1-1-1958

The Measurement of Capacity

Lawrence R. Klein

M. David

COWLES FOUNDATION DISCUSSION PAPER NO. 49

The Measurement of Capacity

By

L. R. Klein

with the assistance of M. David

January 17, 1958

The Measurement of Capacity

By

L. R. Klein with the assistance of M. David

The concept of capacity - plant capacity, industrial capacity or national economic capacity - frequently turns up in economic discussion. It is often referred to as though people generally know its meaning, yet it is an illusive and ill-defined concept. The history of our subject contains sporadic bursts of interest in the matter of capacity, but the present state of treatment of this concept is in need of further development. The present essay attempts to stimulate the debate once again and to set out some new aspects of statistical treatment.

Why measure capacity? Three reasons are proposed for interest in the topic of this essay.

1. A numerical measure of capacity would be a useful addition to our basic collection of statistical series. It may tell us something about underutilization of resources, if used together with a measure of current output, in addition to that told by statistics of unemployment, overtime hours, length of the workweek, etc.
2. A measure of capacity, if obtainable, may be a proxy for a measure of the stock of capital, a concept that provides at least as many difficulties as does capacity. In particular, a capacity measure may enable us to get round the problem of technical progress that bedevils capital measurement.
3. Improved versions of the acceleration theory of investment build on the concept of capacity. Statistical tests of these theories could be made if capacity series were available.

If we think of a production function connecting, let us say for simplicity, capital (k) and labor (n) input to output (x)

$$x = f(n, k),$$

we might define

$$x_c = f(n_c, k_c)$$

as the full-capacity relation, in which all capital facilities are used with the normally available labor force to produce output. In this situation there is no idle capital; hence k_c may be measured as the stock of capital as well as capital input.* In an elementary machine process, it would simply be the number of machines. The capacity measure we seek is, however, x_c. It is in units of output. From the production function we see immediately that capacity output is an index of factor inputs. It is a pure index of capital only if there is a side relation between labor and capital. In the trivial case of linear functions with fixed factor proportions, the results are very simple (and neat).

$$x_c = a\, n_c$$

$$x_c = b\, k_c$$

$$n_c = \frac{b}{a}\, k_c$$

In this case capacity is truly a proxy for the stock of capital. Without even knowing the factor of proportionality, we can say that capacity grows, in percentage terms, as capital grows.

With reference to our first reason for measuring capacity, the simple case would add little to our existing set of statistics. Underutilization of labor, which is already measured extensively, would tell us everything that would be included in an index of the percent utilization of capacity.

* Under conditions of uncertainty some excess capacity may be held in reserve to meet unforeseen fluctuations in demand for output. A standard precautionary reserve could be amalgamated with the measure of full capacity stock of capital.

Recognition of previous work on capacity measurement. The Brookings inquiry into America's Capacity to Produce was perhaps the most ambitious and comprehensive attempt to measure capacity. It was a statistical study concluded in the years before World War II, providing capacity measurements for all the major sectors of the economy. It was basically a pragmatic and empirical study with only secondary contributions of lasting value to the aspects of techniques, method, and theory. It was a highly controversial piece of work. In general, it relied on trade sources for capacity measurement, where these were available, and supplemented them with empirical estimates for different industries based, more or less, on ad hoc judgments by the research investigators.

Input-output studies have made use of capacity estimates. Capital-output ratios are essential ingredients of a dynamic input-output model. They enter as accelerator coefficients, but are better established from the ratio of capital to capacity output than from the ratio of capital to actual output. This is to meet the usual criticism of the acceleration principle in conditions of underutilization of resources.* There is no systematic

* See Anne P. Carter, "Capital Coefficients as Economic Parameters: The Problem of Instability" pp. 287-310. R. T. Bowman and A. Phillips, "Conceptual and Statistical Problems in Estimating Capital Coefficients for Four Metal Fabricating Industries" pp. 347-74. Problems of Capital Formation, Studies in Income and Wealth, Vol. 19. (Princeton: Princeton Univ. Press) 1957.

discussion published on principles of deriving the capacity estimates nor their actual sources.

At the beginning of the war there was a study of capacity carried out for agencies of the government.**

** R. A. Solo, Industrial Capacity in the United States, Office for Emergency Management and Office for Price Administration and Civilian Supply, June, 1941.

For many years, trade associations and other industry sources have published capacity and percent utilization series. The steel ingot figure is familiar to most readers. Similar figures are published for petroleum refining, flour milling, electric power production and other basic products. The sources are brought together and summary statistics derived for a study of the acceleration principle by Hickman.* Individual industry figures

---

* B. G. Hickman, "Capacity Utilization, and the Acceleration Principle" Problems of Capital Formation, Studies in Income and Wealth, Vol. 19 (Princeton: Princeton Univ. Press) 1957. pp. 419-50.

---

have been used by a variety of authors for investigations into the acceleration principle.

The Federal Reserve Board have gone a step further, and gathered together most of the published series on capacity in major industrial materials for combination into a single index with value-added weights.*

---

* Federal Reserve Bulletin. Vol. 43, May, 1957. pp. 509-10

With the addition of electric power capacity and some more obscure series in other lines, the composition of this index could be broadened, but would not, under present conditions, cover more than 10 - 15 percent of industrial output.

The Federal Reserve series is for recent years only and shows a steady rise in capacity since 1950. An exception is the figure for textiles, which shows a decline in capacity since 1954. This is consistent with the published series on the number of active spindles.

Another approach to comprehensive measures of capacity is taken by the Department of Economics of the McGraw-Hill Publishing Co. In their periodic surveys of business investment planning and related matters they include questions on current estimates of capacity, actual output as a fraction of capacity, and desired output as a fraction of capacity. These are all "subjective" estimates supplied on the interviewing form in the respondent's own terms of reference. McGraw-Hill do not attempt to define capacity for the informant. They insist on letting the answers come as they will to the barest wording of the question. In industries with well established and well defined capacity series, they probably get a repetition of the standardized figures. The other replies are unknown as to concept and quality. The McGraw-Hill data are kept up-to-date annually since 1950 - their base year - and cover all the major classifications of the F.R.B. industrial production index. A composite figure for the whole of industry is obtained by forming an average with the same weights as those used in making the production index. Like the Federal Reserve series on capacity for major materials, the McGraw-Hill index shows a gradual trend growth since 1950. The textile component does not move downward as does the corresponding series in the Federal Reserve capacity index.

A useful survey of the entire field, with the exception of the McGraw-Hill material is to be found in a mimeographed report by Zabel and in a summary article based on his larger study. He provides a good bibliography on the whole subject.*

---

* E. Zabel, Concepts and Measurement of Productive Capacity, (mimeographed), Economics Research Project, Princeton Univ. Nov. 1955. "On the Meaning and Use of a Capacity Concept", Naval Research Logistics Quarterly, Vol. 2, Dec. 1955. pp. 237-49.

Proposed methods of measurement. Two types of approach to the measurement problem are suggested. One is pragmatic and one is theoretical. At a pragmatic or empirical level, little or no attention is paid to economic theory. One might accept output levels that engineers claim to be the maximum attainable in a technical sense. Presumably much of the existing trade data on capacity is of this sort. The trouble with this concept is that it pays no attention to cost. With an expensive third shift or other extraordinary outlays output can usually be expanded. "Crash" programs are extreme examples. In this sense, an industry, firm, or small sector viewed in isolation can have a very elastic capacity limit. But capacities for individual sectors prepared in this way may very well not blend properly. We might say that, for a given bill of final demand, they would not satisfy the system of equations defined by an input-output model. Bottlenecks would occur. Even if the engineering technicians for each industry were told to limit their capacity estimates for one or two shifts with normal work complements, usual allowances for maintenance and repair, vacations, and other seasonal regularities it is not evident that full capacity operations in each industry would be mutually compatible. It is tempting and probably useful to construct a series by averaging, with weights, the published technical estimates for different industries as the Federal Reserve Board already does on a limited scale, but this solution has its limitations in addition to the fact that a major share of industry is not included. The same observation applies, of course, to the averaging of the McGraw-Hill series.

Another pragmatic approach is statistical instead of technical, in the engineering sense. A smooth trend curve through peaks of production series is a possible measure. To some extent the Brookings study relied on crude statistical measures of this sort. In industries where we feel that the published capacity data are of fair quality there is a definite suggestion that the trend of capacity runs through, or slightly above, time series peaks. The charts on the steel industry illustrate this point.

In the first place, one must select peaks. Various minor peaks that do not come up to major peaks should not be included. The elimination of these involves matters of judgment. To plot a trend between selected major peaks is also a problem. Linear interpolation is one solution, and interpolation proportional to the time path of investment is another. The accompanying set of charts displays the time pattern of seasonally adjusted major components of the Federal Reserve Index of Industrial Production since 1947. The major peaks are, in most cases, quite prominent. The seasonal adjustment eliminates the possibility of temporary high values of production that cannot be normally maintained. On the same charts, we have interpolated linearly between selected quarterly peaks. Since the data are monthly, while we seek a quarterly series on capacity, at the peak there can be a slight monthly excess over capacity. We have also plotted the interpolation based on investment data. These are quarterly statistics of gross investment by industry groupings. For the same industry and quarterly classifications, we do not have estimates of depreciation; therefore gross instead of net investment data are used. Moreover, the depreciation data are one of the main sources of deficiency in estimates of the stock of capital, a measure that we are trying to avoid by use of our capacity data. On the whole, the statistical estimates look reasonable, but are probably conservative estimates of capacity. It is easier to deal with growing rather than declining periods. In the post-war years most industries grow on balance, but textiles present a different picture and provide the most troublesome estimating problem.

Figure I

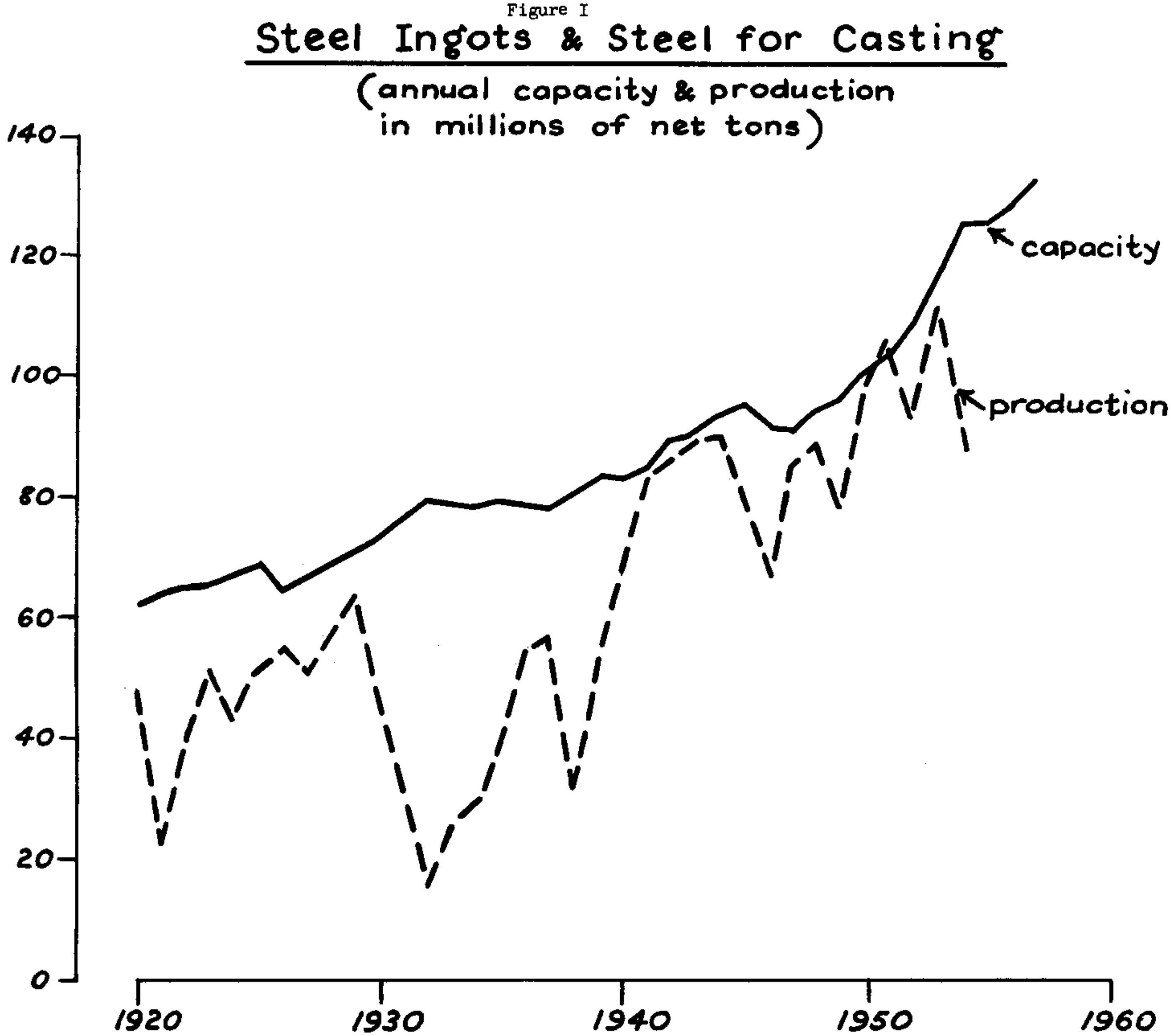

Steel Ingots & Steel for Casting
(annual capacity & production
in millions of net tons)
140
120
100
80
60
40
20
0
1920
1930
1940
1950
1960
capacity
production

Figure II

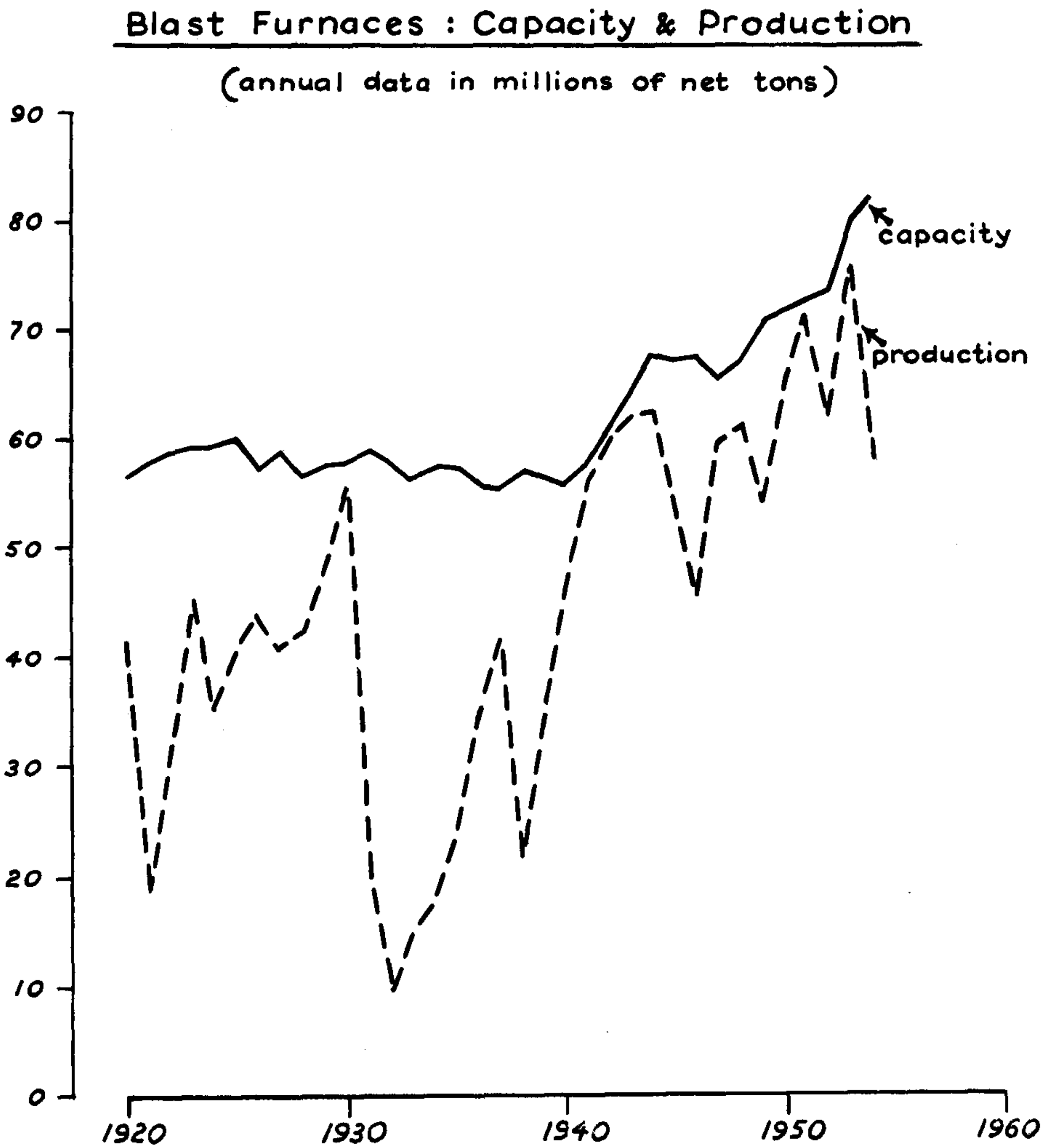

Blast Furnaces : Capacity & Production
(annual data in millions of net tons)
90
80
70
60
50
40
30
20
10
0
1920
1930
1940
1950
1960
capacity
production

Figure III

Food & Beverages

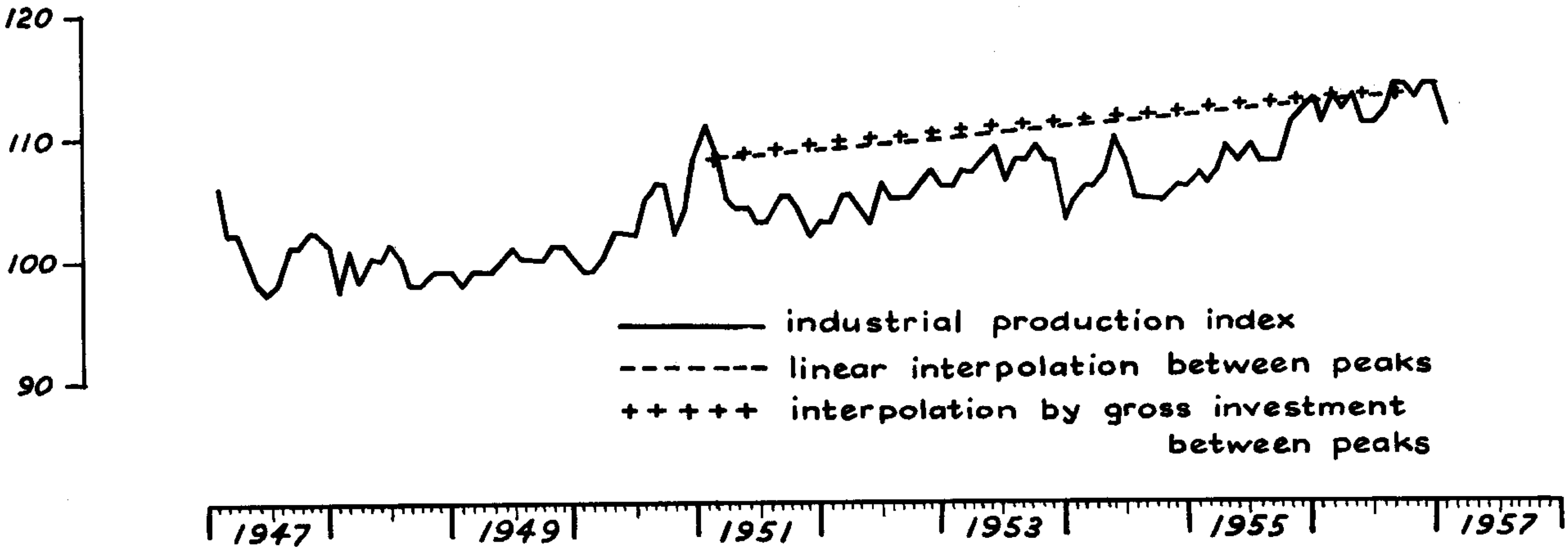


Figure _.

Stone, Clay, & Glass

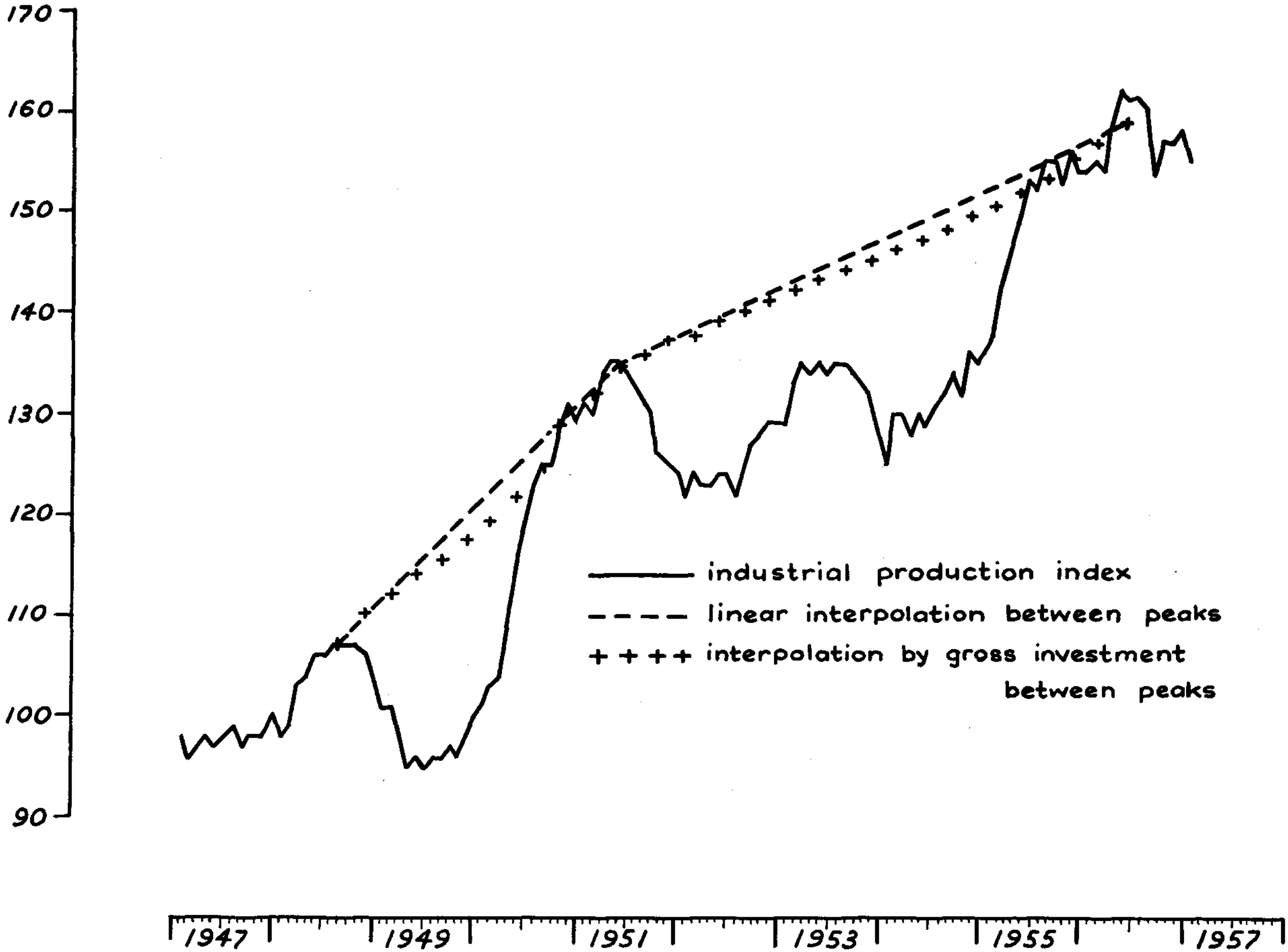


Figure V

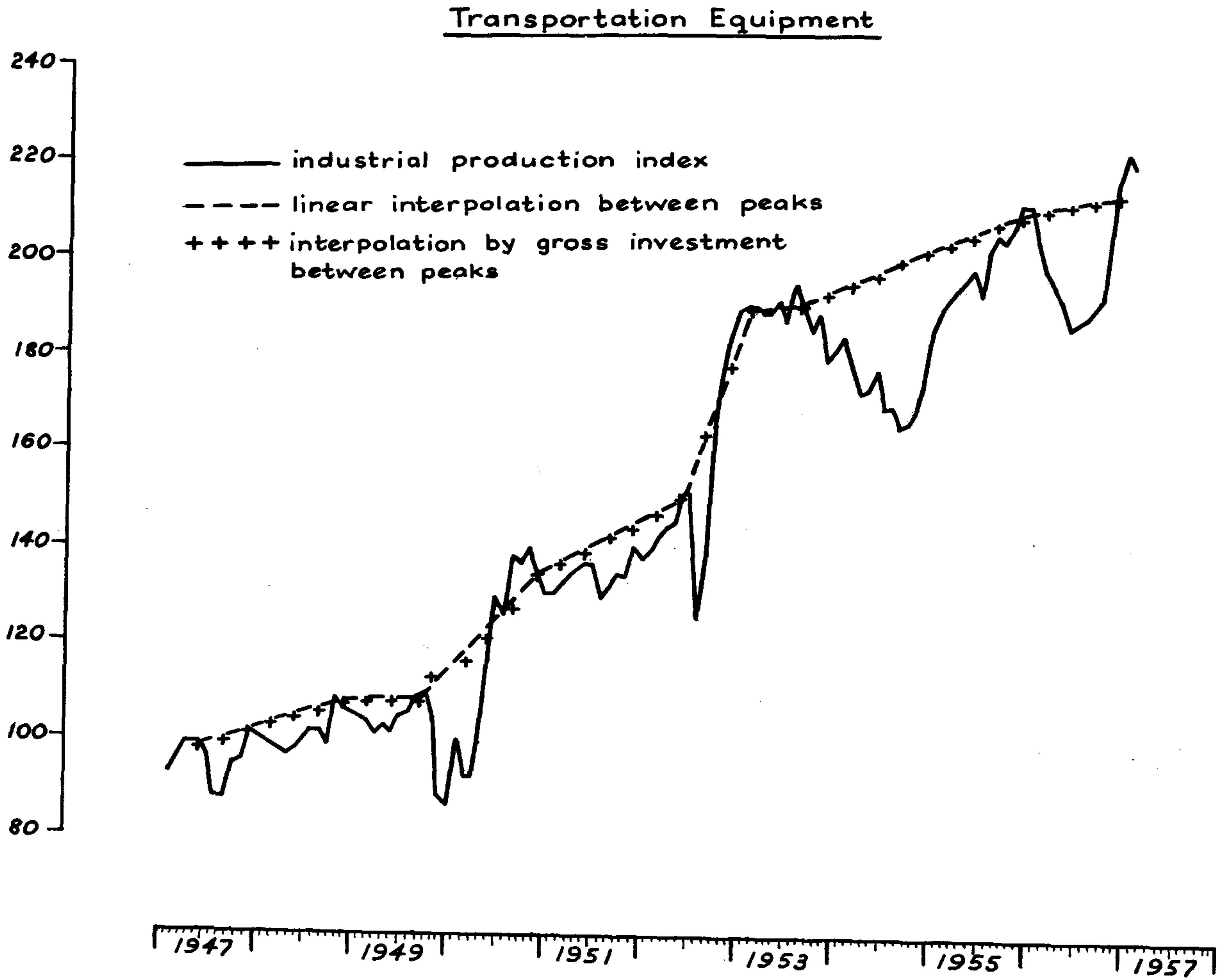

Transportation Equipment
240
220
200
180
160
140
120
100
80
industrial production index
linear interpolation between peaks
interpolation by gross investment between peaks
1947
1949
1951
1953
1955
1957

Figure VI

Paper & Allied Products

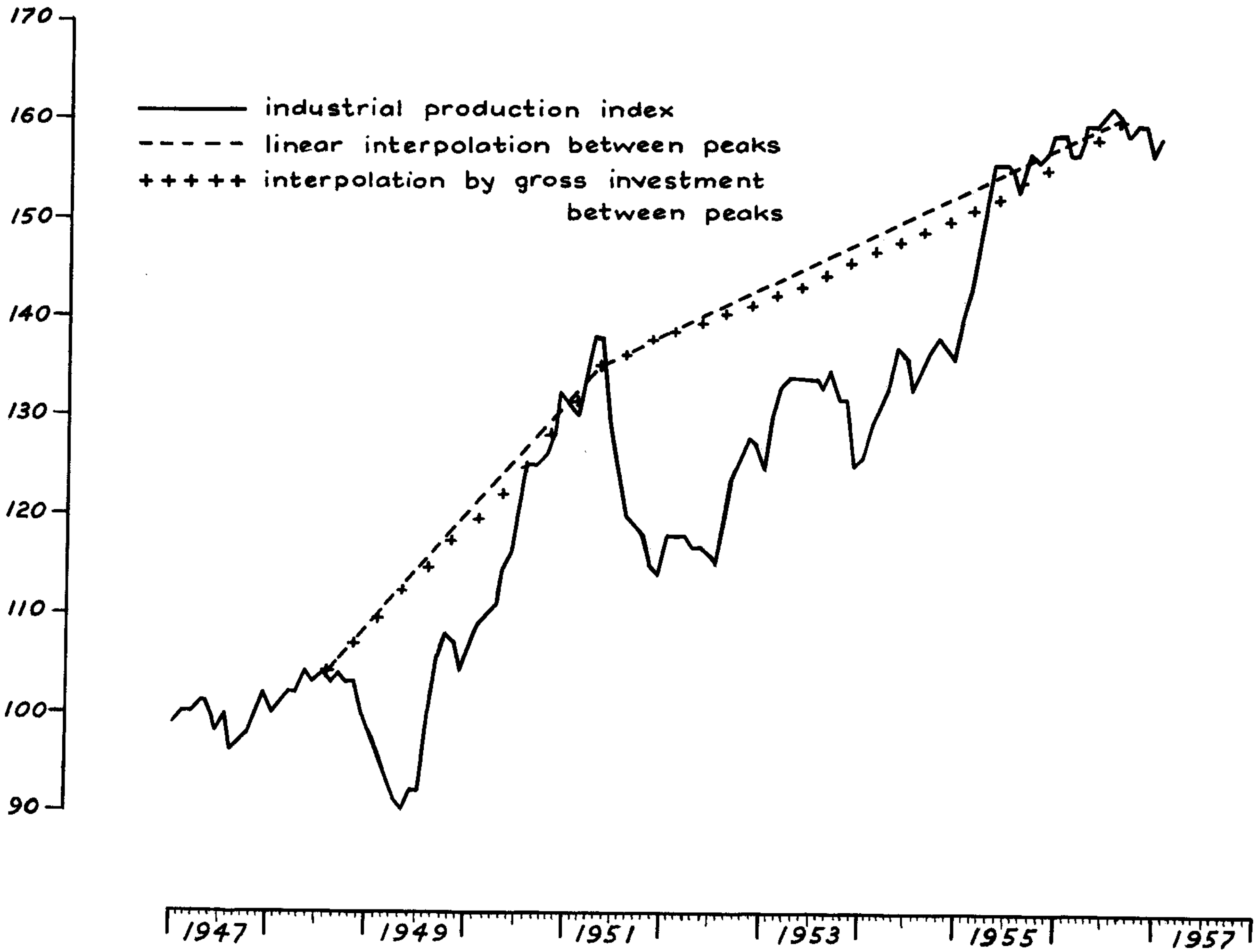


Figure VII

Furniture & Miscellaneous Mfg.

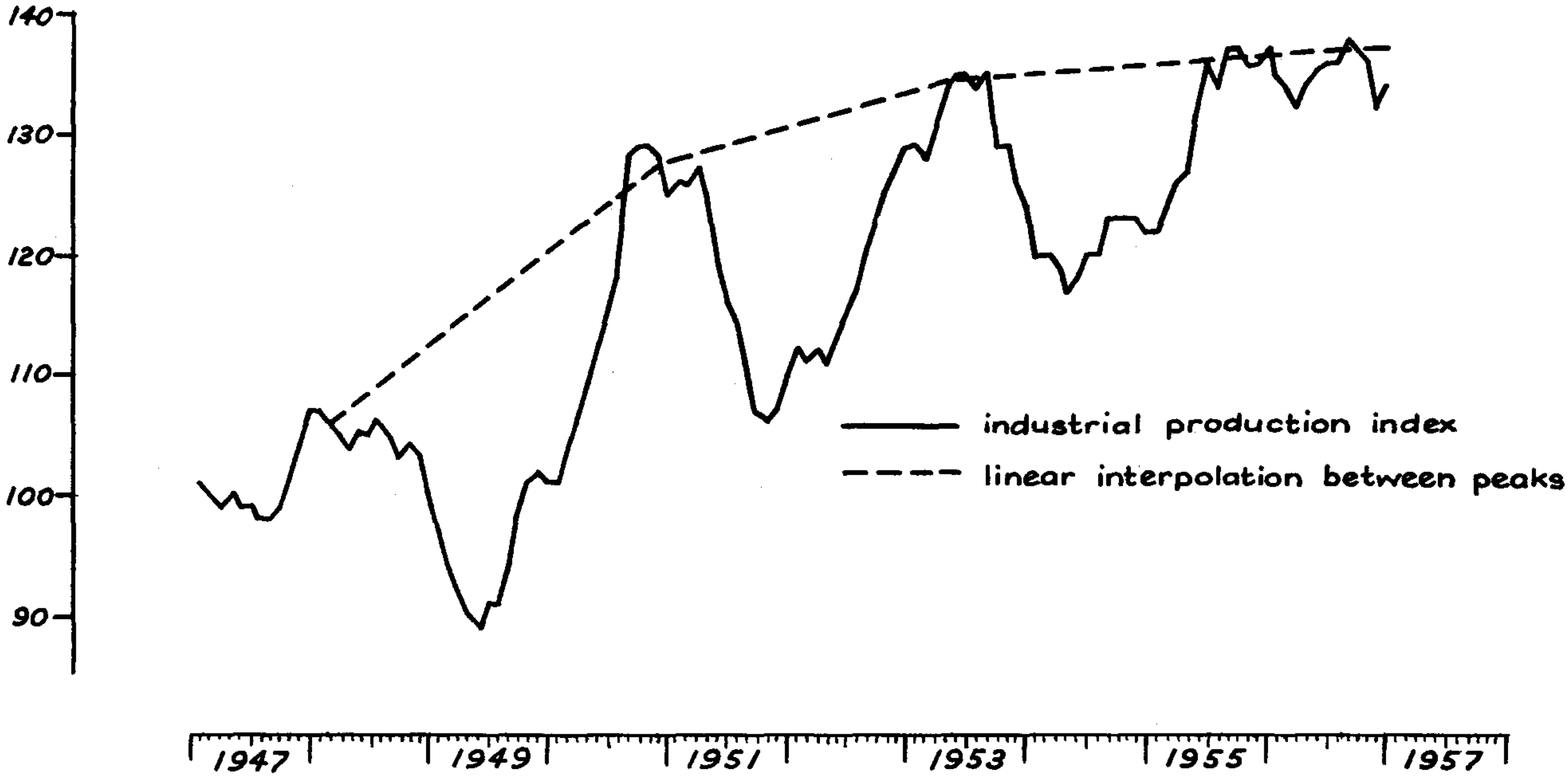


Figure VIII

Electrical Machinery

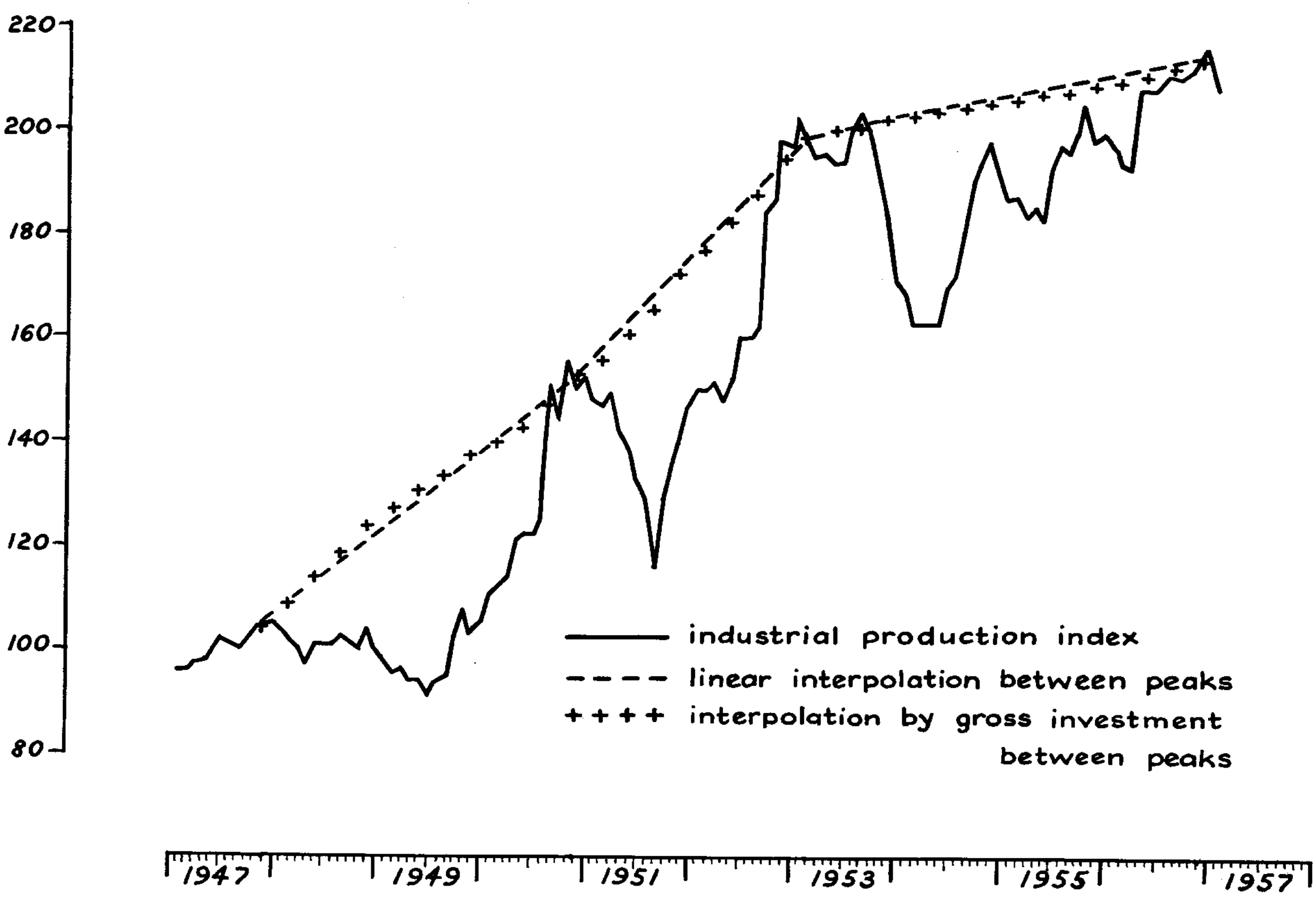


Figure IX

Non-electrical Machinery

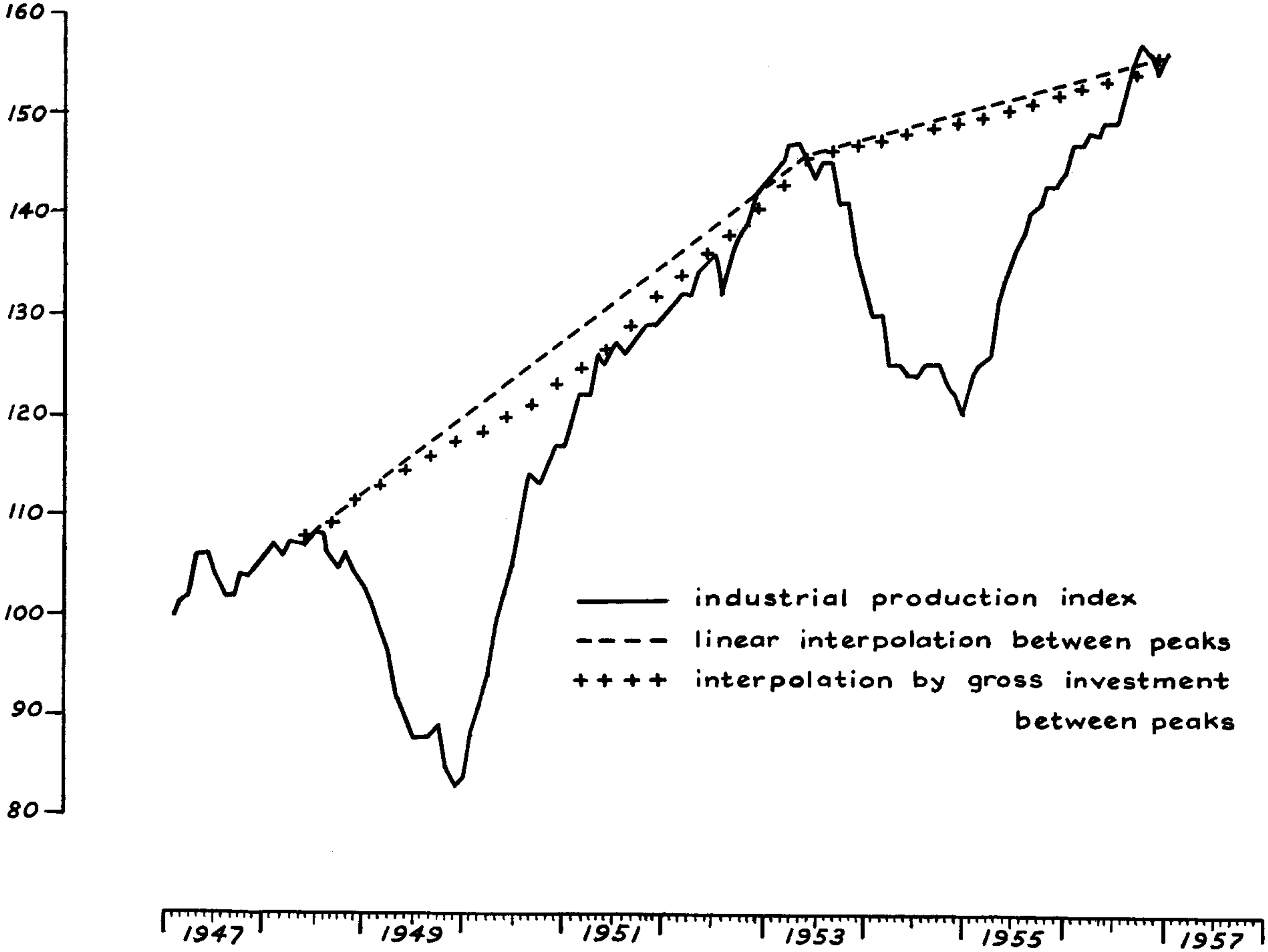
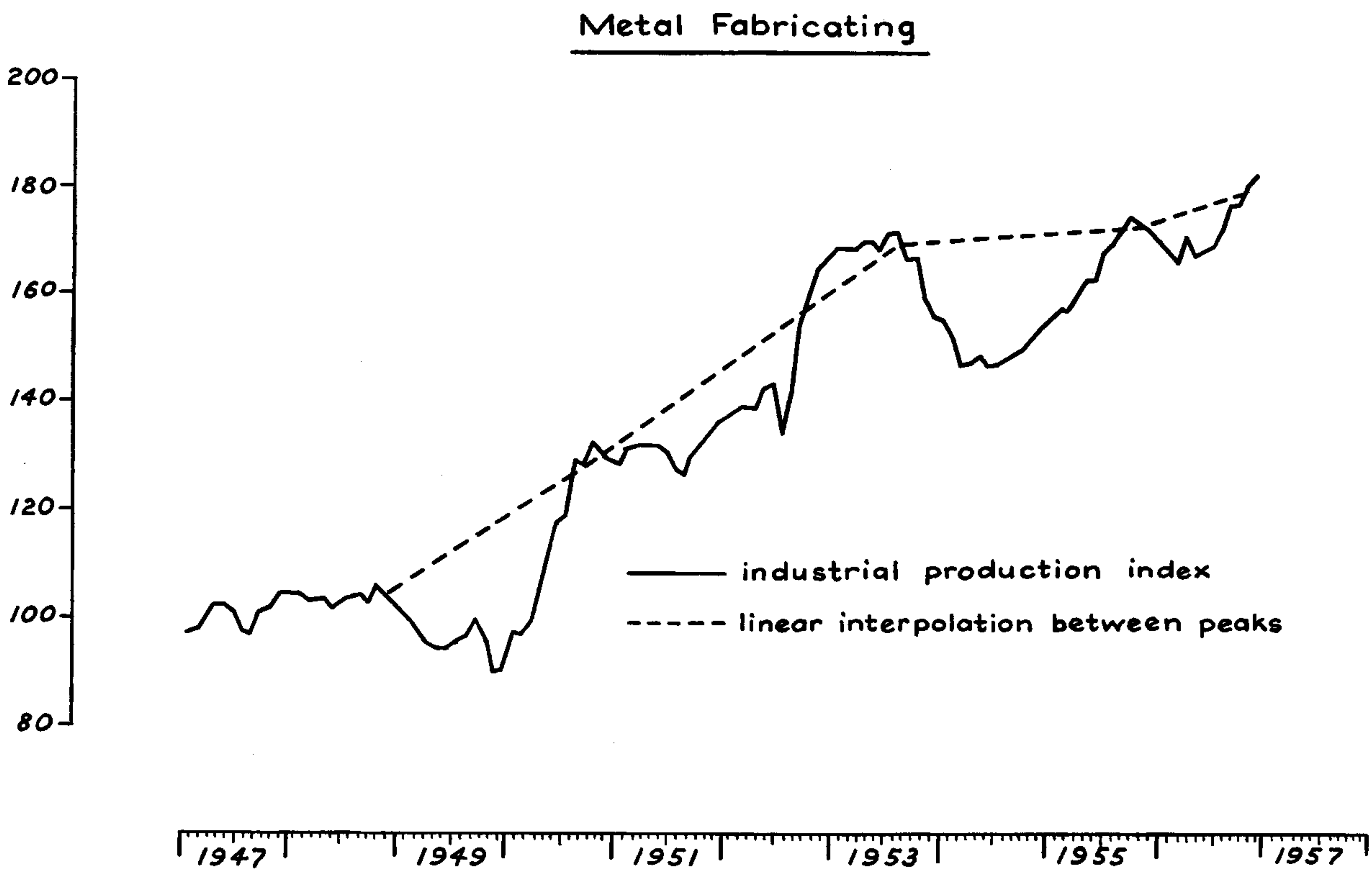


Figure X



Metal Fabricating
200
180
160
140
120
100
80
industrial production index
linear interpolation between peaks
1947
1949
1951
1953
1955
1957

Figure XI

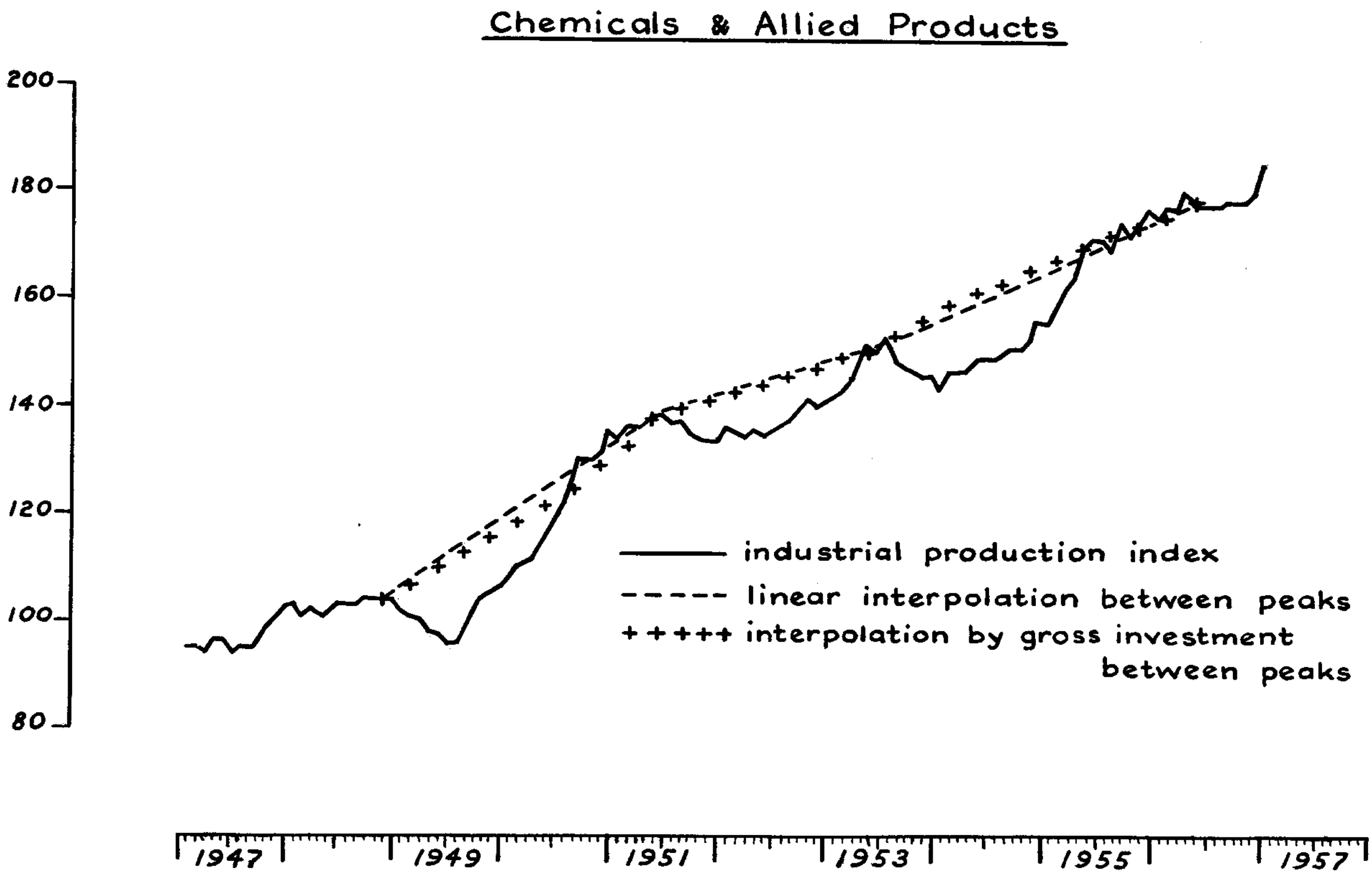

Chemicals & Allied Products
200
180
160
140
120
100
80
industrial production index
linear interpolation between peaks
interpolation by gross investment between peaks
1947
1949
1951
1953
1955
1957

Figure XII

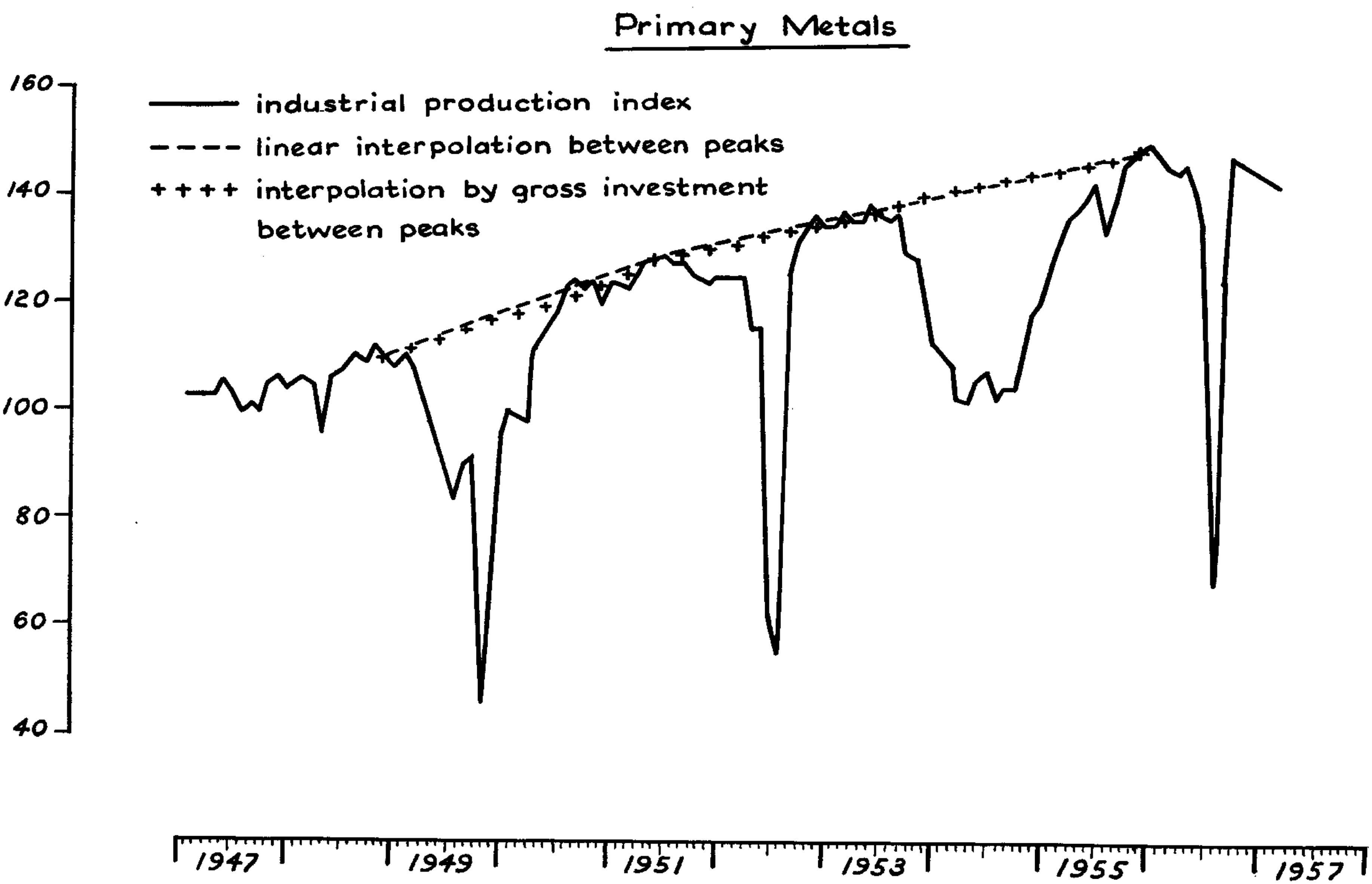

Primary Metals
industrial production index
linear interpolation between peaks
interpolation by gross investment between peaks
160
140
120
100
80
60
40
1947
1949
1951
1953
1955
1957

Figure XIII

Petroleum & Coal Products

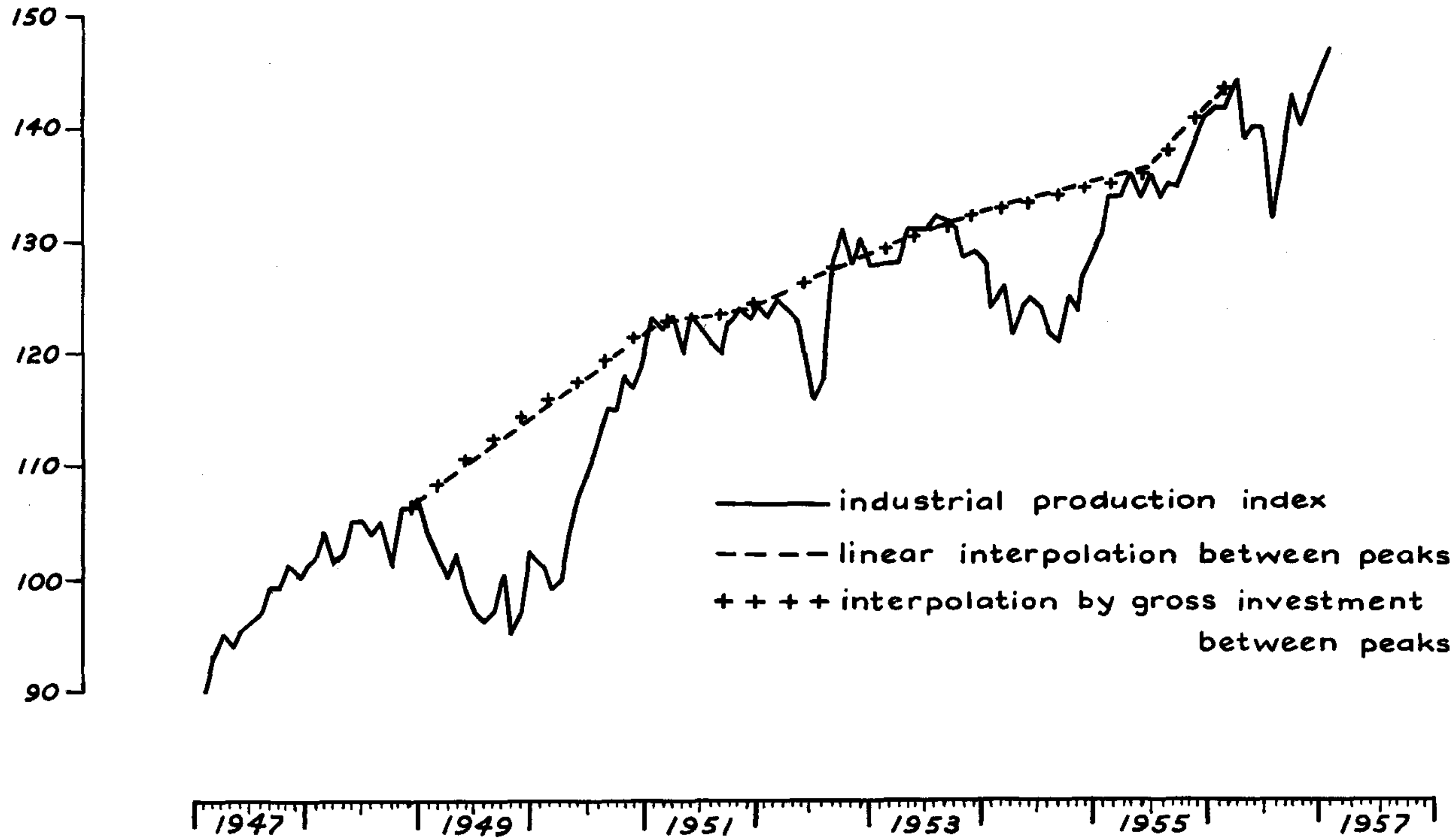


Figure XIV

Rubber Products

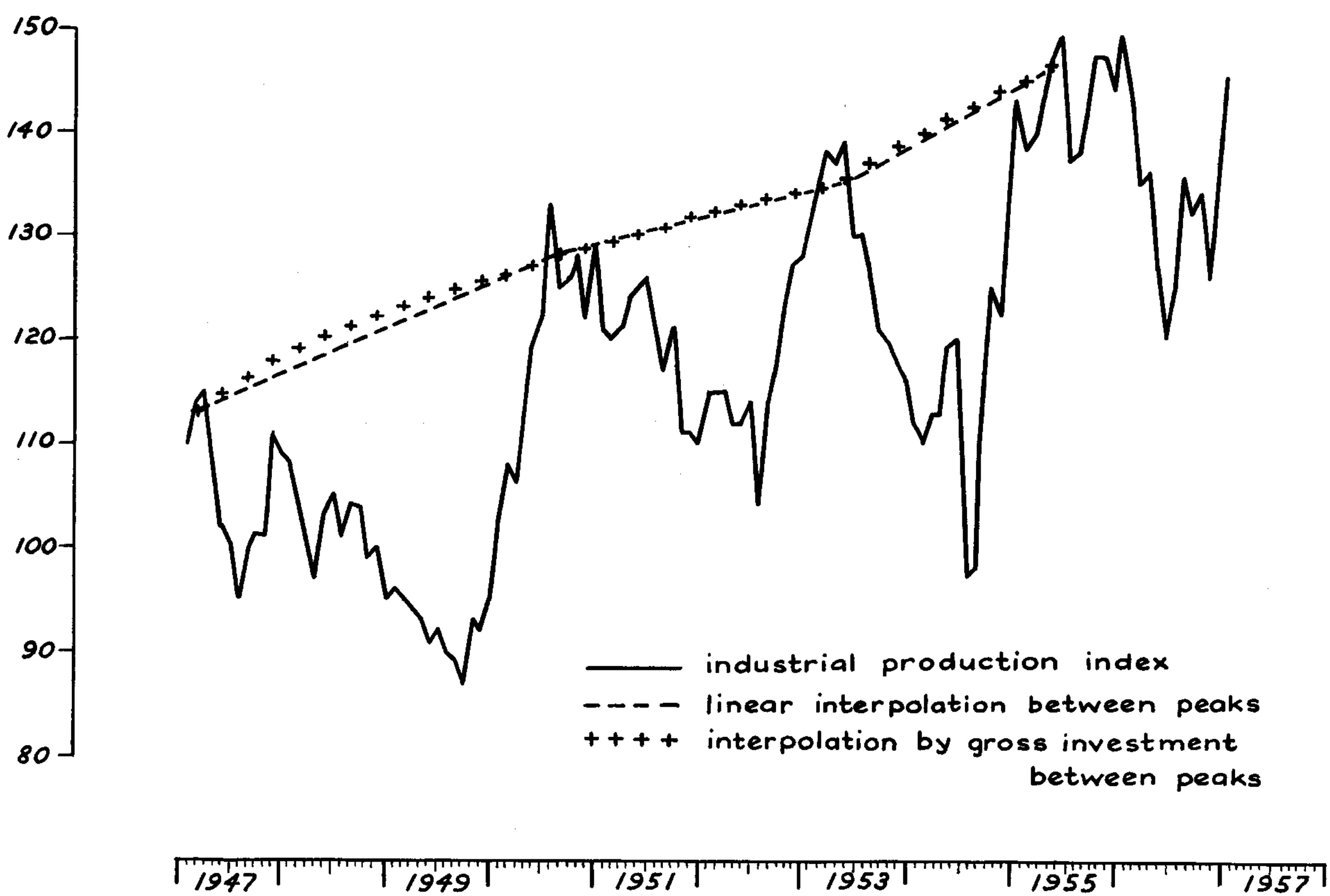


Figure XV

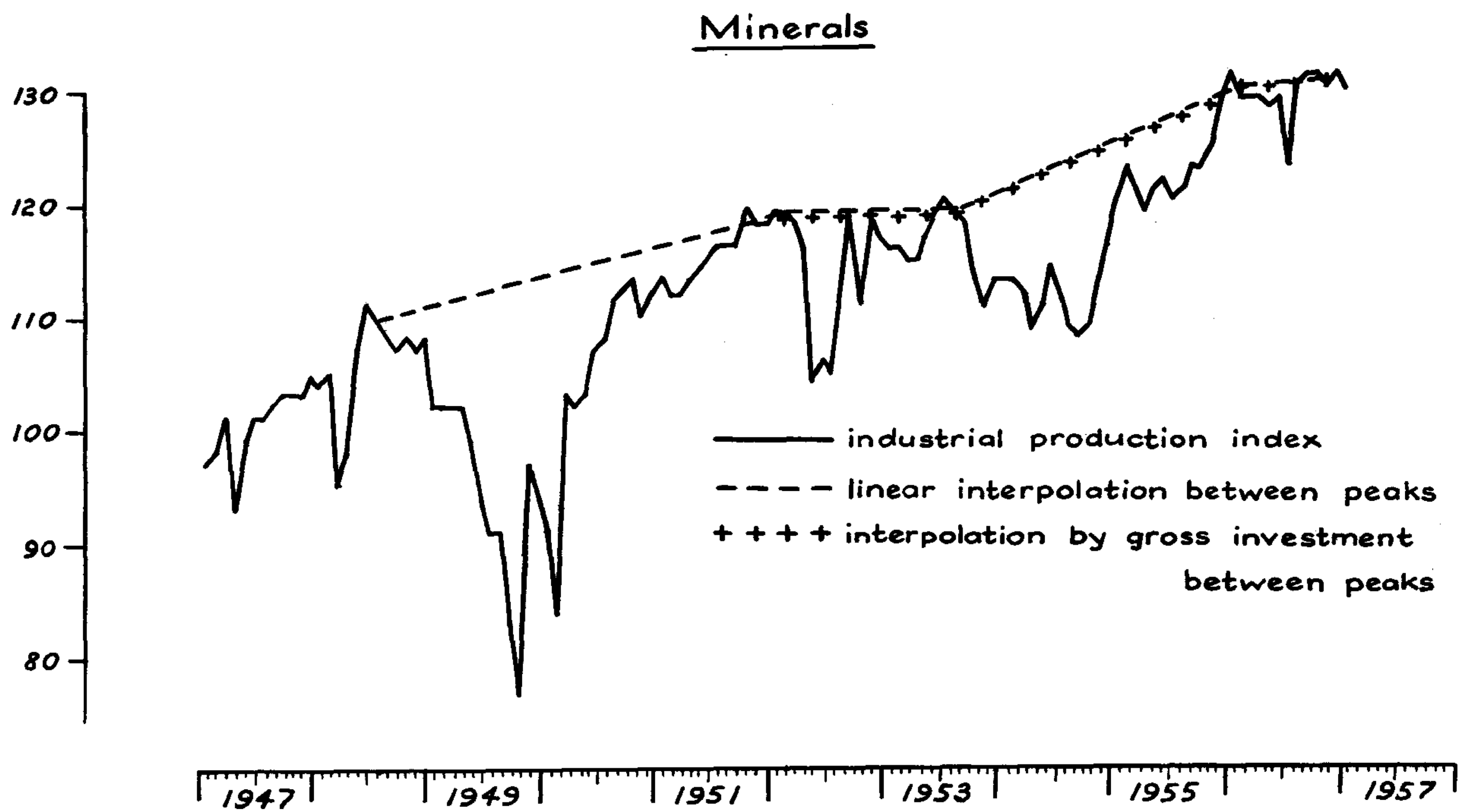
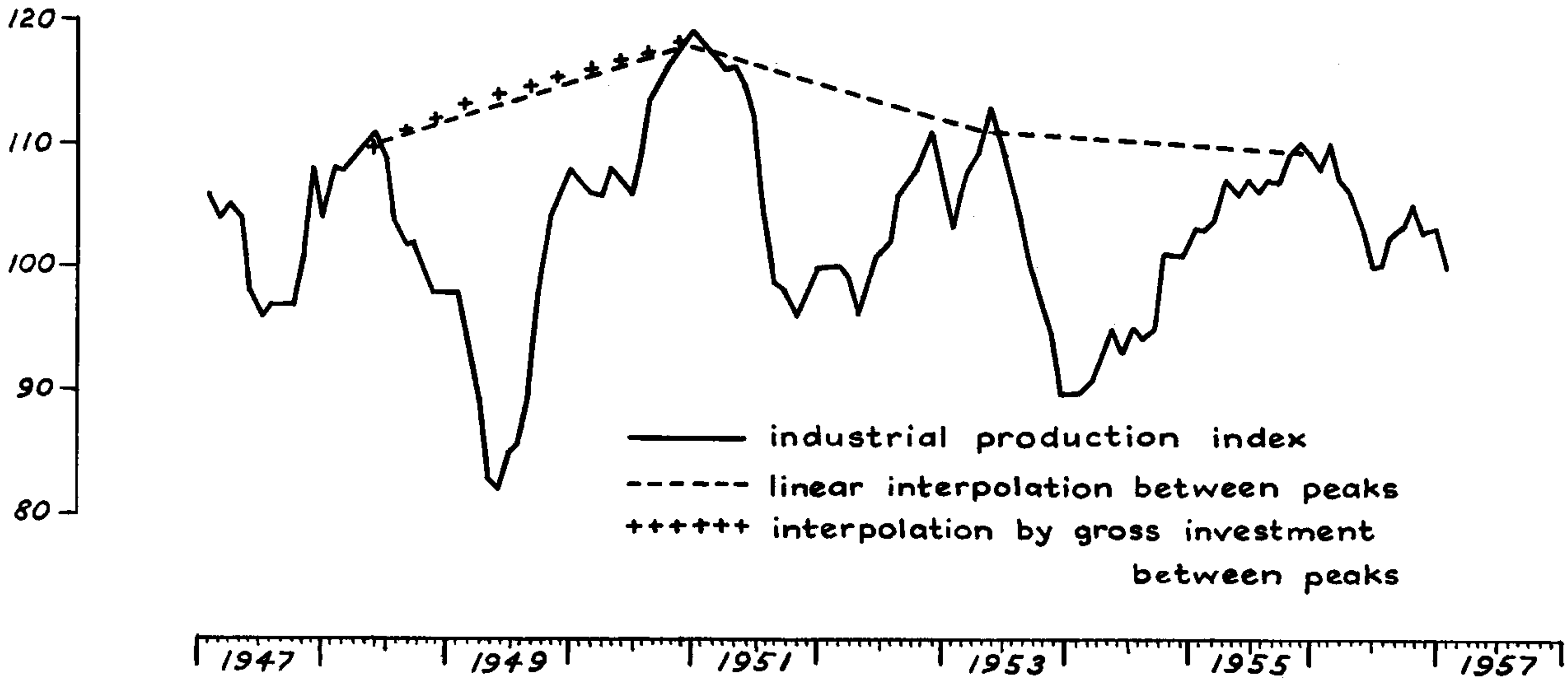

Minerals
130
120
110
100
90
80
1947
1949
1951
1953
1955
1957
industrial production index
linear interpolation between peaks
+ + + + interpolation by gross investment between peaks

Figure XVI

Textile Mill Products



A variation of our statistical procedure would be to select, for each industry, a period in which most indications point to generally full use of resources. This date would vary from sector to sector. For this date, estimate the stock of fixed capital, say, by methods and data like those of Goldsmith.* Let capacity output change in index form by the

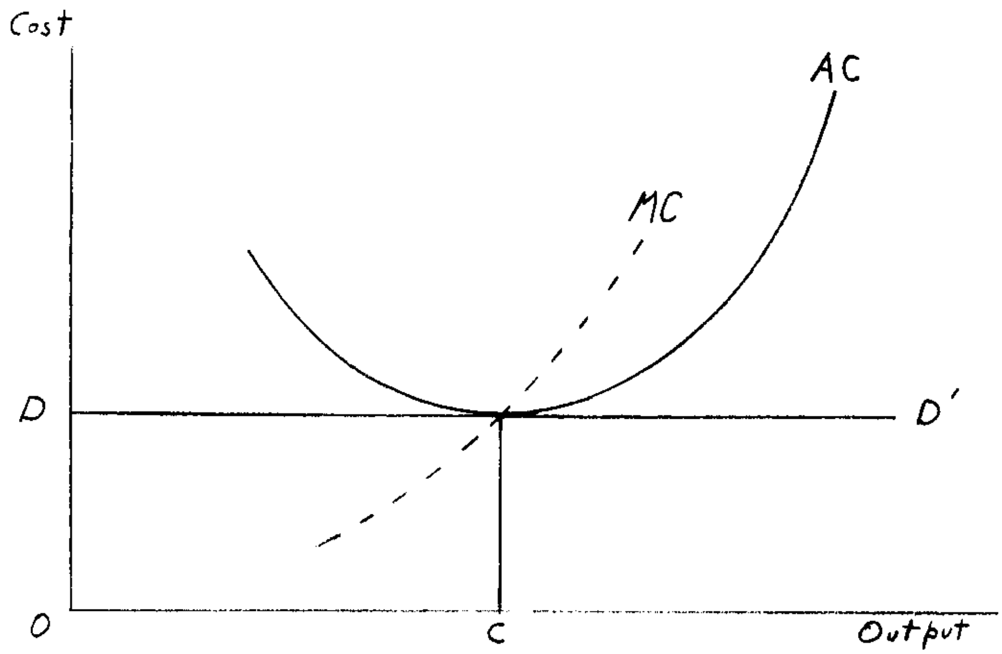
* R. W. Goldsmith, A Study of Savings in the United States. (Princeton: Princeton Univ. Press), 1955.

same number of percentage points that current investment causes the stock of capital to change. This would not, however, help us in developing a proxy measure for the stock of capital that gets round the difficulties raised by technical progress.

Like the other pragmatic measures of capacity, the statistical measures are confronted with the problem of mutual consistency in the Leontief sense. If all or most series peak within a few months of each other, this problem may not be quite as serious as for the other pragmatic measures.

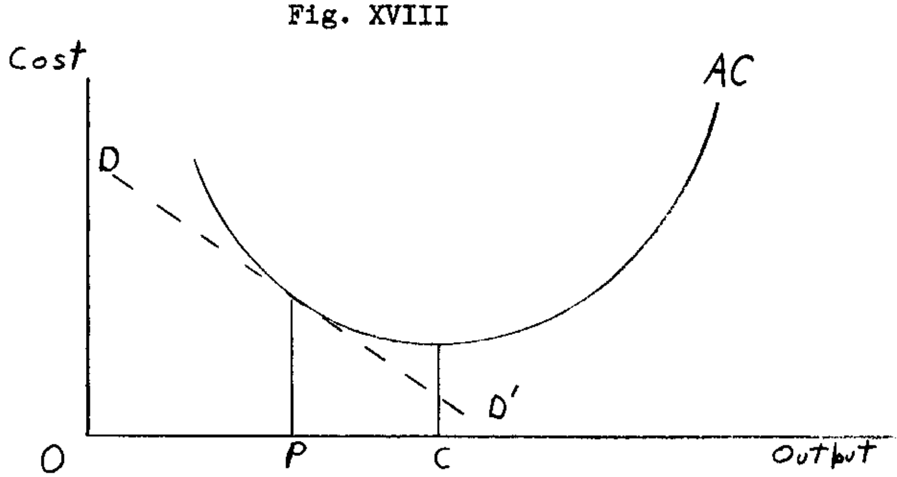
In economic theory full capacity has come to be associated with the concept of competitive equilibrium.

Fig. XVII



In the diagram capacity output is at OC, which gives an equilibrium of no profits with marginal costs = price = marginal revenue. This is a long-run equilibrium, requiring free entry of competitive firms to eliminate profits. Zero profits caused by free entry under imperfect competition would produce a tangency solution with a sloping demand curve at a smaller level of output (OP < OC).

Fig. XVIII

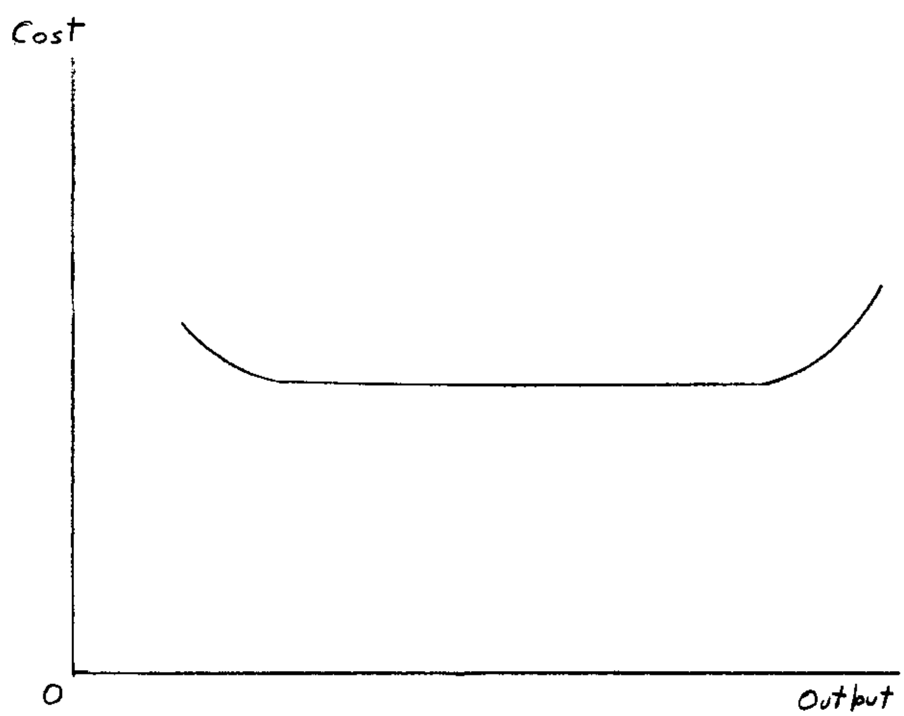


There are refinements of this sort of discussion in the literature of imperfect competition, with possibly different concepts of full capacity, but an argument is made for acceptance of amount OC as a norm for output because such values in all sectors simultaneously would produce some sort of social optimum.* The technical definitions of capacity left a void on the question of costs and therefore are basically indeterminate values. The statistical problem, then, is to determine cost functions for the different industrial sectors of the economy and to compute the output value corresponding to minimum average cost. An alternative possibility would be to work with production instead of cost functions since the former have a higher degree of "autonomy" than the latter and would not change if market values were to adjust , but we are able to reduce the number of dimensions to two variables (except for joint production) and find this added convenience favorable toward the use of cost functions. The analysis is simpler since costs summarize a multiplicity of inputs. From a statistical point of view, we are suggesting fresh computation at frequent intervals. This avoids a major part of the difficulty caused by lack of "autonomy" of the cost function due to fluctuating factor rewards and other input prices.

* We should like to be able to appeal, as well, to the theorems on existence of meaningful solutions to general equilibrium equation systems defining a competitive equilibrium to be assured that such a collection of optimal points for each producing unit would be mutually compatible. Unfortunately this is not generally possible if existing prices differ from those in a competitive market. The theorems on the existence of solutions to the equation systems of competitive equilibrium tell us that there exists a set of prices and wage rates such that an economically meaningful solution is possible. But in passing from existing to competitive prices, cost curves will change. Minimal points on average cost curves depend on relative prices and wage rates. If these change, we should generally expect output corresponding to the minimal point to change.

The empirical investigation of cost functions has led to the broad conclusion of linearity in the relation between total costs and output. If this were the case for total operating costs and if zero output were associated with zero cost, we would find a horizontal average cost curve in contrast to the traditional U-shaped relation familiar in classroom and textbook. The long run cost curve, derived as an envelope of U-shaped short run curves, need not, in principle, have a minimum point. Since we are interested in long term adjustment to competitive equilibrium, we are mainly interested in the long run curve. At most, advocates of linearity in empirical cost curves have conceded that at extreme outputs, either very small or very large, the average cost curve may turn upwards. They consider that the base is broad and flat, with almost indeterminate minimum point.

Fig. XIX

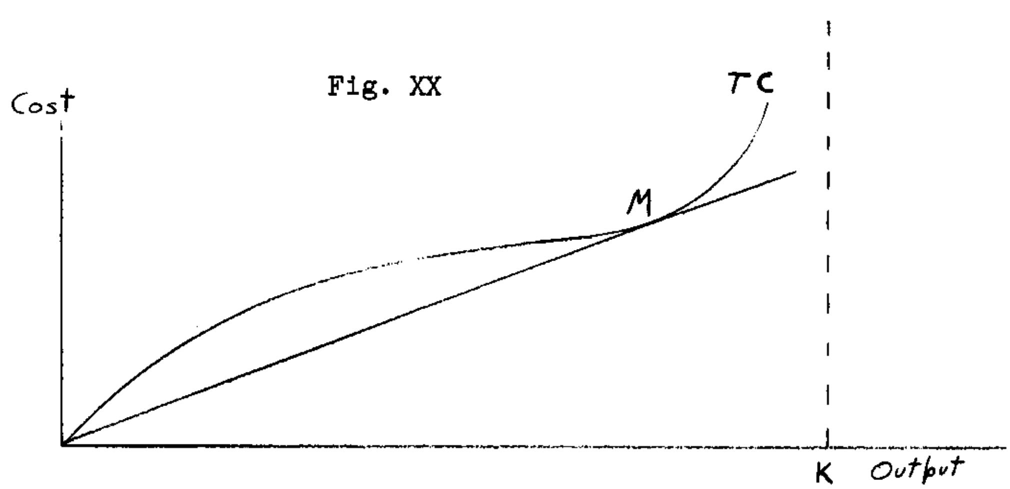


While we shall be dealing with short run components of cost, i.e. current operating costs incurred during an annual period, we shall be trying to estimate the long run cost function. Our sample data will consist of a cross section of accounting records from individual plants or firms, and the relationship estimated from the variation among units during a given short interval of time tends to reflect the long run function. This is a general property of estimation from cross-section data.

The main difficulties in our approach to measurement of capacity on these theoretical considerations is that we are not sure, on a priori grounds, that the long run average cost curve has a minimum and that the tendency towards linearity observed in other studies will blur, if not obliterate, any minimum point.

We shall not use a polynomial expansion for the form of the cost curve because with moderate dispersion it becomes nearly impossible to pick out any delicate degree of curvature. We shall, instead, use a family of cost curves that necessarily have the requisite type of curvature, but in which the degree of curvature may be sharp or gentle. A curve from this family passes through the phases of decreasing, and increasing marginal costs. We choose the "probit" function, which has the form in the diagram below.

Fig. XX



The total cost function is

$$x = \frac{k}{\sqrt{2\pi}\,\sigma}\int_0^c e^{-\frac{(\log t - \mu)^2}{2\sigma^2}}\,\frac{dt}{t}$$

x = output.

c = cost.

Natural logarithms are used throughout

The parameter k is a <u>saturation</u> level of output, an asymptotic value which output cannot exceed no matter what the cost may be. The other two parameters, μ and σ, are parameters of a logarithmic normal distribution. The point of minimum average cost is determined from the coordinates at M, where a ray from the origin is tangent to the total cost curve TC.

In fitting this type of curve to cross-section data, we estimate a microeconomic function pertaining to a typical firm or plant. To derive a capacity estimate for an industry, we must aggregate this function, and this is where the nice properties of this family of functions add to our convenience. Assuming that costs are distributed according to the lognormal law with parameters μ_1 and σ_1, we find*

---

* This follows directly from Theorem 2.2, corollary 2.2b, J. Aitchison and J. A. C. Brown, <u>The Lognormal Distribution</u> (Cambridge Univ. Press) 1957.

$$\bar{x} = \frac{k}{2\pi\,\sigma\sigma_1}\int_0^{\infty}\int_0^{c} e^{-\frac{(\log t - \mu)^2}{2\sigma^2} - \frac{(\log y - \mu_1)^2}{2\sigma_1^2}}\,\frac{dt}{t}\,\frac{dy}{y}$$

$$\bar{x} = \frac{k}{\sqrt{2\pi}\,\sqrt{\sigma^2 + \sigma_1^2}}\int_0^{\bar{c}} e^{-\frac{(\log t - \mu - 1/2\,\sigma_1^2)^2}{2(\sigma^2 + \sigma_1^2)}}\,\frac{dt}{t}$$

From the probit relation we estimate k , μ , and σ . From the marginal distribution of costs, we then estimate μ_1 and σ_1 . From these parameter estimates we are able to construct a relation between industry arithmetic averages belonging to the same family of curves. From the industry average curve we readily find the point at which marginal and average costs are equal, for this defines the point of tangency in diagram XX.

Let us write

$$\mu_2 = \mu + 1/2\sigma_1^2$$

$$\sigma_2^2 = \sigma^2 + \sigma_1^2$$

The industry average relation can be written as

$$\bar{x} = \frac{k}{\sqrt{2\pi}\,\sigma_2} \int_0^{\bar{c}} e^{-\frac{(\log t - \mu_2)^2}{2\sigma_2^2}} \frac{dt}{t}$$

From the transformation

$$u = \frac{\log t - \mu_2}{\sigma_2}$$

we derive

$$\bar{x} = \frac{k}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log \bar{c} - \mu_2}{\sigma_2}} e^{-1/2y^2}\, du = kP\left(\frac{\log \bar{c} - \mu_2}{\sigma_2} : 0,1\right)$$

$$= kP(y)$$

where P is the cumulative normal distribution with zero mean and unit variance. We can also find a convenient expression for marginal cost along the aggregative function.

$$\frac{d\bar{x}}{d\bar{c}} = \frac{k}{\sqrt{2\pi}\ \sigma_2\bar{c}}\ e^{-\frac{(\log \bar{c} - \mu_2)^2}{2\sigma_2^{\ 2}}} = \frac{k}{\sigma_2\bar{c}}\ Z(\frac{\log \bar{c} - \mu_2}{\sigma_2}:\ 0,1) = \frac{k}{\sigma_2\bar{c}} Z(y),$$

where Z is the function of ordinates to the normal distribution with zero mean and unit variance. The point at which average and marginal cost are equal is thus given by

$$\frac{\bar{c}}{\bar{x}} = \frac{d\bar{c}}{d\bar{x}}$$

$$\frac{\bar{c}}{kP(y)} = \frac{\sigma_2\bar{c}}{kZ(y)}$$

$$\frac{Z(y)}{P(y)} = \sigma_2$$

From tables of the normal distribution and its ordinates, values of y can be determined. From this value, we can then estimate $\bar{x}$, $\bar{c}$ associated with minimum average cost.

<u>An Empirical cost-curve for electric power stations</u>. To see whether the suggested approach outlined in the previous section is a feasible avenue towards the construction of capacity series for industries, we choose an example from an industry where excellent data are available and where a well accepted capacity estimate has already been established. The capacities of electric power stations are widely reported and aggregated into an industry total on a regular periodic basis. It would be presumptuous to believe that this figure can be greatly improved upon. Our purpose in this section is to compare the capacity point defined by minimum average cost with the established engineering value to see whether our method may be suggested for the preparation of capacity estimates in other industries where acceptable capacity figures do not exist.

We shall try to select our sample in such a way as to hold constant as many "nuisance" variables as possible in order to pick out the relationship between cost and output from interplant variability.* Our sample consists

---

* For a discussion of the "nuisance" variables, i.e. variables other than scale of output giving rise to interplant cost variation, see W. Isard and J. B. Lansing, "Comparisons of Power Cost for Atomic and Conventional Steam Stations," Review of Economics and Statistics, Vol. XXXI, 1949, pp. 217-26.

---

of plants burning coal exclusively; of conventional construction, and located in the Middle Atlantic and Southern New England States. It is felt that this introduces a great degree of homogeneity in the operating conditions of the several plants of a sample. Moreover the data, taken from a report of the Federal Power Commission, are presented and prepared according to a uniform system of accounting.* Finally the presentation of the data on a

---

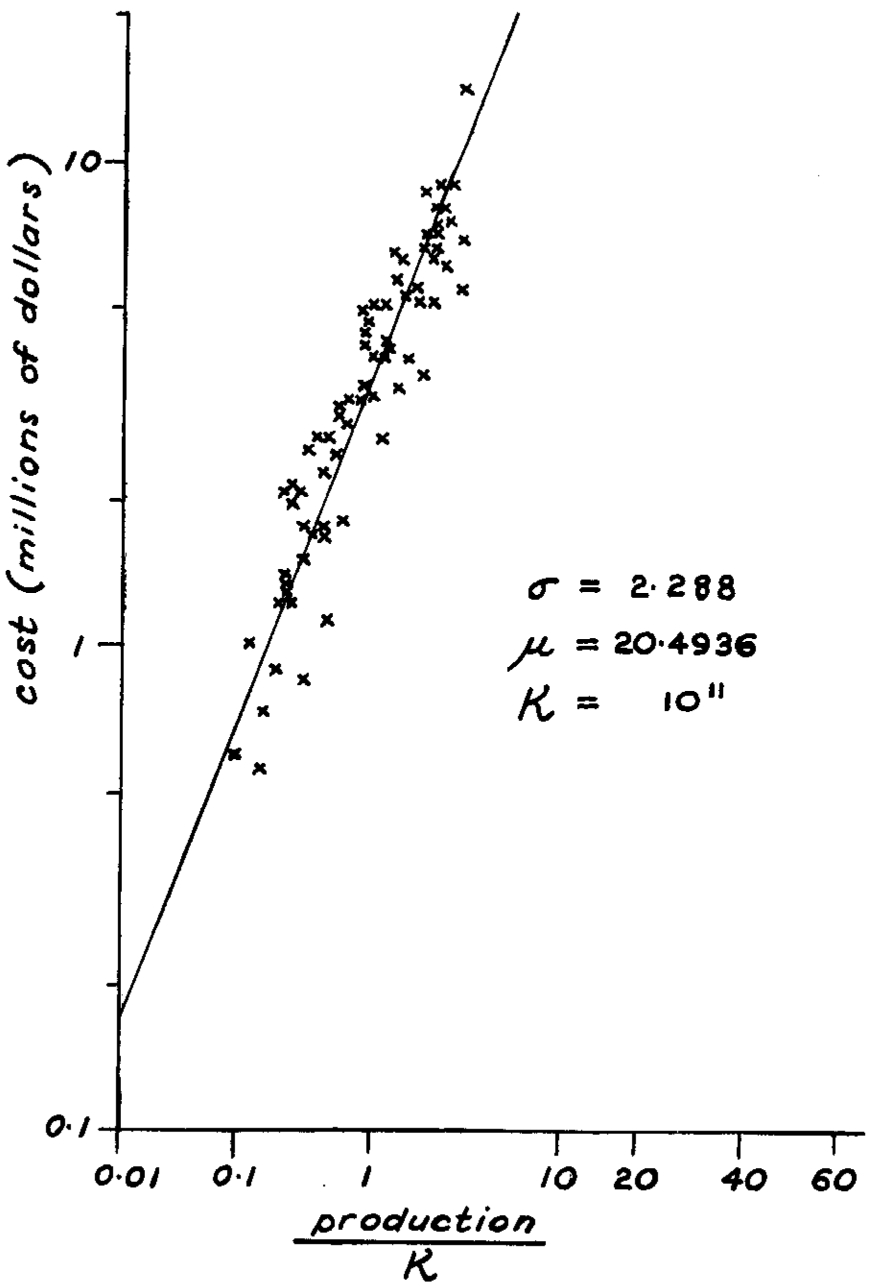
* "Steam Electric Plant Construction Cost and Annual Production Expenses" Federal Power Commission, S - 123, 1955.

---

plant basis is significant. Cost data, and inferences of efficiency of size or capacity from them, should most suitably be on a plant basis. In many industries such data are difficult to uncover, at least in the great detail given for the electric power industry by the Federal Power Commission; therefore, we set out from the most favorable situation.

There are 67 plants in the sample. A graphical estimate of the probit function is obtained from the data plotted in the accompanying diagram. On the vertical axis we have the logarithm of cost, and on the horizontal axis we have a probability scale plotting the normal deviate corresponding to the

Probit Graph for Electric Power Stations



fraction x/k . We first experiment with different values of k to find one that gives a linear scatter of points. We pass a free-hand line through the general drift of points, and determine the parameters μ and σ from

$$\log c_p = \mu - n_p \sigma$$

$$\log c_q = \mu - n_q \sigma$$

p and q are two arbitrarily selected probability values on the horizontal scale. n_p and n_q are the associated normal deviates. Log c_p and log c_q are the logarithms of the corresponding cost values on the curve for these deviates. c_p and c_q are read from the vertical scale.

In the present graph, we estimate

$$x = \frac{10^{11}}{\sqrt{2\pi}\,(2.288)} \int_0^c e^{-\frac{(\log t - 20.494)^2}{2(2.288)^2}} \frac{dt}{t}$$

From the marginal distributions we have

$$\mu_1 = 14.940$$

$$\sigma_1^2 = .713$$

We derive the cost curve of averages from these data by the formulas of the preceding section and find that least cost production occurs at

1,904.0 mill. kwh.

Mean production in the sample is

1,071.2 mill. kwh.

This gives us an operating rate of 56 percent of capacity. The Federal Power Commission records give an alternative measure of capacity termed "net continuous plant capability when not limited by condensor water". This appears to be the nearest concept to our notion of capacity as an equilibrium rate of operation. The ratio of mean production to mean continuous capability in the sample is 57 percent. Thus we conclude that our capacity estimate is not unreasonable in this case.

We shall not say that it is very good because our parameter estimates are crude, being done by free-hand methods.

An empirical cost curve for steel companies. A second example deals with the cost curve for the iron and steel industry, not because the data are of exactly the right sort and easily available, but because this industry has a widely accepted capacity estimate. The Census of Manufactures comes close to providing cost and output data for this and other industries on a plant basis. They present data for establishments. The requisite data on production (value or volume) and costs are collected but nowhere tabulated in just the form needed. We do not find numbers of establishments, mean output, and mean costs by either output or cost size classes. Of course, individual establishment data are not available. From different tables one could piece together value of production, cost of materials and fuel, and wage payments. But they are not properly put together by the classifications needed. Moreover, the steel industry is accorded special treatment in that value added is given without being separated into sales and the cost of materials and fuel. This is presumably due to the vertical integration in the industry.

We, therefore, turned to company data. From the reports of the Iron and Steel Institute we listed companies furnishing capacity data. Our sample then consisted of all such companies for which we could find sales and cost data. We collected accounting reports for companies listed on the New York or American Stock Exchange from the Exchange Libraries. These were supplemented by reports in Moody's Industrials. We were not as fortunate, as in the case of electric power, to have a uniform system of accounts. But from each company report, we tried to extract data on "Net Sales" and "Cost of Sales". Our sample consisted of 39 companies, and we did not have a choice pattern that would control "nuisance" variables. We could not separate firms by type of product, production process, locale or other important characteristics. We know that the accounting data had different meanings from company to company but did our best to eliminate discrepancies. In electric power we had a homogeneous output unit, kwh, but in steel we did not use a physical tonnage variable because it was not reported by all firms in our material sources. Instead we used a value figure, net sales.

For the probit model as used in the previous section on the electric power industry, we plotted the logarithm of company cost for 1954 against the normal deviate corresponding to x/k ; where x represents the value of company sales in 1954, and k is a parameter of the cost function. The line plotted in the accompanying graph is a free hand estimate for $k = 10^{13}$. The equation of the line is

$$x = \frac{10^{13}}{\sqrt{2\pi}\,(4.354)} \int_0^c e^{-\frac{(\log t - 36.704)^2}{2(4.354)^2}} \frac{dt}{t}$$

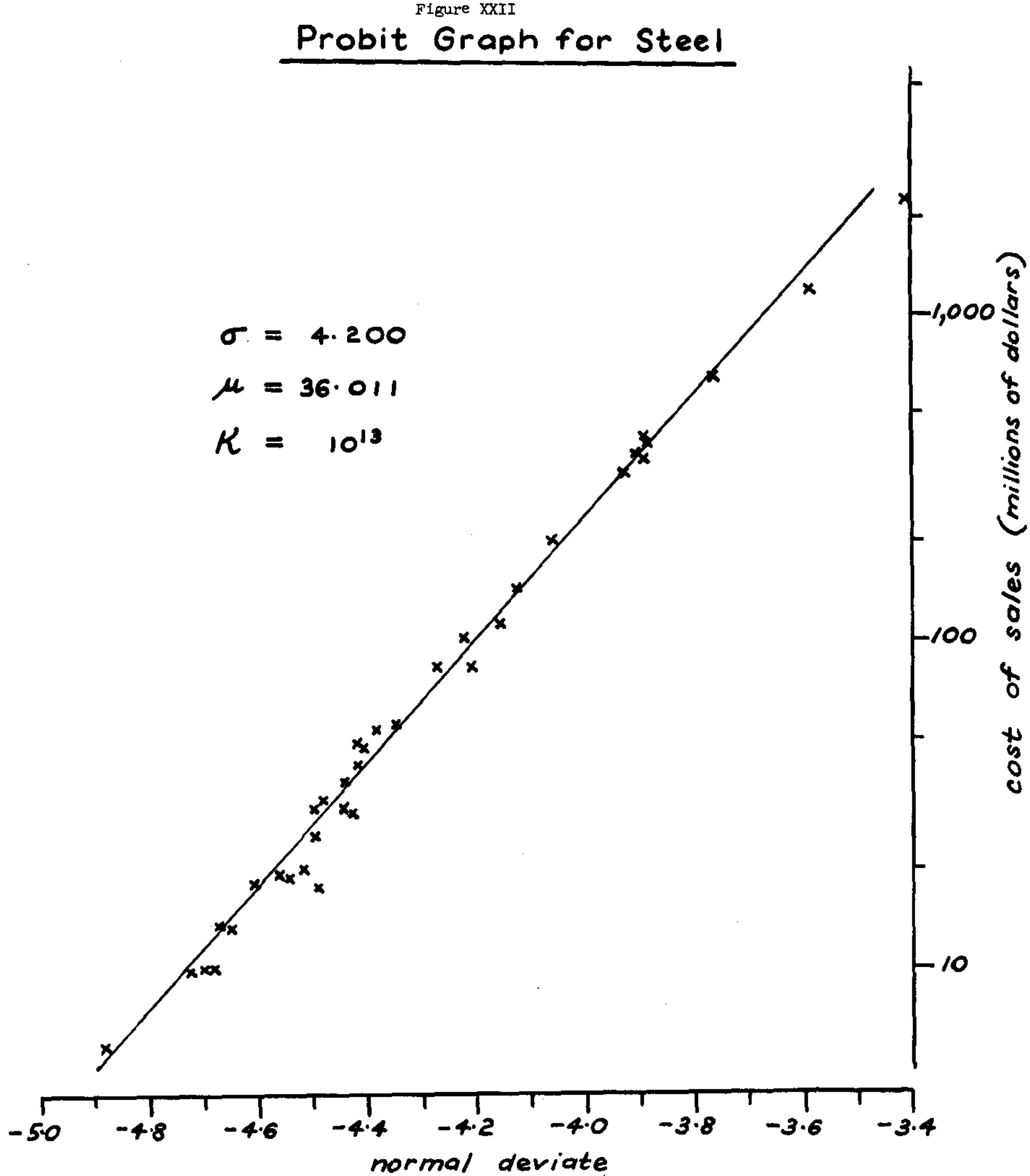
The parameter estimates from the marginal distribution of cost values are

$$\mu_1 = 17.894$$

$$\sigma_1^2 = 2.092$$

The average curve for the entire industry implies a least average cost output (sales) of \$ 59.6 million and associated costs of \$ 46.7 million.

Figure XXII


Probit Graph for Steel
σ = 4·200
μ = 36·011
K = 10^13
1,000
100
10
cost of sales (millions of dollars)
-5·0
-4·8
-4·6
-4·4
-4·2
-4·0
-3·8
-3·6
-3·4
normal deviate

These figures would appear to be far out of line with sample averages of $ 257.6 for sales and $ 190.8 for costs. According to the notions of this paper, it would thus appear that the steel industry is very inefficiently operated at levels far above optimal points. There is perhaps a germ of truth in this observation, but the principal issue in reconciling these averages with the least average cost points is matter of plant *vs*. company data. The company data, plotted as large agglomerations of plants, lead to large sample averages for output and cost even if the fitted line depicts the cost curve faithfully. Industry (a sample) totals are divided by the number of companies, which is much smaller than the number of plants, and give high sample averages because of the extreme dominance of the large companies. The principles underlying the cost curve are on a plant basis, and the fitted cost curve is probably a better representation of the plant-based than the company-based function. The smaller companies in the sample distribution are likely to be companies with one or very few plants, and they lie on the fitted function. The two giants, U. S. Steel and Bethlehem, are distinctly off the curve as low cost producers. We, therefore, suggest interpreting the fitted function as the plant cost function. The derived function for industry averages depends on the parameters of the marginal distribution of costs. In the empirical study, they are estimated from the company distribution and not from the plant distribution; therefore the relationship between average sales and costs is not properly derived.

With all the imperfections in this sample of data, only rough steps may be taken to make a judgment on the comparability of the capacity figures derived from the cost curve and from the published trade data. From data in the Census of Manufactures we find that the total value added for blast furnaces and ingot production was $ 4,666 million in 1954. Wage payments are estimated at $ 1,936 million. The difference between these two figures, $ 2,730 million, gives an industry estimate conceptually comparable to the difference between sales and costs in our company sample. The number of establishments is estimated at about 300 for the main census in 1947. A later figure has not been published. Using this figure for the number of establishments, we find a "profit" figure of about $ 9 million per establishment. In the sample the ratio of costs to sales is .74. Assuming the same ratio for the average establishment, we obtain a corresponding sales value of $ 34 million. This is the figure we compare with the least average cost sales of $ 59.6 million. With these two figures, we estimate output at 57 percent of capacity in 1954. The published trade figures average to 71 percent in 1954. Our figure is in the same neighborhood when adjustment is made for the fact that our sample is based on company data, but the results are not of the same quality as in the previous example.

The roughness of our particular estimates is emphasized if we point out that the results are sensitive to small adjustments in the computed parameters of the cost curve. If the probit line is drawn to pass closer to the points for the two giant companies, which are both low cost producers, the optimal value is increased by as much as 50 percent. This value is less desirable since it takes us further from the concept of a plant cost function, but it does show the sensitivity of the method. In the particular case at hand, the least cost point is ill defined in the sense, shown diagrammatically above, that the average cost curve has a broad flat base.

The curve fitted to the bulk of small company points produces a least average cost point with output far smaller than that of most large companies. This would imply that economies of scale are realized at the plant level and that large agglomerations of plants are not essential for efficiency. This has been put forward in other studies of the steel industry. The main point at variance with this observation is the fact that the two giants have a low ratio of costs to sales. This may be due to the accounting treatment of sales in a vertically integrated company, but there are no satisfactory data for resolving this matter. We, therefore, rely mainly on our cost curve that pays less attention to the plotted points for U. S. Steel and Bethlehem.

The aggregation of capacity. Whether capacity figures for individual industries are estimated from trade sources, by replies to questionnaire surveys, from trends of production, from cost curves, or by other techniques there remains a common problem on the aggregation of the industry figures into a national or other more comprehensive index. Even without deriving a single index, there remains the problem mentioned earlier of mutual compatibility, and this is closely related to the aggregation problem.

The most obvious solution is perhaps to weight each industry's contribution to an overall index by its share in value added.* As mentioned

---

* In appraising the Brookings study, H. Villard raised the question whether the weights should be current shares of value added or shares of value added at full capacity. See H. Villard, "Some Aspects of the Concept of Capacity to Produce" Review of Economic Statistics, Vol. XXI, 1939, pp. 13-20.

earlier, however, this may give rise to bottlenecks and not be capable of realization. A practical method of adjustment to avoid this problem is the following: given a set of capacity estimates for each industry, substitute them into a system of equations defined by an input-output matrix with a full-employment bill of final demand. If the capacity outputs do not satisfy this equation system adjust them so that the sum of squared adjustments are minimized. The minimization is not unrestricted, however. The adjusted values must never aggregate into a larger index of capacity output than the original set before adjustment.

An alternative approach has been suggested by A. Manne. Array the separate industries by the size of their percentage utilization of capacity. The industry (or group of industries) nearest to capacity operation can be raised to full utilization without creating any new resources in the economy and all other industries' outputs can be raised in the same proportion. We shall call this an effortless increase in output. If the original observed output data satisfied an input-output scheme

$$Ax = b,$$

the new derived outputs on the first round of increments will also satisfy it provided that final demand is increased (by the authorities) in the same proportion that all outputs are increased.*

---

* In the input-output model, curves of total cost are linear. Therefore an inconsistency is introduced when using an input-output model in aggregating capacity data derived from minimal points on U-shaped average cost curves.

$$Ap_1x = p_1b$$

A = input-output matrix

x = vector of industry outputs

b = vector of final demand

p_1 = reciprocal of operating rate of industry nearest to full utilization.

On the second round we raise the industry second nearest to full utilization up to its capacity output and raise all other outputs in the same proportion. The second industry and all industries below it in the array can effortlessly have their production at the initial state multiplied by p_2 (the reciprocal of the operating rate in the second industry), but the first industry cannot be increased above its full capacity rate, which it reached on the first round, without some investment. The calculation of the amount of investment needed to bring about a given increase in the first industry's capacity calls for the introduction of an accelerator coefficient of the form

$$I_i = \alpha_i \Delta(x_c)_i,$$

where

I_i = investment in the i-th industry

$(\Delta x_c)_i$ = increment in capacity output of the i-th industry.

α_i = accelerator coefficient of the i-th industry.

On the third round of application of this method, we raise the industry third nearest to full utilization up to its capacity output and raise all other outputs in the same proportion. This step calls forth necessary investment in the first two industries of the original array since both are pushed beyond capacity output. Two accelerator equations, each with different coefficients, in general, must be used.

This process is continued until investment potential is exhausted. In a practical problem of aggregation, the investment potential may be taken as last year's investment (or the highest previous of the past 5 years if last year's was not a record level). This is an attainable capacity. The amount attainable depends on the period being considered. In the shortest run, instantaneously, no investment is permitted, and round one ends the sequence. More investment is permissible over a longer time period, and more rounds are capable of being completed.

Some of the industries enjoying an increase in capacity may be industries producing capital goods, in which case the investment potential is expanded, and attainable capacity is not limited strictly by recent levels of investment.

The case of proportional expansion of all sectors is the simplest to compute, but a generalization is obviously not difficult. Let us consider the first round of expansion more generally. We begin with an initial situation in equilibrium (n x n)

$$Ax_0 = b_0$$

We raise the output of the first industry from x_{10} to $p_1 x_{10}$ and change each of the elements of b_0 by given amounts to $b_0 + \Delta b_0 = b_1$. Delete one equation from the system - that defining the distribution of the first industry's output - and determine the first round outputs from the reduced system [(n-1) x (n-1)]

$$A_{11} x_1 = b_1 - a_1 p_1 x_{10} \quad ,$$

where

A_{11} = minor obtained from A by deleting first row and column of A.

x_1 = vector consisting of $x_{21}, x_{31}, x_{41}, \ldots x_{n1}$ — the first round outputs of the remaining n-1 industries.

b_1 = vector consisting of $b_{20} + \Delta b_{20}, b_{30} + \Delta b_{30}, \ldots b_{n0} + \Delta b_{n0}$.

a_1 = vector consisting of $a_{21}, a_{31}, a_{41}, \ldots a_{n1}$ — n-1 of the elements in the first column of A.

$p_1 x_{10}$ = scalar capacity output of the first industry.

A similarly constructed system of order (n-2)x(n-2) would be used in the second round, and so on.

A simple numerical example has been worked out for the end of 1956, using the McGraw-Hill series on capacity to compare with their aggregate, weighted by shares in value added. In Table I, we have arrayed the industries by degree of utilization (as reported to McGraw-Hill) at the end of 1956, the Federal Reserve industrial production index for December 1956 on a 1950 base, the estimated ratio of investment in 1950 prices to change in the index of capacity on a 1950 production base. The last mentioned terms, the accelerator coefficients, are crudely estimated from the ratios of cumulated gross investment by industry sector, 1951-56, to the cumulated change in capacity over the same period.* Since net

* In some cases where investment data by industry are deficient, the cumulative period covered for estimating the ratio is shorter than 1951 - 1956.

investment data are not available by industry groupings, we were forced to add to the crudeness of our calculations by using gross investment figures.

In Table II we set out the successive rounds of computation. At each stage an industry in the array is brought to capacity and all other outputs are raised in constant proportion. The operating rate at each stage is defined as the ratio of actual aggregate production to actual aggregate production increased by an appropriate factor. The operating rate at each stage is therefore identical with the rate of the industries brought exactly to full utilization at that stage.

Table I

Array of Industries for Aggregation of Capacities and Utilization Rates.

Industry	Percent utilization of capacity end 1956	Index of production Dec. 1956 1950 base	Ratio investment to change in capacity Index on 1950 production base
Iron and steel	98	127	213
Petroleum refining	96	142	310
Paper and products	96	122	67
Nonferrous metals	92	115	38
Stone, clay and glass	90	134	59
Textiles	90	93	111
Rubber and products	88	115	20
Electrical machinery	87	165	28
Nonelectrical machinery	85	149	59
Miscellaneous manufacturing	85	124	208
Metal fabricating	83	123	73
Chemicals	83	148	97
Transportation equipment*	80	348	6
Food and beverages	80	111	150

* Excludes motor vehicles.

Table II

Aggregation of Capacities and Utilization Rates

Industry	Production Indexes and Investment at Each Round						
	Round						
	I	II	III	IV	V	VI	VII
Iron and steel	130*	132	138	141	144	146	150
Petroleum refining		148*	154	158	162	163	167
Paper and products		127*	133	136	139	140	144
Nonferrous metals			125*	128	131	132	135
Stone, clay and glass				149*	152	154	158
Textiles				103*	106	107	109
Rubber and products					131*	132	135
Electrical machinery						190*	194
Nonelectrical machinery							175*
Miscellaneous manufacturing							146*
Metal fabricating							
Chemicals							
Transportation equipment							
Food and beverages							
Required investment (mill 1950 dollars)	nil	426	3,963	6,160	8,862	9,955	13,069
Operating rate	.98	.96	.92	.90	.88	.87	.85

* full capacity.

During 1956, aggregate manufacturing investment totaled $ 15,036 mill. In 1950 prices this deflates to about $ 12.2 bill. At this level of investment, the operating rate, by Table II, should fall between .85 and .87. The McGraw-Hill average is happily at .86, but we should not expect such close agreement in general. The same technique applied to the estimates derived from trend lines through peaks gives a different array of industries and different estimates of utilization rates. In that case, the aggregate rate by the method developed here is 94 percent for the end of 1956. A weighted average, with value-added weights, gives 98 percent. Several industries were operating at or near a peak of their production time series; therefore, figures yielded by this method are higher than the McGraw-Hill series.

The Federal Reserve manufacturing index for December 1956 on a base of 1950 is 132. With a utilization rate of .86 as estimated from the McGraw-Hill data, we would put attainable capacity at 153.

<u>The measurement of capacity and the stock of capital</u>. One of our original reasons for attacking the problem of capacity measurement was to derive, if possible, a proxy measure for the stock of capital. Proper accounting for technical progress is the major obstacle to the derivation of adequate capital measures. Machines, buildings and other capital devices are continually changing in character. Since they last more than one accounting period, at any point of time there is a distribution of capital vintages which are difficult to combine. In actual calculations by the statistician, these problems arise at three stages. First, there is the estimation of an initial value. At this stage, all the different pieces of capital equipment in existence at some point of time must be valued in a common denominator and added

together. Obviously the measurement problem arises in full force here. Second, there is the evaluation of gross capital expenditures for each accounting period. The problem of price-level changes, one of the problems of measurement at this stage, is adequately under control by national income statisticians. Correction for inflationary or deflationary movements in prices does not, however, eliminate the fact that different commodities, as a result of technical progress, are involved in each period's expenditures. Third, we have the intractable problem of estimation of depreciation in order to derive net from gross investment. National income statisticians have made real progress in unscrambling accounting data on depreciation in order to form estimates on a common price level for comparison with gross investment. They have not, however, been able to make appreciable contributions to the specification of the time shape of depreciation (straight line or curvilinear, e.g.) or the appropriate inclusion of figures on obsolescence.

These three sets of difficulties leave the problem of capital measurement unsolved, and we might turn to our analysis of capacity measurement to see whether a usable proxy measure can be obtained. From a purely analytical point of view, we have mentioned already that capacity serves as a capital proxy only in case the production function has a special form, the clearest cut situation being fixed factor proportions and a constant capital -(capacity) output ratio. But even if these *a priori* conditions were to hold, some of the capacity measures would be subject to the same difficulties found for the estimation of capital stock. Engineering estimates or subjective estimates like those provided on the McGraw-Hill surveys may well be independent of capital measures, but the same cannot be said of measures that depend on the use of current investment estimates. The method of trends-through-peaks of production series, if interpolated by investment outlays between peaks, directly depend on capital measures with the same limitations caused by the inability to account for technical progress. With mechanical interpolation between peaks, the method still depends on investment data, as illustrated in the previous section, when an aggregative index is required.

In extending the capacity concept beyond engineering or purely technical considerations into the realm of economics, through the use of cost curves, it is not obvious that the problems connected with capital measurement can be avoided. If operating costs are chosen to exclude depreciation and obsolescence, the problematical aspects of capital measurement do not arise. This is the type of cost figure used in our empirical examples of electric power and steel production. The costs included there are materials, fuel, wages and salaries. These are variable operating costs. To some extent, depreciation depends on intensity of use, and the data on current operating costs should be expanded to include estimates of capital consumption. If this is done, we have not derived a capacity measure independent of the difficulties inherent in the measurement of capital. The full conditions for capacity output, viewed as a normative position, would seem to be best formulated in terms of the long run cost curve consisting of all elements of cost, including depreciation and obsolescence. In terms of this view, we certainly cannot avoid the problems of capital measurement.

Smithies, in a highly suggestive and provocative article, has taken up points that inspired some of the work in the present paper.* His

* A. Smithies, "Economic Fluctuations and Growth", Econometrica, Vol. 25, January, 1957, pp. 1-52.

macroeconomic model consists of three basic equations - a ratchet-type consumption function, an investment function with ratchet and capacity variables, and a capital-capacity equation. The consumption equation is not germane to the present discussion, but the other two are. The investment equation yields a relation between gross capital outlays, current output, highest previous output, and capacity output. This equation is an improvement over the simple orthodox version of the acceleration principle in which net investment is made a linear or proportional function of the change in output.* It is also an improvement

* In this connection see also the discussion by F. Modigliani, comment on the paper by B. G. Hickman, op.cit., pp. 450-63.

over those generalized versions which allow for excess capacity by making investment a linear function of output and the stock of capital. Instead of using the stock of capital, Smithies uses the level of capacity output as a variable in the investment equation. This brings us a step nearer to an "autonomous" relationship and thereby gives a better expression for investment behavior on the demand side. At this stage of his analysis

Smithies has possibly skirted successfully the problems of capital measurement by using capacity as a variable in the investment function. Such was his announced intention. But the system is not closed without the third equation explaining the relationship between the stock of capital and capacity or, as Smithies puts it, between investment and the change in capacity. In this equation, Smithies reintroduces all the difficulties of capital measurement into his model. Had he written it as a capital-capacity relation, this would have been quite evident. Substitution from a simple capital-capacity equation into the investment equation to eliminate the capacity variable would have made his system look nearly the same as the conventional multiplier-accelerator system. In Smithies' model, as he has written it, the change in capacity is a function of gross investment, depreciation, and obsolescence. In essence he has not, by use of such an equation, avoided the problems of capital measurement; he has simply assigned more specific and appropriate roles to the concepts of capacity and capital in the economic process. The measurement problem remains as before.

The Choice among Alternative Measures. In this essay, I have spoken about and made use of several alternative measures of industrial capacity. For a quick set of estimates of high practical value, I would suggest use of the Federal Reserve index (or an extension of it to include more industries) or of the method of trend lines through production peaks. For a more basic treatment of the problem, I would prefer the use of the approach through the estimation of cost functions, but this would involve detailed analysis of each industry throughout the economy. This approach has the effect of bringing economic as well as technical considerations into the problem.

In studying investment behavior, subjective motivation of entrepreneurs is the important consideration, and perhaps the McGraw-Hill data gathered from individual businessmen as expressions of their own estimates of capacity are relevant. Regardless of whether these are good technical estimates and whether they refer to identical concepts of capacity, they may be quite useful in studying investment behavior since they are expressions of what the entrepreneurs think their capacity levels are.