514
raceb	0.1571	4.675	raceb	0.1537	4.576
raceo	0.3351E-01	0.9867	raceo	0.4569E-01	1.321
religceo	0.7927E-01	3.165	religceo	0.8979E-01	3.578
religjsh	0.8883E-01	1.837	religjsh	0.8859E-01	1.822
religpro	0.2620E-01	1.365	religpro	0.2761E-01	1.437
rri	2.512	5.139	rri	2.348	4.794
unionmem	0.8403E-01	4.466	unionmem	0.8562E-01	4.548

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Figure 1: Labor Constraints

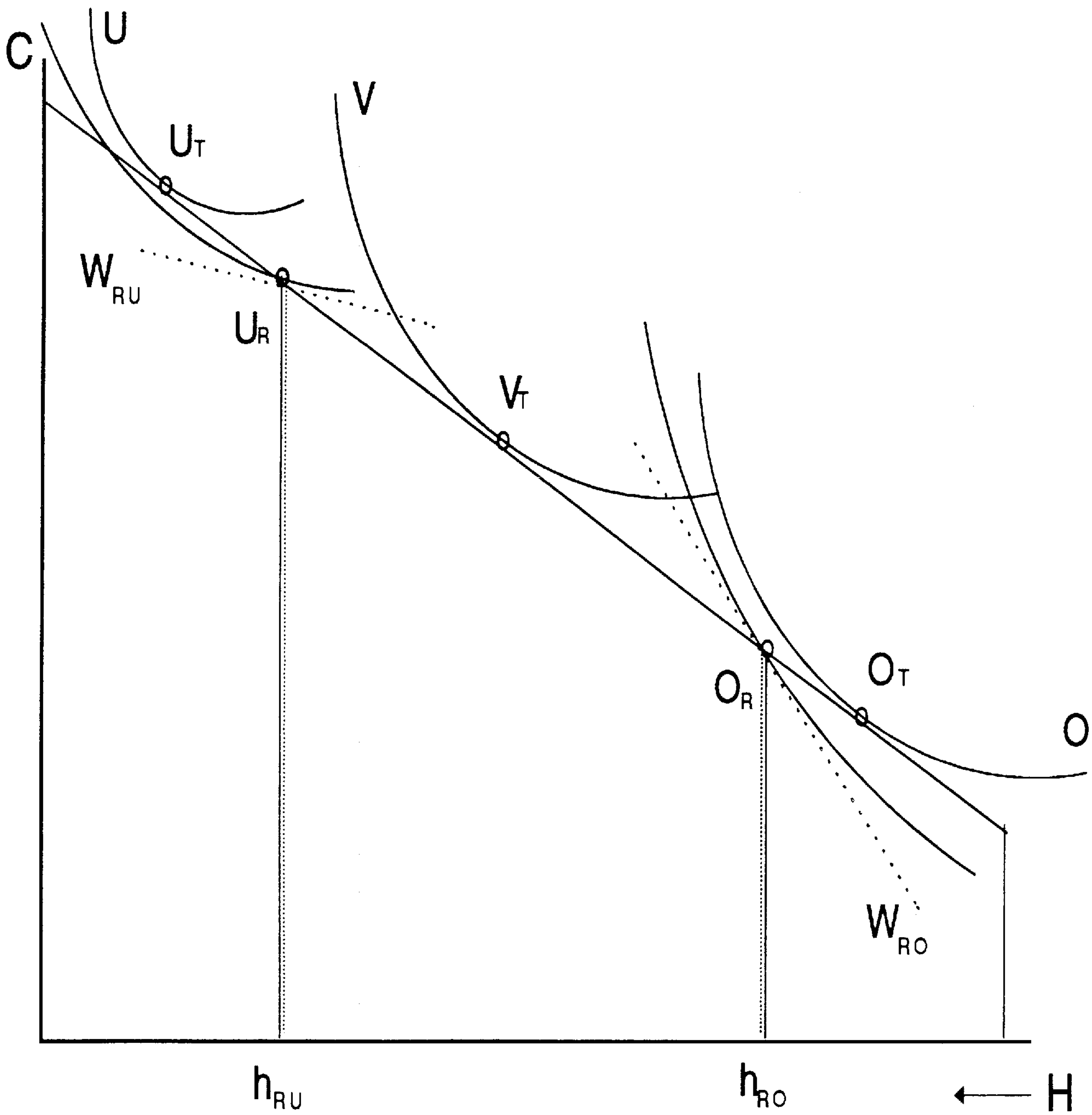


Figure 2: Regimes

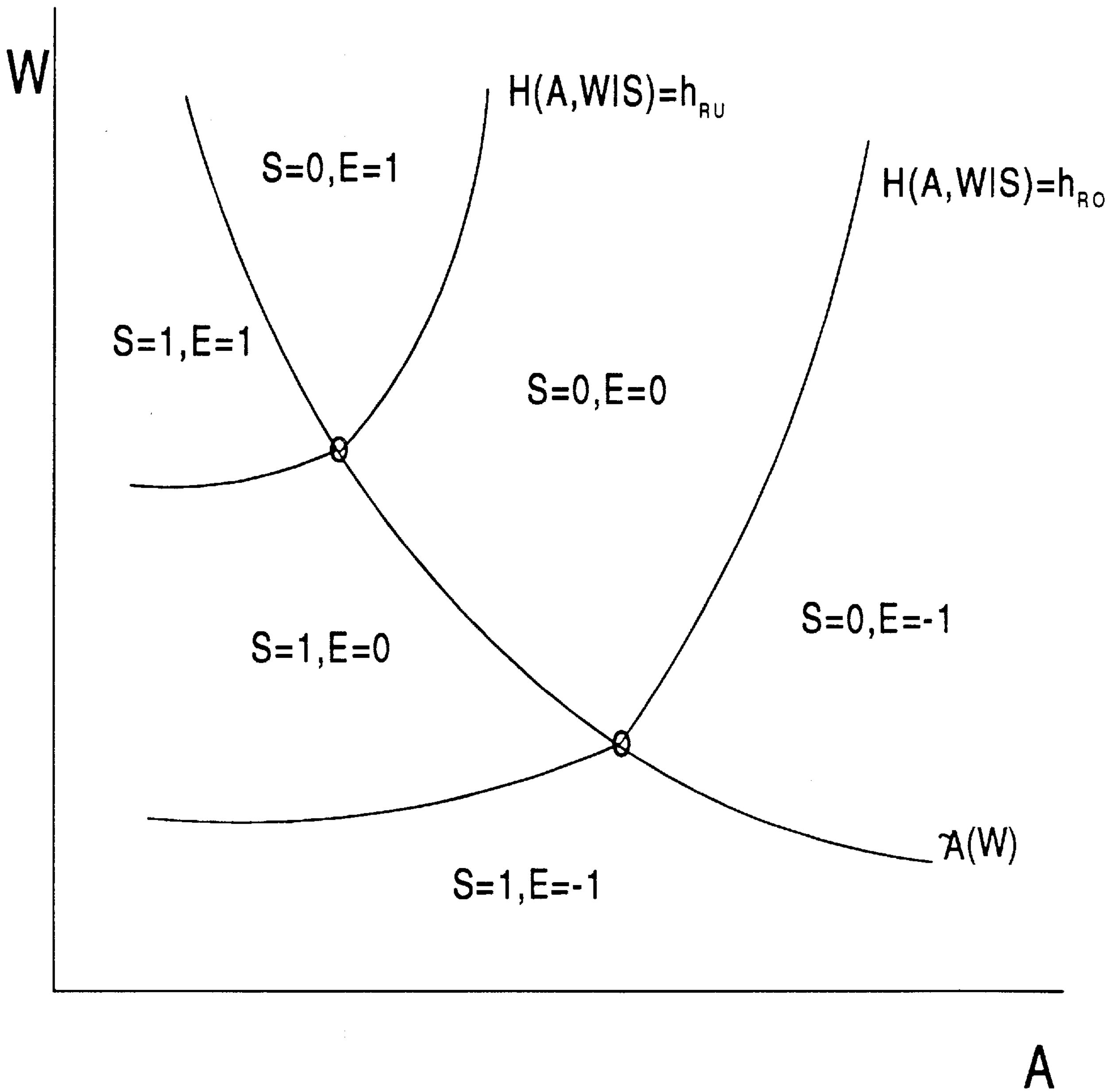
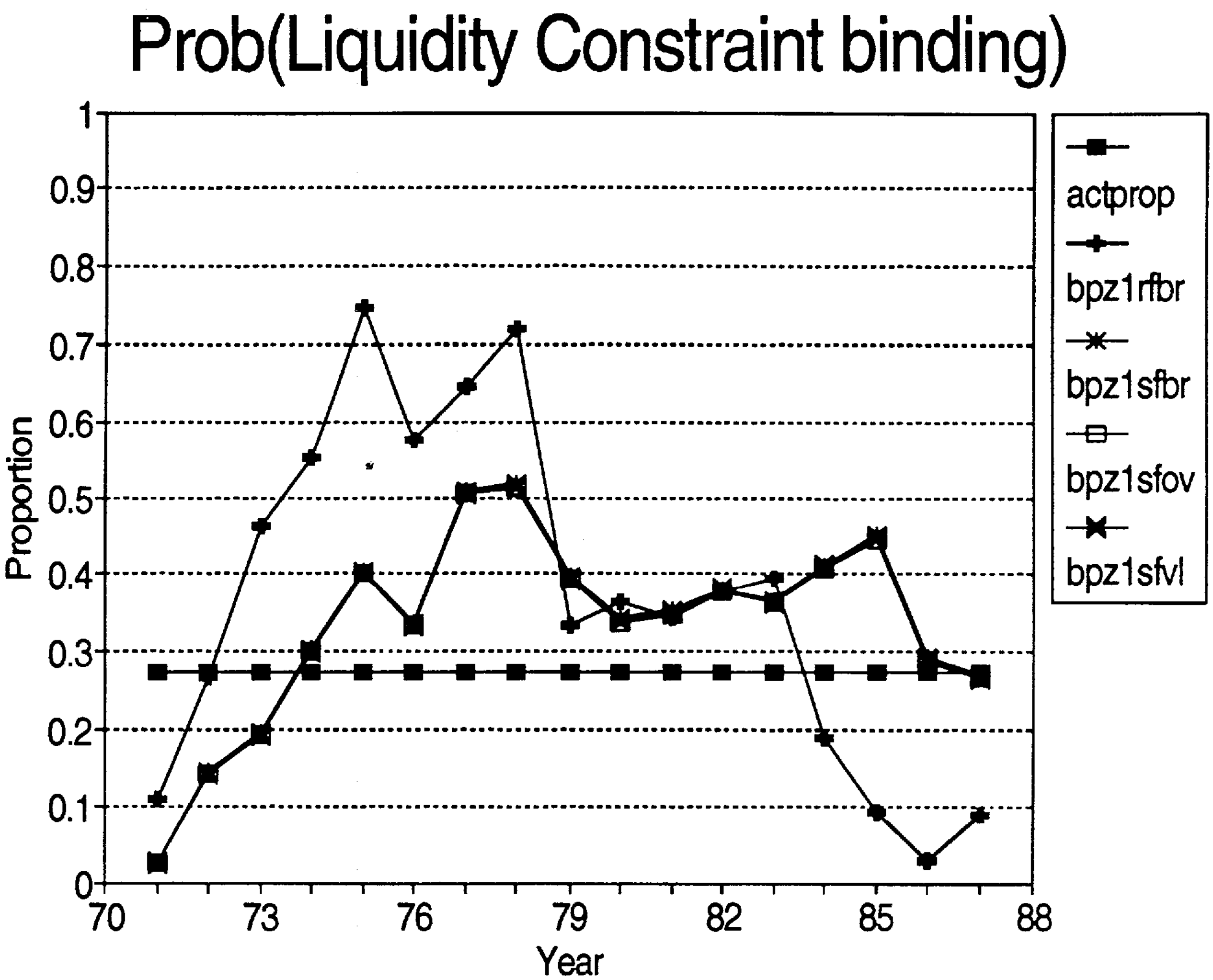


Figure 3: Binary Probit LC=1 Predictions



bpz1rfbr Reduced Form predictions, evaluated at mean values of explanatory variables

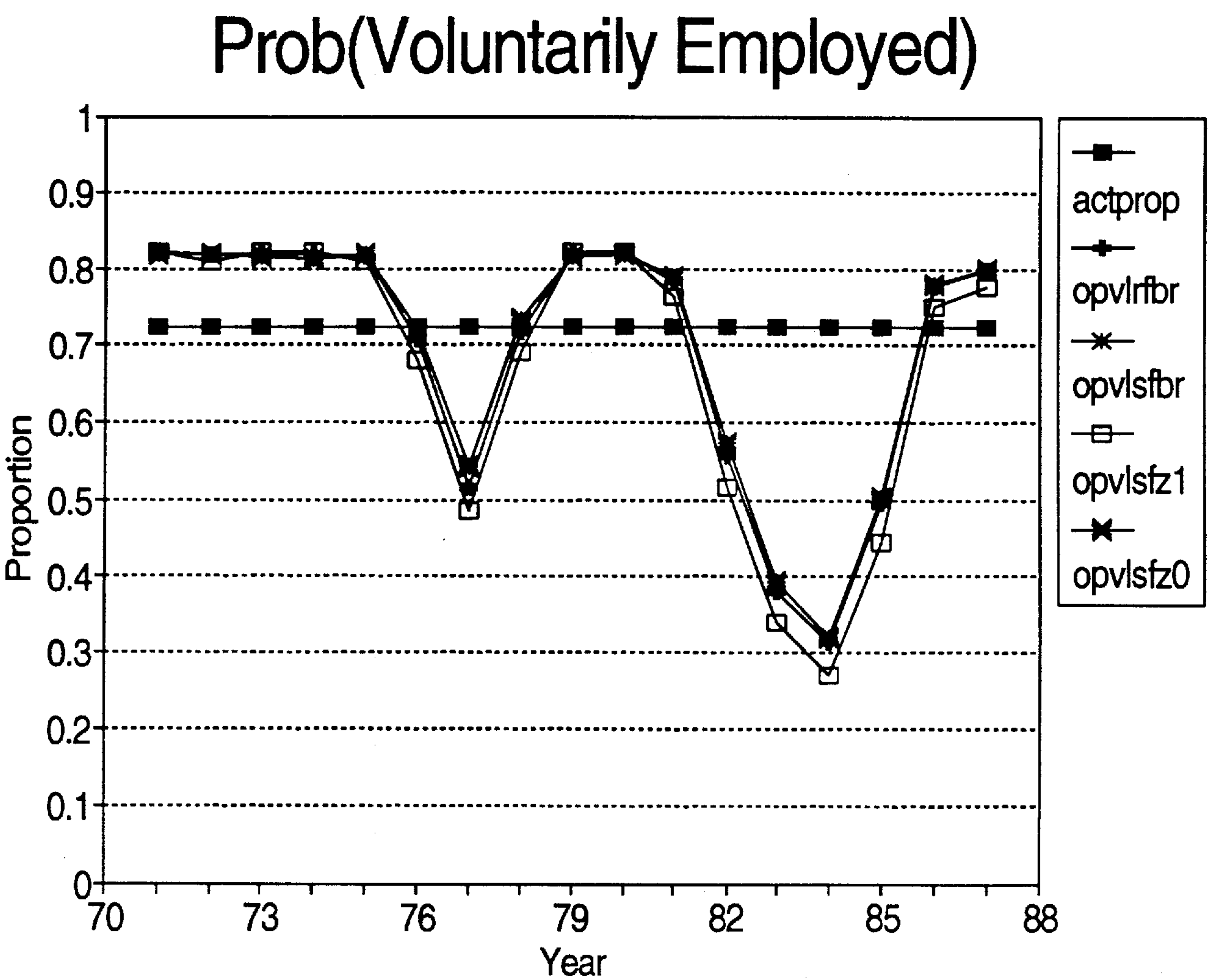
bpz1sfbr Structural Form predictions, evaluated at mean values of explanatory variables

bpz1sfov Structural Form predictions, individual assumed involuntarily overemployed in the past

bpz1sfvl Structural Form predictions, individual assumed voluntarily employed in the past

bpz1sfun Structural Form predictions, individual assumed involuntarily under- or unemployed in the past

Figure 4a: Ordered Probit VL=1 Predictions



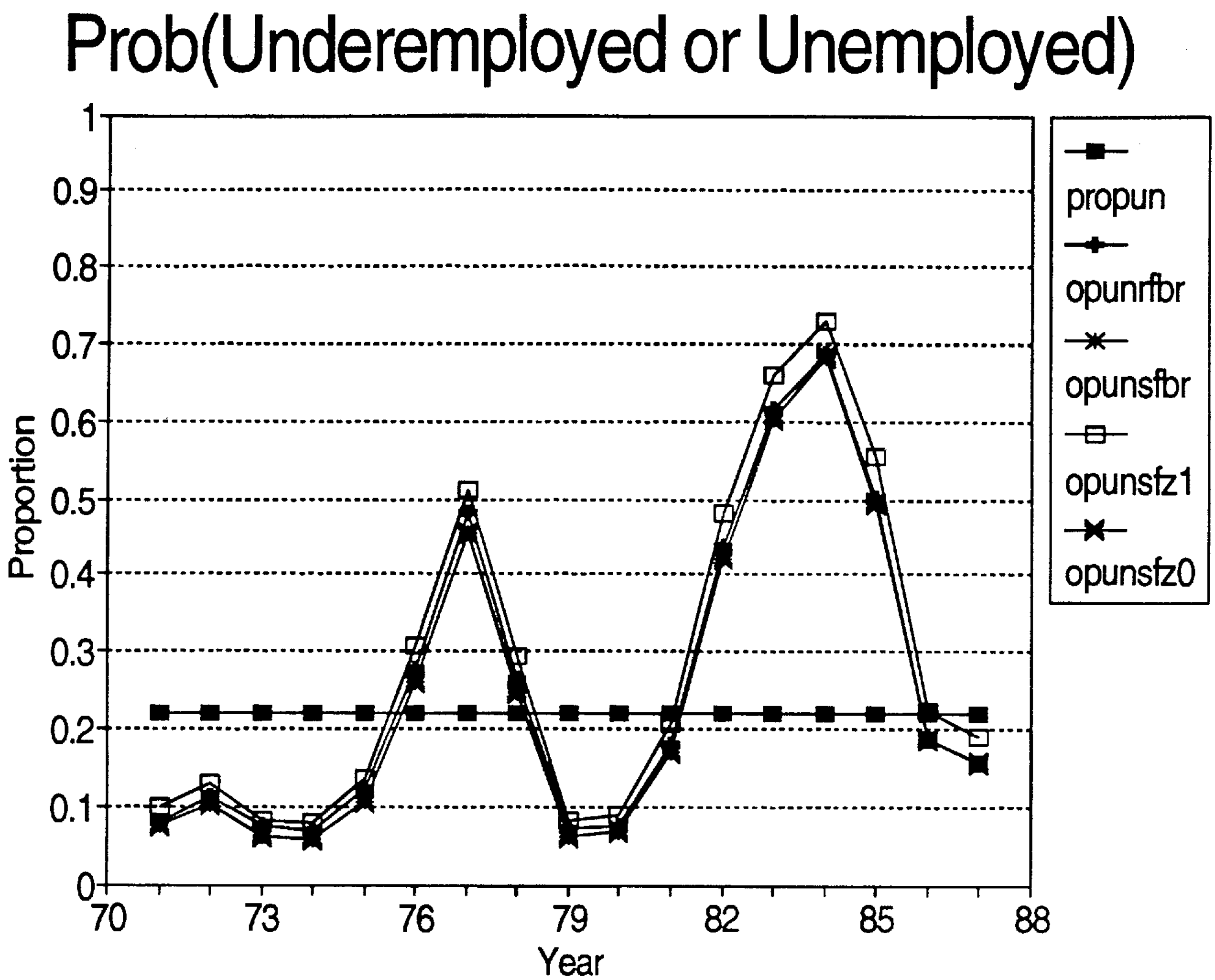
opvlrfbr Reduced Form predictions, evaluated at mean values of explanatory variables

opvlsfbr Structural Form predictions, evaluated at mean values of explanatory variables

opvlsfz1 Structural Form predictions, individual assumed liquidity constrained in the past

opvlsfz0 Structural Form predictions, individual assumed not liquidity constrained in the past

Figure 4b: Ordered Probit UN=1 Predictions



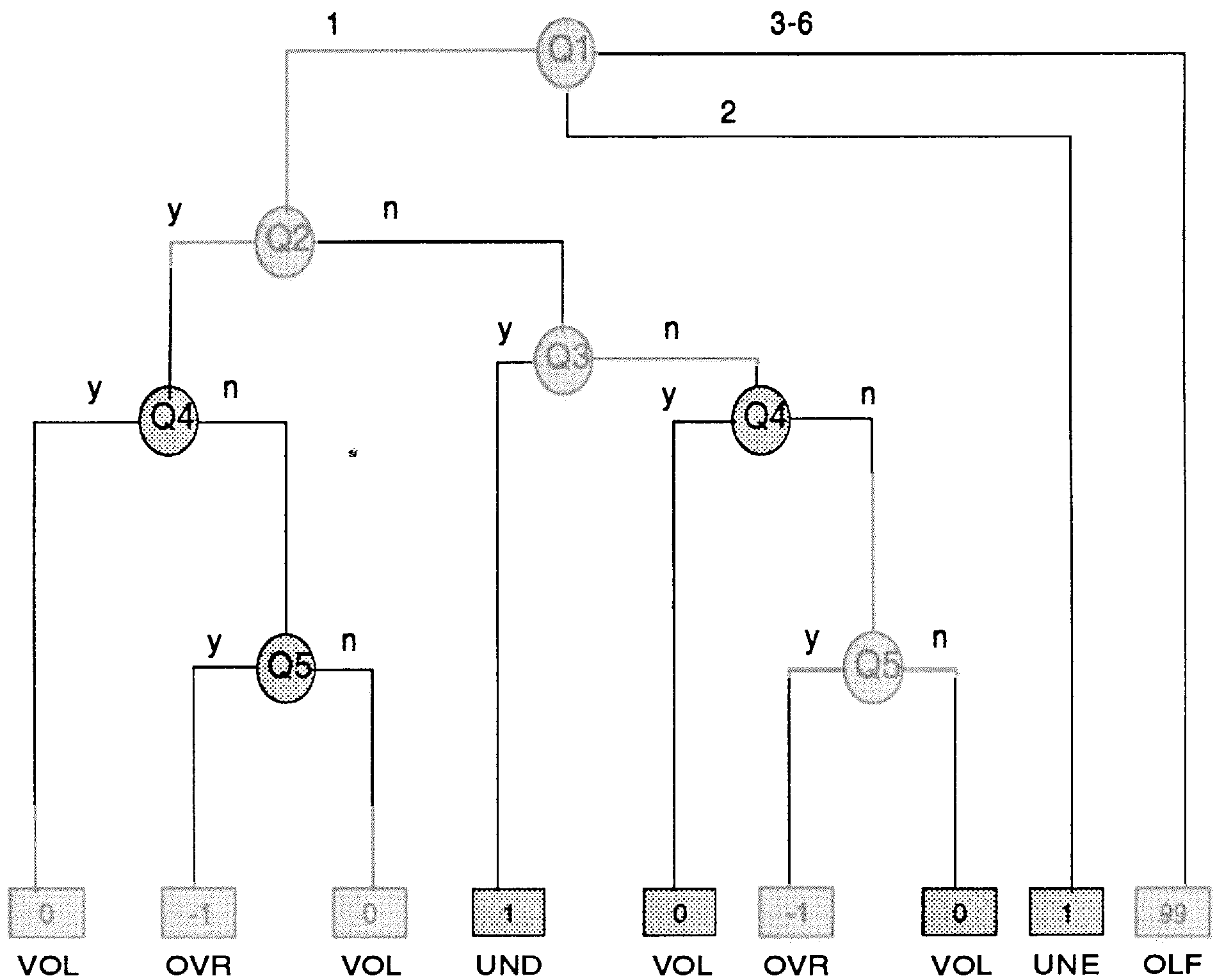
opunrfbr Reduced Form predictions, evaluated at mean values of explanatory variables

opunsfbr Structural Form predictions, evaluated at mean values of explanatory variables

opunsfz1 Structural Form predictions, individual assumed liquidity constrained in the past

opunsfz0 Structural Form predictions, individual assumed not liquidity constrained in the past

Appendix A
Construction of Labour Constraint Indicator, Years 1967-1975



Q1. What is your current employment status? (V3967 in 1975)

1. Working now or temporarily laid off
2. Looking for work, unemployed
3. Retired, permanently disabled
4. Housewife
5. Student
6. Other

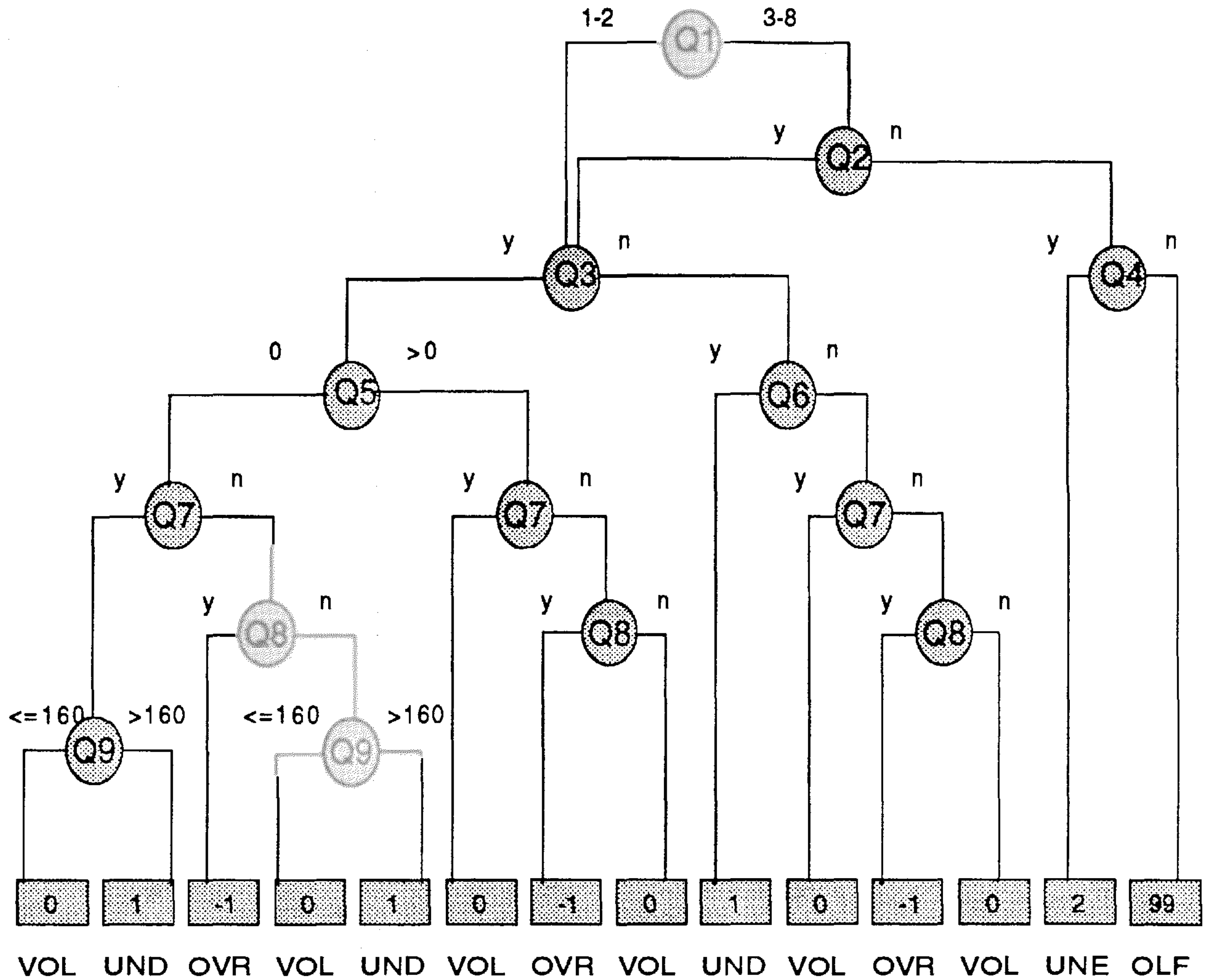
Q2. Could you have worked more hours at (any of) your job(s) this past year? (V4011 in 1975)

Q3. Would you have liked to work more if you could have found more work? (V4012 in 1975)

Q4. Could you have worked less if you had wanted to? (V4013 in 1975)

Q5. Would you have preferred to work less even if you earned less money? (V4014 in 1975)

Appendix A (continued)
Construction of Labour Constraint Indicator, Years 1976-1987



Q1. What is your current employment status? (V14146 in 1987)

1. Working now
2. Temporarily laid off, on sick or maternity leave
3. Looking for work, unemployed
4. Retired
5. Temporarily or permanently disabled
6. Keeping house
7. Student
8. Other

Q2. Are you doing any work for money now at all? (V14148 in 1987)

Q3. Could you have worked more hours at (any of) your job(s) this past year? (V14230 in 1987)

Q4. Have you done anything in the last four weeks to find a job? (V14237 in 1987)

Q5. How much would you have earned per hour? (V14231 in 1987)

Q6. Would you have liked to work more if you could have found more work? (V14234 in 1987)

Q7. Could you have worked less if you had wanted to? (V14232/V14235 in 1987)

Q8. Would you have preferred to work less even if you earned less money? (V14233/V14236 in 1987)

Q9. How many hours of work (if any) did you miss because you were unemployed and looking for work or temporarily laid off? (V13752 in 1987)

Note: Q2 was not included in the 1976 PSID questionnaire. Q4 was included in the 1976 questionnaire and was asked of all individuals responding to Q1 with answers in categories 3-8.