5-1-1957

Theory of Teams: Introduction

Jacob Marschak

COWLES FOUNDATION DISCUSSION PAPER NO. 31

Theory of Teams: Introduction.*

By

Jacob Marschak**

May 28, 1957

---

* Research undertaken by the Cowles Commission for Research in Economics under Contract Nonr-358(01), NR 047-006 with the Office of Naval Research

** Presented at the Seminar on Techniques of Industrial Operations Research, Illinois Institute of Technology, Chicago, June 14, 1957. Acknowledgments are due to Drs. Debreu and Radner of the Cowles Foundation.

Table of Contents.

Theory of Teams: Introduction.

1. SINGLE-PERSON, SINGLE-STAGE DECISIONS: BEST DECISION RULES AND INFORMATION STRUCTURES.

1.1. Concepts and problem. The following list of notations will be presently illustrated on simple examples. It is given at the outset for future reference.

$x = (x_1, \ldots, x_m)$ = external variable(s): a random vector with given probability distribution.

y = information (generally incomplete) about x ; $y = \eta(x)$.

η = "information structure."

a = action based on information y ; $a = \alpha(y)$.

α = decision rule.

ω = payoff function.

$u = \omega(x,a)$ = gross payoff $= \omega(x,\alpha(\eta(x)))$.

$Eu = E\omega(x,\alpha(\eta(x))) = U(\alpha,\eta)$ = expected gross payoff.

$U_\eta = \max_\alpha U(\alpha,\eta) = U(\hat{\alpha}_\eta,\eta)$ = best expected gross payoff for given information structure. This also defines:

$\hat{\alpha}_\eta$: best decision rule for given η .

γ = information cost function.

$\gamma(\eta)$ = information cost.

$V(\alpha,\eta) = U(\alpha,\eta) - \gamma(\eta)$ = expected net payoff.

$\hat{V} = \max_{\alpha,\eta} V(\alpha,\eta) = V(\hat{\alpha},\hat{\eta})$ = best expected net payoff. This also defines:

$\hat{\alpha}$ = best decision rule; $\hat{\eta}$ = best information structure.

Problem: find $\hat{\alpha}$, $\hat{\eta}$, given the payoff function ω and the information cost function γ .

1.1.1. Information Structure. In general, an information structure is a partition of the set of states of nature into subsets; that is, in our notation, a partition of the m-dimensional space of external variables. Information (observation made, message received) is the statement that nature is in one of those subsets. If, under information structure η, the set of states of the world is divided into certain subsets, and if, under η', these same subsets are further partitioned, we say that η' is a "finer" structure then η'. We can suppose that to use a less fine structure is never more expensive than to use a finer one. Note also that η' will contain a greater amount of information in Shannon's sense.

1.2. Special assumptions.

a) η will be a binary number: according as its i-th digit is 1 or 0, information y will include precise knowledge of the value taken by x_i or no knowledge of it. (In a more general case--not treated here--η would give the measure of precision with which each variable is to be observed.)

b) x will be (approximately) normal, with zero mean and with covariances σ_{ij}. Write: $\sigma_i = \sigma_{ii}$; correlation coefficient $\rho_{ij} = \sigma_{ij}/\sigma_i\sigma_j$.

1.3. A case of linear payoff in a bounded action variable.

Let $0 \le a \le 1$; $u = a \cdot \sum_1^m x_i$. Economic interpretation: a = scale of operations, its maximum (the "capacity") being chosen as a unit of scale; let $\mu_i + x_i$ = price of i-th output (if positive) or input (if negative); and $\sum \mu_i = 0$. Then u = profit.

1.3.1. Case m = 2 ; one input, one output.

In this case η has 4 values: (11), (00), (10), (01), to be considered in this sequence:

$\eta = (11)$; $y = (x_1, x_2)$; $\hat{\alpha}(y) = \begin{matrix} 1 \\ 0 \end{matrix}$ if $x_1 + x_2 \begin{matrix} > \\ \leq \end{matrix} 0$.

$U_{11} = E(x_1 + x_2) | (x_1 + x_2 > 0)$.

If φ is the normal density function with variance σ^2 then $\int_0^\infty t\, \varphi(t) dt = \sigma / \sqrt{2\pi}$. Therefore,

(1) $U_{11} = \sqrt{\text{Var}\ (x_1 + x_2)/2\pi} = \sqrt{(\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2)/2\pi}$, where $\rho = \rho_{12}$.

Now turn to $\eta = (00)$. Then y is "no knowledge of values of x_1 or x_2," $\hat{a} = \hat{\alpha}(y)$ is constant, and for any action $\hat{a}$, $E(x_1 + x_2) = 0$, hence

(2) $$U_{00} = 0.$$

Thus $U_{11} \geq U_{00}$; it also follows that, if x_1, x_2 could only be observed together [as in (11)] or not at all [as in (00)], so that $x_1 + x_2$ could be regarded as a single variable, the advantage $U_{11} - U_{00}$ of observing this single variable would be proportional to its standard deviation. If and only if this advantage exceeds the cost of observing the variable, it pays to do so.

Now let $\eta = (10)$; $y = (x_1)$; $E(x_1 + x_2) | x_1 = x_1(1 + \rho \frac{\sigma_2}{\sigma_1})$.

$$E(E(x_1 + x_2) \mid x_1) | x_1 > 0) = (\sigma_1 + \rho\, \sigma_2)/\sqrt{2\pi}$$

If $\sigma_1 + \rho\, \sigma_2 \geq 0$, $\hat{\alpha}(x_1) = \begin{matrix} 1 \\ 0 \end{matrix}$ if $x_1 \begin{matrix} > \\ \leq \end{matrix} 0$; $U_{10} = \frac{\sigma_1 + \rho\, \sigma_2}{\sqrt{2\pi}}$;

If $\sigma_1 + \rho\, \sigma_2 < 0$, $\hat{\alpha}(x_1) = \begin{matrix} 0 \\ 1 \end{matrix}$ if $x_1 \begin{matrix} > \\ \leq \end{matrix} 0$; $U_{10} = -\frac{\sigma_1 + \rho\, \sigma_2}{\sqrt{2\pi}}$.

(3) Summarizing: $U_{10} = |\sigma_1 + \rho\, \sigma_2| / \sqrt{2\pi}$;

Finally let $\eta = (01)$; $y = (x_2)$. Then, by symmetry with (3),

(4) $U_{01} = |\sigma_2 + \rho\, \sigma_1| / \sqrt{2\pi}$

Hence $U_{11} \geq \max(U_{10}, U_{01}) \geq U_{00}$. Moreover, U_{10} is larger (smaller) than U_{01} if σ_1 is larger (smaller) than σ_2 . Without loss of generality, let $0 \leq \sigma_2 \leq \sigma_1 = 1$ (in the above economic interpretation, $\sigma_1 = 1$ implies an appropriate choice of money unit); then

(5) $$U_{11} \geq U_{10} \geq U_{01} \geq U_{00} = 0 \; .$$

On Chart 1, the maximal expected gross profits under each of the four information structures are plotted against the correlation coefficient ρ, given $\sigma_2 = \frac{1}{2}$. The results confirm two intuitive conjectures: 1) that it pays more to get information about a highly volatile variable then about a less volatile one; 2) that, since higher correlation (positive or negative) allows better estimation of a variable from the knowledge of the other one, the advantage of knowing two rather than one variable is small when the absolute value of the correlation coefficient is low.

The latter consideration might suggest that the gross expected payoffs, as well as the differences between them (i.e., the comparative advantages of the information structures) are symmetrical functions of the correlation coefficient, with extrema (if any) at $\rho = 0$. This is not so. The asymmetry (seen in Chart 1) can be made plausible in terms of the economic interpretation, by the fact that a high positive correlation between output price and input price (i.e., in our notation, a high negative correlation between x_1 and x_2) tends to make the average of positive profits low; and it is the positive profits that one tries to make, (by putting $a = 1$), and the negative ones, to avoid (by putting $a = 0$) .

To compare the best net expected profits, we must make assumptions about information cost. Let c = cost of being informed about any one of the two variables: thus, $\gamma(00) = 0$, $\gamma(10) = c = \gamma(01)$, $\gamma(11) = 2c$. Then, by (5), the information structure (01) is inadmissible. The comparison between

(6) $$V_{11} = U_{11} - 2c \; ; \; V_{10} = U_{10} - c \; ; \quad \text{and} \quad V_{00} = U_{00} \; ,$$

for given ρ and σ_2, results (for $c = 0.1$) in Chart 2. We see that, under our assumptions, it pays to observe both variables only if the smaller of the variances (σ_2) is not too low, and the absolute value of the correlation coefficient not too high; but high variances paired with high negative correlation make it best to close shop (or, with a slightly different economic interpretation, to proceed in a routine manner, using a constant scale of operation without paying attention to external variations) !

1.3.2 Case $m > 2$. In this general case, each of the 2^m information structures is defined by picking out the (possibly empty) subset S consisting of those variables that are to be observed. After computing, for each S, the expectation

$$E \sum_{i=1}^{m} x_i \mid \left(\sum_{j \in S} x_j > 0 \right) ,$$

(an exercise in multivariate regressions) one finds the best decision rule for each S, and computes the corresponding gross expected profit.

2. n-PERSON SINGLE-STAGE TEAM

2.1 Concepts and problem. The previous concepts are generalized by re-interpreting the symbols η, y, α, a as n-tuples; the subscript i will refer to an action variable a_i (a component of a), controlled by the team member i. Then

$y_i = \eta_i(x)$ = information of i-th member

η_i = information structure for i-th member

$\eta = (\eta_1, \ldots, \eta_n)$ = team information structure

α_i = decision rule for i-th member

$a_i = \alpha_i(y_i)$ = action of i-th member

$$\alpha = (\alpha_1, \ldots, \alpha_n) = \text{team decision rule}$$

$$a = (a_1, \ldots, a_n) = \text{team action}$$

2.2. Additive teams. If it is possible to decompose the gross payoff into additive components

$$u(x,a) = \sum_1^n u_i(x, a_i) \ ,$$

then we say the team is additive, and there is "no interaction" between its members. In this case the only reason for communication between members can lie in the fact that some members have better access to certain types of information than others. In this case, it is easy to apply the principles developed in Section 1. Let c be the cost of establishing a communication link; let η_i' and η_1, respectively, be two information structures for the i-th member, with and without that link. Then the communication link with a cost c is only worth establishing if

$$\max_{\alpha_i} U_i(\alpha_i, \eta_i') - \max_{\alpha_i} U(\alpha_i, \eta_i) > c \ ,$$

where $U_i = Eu_i$.

2.3. Information matrix for a team. If we assume as before that the information structure η_i for the i-th member is a binary number, the information structure for the team becomes a matrix $\eta = || \eta_{ij} ||$ where $\eta_{ij} = 1$ or 0 according as the i-th member does or does not receive information on the external variable x_j.

Note that, in general, an action variable a_i ascribed to the i-th partner may itself be a bundle of physically described actions; and similarly with the external variables.

2.4. Information costs.

2.4.1. Fixed and variable costs. The fixed information costs (which may be also called the cost of the network) are not only the installment costs of physical equipment for information and communication but also, more essentially, the executive salaries paid on long-term contracts to the extent to which the time is taken by gathering and exchanging information. [The rest of executives' salaries is paid for decision making, and we might well add a cost of decision making $\delta(\alpha)$, to the cost of information $\gamma(\eta)$ in our conceptual scheme.] We may occasionally distinguish the cost of each observation post and the cost of each communication link.

The variable costs are those costs depending on the degree of use of the equipment and of the long-term personnel: these include the salaries on short-term contracts.

2.4.2. Information cost as a function of "fineness." A "finer" information structure in the sense of 1.1.1 can never be less costly than a coarser one. This statement is weaker than the following one: the larger the amount of information the larger the cost. For, the case when one information structure is finer then another is more special than the case of two differing information amounts.

2.5. The case of co-specialization of action and information. In this important case, there is a one-to-one correspondence between action-variables and external variables, in the following sense: the observation cost of the i-th external variable to the i-th partner is smaller than it is to any other partner. Thus each member is a "specialist" in a particular external variable.

In this case, the number of admissible information structures is reduced. For $n=2$, denote each network by a pair of symbols, of which the first refers to $i=1$, and the second to $i=2$. The symbol $\square$ will mean "no observation performed," and x will mean "observation performed." An arrow $\longrightarrow$ denotes communication; a double arrow $\longleftrightarrow$ means two-way communication. Then, of the $2^4 = 16$ possible information structures, only those generated by the 9 following networks remain admissible

1) □ □ 2) x □ 3) □ x 4) x x

5) x ⟶ □ 6) □ ⟵ x 7) x ⟶ x 8) x ⟵ x 9) x ⟷ x

2.6. Linear additive team. Its payoff is

$$u = \sum_{i=1}^{n} u_i \ , \ u_i = a_i \sum_{i=1}^{m} x_j \ ; \ 0 \leq a_i \leq 1 \ , \ i = 1,\ldots,n$$

Because of decomposability, the expected gross payoff for a given information matrix $\eta = ||\eta_{ij}||$, is obtained by computing $\max_{\alpha_i} Eu_i(\alpha_i,\eta_i)$ for each row (following 1.3.2.), and adding over all rows. (We still assume normal distribution of x, for simplicity).

2.6.1. The case n=2 , with co-specialization. Assume further for illustration's sake that the cost of each observation post = cost of each communication link = $c > 0$. Then the net expected payoffs of the nine networks listed in (2.5) are as follows:

1) 0 ; 2) $U_{10} - c$; 3) $U_{01} - c$; 4) $U_{10} + U_{01} - 2c$;

5) $2U_{10} - 2c$; 6) $U_{01} - 2c$; 7) $U_{10} + U_{11} - 3c$; 8) $U_{11} + U_{01} - 3c$;

9) $2U_{11} - 4c$.

If, as in 1.3.1, we let (without loss of generality) $\sigma_2 \leq \sigma_1$, then $U_{01} \leq U_{10}$, and there remain the net payoffs of only three admissible networks:

1) $V(\square\ \square) = 0$; 5) $V(x \longrightarrow \square) = 2(U_{10} - c)$; 9) $V(x \longleftrightarrow) = 2(U_{11} - 2c)$.

Clearly, the conditions for any one of these three networks to dominate the other two are identical with the conditions, developed in 1.3.1, for any of the 3 single-person information structures (00), (10), (11) to dominate the other two.

Hence, Chart 2 can again be used, with the 2-person symbols

$\square\ \square$ $\qquad$ $x \longrightarrow \square$ $\qquad$ $x \longleftrightarrow x$

replacing, respectively, the 1-person information structure symbols

(00) $\qquad$ (10) $\qquad$ (11) .

2.7. <u>A necessary condition.</u> Looking back at the best decision rules α_1 , α_2 derived and used in our team example, we see that they satisfy simultaneously the following conditions: α_1 is that function of y_1 which maximizes the conditional expectation $Eu(x_1,x_2,\alpha_1(y_1),a_2) \mid y_1$; and α_2 is the function of y_2 that maximizes the conditional expectation $Eu(x_1,x_2,a_1,\alpha_2(y_2)) \mid y_2$. Radner has proved that this necessary condition applies to a team with <u>any</u> payoff.

2.8. <u>Decision Skills</u>. We can conceive of a person's decision skill as a set of <u>past</u> values of the external variable, retained in his memory; or, more generally, as a set of <u>constants</u> known to him. If the "organizer" of the team does not have the same knowledge, the precise determination of the decision function will be left to each member. For example, let us modify the payoff function just used, $u = (a_1 + a_2)(x_1 + x_2)$, into a new function, $u = (a_1 + a_2)(x_1 - \xi_1 + x_2 - \xi_2)$, where the constant ξ_i is known to the i-th partner. Suppose the network is $(x \;\; x)$.

The decision rule,

$$\hat{\alpha}_i(x_i) = \begin{matrix} 1 \\ 0 \end{matrix} \text{ if } x_i \begin{matrix} > \\ \leq \end{matrix} 0 ,$$

will then be replaced by

$$\hat{\alpha}_i(x_i) = \begin{matrix} 1 \\ 0 \end{matrix} \text{ if } x_i \begin{matrix} > \\ \leq \end{matrix} \xi_i ;$$

the constant ξ_i, known to the i-th partner, is possibly unknown to the team's "organizer." The decision rule is then prescribed to each member only in a form that is not fully specified. It is for the member himself to specify it fully.

However, if the organizer has to choose the optimal network, he will have to assign at least a probability distribution to the constants ξ_1, ξ_2. Stated in this form, the problem is reduced to the original one.

2.9. Authority: Communication of Decision vs. Communication of Events.

In the example of 2.6., the asymmetry of the network with respect to the two partners arises only from a difference in the variances of x_1 and x_2: if $\sigma_1 > \sigma_2$ then it is more useful to let the value of x_1 be told to the second partner by the first, than to let the value of x_2 be told to the first partner by the second. More of the talking should be done by the man who has access to relatively uncertain events than by the man who observes relatively certain ones.

Is this perhaps the reason why the tribal priest (in charge of predicting weather, thus setting the day for planting crops) has more "power" than the women (in charge of making pottery on the basis of rather constant properties of clay)? It may be objected, however, that "power" does not usually consist in the role of communicating events (x_1, in our case).

We may now add, on the basis of 2.7., and using the modified payoff example presented there: it is the high variance of $(x_i - \xi_i)$ that makes the i-th partner the communicating one. But the variance of ξ_i merely measures the ignorance of the organizer about the value of ξ_i known precisely to the i-th member. That is, the latter must possess a "special," "expert" knowledge.

However, even this "expert" reporting will be felt by many as not coinciding with "power," or "leadership." The usual form in which a "leader" communicates is in telling of his decision (or that part of it relevant to the subordinate's action), not of the state of external variables. Moreover, the difference in skills is not essential: it is easy to give examples of operations in which one person gives signals to be followed by the other, and where such division of functions is useful even if no different skills are required: as when, for example, one person gets out of the car to help the other park it, by signalling the needed movements.

In our example of 2.5., with network $x \longrightarrow \square$ (and with $\sigma_1 > \sigma_2$), it is sufficient for the second member to learn from the first, not the precise value of x_1 but only whether it is positive or negative; and this can be inferred from the first person's decision (viz., $a_1 = 1$ or 0 according as $x_1 >$ or ≤ 0). Thus while the amount of information to the second person is decreased, the payoff remains the same. Now, it is cheaper to communicate when there are only two possible events to tell about, than when there are more than two (see 1.1.1 above). Therefore, it will be more advantageous, in the case under discussion, if the first member tells of his decision than if the tells the value of x_1 .

[One can also introduce the concept of "communication skill": the knowledge, by member 1, of how best to map <u>his</u> information into the information for member 2: that is, which "code" is cheapest and best understood.]

In the previous example, with the network $x \longrightarrow \square$, the communication of the value of x_1 contributed to the payoff exactly as much as the (presumably cheaper) communication of the decision a_1 . Therefore, the latter method was clearly preferable. In general, the choice will not be so simple. By communicating a smaller amount of information, the payoff may be diminished. If it is diminished by less than the diminution of the cost of communication, the cheaper form of communication is preferable; in particular, the "authoritative" way may be the preferable one. As an example, consider the network $x \longrightarrow x$ and compare the expected payoff produced by member 2 if he observes x_2 and, in addition, receives information: (I) on the value of x_1 or (II) on the action a_1 and therefore on the sign of x_1 . In case (I), the expected payoff is the quantity U_{11} computed in equation (1), Sec. 1.3.1. If we assume, for simplicity $\sigma_1 = \sigma_2 = 1$, and $\rho = 0$, then $U_{11} = 1/\sqrt{\pi}$. What is, under the same assumptions, the expected payoff in case (II)? In this case, the second member's best decision rule is as follows: "$a_2 = 1$ if and only if either $a_1 = 1$ and $x_2 + (Ex_1 \mid x_1 > 0) > 0$ or $a_1 = 0$ and $x_2 + (Ex_1 \mid x_1 < 0) > 0$."
This yields an expected payoff

$$U'_{11} = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{1}{4\pi}\right)$$

The difference $U_{11} - U'_{11}$ (= about 0.38) is the information cost saving that would have to be achieved by the "authoritative" method (communication of decision) in order to justify its use, instead of communicating full information about the external variable x_1 .

2.10. Non-additive teams.

2.10.1. A non-smooth example: Consider the production and the sales department of a firm. To each production level corresponds a certain action, of the production department--purchase of raw materials, etc.--and a certain expenditure depending on external variables such as the prices of raw materials. To each sales level corresponds a certain expenditure of the sales department for promotion, etc., depending on external variables such as the prices of media and the state of the market. Denote the value produced by a_1 dollars, and the value sold (i.e., promised to be delivered to customers) by a_2 . Let the corresponding expenditures be $f_1(a_1,x_1)$ dollars and $f_2(a_2,x_2)$ dollars respectively, where the parameters x_1,x_2 are the relevant external random variables. If the commodity is perishable (no storage possible) the profit (payoff) is $\omega = \min(a_1,a_2) - f_1(a_1,x_1) - f_2(a_2,x_2)$. We can assume each f_i differentiable, with $\partial f_i/\partial a_i > 0$. If, moreover, $\partial^2 f_i/\partial a_i^2 > 0$ (increasing marginal cost of production as well as of sales), Chart 3 represents the lines of equal profit, for varying pairs a_1, a_2 and a fixed pair of values of x_1 and x_2 . Maximum profit is reached at a certain point of the line (a "ridge") $a_1 = a_2$.

Clearly, the payoff is non-decomposable into two separate payoffs, one to each department. This non-additive nature of the payoff can also be expressed by saying that there is interaction between the actions a_1 and a_2: At least at some points, the effect of an increase in a_1, with a_2 unchanged, does depend on the size of a_2. True, there is no such effect as long as a_1 both before and after the increase remains say, below a_2: changing from, say a_1^o to $a_1^o + h < a_2$. For then the increase in the payoff will be equal to $f_1(a_1^o + h) - f_1(a_1^o)$, hence it will be independent of a_2. However, if $a_1^o < a_2 \leq a_1^o + h$, the payoff increases by $(a_2 - a_1^o) + f(a_1^o + h)$; and this quantity does depend on a_2.

In a modified form, this model will be closely analysed in R. Radner's paper presented at this seminar.

2.11. <u>A Smooth Model with a constant interaction coefficient</u>. If the payoff is twice-differentiable, the interaction coefficient between two action variables a_i, a_j can be defined simply as $\partial^2 u/\partial a_i \partial a_j$. We shall consider, in particular, the case of a payoff quadratic in the action variables, and with $\partial^2 u/\partial a_i \partial a_j$ independent of the x's. Then the interaction coefficient is a constant. As an example (with $n = 2$) consider

$$u = - a_1^2 - a_2^2 + 2q\, a_1 a_2 - a_1 x_1 - a_2 x_2 + u_o;\ 0 < q < 1 .$$

This payoff function may be given the following economic interpretation: u is the profit of a firm, if x_1 and x_2 are the deviations of the prices of the two inputs from their means; a_1 and a_2 are the inputs, measured in appropriate units from appropriate origins (see below); u_o is then a linear combination of x_1 and x_2, independent of a_1 and a_2. The expression $(- a_1 - a_2^2 + 2q\, a_1 a_2)$ gives the physical output (up to a constant term) and also the money revenue of the firm, if the price of an output unit is constant (=1). The lines of equal profit are given on <u>Chart 4</u>. They are ellipses (circles when $q = 0$).

Note, that in the assumed payoff function - unlike that of the payoff previously discussed in 2.7 - $\partial^2 u/\partial a_i \partial x_j = 0$ $(i \neq j)$: the effect of changing the i-th input does not depend on the price of the j-th input. Hence the reason for communication which existed in that earlier example does not exist in the present one. This will make it possible to isolate the role of the interaction coefficient in determining the best network.

Radner has shown that in the "smooth" case, the necessary condition of 2.7 is also sufficient; and that in the case of quadratic payoff functions and normally distributed external variables the best decision functions (given the information structure) are linear in the x's. It suffices therefore to maximize simultaneously the two conditional expectations $Eu|y_1$ and $Eu|y_2$ with respect to the unknown coefficients.

Since the (rather simple) role of the ratio σ_2/σ_1 was already discussed in a previous example, we shall assume simply $\sigma_1 = \sigma_2$, and concentrate on the role of ρ and q. We shall study again a case of co-specialization (as in 2.6.1). These assumptions reduce the problem to the choice among six networks; for each of these, the optimal decision rules and the best gross payoff can be read off <u>CHART 5</u>. Note that the first and last lines of the chart convey the meaning of the origins from which a_1 and a_2 were measured. These origins are the "normal" optimal inputs in the following sense: when the prices of the inputs are not known, or whether they are at their average levels (so that $x_1 = 0 = x_2$), then the best inputs are at their origins.

Chart 6 is derived from the gross expected payoffs of Chart 5, and the additional assumption that each observation post and each communication link has a cost $c = \frac{1}{3}$.

3. MULTI-STAGE TEAMS.

3.1. A multi-stage single-person problem as a team problem. Denote by a_i the person's action on the i-th "day," based on the information on a single external variable received on this and each of the previous days, beginning with day 1. If the information received on the j-th day is x_j , the action rules (the "strategy") are $\alpha_1, \alpha_2, \ldots$:

$$a_1 = \alpha_1(x_1);\ a_2 = \alpha_1(x_1, x_2); \ldots;\ a_m = \alpha_m(x_1, \ldots, x_n) ,$$

if the planning has a horizon of m days. The strategy $\alpha = (\alpha_1, \ldots, \alpha_m)$ can be chosen as if the person were a team, with a triangular information matrix (compare 2.3)

$$\begin{pmatrix} 1 & 1 & 1 & \ldots & 1 \\ 0 & 1 & 1 & \ldots & 1 \\ 0 & 0 & 1 & \ldots & 1 \\ - & - & - & - & - \\ 0 & 0 & 0 & \ldots & 1 \end{pmatrix}$$

where the i-th row corresponds to the action on the i-th "day," and the j-th column corresponds to the information on that "day." Such information matrix assumes perfect memory. It can be modified if, for example, memory lasts for at most $k(< m)$ "days."

It is also obvious how the matrix will be modified if certain external variables are observed on certain days and other variables, on others. It remains true under all these modifications, that, for a given information matrix, the best strategy and the resulting expected payoff can be computed entirely as in section 2.

3.2. Extension to n persons. Feedback. This extension is also obvious. We can also (as in Section 2.9) allow for communication of decisions, including lagged ones. Also, we can now allow for "feedbacks," i.e., the communications about the changes in the state of nature resulting from past action.

3.3. Information cost in multi-stage teams. In addition to the number of links, observation posts, and the number of decision makers--see 2.4.1 above--a further cost factor can now be added: the number of communication pulses, or some other measure of time needed to convey information to a given member of the team. Another, and more fundamental approach to the cost of delays between external events and team decisions, would involve the probability distribution of external variables over time: the damage due to delay is the larger the more probable is a large difference between the state of the world at the time of an event and the time of a decision based on it. Hence, the larger the variability of an external variable the more important it is to diminish the delay of responding to it, and hence to diminish the number of "pulses."

Chart 3

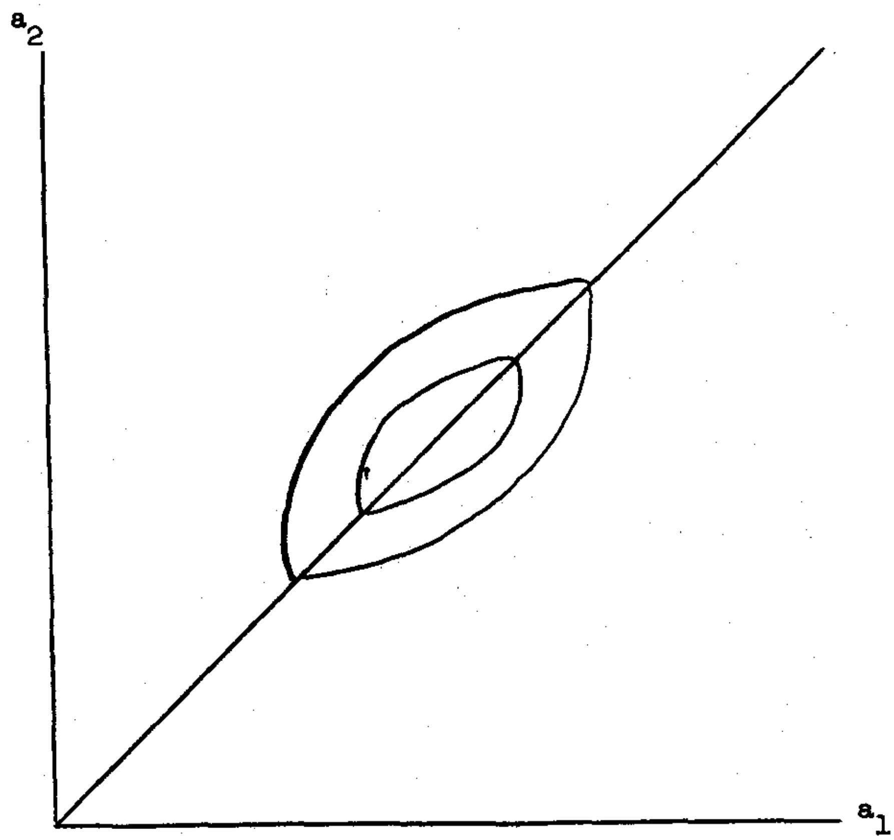


Chart 3. Lines of equal profit.
Profit function: $\min(a_1,a_2) - f_1(a_1) - f_2(a_2)$.

Chart 4

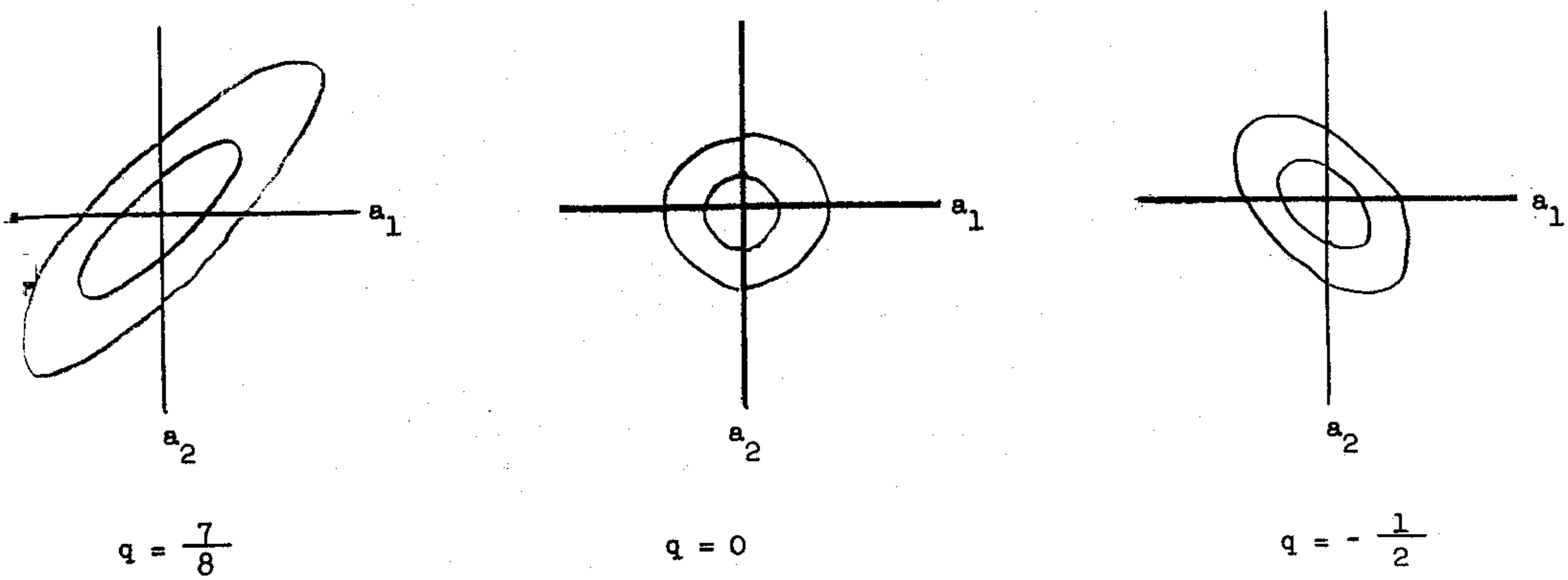


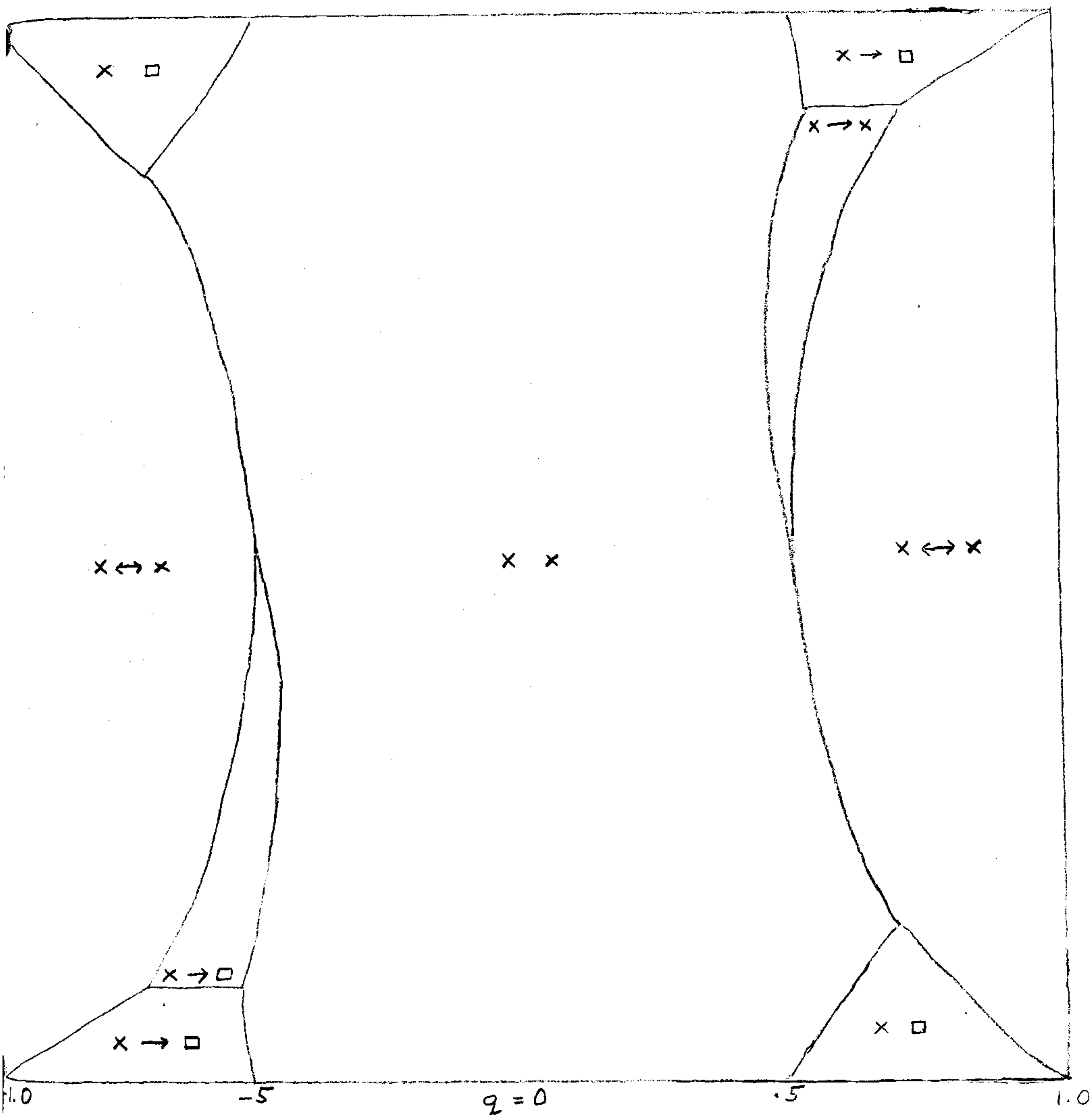
Chart 4. Lines of equal profit. Quadratic profit function, with $-1 < q < 1$. Maximum profit is at the origin.

Chart 5

ork	Best decision rule $\hat{\alpha}_1$: Coefficient of x_1	x_2	Best decision rule $\hat{\alpha}_2$: Coefficient of x_1	x_2	Best expected gross payoff
□	0	0	0	0	0
□	$-\frac{1}{2}$	0	0	0	$\frac{1}{4}$
□	$-\frac{1+\rho q}{2(1-q^2)}$	0	$-\frac{q+\rho}{2(1-q^2)}$	0	$\frac{1+2\rho q+\rho^2}{4(1-q^2)}$
x	$-\frac{1}{2(1-\rho q)}$	0	0	$-\frac{1}{2(1-\rho q)}$	$\frac{(1+\rho q)}{2(1-\rho^2 q^2)}$
x	$-\frac{1+\rho q}{2(1-q^2)}$	0	$-\frac{2(1+q\rho)}{2(1-q^2)}$	$-\frac{1}{2}$	$\frac{2(1+\rho q)-q^2(1-\rho^2)}{4(1-q^2)}$
x	$-\frac{1}{2(1-q^2)}$	$-\frac{q}{2(1-q^2)}$	$-\frac{q}{2(1-q^2)}$	$-\frac{1}{2(1-q^2)}$	$\frac{1+\rho q}{2(1-q^2)}$

Chart 5. Quadratic team with $\sigma_1 = \sigma_2 = 1$. Of the nine admissible networks six are shown. For the other three (□ x, □ ← x, x ← x), the entries can be obtained by interchanging the subscripts 1 and 2.

Chart 6. Choice of networks for quadratic payoff function.



Regions of (q, ρ)-plane in which each network is best.
q = interaction between inputs.
ρ = correlation between input prices.
Variance of each input price = 1.
Cost of each observation post or communication link = 1/3.