11-1-2005

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November 2005
Revised June 2009

COWLES FOUNDATION DISCUSSION PAPER NO. 1539R

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A Credit Mechanism for Selecting a Unique Competitive Equilibrium*

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November 27, 2006

Abstract

The enlargement of the general-equilibrium structure to allow for default subject to appropriate credit limits and default penalties results in a construction of a simple mechanism for a credit using society. We show that there generically exists a price-normalizing bundle that determines a credit money along with appropriate credit limits and default penalties for the credit mechanism to select a unique competitive equilibrium (CE). With some additional conditions, a common credit money can be applied such that any CE can be a unique selection by the credit mechanism with appropriate credit limits default penalties for the traders. This will include a CE with the minimal cash flow property. Such CEs are special for the reason that they minimize the need for a substitute-for-trust (i.e. money) in trade.

KEYWORDS: Competitive equilibrium, credit mechanism, marginal utility of income, IOU, welfare economics. (JEL Classification D5, C72, E4)

1 Introduction

The problem of finding the most general conditions required to guarantee a unique CE in a general-equilibrium system is complex and challenging mathematically.

*This version is based on Cowles Foundation Discussion Paper NO. 1539. We wish to thank Don Brown for helpful comments.

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By enlarging the problem an approach is proposed that both offers a solution and facilitates an interesting selection.

The general-equilibrium model does not utilize credit or financial institutions because trust is implicitly perfect. All trade is balanced at the end of the market. It is as if each individual at the start of the model has available implicitly a credit line equal to the worth of the individual's initial wealth at the final market price. When trust is imperfect and credit is introduced, however, a mechanism is needed to determine the worth to an individual at the end of trade. This includes the possibility of having credit left over and the cost or penalty for ending up in debt.

We consider a credit mechanism for a single period exchange economy that involves a credit money together with a credit limit and a per-unit default penalty for each trader, such that ending up with net credit is worthless for the trader while the default penalty is levied against him for ending as a net debtor.[1] To see how to implement the mechanism, consider, for specificity, that trade is in banknotes which are provided by a mutual bank. Before trading begins, traders exchange personal IOUs for banknotes with the bank charging them an interest rate of zero. Each trader may exchange personal IOUs for banknotes for up to a certain exogenously specified total amount. After all traders have received their incomes, they go to the bank to settle up all outstanding credit.[2]

Given a credit mechanism, whether it is optimal for a trader to over spend at prevailing prices depends on the size of the per-unit default penalty relative to his marginal utility of income. In a general-equilibrium model, a trader's marginal utility of income equals the Lagrangian multiplier associated with his utility maximization problem; hence, it depends endogenously on the prices. In particular, scaling up all the prices scales down the trader's marginal utility of income by equal amount. It follows that the marginal utility of income of a trader enjoys a degree of freedom unless prices are normalized.

We are interested in a price normalization that calls for the same value of some commodity bundle with a positive quantity of each good. We refer to this bundle

[1]For example, default penalties may be in the form of asset confiscation from the debtors or jail sentences or other societal punishments.

[2]One way to play the model in a classroom is at the beginning to give each student a large stack of banknotes and inform him that at the end of the game, after he has bought and received income from selling, he has to return exactly the amount he started with initially or he will have to pay a default penalty.

For discussions on various credit mechanisms for the competitive model, the reader is referred to Shubik (1999).

as a price-normalizing bundle. When the total endowment bundle of the economy is used as the price-normalizing bundle, all normalized price systems yield the same value of the economy's total wealth.

We begin investigation of the credit mechanism with a useful property of general-equilibrium analysis. Namely, under some mild conditions, a CE for an economy corresponds to a saddle-point of a Lagrangian function for each trader. This saddle-point characterization of a CE has useful applications for price normalization and for the design of default penalties towards the selection of a unique CE, as well as for the study of the already familiar welfare properties of CE allocations.

We show that given a CE not equal to any non-negative linear combination of the other CEs, a finite iterated process generates a price-normalizing bundle that determines a credit money along with an appropriate credit limit and default penalty for each trader for the given CE to be a unique selection by the credit mechanism. The existence of a CE satisfying the above independence condition is generic. With some additional conditions, a uniform credit money can be applied such that any CE can be a unique selection by the credit mechanism with appropriate credit limits and default penalties for the traders. This will include a CE with the minimal cash flow property. Such CEs are special for the reason that they minimize the need for a substitute-for-trust (i.e. money) in trade.[3]

Our results can be extended to production economies via Rader's equivalence principle. For this reason, we confine analysis to pure exchange economies in a large part of the paper. The rest of the paper is organized as follows. The next section briefly discusses saddle-point characterization of a CE and its applications. Section 3 presents results for pure exchange economies. Section 4 discusses extensions to

[3] Allen, Dutta, and Polemarchakis (2002) consider CE selections of a different nature. They begin with a random selection, which is a probability distribution over the set of CEs. Such a distribution introduces an additional source of uncertainty. They then consider an enlargement of the general-equilibrium structure to allow traders to insure against such additional uncertainties by opening asset markets contingent on the CEs prior to spot market trading. The problem of the multiplicity of the CEs, however, cannot be resolved with such asset markets (see Proposition 1 in Allen, et al 2002).

By taking expectation with respect to the implied distribution over the CEs by a random selection, a reallocation of the endowments is obtained. Taking the bundles in the reallocation as traders' new endowments, a new set of CEs will be obtained to which the random selection can be repeated. This way, an iterative process is established. Allen, et al (2002) show that the process converges to an allocation of endowments that implies a unique CE (see Proposition 2 in their paper). Note, however, this iterative process is of non-tâtonnement nature and is, therefore, different from the iterative process of price normalization in the present paper.

economies with production and section 5 concludes the paper.

2 Saddle-Point Characterization of Competitive Equilibria

Consider an exchange economy $\mathcal{E} = \{X^i, u^i, a^i\}_{i=1}^n$ with trader i's consumption set X^i, utility function u^i, and endowment a^i. We assume *A1:* $X^i = \Re^m_+$; *A2:* u^i is continuous and concave; *A3:* $a^i \in \Re^m_+$ with $a^i \neq 0$; and *A4:* For each $1 \leq h \leq m$, there is a trader i such that $u^i(x^i + \delta e^h) > u^i(x^i)$ for all $x^i \in \Re^m_+$ and for all $\delta > 0$, where $e^h \in \Re^m$ with $e^h_h = 1$ and $e^h_k = 0$ for all $k \neq h$.[4] These are familiar assumptions in general-equilibrium analysis. A CE for economy $\mathcal{E}$ is a pair $(\bar{x}, \bar{p})$ with allocation $\bar{x} = (\bar{x}^1, \cdots, \bar{x}^n)$ and price vector $\bar{p}$ such that $\bar{x}^i$ solves

$$\begin{array}{ll} \text{maximize} & u^i(x^i) \\ \text{subject to} & (i)\ \bar{p} \cdot (a^i - x^i) \geq 0 \\ & (ii)\ x^i \in \Re^m_+ \end{array} \qquad \text{(Utility Maximization)} \qquad (1)$$

for all i and

$$\sum_{i=1}^n \bar{x}^i = \sum_{i=1}^n a^i. \qquad \text{(Market Clearance)} \qquad (2)$$

2.1 A Saddle Point Characterization

To apply the Saddle-Point Theorem, notice first that due to A4, all CE prices are positive. The saddle-point characterization of CEs in Theorem 1 below is well-known:[5]

Theorem 1 (Saddle-Point Characterization) *Let $\mathcal{E} = \{X^i, u^i, a^i\}_{i=1}^n$ be an exchange economy satisfying A1-A4. Then, a pair $(\bar{x}, \bar{p}) \in \Re^{mn}_+ \times \Re^m_{++}$ is a CE if and only if $\bar{x}$ satisfies (2) and there exists a vector $\bar{\lambda} \in \Re^n_{++}$ such that for all i the triplet $(\bar{x}^i, \bar{p}, \bar{\lambda}^i)$ satisfies*

$$u^i(x^i) + \bar{\lambda}^i \bar{p} \cdot (a^i - x^i) \leq u^i(\bar{x}) + \bar{\lambda}^i \bar{p} \cdot (a^i - \bar{x}^i) \leq u^i(\bar{x}^i) + \lambda^i \bar{p} \cdot (a^i - \bar{x}^i) \qquad (3)$$

for all $x^i \in \Re^m_+$ and for all $\lambda^i \in \Re_+$.

[4] For any positive integer q, $\Re^q_+$ denotes the non-negative orthant of the q-dimensional Euclidean space and $\Re^q_{++}$ denotes the subset of $\Re^q_+$ containing vectors in $\Re^q_+$ all with positive components.

[5] A proof can be established by applying the saddle-point characterization of solutions of a non-linear programming problem. See Takayama (1985, p. 75) for the saddle-point characterization.

Notice that condition (3) is equivalent to $(\bar{x}^i, \bar{\lambda}^i)$ being a saddle-point for the Lagrangian of the utility maximization problem (1). When a triplet $(\bar{x}, \bar{p}, \bar{\lambda})$ satisfies (2) and (3), we call it *a competitive triplet* and we call $\bar{x}$, $\bar{p}$, and $\bar{\lambda}$, respectively, a *competitive allocation*, a *competitive price vector*, and a *competitive multiplier vector*.

Two applications of Theorem 1 are relevant to the rest of the paper. Corollary 1 below shows that a competitive allocation maximizes a weighted welfare function with the welfare weights equal to the reciprocals of the associated competitive multipliers. Corollary 2 shows that under some additional conditions, there is a one-to-one correspondence between competitive equilibria and competitive multiplier vectors. Corollary 1 is familiar and follows easily from Theorem 1.

Corollary 1 *Assume* $\mathcal{E} = \{X^i, u^i, a^i\}_{i=1}^n$ *satisfies A1-A4. If* $(\bar{x}, \bar{p}, \bar{\lambda}) \in \Re_+^{mn} \times \Re_{++}^m \times \Re_{++}^n$ *is a competitive triplet for* $\mathcal{E}$*, then* $\bar{x}$ *solves the weighted welfare maximization problem:*

$$\begin{array}{ll} \textit{maximize} & \sum_{i=1}^n \frac{1}{\lambda^i} u^i(x^i) \\ \\ \textit{subject to} & (i) \sum_{i=1}^n (a^i - x^i) \geq 0 \\ & (ii)\ x^i \in \Re_+^m, i = 1, 2, \cdots, n. \end{array} \tag{4}$$

Corollary 2 *Assume* $\mathcal{E} = \{X^i, u^i, a^i\}_{i=1}^n$ *satisfies A1 and A4. Assume further*

A2′: u^i is continuously differentiable and strictly concave;

A5: CE allocations are all interior allocations.[6]

Then, there is a one-to-one correspondence between CEs and competitive multiplier vectors for economy $\mathcal{E}$*.*

Proof. Let $(\bar{x}, \bar{p})$ be a CE. Then, by A5, $\bar{x}^i \in \Re_{++}^m$ for all i. Consequently, by A2′ and by the Kuhn-Tucker conditions, competitive multiplier vectors $\bar{\lambda} \in \Re_+^n$ that correspond to $(\bar{x}, \bar{p})$ satisfy [7]

$$\bigtriangledown u^i(\bar{x}^i) = \bar{\lambda}^i \bar{p},\ \ i = 1, 2, \cdots, n.$$

[6] A sufficient condition to guarantee the interiority of the CE allocations is for all i, $u^i(x^i) > u^i(y^i)$ whenever x^i is an interior bundle and y^i is a corner bundle. The reason is that all CE prices are strictly positive under A4 and hence the value of each trader's endowment at these prices are positive.

[7] Here $\bigtriangledown u^i(\bar{x}^i)$ denotes the gradient of u^i at $\bar{x}^i$.

The uniqueness of the associated competitive multiplier vector follows from the above equation.

Conversely, with the strict concavity of utility functions, problem (4) has a unique solution given competitive multiplier vector $\bar{\lambda}$. It thus follows from Corollary 1 that $\bar{\lambda}$ determines a unique competitive allocation $\bar{x}$. Since, by A5, $\bar{x}^i \in \Re^m_{++}$ for all i, it follows from the Kuhn-Tucker conditions for problem (4) that there is a Lagrangian multiplier vector $\bar{p} \in \Re^m_{++}$ such that

$$\bar{p} = \sum_{i=1}^{n} \frac{1}{\bar{\lambda}^i} \bigtriangledown u^i(\bar{x}^i).$$

Thus, the Lagrangian multiplier vector for problem (4) is unique. Since the price vector associated with competitive allocation $\bar{x}$ is necessarily a Lagrangian multiplier vector for problem (4), it is thus unique. ■

The one-to-one correspondence in Corollary 2 implies that the products $\bar{\lambda}^i\bar{p}$, $i = 1, 2, \cdots, n$, are uniquely determined in every CE under the conditions in the corollary.

3 Price Normalization and Selection of CEs

Competitive equilibrium prices are homogeneous of degree zero. Consequently, prices can be normalized without changing competitive allocations. When normalizing the prices, however, economists rarely consider the effects of price normalization on competitive multipliers, hence on marginal utilities of income of the traders. In what follows it will become clear that those effects are important for the construction of a credit mechanism.

3.1 A Credit Mechanism

Competitive multipliers are known as marginal utilities of income. By Corollary 2, normalizing the prices would change these marginal utilities accordingly. To consider the possibility of selecting a unique CE using a credit mechanism, the sizes of the per-unit default penalties relative to the marginal utilities of income at CEs turn out to be essential.

Suppose that traders use banknotes to buy or sell goods. A bank provides all traders with banknotes with zero interest. Before trading begins, traders exchange personal IOUs for banknotes. Let $C^i_{m+1} \subseteq \Re_+$ denote the set of quantities of the banknotes that trader i can obtain with his personal IOUs. The maximum quantity in this set is the credit limit the bank provides to trader i. After he has bought and received income from selling, trader i settles up all outstanding credit with the bank. It is of no value to him to end as a net creditor, while he will be penalized for ending as a net debtor.

Let $P \subseteq \Re^m_+$ denote a compact set of normalized price vectors with $P \cap \Re^m_{++} \neq \emptyset$. Define value V by

$$V = \max \left\{ p \cdot \sum_{i=1}^{n} a^i \mid p \in P \right\}.$$

Now let $\bar{p} \in P$ be any competitive price vector. By A4, $\bar{p} \in \Re^m_{++}$. Since $a^i \in \Re^m_+$ and $a^i \neq 0$ by A1 and A3, we have $\bar{p} \cdot a^i < \bar{p} \cdot \sum_{j=1}^{n} a^j$. This inequality together with the above definition of V implies that $\bar{p} \cdot a^i < V$ for all i.

Set $C^i_{m+1} = [0, V]$ for all i. This means that each trader i can obtain credit more than he could repay with the worth of his endowment at any normalized price vector in P. However, trader i pays a penalty of $\mu^i > 0$ for each unit of debt he is unable to repay. Let $\mu = (\mu^i)$ denote the vector of these per-unit penalties.

The preceding credit limits and default penalties together with the requirement that trade be in banknotes (credit money) results in a credit mechanism that transforms economy $\mathcal{E}$ into $\mathcal{E}_\mu$ in which trader i has utility function

$$u^i(x^i) + \mu^i \min[x^i_{m+1}, 0],$$

where x^i_{m+1} denotes the amount of excess credit. It is the amount of debt when $x^i_{m+1} < 0$ or credit left over when $x^i_{m+1} > 0$.

In summary, a credit mechanism as outlined above is completely characterized by specifying a price normalization P for determining a credit money and for each i, a credit limit C^i_{m+1} and a per-unit penalty μ^i. In short, a credit mechanism is denoted by $\Gamma = \{P, \{C^i_{m+1}, \mu_i\}_{i \in N}\}$.

Definition 1 *Let $\mathcal{E} = \{X^i, u^i, a^i\}_{i\in N}$ be an exchange economy. The credit mechanism Γ selects a CE, $(\hat{x}, \hat{p})$, of $\mathcal{E}$ if $(\hat{x}, \hat{x}_{m+1}, \hat{p})$ is a CE for $\mathcal{E}_\mu$ with $\hat{x}_{m+1} = (\hat{x}^1_{m+1}, \cdots, \hat{x}^n_{m+1})$ and $\hat{x}^i_{m+1} = 0$ for all i.*

The following theorem provides a connection of the CEs of $\mathcal{E}_\mu$ with those of $\mathcal{E}$.

Theorem 2 *Let $\mathcal{E} = \{X^i, u^i, a^i\}_{i\in N}$ be an exchange economy. Assume $\mathcal{E}$ satisfies A1-A4. Assume further prices are normalized so that the resulting normalized price* *vectors consist of a compact set $P \cap \Re^m_{++} \not= \emptyset$. If $((x^*, x^*_{m+1}), p^*)$ is a CE for $\mathcal{E}_\mu$, then $x^{*i}_{m+1} = 0$ for all i and for some competitive multiplier vector $\lambda^* \in \Re^n_{++}$ with $\lambda^* \leq \mu$, (x^*, p^*, λ^*) is a competitive triplet for $\mathcal{E}$.*

Proof. Let $((x^*, x^*_{m+1}), p^*)$ be a CE for $\mathcal{E}_\mu$. Since being a net creditor is worthless and since traders are price-taking, we conclude that for all $i \in N$, $x^{*i}_{m+1} \leq 0$. This implies that (x^{*i}, x^{*i}_{m+1}) solves

$$\begin{array}{ll} \text{Maximize} & u^i(x^i) + \mu^i x^i_{m+1} \\ \text{Subject to} & p^* \cdot a^i - p^* \cdot x^i \geq x^i_{m+1}, \\ & x^i \in \Re^m_+,\ p^* \cdot a^i - V \leq x^i_{m+1} \leq 0. \end{array}$$

By the saddle-point characterization, there exists a number $\lambda^{*i} \geq 0$ such that the triplet $(x^{*i}, x^{*i}_{m+1}, \lambda^{*i})$ satisfies

$$\begin{array}{l} u^i(x^i) + \mu^i x^i_{m+1} + \lambda^{*i}(p^* \cdot a^i - p^* \cdot x^i - x^i_{m+1}) \\ \leq \\ u^i(x^{*i}) + \mu^i x^{*i}_{m+1} + \lambda^{*i}(p^* \cdot a^i - p^* \cdot x^{*i} - x^{*i}_{m+1}) \\ \leq \\ u^i(x^{*i}) + \mu^i x^{*i}_{m+1} + \lambda^{i}(p^* \cdot a^i - p^* \cdot x^{*i} - x^{*i}_{m+1}) \end{array} \tag{5}$$

for all $x^i \in \Re^m_+$, all $p^* \cdot a^i - V \leq x^i_{m+1} \leq 0$, and for all $\lambda^i \in \Re_+$.

The non-satiation of u^i together with the first inequality in (5) implies $\lambda^{*i} > 0$. This in turn with the second inequality in (5) implies

$$p^* \cdot x^{*i} + x^{*i}_{m+1} = p^* \cdot a^i. \tag{6}$$

Since $\sum_{i \in N} x^{*i} = \sum_{i \in N} a^i$ and since $x^{*i}_{m+1} \leq 0$ for all i, it follows from (6) that $x^{*i}_{m+1} = 0$ for all i.

Since $x^{*i}_{m+1} = 0$, by (5), $(x^{*i}, p^*, \lambda^{*i})$ satisfies (3). Thus, (x^*, p^*, λ^*) is a competitive triplet for $\mathcal{E}$. Next, by taking x^i to be x^{*i} in (5), the first inequality implies

$$(\mu^i - \lambda^{*i}) x^i_{m+1} \leq 0,\ \forall x^i_{m+1} \in [p^* \cdot a^i - V, 0].$$

Since $p^* \cdot a^i < V$, the condition $\lambda^{*i} \leq \mu^i$ follows from the above inequality. ■

When a competitive multiplier vector $\bar{\lambda}$ associated with a CE is such that $\bar{\lambda}_i > \mu_i$ for some i, the per-unit penalty on trader i is not severe enough, in the sense that on the margin i gains from being in debt. When this occurs, the budget constraint will be violated. Since no one ends as a net creditor, the market for commodities will be imbalanced. Hence, such a CE cannot survive the credit mechanism. A direct application of Theorem 2 implies:

Corollary 3 *Let $\mathcal{E} = \{X^i, u^i, a^i\}_{i \in N}$ be an exchange economy. Assume $\mathcal{E}$ satisfies A1-A4. Assume further prices are normalized so that the resulting normalized price vectors consist of a compact set $P \cap \Re^m_{++} \neq \emptyset$. Then, only those CEs of $\mathcal{E}$ with multiplier vectors $\lambda \leq \mu$ are selected by the credit mechanism.*

In the sequel we consider price normalization of the form

$$P = \left\{ p \in \Re^m_+ \mid p \cdot r \equiv \text{constant} \right\}$$

for some price-normalizing bundle $r \in \Re^m_{++}$. Without loss of generality, we may take the constant to be 1. Given a CE, Corollary 3 shows that a sufficient condition for it to be uniquely selected is that there exits a price-normalizing bundle, under which the competitive multiplier vector associated with it does not dominate the competitive multiplier vector of every other CE. With such a price-normalizing bundle, Corollary 3 implies that the given CE is the unique selection for the credit mechanism with a *non-discriminatory default penalty* equal to the maximum multiplier of the associated competitive multiplier vector.

3.2 Price Normalization for Selection of a Unique CE

In this subsection, we show that in general a price-normalizing bundle exists with which default penalties can be specified for the credit mechanism to select a unique CE. To this end, we confine attention to exchange economies with *finitely many* CEs. This is not too restrictive because the property of having finitely many CEs is *generic*.[8] We will apply the following theorem of the alternative for matrices.

Theorem 3 (Theorem of the Alternative for Matrices) *Let $A = (a_{ij})$ be a $m \times k$ matrix. Then, either (i) or (ii) must hold:*

(i) The origin $0 \in \Re^m$ is contained in the convex hull of the k vectors

$$a_j = (a_{1j}, a_{2j}, \cdots, a_{mj}),\ \ j = 1, 2, \cdots, k$$

and the m unit vectors $e_i \in \Re^m$ with

$$e_{ij} = \begin{cases} 1 & if\ i = j; \\ 0 & if\ i \neq j. \end{cases}$$

[8] A technical problem with there being infinitely many CEs is that we are no longer able to apply the result on the alternative for matrices as stated in Theorem 3 below.

(ii) There exists a vector $r \in \Re^m_{++}$ such that $a_j \cdot r > 0$ for $j = 1, 2, \cdots, k$.

Proof. See Lemma II.4.3 in Owen (1982, pp. 17-18) for a proof. ■

Let $(\hat{x}, \hat{p})$ be a CE for economy $\mathcal{E}$. Assume that the price vector $\hat{p}$ is not equal to any non-negative linear combination of the competitive price vectors associated with the other CEs.[9] Now consider an iterative process that begins from the price normalization with price-normalizing bundle $e = (1, 1, \cdots, 1) \in \Re^m$. This is the familiar price normalization that results in normalized price vectors that form the price simplex

$$\Delta^m = \Big\{p \in \Re^m_+ | \sum_{h=1}^{m} p_h = 1\Big\}.$$

Let the other CEs of $\mathcal{E}$ be indexed as $(\hat{x}(j), \hat{p}(j))$, $j = 1, 2, \cdots, k$.

Lemma 1 *Assume A1, A2', and A3-A5 are satisfied. Assume further $\hat{p} \neq \sum_{j=1}^{k} \alpha_j \hat{p}(j)$ for any $\alpha_j \geq 0$, $j = 1, 2, \cdots, k$. Then, $0 \in \Re^m$ is not in the convex hull of vectors $\{\frac{\hat{p}(j)}{\hat{p}(j)\cdot e} - \frac{\hat{p}}{\hat{p}\cdot e}\}_{j=1}^{k} \cup \{e_i\}_{i=1}^{m}$.*

Proof. Suppose on the contrary that 0 is in the convex hull. Then, there exist non-negative weights $\alpha_1, \cdots, \alpha_k$ and $\beta_1, \beta_2, \cdots, \beta_m$ such that

$$\sum_{j=1}^{k} \alpha_j + \sum_{i=1}^{m} \beta_i = 1 \text{ and } \sum_{j=1}^{k} \alpha_j \Big[\frac{\hat{p}(j)}{\hat{p}(j) \cdot e} - \frac{\hat{p}}{\hat{p} \cdot e}\Big] + \sum_{i=1}^{m} \beta_i e_i = 0. \tag{7}$$

Taking the inner product with e on both sides of (7) yields $\sum_i \beta_i = 0$. This together with $\beta_i \geq 0$ for all i implies each $\beta_i = 0$. Consequently, by (7), $\hat{p} = \sum_j \alpha_j \frac{\hat{p}\cdot e}{\hat{p}(j)\cdot e} \hat{p}(j)$, which contradicts the assumption that $\hat{p}$ is not equal to any non-negative linear combination of price vectors $\hat{p}(j)$ for all j. ■

Set

$$p^1 = \frac{\hat{p}}{\hat{p} \cdot e} \text{ and } p^1(j) = \frac{\hat{p}(j)}{\hat{p}(j) \cdot e}.$$

Lemma 1 and Theorem 3 together imply that under the assumptions in Lemma 1, there exists a bundle in $\Re^m_{++}$ which we denote by r^1 such that

$$\Big[\frac{\hat{p}(j)}{\hat{p}(j) \cdot e} - \frac{\hat{p}}{\hat{p} \cdot e}\Big] \cdot r^1 > 0.$$

[9] This is equivalent to the condition that when normalized to be in the price simplex, $\frac{\hat{p}}{\hat{p}\cdot e}$ is not in the convex hull of $\frac{\hat{p}(j)}{\hat{p}(j)\cdot e}$, $j = 1, 2, \cdots, k$.

Equivalently,

$$p^1 \cdot r^1 < p^1(j) \cdot r^1, \; j = 1, \cdots, k. \tag{8}$$

Taking r^1 to be the price-normalizing bundle, we can normalize p^1 and $p^1(j)$ to

$$p^2 = \frac{p^1}{p^1 \cdot r^1} \text{ and } p^2(j) = \frac{p^1(j)}{p^1(j) \cdot r^1}. \tag{9}$$

Lemma 2 *Assume conditions in Lemma 1 are satisfied. Then, there exists* $r^1 \in \Re^m_{++}$ *such that (8) is satisfied and* $0 \in \Re^m$ *is not in the convex hull of the vectors* $\{\frac{p^1 \cdot r^1}{\hat{p}^1(j) \cdot r^1} p^2(j) - p^2\}_{j=1}^k \cup \{e_i\}_{i=1}^m$.

Proof. Suppose on the contrary for any r^1 satisfying (8), there exist non-negative weights $\alpha_1, \cdots, \alpha_k$ and $\beta_1, \beta_2, \cdots, \beta_m$ such that

$$\sum_{j=1}^k \alpha_j + \sum_{i=1}^m \beta_i = 1 \text{ and } \sum_{j=1}^k \alpha_j \left[\frac{p^1 \cdot r^1}{p^1(j) \cdot r^1} p^2(j) - p^2 \right] + \sum_{i=1}^m \beta_i e_i = 0. \tag{10}$$

By (9), $p^2 \cdot r^1 = 1$ and $p^2(j) \cdot r^1 = 1$ for all j. By taking the inner product with r^1 on both sides of the second equation in (10), we get

$$\sum_{j=1}^k \alpha_j \left[\frac{p^1 \cdot r^1}{p^1(j) \cdot r^1} - 1 \right] + \sum_{i=1}^m \beta_i r_i^1 = 0. \tag{11}$$

Since scaling down r^1 does not change the ratios $p^1 \cdot r^1 / \hat{p}(j) \cdot r^1$ neither does it change inequality (8), the weights α_j, β_i in (10) remain unchanged as r^1 is scaled down. However, as r^1 is scaled down, the second term on the left-hand-side of (11) approaches to zero while the first sum stays constant. By (8),

$$\frac{p^1 \cdot r^1}{p^1(j) \cdot r^1} - 1 < 0, \; j = 1, 2, \cdots, k$$

and by (10), $\alpha_j > 0$ for at least one j. It follows that the first sum on the left-hand-side of (11) remains constant and negative as r^1 is scaled down. This establishes the desired contradiction. ■

By Theorem 3 and Lemma 2, there exists a bundle $r^2 \in \Re^m_{++}$ such that

$$p^2 \cdot r^2 < \frac{p^1 \cdot r^1}{p^1(j) \cdot r^1} p^2(j) \cdot r^2, \; j = 1, \cdots, k. \tag{12}$$

Taking r^2 to be the normalizing bundle, p^2 and $p^2(j)$ can be normalized to

$$p^3 = \frac{p^2}{p^2 \cdot r^2} \text{ and } p^3(j) = \frac{p^2(j)}{p^2(j) \cdot r^2}. \tag{13}$$

By (8) and (12),

$$\frac{p^2 \cdot r^2}{p^2(j) \cdot r^2} < \frac{p^1 \cdot r^1}{p^1(j) \cdot r^1} < 1.$$

Hence, it follows from (8), (12), and (13) that the proof of Lemma 2 can be adapted to show the existence of a bundle $r^3 \in \Re^m_{++}$ such that

$$p^3 \cdot r^3 < \frac{p^2 \cdot r^2}{p^2(j) \cdot r^2} p^3(j) \cdot r^3.$$

To iterate this process inductively, suppose that for $t \geq 2$, $p^1, \cdots, p^{t-1}, p^t$ and $p^1(j), \cdots, p^{t-1}(j), p^t(j)$ for all j have been determined such that for some strictly positive bundles $r^1, \cdots, r^t \in \Re^m_{++}$

$$\frac{p^{\tau-1} \cdot r^{\tau-1}}{p^{\tau-1}(j) \cdot r^{\tau-1}} < 1 \text{ and } p^\tau \cdot r^\tau < \frac{p^{\tau-1} \cdot r^{\tau-1}}{p^{\tau-1}(j) \cdot r^{\tau-1}} p^\tau(j) \cdot r^\tau \tag{14}$$

for $j = 1, \cdots, k$ and $\tau = 2, \cdots, t$. Using r^t as the normalizing bundle, we can normalize p^t and $p^t(j)$ to

$$p^{t+1} = \frac{p^t}{p^t \cdot r^t} \text{ and } p^{t+1}(j) = \frac{p^t(j)}{p^t(j) \cdot r^t}, \; j = 1, 2, \cdots, k. \tag{15}$$

By (14),

$$\frac{p^t \cdot r^t}{p^t(j) \cdot r^t} < 1$$

for all j. It follows that the proof of Lemma 2 can be adapted to prove Lemma 3 below.

Lemma 3 *Assume conditions in Lemma 1 are satisfied. Assume further for $t \geq 2$, $r^1, \cdots, r^{t-1}, r^t$, $p^1, \cdots, p^{t-1}, p^t$, and $p^1(j), \cdots, p^{t-1}(j), p^t(j)$, for all j, have been determined such that (14) is satisfied. Then, bundle $r^t \in \Re^m_{++}$ can be so chosen that $0 \in \Re^m$ is not in the convex hull of the vectors*

$$\left\{ \frac{p^t \cdot r^t}{p^t(j) \cdot r^t} p^{t+1}(j) - p^{t+1} \right\}_{j=1}^k \cup \{e_i\}_{i=1}^m.$$

By Lemma 3 and Theorem 3, there exists a strictly positive bundle $r^{t+1} \in \Re^m_{++}$ such that

$$p^{t+1} \cdot r^{t+1} < \frac{p^t \cdot r^t}{p^t(j) \cdot r^t} p^{t+1}(j) \cdot r^{t+1}, \ j = 1, 2, \cdots, k.$$

Thus, by induction, the process can be iterated for any finite number of times. We now show that the process leads to the existence of a price-normalizing bundle in $\Re^m_{++}$ with which the competitive multiplier vector associated with $(\hat{x}, \hat{p})$ is the smallest.

Theorem 4 *Assume A1, A2', and A3-A5 are satisfied. Let* $(\hat{x}, \hat{p})$ *and* $(\hat{x}(j), \hat{p}(j))$, $j = 1, 2, \cdots, k$ *be the CEs for* $\mathcal{E}$*. Assume* $\hat{p} \not\models \sum_{j=1}^{k} \alpha_j \hat{p}(j)$ *for any* $\alpha_j \geq 0$ *for all* j*. Then, there exists a price-normalizing bundle* $\hat{r} \in \Re^m_{++}$ *with which the associated competitive multiplier vector with* $(\hat{x}, \frac{\hat{p}}{\hat{p} \cdot \hat{r}})$ *is strictly dominated by the associated competitive multiplier vector with* $(\hat{x}(j), \frac{\hat{p}(j)}{\hat{p}(j) \cdot \hat{r}})$*, for* $j = 1, 2, \cdots, k$.

Proof. Denote the competitive multiplier associated with $(\hat{x}, \hat{p})$ by $\hat{\lambda}$ and that with $(\hat{x}(j), \hat{p}(j))$ by $\hat{\lambda}(j)$ for $j = 1, 2, \cdots, k$. The process of iterated price normalization generates sequences $\{r^t\}$, $\{p^t\}$, and $\{p^t(j)\}$ for all j that satisfy (14) and (15) with $r^0 = e$, $\hat{p}^0 = \hat{p}$, and $\hat{p}^0(j) = \hat{p}(j)$. Since the CE allocations remain unchanged throughout the iteration process, it follows from Corollary 2 and (15) that

$$\lambda^{t+1} = (p^t \cdot r^t)\lambda^t \text{ and } \lambda^{t+1}(j) = (p^t(j) \cdot r^t)\lambda^t(j), \ \forall t, \forall j. \tag{16}$$

By iterated substitutions, it follows from (16)

$$\lambda^t = \Big(\prod_{\tau=1}^{t} p^{\tau-1} \cdot r^{\tau-1}\Big)\hat{\lambda} \text{ and } \lambda^t(j) = \Big(\prod_{\tau=1}^{t} p^{\tau-1}(j) \cdot r^{\tau-1}\Big)\hat{\lambda}(j), \ j = 1, 2, \cdots k.$$

Consequently,

$$\lambda^t << \lambda^t(j) \text{ if and only if } \Big(\prod_{\tau=1}^{t} \frac{p^{\tau-1} \cdot r^{\tau-1}}{p^{\tau-1}(j) \cdot r^{\tau-1}}\Big)\hat{\lambda} << \hat{\lambda}(j). \tag{17}$$

By (14),

$$\prod_{\tau=1}^{t} \frac{p^{\tau-1} \cdot r^{\tau-1}}{p^{\tau-1}(j) \cdot r^{\tau-1}} < \Big(\frac{\hat{p} \cdot e}{\hat{p}(j) \cdot e}\Big)\Big(\frac{\hat{p} \cdot r^1}{p^1(j) \cdot r^1}\Big)^{t-1}. \tag{18}$$

Since, by (8), $\frac{\hat{p}\cdot r^1}{p^1(j)\cdot r^1} < 1$, the right-hand-side of (18) approaches to 0 as the number of iterations gets large. Consequently, since $\hat{\lambda}(j) >> 0$ for all j, there exists a positive integer $\hat{t}$ such that (17) holds for $t \geq \hat{t}$. Set $\hat{r} = r^{\hat{t}}$. The proof is completed by noticing that from (15)

$$p^t = \frac{\hat{p}}{\hat{p}\cdot r^t} \text{ and } p^t(j) = \frac{\hat{p}(j)}{\hat{p}(j)\cdot r^t}$$

for all $t \geq 0$. Hence, p^t and $p^t(j)$ are obtained from normalizing $\hat{p}$ and $\hat{p}(j)$ through price-normalizing bundle r^t, respectively. ■

Corollary 3 and Theorem 4 together imply the following selection of a unique CE:

Corollary 4 (Selection of a Unique CE) *Assume A1, A2', and A3-A5 are satisfied. Then, there exists a price-normalizing bundle $\hat{r}$ with which the credit mechanism selects a unique CE with some non-discriminatory default penalty.*

Proof. Let e be the price-normalizing bundle. Then, the set of normalized competitive price vectors is a compact subset of the price simplex Δ^m. Thus, by the Krein-Milman Theorem, it has an extreme point.[10] Let $(\hat{x}, \hat{p})$ be a CE such that $\frac{\hat{p}}{\hat{p}\cdot e}$ is an extreme point in the set of normalized competitive price vectors. Then, $\hat{p}$ cannot be equal to any non-negative linear combination of other CE price vectors. By Theorem 4, there exists a price-normalizing bundle $\hat{r} \in \Re^m_{++}$ such that the competitive multiplier vector associated with $(\hat{x}, \frac{\hat{p}}{\hat{p}\cdot\hat{r}})$ is strictly dominated by the competitive multiplier vector associated with every other CE under price normalization through $\hat{r}$. Consequently, by Corollary 3, with price-normalizing bundle $\hat{r}$ $(\hat{x}, \frac{\hat{p}}{\hat{p}\cdot\hat{r}})$ is a unique selection for the credit mechanism with default penalties all equal to the maximum multiplier of the competitive multiplier vector associated with it. ■

3.2.1 Selection with a Uniform Price-Normalizing Bundle

In this subsection we consider price-normalizing bundles, with which the credit mechanism uniquely selects any CE with appropriate default penalties. By Theorem 2, such unique selections require that the resulting competitive multiplier vectors do not dominate each other. We assume:

[10] See Royden (1968, p. 207).

A6: There exists a bundle $r \in \Re^m_{++}$ such that for any two interior Pareto optimal allocations $\bar{x}$ and $\hat{x}$,

$$\begin{array}{l} \bigtriangledown u^i(\bar{x}^i) \cdot r > \bigtriangledown u^i(\hat{x}^i) \cdot r \text{ for some } i \\ \Longrightarrow \\ \bigtriangledown u^j(\bar{x}^j) \cdot r < \bigtriangledown u^j(\hat{x}^j) \cdot r \text{ for some } j. \end{array}$$

The inner product of the gradient $\bigtriangledown u^i(x^i)$ with bundle r is the *directional derivative* of u^i at x^i in direction r. It measures the instantaneous rate of change in trader i's utility caused by a change from bundle x^i to bundle $x^i + \delta r$ for some small number $\delta > 0$, that is, $\bigtriangledown u^i(x^i) \cdot r$ is the limit of

$$\frac{u^i(x^i + \delta r) - u^i(x^i)}{\delta}$$

as $\delta > 0$ approaches to zero. The following two examples all have three interior CEs and all satisfy A6.

Example 1: (Shapley and Shubik 1977) There are two goods and two traders with endowments $a^1 = (40, 0)$, $a^2 = (0, 50)$ and utility functions $u^1(x^1) = x^1_1 + 100(1 - e^{-x^1_2/10})$, $u^2(x^2) = 110(1 - e^{x^2_1/10}) + x^2_2$ on $\Re^2_+$. Traders 1 and 2 are respectively named Ivan and John in Shapley and Shubik (1977); goods 1 and 2 are respectively called rubles and dollars in their paper. There are three interior CEs in this economy. Furthermore, the interior Pareto optimal allocations satisfy

$$x^2_2 = x^2_1 + 50 - 10 \ln 110. \tag{19}$$

Notice $\bigtriangledown u^1(x^1) = (1, 10e^{-x^1_2/10})$ and $\bigtriangledown u^2(x^2) = (11e^{-x^2_1/10}, 1)$. Equation (19) implies that to be Pareto optimal, trader 2's consumption of good 2 increases with his consumption of good 1. Since trader 1's marginal utility of good 1 is constant and his marginal utility of good 2 is decreasing while trader 2's marginal utility of good 2 is constant and his marginal utility of good 1 is decreasing, this example satisfies A6 for any bundle $r \in \Re^2_{++}$.

Example 2: (Mas-Colell, Whinston, and Green 1995, pp. 521-522) There are two goods and two traders with endowments $a^1 = (2, r)$, $a^2 = (r, 2)$, $r > 0$, and utility functions $u^1(x^1) = x^1_1 - \frac{1}{8}(x^1_2)^{-8}$, $u^2(x^2) = -\frac{1}{8}(x^2_1)^{-8} + x^2_2$ on $\Re^2_{++}$. There are three interior CEs in this economy. Furthermore, the interior Pareto optimal allocations satisfy

$$x^2_1 x^1_2 = 1. \tag{20}$$

Notice $\bigtriangledown u^1(x^1) = (1, (x_2^1)^{-9})$ and $\bigtriangledown u^2(x^2) = ((x_1^2)^{-9}, 1)$. Equation (20) implies that to be Pareto optimal, any increase in trader 1's consumption of good 2 leads to a decrease in trader 2's consumption of good 1. Since trader 1's marginal utility of good 1 is constant and his marginal utility of good 2 is decreasing while trader 2's marginal utility of good 2 is constant and his marginal utility of good 1 is decreasing, this example satisfies A6 for any bundle $r \in \Re^2_{++}$.

We now establish the non-dominance of the traders' marginal utilities of income at CEs.

Theorem 5 (Non-Dominance) *Assume $\mathcal{E} = \{X^i, u^i, a^i\}_{i=1}^n$ satisfies A1, A2′, and A4-A6. Let r be a bundle as in A6. Then, for any two competitive triplets $(\bar{x}, \bar{p}, \bar{\lambda})$ and $(\hat{x}, \hat{p}, \hat{\lambda})$, $\bar{p} \cdot r = \hat{p} \cdot r$ implies that $\bar{\lambda} \geq \hat{\lambda}$ and $\bar{\lambda} \not= \hat{\lambda}$ cannot hold.*

Proof. Suppose on the contrary that there are two competitive triplets $(\bar{x}, \bar{p}, \bar{\lambda})$ and $(\hat{x}, \hat{p}, \hat{\lambda})$ with $\bar{p} \cdot r = \hat{p} \cdot r$ and

$$\bar{\lambda} \geq \hat{\lambda} \text{ and } \bar{\lambda} \not= \hat{\lambda}. \tag{21}$$

Since $r \in \Re^m_{++}$ and since $\bar{p} \cdot r = \hat{p} \cdot r > 0$, (21) implies

$$\bar{\lambda}^i \bar{p} \cdot r \geq \hat{\lambda}^i \hat{p} \cdot r$$

for all i and

$$\bar{\lambda}^j \bar{p} \cdot r > \hat{\lambda}^j \hat{p} \cdot r$$

for at least one j. However, by A2′, A5, and the Kuhn-Tucker conditions, $\bar{\lambda}^k \bar{p} = \bigtriangledown u^k(\bar{x}^k)$ and $\hat{\lambda}^k \hat{p} = \bigtriangledown u^k(\hat{x}^k)$ for all k. The above dominance of vector $(\bar{\lambda}^i \bar{p} \cdot r)$ over vector $(\hat{\lambda}^i \hat{p} \cdot r)$ then contradicts A6. ■

Combining Theorem 5 with Corollary 3, we can now establish:

Corollary 5 (Selection of a Unique CE) *Let $\mathcal{E} = \{X^i, u^i, a^i\}_{i \in N}$ be an exchange economy. Assume $\mathcal{E}$ satisfies A1, A3, A2′, and A4-A6. Then, under the price normalization that results in the set of normalized price vectors*

$$P = \left\{ p \in \Re^m_+ \mid p \cdot r \equiv 1 \right\}$$

with bundle r as in A6, every CE, $(\bar{x}, \bar{p})$, of $\mathcal{E}$ is a unique selection by the credit mechanism with default penalties $\mu = \bar{\lambda}$, where $\bar{\lambda}$ is the competitive multiplier associated with $(\bar{x}, \bar{p})$.

We end this section with an example to demonstrate the total cash flows and the Lagrangian multipliers in CEs.

Example 3: Consider the 2-person economy of Shapley and Shubik (1977). As we showed in Example 1, for this economy A6 is satisfied even when we replace "for some bundle $r \in \Re^2_{++}$" with "for all bundles $r \in \Re^2_{++}$". Thus, we can choose $r = a^1 + a^2$ to be a price-normalizing bundle. If we normalize the prices by the condition $p \cdot r = 1,000$, so that the economy's total wealth is always 1,000, then the competitive price vectors, competitive multiplier vectors, total cash flows are as in the following table, all with a two-digit decimal rounding off:

	x^*	p^*	λ^*	TW	TCF
CE1	((32.26, 39.26), (7.74, 10.74))	(3.4, 17.27)	(0.29, 0.06)	1000	704.34
CE2	((13.17, 20.18), (26.83, 29.82))	(12.9, 9.68)	(0.08, 0.1)	1000	541.45
CE3	((3.22, 10.23), (36.78, 39.77))	(18.5, 5.19)	(0.05, 1.9)	1000	733.52

In this table, TW stands for the total wealth of the economy and TCF for the total cash flow. The cash flow required from trader i at prices p_1, p_2 and bundle x^i is given by $p_1 \max\{0, x^i_1 - a^i_1\} + p_2 \max\{0, x^i_2 - a^i_2\}$ and the total cash flow required in a CE is the sum of the cash flows required from both traders at their respective equilibrium bundles and the equilibrium prices. Notice that the middle CE (CE2) is the only minimum cash flow CE. To uniquely select it, we can set the per-unit default penalties equal to the traders' competitive multipliers 0.08 and 0.1. Alternatively, we can also choose a non-discriminatory per-unit default penalty equal to 0.1. In fact, it follows from the proof of Theorem 3 that any non-discriminatory per-unit default penalties between 0.1 and the next highest maximum competitive multiplier which is equal to 0.19 would work.

4 Selection with Production

An *economy* with l goods, n consumers, and household production is an array $\mathcal{E} = \{(X^i, u^i, a^i, Y^i)\}_{i \in N}$, where N is the consumer set, $X^i \subseteq \Re^m$ is the consumption set of consumer i, u^i is i's utility function, a^i is his endowment bundle, and $Y^i \subseteq \Re^m$ is his household production possibility set.[11] An element y^i in Y^i represents a

[11] This model of an economy was considered in Hurwicz (1960), Rader (1964), Shapley (1973), Billera (1974), among others.

production plan that i can carry out. As usual, inputs into production appear as negative components of y^i and outputs as positive components. For all $i \in N$, X^i and Y^i are closed and convex.

4.1 Competitive Allocations

With household production, a production plan changes a consumer's initial endowment before trading. Hence, the selection of a production plan by an individual is guided by utility maximization instead of profit maximization. However, with price-taking traders, utility maximization implies profit maximization.

Definition 2 *A CE for economy $\mathcal{E} = \{(X^i, u^i, a^i, Y^i)\}_{i \in N}$ is a point*

$$((x^{*i}, y^{*i})_{i \in N}, p^*) \in (\Pi_{i \in N}(X^i \times Y^i)) \times \Re^m_+$$

such that

(i) For $i \in N$, $p^* \cdot x^{*i} = p^* \cdot a^i + p^* \cdot y^{*i}$ and $u^i(x^i) > u^i(x^{*i})$ implies $p^* \cdot x^i > p^* \cdot a^i + p^* \cdot y^i$ for all $y^i \in Y^i$;

(ii) $\sum_{i \in N} x^{*i} = \sum_{i \in N} a^i + \sum_{i \in N} y^{*i}$.

4.2 Arrow-Debreu Economy

In the Arrow-Debreu model of an economy with $m < \infty$ goods, there are a set, N, of finitely many consumers with consumer $i \in N$ characterized by the triplet (X^i, u^i, a^i) and a set, J, of producers with producer $j \in J$ characterized by a production possibility set Y^j. In addition, each consumer i is also endowed with a relative share θ_{ij} of firm j's profit (see Arrow and Debreu 1954, Debreu 1959). Symbolically, an *Arrow-Debreu economy* is an array $\mathcal{E} = \{\{(X^i, u^i, a^i)\}_{i \in N}, \{Y^j\}_{j \in J}, \{\theta_{ij}\}_{i \in N, j \in J}\}$. For all $i \in N$ and all $j \in J$, X^i and Y^j are closed and convex.

Definition 3 *A CE for $\mathcal{E} = \{\{(X^i, u^i, a^i)\}_{i \in N}, \{Y^j\}_{j \in J}, \{\theta_{ij}\}_{i \in N, j \in J}\}$ is a point*

$$((x^{*i})_{i \in N}, (y^{*j})_{j \in J}, p^*) \in (X_{i \in N} X^i) \times (X_{j \in J} Y^j) \times \Re^m_+$$

such that

(i'a) For $i \in N$, $p^* \cdot x^{*i} = p^* \cdot a^i + \sum_{j \in J} \theta_{ij} p^* \cdot y^{*j}$ and $u^i(x^i) > u^i(x^{*i})$ implies $p^* \cdot x^i > p^* \cdot a^i + \sum_{j \in J} \theta_{ij} p^* \cdot y^{*j}$;

(i′b) For $j \in J$, $p^* \cdot y^{*j} \geq p^* \cdot y^j$, for $y^j \in Y^j$;

(ii′) $\sum_{i \in N} x^{*i} = \sum_{i \in N} a^i + \sum_{j \in J} y^{*j}$.

The relative shares θ_{ij} may be interpreted as representing private proprietorships of the production possibilities and facilities. With this interpretation, we can think of consumer i as owning the technology set $\theta_{ij} Y_j$ at his disposal in firm j. Consequently, we may think of consumer i as owning the following production possibility set in the Arrow-Debreu economy:

$$\tilde{Y}^i = \sum_{j \in J} \theta_{ij} Y_j. \tag{22}$$

We denote elements in $\tilde{Y}^i$ by $\tilde{y}^i = \sum_{j \in J} \theta_{ij} y^{ij}$ for some $y^{ij} \in Y^j$, $j \in J$. The reader is referred to Rader (1964, pp. 160–163) and Nikaido (1968, p. 285) for a justification of this understanding of the consumers' ownership shares. With equation (22), the Arrow-Debreu economy $\mathcal{E}$ is converted into an economy with household production which we denote by $\tilde{\mathcal{E}} = \{(X^i, u^i, a^i, \tilde{Y}^i)\}_{i \in N}$.

Rader showed that an Arrow-Debreu economy $\mathcal{E}$ with convex production possibility sets is equivalent to economy $\tilde{\mathcal{E}}$, in the sense that the competitive allocations are the same across the two economies (see Rader 1964, pp. 160–163):

Theorem 6 *Let $\mathcal{E} = \{\{(X^i, u^i, a^i)\}_{i \in N}, \{Y^j\}_{j \in J}, \{\theta_{ij}\}_{i \in N, j \in J}\}$ be an Arrow-Debreu economy and let $\tilde{\mathcal{E}} = \{(X^i, u^i, a^i, \tilde{Y}^i)\}_{i \in N}$ with $\tilde{Y}^i$ given in (22). Then, for any CE $((x^{*i})_{i \in N}, (y^{*j})_{j \in J}, p^*)$ of $\mathcal{E}$, there are production plans $\tilde{y}^{*i} \in \tilde{Y}^i$, $i \in N$, such that $((x^{*i}, \tilde{y}^{*i})_{i \in N}, p^*)$ is a CE of $\tilde{\mathcal{E}}$. Conversely, for any CE $((x^{*i}, \tilde{y}^{*i})_{i \in N}, p^*)$ of $\tilde{\mathcal{E}}$, there are production plans y^{*j}, $j \in J$, such that $((x^{*i})_{i \in N}, (y^{*j})_{j \in J}, p^*)$ is a CE of $\mathcal{E}$.*

4.3 *Rader's Equivalence Principle*

Rader (1964) considers how to transform an economy with household production into an exchange economy using *induced preferences.* He shows that all the properties pertaining to the consumers' characteristics in a production economy go over to the induced exchange economy. Furthermore, the CEs of the original economy and those of the induced exchange economy are equivalent (see Rader 1964, pp. 155-57). It follows that our credit mechanism and results in the previous sections can be extended to a production economy via its induced exchange economy.

5 Conclusion

In this paper we investigated the possibilities to enlarge the general-equilibrium structure by allowing default subject to appropriate credit limits and penalties. The enlargement of the general equilibrium structure results in a construction of a simple credit mechanism for a credit using society to select a unique CE.

The implementation of the credit mechanism involves a bank providing banknotes that traders use as a direct and anonymous means of payment. The traders exchange personal IOUs for banknotes with exogenously specified credit lines at the beginning, and they settle up all outstanding credits with the bank at the end of the market. Under the credit mechanism, ending as a net debtor is penalized while ending as a net creditor is worthless.

Given price normalization and default penalties, we characterized the CEs that will be selected by the credit mechanism. They are those CEs with traders' marginal utilities of income dominated by the corresponding per-unit default penalties. Applying this result, we showed that in general price normalization exists under which the credit mechanism selects a unique CE with some non-discriminatory default penalty. Furthermore, with the additional condition that for some bundle with a positive quantity of each good, the derivatives of the traders' utility functions at each Pareto optimal allocation along the direction represented by the bundle do not dominate those at every other Pareto optimal allocation, price normalization calling for an equal value of the bundle guarantees that any CE can be a unique selection for the credit mechanism with appropriate default penalties.

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