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Why does Bad News Increase Volatility and Decrease Leverage?

Ana Fostel * John Geanakoplos†‡

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Abstract

A recent literature shows how an increase in volatility reduces leverage. However, in order to explain pro-cyclical leverage it *assumes* that bad news increases volatility, that is, it assumes an inverse relationship between first and second moments of asset returns. This paper suggests a reason why bad news is more often than not associated with higher future volatility. We show that, in a model with endogenous leverage and heterogeneous beliefs, agents have the incentive to invest mostly in technologies that become more volatile in bad times. Agents choose these technologies because they can be leveraged more during normal times. Together with the existing literature this explains pro-cyclical leverage. The result also gives a rationale to the pattern of volatility smiles observed in the stock options since 1987. Finally, the paper presents for the first time a dynamic model in which an asset is endogenously traded simultaneously at different margin requirements in equilibrium.

Keywords: Collateral, Endogenous Leverage, VaR, Volatility, Volatility Smile.
JEL Codes: D52, D53, E44, G01, G11, G12

1 Introduction

After the recent financial crisis there is almost universal agreement that leverage is *pro-cyclical:* leverage is high during normal times and low during anxious or crisis times. Figures 1 and 2, taken from Geanakoplos (2010b), display leverage and asset prices for the housing market and for AAA Securities from 1998-2009. They both show that leverage is pro-cyclical: prices rise as leverage increases, and prices fall

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as leverage decreases. In particular, both leverage and prices collapsed during the recent financial crisis. This has also been documented by Adrian and Shin (2009) and Gorton and Metrick (2010).

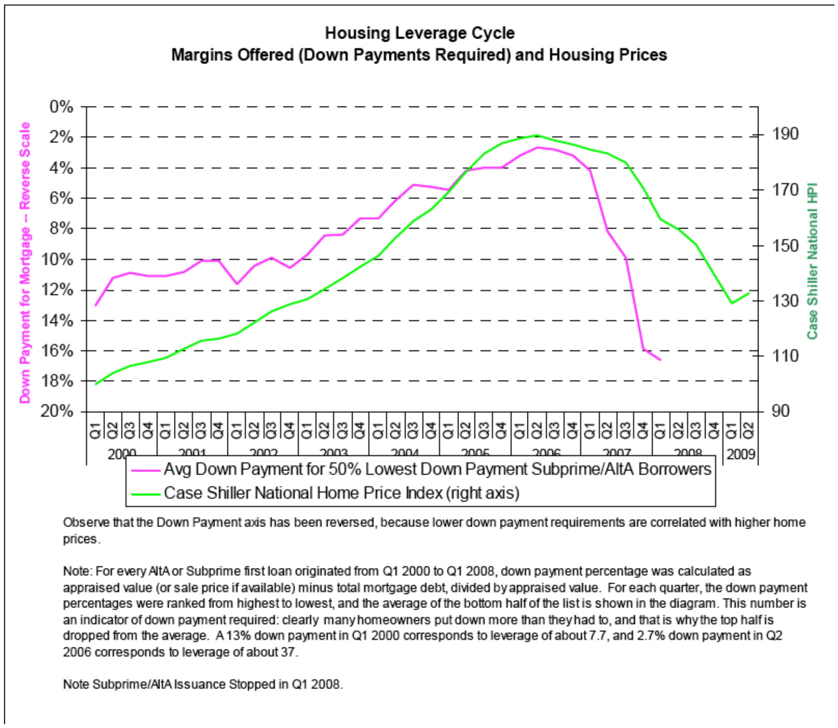

Figure 1: Pro-cyclical leverage: Housing.

A recent theoretical literature has gone quite far in explaining how leverage is influenced by volatility in equilibrium, and why there is a positive relationship between leverage and asset prices. For example, Geanakoplos (1997, 2003, 20010a) shows how supply and demand determine equilibrium leverage and why higher tail volatility reduces leverage. In his model higher leverage increases asset prices. He suggested (in 2003) that big crises occur when bad news is of a particular kind he called "scary bad news", because the news raises tail volatility, as well as decreasing expectations, and hence reduces leverage. Prices then decline not only because of the lower expectations, but also because of the lower leverage.[1] A similar story has been told in Brunnemeier and Pedersen (2009). Geanakoplos has called this amplification mechanism *the Leverage Cycle*.[2] Fostel-Geanakoplos (2008) extended it further to many assets and adverse selection.

[1]Prices also decline because the optimists, who leverage up in the ebullient phase of the cycle, go disproportionately bankrupt when bad news comes and prices start to fall.

[2]As opposed to Credit Cycles from the more classical literature in Macroeconomics (such as Kiyotaki and Moore (1997) and Bernanke and Gertler (1997), which refers to the feedback and comovement between borrowing and prices, ignoring changes in their ratio, that is, ignoring changes in leverage.

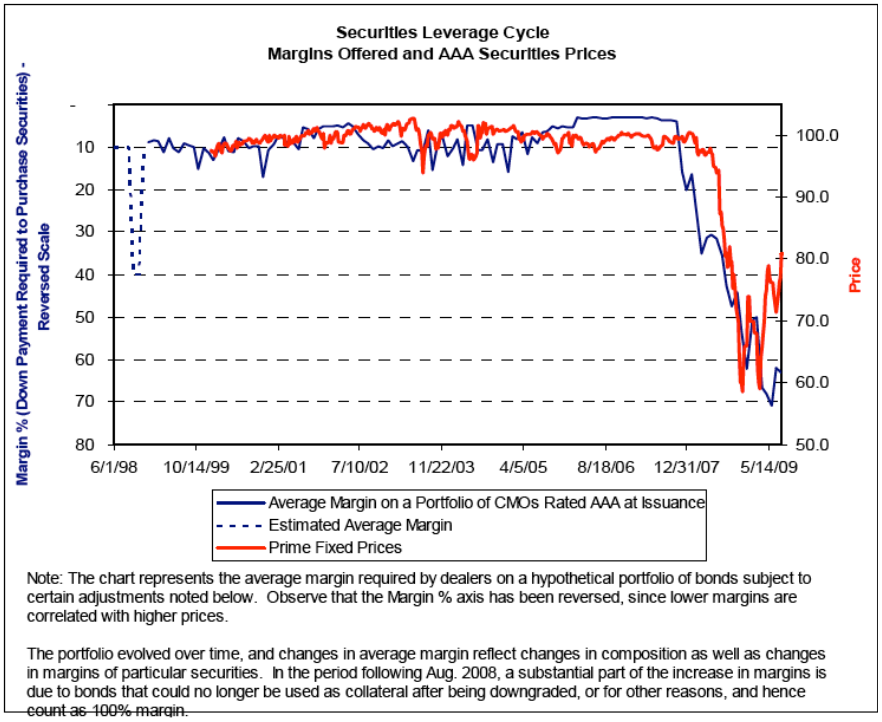

Figure 2: Pro-cyclical leverage: AAA Securities.

The leverage cycle mechanism essentially *assumes* that bad news is associated with high volatility, so that there is an inverse relationship between first moments (expected future payoffs) and second moments (volatility of payoff). This assumption that bad news, at least very bad news, is associated with very high volatility seems quite plausible. Figure 3 shows the history of the VIX index (the Chicago Board Options Exchange Volatility Index) a popular measure of the implied volatility of SP 500 index options. A high value corresponds to a more volatile market and therefore more costly options. Often referred to as the fear index, it represents one measure of the market's expectation of volatility over the next 30 day period. We clearly see that the index was very high during the recent financial crisis implying that bad news indeed came associated with high volatility.

Without a theory that explains why bad news induces high volatility we are only half way in explaining the pro-cyclical pattern of leverage observed in the data. The main contribution of this paper is to shed light on this missing link and hence to more fully understand the relationship between news, volatility and leverage. We show that in a model with endogenous leverage and heterogeneous beliefs, agents have the incentive to invest mostly in technologies that become more volatile in bad times. Agents choose these technologies because they can be leveraged more during normal times. In this sense, the paper "closes" the leverage cycle models.

More precisely, we consider a family of projects (assets) k such that every agent

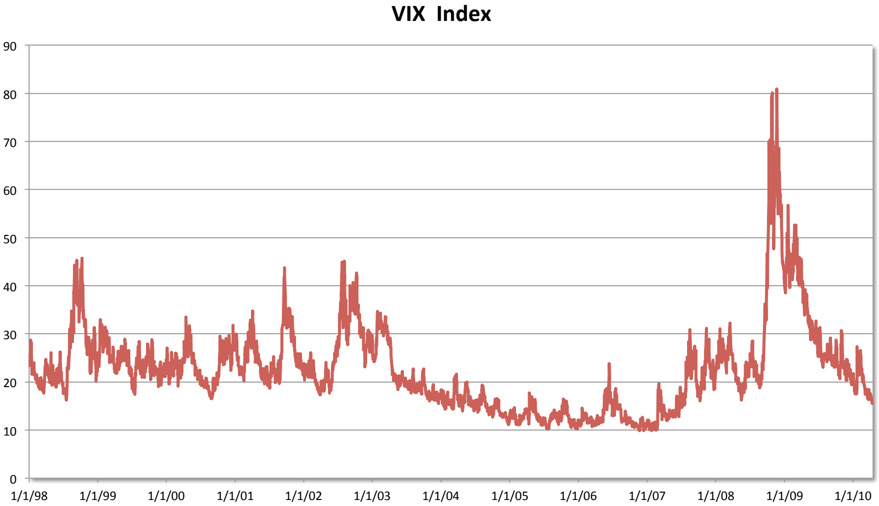

Figure 3: VIX index.

h believes every project has the same probability Q^h of ultimate success U and probability $1 - Q^h$ of ultimate failure D in the last period. Without loss of generality, we suppose the projects pay off 1 if they succeed, and $R < 1$ if they fail. In an intermediate period agents get good news u about the projects, which raises their probabilities of success to $q^h_{uU}(k) > Q^h$, or they get bad news d, which lowers their probabilities of success to $q^h_{dU}(k) < Q^h$. Projects k are characterized by their probabilities of good news $q^h_u(k)$, as well as by $q^h_{uU}(k)$ and $q^h_{dU}(k)$, where $q^h_u(k)q^h_{uU}(k) + (1 - q^h_u(k))q^h_{dU}(k) = Q^h$. It is important to the model to have heterogeneity among agents. For simplicity we suppose that the agents are risk neutral and perfectly patient, and are ordered according to their optimism, so that when $i > h$, $(q^i_u(k), q^i_{uU}(k), q^i_{dU}(k)) > (q^h_u(k), q^h_{uU}(k), q^h_{dU}(k))$. We assume a continuum of agents $h \in (0, 1)$.

In "Post-Bad News Volatile projects" (from now on BV), bad news comes associated with an increase in future payoff volatility. Extreme BV projects are characterized by $q^h_{uU}(k) = 1$ for all h, so that everyone agrees all the volatility comes after bad news. In "Post-Good News Volatile projects" (from now on GV) good news induces higher future payoff volatility. Extreme GV projects are characterized by $q^h_{dU}(k) = 0$ for all h, so that everyone agrees all the volatility comes after good news.

Three BV examples of bad news inducing higher volatility are: i) an airline announces that the plane is now expected to be ten minutes late, which makes people worry it will be an hour late, ii) a bank announces it has lost \$5 billion, which makes investors fear another \$20 billion may follow, and iii) subprime delinquencies shoot up from 2% to 5%, which makes people worry they may go up to 30%. A GV example of good news inducing higher volatility might be that after a presidential candidate wins a crucial primary he may become president or be destroyed by a hitherto unknown scandal. Similarly, after a biotech company patents a new drug, its profits may soar, or side effects from the drug may force the whole research effort to be scratched. Notice that in the three BV examples each piece of bad news reveals only a little information about expected outcomes but creates a lot of uncertainty, while in the GV examples it is the good news that raises expected outcomes a little but creates much more volatility.

Which projects will be chosen to be produced in equilibrium, and therefore what are the equilibrium fluctuations in volatility and leverage as good news or bad news arrives? In Section 2 we begin to answer this question by describing how the economy will trade the projects, and at what prices, assuming that K of them have already been chosen. In the model, agents can use these projects (assets) as collateral to borrow money. Agents are presented with a menu of one-period non-contingent promises, each collateralized by one unit of asset (or project). The leverage an agent uses to buy an asset is defined by the total value of all the promises he makes using the asset as collateral divided by the total value of asset he holds. Leverage becomes endogenous because in equilibrium agents may choose different non-contingent promises from the menu.

In Section 3 we specialize to the case where exactly one of the projects has already been chosen. Propositions 1 and 2 show that then equilibrium exists and is unique and that leverage is endogenously determined in equilibrium and corresponds to the "Value at risk equal zero" rule (*VaR=0*). Each buyer uses the asset as collateral to promise the value of the asset in the worst case scenario in the next period, that is borrowing as much as possible while preventing default from occurring in equilibrium. (We call this the maxmin promise). The key assumption in the proposition is that the tree is binary. Another important ingredient in the proof is the continuum of distinct risk neutral agents. This allows us to find a marginal buyer who partitions the set of agents into "optimists" who want to leverage as much as possible and "pessimists" who do not want to compete with the optimists for any risky portfolio and who therefore end up holding no risk at all.

In propositions 3 to 5 we show that: i) the initial prices of all the extreme GV projects are the same, and lower than all other projects, ii) the highest initial priced project is always an extreme BV project iii) initial leverage is higher in extreme BV projects than in extreme GV projects and iv) leverage is pro-cyclical in extreme BV projects and counter-cyclical in all the others.

Why do the projects have such different prices and leverage characteristics in equilibrium despite their identical final payoff distribution?

Extreme BV projects can be leveraged more at time 0 than extreme GV projects due to the type of bad news. Given the endogenous *VaR=0* rule, the maximum that agents can promise is the worst case scenario in the immediate future: the price of the project after bad news. But in extreme BV projects the price does not fall much after bad news precisely because bad news is little informative. By contrast, bad news in extreme GV projects is very informative, drastically lowering the equilibrium promise at time 0.

Extreme BV projects are more valuable than extreme GV projects at the beginning because they can be leveraged more. A higher borrowing capacity implies that all the assets in the economy can be bought by fewer investors. Since there is a continuum of buyers with continuously decreasing valuations, the marginal buyer then has a more optimistic asset valuation. This raises the project's price. The reason some extreme BV projects are more valuable than *all* other projects is a bit more subtle and is explained in detail in the proof. Essentially the former give optimists the incentive to leverage at time 0, effectively betting on uU, rather than waiting until after the news to leverage and betting on both uU and dU. Again this raises the marginal buyer at time 0 and so raises the price.[3]

Finally, an implication of the *VaR = 0* rule is that all projects other than extreme BV projects exhibit counter-cyclical leverage in equilibrium. Every project is worth the most after good news u, but as long as every agent still thinks D is possible, the same minimum promise of R will be the only traded promise at u and d. Hence the value ratio of promise to collateral will be least just when the price of the asset is highest. Only in extreme BV projects will leverage be pro-cyclical, because in those projects agents do not think D is possible after u and hence the only traded promise shoots up to 1 after good news. Since there are comparatively few of these pro-cyclical projects, we have biased our search for the equilibrium projects in favor of projects that will exhibit counter-cyclical leverage. A more general setting might have allowed for the worst case scenario to become much worse after bad news. That would have explained why leverage goes down after bad news. But we have instead assumed that the same two outcomes of 1 and R are feasible after both good news and bad news. Despite tying our hands, we find that the highest priced project displays pro-cyclical leverage and counter-cyclical volatility.

We close Section 3 by showing that the initial price of the extreme BV project is higher still if we extend the number of periods, so that at each period either good news arrives, in which case the project is worth 1 for sure, or bad news arrives, in

[3] Given an arbitrary project that is not extreme BV, it is possible to find an extreme BV project such that every agent's beliefs conditional on bad news d are the same, (so that the price after bad news is the same and hence just as much can be borrowed in equilibrium at time 0) and for which $(q_u^i q_{uU}^i / q_d^i q_{dU}^i)/(q_u^h q_{uU}^h / q_d^h q_{dU}^h)$ has risen for all $i > h$. This makes it more attractive for an optimist to buy the asset at time 0 by leveraging, rather than waiting to buy the asset after news has arrived, and thus gives the extreme BV project a higher initial price.

which case one needs to wait another period for further good or bad news. In this case, extreme BV projects represent the situation in which crises develop slowly.

In Section 4 we move on to answer a more difficult question. Suppose each agent owns a technology that can transform his labor into any portfolio (a, b) of two projects, an extreme BV and an extreme GV, such that $a + b = 1$. Now both projects can co-exist within the same economy and hence the analysis becomes more difficult. We make the analysis even richer by allowing news about the projects to be independent, so time 0 is followed by four possible pieces of news, and the tree is no longer binary.

Unlike before, *VaR=0* is not the only contract traded in equilibrium at time zero. Now, two non-contingent promises will be actively traded in equilibrium for each asset: a risk-less promise and a risky one that defaults in the worst state. Each contract has an associated leverage, and *asset leverage* is defined as the average leverage over all the traded contracts that use the asset as collateral. Two new things appear in this extended model (that were not in the baseline model with one asset) which are more in tune with what we observe in the real world. First, there is default in equilibrium and second the same asset is traded simultaneously at different margin requirements by different investors.

We numerically show that all agents choose mainly the BV project. In equilibrium in our simulation all agents choose to invest their labor in a portfolio with a 70% share of the BV project.[4] Moreover, both projects display the same leverage patterns as when considered separately, i.e. the extreme BV project is leveraged more than the exreme GV project and leverage is pro-cyclical in the BV project and counter-cyclical in the GV project. Most of the time when we observe bad news about a project we will observe high volatility and low leverage, explaining the leverage cycle stylized facts above. We emphasize that this preference for BV technologies relies uniquely on a liquidity channel. Agents in this economy are risk-neutral. Agents choose mainly BV technologies because they can borrow more at the initial period.

This result suggests an explanation of why crises develop slowly: agents have an incentive to choose projects in which bad news is little informative, because those are the most valuable in the initial period. It is worth remembering that the subprime crisis of 2007-9 developed very slowly over two and a half years. Over the first year and a half most pundits maintained that the crisis would turn out to be minor, even though mortgage security prices and housing prices were steadily declining.

Finally, this result also suggests an explanation for the observed "Volatility Smile" in stock options. This refers to the fact that implied volatility has a negative relationship with the strike price, so volatility decreases as the strike price increases. Hence,

[4]This result is robust to different choices of parameter values as shown in the appendix. The proportion of labor invested in the BV technology is never less than 70%.

bad news comes (or are assumed to come) with high volatility. This pattern has existed for the majority of equities only after the stock market crash of 1987. This has led some economist like Bates (2000) and Rubinstein (1995) to explain volatilites smiles by "crashophobia". Traders were concerned about the possibility of another crash and they priced options accordingly. Our result provides a completely different explanation. Our agents are perfectly rational, they endogenously chose projects associated with volatile bad news since they can leverage more with them.

Our paper is most closely related to Geanakoplos (2003), which (in our language) analyzed the leverage cycle in the context of an extreme BV example. Our paper is related to a literature on collateral and credit constraints as in Bernanke, Gertler and Gilchrist (1999), Caballero and Krishnamurthy (2001), Fostel and Geanakoplos (2008a), Holmstrom and Tirole (1997), Kiyotaki and Moore (1997) and Shleifer and Vishny (1992). More closely, our paper is related to a literature on leverage as in Araujo, Kubler and Schommer (2009), Acharya and Viswanathan (2009), Adrian and Shin (2009), Brunnermeier and Pedersen (2009), Cao (2010), Fostel and Geanakoplos (2008b and 2010), Geanakoplos (1997, 2003 and 2010a), Gromb and Vayanos (2002) and Simsek (2010). It is also related to work that studies the asset price implications of leverage as Hindy (1994), Hindy and Huang (1995) and Garleanu and Pedersen (2009). Some of these papers focus on *investor-based leverage* as in Acharya and Viswanathan (2009), Adrian and Shin (2009) and Gromb and Vayanos (2002), and others like Brunnermeier and Pedersen (2009), Cao (2010), Fostel and Geanakoplos (2008b and 2010), Geanakoplos (1997, 2003 and 20010a) and Simsek (2010) focus on *asset-based leverage*. Not all these models present a theory of endogenous leverage; most of them assume a *VAR=0* rule and study the cyclical properties of leverage as well as its asset pricing implications. In Acharya and Viswanathan (2009) and Adrian and Shin (2009) the endogeneity of leverage relies on asymmetric information and moral hazard problems between lenders and borrowers. In Araujo et. al (2009), Cao (2010), Geanakoplos (1997, 2003, 2010a), Fostel-Geanakoplos (2008b) and Simsek (2010) endogeneity does not rely on asymmetric information, rather financial contracts are micro founded by a collateralized loan market. However, while all of these papers related low leverage with high volatility, none of them explain or endogenize the type of bad news, but rather *assume* that bad news comes with an increase in volatility. Furthermore, our paper is the first model to solve fully for endogenous leverage in a dynamic economy with a continuum of agents and more than two successor states. Geanakoplos (1997) showed how to make leverage endogenous by defining a contract as an ordered pair (promise, collateral) and requiring that every contract be priced in equilibrium, even if it is not actively traded. In Geanakoplos (1997, 2003, 2010a) and Fostel-Geanakoplos (2008b) only one contract is traded. Araujo et.al (2009) gives a two period example of an asset which is used as collateral in two different actively traded contract. Finally, Bloom (2009), provides a theory of why high volatility creates recessions. The main channel in his paper is risk aversion.

2 A General Equilibrium Model of Endogenous Leverage

2.1 Time and Uncertainty

The model is a finite-horizon general equilibrium model, with time $t = 0, \cdots, T$. Uncertainty is represented by a tree of date-events or states $s \in S$, including a root $s = 0$. Each state $s \neq 0$ has an immediate predecessor s^*, and each non-terminal node $s \in S \backslash S_T$ has a set $S(s)$ of immediate successors. Each successor $\tau \in S(s)$ is reached from s via a branch $\sigma \in B(s)$; we write $\tau = s\sigma$. We denote the time of s by the number of nodes $t(s)$ on the path from 0 to s^*.

2.2 Financial Contracts and Collateral

A financial contract (A, C) consists of both a promise, A, and collateral backing it, C. Collateral consists of durable goods, which will be called assets. The lender has the right to seize as much of the collateral as will make him whole once the loan comes due, but no more.

Suppose there is a single storable consumption good c and $k = 1, ..., K$ assets which pay dividends d_s^k in each state s. We take the consumption good as numeraire and denote the price of asset k in each state as p_s^k. We will focus on one-period non-contingent contracts. Contract j_s^k is of the form $(j \cdot \widetilde{1}_s, 1_k)$, where $\widetilde{1}_s \in R^{S(s)}$ stands for the vector of ones with dimension equal the number of successors of s and 1_k stands for one unit of asset k. Hence, contract j_s^k promises j units of consumption good in each successor state of s and the promise is backed by one unit of asset k. Contract $j_s^k \in J_s^k$ where J_s^k is the set of all contracts at state s that use as collateral one unit of asset k. Finally, $J_s = \bigcup_k J_s^k$ and $J = \bigcup_{s \in S \backslash S_T} J_s$.

The price of contract j_s^k in state s is π_s^{jk}. An investor can borrow π_s^{jk} today by selling contract j_s^k in exchange for a promise of j tomorrow. Since the maximum a borrower can lose is his collateral if he does not honor his promise, the actual delivery of contract j_s^k in states $\tau \in S(s)$ is $min\{j, p_\tau^k + d_\tau^k\}$. If the collateral is big so that $j \leq p_\tau^k + d_\tau^k \; \forall \tau \in S(s)$, then the contract will not default. In that case its price defines a riskless rate of interest $(1 + r_s^{jk}) = \frac{j}{\pi_s^{jk}}$.

The margin requirement m_s^{jk} associated to contract j_s^k in state s is given by

$$m_s^{jk} = \frac{p_s^k - \pi_s^{jk}}{p_s^k} \tag{1}$$

Leverage associated to contract j_s^k in state s is the inverse of the margin, $1/m_s^{jk}$ and the *Loan-to-Value* (LTV) associated to contract j_s^k in state s is $1 - m_s^{jk}$.

We define the *asset loan-to-value* for asset k, as the trade-value weighted average of $1 - m_s^{jk}$ across all contracts actively traded in equilibrium that used asset k as collateral.[5]

2.3 Production

Each investor h has an endowment of the consumption good and labor, denoted by $e_s^h \in R_+$ and $l_s^h \in R_+$ in each state $s \in S$. We assume that the consumption good and labor are present at time 0, $\sum_{h\in H} e_0^h > 0, \sum_{h\in H} l_0^h > 0$.

Every agent has direct access to two types of constant-returns-to-scale production processes in the model: an inter-period and a within-period production. The inter-period production is a simple way to model consumption good durability in the economy. A unit of consumption warehoused in state s yields one unit of consumption in all successors states. There is no depreciation.

The second type of production, the within-period production, transforms labor, l, into a portfolio of assets to be chosen by the investor in the set $Z_s^h = \{(z_s^1, ..., z_s^K) \in R_+^K : z_s^1 + ... + z_s^K \leq l_s^h\}$. Any investor can use his l_s^h units of labor to produce any combination of assets.

2.4 Utility

The von-Neumann-Morgenstern expected utility of each investor $h \in H$ is characterized by a Bernoulli utility, u^h, a discounting factor, δ^h and subjective probabilities, q^h. We assume that the Bernoulli utility function for consumption in each state $s \in S$, $u^h : R_+ \rightarrow R$, is differentiable, concave, and monotonic. Agent h assigns subjective probability q_s^h to the transition from s^* to s; naturally $q_0 = 1$. Letting $\bar{q}_s^h$ be the product of all $q_{s'}^h$ along the path from 0 to s, we have

$$U^h = \sum_{s\in S} \bar{q}_s^h (\delta^h)^{t(s)} u^h(c_s) \tag{2}$$

2.5 Budget Set

Given asset and contract prices $((p_s^k, \pi_s^{jk}), s \in S, j_s^k \in J_s^k)$, each agent $h \in H$ decides what assets to produce, z_s, consumption, c_s, warehousing, w_s, asset holdings, y_s, and contract sales (borrowing) $\varphi_{j_s^k} > 0$, and purchases (lending), $\varphi_{j_s^k} < 0$, in order to maximize utility (2) subject to the budget set defined by

[5] For a detailed description see Fostel-Geanakoplos (2010)

$$B^h(p,\pi) = \{(z,c,w,y,\varphi) \in R_+^{SK} \times R_+^S \times R_+^S \times R_+^{SK} \times (R^{J_s})_{s\in S\setminus S_T} : \forall s$$

$$(c_s + w_s - e_s^h - w_{s*}) + \sum_k p_s^k(y_s^k - y_{s^*}^k - z_s^k) \leq$$

$$\sum_k y_{s^*}^k d_s^k + \sum_{j_s^k \in J_s} \varphi_{j_s^k} \pi_s^{jk} - \sum_{j_{s*}^k \in J_{s*}} \varphi_{j_{s*}^k} min(p_s^k + d_s^k, j);$$

$$z_s \in Z_s^h;$$

$$\sum_{j_s^k \in J_s^k} max(0, \varphi_{j_s^k}) \leq y_s^k, \forall k\}$$

In each state s, expenditures on consumption and warehousing minus endowments and storage, plus total expenditures on assets minus asset holdings carried over from the last period and asset output from the within-period technology, can be at most equal to total asset deliveries plus the money borrowed selling contracts, minus the payments due at s from contracts sold in the previous period.[6] Within-period production is feasible. Finally, those agents who borrow must hold the required collateral.

Let us emphasize two important things. First, notice that there is no sign constraint on $\varphi_{j_s^k}$: a positive (negative) $\varphi_{j_s^k}$ indicates the agent is selling (buying) contracts or borrowing (lending) π_s^{jk}. Second, notice that we are assuming that short selling of assets is not possible.

2.6 Collateral Equilibrium

A *Collateral Equilibrium* in this economy is a set of asset prices and contract prices, production and consumption decisions, and financial decisions on assets and contract holdings $((p,\pi),(z^h,c^h,w^h,y^h,\varphi^h)_{h\in H}) \in (R_+^K \times R_+^{J_s})_{s\in S\setminus S_T} \times (R_+^{SK} \times R_+^S \times R_+^S \times R_+^{SK} \times (R^{J_s})_{s\in S\setminus S_T})^H$ such that $\forall s$

1. $\sum_{h\in H}(c_s^h + w_s^h - e_s^h - w_{s*}^h) = \sum_{h\in H} y_{s^*}^h d_s$
2. $\sum_{h\in H}(y_s^h - y_{s^*}^h - z_s^h) = 0$
3. $\sum_{h\in H} \varphi_{j_s^k}^h = 0, \forall j_s^k \in J_s$
4. $(z^h, c^h, w^h, y^h, \varphi^h) \in B^h(p,\pi), \forall h$
5. $(z,c,w,y,\varphi) \in B^h(p,\pi) \Rightarrow U^h(c) \leq U^h(c^h), \forall h$

Markets for consumption, assets and promises clear in equilibrium and agents optimize their utility in their budget set. As shown by Geanakoplos and Zame (1997), equilibrium in this model always exists under the assumptions we have made so far.

[6] We take $y_{0*}^h = 0$.

3 News and Asset Prices and Leverage

3.1 A baseline Economy

In this section we assume that there is only one asset. Throughout the paper we consider assets and projects as synonyms. Suppose there are three periods, $t = 0, 1, 2$. The single asset, Y, delivers only at the final period. We assume that state 0 has two successors U, for up, and D, for down, representing good and bad news respectively. Each of these states $s \in \{U, D\}$ has at most two successors sU and/or sD, at which the asset pays 1 or $R < 1$, respectively. Thus the set of states is $S \subseteq \{0, U, D, UU, UD, DU, DD\}$. Figure 4 depicts a tree consistent with this description.

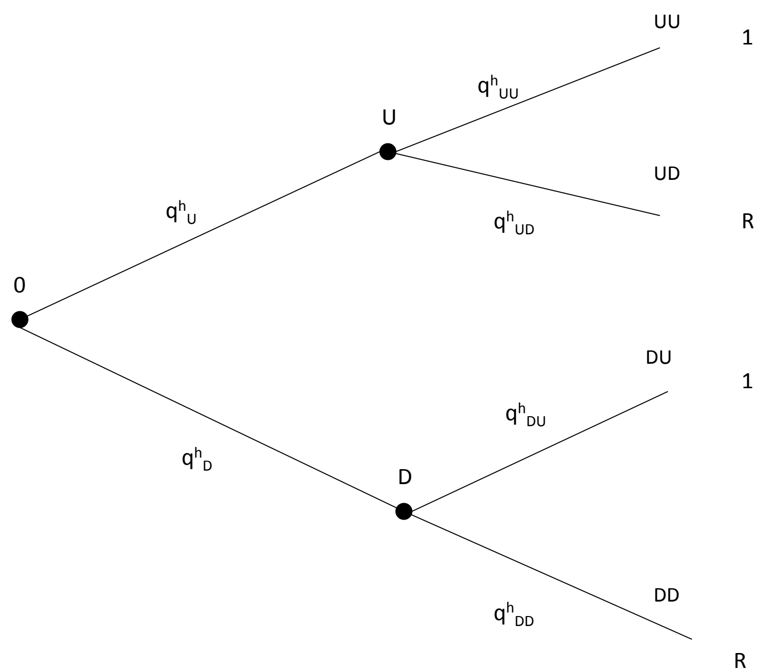

Figure 4: Asset payoff description.

There is a continuum of heterogenous agents indexed by $h \in H = (0, 1)$. The only source of heterogeneity is in the subjective probabilities q_s^h that agent h believes measures the likelihood of moving from s^* to s, where q_s^h is a continuous function of h, for each fixed $s \in S$. If state s exists in the tree, then we suppose that $q_s^h > 0$ for all h. (If state s does not exist in the tree, then for brevity we sometimes refer to q_s^h anyway, where we mean $q_s^h = 0$ for all h.) U can be interpreted as good news since we assume that

$$q_{UU}^h > q_{DU}^h, \forall h \tag{3}$$

i.e., the probability of full payment after U is higher than after D.

We assume the higher the h, the more optimistic the agent is about all aspects of the future. So, whenever $h > h'$, $q_U^h > q_U^{h'}$ and, provided s has two successors, $q_{sU}^h > q_{sU}^{h'}$ for $s \in \{U, D\}$, and, if DU exists in the tree, then $\frac{\bar{q}_{UU}^h}{\bar{q}_{DU}^h} \equiv \frac{q_{0U}^h q_{UU}^h}{q_{0D}^h q_{DU}^h} > \frac{q_{0U}^{h'} q_{DU}^h}{q_{0D}^{h'} q_{DU}^{h'}} \equiv \frac{\bar{q}_{UU}^{h'}}{\bar{q}_{DU}^{h'}}$. The last inequality means that the more optimistic the agent, the more likely he thinks the payoff of 1 is reached via the UU route as opposed to the DU route. We shall refer to all these conditions as the *Optimism Assumption*.

Agents are risk neutral and do not discount the future. They start at $t = 0$ with an endowment of 1 unit of the consumption good and 1 unit of labor. More formally, $U^h = \sum_{s \in S} \bar{q}_s^h c_s$, $e_0^h = 1$ and $e_s^h = 0, s \neq 0$, and $l_0^h = 1$ and $l_s^h = 0, s \neq 0$. In this baseline economy with one asset it is clear that in equilibrium every investor will transform his labor into one unit of the asset at time 0.

3.2 Projects

We consider a family of projects (assets) k such that every agent h believes every project has the same probability Q^h of ultimate success (UU or DU) and probability $1 - Q^h$ of ultimate failure (UD or DD) in the last period. In the intermediate period agents get good news U, which raises their probabilities of success to $q_{UU}^h(k) > Q^h$, or they get bad news D, which lowers their probabilities of success to $q_{DU}^h(k) < Q^h$. Projects k are characterized by the probabilities $(q_U^h(k), q_{UU}^h(k), q_{DU}^h(k))$, where $q_U^h(k) q_{UU}^h(k) + (1 - q_U^h(k)) q_{DU}^h(k) = Q^h$. We ask and answer the question: which of these projects k has the highest equilibrium price at 0, and does that project display pro-cyclical or counter-cyclical leverage and volatility?

Consider three extreme families of projects. The first one is described in figure 5. If state U is reached in the second period, uncertainty is completely resolved since the asset pays for sure 1 at the end. However, if D is reached, uncertainty remains. In fact, D is bad news, but of the sort that not only decreases the expected asset payoff compared with U but also increases final payoff volatility. This kind of project represents the situation in which each piece of bad news is not very informative and induces high future volatility. We call it an extreme "Post-Bad News Volatility" project, or extreme BV for short.[7]

The second one is described in figure 6. We call this type extreme "Post-Good News Volatility" projects, or extreme GV for short. If D is reached, all uncertainty is resolved and the asset pays R for sure. However, if U is reached uncertainty remains. Extreme GV projects represent the situation in which each piece of good news, as opposed to bad news as in the extreme BV projects, is not very informative and induces high future volatility.

[7] This is the example in Geanakoplos (2003, 2010a).

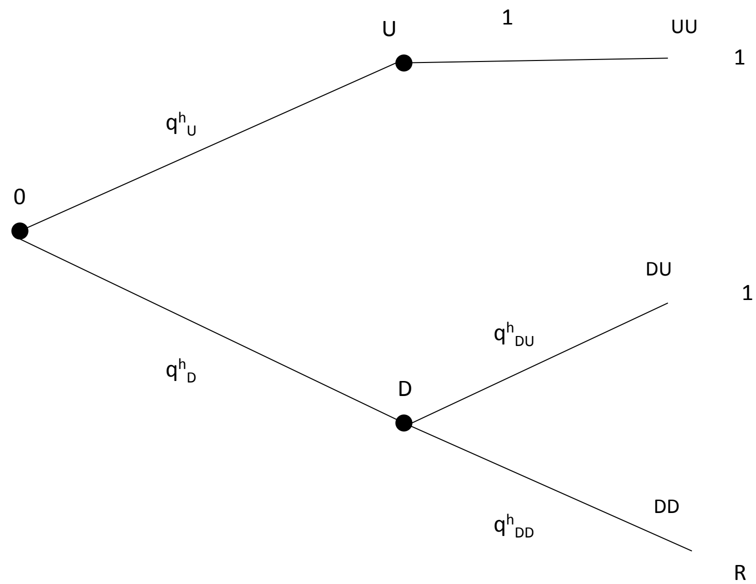

Figure 5: Extreme BV Project.

Thirdly, consider the "two-period" projects shown in Figure 7, in which U is followed by UU for sure, and D is followed by DD for sure. These projects are all equivalent to a two-period tree in which 0 is followed immediately by UU with probability Q^h and by DD with probability $1-Q^h$. Needless to say, the vast majority of the projects fall into none of these three extreme families.

Propositions 1 and 2 show that for every project, equilibrium exists and is unique and that leverage is endogenously determined in equilibrium and corresponds to the "Value at risk equal zero" rule (VaR=0). Each buyer uses the asset as collateral to promise the value of the asset in the worst case scenario in the next period, that is borrowing as much as possible while preventing default from occurring in equilibrium. (We call this the maxmin promise). In propositions 2 to 5 we show that: i) the initial prices of all extreme GV projects are the same as the two period project, and lower than all other projects, ii) the highest initial priced project is always an extreme BV project iii) initial leverage is higher in extreme BV projects than in extreme GV projects and iv) leverage is pro-cyclical in extreme BV projects and counter-cyclical in all the others.

In the remainder of Section 3 we will prove these results and show numerical simulations for a fixed families of probabilities.

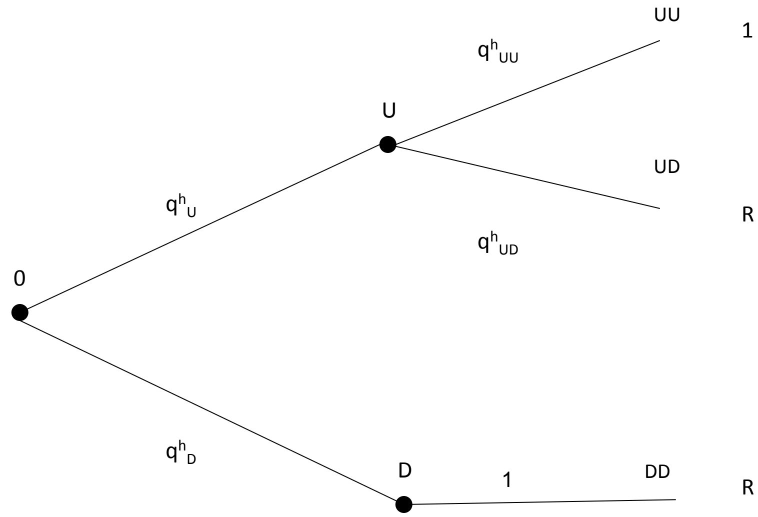

Figure 6: Extreme GV Project.

3.3 Endogenous Leverage

Proposition 1 shows that agents will never default in equilibrium, that is, they only trade *VaR=0* contracts. In fact, the proposition proves something stronger, that only one contract is traded: the maxmin contract.

Proposition 1

Suppose that in equilibrium the max min contract $j_s^* = min_{\tau \in S(s)}\{p_\tau + d_\tau\}$ *is available to be traded, that is* $j_s^* \in J_s$ *for every non-terminal state* s*. Then* j_s^* *is the only contract traded in state* s*, and the risk-less interest rate is equal to zero,* $\pi_s^{j_s^*} = j_s^*$.

Furthermore, $p_U > p_0 > p_D$*. At each state* s *that has two successors, there is a marginal buyer* h_s *such that all agents* $h > h_s$ *buy the asset and sell* j_s^**, and all agents* $h < h_s$ *buy* j_s^* *and/or hold the consumption good. Finally,* $h_0 > h_D$*, if* D *has two successors, and* $h_0 > h_U = h_D$*, provided that* U *and* D *each have two successors.*

Proof: Without loss of generality we only consider contracts in state s with $j \leq max_{\tau \in S(s)}\{p_\tau + d_\tau\}$ since bigger promises are equivalent.

1. All riskless rates are non-positive. If $0 < j \leq j_s^*$ then $\pi_s^j \geq j$.

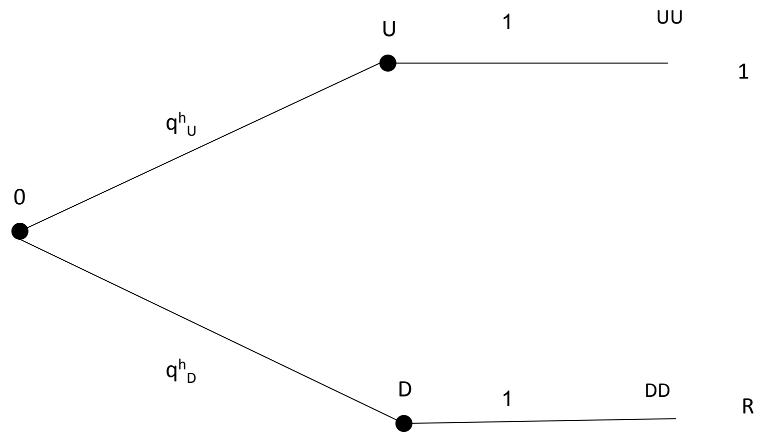

Figure 7: “Two-period” Project.

Consider first the state $s = 0$, where we know the endowment of consumption good is non-zero. Somebody has to hold a positive amount of the consumption good at the end of period $s = 0$, either to consume or to inventory. But if $\pi^j_s < j$ they would have done better investing $\epsilon\pi^j_s$ in contract j and receiving ϵj in the next period giving them a higher utility since there is no discounting, a contradiction. Consider now state $s = U$ and suppose $\pi^j_s < j$. No agent would consume his consumption good at $s = 0$, because he could do better inventorying it into states U and D, eating if in D and buying contract j in state S, and then consuming even more at UU and UD. Hence agents would enter state s with consumption good, but that leads to a contradiction as before. The same argument applies to $s = D$.

2. Observable riskless rates are zero. If $0 < j \leq j^*_s$ is traded in equilibrium, then $\pi^j_s = j$ and $\pi^{j^*_s}_s = j^*_s$.

   Nobody would buy j if $\pi^j_s > j$, since he could do better by inventorying, so $\pi^j_s = j$. The seller of j could have sold $\frac{j}{j^*_s}$ units of j^*_s instead (and used less collateral). If he chose not to do so, then $\frac{\pi^{j^*_s}_s}{j^*_s} \leq \frac{\pi^j_s}{j} = 1$, so $\pi^{j^*_s}_s = j^*_s$.

3. If j with $p_{sU} + d_{sU} > j > j^*_s$ is traded in equilibrium, then $\pi^{j^*_s}_s = j^*_s$. Letting $a_s = \frac{p_s - j^*_s}{p_{sU} + d_{sU} - j^*_s}$ and $b_s = 1 - a_s$, then $\pi^j_s = a_s j + b_s j^*_s$. (The analogous conclusion

would hold if $p_{sD} + d_{sD} > j > j_s^*$.)

Contract j pays fully in the up state, but defaults and pays only $j_s^* = p_{sD} + d_{sD}$ in the down state. The seller of the contract must have put up the collateral of one unit of the asset, and therefore is effectively buying an Arrow security in the U state, paying a price per dollar of

$$\bar{a}_s = \frac{p_s - \pi_s^j}{p_{sU} + d_{sU} - j}$$

The *seller* of contract j could instead have acquired U Arrow securities by buying the asset while borrowing $\pi_s^{j_s^*}$, that is making the riskless promise j_s^*. Hence

$$\frac{1}{\bar{a}_s} = \frac{p_{sU} + d_{sU} - j}{p_s - \pi_s^j} \geq \frac{p_{sU} + d_{sU} - j_s^*}{p_s - \pi_s^{j_s^*}} \geq \frac{p_{sU} + d_{sU} - j_s^*}{p_s - j_s^*} = \frac{1}{a_s}$$

The *buyer* of contract j could have instead inventoried j_s^* consumption goods and bought $(j - j_s^*)$ U Arrow securities via the risky promise as above, hence it must be that

$$\pi_s^j \leq j_s^* + (j - j_s^*)\frac{p_s - \pi_s^j}{p_{sU} + d_{sU} - j}$$

and hence that

$$\frac{(j - j_s^*)}{\pi_s^j - j_s^*} \geq \frac{p_{sU} + d_{sU} - j}{p_s - \pi_s^j}$$

It follows that all the previous inequalities must be equalities, otherwise we would have[8]

$$\frac{(j - j_s^*) + p_{sU} + d_{sU} - j}{\pi_s^j - j_s^* + p_s - \pi_s^j} > \frac{p_{sU} + d_{sU} - j_s^*}{p_s - j_s^*}$$

a contradiction.

Thus if contract j is traded, then $\pi_s^{j_s^*} = j_s^*$ and $\pi_s^j = a_s j + b_s j_s^*$.

4. $\pi_s^{j_s^*} = j_s^*$

If $\pi_s^{j_s^*} > j_s^*$, any agent who ends up holding some of the asset would be foolish not to borrow. At worst the agent uses ϵ units of the asset as collateral to sell ϵ units of contract j_s^*, then inventories $\pi_s^{j_s^*}$ and pays back j_s^*, getting extra utility for nothing. From (2) and (3), no matter which contract j he is borrowing on, $\pi_s^{j_s^*} = j_s^*$.

[8] We make use of the arithmetic property that if $a, b, c, d > 0$, and $\frac{a}{b} > \frac{c}{d}$ then $\frac{a+c}{b+d} > \frac{c}{d}$.

5. If $s \in \{U, D\}$ has two successors, then any portfolio that any agent h would want to hold delivers (c_{sU}, c_{sD}), with $c_{sU} \geq c_{sD}$ and costs $a_s c_{sU} + b_s c_{sD}$, where $a_s = \frac{p_s - j_s^*}{p_{sU} + d_{sU} - j_s^*}$, and $b_s = 1 - a_s$.

   Any feasible portfolio payoff (c_{sU}, c_{sD}) requires $c_{sU} \geq c_{sD}$. The cheapest way to buy those payoffs is to inventory c_{sD} units of the consumption good and to buy $c_{sU} - c_{sD}$ units of the U Arrow security via the purchase of the asset borrowing using contract j_s^*.

6. If $s \in \{U, D\}$ has two successors, then the only contract traded is the maxmin contract j_s^*. Moreover, defining the "marginal buyer" as the unique h_s such that $q_{sU}^{h_s} = a_s$, all agents $h > h_s$ simply buy the asset and sell j_s^*, and all agents $h < h_s$ simply inventory the consumption good and/or buy j_s^*.

   Let H_s be the set of all traders with

$$\frac{q_{sU}^h}{q_{sD}^h} > \frac{a_s}{b_s}$$

   and let I_s be the set of all traders with

$$\frac{q_{sU}^h}{q_{sD}^h} < \frac{a_s}{b_s}$$

   Since every risk neutral agent h wants to hold a portfolio that maximizes his return per dollar

$$\mu_s^h = \frac{q_{sU}^h c_{sU} + q_{sD}^h c_{sD}}{a_s c_{sU} + b_s c_{sD}}$$

   it is evident that agents $h \in H_s$ will only buy the U Arrow securities and agents $i \in I_s$ will only hold portfolios with payoffs $c_{sU} = c_{sD}$. In particular, none of them will buy the contracts j that involve default in the bad state. Since by our increasing optimism assumption, there is exactly one (measure zero) agent h_s with $\frac{q_{sU}^{h_s}}{q_{sD}^{h_s}} = \frac{a_s}{b_s}$, we conclude that there is no default (up to measure zero) in equilibrium, confirming the *VaR=0* rule. It follows that no agent $i \in I_s$ will hold any of the asset. Hence, no $i \in I_s = \{h \in (0,1) : h < h_s\}$ would be able to sell any contracts. All the asset will be held by agents $h \in H_s = \{h \in (0,1) : h > h_s\}$, but since they only want to hold the U Arrow security, they must all be buying the asset via selling the maxmin contract. In short, the maxmin contract is the only contract sold in equilibrium. Note that for $h \in H_s$, $\mu_s^h = q_{sU}^h / a_s$ and for $i \in I_s$, $\mu_s^i = q_{sU}^i + q_{sD}^i = 1$ In short, $\mu_s^h = \max\{1, q_{sU}^h / a_s\}$.

7. $p_U > p_D$.

   If U has just one successor, then $p_U = 1 > p_D$. If D has just one successor, then $p_U > R = p_D$. Suppose both $s \in \{U, D\}$ have two successors. By step 6 only agents in $I_s = [0, h_s)$ consume in state sD, which they do by saving all their wealth at state s. If $p_U \le p_D$, then by step 6 ($p_s = q^{h_s}_{sU} 1 + q^{h_s}_{sD} R$) and the optimism assumption, we would need $h_D > h_U$. Furthermore, every agent $h \in (0,1)$ would have at least as much wealth at $s = D$ as he does at $s = U$. But that would be a contradiction, since the total supply of consumption goods is the same $1 + R$ at UD and DD.

8. Any portfolio that any agent h would want to hold at state 0 delivers (c_U, c_D), with $c_U \ge c_D$ and costs $a_0 c_U + b_0 c_D$, where $a_0 = \frac{p_0 - j_0^*}{p_U - j_0^*}$ and $b_0 = 1 - a_0$. The only contract traded is the maxmin contract j_0^*. Moreover, there is a "marginal buyer" h_0 who is indifferent between buying the asset or holding money at state 0. All agents $h > h_0$ simply buy the asset and sell j_0^*, and all agents $h < h_0$ simply buy j_0^* and/or hold the consumption good.

   Because $p_U > p_D$, the description of equilibrium in period $s = 0$ is completely analogous to the previous cases, except that now we must replace $q^h_{ss'}$ with $q^h_{ss'} \mu^h_{s'}$. The identical proof goes through provided that we can show that the utility agent h gets from the cash flows $c_U > c_D$ is continuous and strictly increasing in h. That follows if whenever $h > i$,

$$\frac{q^h_{0U} \mu^h_U}{q^h_{0D} \mu^h_D} > \frac{q^i_{0U} \mu^i_U}{q^i_{0D} \mu^i_D}$$

   or equivalently if

$$\frac{q^h_{0U} \max\{1, q^h_{UU}/q^{h_U}_{UU}\}}{q^h_{0D} \max\{1, q^h_{DU}/q^{h_D}_{DU}\}}$$

   is increasing in h. For $h \ge h_D$, this means

$$\frac{q^h_{0U} q^h_{UU}/q^{h_U}_{UU}}{q^h_{0D} q^h_{DU}/q^{h_D}_{DU}}$$

   is increasing in h, which follows from the optimism assumption (since $q^{h_U}_{UU}$ and $q^{h_D}_{DU}$ are fixed as h varies). For $h_D \ge h \ge h_U$, this means

$$\frac{q^h_{0U} q^h_{UU}/q^{h_U}_{UU}}{q^h_{0D}}$$

   which is increasing in h since q^h_{0U} and q^h_{UU} are increasing, and q^h_{0D} is decreasing in h. For $h_U \ge h$, this means

$$\frac{q^h_{0U}}{q^h_{0D}}$$

   which is definitely increasing.

9. Furthermore, $p_U > p_0 > p_D$. If D has two successors, then $h_0 > h_D$, and if both U and D have two successors, then $h_0 > h_U = h_D$. If U has two successors and D has one successor, then $h_0 = h_U$.

The marginal buyer h_0 must be indifferent between the asset and the consumption good. Since p_0 invested in the consumption good yields p_0 in both states U and D, we must have $p_U > p_0 > p_D$. Since all the buyers $h \in (h_0, 1)$ borrow $p_D \geq R$ at 0, they each owe $p_D \geq R$ at U and D. If D has two successors, then $p_D > R$ and the most any agent can borrow at D is R. Hence all the agents $h \in (h_0, 1)$ go completely bankrupt at D and the marginal buyer $h_D < h_0$. If in adddition U has two successors, then the most that can be borrowed at U is also R. Hence again the agents $h \in (h_0, 1)$ are forced to sell some of their assets, and the marginal buyer $h_U < h_0$. But then every agent $h \in (0, h_0) \supset [(0, h_U) \cup (0, h_D)]$ has the same wealth $1 + p_0$ at U and at D. In order for consumption demand to equal consumption supply at UD and DD, we must then have $h_U = h_D = (1 + R)/(1 + p_0)$. If U has two successors and D has one successor, then $h_D = R$ and the agents $h \in (h_0, 1)$ can just roll over their loans at U and keep their assets, so $h_0 = h_U = (1 + R)/(1 + p_0)$.■

As discussed before, *leverage* is endogenously determined in equilibrium. In particular, the proposition *derives* the conclusion that although all contracts will be priced in equilibrium, the only contract actively traded is the maxmin contract, which corresponds to the Value at Risk equal zero rule *assumed* by many other papers in the literature.

Geanakoplos (2003) proved a similar proposition for a special case corresponding to the extreme BV economy. Proposition 1 is more general and encompasses all other economies characterized by binary trees we will consider in this paper.

The key assumption in the proposition is that the tree is binary. (This implies that the maxmin promise plus the U Arrow security, obtained by buying the asset while selling the maxmin contract, positively spans the set of feasible portfolios payoffs.) Another important ingredient in the proof is the continuum of distinct risk neutral agents. This allows us to find a marginal buyer who partitions the set of agents into "optimists" who want to leverage as much as possible and "pessimists" who do not want to compete with the optimists for any risky portfolio and who therefore end up holding no risk at all.

The reader can easily check that the key to the binary assumption is that the asset has two distinct payoffs in the immediate successors states of every node. We could have derived *Var = 0* even if there were three successors states, or even multiple assets, provided that each asset had exactly two distinct payoffs in the following period. The proof also did not depend on there being two terminal payoffs 1 and R. There could just as well have been four final payoffs, a different one for each terminal node. It might be natural to assume that the worst terminal payoff after D is far worse than the worst terminal payoff after U: bad news makes a disaster possible. In that case, it is clear from *Var = 0* that the maxmin promise at D would be much smaller than the maxmin promise at 0 and U, and hence leverage would be pro-cyclical in all projects. But we shall not take this route. We shall tie our hands and assume that there are only two possible terminal outcomes, 1 and R, but we shall prove that leverage is nonetheless pro-cycical in the highest priced projects.

3.4 Equilibrium and Uniqueness

We use proposition 1 to describe a system of equations that characterizes equilibrium. First we deal with the case in which each $s \in \{U, D\}$ has two successors. The system has six equations and six unkowns $p_0, p_U, p_D, h_0, h_U, h_D$.

As was shown in proposition 1, at each state s there will be a *marginal buyer*, h_s, who will be indifferent between buying or selling Y. All agents $h > h_s$ will buy all they can afford of Y, i.e., they will sell all their endowment of the consumption good and borrow to the max using Y as collateral. On the other hand, agents $h < h_s$ will sell all their endowment of Y and lend to the more optimistic investors. Equating demand and supply, or equivalently, expenditures and revenues, provides us with the first three equations in our system.

At $s = 0$ aggregate revenue from sales of the asset is given by p_0.[9] On the other hand, aggregate expenditure on the asset is given by $(1 - h_0)(1 + p_0) + p_D$. The first term is total income (endowment plus revenues from asset sales) of buyers $h \in [h_0, 1)$. The second term is borrowing, which from proposition 1 is p_D. Equating we have

$$p_0 = (1 - h_0)(1 + p_0) + p_D \tag{4}$$

Let $s \in \{U, D\}$ have two successors sU and sD. Total revenue from asset sales must equal total expenditure on asset purchases. This gives us

$$p_s = (p_s - p_D) + (h_0 - h_s)(p_0 + 1) + R \tag{5}$$

The first term on the RHS is the income after debt repayment of those holding the asset from period 0. The second term is the income of the new buyers $h \in [h_s, h_0)$, carried over from period 0. The last term is new borrowing.[10]

[9] All asset endowments and production add to 1 and without loss of generality are put up for sale even by those who buy it.

[10] Notice that since D has two successors, $p_D > R$. All the agents $h \in [h_0, 1)$ will be forced to sell off all their assets even though they think the price p_D is well below the value they would be willing to pay if they had the money. At U the original buyers $h \in [h_0, 1)$ can only borrow R, which is less than the p_D they owe, so they will not be able to roll over all their loans without selling some assets. Even though the traders $h \in [h_0, 1)$ think the asset is underpriced at p_U, and even though the news is good, tightening margins force them to sell. Thus fire sales can take place in equilibrium at both U and D. If s has just one successor then any one agent can buy all the assets since leverage is 100%. Fire sales do not occur in that case.

The next equations state that the price at $s \in \{U, D\}$ is equal to the marginal buyer's valuation of the asset's future payoff.

$$p_s = q_{sU}^{h_s} 1 + q_{sD}^{h_s} R \tag{6}$$

The last equation equates the marginal utility to h_0 of one dollar to the marginal utility of using one dollar to purchase Y at $s = 0$:

$$\frac{q_U^{h_0} p_U (q_{UU}^{h_0}/q_{UU}^{h_U}) + q_D^{h_0} p_D (q_{DU}^{h_0}/q_{DU}^{h_D})}{p_0} = \frac{q_U^{h_0} 1 (q_{UU}^{h_0}/q_{UU}^{h_U}) + q_D^{h_0} 1 (q_{DU}^{h_0}/q_{DU}^{h_D})}{1} \tag{7}$$

Notice that payoffs on both sides of the equation are weighted by the ratio $(q_{sU}^{h_0}/q_{sU}^{h_s})$ for $s \in \{U, D\}$. If agent h_0 reaches state $s \in \{U, D\}$ with a dollar he will want to leverage his wealth to the max to purchase Y.[11] This will result in a gain per dollar of $\frac{q_{sU}^{h_0}(1-R)}{p_s - R} = \frac{q_{sU}^{h_0}(1-R)}{q_{sU}^{h_s} 1 + q_{sD}^{h_s} R - R} = \frac{q_{sU}^{h_0}}{q_{sU}^{h_s}}$.[12] Hence the marginal utility of a dollar at time 0 is given by the probability of reaching U times the dollar times the marginal utility given above plus the analogous expression for reaching D. This explains the RHS of equation (8). The LHS has exactly the same explanation once we realize that the best action for the h_0 at $s \in \{U, D\}$ is to sell the asset and use the cash to buy more of it on margin. This gives six equations in six unknowns.

If s has a unique successor, then the last equation must be modified by replacing $(q_{sU}^{h_0}/q_{sU}^{h_s})$ by 1 and dropping the variable h_s. Furthermore, the equation in (5) and the equation in (6) corresponding to state s, are replaced with one simple equation $p_s = 1$(if $s = U$) or $p_s = R$ (if $s = D$).

Next we prove existence and uniqueness.

Proposition 2

Equilibrium exists and is unique in the baseline economy.

Proof: Consider first the system of six equations, when each state $s \in \{U, D\}$ has two successors. We shall now reduce the six equilibrium conditions into one equation $F(h) = 0$. We proceed to define F. In accordance with step 9 of proposition 1, let $h_U = h_D = h$. For $h \in (0, 1)$ let $p_0(h) = \frac{1+R}{h} - 1$. Thus we already know that $p_0(h)$ declines as h increases. Define $p_U(h) = q_{UU}^h 1 + q_{UD}^h R$ and $p_D(h) =$

[11] Agents are perfectly rational and forward looking. There are other options at $s = D$, like eating the good, storing it or buying Y unleveraged, but these are all dominated strategies in equilibrium.

[12] Another way of understanding the same is to notice that buying Y on margin at s is equivalent to buying the Arrow security that pays only at up (since at down the net payoff is zero). The price of this security is given by $q_{sU}^{h_s}$, the marginal buyer's valuation. Hence, with a dollar, h_0 can buy $1/q_{sU}^{h_s}$ units which are worth $(q_{sU}^{h_0}/q_{sU}^{h_s})$, explaining the ratio.

$\min\{q^h_{DU}1 + q^h_{DD}R, p_0(h)\}$. From equation (4), we have $1 - h_0(h) = \frac{p_0(h)-p_D(h)}{1+p_0(h)}$, or $h_0(h) = \frac{1+p_D(h)}{1+p_0(h)}$ or $h_0(h) = \frac{1+p_D(h)}{1+R}h$. If $p_0(h) > p_D(h)$, then $p_D(h)$ is increasing in h. Hence $h_0(h)$ is increasing in h if $p_0(h) > p_D(h)$.

Let $F(h) = \frac{q_U^{h_0(h)} p_U(h) q_{UU}^{h_0(h)}/q^h_{UU} + q_D^{h_0(h)} p_D(h) q_{DU}^{h_0(h)}/q^h_{DU}}{p_0(h)} - \frac{q_U^{h_0(h)} 1 q_{UU}^{h_0(h)}/q^h_{UU} + q_D^{h_0(h)} 1 q_{DU}^{h_0(h)}/q^h_{DU}}{1}$. We will show that at any point $h \in (0,1)$ where $F(h) = 0$, F is increasing in h. Note first that as h increases, $p_0(h)$ decreases, and this causes F to increase. Next, note from the preceding paragraph that at any $h \in (0,1)$, $p_U(h) > p_D(h)$. Hence at $F(h) = 0$, $p_U(h)/p_0(h) > 1 > p_D(h)/p_0(h)$. Hence, $h_0(h)$ increases when h increases in a neighborhood of $F(h) = 0$. By the optimism assumption this means $q_U^{h_0(h)} q_{UU}^{h_0(h)} / q_D^{h_0(h)} q_{DU}^{h_0(h)}$ increases, which (by the previous inequalities) has the effect of increasing $F(h)$. Finally, $\frac{p_U(h)/q^h_{UU}}{p_0(h)} - \frac{1}{q^h_{UU}} = \frac{[q^h_{UU}1 + q^h_{UD}R]/q^h_{UU}}{p_0(h)} - \frac{q^h_{UU} + q^h_{UD}}{q^h_{UU}} = (\frac{1}{p_0(h)} - 1) + (\frac{R}{p_0(h)} - 1)\frac{q^h_{UD}}{q^h_{UU}}$. This is increasing in h because $\frac{R}{p_0(h)} < 1$. Exactly the same argument can be used to show that $\frac{p_D(h)/q^h_{DU}}{p_0(h)} - \frac{1}{q^h_{DU}} = (\frac{1}{p_0(h)} - 1) + (\frac{R}{p_0(h)} - 1)\frac{q^h_{DD}}{q^h_{DU}}$ is increasing in h. Thus we have shown that indeed $F(h)$ is increasing in h in a neighborhood of $F(h) = 0$. This and the continuity of F proves that there is at most a unique h with $F(h) = 0$, and hence that equations (4)-(7) have at most one solution.

Notice that as $h \to 0$, $p_0(h) \to \infty$, so $F(h)$ must become negative. But when $h = 1$, $p_0(h) = R = p_D(h) < p_U(h)$, so $F(1) > 0$. Since F is continuous, there must be an $h \in (0,1)$ with $F(h) = 0$. This completes the proof in the case where each $s \in \{U, D\}$ has two successors. If exactly one $s \in \{U, D\}$ has two successors, the proof can be handled almost the same way.

If both U and D have a single successor, then the proof is modified by defining the equation F in the single variable h_0 as follows. As before, define $p_0(h_0) = \frac{1+R}{h_0} - 1$. Now define $F(h) = \frac{Q^{h_0}1 + (1-Q^{h_0})R}{p_0(h_0)} - 1$. Raising h_0 near where $F(h_0) = 0$ lowers $p_0(h_0)$ and raises Q^{h_0}, both of which increase F. Hence for the reasons above $F(h_0) = 0$ has a unique solution.■

3.5 Asset Prices and Leverage

In this section we present results that characterize prices and leverage of different projects.

Proposition 3

Only extreme BV *projects generate pro-cyclical leverage; all other projects (except the trivial two-period projects) generate counter-cyclical leverage.*

Proof: By proposition 1, buying 1 unit of Y *on margin* at state s means: selling a promise of $\min_{\tau \in S(s)}[p_\tau + d_\tau]$ using that unit of Y as collateral, and paying $(p_s - \min_{\tau \in S(s)}[p_\tau + d_\tau])$ in cash. The Loan to Value (LTV) of Y at s is, $LTV_s = \frac{\min_{\tau \in S(s)}[p_\tau + d_\tau]}{p_s}$. If $s \in \{U, D\}$ has only one successor sU, then s must be good news and so $s = U$. Moreover, every agent will agree on $q^h_{sU} = q^h_{UU} = 1$ and so in equilibrium we must have $p_U = d_{UU} = 1$ and therefore $LTV_U = 1/1 = 100\%$. If we are not in the trivial two-period model, there will be still uncertainty remaining at $s = D$, i.e. $s = D$ has two successors, so $R < p_D < 1$ and hence $LTV_D = R/p_D < 100\% = LTV_U$. Hence, leverage is pro-cyclical. In the other extreme case, if $s \in \{U, D\}$ has only one successor sD, then $s = D$, $q^h_{sD} = q^h_{DD} = 1$, $p_D = d_{DD} = R$ and therefore $LTV_U = R/R = 100\%$. Since there will be still uncertainty remaining at $s = U$, i.e. $s = U$ has two successors, then $R < p_U < 1$ and hence $LTV_U = R/p_U < 100\% = LTV_D$. Hence leverage is counter-cyclical. Every project in which both U and D have two successors gives rise to counter-cyclical leverage because $p_U > p_D$ and hence $LTV_U = R/p_U < R/p_D = LTV_D$. ■

The result is a direct consequence of the *VaR = 0* rule. Every project is worth the most after good news U but as long as every agent still thinks D is possible, the minimum promise of R will be the only traded promise at U. Hence the value ratio of promise to collateral will be least just when the price of the asset is highest.

Proposition 4 shows that every extreme GV project has the same price, which is lower than the price of every other project. Finally, proposition 5 shows that the highest priced projects are always exclusively extreme BV projects.

Proposition 4

Every extreme GV *project has the same initial price and leverage as the two-period model, and these are lower than the initial price and leverage of every other project.*

Proof: From the proof of proposition 1 it is evident that the initial price p_0 is the same as in the trivial two-period project. As we saw in the proof of proposition 2, in the trivial two-period model $p_0(h_0) = \frac{1+R}{h_0} - 1 = Q^{h_0}1 + (1 - Q^{h_0})R$.

Consider now any other project in which at least one $s \in \{U, D\}$ has two successors and a marginal buyer $\bar{h}$. We know from proposition 1 that the initial price $p_0(\bar{h}) = \frac{1+R}{\bar{h}} - 1 > Q^{\bar{h}}1 + (1 - Q^{\bar{h}})R$. The first equality is the familiar equality derived in step 9 of proposition 1. The strict inequality holds because by proposition 1 the marginal utility to $\bar{h}$ of holding the consumption good at 0 is 1 (since the price of Y at U and D is equal to its expected payoffs according to $\bar{h}$, that is $\mu^{\bar{h}}_U = \mu^{\bar{h}}_D = 1$) and because $\bar{h}$ strictly prefers not to buy Y at 0. Thus if $\bar{h} < h_0$, then $p_0(\bar{h}) = \frac{1+R}{\bar{h}} - 1 > \frac{1+R}{h_0} - 1 = p_0(h_0)$. But by the optimism assumption, if $\bar{h} \geq h_0$, then $p_0(\bar{h}) > Q^{\bar{h}}1 + (1 - Q^{\bar{h}})R \geq Q^{h_0}1 + (1 - Q^{h_0})R = p_0(h_0)$. Either way, $p_0(\bar{h}) > p_0(h_0)$.

We now turn to initial leverage, which is $\frac{R}{p_0}$ in the two-period model (and in any extreme GV project) and $\frac{\bar{p}_D}{\bar{p}_0}$ in the other project. Suppose $\frac{R}{p_0} \geq \frac{\bar{p}_D}{\bar{p}_0}$. Then the down-payment would be strictly less in the two-period project, while the payoff $1 - R > 1 - \bar{p}_D$ would be strictly more. Hence in order for the marginal buyer in each economy to be indifferent between the project and money, $h_0 < \bar{h}_0$. But that leads to a contradiction since then from the supply equals demand equation for each economy, $h_0 = \frac{(1-h_0)}{p_0} + \frac{R}{p_0} > \frac{(1-\bar{h}_0)}{\bar{p}_0} + \frac{\bar{p}_D}{\bar{p}_0} = \bar{h}_0.\blacksquare$

According to proposition 4, the extreme GV projects have the lowest initial prices of all. In proposition 5 we show that some extreme BV project has the highest price of all, provided we confine our attention to projects satisfying one more optimism assumption.

Proposition 5

Let $q_s^h > 0$ be the probabilities in a non-extreme project in the baseline economy satisfying the optimism conditions and the condition that $\bar{q}_{UD}^h/\bar{q}_{DD}^h$ is weakly decreasing in h.[13] *Then, there is another set of probabilities q_s^{Bi} that give rise to a corresponding extreme* BV *economy with $p_0^B > p_0$. It follows that among projects satisfying the additional optimism assumption, only an extreme* BV *project gives the maximal initial price.*

Proof: We will make use of the following lemma.

Lemma:

Let $q_s^h(0) > 0$ be probabilities for an extreme BV. Let the weakly declining function $t : (0,1) \to (0,1)$ define probabilities $q_s^h(t)$ by the terminal probabilities

$$
\begin{aligned}
\bar{q}_{UU}^h(t) &= \bar{q}_{UU}^h(0) + t_h \bar{q}_{DU}^h(0) \\
\bar{q}_{UD}^h(t) &= \bar{q}_{UD}^h(0) + t_h \bar{q}_{DD}^h(0) \\
\bar{q}_{DU}^h(t) &= \bar{q}_{DU}^h(0) - t_h \bar{q}_{DU}^h(0) \\
\bar{q}_{DD}^h(t) &= \bar{q}_{DD}^h(0) - t_h \bar{q}_{DD}^h(0)
\end{aligned}
$$

obtained by moving t_h of agent h's probability from DU and DD to UU and DU, respectively. Then the $q_s^h(t)$ also satisfy the optimism assumption. Moreover the unique equilibrium initial price $p_0(0)$ of the original extreme BV economy is greater than the unique equilibrium price $p_0(t)$.

Proof of lemma:

Notice that for all h, $\bar{q}_{UU}^h(t) + \bar{q}_{DU}^h(t) = \bar{q}_{UU}^h(0) + \bar{q}_{DU}^h(0) = Q^h$ and $\bar{q}_{UD}^h(t) + \bar{q}_{DD}^h(t) = \bar{q}_{UD}^h(0) + \bar{q}_{DD}^h(0) = 1 - Q^h$ and $\frac{\bar{q}_{DU}^h(t)}{\bar{q}_{DD}^h(t)} = \frac{\bar{q}_{DU}^h(0)}{\bar{q}_{DD}^h(0)}$. Notice that for $i > h$, $\frac{\bar{q}_{UU}^i(t)}{\bar{q}_{DU}^i(t)} > \frac{\bar{q}_{UU}^h(t)}{\bar{q}_{DU}^h(t)}$ and $\frac{\bar{q}_{DU}^i(t)}{\bar{q}_{DU}^h(t)} \geq \frac{\bar{q}_{DU}^i(0)}{\bar{q}_{DU}^h(0)}$. Fix the marginal buyer h at D at the equilibrium level for the original extreme BV economy $q_s^h(0)$. Following the proof of proposition

[13] The extra assumption guarantees that the higher is h, the more likely an outcome of R came from DD rather than UD.

2, note that p_D does not depend on t because $\frac{\bar{q}^h_{DU}(t)}{\bar{q}^h_{DD}(t)} = \frac{\bar{q}^h_{DU}(0)}{\bar{q}^h_{DD}(0)}$. Hence h_0 is a function of h alone. Consider the expression $F(h,t)$, where $F(h,t) = \bar{q}^{h_0(h)}_{UU}(t)[(\frac{1}{p_0(h)}-1)+(\frac{R}{p_0(h)}-1)\frac{q^h_{UD}(t)}{q^h_{UU}(t)}]+\bar{q}^{h_0(h)}_{DU}(t)[(\frac{1}{p_0(h)}-1)+(\frac{R}{p_0(h)}-1)\frac{q^h_{DD}(t)}{q^h_{DU}(t)}]$. Then $F(h,t) = (\frac{1}{p_0(h)}-1)(\bar{q}^{h_0(h)}_{UU}(t)+\bar{q}^{h_0(h)}_{DU}(t))-(1-\frac{R}{p_0(h)})[\bar{q}^{h_0(h)}_{UU}(t)\frac{\bar{q}^h_{UD}(t)}{\bar{q}^h_{UU}(t)}+\bar{q}^{h_0(h)}_{DU}(t)\frac{\bar{q}^h_{DD}(t)}{\bar{q}^h_{DU}(t)}] = (\frac{1}{p_0(h)}-1)(\bar{q}^{h_0(h)}_{UU}(t)+\bar{q}^{h_0(h)}_{DU}(t))-(1-\frac{R}{p_0(h)})[\frac{\bar{q}^{h_0(h)}_{UU}(t)}{\bar{q}^h_{UU}(t)}\bar{q}^h_{UD}(t)+\frac{\bar{q}^{h_0(h)}_{DU}(t)}{\bar{q}^h_{DU}(t)}\bar{q}^h_{DD}(t)]$. We wish to show that $F(h,t) < 0$ for $t > 0$. Since $(\bar{q}^{h_0(h)}_{UU}(t) + \bar{q}^{h_0(h)}_{DU}(t)) = Q^{h_0(h)}$ is independent of t, and since $(1 - \frac{R}{p_0(h)}) > 0$, we must show that $G(h,t) > G(h,0)$, where $G(h,t) = \frac{\bar{q}^{h_0(h)}_{UU}(t)}{\bar{q}^h_{UU}(t)}\bar{q}^h_{UD}(t) + \frac{\bar{q}^{h_0(h)}_{DU}(t)}{\bar{q}^h_{DU}(t)}\bar{q}^h_{DD}(t)$. Recall that $h_0(h) > h$, hence $\frac{\bar{q}^{h_0(h)}_{DU}(t)}{\bar{q}^h_{DU}(t)} \geq \frac{\bar{q}^{h_0(h)}_{DU}(0)}{\bar{q}^h_{DU}(0)}$. Moreover, $\bar{q}^h_{UD}(0) = 0$. At any (h,t), $\frac{\bar{q}^{h_0(h)}_{UU}(t)}{\bar{q}^h_{UU}(t)} > \frac{\bar{q}^{h_0(h)}_{DU}(t)}{\bar{q}^h_{DU}(t)}$, so $G(h,t) > G(h,0)$ because $\bar{q}^h_{UD}(t) + \bar{q}^h_{DD}(t) = \bar{q}^h_{UD}(0) + \bar{q}^h_{DD}(0)$. Thus we have shown $F(h,t) < 0$. Hence as in the proof of proposition 2, there must be $h(t) > h$ with $F(h(t),t) = 0$. But then by the familiar formula for the initial price given in (9) of proposition 1 and in proposition 2, $p_0(h(t)) < p_0(h)$. This concludes the proof of the lemma.

To prove proposition 5, notice that given any non-extreme project q^h_s, we can find an extreme BV project defined by $q^h_{DD}(0) = q^h_{UD} + q^h_{DD}$ and $q^h_{DU}(0) = q^h_{UD}\frac{q^h_{DU}}{q^h_{DD}} + q^h_{DU}$ and a weakling decreasing function t_h (defined by $t_h = \frac{q^h_{UD}}{q^h_{DD}(0)}$) so that the original project corresponds to project t in the lemma. ■

The idea of the proof is as follows. Given an arbitrary project that is not extreme BV, it is possible to find an extreme BV project such that every agent's beliefs conditional on bad news d are the same, (so that the price after bad news is the same and hence just as much can be borrowed in equilibrium at time 0) and for which $(q^i_u q^i_{uU}/q^i_d q^i_{dU})/(q^h_u q^h_{uU}/q^h_d q^h_{dU})$ has risen for all $i > h$. This makes it more attractive for an optimist to buy the asset at time 0 by leveraging, rather than waiting to buy the asset after news has arrived, and thus gives the extreme BV project a higher initial price.

3.6 Numerical Simulations

In this section we present numerical simulations in order to develop more intuition for all the previous results.

3.6.1 Three-period economy

We simulate equilibrium now in the two extreme cases of BV and GV. To fix ideas, suppose that in every project, the probability according to h of final good output 1 is

$$Q^h = 1 - (1-h)^2 = q_U^h q_{UU}^h + (1-q_U^h) q_{DU}^h \tag{8}$$

For the extreme BV economy we take $q_U^h = q_{DU}^h = h$, and for the extreme GV project we take $q_U^h = q_{UU}^h = \sqrt{1-(1-h)^2}$.

We first solve the system of equations described in section 3.4 to find the equilibrium in the extreme BV project. Figure 8 shows equilibrium prices, marginal buyers and leverage for $R = .2$.

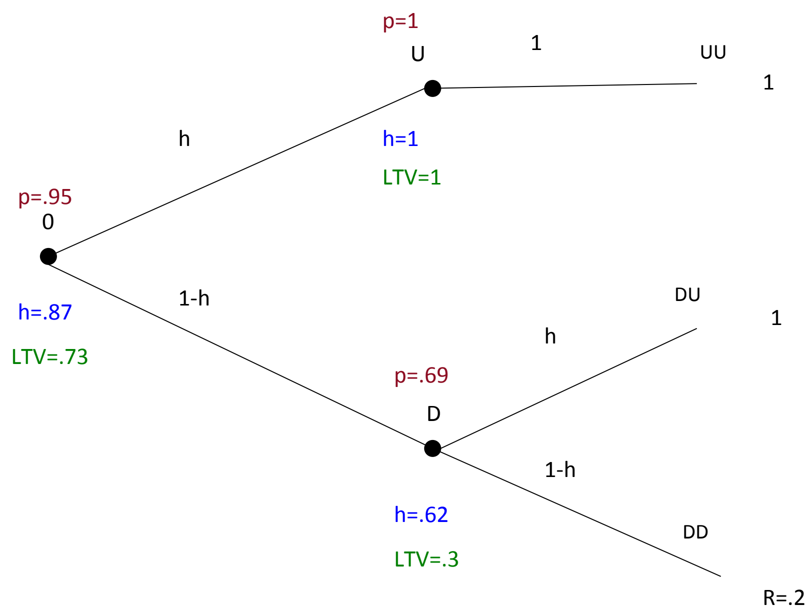

Figure 8: Extreme BV Equilibrium for $R = .2$.

The first observation is that the price of the asset falls from 0 to D, from .95 to .69, a fall of 27%. The marginal buyer at $t = 0$, $h = .87$, thinks at the beginning that there is a probability of 1.69% of reaching the disaster state DD, but once D is reached this probability rises to 13%. This would imply a fall in the price of only 9%. So why is the crash of 27% so much bigger than the bad news of 9%? There are three reasons for the crash.

First, as we just saw, is the presence of bad news. The second reason is that after bad news, the leveraged investors lose all their wealth: the value of the asset at D is exactly equal to their debt, so they go bankrupt. Therefore even the topmost buyer at D is below the marginal buyer at 0. Third, with the arrival of bad news, leverage goes down (margins go up), from $LTV_0 = .73$ to $LTV_D = .3$, so more buyers are needed at D than at 0. Thus the marginal buyer at D is far below the marginal buyer at 0: $h_D = .62 < .87$. The asset falls so far in price at D because every agent values it less and because the marginal buyer is so much lower. This phenomenon was called the *Leverage Cycle* by Geanakoplos (2003) and extended further to many assets and adverse selection by Fostel-Geanakoplos (2008).

We solve next the system of equations described in section 3.4 to find the equilibrium in the extreme GV project. Figure 9 shows equilibrium prices, marginal buyers and leverage for $R = .2$.

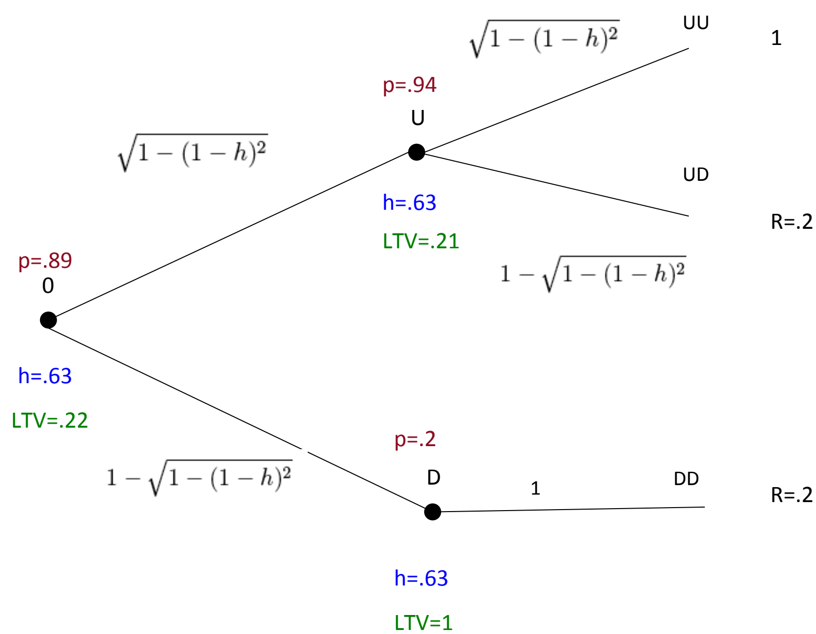

Figure 9: Extreme GV Equilibrium for $R = .2$.

In equilibrium, the asset price collapses from .89 all the way to .2 given the imminent nature of the disaster once D has been reached. It goes up at U to .94. The marginal buyer at $t = 0$ and $t = U$ is the same, so optimists roll-over their debt once they reach U.

3.6.2 Long-run analysis

We extend our previous examples to an N horizon economy. We maintain the same terminal probabilities for outcomes 1 and R, independent of N, with constant probabilities of U throughout each tree. The extreme BV and extreme GV projects are described in figure 10. In the extreme BV project, as before, the imminent occurrence of the bad final outcome R is pushed until the very end, and bad news comes in small drops with an associated higher future volatility. On the other hand, in the extreme GV project, good news, instead of bad news, has the property of revealing little information and inducing high volatility. We calculate the equilibrium for each project separately. The system of equations that characterizes the equilibrium in each project and the equilibrium values are described in detailed in Appendix 6.1. They are the natural (though not obvious) extension of the three period case. The prices and leverage are noted at some of the nodes for $N = 10$ in figure 10.

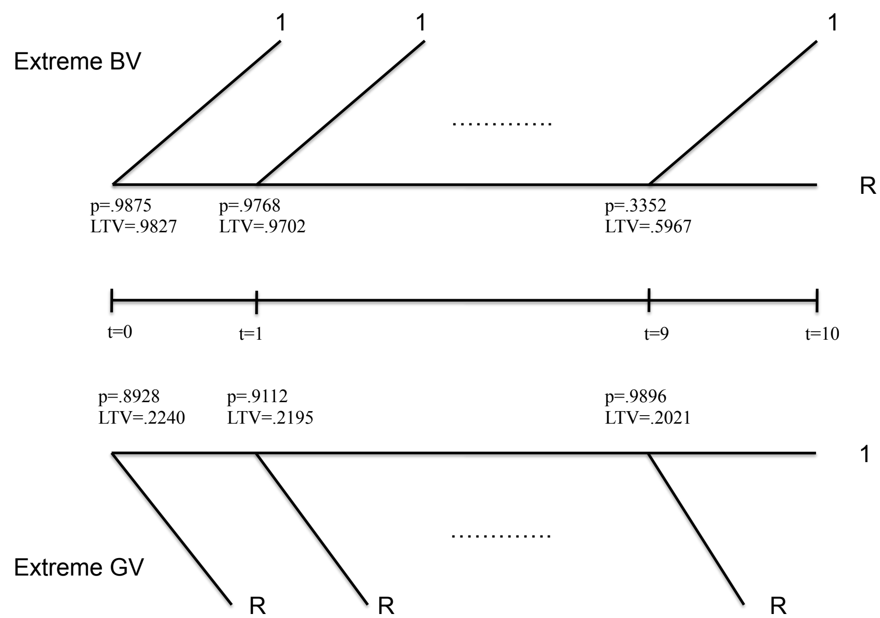

Figure 10: Prices and leverage for extreme BV and extreme GV projects, $N = 10$ periods for $R = .2$.

Figure 10 shows that the results of previous sections hold even in longer horizon economies. The price of the extreme BV project is higher than the price in the extreme GV project and leverage is pro-cyclical in the extreme BV project and counter-cyclical in the extreme GV project. In fact, the longer the horizon the bigger the gap in initial prices. The gradually unfolding descent in the N period extreme BV project is reminiscent of the slowly building crisis of 2007-2009.

4 Does Bad News Come With High Volatility?

In this section we move on to answer a more difficult question: if agents have the opportunity to use their labor to produce any *combination* of the two type of projects, extreme BV and extreme GV, which combination would they choose in equilibrium? The question is made still more difficult because we assume that news about the projects are independent, requiring four successors of the initial node. We thus get a good robustness check of our binary tree conclusions.

It is very tempting to jump to the conclusion that all agents will choose the extreme BV project since it has a higher price at the beginning as shown in Section 3. Unfortunately, this answer is incorrect. Further inspection reveals that once everyone else has chosen the extreme BV project, it becomes profitable for any one agent to produce the extreme GV project. To solve the problem we need to appeal to the full force of the multiple asset and multiple states model described in section 2.

Suppose there are two assets, X and Y, with independent payoffs. Asset X corresponds to the extreme BV project and asset Y to the extreme GV project. Their probabilities are as defined in the numerical simulations in Section 3.6. The joint tree of payoffs is described in figure 11. Note that state $s = 0$ now has four successors. For example, the state (U, U) in the intermediate period corresponds to the situation in which X (BV) and Y (GV) receive good news. The probability of such event for agent h is $h\sqrt{1-(1-h)^2}$.

Agents are as in the baseline economy in section 3. They can transform their unit of labor into a portfolio of different projects at $t = 0$. The within-period technology is given by $Z_0^h = \{(z_0^X, z_0^Y) \in R_+^2 : z_0^X + z_0^Y = 1\}$, where z_0^X is the share of X (BV project) and z_0^Y the share of Y (GV project).

Figure 11 shows the equilibrium prices at each node for both assets, extreme BV and extreme GV, respectively for $R = .2$. At equilibrium, all agents choose to produce the same mix $z_0^X = .7$ and $z_0^Y = .3$. But how did we find equilibrium?

4.1 Endogenous Leverage

Before moving on to solve the model, let us go back to the question of endogenous leverage. By proposition 1, *Var = 0* holds for the intermediate states $s \in \{UU, UD, DU, DD\}$, since for each asset there are at most two distinct successor payoff values. Hence, the only contract traded in all intermediate states is the one that prevents default in equilibrium as in Section 3.

However, the situation is different at time 0 since there are four successor states in $S(0)$ with three distinct successor payoff values for each asset[14], and therefore it is

[14] X's price is 1 at UU and UD and Y's price is R at UD and DD.

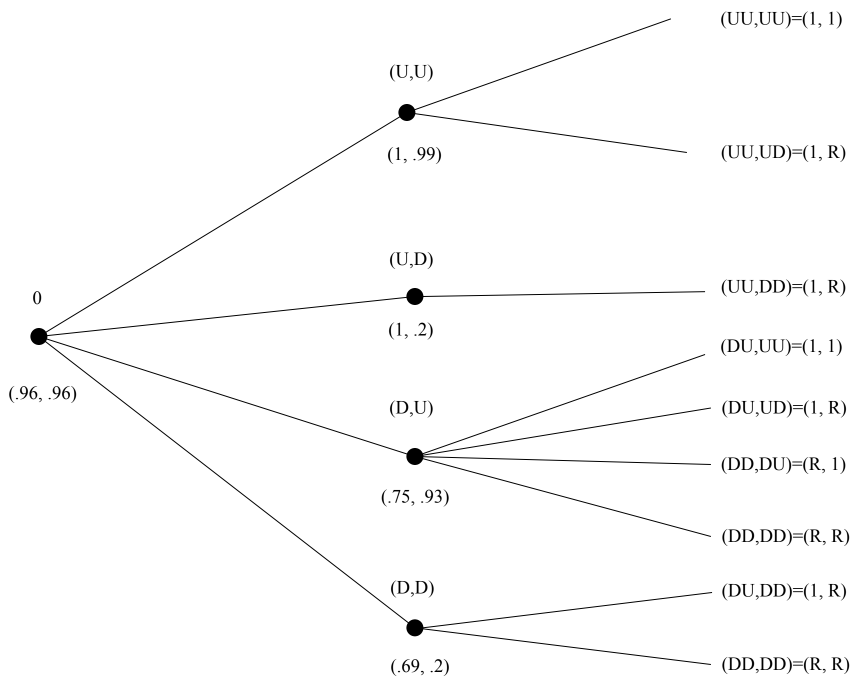

Figure 11: Joint extreme BV and extreme GV economy. Equilibrium prices for $R = .2$.

not possible to appeal to the result anymore. In fact, as we show next, for each asset two types of contracts will be traded in equilibrium: one that promises the worst-case scenario and another that promises the middle-case scenario. While the first one is risk-less as before, the second one is not since it defaults in the worst state. In this model, not only is there default in equilibrium, but also the same asset is traded simultaneously with different margin requirements by different investors. Araujo et.al. (2009) and Fostel-Geanakoplos (2010) displayed the same phenomenon in a two period model. In the following section we show for the first time that multiple margins can emerge in equilibrium in a dynamic setting. The dynamic setting is more difficult because the payoffs of the risky bonds are endogenous.

4.2 Procedure to find the equilibrium

This section describes in detail the procedure to compute the equilibrium. The first thing we do is find an equilibrium for any fixed $z_0^X, z_0^Y = 1 - z_0^X$. Then using the fact that the two asset prices at the beginning must be equal in a genuine equilibrium[15],

[15] In general equilibrium all assets are put to sale first, if one asset had a higher price, investors would invest all of their labor into that asset, sell it and buy the other.

we find the z_0^X that precisely accomplishes that.[16]

Given price expectations, buying an asset on margin using a financial contract defines a down-payment at time 0 and a profile of net payoffs in the future. In this sense, we can think of nine different securities at time 0, six risky and three risk-less: i) buying X on margin using the risky bond (the one that promises p_{DU}^X), ii) buying X on margin using the risk-less bond (which promises the smaller amount p_{DD}^X), iii) buying Y on margin using the risky bond (the one that promises p_{DU}^Y), iv) buying Y on margin using the risk-less bond (which promises the smaller amount p_{DD}^Y), v) the risky bond that promises p_{DU}^X, vi) the risky bond that promises p_{DU}^Y, vii) the risk-less bond that promises p_{DD}^X, viii) the risk-less bond that promises p_{DD}^Y and ix) warehousing.

In equilibrium the riskless interest rate will be zero, as before, hence all the riskless bonds will be priced equal to their respective promise. In addition to z_0^X and z_0^Y we need to find the value of 20 variables:

- Asset prices: $p_0^X, p_0^Y, p_{UU}^Y, p_{DU}^X, p_{DU}^Y, p_{DD}^X$.[17]
- Risky bond prices at $s = 0$: π^X, π^Y, where π^k is the price of the bond that promises p_{DU}^k in all successors states in the future.
- Asset marginal buyers: $h_M^X, h_M^Y, h_m^X, h_m^Y, h_{UU}^Y, h_{DU}^X, h_{DU}^Y, h_{DD}^X$, where $h_M^k(h_m^k)$ corresponds to the marginal buyer of the k asset leveraging with the risky (risk-less) bond.
- Risky bond marginal buyers: h^{BX}, h^{BY}.
- Asset purchases at $s = 0$ leveraging with the risky bond: y^X, y^Y.

Following the same idea as in Section 3, we guess a regime, consisting of a ranking of the securities. Then for every consecutive pair of securities, we find a marginal buyer that is indifferent between the two. This defines a system of equations. Once we get a solution we need to check: first, that $p_{DU}^X > p_{DD}^X$, so that prices are consistent with our guess about which bonds are risky and riskless on X, second, that $p_{UU}^Y > p_{DU}^Y$, so that prices are consistent with our guess about which bonds are risky and riskless on Y, and finally, that each regime is genuine, i.e. all the marginal agents strictly prefer their pair of securities to all the others, and all agents in between consecutive marginal agents strictly prefer just one security.

We now describe the regimes at each node. Figure 12 shows a graphical illustration of them and of the equilibrium values of all marginal buyers.

[16]Hopefully if we start with a good guess of z_0^X near the true value we will be able to shift z_0^X until prices are equal without changing the equilibrium regime by continuity.

[17]Notice that some prices are obvious, X's price equals 1 for sure at UU and UD, whereas Y's price is R at UD and DD. It is also clear that at UD all uncertainty is resolved and there is no more trade.

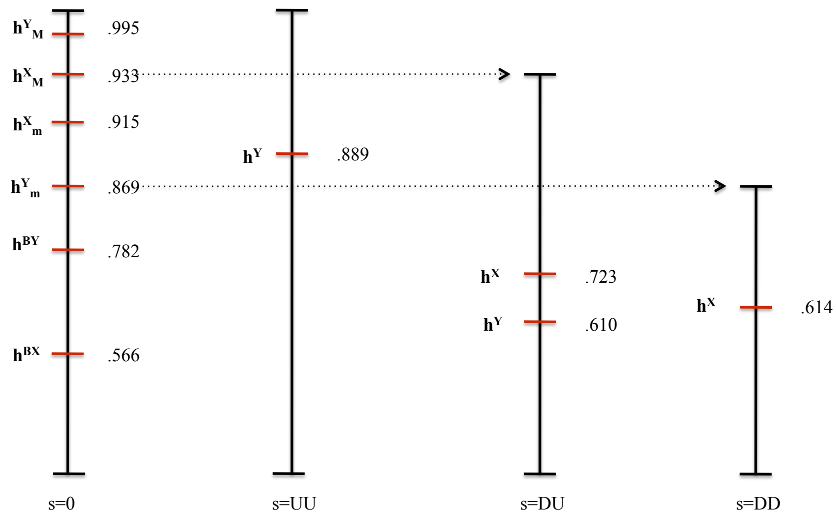

Figure 12: Equilibrium Regimes for $R = .2$.

At $s = 0$, the order is the following. $h^Y_M > h^X_M > h^X_m > h^Y_m > h^{BY} > h^{BX}$. All $h > h^Y_M$ buy Y, sell X and promise p^Y_{DU}. $h^Y_M > h > h^X_M$ buy X, sell Y and promise p^X_{DU}. $h^X_M > h > h^X_m$ buy X, sell Y and promise p^X_{DD}. $h^X_m > h > h^Y_m$ buy Y, sell X and promise R. $h^Y_m > h > h^{BY}$ sell both assets and buy the BY bond (lend in the risky market collateralized by Y). $h^{BY} > h > h^{BX}$ sell all assets and buy the BX bond (lend in the risky market collateralized by X). Finally, $h < h^{BX}$ sell everything, hold risk-less securities (so lend in the risk-less markets).

At $s = UU$ there is only trade on asset Y, and the marginal buyer is such that $h^X_m > h^Y_{UU} > h^Y_m$. As before, all $h > h^Y_{UU}$ buy Y and promise R. Below lend and buy X.

At $s = DU$, there is trade in both assets, and the marginal buyers are such that $h^{BY} > h^X_{DU} > h^Y_{DU} > h^{BX}$. $h > h^X_M$ go bankrupt since they promise exactly what they own. $h > h^X_{DU}$ buy X and promise R. $h^X_{DU} > h > h^Y_{DU}$ buy Y and promise R. All $h < h^Y_{DU}$ lend.

At $s = DD$ there is only trade on asset X and the marginal buyer is such that $h^{BY} > h^X_{DD} > h^{BX}$. All $h > h^Y_m$ are out of business either because they default or they have no money left. $h > h^X_{DD}$ buy X and promise R. $h < h^X_{DD}$ lend.

We calculate the equilibrium values and finally check the assumed regime is a genuine equilibrium. The system of equations used to solved for the equilibrium is presented in appendix 6.2.

4.3 Agents Prefer the Extreme *BV* Project

All equilibrium values listed in figures 11 and 12 are consistent with the assumed regimes and prices as discussed in appendix 6.2. The most important thing to observe is that $z_0^X = .7$, this is, all agents choose to invest their labor in a portfolio with a 70% share of the extreme BV project. Or equivalently, 70% of the economy invests in extreme BV projects when given the opportunity to choose. The consequence of this is that, since we assumed that the two projects were independent, 70% of the time when bad news occurs they will be of the volatile type, and we will observe pro-cyclical leverage. This result is robust to any choice of the parameter R as discussed in appendix 6.3.

4.4 Endogenous Leverage Reconsidered

When the asset could take on at most two immediate successor values, equilibrium determines a unique actively traded promise and hence leverage. With three or more successor values, we cannot expect a simple promise. But equilibrium still determines the average leverage used to buy each asset.

Equilibrium leverage is presented in table 1. There are eight securities in total, six risky securities and two risk-less securities (without considering warehousing). Columns 2 and 3 show the holdings and value of such holdings for each of the securities. Most importantly, column 4 shows the LTV of each of the four traded contracts. As was expected, LTV is higher for the risky contracts (they have a higher promise) for both assets. Finally, column 5 shows the LTV for each asset. Whereas the LTV for BV is .76, it is only .6 for GV. As defined in section 2, asset LTV is a weighted average. For example the LTV for BV is obtained from the total amount borrowed using all contracts, $.423 + .091$ divided by the total value of collateral, $.966 \times .695$.

As in Section 3, BV can be leveraged more than the GV. Second, also as before, leverage in BV is pro-cyclical while it is counter-cyclical in GV. Third, notice that even though both projects have the same initial price in equilibrium, for both assets the price is higher than in Section 3 (.966 versus .95 for BV and .89 for GV). The main reason for this difference is that now with a different tree, more contracts are traded in equilibrium, not only the risk-less one. Both assets can be leveraged more now using risky contracts which promise more (and hence default as well). Whereas there is not so much difference between the minimum promise and the medium promise for BV (.691 and .754) this difference is significant for GV (.2 and .936). For a precise discussion between leverage and asset prices see FG (2010).

Table 1: Equilibrium Contract and Asset Leverage for $R = .2$.

Leverage at s=0

Security	Holdings	Holdings Value	Contract LTV	Asset	Asset LTV
Y lev Medium	0.186	0.180	0.947	X (GV)	0.766
X lev Medium	0.563	0.544	0.778		
X lev Min	0.132	0.128	0.715	Y (BV)	0.660
Y lev Min	0.119	0.115	0.207		
Y risky bond	0.186	0.171			
X risky bond	0.563	0.423			
Y riskless bond	0.119	0.024			
X riskless bond	0.132	0.091			

Leverage at intermediate nodes

	UU	UD	DU	DD
X (BV)	1.000	1.000	0.267	0.290
Y (GV)	0.202	1.000	0.215	1.000

So, why did agents choose extreme BV more? The simple reason is that BV can be leveraged more at the beginning. So the most optimistic agents will choose extreme BV. However, as soon as less optimistic people opt for volatile bad news projects, its price will start to decline and the extreme GV project will start to become attractive to other investors. This process will continue until prices are equal in equilibrium.

Our main result also suggests an explanation for the observed "Volatility Smile" in stock options. This refers to the fact that implied volatility has a negative relationship with the strike price, so volatility decreases as the strike price increases. Hence, bad news comes (or are assumed to come) with high volatility. The pattern has existed for equities only after the stock market crash of 1987. This has led some economist like Bates (2000) and Rubinstein (1995) to explain volatilites smiles by "crashophobia". Traders are concerned about the possibility of another crash and they price options accordingly. Our result provides a completely different explanation. Our agents are perfectly rational, they endogenously choose projects associated with volatile bad news since they can leverage more with them. For 70% of the projects in our economy, their volatility goes up after their price falls.

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6 Appendix

6.1 Extreme BV and Extreme GV Projects: Long Run Analysis.

Notice that since the final probability of disaster is constant (regardless of N), the probability of bad news in period k is given by $(1-h_k)^{2/k}$.

- $p_{N+1} = R$
- $p_N = (1-(1-h_N)^{2/N}) + (1-h_N)^{2/N} R$
- $h_{N-1} = \frac{h_N(1+p_N)}{1+p_{N+1}}$

- $p_{N-1} = \frac{(1-(1-h_{N-1})^{2/N})+(1-h_{N-1})^{2/N}\frac{(1-(1-h_{N-1})^{2/N})}{(1-(1-h_N)^{2/N})}p_N}{(1-(1-h_{N-1})^{2/N})+(1-h_{N-1})^{2/N}\frac{(1-(1-h_{N-1})^{2/N})}{(1-(1-h_N)^{2/N})}}$

- $h_{N-2} = \frac{h_{N-1}(1+p_{N-1})}{1+p_N}$

  $\vdots$

- $p_1 = \frac{(1-(1-h_1)^{2/N})+(1-h_1)^{2/N}\frac{(1-(1-h_1)^{2/N})}{(1-(1-h_2)^{2/N})}p_2}{(1-(1-h_1)^{2/N})+(1-h_1)^{2/N}\frac{(1-(1-h_1)^{2/N})}{(1-(1-h_2)^{2/N})}}$

- $h_0 = \frac{h_1(1+p_1)}{1+p_2} = 1$

We use the fact that the marginal buyer rollover his debt at every node to build up the system and then verify that the guess is correct. Notice that the probability of good news in period k is given by $(1-(1-h_k)^2)^{1/N}$.

- $p_1 = ((1-(1-h_k)^2)^{1/N})^N + (1-((1-(1-h_k)^2)^{1/N})^N)R$
- $p_1 = \frac{(1-h_1)+R}{h_1}$

  $\vdots$

- $p_k = ((1-(1-h_k)^2)^{1/N})^{N-k} + (1-((1-(1-h_k)^2)^{1/N})^{N-k})R$

Tables 2 and 3 present all the equilibrium values.

6.2 System of Equations in Section 4.

The system of equations is conceptually an extension of the system in Section 3. In every state supply equals demand for all the securities. Also marginal buyers are determined by an indifference condition between investing in two different securities. As before, all marginal utility of a dollar invested in any security is weighted by the marginal utility of future actions in each state. Equations (a)-(l) corresponds to state $s=0$. Equations (m)-(n) to state $s=UU$. Equations (o)-(r) to state $s=DU$ and the rest to state $s=DD$.

Notation: q_s^h is the probability of state s by buyer h.

1. $y^Y = \frac{(1-h_M^Y)+\alpha p_1^X(1-h_M^Y)+(1-\alpha)p_1^Y(1-h_M^Y)}{p_1^Y-\pi^Y}$
2. $y^X = \frac{(h_M^Y-h_M^X)+(1-\alpha)p_1^Y(h_M^Y-h_M^X)+\alpha p_1^X(h_M^Y-h_M^X)}{p_1^X-\pi^X}$
3. $(\alpha h_m^X+\alpha(1-h_M^Y)-y^X) = \frac{(h_M^X-h_m^X)+(1-\alpha)p_1^Y(h_M^X-h_m^X)+\alpha p_1^X(h_M^X-h_m^X)}{p_1^X-p_{DD}^X}$

Table 2: BV equilibrium N=10, R=.2.

Period	Mrg buyer	Price bad state	Price good state	Leverage bad state	Leverage good state
0	0.9914	0.9875		0.9827	
1	0.9768	0.9704	1.0000	0.9702	1.0000
2	0.9547	0.9415	1.0000	0.9534	1.0000
3	0.9244	0.8976	1.0000	0.9327	1.0000
4	0.8856	0.8372	1.0000	0.9081	1.0000
5	0.8394	0.7603	1.0000	0.8791	1.0000
6	0.7870	0.6684	1.0000	0.8441	1.0000
7	0.7301	0.5642	1.0000	0.7995	1.0000
8	0.6718	0.4511	1.0000	0.7431	1.0000
9	0.6038	0.3352	1.0000	0.5967	1.0000
10		0.2000	1.0000		

Table 3: GV equilibrium N=10, R=.2.

Period	Mrg buyer	Price good state	Price bad state	Leverage good state	Leverage bad state
0	0.6340	0.8928		0.2240	
1	0.6340	0.9112	0.2000	0.2195	1.0000
2	0.6340	0.9205	0.2000	0.2173	1.0000
3	0.6340	0.9300	0.2000	0.2151	1.0000
4	0.6340	0.9396	0.2000	0.2129	1.0000
5	0.6340	0.9494	0.2000	0.2107	1.0000
6	0.6340	0.9592	0.2000	0.2085	1.0000
7	0.6340	0.9692	0.2000	0.2064	1.0000
8	0.6340	0.9793	0.2000	0.2042	1.0000
9	0.6340	0.9896	0.2000	0.2021	1.0000
10		1.0000	0.2000		

4. $((1-\alpha)h_m^Y + (1-\alpha)(h_M^Y - h_m^X) - y^Y) = \frac{(h_m^X - h_m^Y) + \alpha p_1^X (h_m^X - h_m^Y) + (1-\alpha) p_1^Y (h_m^X - h_m^Y)}{p_1^Y - R}$

5. $((1-\alpha)(1-h_M^Y) + y^Y) = \frac{(h_m^Y - h^{BY})(1+\alpha p_1^X + (1-\alpha)p_1^Y)}{\pi^Y}$

6. $(\alpha(h_M^Y - h_M^X) + y^X) = \frac{(h^{BY} - h^{BX})(1+\alpha p_1^X + (1-\alpha)p_1^Y)}{\pi^X}$

7. $\frac{q_{UU}^{h_M^Y}(p_{UU}^Y - p_{DU}^Y)}{p_1^Y - \pi^Y} \frac{\sqrt{1-(1-h_M^Y)^2}(1-R)}{p_{UU}^Y - R} = \frac{q_{UU}^{h_M^Y}(1-p_{DU}^X)}{p_1^X - \pi^X} \frac{\sqrt{1-(1-h_M^Y)^2}(1-R)}{p_{UU}^Y - R} + \frac{q_{UD}^{h_M^Y}(1-p_{DU}^X)}{p_1^X - \pi^X}$

8. $\frac{q_{UU}^{h_M^X}(1-p_{DU}^X)}{p_1^X - \pi^X} \frac{\sqrt{1-(1-h_M^X)^2}(1-R)}{p_{UU}^Y - R} + \frac{q_{UD}^{h_M^X}(1-p_{DU}^X)}{p_1^X - \pi^X} =$

$= \frac{q_{UU}^{h_M^X}(1-p_{DD}^X)}{p_1^X - p_{DD}^X} \frac{\sqrt{1-(1-h_M^X)^2}(1-R)}{p_{UU}^Y - R} + \frac{q_{UD}^{h_M^X}(1-p_{DD}^X)}{p_1^X - p_{DD}^X} + \frac{q_{DU}^{h_M^X}(p_{DU}^X - p_{DD}^X)}{p_1^X - p_{DD}^X} \frac{h_M^X(1-R)}{p_{DU}^X - R}$

9. $\frac{q_{UU}^{h_m^X}(1-p_{DD}^X)}{p_1^X - p_{DD}^X} \frac{\sqrt{1-(1-h_m^X)^2}(1-R)}{p_{UU}^Y - R} + \frac{q_{UD}^{h_m^X}(1-p_{DD}^X)}{p_1^X - p_{DD}^X} + \frac{q_{DU}^{h_m^X}(p_{DU}^X - p_{DD}^X)}{p_1^X - p_{DD}^X} \frac{h_m^X(1-R)}{p_{DU}^X - R} =$

$= \frac{q_{UU}^{h_m^X}(p_{UU}^Y - R)}{p_1^Y - R} \frac{\sqrt{1-(1-h_m^X)^2}(1-R)}{p_{UU}^Y - R} + \frac{q_{DU}^{h_m^X}(p_{DU}^Y - R)}{p_1^Y - R} \frac{h_m^X(1-R)}{p_{DU}^X - R}$

10. $\frac{q_{UU}^{h_m^Y}(p_{UU}^Y - R)}{p_1^Y - R} + \frac{q_{DU}^{h_m^Y}(p_{DU}^Y - R)}{p_1^Y - R} \frac{h_m^Y(1-R)}{p_{DU}^X - R} =$

$= \frac{q_{UU}^{h_m^Y} p_{DU}^Y + q_{UD}^{h_m^Y} R + q_{DU}^{h_m^Y} p_{DU}^Y \frac{h_m^Y(1-R)}{p_{DU}^X - R} + q_{DD}^{h_m^Y} R \frac{h_m^Y(1-R)}{p_{DD}^X - R}}{\pi^Y}$

11. $\frac{q_{UU}^{h^{BY}} p_{DU}^Y + q_{UD}^{h^{BY}} R + q_{DU}^{h^{BY}} p_{DU}^Y \frac{h^{BY}(1-R)}{p_{DU}^X - R} + q_{DD}^{h^{BY}} R \frac{h^{BY}(1-R)}{p_{DD}^X - R}}{\pi^Y} =$

$= \frac{q_{UU}^{h^{BY}} p_{DU}^X + q_{UD}^{h^{BY}} p_{DU}^X + q_{DU}^{h^{BY}} p_{DU}^X \frac{h^{BY}(1-R)}{p_{DU}^X - R} + q_{DD}^{h^{BY}} p_{DD}^X \frac{h^{BY}(1-R)}{p_{DD}^X - R}}{\pi^X}$

12. $\frac{q_{UU}^{h^{BX}} p_{DU}^X + q_{UD}^{h^{BX}} p_{DU}^X + q_{DU}^{h^{BX}} p_{DU}^X + q_{DD}^{h^{BX}} p_{DD}^X}{\pi^X} = 1$

13. $\frac{\sqrt{1-(1-h_{UU}^Y)^2}(1-R)}{p_{UU}^Y - R} = 1$

14. $(1-\alpha) = \frac{(p_{UU}^Y - p_{DU}^Y)((1-\alpha)(1-h_M^Y) + y^Y) + (1-p_{DU}^X)(\alpha(h_M^Y - h_M^X) + y^X)}{p_{UU}^Y - R} +$

$\frac{(1-p_{DD}^X)(\alpha(h_M^X - h_m^X) + (\alpha h_m^X + \alpha(1-h_M^Y) - y^X))(h_M^X - h_{UU}^Y)/(h_M^X - h_m^X)}{p_{UU}^Y - R}$

15. $\frac{h_{DU}^X(1-R)}{p_{DU}^X - R} = \frac{\sqrt{1-(1-h_{DU}^X)^2}(1-R)}{p_{DU}^Y - R}$

16. $\frac{\sqrt{1-(1-h_{DU}^Y)^2}(1-R)}{p_{DU}^Y - R} = 1$

17. $\alpha = \frac{(p_{DU}^X - p_{DD}^X)(\alpha(h_M^X - h_m^X) + (\alpha h_m^X + \alpha(1-h_M^Y) - y^X)) + (p_{DU}^Y - R)((1-\alpha)(h_m^X - h_m^Y)}{p_{DU}^X - R} +$

$\frac{((1-\alpha)h_m^Y + (1-\alpha)(h_M^Y - h_m^X) - y^Y))}{p_{DU}^X - R} +$

$\frac{p_{DU}^Y((1-\alpha)(1-h_M^Y) + y^Y)(h^{BY} - h_{DU}^X)/(h^{BY} - h^{BX})}{p_{DU}^X - R}$

18. $(1-\alpha) = \frac{(h^X_{DU}-h^Y_{DU})/(h^{BY}-h^{BX})p^Y_{DU}((1-\alpha)(1-h^Y_M)+y^Y)}{p^Y_{DU}-R}$

19. $\frac{h^X_{DD}(1-R)}{p^X_{DD}-R} = 1$

20. $\alpha = \frac{R((1-k=\alpha)(1-h^Y_M)+y^Y)+\frac{h^{BY}-h^X_{DD}}{h^{BY}-h^{BX}}p^X_{DD}(\alpha(h^Y_M-h^X_M)+y^X)}{p^X_{DD}-R}$

All the values listed in figures 8 and 9 are consistent with the assumed regimes and prices as discussed in section 4.2. It turns out also that this equilibrium is genuine in the sense that all agents' decisions are optimal. The risky bond prices at date 0 are $\pi^X = .7521$ on a promise of .7548, corresponding to an interest rate of .36% and $\pi^Y = .9156$ on a promise of .9366, corresponding to an interest rate of 2.3%. The most leveraged asset purchases at date 0 are $y^X = .520$ and $y^Y = .184$. The verification that each agent is indeed maximizing is available upon request.

6.3 Robustness Analysis.

Table 6 presents the proportion invested in the extreme BV project (α) and leverage for each project at $s = 0$ for a grid of values of R, the key parameter in our simulations. We can see that the two properties, that $\alpha > .5$ (so that investors invest mostly in the BV technology) and that initial leverage higher in extreme BV than in extreme GV, are valid for values of R other than .2 considered in the main text. The grid presents values up to $R = .6$. For values larger than $R = .7$ the equilibrium regime discussed in section 4.2 is not genuine anymore. Two contracts are still traded for the extreme BV project, but only the riskless contract is traded for the extreme GV project. It is obvious that for higher values of R, the extreme BV project will be leveraged even more and hence our result is clearly true.

Table 4: Robustness Section 4.

R	**α**	**price**	$\mathbf{LTV_{BV}}$	$\mathbf{LTV_{GV}}$
0.2	0.6950	0.9664	0.7662	0.6596
0.3	0.7120	0.9800	0.8135	0.5968
0.4	0.7280	0.9891	0.8582	0.5691
0.5	0.7450	0.9947	0.8978	0.5984
0.6	0.7600	0.9978	0.9323	0.6429