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IDENTIFICATION IN DIFFERENTIATED PRODUCTS MARKETS

By

Steven T. Berry and Philip A. Haile

August 2015

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Identification in Differentiated Products Markets

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Abstract

Empirical models of demand for—and, often, supply of—differentiated products are widely used in practice, typically employing parametric functional forms and distributions of consumer heterogeneity. We review some recent work studying identification in a broad class of such models. This work shows that parametric functional forms and distributional assumptions are not essential for identification. Rather, identification relies primarily on the standard requirement that instruments be available for the endogenous variables—here, typically, prices and quantities. We discuss the kinds of instruments needed for identification and how the reliance on instruments can be reduced by nonparametric functional form restrictions or better data. We also discuss results on discrimination between alternative models of oligopoly competition.

1 Introduction

Over the last twenty years there has been an explosion of empirical work using models of differentiated products demand and supply that build on Berry et al. (1995) ("BLP"). Initially this work focused on traditional topics in industrial organization such as market power, mergers, or the introduction of new goods. But as Table 1 suggests, these models are being applied to an increasingly broad range of markets and questions in economics. Of course, a quantitative understanding of demand and/or supply is essential to answering many positive and normative questions in a wide range of markets, and most markets involve differentiated goods. Empirical models following BLP are particularly attractive for such applications because they allow for both heterogeneity in consumer preferences and product-level unobservables. The former is essential for accurately capturing consumer "substitution paterns" (own- and cross-price demand elasticities) while the latter makes explicit the source of the endogeneity problems that we know arise in even the most elementary supply and demand settings.

Table 1: Example Markets and Topics

Topic	**Example Papers**
transportation demand	McFadden et al. (1977)
market power	Berry et al. (1995), Nevo (2001)
mergers	Nevo (2000), Capps et al. (2003)), citeFan
welfare from new goods	Petrin (2002), Eizenberg (2011)
network effects	Rysman (2004), Nair et al. (2004)
product promotions	Chintagunta & Honoré (1996), Allenby & Rossi (1999)
environmental policy	Goldberg (1998)
vertical contracting	Villas-Boas (2007), Ho (2009)
equilibrium product quality	Fan (2013)
media bias	Gentzkow & Shapiro (2010))
asymmetric info & insurance	Cardon & Hendel (2001), Bundorf et al. (2012), Lustig (2010)
trade policy	Goldberg (1995), Berry et al. (1999), Goldberg & Verboven (2001)
residential sorting	Bayer et al. (2007)
voting	Gordon & Hartmann (2013)
school choice	Hastings et al. (2010), Neilson (2013)

Until recently, however, identification of BLP-type models was not well understood. One often encountered informal speculation—either that the models were identified only through functional form restrictions, or that certain model parameters were identified by certain moments of the data. But there was no formal analysis applicable to this class of models. Here we discuss some recent work on this topic, focusing primarily on results presented in Berry et al.

(2013), Berry & Haile (2014), and Berry & Haile (2010).[1] This work considers nonparametric generalizations of the BLP model and provides sufficient conditions for identification of demand, identification of firms' marginal costs and cost functions, and discrimination between alternative models of firm behavior.

Broadly speaking, these papers show that when it comes to identification, there is nothing special about BLP-type models. Even in a fully nonparametric specification, the primary requirement for identification is the availability of instruments providing sufficient exogenous variation in prices and quantities. These results both provide comfort—that the functional forms and distributional assumptions used for approximation in finite samples are not essential—and clarify which kinds of instrumental variables can suffice.

In this review we give an overview of some key ideas and results from this recent work. We also discuss practical implications for applied work, focusing in particular on the types of instruments yielding identification of the most flexible models, as well as tradeoffs between the flexibility of the model (e.g., between alternative weak separability restrictions), the types of data available (e.g., market-level vs. individual-level), and the demands on instrumental variables.

1.1 A Starting Point: The BLP Demand Model

On the demand side, BLP builds on classic models of demand with endogenous prices, on the rich literature discussing discrete choice models of demand (as in McFadden (1981)) as well as on a few earlier pioneering empirical works on differentiated product markets in equilibrium, especially Bresnahan (1981, 1987). One variation of this model (see Berry et al. (1999)) posits that consumer i's conditional indirect utility for good j in market t can be written as

$$v_{ijt} = x_{jt}\beta_{it} - \alpha_{it}p_{jt} + \xi_{jt} + \epsilon_{ijt}. \tag{1}$$

Here the vector x_{jt} represents observed exogenous product characteristics, ξ_{jt} is an unobserved product/market characteristic, and p_{jt} is the endogenous characteristic price.[2]

In addition to an i.i.d. idiosyncratic product/consumer "match value" ϵ_{ijt}, there are two types of unobservables in (1), both essential. One is the vector $(\beta_{it}, \alpha_{it})$ of random coefficients on x_{jt} and p_{jt}. These random coefficients allow heterogeneity in preferences that can explain why consumers tend to substitute

[1] We will also make some use of a recent literature on the identification of simultaneous equations, as represented by Matzkin (2008), Matzkin (2015) and Berry & Haile (2015).

[2] For simplicity we discuss only the usual case in which price is the (only) endogenous product attribute. The arguments extend to the case of other endogenous attributes, although when there is more than one endogenous characteristic per good, the number of instruments required will be greater.

among "similar" products (those close in (x_t, p_t)-space) when prices or other attributes of the choice set change. The second is the product/market-level unobservable ξ_{jt}. The presence of these demand shocks is not only realistic (typically we don't observe all relevant attributes a product or market) but also generates the familiar problem of price endogeneity. That is, the price of a good in a given market will typically be correlated with the latent demand shocks (those associated with *all* goods) in that market. Such correlation is implied, for example, by standard oligopoly models when firms know the realizations of these shocks or, more generally, the elasticities of the demand system. It is this combination of rich consumer heterogeneity through latent taste shocks and endogeneity through product/market-specific demand shocks that distinguishes the BLP framework, and upon which the nonparametric models of Berry & Haile (2014) and Berry & Haile (2010) build.

The parametric BLP model already raises a number of difficult questions about identification. One is how to handle the problem of price endogeneity. Instrumental variables will typically be needed. Classic instruments for prices are exogenous cost shifters. Berry et al. (1995) made limited use of cost shifters, combining these with instruments (characteristics of competing products—the so-called "BLP instruments") that economic theory tells us will shift equilibrium markups (in addition to shifting market shares conditional on prices). Following Hausman (1996), Nevo (2001) emphasized proxies for cost shifters (prices of the same good in other markets), while Berry et al. (1999) added a range of cost shifters like exchange rates. A more subtle point involves identification of the "substitution" parameters—those governing the distribution of the random coefficients. Intuition suggest that these parameters would be identified by exogenous changes in choice sets (e.g., additions and removals of choices), since these would directly reveal the extent to which consumers tend to substitute to "similar" goods. But such exogenous variation often is limited if present at all. So can more continuous changes in choice sets—say, variation in prices of a fixed set of goods—do the job? Are the kinds of instruments used in practice sufficient to identify these parameters without relying on distributional assumptions? What about models with less parametric structure on consumer preferences? Can anything be gained by combining the supply and demand model in a single system (see, e.g., Bresnahan (1987), Berry et al. (1995))? Does consumer-level demand data help? Although there are many empirical papers that follow on the BLP parametric form, these identification questions remained without a formal answer for many years.

1.2 Related Literatures

We have already mentioned some key papers that motivate the work we review, but we pause here to highlight a few others. A huge literature on applied discrete choice demand goes back at least to McFadden (1974), with many later contributions. On the supply side, the idea of estimating marginal costs by using first-order conditions for imperfectly competitive firms goes back to Rosse (1970), continues through the "New Empirical IO" literature of the 1980s (see Bresnahan (1989)) and has become a standard tool of the empirical IO (and auctions) literatures. The literature on discriminating between different modes of oligopoly competition dates at least to Bresnahan (1982) and Lau (1982).

In addition, there is an extensive pre-existing literature on the identification of semi/nonparametric discrete choice models. Perhaps surprisingly, these papers do not cover models with the key features (consumer heterogeneity and endogeneity through product-level unobservables) of the BLP framework. A number of papers consider rich taste heterogeneity, but without endogeneity. Examples include Ichimura & Thompson (1998) and Briesch et al. (2010). Other papers on the nonparametric identification of discrete choice models without endogeneity concerns include Manski (1985), Manski (1987), Manski (1988), Matzkin (1992), Matzkin (1993), Magnac & Maurin (2007), and Fox & Gandhi (2011).

Another set of papers (e.g., Blundell & Powell (2004)) discuss control function techniques for dealing with endogeneity. However, control function techniques apply to triangular (or recursive) systems of equations, not to a fully simultaneous system that arises in a model of differentiated products supply and demand (see also Blundell & Matzkin (2014)).

Yet another set of papers considers endogeneity involving correlation between a choice characteristic and unobservables at the individual ("consumer" or "worker") level. One example here is Lewbel (2000), who models a conditional indirect utility function along the lines of

$$v_{ij} = x_{ij}\beta + \epsilon_{ij} \quad \epsilon_{ij} \sim F\left(\cdot|x_{ij}\right) \tag{2}$$

Here, observed x variables can shift the distribution of ϵ at the consumer level. However, the typical counterfactual of interest in a differentiated products context involves holding both market-level unobservables and consumer tastes fixed while letting the distribution of utilities change with the value of an observed endogenous variable (e.g. price). This requires a different framework. Other papers that consider endogeneity in discrete choice models, but in a framework different from the differentiated products framework we discuss include (among others) Honoré & Lewbel (2002), Hong & Tamer (2003), Altonji & Matzkin (2005), Lewbel (2006), Hoderlein (2009) and Petrin & Gandhi (2011).

1.3 The Path Ahead

Looking ahead to the remainder of the paper, we first review some familiar parametric demand models in order to motivate key ideas behind our approach. We then discuss identification of demand when one has only market level data, as in the original BLP paper. This is followed by a discussion of the gains that can be obtained from observing individual-level("micro") data. Micro data does not eliminate the endogeneity problem, but it can substantially reduce the number of instruments required for identification. We then turn to identification of the supply side, including results that demonstrate constructive joint identification of marginal costs and demand, as well as results on discrimination between alternative static oligopoly models.

2 Insights from Parametric Models

Our approach to the nonparametric identification problem builds on insights from familiar parametric models. Consider a standard multinomial logit model (incorporating product-specific unobservables) where consumer i's conditional indirect utility from product $j > 0$ in market t is

$$v_{ijt} = x_{jt}\beta - \alpha p_{jt} + \xi_{jt} + \epsilon_{ijt} \tag{3}$$

and the conditional indirect utility from the outside good ("good 0") is normalized to $v_{i0t} = \epsilon_{i0t}$. Letting

$$\gamma_{jt} = x_{jt}\beta - \alpha p_{jt} + \xi_{jt},$$

choice probabilities (or "market shares") are given by the well known formula

$$s_{jt} = \frac{e^{\gamma_{jt}}}{1 + \sum_k e^{\gamma_{kt}}}. \tag{4}$$

While each share depends on the entire vector $(\gamma_{1t}, \ldots, \gamma_{Jt})$, the share of good j is strictly increasing in γ_{jt}. Thus, the "mean utility" γ_{jt} has the flavor of a "quality index" that has both an observed and unobserved component.

As is well known, the relationship (4) can be inverted to express each index as a function of the market share vector: $\gamma_{jt} = ln(s_{jt}) - ln(s_{0t})$. Remembering the definition of the index γ_{jt}, we have

$$ln(s_{jt}) - ln(s_{0t}) = x_{jt}\beta - \alpha p_{jt} + \xi_{jt}.$$

As noted in Berry (1994), this now looks like a regression model with an endogenous covariate p_{jt}. To identify the model parameters (α, β) one needs an instrument for price. To look even more like what we do later, partition x_{jt} as $\left(x_{jt}^{(1)}, x_{jt}^{(2)}\right)$ where $x_{jt}^{(1)}$ is a scalar. Then condition on $x_{jt}^{(2)}$ and rewrite (2) as

$$x_{jt}^{(1)} + \tilde{\xi}_{jt} = \frac{1}{\beta}\left(ln(s_{jt}) - ln(s_{0t})\right) + \frac{\alpha}{\beta} p_{jt}, \tag{5}$$

with $\tilde{\xi}_{jt} = \xi_{jt}/\beta$. Note that the right-hand side of this expression is a tightly parameterized function of shares and prices while the left-hand side is just a linear index of $x_{jt}^{(1)}$ and the (rescaled) unobserved characteristic.

More complicated models yield this same general form, only with less restrictive functions of shares and prices on the right-hand side. For example, if we follow the same steps for the two-level nested logit model one can obtain (see Berry (1994)) the equation

$$\ln(s_{jt}) - \ln(s_{0t}) = x_{jt}\beta - \alpha p_{jt} + (1-\lambda)\ln(s_{j|g,t}) + \xi_{jt}.$$

where $s_{j|g,t}$ denotes the share of good j within its nest (or "group") g. This again looks like a regression model, but now identification requires instruments not only for p_{jt} but also for the endogenous share $\ln(s_{j|g,t})$. Loosely speaking, the need for this "extra" instrument reflects the new parameter λ, which (compared to the multinomial logit) gives more flexibility to the patterns of substitution permitted by the model. Again, fix $x_{jt}^{(2)}$ and rewrite this as

$$x_{jt}^{(1)} + \tilde{\xi}_{jt} = \frac{1}{\beta}\left(ln(s_{jt}) - ln(s_{0t}) - (1-\lambda)\ln(s_{j/g,t})\right) + \frac{\alpha}{\beta}p_{jt}, \tag{6}$$

yielding a linear index on the left and a more complicated function of price and shares on the right.

Finally, consider the BLP model and assume that there is one element of the product characteristic vector, $x_{jt}^{(1)}$, that does not have a random coefficient. Then the model can once again be rewritten with a non-random linear index $x_{jt}^{(1)}\bar{\beta}_0^{(1)} + \tilde{\xi}_{jt}$ for each product. Although there is no longer a closed form for the inverse of the system of market shares, BLP show that such an inverse exists (and provide a computational algorithm). To make a connection to equations (5) and (6), we can write this inverse as

$$x_{jt}^{(1)} + \tilde{\xi}_{jt} = \frac{1}{\bar{\beta}_0^{(1)}}\tilde{\delta}_j\left(s_t, p_t, x_t^{(2)}, \theta\right). \tag{7}$$

Compared to (5) and (6), the right-hand side of (7) is now a more complicated function of prices and market shares, all of which are endogenous. For identification of the parameters in this equation, one needs instruments for prices, and additional instruments for the market shares that appear in the inverse function $\bar{\delta}_j$. In the literature, the latter set of instruments are often described loosely as instruments identifying the "nonlinear" parameters of the model—those determining how the random coefficients affect substitution patterns.

Our discussion of each of these examples featured three key components:

1. an *index* of each good's unobserved characteristic and an observed characteristic,

2. an *inversion* mapping observed market shares to each index, and

3. a set of *instrumental variables* for the prices and market shares appearing in the inverse function.

Our approach to nonparametric identification will make use of this same sequence of features.

3 Identification of Demand From Market Level Data

3.1 Demand Model

We begin by sketching the discrete choice demand model of Berry & Haile (2014). In each market t there is a continuum of consumers. In practice markets are typically defined by time and/or geography, although one can use observed consumer characteristics to define markets more narrowly. Formally, a market is defined by $\{J_t, \chi_t\}$, where J_t denotes the number of "inside goods" available. The "outside good"—the option to purchase none of the modeled goods—is always available as well and indexed as "good 0." The matrix χ_t describes all product and market characteristics (observed and unobserved).[3] We henceforth condition on the number of goods available, setting $J_t = J$ for all t. In each market, each consumer i chooses one good $j \in \{0, 1, \ldots, J\}$.

Following the BLP notation above,

- $x_{jt} \in \mathbb{R}^K$ is a vector of exogenous observables,
- $p_{jt} \in \mathbb{R}$ is a vector endogenous observables (prices), and
- $\xi_{jt} \in \mathbb{R}$ is a market/choice-specific unobservable.

We let $x_t = (x_{1t}, \ldots, x_{Jt})$, $p_t = (p_{1t}, \ldots, p_{Jt})$ and $\xi_t = (\xi_{1t}, \ldots, \xi_{Jt})$. Thus, the characteristics of the market are $\chi_t = (x_t, p_t, \xi_t)$.

Demand is derived from a random utility model. Each consumer i in market t has preferences characterized by a vector of conditional indirect utilities (henceforth "utilities") for the available goods:

$$v_{i0t}, v_{i1t,} \ldots, v_{iJt}.$$

The utility v_{i0t} of the outside good is normalized to zero. From the econometrician's perspective, for each consumer i, $(v_{i1t,} \ldots, v_{iJt})$ is a vector of random variables drawn from a joint distribution $F_v(\cdot|\chi_t)$.

Thus far the model is extremely general, placing no restriction on how $F_v(\cdot|\chi_t)$ depends on χ_t. Implicitly we have imposed one significant assumption

[3] We use the terms "good," "product," and "choice" interchangeably.

on the demand model by restricting the jt-level unobservable ξ_{jt} to be a scalar. This is standard in practice and appears difficult to relax without data on multiple decisions (e.g., panel data, ranked data, or repeated choice data) or distinct sub-markets (subpopulations of consumers) assumed to share the same demand shocks (see, e.g., Athey & Imbens (2007)).

We further restrict the model with an index structure. In particular, we partition the exogenous observed characteristics into two parts, $x_{jt} = \left(x_{jt}^{(1)}, x_{jt}^{(2)}\right)$, with $x_{jt}^{(1)} \in \mathbb{R}$. For each good j we then define an index

$$\delta_{jt} = \delta_j \left(x_{jt}^{(1)}, \xi_{jt}\right), \tag{8}$$

for some (possibly unknown) function δ_j that is strictly increasing in ξ_{jt}. We will require that the joint distribution $F_v\left(\cdot|\chi_t\right)$ depend on $x_{jt}^{(1)}$ and ξ_{jt} only through the index δ_{jt}; i.e., letting $\delta_t = \{\delta_{1t}, \ldots, \delta_{Jt}\}$,

$$F_v(\cdot|\chi_t) = F_v(\cdot|\delta_t, x_t^{(2)}, p_t). \tag{9}$$

This is a *weak separability* condition. Although it places no restriction on how $(\delta_t, x_t^{(2)}, p_t)$ together alter the joint distribution of utilities, it requires that the marginal rate of substitution between $x_{jt}^{(1)}$ and ξ_{jt} be invariant to $(x_t^{(2)}, p_t)$. This condition is satisfied in all of our empirical examples. However, whereas those examples specified utilities that are separable in the index, this is not required by (9).

Henceforth we condition on $x_t^{(2)}$, suppress it in the notation when possible, and let x_{jt} denote $x_{jt}^{(1)}$. The model is then completely general in $x_t^{(2)}$. For the results of this section, we will also assume that each δ_j is linear:

$$\delta_{jt} = x_{jt}^{(1)}\beta_j + \xi_{jt}, \tag{10}$$

with each β_j normalized to one without loss (this merely defines the units of ξ_{jt}). Berry & Haile (2014) discuss identification with a nonseparable index, and we discuss the case of a nonlinear but separable index below.

As usual, market level demand is derived from the conditional (on χ_t) distribution of random utilities. Each consumer chooses the product giving the highest utility, yielding choice probabilities (market shares)

$$s_{jt} = \sigma_j\left(\chi_t\right) = \sigma_j\left(\delta_t, p_t\right) = \Pr\left(\arg\max_{j \in \mathcal{J}}\ v_{ijt} = j | \delta_t, p_t\right) \quad j = 1, \ldots, J. \tag{11}$$

The functions σ_j fully characterize the demand system. We observe (s_t, x_t, p_t). If we also observed each ξ_{jt} we would directly observe the demand functions $(\sigma_1, \ldots, \sigma_J)$; i.e., identification of demand would be trivial. Thus, the main challenge to nonparametric identification of demand is endogeneity arising through the unobservables ξ_{jt}. In fact, the challenge here is more complex than in many familiar economic settings because each of the endogenous variables p_{jt} and s_{jt} (price and quantity) depends on all J demand shocks $(\xi_{1t}, \ldots, \xi_{Jt})$. Standard IV and control function approaches cannot be applied here because prices and quantities are determined in a fully simultaneous system. Our approach to overcoming these challenges builds on the "index-inverse-instruments" intuition developed above for the parametric examples. We have already described the index restriction. We next discuss the inversion of market shares.

3.2 Inversion

The goal of the inversion step is to show that, given any vector of prices and nonzero market shares, there is a unique vector of indices δ_t that can rationalize this observation with the demand system (11). Berry et al. (2013) demonstrate this invertibility under a condition they call "connected substitutes."[4] This condition has two parts. The first is that all goods are weak gross substitutes with respect to the indices; i.e., for $k \neq j$, $\sigma_k(\delta_t, p_t)$ is weakly decreasing in δ_{jt} for all $(\delta_t, p_t) \in \mathbb{R}^{2J}$. This is little more than a monotonicity assumption implying that δ_{jt} is "good" (e.g., an index of product "quality") at least on average across consumers. A sufficient condition is that, as in the parametric examples, higher values of δ_{jt} raise the utility of good j without affecting the utilities of other goods.

The second part of the Berry et al. (2013) connected substitutes condition is "connected strict substitution." This condition requires that starting from any good j there be a chain of substitution leading to the outside good. In a multinomial logit model (or any discrete choice model like the BLP model where additive shocks to utilities have full support) all goods strictly substitute directly to the outside good (all else equal, an increase in the quality of good j causes the market share of the outside good to fall), so the condition is satisfied. In a classic model of pure vertical differentiation, each good strictly substitutes only with its immediate neighbors in the quality hierarchy, but those goods substitute to their neighbors and so forth. Starting from any good $j > 0$, there will therefore be a path from one neighbor to the next that eventually links j to the outside good. Thus, in the vertical model, the connected strict substitution requirement is again satisfied.

Formally, *a good j substitutes to good k at (δ_t, p_t)* if $\sigma_k(\delta_t, p_t)$ is strictly decreasing in δ_{jt}. We represent the pattern of strict substitution with a matrix $\Sigma(\delta_t, p_t)$ that has entries

$$\Sigma_{j+1,k+1} = \begin{cases} 1\{\text{good } j \text{ substitutes to good } k \text{ at } x\} & j > 0 \\ 0 & j = 0. \end{cases}$$

[4] Their result applies outside the discrete choice framework studied here as well.

The second part of the connected substitutes condition is that for all (δ_t, p_t) in their support, the directed graph of $\Sigma(\delta_t, p_t)$ has, from every node $k \neq 0$, a directed path to node 0.[5]

Berry et al. (2013) discuss a range of examples and point out that when δ_{jt} is a quality index, it is extremely hard to break the connected substitutes assumption in a discrete choice setting. Weak substitutes is a direct implication of weak monotonicity in a discrete choice model, and connected strict substitution then requires only that there is no strict subset of the goods that substitute only among themselves. That is to say, all the goods must "belong" in one demand system.

Given connected substitutes, Berry et al. (2013) prove existence of the inverse market share function. That is, for all j there is a function σ_j^{-1} such that

$$\delta_{jt} = \sigma_j^{-1}(s_t, p_t) \tag{12}$$

for all (s_t, p_t) in their support.

We will not discuss the proof, but point out that this result offers an alternative to the classic invertibility results of Gale & Nikaido (1965) or Palais (1959), which can be difficult to apply to demand systems. Because Berry et al's result can be applied outside the discrete choice setting considered here, an interesting question is whether this can be used as the starting point for studying identification of demand in other settings as well.

Unlike the inversion results for the parametric examples, the invertibility result of Berry et al. (2013) is not a characterization (or computational algorithm) for the inverse. Nonetheless, having established that the inverse exists, we can move on to its identification using instrumental variables. Note that once the inverse is known, so is the value of each index δ_{jt} and, therefore, each demand shock ξ_{jt}. Thus, once we demonstrate identification of the inverse, it is as if there were no unobservables—a situation in which (as discussed above) identification of the demand system is trivial.

3.3 Instruments: Identifying the Inverse Demand System

The final step of the argument involves using instrumental variables to identify the inverse market share functions. Recalling that we have let

$$\delta_{ij} = x_{jt} + \xi_{jt}$$

we can rewrite (12) as

$$x_{jt} = \sigma_j^{-1}(s_t, p_t) - \xi_{jt}. \tag{13}$$

[5] One can of course define the connected substitutes condition in terms of substitution in response to changes in a product characteristic other than the index—for example, price.

This equation bears close resemblance to the standard nonparametric regression model studied by Newey & Powell (2003). They considered identification of the model

$$y_i = \Gamma(x_i) + \epsilon_i, \tag{14}$$

using instrumental variables w_i that satisfy a standard exclusion restriction that $E[\epsilon_i|\mathrm{w}_i] = 0$ a.s. They show that the analog of the classic rank condition for identification of a linear regression model is a standard "completeness" condition (Lehmann & Scheffe (1950, 1955)), namely that if $B(\cdot)$ is a function with finite expectation satisfying $E[B(x_i)|\mathrm{w}_i] = 0$ almost surely, then $B(x_i) = 0$ almost surely. Like the classic rank condition, completeness requires that the instruments shift the endogenous variables around "enough;" further, like the rank condition, completeness is not only sufficient but also necessary for identification of the unknown regression function Γ.

Equations (13) and (14) are not quite equivalent. Whereas the endogenous variable Y_i appears alone on the left in (14), in (13) all endogenous variables enter through the unknown function σ_j. Further, in (13) the exogenous observable x_{jt}—which we will see is an essential instrument—is not excluded. Nonetheless, Berry & Haile (2014) show that the proof used by Newey & Powell (2003) can be used to establish identification of (14) under analogous exclusion and completeness conditions. In particular, let w_t now denote a set of observables and suppose that (i) $E[\xi_{jt}|x_t, \mathrm{w}_t] = 0$ a.s., and (ii) for all functions $B(s_t, p_t)$ with finite expectation,[6] if $E[B(s_t, p_t)|x_t, \mathrm{w}_t] = 0$ a.s. then $B(s_t, p_t) = 0$ a.s. Then each of the inverse functions $\sigma_j(\cdot)$ is identified. Given (13) each unobserved product characteristic ξ_{jt} is then identified, and identification of the demand system (11) follows immediately.

We discuss the availability of instruments below. However, it is worth pausing to observe that this result provides a reassuringly "dull" answer to the question of what ensures identification of demand in a BLP-type model: instruments for the endogenous variables. Because the model involves discrete choice, rich consumer heterogeneity, and a simultaneous system (e.g., all prices and quantities depend on all J demand shocks), the best possible hope is that identification would be obtained under same IV conditions needed for identification of a homogeneous regression model with a separable scalar error. This is what is shown by Berry & Haile (2014).

3.4 Discussion

3.4.1 Demand versus Utility

The demand functions σ_j are the main features of interest on the consumer side of the model. These functions determine all demand counterfactuals (up

[6] Other types of completeness assumptions (see, e.g., Andrews (2011)) can also suffice.

to extrapolation/interpolation, as usual). Demand is the input from the consumer model needed to estimate marginal costs, to test models of supply, to simulate a merger, and so forth. However, demand is not necessarily enough to identify changes in consumer welfare. As usual, valid notions of aggregate welfare changes can be obtained under an additional assumption of quasilinearity. Welfare changes (changes in consumer surplus) are then identified whenever demand is.

Berry-Haile-market also show how to extend identification of demand to identification of the joint distribution of random utilities, $F_v(\cdot|\chi_t)$ under standard quasi-linearity and support conditions. However, given identification of demand and consumer surplus, there is often little gain from identifying a particular distribution of random utilities consistent with demand.

3.4.2 Instrumental Variables

Although it is encouraging that essentially "all" one needs for identification is a suitable set of instruments, this leads to the important question of what kinds of observables can play the role of these instruments in practice. One important observation is that for the completeness condition above to hold the instruments must have dimension no less than $2J$—i.e., instruments for all $2J$ endogenous variables (s_t, p_t) are needed. This should not be surprising given the discussion of the parametric examples. Except in the most restrictive of these models, we required instruments not only for prices but for some quantities (market shares) as well. Formally, this requirement reflects the fact that in a multi-good setting the quantity demanded of a given good depends on the endogenous prices of all goods as well as the demand shocks associated with all goods.[7] Inverting the demand system is a valuable "trick" producing a system with only one structural error (demand shock) per equation (see (13)). But in general the resulting inverted demand equations each depend on all prices and quantities. Identification of the inverse demand function requires independent exogenous variation in each of these variables.

Excluded cost shifters (e.g., input prices) are the classic instruments for identifying demand. Proxies for cost shifters or for marginal costs themselves (if these proxies are properly excluded from demand) can of course substitute. Hausman (1996) proposed a particular type of proxy for the marginal cost of good j in market t: the prices of good j in other markets t' (see also Nevo (2000)). Another type of candidate instrument is an excluded shifter of firm markups. One possibility involves demographics of other markets falling in the same "pricing zone" (see, e.g., Gentzkow & Shapiro (2010) or Fan (2013)). The availability and suitability of these different types of instruments varies across

[7] Contrast this with a classical supply and demand model, where the quantity demanded depends on only one endogenous price and one demand shock.

applications. Excluded cost shifters are sometimes available, either at the firm or product level. "Hausman instruments" can always be constructed from a market-level data set but have been controversial: in some applications the exclusion restriction may be difficult to defend, and in others the instruments may have little cross-market variation. Instruments based on demographics of other markets require at least market-level demographic data and a supply-side connection (like zonal pricing) between the price in one market and the demographics of another.

But even when one has suitable instruments in one or more of these classes, the analysis above reveals something important: such instruments cannot suffice on their own in the model above. This is because *costs and markups affect the endogenous variables* (s_t, p_t) *only through prices.*

This point may become more clear by considering the identification problem when all price variation is exogenous (mean independent of ξ_t). With exogenous prices, p_t can be treated as an element of $x_t^{(2)}$ and held fixed for the purposes of studying identification. The inverted demand equations then take the form (cf. (13))

$$x_{jt} = \sigma_j^{-1}(s_t) - \xi_{jt}. \tag{15}$$

Each of these equations involves the J-dimensional endogenous variable s_t. Identification can still be obtained following the argument above, but this requires instruments for s_t. Excluded shifters of costs or markups cannot do the job because they only alter shares through prices (which are fixed).

This raises the question of what remaining instruments, excluded from the inverse market share function, can shift s_t in (13). The exogenous product characteristics x_t are the only available instruments, and they are of the correct dimension. Thus, our result on the identification of demand requires J dimensional instruments w_t affecting prices (costs or markups), plus the product characteristics that enter the demand indices δ_{jt}. Strictly speaking, the identification result provides sufficient conditions; however, given the presence of $2J$ endogenous variables, it seems unlikely that one could avoid the need for exogenous variation of dimension $2J$ without additional data or structure.

Some intuition about the role of the instruments x_t can be seen by first recalling that we have inverted the market share function, placing the shares on the right-hand side instead of the indices δ_t. Holding prices fixed (with other instruments for prices), by the implicit function theorem the derivative matrix of market shares with respect to the quality indices is the (matrix) inverse of the inverse market share functions with respect to shares. Thus, identifying the effect of shares in the inverse demand system (13) is equivalent to identifying the effect of the "quality" index δ_t on market shares. The x_t vector directly shifts the indices δ_t, so it is not surprising that these are appropriate instruments for market shares in the inverse demand function. Further, without the index structure that links the effects of ξ_{jt} to the effects of x_{jt}, it is not clear how one could quantify the effects that a changes in unobservables have on demand. With the linear index structure this link is particularly clear: a unit increase in ξ_{jt} has the same effects as a unit increase in x_{jt}.

This analysis also clears up some possible confusion about the excludability of the BLP instruments. On one hand, it natural to ask how (whether) demand shifters could be properly excluded from demand. On the other hand, it is clear that characteristics of goods $k \neq j$ are excluded from the *utility* of good j, and the parametric examples suggest that perhaps this is enough. We can see in the analysis above that it is the index restriction which makes the exogenous variables x_t available as instruments (rather than being fixed like $x_t^{(2)}$) and which yields a system of inverse demand equations in which each x_{jt} appears in only one equation. Thus, as suggested above, the index restriction and the BLP instruments are both central to the identification result above.

We also see that the BLP instruments are not sufficient on their own in this model. They provide the required independent variation in s_t, but something else must provide the independent exogenous variation in p_t.[8] This requires additional instruments for costs or markups. The importance of combining both types of variation has been recognized in much of the applied literature. For example, Berry et al. (1995) utilized both the "BLP instruments" and proxies for marginal cost, and Berry et al. (1999) added further cost instruments. The importance of having both types of instruments, even in a parametric setting, is supported by recent simulations in Reynaert & Verboven (2014). The results of Berry & Haile (2014) may explain why: both types of instruments appear to be essential, at least without additional data or structure.

3.4.3 Nonparametric Functional Form Restrictions

We saw in the parametric examples that the instrumental variables requirements were milder than those for the fully nonparametric case. In fact, the parametric structure not only reduced a completeness condition to a standard rank condition, but also cut the number of instruments required. This suggests a potentially important role for functional form restrictions in substituting for sources of exogenous variation. This may be a concern in some applications, where one suspects that functional form, rather than exogenous variation, is pinning down demand parameters. But this tradeoff may be advantageous in other cases: it is sometimes difficult to find J-dimensional cost shifters (or proxies) that can be excluded from demand, and further exploration reveals that even fairly modest restrictions on the demand system can yield substantial reductions in the instrumental variables requirements.

[8] cf. Armstrong (2015). The vector x_t typically will alter elasticities and therefore markups (prices). However, our formal argument relies on the direct effect of x_t on shares, holding prices fixed.

To illustrate one such possibility, consider a stronger weak separability requirement where price is included in the index δ_{jt} along with x_{jt} (i.e., $x_{jt}^{(1)}$) and ξ_{jt}:

$$\delta_{jt} = x_{jt}\beta - \alpha p_{jt} + \xi_{jt}. \tag{16}$$

This specification substantially generalizes versions of the BLP model in which, as often assumed in applied work, price and at least one exogenous observable enter the utility function without a random coefficient. Without loss of generality, we can choose the units of ξ_{jt} by setting $\alpha = 1$. The equations of the inverse demand system then have the form

$$x_{jt}\beta - p_{jt} + \xi_{jt} = \sigma_j^{-1}(s_t)$$

or

$$p_{jt} = -\sigma_j^{-1}(s_t) + x_{jt}\beta - \xi_{jt}.$$

This takes the form of a classic inverse demand function with price on the left hand side and quantities (here, shares) on the right. In this case, because shares are the only endogenous variables on the right, the BLP instruments x_{-jt} can suffice alone: instruments for costs or markups would still be useful in practice but would not be necessary for identification.

There are many other possibilities for imposing functional form restrictions that reduce the instrumental variable requirements. These include a variety of nonparametric restrictions such as symmetry (exchangeability), various nesting structures, etc., that can often be motivated by economics or features of the market being studied.

4 Gains from Micro Data

Better data can also relax the instrumental variables requirements. Here we consider "micro data" that matches individual choices to consumer/choice-specific observables

$$z_{it} = (z_{i1t}, \ldots, z_{iJt}),$$

where each $z_{ijt} \in \mathbb{R}$.[9] Often these consumer/choice-specific variables will involve interactions between consumer attributes (say d_{it}) and product attributes ($x_t^{(1)}$ or $x^{(2)}$ in the notation of the previous section.) For example, many applications have utilized interactions of consumer and choice locations to create measures of consumer$\times$product-specific distances

[9] Other consumer-level observables can be accommodated in a fully flexible way by treating these as characteristics (elements of x_t) of distinct markets. This would permit a different vector of unobservables ξ_t for consumers with different observable characteristics. For example, an unobserved product characteristic viewed as desirable by one demographic group could be viewed as undesirable by another.

- to different modes of transport (McFadden et al. (1977)),
- to different hospitals (Capps et al. (2003)),
- to different schools (Hastings et al. (2010)), and
- to different retailers (Burda et al. (2008)).

Other examples of observables that might play the role of z_{it} include

- exposure to product-specific advertising (Ackerberg (2003)),
- family size×car size (Goldberg (1995), BLP),
- consumer-newspaper political match (Gentzkow & Shapiro (2010)),
- household-neighborhood demographic match (Bayer & Timmins (2007)).

As before, Berry & Haile (2010) start from an extremely flexible random utility model where

$$(v_{i1t}, \ldots, v_{iJt}) \sim F_v(\cdot|\chi_{it}), \tag{17}$$

and now

$$\chi_{it} = (z_{it}, x_t, p_t, \xi_t). \tag{18}$$

Consumer utility maximization yields choice probabilities

$$\sigma_j(\chi_{it}) = \sigma_j(z_{it}, x_t, p_t, \xi_t) \quad j = 1, \ldots, J. \tag{19}$$

Let $\sigma(z_{it}, x_t, p_t, \xi_t) = (\sigma_1(z_{it}, x_t, p_t, \xi_t), \ldots, \sigma_J(z_{it}, x_t, p_t, \xi_t))$.

Within a given market, (x_t, p_t, ξ_t) are fixed, but choice probabilities vary with the value of z_{it}. Let $\mathcal{Z}_t$ denote the support of z_{it} in market t. For each $\hat{z} \in \mathcal{Z}_t$ we observe the conditional choice probability vector $s_t(\hat{z}) = (s_{1t}(\hat{z}), \ldots, s_{Jt}(\hat{z}))$, where each

$$s_{jt}(\hat{z}) = \sigma_j(\hat{z}, x_t, p_t, \xi_t)$$

gives the market share of good j among $\hat{z}$-type consumers, given the true value of the vector (x_t, p_t, ξ_t) in market t. Let

$$\mathcal{S}_t = s_t(\mathcal{Z}_t).$$

denote the set of all conditional choice probabilities in market t generated by some $\hat{z} \in \mathcal{Z}_t$. Of course, for every $s \in \mathcal{S}_t$, there is at least one vector $z_t^*(s) \in \mathcal{Z}_t$ such that

$$s_t(z_t^*(s)) = s. \tag{20}$$

The identification arguments in Berry & Haile (2010) rely heavily on the within-market variation in choice probabilities that results from variation in the consumer characteristics z_{it}. Just as in the case of market-level data, the primary challenge is uncovering the value of the latent demand shocks ξ_{jt}; once the demand shocks are known, identification of demand is trivial. Berry & Haile (2010) obtain identification using two different approaches (applied to slightly different models), each proceeding in two steps. First, within-market variation in z_{it} is used to construct, for each jt combination, a scalar variable that is a function of prices and the scalar unobservable ξ_{jt}. Second, standard instrumental variables arguments are applied to obtain identification of this function and, therefore the value of each structural error ξ_{jt}. Both approaches lead to results allowing identification of demand with weaker instrumental variables requirements than those we required without micro data.

4.1 Identification with a Common Choice Probability

4.1.1 Index and Inversion

The first approach utilizes an index restriction very similar to that used in the case of market-level data. Let

$$\lambda_{ijt} = g_j(z_{ijt}) + \xi_{jt},$$

where g_j is an unknown strictly increasing function. Assume now that z_{it} and ξ_t affect the joint distribution of utilities only through the indices; i.e., letting $\lambda_{it} = (\lambda_{i1t}, \ldots, \lambda_{iJt})$, assume

$$F_v\left(\cdot|\chi_{it}\right) = F_v\left(\cdot|\lambda_{it}, x_t, p_t, \right).$$

With this restriction, we can write the components of the demand system (19) as

$$\sigma_j\left(\lambda_{it}, x_t, p_t\right) \quad j = 1. \ldots, J.$$

For the remainder of this section, we condition on x_t—treating this in a completely general fashion as we did $x_t^{(2)}$ in the previous section—and drop it from the notation.

Berry & Haile (2010) require sufficient variation in z_{it} to generate a *common choice probability* vector $\bar{s}$. That is, they require that there be some choice probability vector $\bar{s}$ such that

$$\bar{s} \in \mathcal{S}_t \quad \forall t. \tag{21}$$

Because choice probabilities conditional on z_{it} are observable, this assumption is directly verifiable from data. Further, full support for z_{it} is not required unless the support of the unobservable ξ_t is itself $\mathbb{R}^J$. In many industries, unobserved tastes vary across markets but not to an extreme degree, so the variation in z_{it} required need not be extreme. In particular, z_{it} need only move the choice probabilities to the common vector $\bar{s}$ in every market; no probability needs to be driven to zero or to one, in contrast to the requirements of "identification at infinity" arguments. Indeed, such arguments typically use assumptions implying that *every* $\bar{s}$ in the J-simplex is a common choice probability (i.e., all such $\bar{s}$ satisfy (21)).

A key insight underlying Berry & Haile's first approach is that variation in the value of $z_t^*(\bar{s})$ (recall (20)) across markets can be linked precisely to variation in the vector of demand shocks ξ_t. Assuming that the demand system satisfies connected substitutes (now with respect to the indices λ_{ijt} instead of δ_{jt}), the demand system can be inverted: for any choice probability vector s and any price vector p_t there is at most one vector of indices λ_{it} such that $\sigma_j(\lambda_{it}, p_t) = s_j$ for all j. Of particular interest is the inverse at the common choice probability $\bar{s}$. Since $\bar{s} \in \mathcal{S}_t$ for every market t, the invertibility result of Berry et al. (2013) ensures that for each market there is a unique $z_t^*(\bar{s}) \in \mathcal{Z}_t$ such that $s_t(z_t^*(\bar{s})) = \bar{s}$. Thus, we may write

$$g_j\left(z_{jt}^*(\bar{s})\right) + \xi_{jt} = \sigma_j^{-1}(\bar{s}, p_t). \tag{22}$$

4.1.2 Instruments

Berry & Haile (2010) provide conditions ensuring that each function g_j is identified. For simplicity we take this as given for the remainder of this section. This is equivalent to focusing on the case of a linear index structure

$$\lambda_{ijt} = z_{ijt}\beta_j + \xi_{jt},$$

where each β_j can be normalized to one without loss (setting the scale of the unobservable ξ_{jt}). This allows us to rewrite (22) as

$$z_{jt}^*(\bar{s}) = \sigma_j^{-1}(\bar{s}, p_t) - \xi_{jt}. \tag{23}$$

This equation takes exactly the form of the Newey-Powell nonparametric regression model (14), with $z_{jt}^*(\bar{s})$ playing the role of the dependent variable, $\sigma_j^{-1}(\bar{s}, p_t)$ the unknown regression function, and ξ_{jt} the error term that is correlated with the endogenous vector p_t. Using this insight and now exploiting cross-market variation, identification of each function $\sigma_j^{-1}(\cdot)$ (and therefore each structural error ξ_{jt}) follows immediately from the result of Newey & Powell (2003) discussed above, given excluded instruments $\mathrm{w_t}$ for p_t that satisfy the standard completeness condition. With each ξ_{jt} known, identification of demand is immediate.

Here, as in the case of market-level data, we require instruments for p_t. However, we no longer need instruments for s_t, which is held fixed at $\bar{s}$ in (23). To suggest why, remember that in the case of market-level data we relied on instruments that provided variation in quantities (shares) even when the structural errors ξ_t and prices p_t were held fixed. We avoid this need by exploiting within-market variation in z_{it}. Within a market, ξ_t and p_t are held fixed by construction while variation in z_{it} provides variation in quantities. Thus, only instruments for p_t are needed when we move to arguments that rely on cross-market variation.

Cost shifters and proxies are, as usual, important candidate instruments for p_t. In addition, in this micro data context, one could also use features of the market-level distribution of z_t as instruments for price. For example, if a market as a whole features unusually many high-income consumers, prices in this market may be unusually high. This type of instrument is a variant of the "characteristics of nearby markets" discussed in the case of market data, although here "nearby consumers" can replace "nearby markets." Berry & Haile (2010) call these "Waldfogel" instruments, after the insight in Waldfogel (2003) that choice sets (including prices) naturally vary with the local distribution of consumer types. To see why such instruments may be excludable, recall that with micro data the observed choice probabilities already condition on the consumer's own z_{it}. So the exclusion restriction is a requirement that there be no demand spillovers (or sorting affects) that would require putting "attributes of other consumers in the market" in the micro-level demand system.

Note that the BLP instruments are not available as instruments here: we have conditioned on the entire x_t. Indeed, we have not imposed any condition requiring that "rival characteristics" x_{-jt} be excluded from the shifters of v_{ijt}. Such a restriction can restore the availability of the BLP instruments, and other functional form restrictions can reduce the number of instruments required even further.

To illustrate this, first suppose that we add p_t to the index $\lambda_{jt}(z)$, so that

$$\lambda_{ijt}(z) = z_{ijt} - \alpha p_{jt} + \xi_{jt}. \tag{24}$$

In this case, equation (23) becomes

$$z^*_{jt}(\bar{s}) = \sigma_j^{-1}(\bar{s},) + \alpha p_{jt} - \xi_{jt}. \tag{25}$$

The first term $\sigma_j^{-1}(\bar{s},)$ on the right is a constant; thus, we have a regression model with just one endogenous variable. Alternatively, we could put x_t (or some subset of $(x_t^{(1)}, x_t^{(2)})$) into the index:

$$\lambda_{ijt}(z) = z_{ijt} + x_{jt}\beta + \xi_{jt}.$$

This which would make the "characteristics of other products" (the BLP instruments) available as instruments for price, at the cost of imposing restrictions on the way that at least some of the x_t's shift the distribution of utilities.

4.2 Utility Linear in z with Large Support

Berry & Haile (2010) consider a second approach using a slightly different specification in which $(z_{i1t}, \ldots, z_{iJt})$ serve as a "special regressors" (e.g., Lewbel (1998, 2000)). This approach requires that $(z_{i1t}, \ldots, z_{iJt})$ have large support (sufficient support to move choice probabilities to all points on the simplex). This is a strong assumption, but special regressors with large support have been used at least since Manski (1975, 1985) as a useful way to explore how far exogenous variation in covariates can go toward eliminating the need for distributional assumptions in discrete choice models. Berry & Haile (2010) take the same approach in order to explore how far variation in the demand shifters z_{it} can go in reducing the need for excluded instruments.

In the simplest version of this second approach,[10] Berry & Haile (2010) assume that utilities take the form

$$v_{ijt} = \lambda_{ijt} + \mu_{ijt} \quad j = 1, \ldots, J. \tag{26}$$

Here

$$\lambda_{ijt} = z_{ijt} + \xi_{jt} \tag{27}$$

as before, and the random variables $(\mu_{i1t}, \ldots, \mu_{i1t})$ have a joint distribution that does not depend on λ_t:[11]

$$F_\mu\left(\cdot|\chi_t\right) = F_\mu\left(\cdot|x_t, p_t\right). \tag{28}$$

In addition, Berry & Haile (2010) impose the standard condition that the utility from good j is unaffected by the observed characteristics of other goods. With (28) this implies

$$Pr(\mu_{ijt} < c\ |x_t, p_t, \xi_t) = Pr(\mu_{ijt} < c\ |x_{jt}, p_{jt}) \tag{29}$$

This specification is a nonparametric generalization of standard random coefficients models, where the utility from a good is a random function of its characteristics and price.[12]

Under the large support assumption

$$\text{supp } z_{it}|\left(x_t, p_t, \xi_t\right) = \mathbb{R}^J, \tag{30}$$

[10]As above, this version can be generalized to allow each z_{ijt} to enter through an unknown monotonic function $g_j(\cdot)$. Below we discuss another variation that treats ξ_t more flexibly.

[11]If F_μ depended freely on λ_t, what appears to be an additive separability restriction in (26) would actually have no content: the distribution of v_{it} could still change freely with λ_t.

[12]For example, one obtains this structure by assuming that $v_{ijt} = z_{ijt} + v(x_{jt}, p_{jt}, \xi_{jt}, \theta_{it})$ for all j where θ_j is an infinite-dimensional random parameter whose distribution does not depend on χ_{it}.

identification can be shown in two simple steps. First, by standard arguments, within-market variation in z_{it} traces out the joint distribution of $(\mu_{i1t}+\xi_{1t},\ldots,\mu_{iJt}+\xi_{Jt})$ separately for each market. This joint distribution also reveals, for each j, the marginal distribution of $\mu_{ijt}+\xi_{jt}$ in each market t. The second step then treats a functional of this distribution as the dependent variable in a nonparametric IV regression model. For example, by (29), for a given j the mean (or any quantile) of this marginal distribution takes the form

$$\delta_{jt} = h_j(x_{jt}, p_{jt}) + \xi_{jt} \tag{31}$$

where h_j is an unknown function. Once again, we can condition on x_{jt} without loss, yielding an equation taking the form of the regression model studied by Newey & Powell (2003). Since the "dependent variable" δ_{jt} can be treated as observed (by the first step), identification of the function h_j (and, then, the residuals ξ_{jt}) requires instruments satisfying a standard completeness condition. Because there is now only *one* price entering h_j, identification can be obtained with only a single instrument. Further, in this case all the instrument types discussed above are candidates; in particular, the BLP instruments x_{-jt} were not fixed in the derivation of (31), so they are available as instruments. In addition, since (31) requires only one instrument at a time, a single market-level instrument (e.g., a single "Waldfogel instrument") could potentially suffice.

The model above can be generalized by dropping the requirement that utilities be separable in the demand shocks ξ_{jt}. Consider the specification

$$v_{ijt} = z_{ijt}\beta + \tilde{\mu}_{ijt}$$

with the joint distribution of $\tilde{\mu}_{it}$ now permitted to depend on ξ_t:

$$F_\mu\left(\cdot|\chi_t\right) = F_\mu\left(\cdot|x_t, p_t, \xi_t\right)$$

Under an appropriate monotonicity condition—for example, let μ_{ijt} be stochastically increasing in ξ_{kt} when $j=k$ but independent of ξ_{kt} otherwise—repeating the analysis above yields an equation taking the form of a nonparametric IV regression with a nonseparable error. For example, Berry & Haile (2010) shows that under appropriate assumptions the mean of the marginal distribution of $\tilde{\mu}_{ijt}$ would take the form

$$\delta_{jt} = h_j(x_{jt}, p_{jt}, \xi_{jt}) \tag{32}$$

where h_j is strictly increasing ξ_{jt}. Chernozhukov & Hansen (2005) have consider identification of such a nonseparable nonparametric regression model under modified "completeness" conditions that (at least in the case of a continuous endogenous dependent variable) are somewhat harder to interpret than the standard completeness condition of Newey & Powell (2003) that suffices for (31). However, whether we use the model with separable ξ_{jt} leading to (31) or the more general model leading to (32), this second class of ("separable in z") models studied in Berry & Haile (2010) has the advantage of minimizing the number of instruments required while maximizing the availability of candidate instruments.

5 Supply

Many positive and normative questions regarding differentiated products markets require a quantitative understanding of "supply" as well as demand. In a static oligopoly context, the primitives of a supply model are marginal cost functions and a specification of oligopoly competition. Because reliable marginal cost data are seldom available, there is a long tradition in the empirical industrial organization literature of estimating marginal costs using firm first-order conditions.

A very early example is Rosse (1970), which considered a model of monopoly newspaper publishers, and the key insight of this literature is particularly transparent in the case of monopoly. Marginal revenue is a function of observed quantity, observed price, and the slope of demand. If the slope of demand is already known, marginal cost is revealed directly as equal to marginal revenue. Rosse (1970) considered joint estimation of demand and marginal cost parameters from a combination of demand equations and monopoly first-order conditions. Elements of this approach are carried over to oligopoly models in the "New Empirical Industrial Organization" literature (see, e.g., Bresnahan (1981, 1989)). That literature also asks whether the hypothesis of a particular form of oligopoly competition is falsifiable. The widely used approach of BLP involves combining estimates of demand parameters with first-order conditions characterizing equilibrium in a given oligopoly model (typically multi-product oligopoly price setting) to solve for marginal costs and parameters of marginal cost functions.

Below we discuss nonparametric identification results that build on these earlier insights.[13] We discuss identification of marginal costs, identification of marginal cost functions, and discrimination between particular models of oligopoly competition. For the remainder of this section we do this treating demand as known, reflecting the results above on identification of demand. However, in the following section we discuss results obtained by treating demand and supply explicitly as a fully simultaneous system.

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[13]There are important questions about identification when firms make discrete supply decisions—e.g., entry or introduction of new products—as well as identification of dynamic oligopoly models. These are beyond the scope of this review.

5.1 Identification of Marginal Costs with a Known Oligopoly Model

When the model of oligopoly competition is known, identification of equilibrium marginal costs follows almost immediately from identification of demand. Let mc_{jt} denote the equilibrium marginal cost of good j in market t.[14] Given the connected substitutes condition on demand (now with respect to prices), Berry & Haile (2014) point out that for a wide variety of static oligopoly models there exists a *known* function ψ_j such that in equilibrium

$$mc_{jt} = \psi_j \left(s_t, M_t, D_t(s_t, p_t), p_t\right), \tag{33}$$

where $D_t \left(s_t, p_t\right)$ is the matrix of partial derivatives $\frac{\partial \sigma_k}{\partial p_\ell}$. Here M_t is the scalar number of potential consumers in market t ("market size"), so that the level of demand is $q_{jt} = M_t s_{jt}$.

If the function ψ_j is known, then mc_{jt} is directly identified from (33) once the demand derivatives D_t are identified. Since most standard oligopoly models imply a known functional form for ψ_j, identification of the level of marginal cost then follows directly from the identification of demand.

In the simple case of monopoly, ψ_j is just the standard marginal revenue function. More generally, the function ψ_j can be interpreted as a "generalized product-specific marginal revenue" function. For example, in the case of single-product firms setting prices in a static Nash equilibrium, firm j's first-order condition can be written

$$mc_{jt} = p_{jt} + \frac{1}{\partial s_{jt}/\partial p_{jt}} M_t s_{jt}. \tag{34}$$

The right hand side of (34) is the marginal revenue that stems from a price decrease sufficient to increase quantity by one unit, holding constant the prices of other products. With multi-product firms and simultaneous price setting (as in BLP), each function ψ_j is obtained by solving a simultaneous system of first-order conditions. The vector of marginal costs for all goods in market t is given by

$$mc_t = p_t + \Delta_t^{-1} s_t, \tag{35}$$

where Δ_t is a matrix with (j,k)-entry equal to $\partial s_{kt}/\partial p_{jt}$ when goods j and k are produced by the same firm, and zero otherwise.

5.2 Identification of Marginal Cost Functions

Equation (33) yields the immediate identification of the level of market costs mc_{jt} whenever the right-hand side of the equation is known. However, un-

[14] In general this is the marginal cost of good j in market t conditional on the observed values of all relevant quantities.

less marginal costs are constant, many counterfactuals of interest will require knowing how costs vary with output.

Typically one might specify a marginal cost function of the form

$$mc_{jt} = c_j\left(q_{jt}, \mathrm{w}_{jt}\right) + \omega_{jt} \tag{36}$$

where w_{jt} and ω_{jt} are observed and unobserved cost shifters, respectively. With the values of mc_{jt} known, equation (36) takes the form of a standard separable nonparametric regression, as in Newey & Powell (2003). Identification of the unknown function c_j requires an instrument for the endogenous variables in (36). A natural assumption is that the cost shifters of all the firms are mean independent of each ω_{jt}. In that case, a single excluded instrument—for the endogenous quantity q_{jt}—can suffice.

Natural instruments for q_{jt} include the exogenous cost shifters w_{-jt} of other goods. Other potential instruments are also likely to be available as well. Demand shifters of own and rival products or firms are all possible instruments, if they are properly excluded from own-product marginal costs. Market-level factors that influence the level of demand (like total population and the population of various demographic groups) are other possible instruments.

Note that, given identification of demand, the problem of finding an appropriate number of instruments is much easier on the cost side than on the demand side. In the simplest case, there is only one endogenous variable in the marginal cost function but possibly many available cost and demand instruments. This availability of instruments means that richer specifications of the marginal cost functions may be accommodated. For example, in some applications one may wish to allow production spillovers across goods produced by the same firm. This could introduce the entire vector Q_j of a firm's own quantities (all endogenous) as arguments of the marginal cost function c_j. But in many cases there will be excluded instruments available of dimension even larger than that of Q_j.

5.3 Identifying Cost Shocks with an Unknown Oligopoly Model

Berry & Haile (2014) show that one need not specify the oligopoly model in order to identify the latent shocks to marginal costs, ω_{jt}. This can be directly useful because it allows identification of a reduced form for equilibrium prices that can be used to answer some kinds of counterfactual questions without specifying the form of oligopoly competition. Identification of the latent cost shocks also leads to the most straightforward and powerful of their results on discrimination between alternative oligopoly models, a topic we take up in the following section.

Here we continue to assume that equation (34) holds but allow the form of the function ψ_j to be unknown. Thus, we relax the assumption of a known oligopoly model and require only that some relation of the form (34) exist. To allow this relaxation, we will require an index structure on the marginal cost function similar to one exploited previously on the demand side.[15] Partition the cost shifters as $\left(\mathrm{w}_{jt}^{(1)}, \mathrm{w}_{jt}^{(2)}\right)$, with $\mathrm{w}_{jt}^{(1)} \in \mathbb{R}$, and define the cost indices

$$\kappa_{jt} = \mathrm{w}_{jt}^{(1)} \gamma_j + \omega_{jt} \quad j = 1, \ldots, J \tag{37}$$

where each parameter γ_j may be normalized to one without loss. Suppose now that the cost shifter $\mathrm{w}_{jt}^{(1)}$ and the cost shock ω_{jt} enter marginal costs only through the index. In particular, assume that each marginal cost function $c_j\left(q_{jt}, \mathrm{w}_{jt}, \omega_{jt}\right)$ can be rewritten as

$$c_j\left(q_{jt}, \kappa_{jt}, \mathrm{w}_{jt}^{(2)}\right)$$

where c_j is strictly increasing in κ_{jt}.

Now fix $\mathrm{w}_{jt}^{(2)}$ for all jt, drop it from the notation, and let w_{jt} represent $\mathrm{w}_{jt}^{(1)}$. Berry & Haile (2014) show that the supply side first-order conditions can be inverted to obtain relations of the form

$$\kappa_{jt} = \pi_j^{-1}\left(s_t, p_t\right), \tag{38}$$

where each π_j^{-1} is an unknown function.[16] Rewriting this as

$$\mathrm{w}_{jt} = \pi_j^{-1}\left(s_t, p_t\right) - \omega_{jt} \tag{39}$$

we obtain an equation taking exactly the form of the inverted demand equations (13) above. Identification (of the functions π_j^{-1} and, therefore, each ω_{jt}) can thus be obtained using the same extension of Newey & Powell (2003) relied on in section 3.3, now with $\left(x_t, \mathrm{w}_t\right)$ as the instruments.

Because here we have fixed not only $x_t^{(2)}$ but also $\mathrm{w}_t^{(2)}$, the instrumental variables requirement is more demanding than that for the parallel result on identification of demand. In particular, the demand shifters x_t (i.e., $x_t^{(1)}$) must now be excluded from marginal costs. In some applications product characteristics will vary without affecting firm costs due to technological constraints (e.g., satellite television reception in Goolsbee & Petrin (2004)) or other exogenous market-specific factors (climate, topography, transportation network).

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[15]This structure is also used below when we discuss identification of demand and supply within a single simultaneous system.

[16]This function is the composition of the inverse (with respect to κ_{jt}) of the marginal cost function and the "generalized product-specific marginal revenue" function ψ_j appearing in the first-order condition for good j. We use the notation π^{-1} as a reminder that this relation is obtained by inverting the equilibrium map to express the cost index κ_{jt} in terms of the market outcomes $\left(s_t, p_t\right)$.

In other cases, product characteristics that shift demand may alter fixed costs rather than marginal costs—-for example, quality produced through research and development, a product's geographic location, or product-specific advertising (e.g., Goeree (2008)). Other examples of instruments that can play the role of x_t here involve interactions between market and product characteristics. For example, BLP point out that conditional on the product characteristic "miles per gallon" (which might affect marginal cost) , "miles per dollar" is a demand shifter that is properly excluded from the marginal cost function.

Note that given identification of the inverted demand functions σ_j^{-1} and the functions π_j^{-1}, total differentiation of the system

$$\begin{aligned} x_{jt} + \xi_{jt} &= \sigma_j^{-1}(s_t, p_t) \quad \forall j, \\ \mathrm{w}_{jt} + \omega_{jt} &= \pi_j^{-1}(s_t, p_t) \quad \forall j \end{aligned} \tag{40}$$

will reveal the response of quantities and prices to the exogenous shifters (x_t, w_t). This is true even when the interpretation of π_j^{-1} is not yet established via appeal to some particular oligopoly model. In section 6 we will return to this system of simultaneous equations to consider a different approach to identification.

5.4 Discrimination Between Models of Firm "Conduct"

An important early literature in industrial organization explored discrimination between alternative models of firm "conduct" (see, *e.g.*, Bresnahan (1989) and references therein). This literature was developed on the important insight in Bresnahan (1982) that "rotations of demand" (changes in market conditions that alter the slope of demand but leave the equilibrium quantity unchanged) can alter equilibrium markups differently in different models of supply. Thus, the price response to such rotations might be used to infer the true model. A formalization of this intuition was developed in Lau (1982), although the result was limited to "conjectural variations" models with nonstochastic demand and cost, homogeneous goods, and symmetric firms. Berry & Haile (2014) showed that none of these limitations is essential, and that Bresnahan's intuition can be generalized to provide an approach for discriminating between alternative models in a much less restrictive setting. Moreover, in the differentiated products context, there are many different types of variation that can be exploited, not just the rotations of market demand considered by Bresnahan (1982) and Lau (1982).

The simplest results of this type in Berry & Haile (2014) take advantage of the results in the previous section. Suppose that we have already identified each cost index κ_{jt} as well as the demand derivatives in each market. Consider again the first-order condition

$$c_j(q_{jt}, \kappa_{jt}) = \psi_j(s_t, M_t, D_t(s_t, p_t), p_t) \tag{41}$$

For a the function ψ_j implied by a hypothesized model of oligopoly competition. An implication of this hypothesis is that all observations (jt combinations) with the same values of q_{jt} and κ_{jt} must have the same value for the right-hand side of equation (41) as well. If we misspecify the model, this restriction will often fail: different values of marginal cost will be required to rationalize the data, even though all shifters of marginal costs have be held fixed. Such a finding will falsify a misspecified oligopoly model.

As the simplest example, suppose that there is a single producer setting prices and quantities as a profit-maximizing monopolist. If we mistakenly hypothesize that the firm prices at marginal cost, this would imply the model

$$c_j\left(q_{jt}, \kappa_{jt}\right) = p_{jt} \tag{42}$$

(i.e., $\psi_j\left(s_t, M_t, D_t\left(s_t, p_t\right), p_t\right) = p_{jt}$). This hypothesis would be falsified if (for example) there are two markets with the same $(q_{jt}, \kappa_{jt}, p_{jt})$ but different demand derivatives—i.e., if there is a rotation in demand. However, rotations of demand just one example of the kind of variation that can reveal misspecification of the oligopoly model. In general a model can be falsified whenever there is variation between two markets t and t' or products j and j' that (i) leaves the relation between quantities and marginal cost unchanged (i.e., $c_j(\cdot, \kappa_{jt}) = c_{j'}(\cdot, \kappa_{j't'})$, (ii) yields $q_{jt} = q_{j't'}$, but (iii) would imply a change in at least one of these equilibrium quantities under the false model. Even if preferences and market size do not change (demand is fixed), one can obtain a contradiction under a misspecified model from variation in the number of competing firms, the set of competing goods, observed or unobserved characteristics of competing products, or observed/unobserved cost shifters of competitors. Berry & Haile (2014) provide several examples. They also show how the insights described here can be generalized to apply without requiring identification of the marginal cost shocks and, therefore, without the cost-side index structure relied on in section 5.3.

6 Identification in a Simultaneous System

Although the main challenge to identification in differentiated products markets is the simultaneous determination of prices and quantities, our approach in the identification results presented above was sequential. We began by demonstrating identification of demand without specifying a model of supply; then, given knowledge of demand, we showed how the primitives of supply could be identified. An alternative is to directly consider identification of the fully simultaneous model of differentiated products demand and supply.

Berry & Haile (2014) explore such an approach, using the same kind of index structure (on both the demand and cost sides) relied on in section 5.3 above. In particular, they consider identification of the system of $2J$ equations in (40). These equations exhibit what Berry & Haile (2015) call a "residual index structure," as each structural error enters the model through an index that also depends on an equation-specific observable. This class of models was introduced by Matzkin (2008) and is also studied by Matzkin (2015) and in Berry & Haile (2014, 2011).

The results in this literature show a tradeoff between [i] support conditions on the instruments (here $x_t, \mathrm{w_t}$)) and [ii] shape restrictions on the joint density of the unobservables (here (xi_t, ω_t)). At one extreme, Berry & Haile (2014, 2015) show identification of the system of equations (40) under an assumption that the instruments have large support, but with no restriction on the density of unobservables. At the other extreme, Berry & Haile (2015) shows that even with arbitrarily small support for the instruments, *failure* of identification occurs only under strong restrictions on the joint density of unobservables. These results can be extended, appealing to results from Matzkin (2008) or Berry & Haile (2015), to models of the form

$$
\begin{aligned}
g_j(x_{jt}) + \xi_{jt} &= \sigma_j^{-1}(s_t, p_t) \quad \forall j \\
h_j(w_{jt}) + \omega_{jt} &= \pi_j^{-1}(s_t, p_t) \quad \forall jv
\end{aligned}
$$

where the functions g_j and h_j are unknown strictly increasing functions.

The simultaneous equations approach contrasts with the approaches in the previous sections that rely on completeness conditions to establish identification. A disadvantage of identification results relying on completeness conditions is a lack of transparency about the kinds of assumptions on economic primitives that will generate completeness. A related point is that identification proofs relying on completeness conditions provide little insight about *how* the observables uniquely determine the primitives; formally, identification is proved by contradiction. In contrast, the results from the simultaneous equations literature rely on explicit conditions on the primitives of the supply and demand model and permit constructive identification proofs.

7 Conclusion

We have reviewed recent nonparametric identification results for models of discrete choice demand and oligopoly supply that are widely used in empirical economics. Our results show that identification of these models relies primarily on the presence of sufficient exogenous sources of variation in prices and quantities—i.e., on the standard requirement of adequate instruments. With market-level data, these results require instruments for all prices and quantities: exogenous shifters of costs or markups as well as the "BLP instruments" that, besides altering markups, shift market shares conditional on prices. Consumer level "micro data" can eliminate the need for market share instruments, and some special cases require only a single exogenous price shifter to identify demand. Given identification of demand, identification of marginal costs requires no additional instruments, and identification of firms' marginal cost functions can be attained with as few as one excluded instrument that shifts equilibrium quantities across markets. We also show that the standard practice of specifying a form of competition on the supply side generally leads to falsifiable restrictions that can be used to discriminate between alternative models of firm conduct. Finally, a simultaneous equations approach to supply and demand allows for constructive proofs of identification under various combinations of support and density conditions.

We believe these nonparametric identification results are of use to applied researchers regardless of whether they use nonparametric *estimation* techniques. Economists have long recognized the distinction between identifiability of ("within") a model and properties of any particular estimator (e.g., Koopmans (1945, 1950), Hurwicz (1950), Koopmans & Reiersol (1950)). Because economic theory typically delivers few implications regarding functional forms or distributions of unobservables, it seems unambiguously good that identification hold without such assumptions (see, e.g., Matzkin (2013)). On the other hand, because every data set is finite, going beyond mere description of the data requires reliance on approximation techniques. Indeed, in practice the difference between parametric and nonparametric estimators is frequently not a matter of whether one estimates a finite set of parameters but of how one constructs standard errors (see, e.g., Horowitz (2011, 2014)). Even when estimation relies on parsimonious parametric specifications, a nonparametric identification result limits the essential role of functional form and distributional assumptions to the unavoidable jobs of approximation, extrapolation, and compensation for the gap between the exogenous variation available in practice and the idealized variation that would allow one to definitively distinguish between all nonparametric structures permitted by the model.

The primary message of the results reviewed here is that identification holds under precisely the kinds of instrumental variables conditions required for more familiar (e.g., regression) models. Thus, as usual, the key identification question in applied work concerns the availability of suitable instruments. The differentiated products setting points to a variety of potential instruments, and our results clarify the types of instruments that suffice in different settings. They also reveal some tradeoffs between instrumental variables requirements, the flexibility of the underlying model, and the kind of consumer-level data relied upon.

References

Ackerberg D. 2003. Advertising learning and customer choice in experience good markets: A structural empirical examination. *International Economic Review* 44:1007–1040

Allenby GM, Rossi PE. 1999. Marketing models of consumer heterogeneity. *Journal of Econometrics* 89:57–78

Altonji J, Matzkin RL. 2005. Cross-section and panel data estimators for nonseparable models with endogenous regressors. *Econometrica* 73:1053–1102

Andrews DWK. 2011. Examples of l^2-complete and boundedly complete distributions. Discussion Paper 1801, Cowles Foundation, Yale University

Armstrong TB. 2015. Large market asymptotics for differentiated product demand estimators with economic models of supply. Tech. rep., Yale University

Athey S, Imbens GW. 2007. Discrete choice models with multiple unobserved choice characteristics. *International Economic Review* 48:1159–1192

Bayer P, Ferreira F, McMillan R. 2007. A unified framework for measuring preferences for schools and neighborhoods. *Journal of Political Economy* 115:588–638

Bayer P, Timmins C. 2007. Estimating equilibrium models of sorting across locations. *The Economic Journal* 117:353–374

Berry S. 1994. Estimating discrete choice models of product differentiation. *RAND Journal of Economics* 23:242–262

Berry S, Levinsohn J, Pakes A. 1995. Automobile prices in market equilibrium. *Econometrica* 60:889–917

Berry S, Levinsohn J, Pakes A. 1999. Voluntary export restraints on automobiles: Evaluating a strategic trade policy. *American Economic Review* 89:189–211

Berry ST, Gandhi A, Haile PA. 2013. Connected substitutes and invertibility of demand. *Econometrica* 81:2087–2111

Berry ST, Haile PA. 2010. Nonparametric identification of multinomial choice demand models with heterogeneous consumers. Discussion Paper 1718, Cowles Foundation, Yale University

Berry ST, Haile PA. 2011. Identification in a class of nonparametric simultaneous equations models. Discussion Paper 1787R, Cowles Foundation at Yale

Berry ST, Haile PA. 2014. Identification in differentiated products markets using market level data. *Econometrica* 82:1749–1797

Berry ST, Haile PA. 2015. Nonparametric identification of simultaneous equations models with a residual index structure. Tech. rep., Yale University

Blundell R, Matzkin RL. 2014. Control functions in nonseparable simultaneous equations models. *Quantitative Economics* 5:271–295

Blundell RW, Powell JL. 2004. Endogeneity in semiparametric binary response models. *Review of Economic Studies* 71:655–679

Bresnahan T. 1981. Departures from marginal cost pricing in the american automobile industry. *Journal of Econometrics* 17:201–227

Bresnahan T. 1982. The oligopoly solution concept is identified. *Economics Letters* 10:82–97

Bresnahan T. 1987. Competition and collusion in the american automobile oligopoly: The 1955 price war. *Journal of Industrial Economics* 35:457–482

Bresnahan T. 1989. Empirical studies of industries with market power, In *The Handbook of Industrial Organization*, eds. R Schamlensee, R Willig, no. 10 in Handbooks in Economics. North-Holland

Briesch RA, Chintagunta PK, Matzkin RL. 2010. Nonparametric discrete choice models with unobserved heterogeneity. *Journal of Business and Economic Statistics* 28:291–307

Bundorf K, Levin J, Mahoney N. 2012. Pricing and welfare in health plan choice. *American Economic Review* 102:3214–3248

Burda M, Harding M, Hausman J. 2008. A bayesian mixed logit-probit model for multinomial choice. Tech. rep., M.I.T.

Capps C, Dranove D, Satterthwaite M. 2003. Competition and market power in option demand markets. *RAND Journal of Economics* 34:737–763

Cardon J, Hendel I. 2001. Asymmetric information in health care and health insurance markets" evidence from the national medical expenditure survery. *RAND Journal of Economics* 32:408–427

Chernozhukov V, Hansen C. 2005. An iv model of quantile treatment effects. *Econometrica* 73:245–261

Chintagunta PK, Honoré BE. 1996. Investigating the effects of marketing variables and unobserved heterogeneity in a multinomial probit model. *International Journal of Marketing Research* 13:1–15

Eizenberg A. 2011. Upstream innovation and product variety in the united states home pc market. Tech. rep., Hebrew University

Fan Y. 2013. Ownership consolidation and product characteristics: A study of the u.s. daily newspaper market. *American Economic Review* 103:1598–1628

Fox J, Gandhi A. 2011. Identifying demand with multi-dimensional unobservables: A random functions approach. Tech. rep., University of Wisconsin

Gale D, Nikaido H. 1965. The jacobian matrix and global univalence of mappings. *Mathematische Annalen* 159:81–93

Gentzkow M, Shapiro J. 2010. What drives media slant? evidence from u.s. newspapers. *Econometrica* 78:35–71

Goeree MS. 2008. Limited information and advertising in the us personal computer industry. *Econometrica* 76:1017–1074

Goldberg P. 1998. The effects of the corporate average fuel economy standards in the automobile industry. *Journal of Industrial Economics* 46:1–33

Goldberg P, Verboven F. 2001. The evolution of price dispersion in the european car market. *The Review of Economics Studies* 68:811–848

Goldberg PK. 1995. Product differentiation and oligopoly in international markets: The case of the u.s. automobile industry. *Econometrica* 63:891–951

Goolsbee A, Petrin A. 2004. The consumer gains from direct broadcast satellites and the competition with cable tv. *Econometrica* 72:351–381

Gordon B, Hartmann WR. 2013. Advertising effects in presidential elections. *Marketing Science* 32:19–35

Hastings J, Kane T, Staiger D. 2010. Heterogeneous preferences and the efficacy of public school choice. Tech. rep., Brown University

Hausman JA. 1996. Valuation of new goods under perfect and imperfect competitioin. In *The Economics of New Goods*, eds. TF Bresnahan, RJ Gordon, chap. 5. Chicago: University of Chicago Press, 209–248

Ho K. 2009. Insurer-provider networks in the medical care market. *American Economic Review* 99:393–430

Hoderlein S. 2009. Endogenous semiparametric binary choice models with heteroskedasticity. Working Paper 747, Boston College

Hong H, Tamer E. 2003. Endogenous binary choice model with median restrictions. *Economics Letters* 80:219–225

Honoré BE, Lewbel A. 2002. Semiparametric binary choice panel data models without strictly exogenous regressors. *Econometrica* 70:2053–2063

Horowitz JL. 2011. Applied nonparametric instrumental variables estimation. *Econometrica* 79:347–394

Horowitz JL. 2014. Ill-posed inverse problems in economics. *Annual Review of Ecnomics* 6:21–51

Hurwicz L. 1950. Generalization of the concept of identification. In *Statistical Inference in Dynamic Economic Models, Cowles Commission Monograph 10*, ed. TC Koopmans. New York: John Wiley and Sons, 245–257

Ichimura H, Thompson TS. 1998. Maximum likelihood estimation of a binary choice model with random coefficients of unknown distribution. *Journal of Econometrics* 86:269–95

Koopmans TC. 1945. Statistical estimation of simultaneous economic relations. *Journal of the American Statistical Association* 40:448–466

Koopmans TC, ed. 1950. Statistical inference in dynamic economic models, cowles commission monograph 10. New York: John Wiley and Sons

Koopmans TC, Reiersol O. 1950. The identification of structural characteristics. *The Annals of Mathematical Statistics* 21:165–181

Lau L. 1982. On identifying the degree of competitiveness from industry price and output data. *Economics Letters* 10:93–99

Lehmann EL, Scheffe H. 1950. Completeness, similar regions, and unbiased estimation. *Sankhya* 10:305–340

Lehmann EL, Scheffe H. 1955. Completeness, similar regions, and unbiased estimation. *Sankhya* 15:219–236

Lewbel A. 1998. Semiparametric latent variable model estimation with endogenous or mismeasured regressors. *Econometrica* 66:105–121

Lewbel A. 2000. Semiparametric qualitative response model estimation with unknown heteroscedasticity or instrumental variables. *Journal of Econometrics* 97:145–177

Lewbel A. 2006. Simple endogenous binary choice and selection panel model estimators. Working Paper 613, Boston College

Lustig J. 2010. The welfare effects of adverse selection in privatized medicare. Tech. rep., Boston University

Magnac T, Maurin E. 2007. Identification and information in monotone binary models. *Journal of Econometrics* 139:76–104

Manski CF. 1975. Maximum score estimation of the stochastic utility model of choice. *Journal of Econometrics* 3:205–228

Manski CF. 1985. Semiparametric analysis of discrete response: Asymptotic properties of the maximum score estimator. *Journal of Econometrics* 27:313–333

Manski CF. 1987. Semiparametric analysis of random effects linear models from binary panel data. *Econometrica* 55:357–362

Manski CF. 1988. Identification of binary response models. *Journal of the American Statitical Association* 83:729–738

Matzkin RL. 1992. Nonparametric and distribution-free estimation of the binary choice and threshold crossing models. *Econometrica* 60:239–270

Matzkin RL. 1993. Nonparametric identification and estimation of polychotomous choice models. *Journal of Econometrics* 58:137–168

Matzkin RL. 2008. Identification in nonparametric simultaneous equations. *Econometrica* 76:945–978

Matzkin RL. 2013. Nonparametric identification in structural economic models. *Annual Review of Economics* 5:457–486

Matzkin RL. 2015. Estimation of nonparametric models with simultaneity. *Econometrica* 83:1–66

McFadden D. 1974. Conditional logit analysis of qualitative choice behavior. In *Frontiers of Econometrics*, ed. P Zarembka. New York: Academic Press

McFadden D. 1981. Econometric models of probabilistic choice. In *Structural Analysis of Discrete Data with Econometric Applications*, eds. C Manski, D McFadden. Cambridge, MA: MIT Press

McFadden D, Talvitie A, Associates. 1977. Demand model estimation and validation. Berkeley CA: Institute of Transportation Studies

Nair H, Chintagunta P, Dube JP. 2004. Empirical analysis of indirect network effects in the market for personal digital assistants. *Quantitative Marketing and Economics* 2:23–58

Neilson C. 2013. Targeted vouchers, competition among schools, and the academic achievement of poor students. Tech. rep., Yale University

Nevo A. 2000. Mergers with differentiated products: The case of the ready-to-eat cereal industry. *RAND Journal ofEconomics* 31:395–421

Nevo A. 2001. Measuring market power in the ready-to-eat cereal industry. *Econometrica* 69:307–42

Newey WK, Powell JL. 2003. Instrumental variable estimation in nonparametric models. *Econometrica* 71:1565–1578

Palais RS. 1959. Natural operations on differential forms. *Transactions of the American Mathematical Society* 92:125–141

Petrin A. 2002. Quantifying the benefits of new products: The case of the minivan. *JPE* 110:705–729

Petrin A, Gandhi A. 2011. Identification and estimation in discrete choice demand models when endogeneous variables interact with the error. Tech. rep., University of Minnesota

Reynaert M, Verboven F. 2014. Improving the performance of random coefficients demand models: The role of optimal instruments. *Journal of Econometrics* 179:83 – 98

Rosse JN. 1970. Estimating cost function parameters without using cost function data: An illustrated methodology. *Econometrica* 38:256–275

Rysman M. 2004. Competition between networks: A study of the market for yellow pages. *Review of Economic Studies* 71:483–512

Villas-Boas SB. 2007. Vertical relationships between manufacturers and retailers: Inference with limited data. *Review of Economic Studies* 74:625–652

Waldfogel J. 2003. Preference externalities: An empirical study of who benefits whom in differentiated-product markets. *RAND Journal of Economics* 34:557–68