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The Optimal Concentration of Creditors*

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Abstract

There are situations in which dispersed creditors (e.g., public creditors) have more difficulties and higher costs when collecting their claims in financial distress than concentrated creditors (e.g., banks). Under this assumption, our model predicts that measures of debt concentration relate [a] positively to creditors' chosen aggregate debt collection expenditures; [b] positively to management's chosen expenditures to avoid paying; [c] positively to total net litigation costs/waste in financial distress; and [d] positively to accomplished claim recovery by creditors (to which we present some preliminary favorable empirical evidence). Under additional assumptions, measures of debt concentration relate [e] positively to intrinsic firm quality; [f] positively to creditor monitoring and negatively to managerial waste; [g] positively to optimal continuation/discontinuation choices; [h] negatively to issuing marketing expenses. In a signaling model, when concentration alone is not a sufficient signal, firms choose the ultimately concentrated debt (i.e., a house bank) and have to pay a high interest.

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I Introduction

Coordination failure among multiple claimants, be they creditors or owners, is a subject well-studied in the academic literature. Such coordination failures can lead to takeover failures (Grossman and Hart (1980)) or bank-runs (Diamond and Dybvig (1983), Obstfeld (1996); Morris and Shin (1998); Morris and Shin (1999)), or generally reduce the probability of successful renegotiation to a proposed reorganization plan when renegotiation requires simultaneous assent by many claimants (Preece and Mullineaux (1996); Hege (1997); Bergloef, Roland, and von Thadden (2000)). In many of these models, the coordination failures aid the dispersed claimants. In a sense, claimants cooperation has to be purchased with an offer that is attractive enough for each and every claimant to choose to collaborate. Thus, coordination failure can suggest that dispersed creditors or owners can receive higher settlements than their hypothetically more concentrated but otherwise identical counterparts.

Zingales (1995) uses this insight to show that an entrepreneur may prefer to sell a firm to dispersed owners in an IPO, who in turn can later obtain a higher price for the shares from a potential acquiror than this entrepreneur could have obtained by herself.[1] In Bolton and Scharfstein (1996), the paper most interested in the optimal concentration of creditors and thus most similar in goals to our own paper, coordination failure is used to explicitly derive an optimal number of creditors: in financial distress, two creditors can extract more surplus than one creditor. This can deter inappropriate (strategic) default by management. However, coordination problems can also make two creditors less likely to facilitate corporate continuation when it is optimal. The optimal number of creditors is thus a coordination tradeoff designed to approximate optimal termination/continuation.[2]

Yet, it is possible to draw a different conclusion from the fact that dispersed creditors cannot easily coordinate. Dispersed creditors are first and foremost un-

[1] Of course, it could be that being public raises the probability that this firm will appear on the radar screen of potential acquirors.

[2] Rajan (1992), Repullo and Suarez (1998), and others consider the tradeoff between a concentrated creditor's ability to collect information and decide intelligently, and his worse ability to negotiate a better settlement due to lower concentration.

able to be proactive. Thus, even though they are at an advantage when positive assent to a relief plan is required from every creditor, they are at a *disadvantage* when active opposition to management's relief plan is required. In this case, mutual free-riding incentives weakens the overall outcome for dispersed claimants. A good example of how dispersion can facilitate bondholder expropriation are Gertner and Scharfstein (1991) and Bernardo and Talley (1996), in which management can use exchange offers to expropriate wealth from uncoordinated creditors.

Our paper begins with a motivating pilot study from the New York State bankruptcy court. In a sample of 63 bankruptcies, and adjusting for firm size, secured creditor concentration helps secured creditors, and unsecured creditor concentration hurts secured creditors in avoiding APR violations in favor of unsecured creditors.

Our model assumes that creditors must pro-actively seek to enforce their claims. Our model is based on a *conflict game*, in which more proactive claimants can achieve better outcomes for themselves. Conflict theory can be viewed as a reduced-form method of modeling negotiations, which sidesteps the usually complex, often asymmetric information games which underlie formal models of bargaining derived from first principles (e.g., Rubinstein (1982)). In exchange for a certain ad-hocness in the specification of how legal effort aids outcomes, conflict models sometimes offer a more realistic description of empirical bargaining outcomes. Their reduced form of modeling can provide interesting and relatively easily empirically testable predictions. Conflict theory is by now an accepted mainstream method of modeling (e.g., Hirshleifer (1978), Hirshleifer (1989), Hirshleifer (1991), Welch (1997), Rajan and Zingales (1998), Bernardo, Talley, and Welch (2000) and many others).

In our model, multiple creditors have to negotiate with the entrepreneur in case of financial distress. Our paper uses "management," "equity", and "entrepreneur" interchangeably. Collection costs can stem from the costs of filing a claim, following up through the bankruptcy process, investigating the firm's true resources, communicating and negotiating with and pressuring management, hiring lawyers, bringing motions to the court, etc. Management can establish procedures which make it difficult for its creditors to prove and recover their claims, or hire lawyers to outright oppose APR (absolute priority rule). Indeed, a casual perusal of bankruptcy records shows that it is not difficult to find examples of creditors who did not find it in their interest to go through the legal hoops necessary even to file, much less to reclaim relatively modest claims in Chapter 11.[3] Further, civil liability claims are commonly dismissed by the bankruptcy court altogether.

Such APR violations are consistent with our model, in which a larger number of creditors suffers from more mutual free-riding, which in turn compromises their ability to collect on their claims. De facto, our model argues that, given a fixed level of debt, a distressed firm with a million uncoordinated small creditors is less likely to be forced to pay its obligations than a firm with one creditor or a firm with creditors that have a coordinating organ (e.g., a trustee for financial bonds). Although we are thinking of idiosyncratic, small credit (such as *small* trade credit [Biais and Gollier (1997), Petersen and Rajan (1997)]) as a good application for our model, our model can also apply to public debt which is not fully coordinated or even civil legal claims brought by product customers and other stakeholders.

The differences in collection ability allow us to derive an optimal concentration of creditors. An entrepreneur who chooses a large number of creditors *ex-ante* assures herself of better bargaining ability against creditors in case of financial distress *ex-post*. Yet, in equilibrium, this costs the entrepreneur a higher interest rate when raising the debt *ex-ante*.[4] In contrast, an entrepreneur who chooses a single creditor *ex-ante* will be forced to extensively (and expensively) negotiate with this creditor in case of financial distress, and this creditor will likely be relatively more successful in enforcing her claim. Yet, in equilibrium, such an entrepreneur will also enjoy a lower interest rate when raising the debt *ex-ante*.

In our model, the *ex-ante* number of creditors determines both the *ex-post* distribution of cash flows in distress *and* the socially inefficient costs of claim collection.

[3] An alternative to the conflict game would be to model collection costs of creditors directly, and show that some effort by the firm would lead such creditors to not incur the costs of attempting recovery. The implications of such a model are the same as those that we stress in our own paper.

[4] In the case of customers who can recognize their lower ability to file civil suit in case the product turns out to be defective, in lower product prices for an equally good product.

The model shows that measures of debt dispersion (the number of creditors) correlates positively with the entrepreneur's retention of the firm in bankruptcy (fewer creditors ⇒ worse outcome for management in financial distress), and negatively with the in-equilibrium claims collection costs (fewer creditors ⇒ more collection efforts, costs, and waste).

In our simplest framework, the only deadweight cost of credit is the in-equilibrium spending on conflict. Thus, by itself, this "number of creditors" tradeoff in financial distress—in which more creditors in financial distress have lesser ability to wrestle the firm from management—has a first-best outcome, in which the number of creditors is infinitely large. No collection costs would be incurred in financial distress, and perfectly dispersed creditors receive proper ex-ante compensation (higher interest rates) for their perfect ex-post expropriation.

To solve an "optimal capital structure" model in which at least some firms find it in their interest to choose a small number of creditors, there must be an offsetting advantage to having fewer creditors. There are at least four applicable mechanisms that can be invoked:

Signaling Firms may know whether they are of high-quality or low-quality. A firm that chooses fewer creditors signals its higher confidence that it will not go bankrupt and incur ex-post waste.

Fewer Creditors ⇒ Higher Inferred Firm Quality

Agency Management may be better kept in check by fewer creditors. Such creditors have an incentive to invest more in monitoring activity even if the firm is not in distress.

Fewer Creditors ⇒ Better Creditor Monitoring

Optimal Continuation/Termination Fewer coordinated creditors can respond better to make an intelligent decision of whether a firm in distress should continue to operate.

$$\text{Fewer Creditors} \Rightarrow \text{Better Termination Choices}$$

This is the mechanism in Bolton and Scharfstein (1996) (which however offers diametrically opposite empirical implications to our own paper with respect to the number of creditors). In a sense, this mechanism can be considered similar to value-enhancing agency monitoring, but *after* the firm enters financial distress.

Simple Transaction Costs It may be more expensive to market debt claims to multiple creditors than it is to market them to just a few creditors.

$$\text{Fewer Creditors} \Rightarrow \text{Lower Marketing Costs}$$

Indeed, when our model is applied to product market liabilities/claims, it may be exceedingly expensive for the firm to alter its market from few product purchasers (imposing high distress costs) to just a few purchasers (with lower distress costs).

Naturally, these factors may be simultaneously at work. For example, a single firm may balance the costs of fewer creditors (i.e., incurring in-equilibrium conflict waste) against a better inference about its quality drawn by the market, against better efficient monitoring of managerial choices, and against a desire to *ex-ante* commit to sometimes (and optimally) hand the firm to creditors in financial distress.

The model is kept deliberately simple, if only to avoid deceiving the reader into believing that more algebraic generality could purchase significant more empirical realism. Corporate finance models are often highly stylized and serve primarily to show that the economic intuition is internally consistent—and our model is no exception. Real life corporate decisions are complex and doubtlessly determined by many unmodeled factors. In particular, the reader should recognize that there can also be many situations in which dispersed creditors have *more* power, not less power, e.g. as in Bolton and Scharfstein (1996). In the end, it will be up to the empirical evidence to measure whether and when dispersion weakens creditors and when it strengthens creditors. Fortunately, creditor concentration is relatively easy to measure empirically, which should permit empiricists to relatively easily distinguish between the predictions of these models.

At times, we identify the most concentrated debt, i.e., a single creditor, as a (house) bank. Although banks doubtlessly perform other functions, they do tend to assume debt in a more concentrated fashion than public creditors. Thus, although simplistic , it is quite likely that lack of dispersion is a good characterization of *one* of the differences between public creditors and banks. In the signaling version of our paper, when concentration (the most efficient signal) is exhausted, creditors resort to paying excess rents to banks to assure separation. Thus, this version of our model predicts that bank debt carries higher *expected* (not promised!) yields higher than public debt.

Welch (1997) is closest to our paper, at least in its approach and technology. It models the conflict between existing bank debt and public debt and comes to the conclusion that *if* a company has already issued both kinds of debt, and it now must decide which to make senior, it is the bankdebt which should be the senior security. In Welch (1997), there is neither a role for equity, nor a role for multiple creditors with *equal* fighting ability, nor explicit free-riding among creditors of equal seniority, nor an endogenous determination of the number of creditors or type of credit or excess interest rate. Indeed, Welch (1997) does not even consider the entrepreneur's choice *between* bank debt *and* public debt.

We shall now proceed as follows: Section II describes our pilot study, in which we find that creditor concentration helped creditors in a particular situation. Section III describes the conflict game played between N creditors and management in financial distress. This section solves the dynamic optimization from the perspective of management. The result of this section is that there is a monotonically positive relation between debt concentration and in-equilibrium waste. Section IV grafts onto this base model a signaling case in which higher-quality managers signal their confidence by choosing fewer creditors. We also show that after concentration signaling is exhausted (i.e., the firm chooses only 1 creditor, a "house" bank), entrepreneurs must resort to yield signaling. Section V outlines variant models (agency, continuation, marketing) that similarly lead to an interior optimal creditor concentration. Section VI discusses our empirical implications, contrasts them with Bolton and Scharfstein (1996), and describes some evidence that is relevant to our argument. Section VII concludes briefly.

II A Pilot Study: Does Concentration or Dispersion Aid Recovery?

In Bolton and Scharfstein (1996), concentration hurts creditors. Because our model will be consistent with the opposite implication (that concentration helps creditors), we decided to conduct a pilot study.

We collected bankruptcy cases from the Southern District of New York (Manhattan, Poughkeepsie, and White Plains). The full text is available through the Public Access to Court Electronic Records (PACER), an electronic public access service that allows users to obtain case and docket information. We identified all the closed Chapter 11 cases in the district as of August 2001, for which data on firm characteristics, creditors' claims, and Chapter 11 resolution was available. We hand-coded this information. Our data sample differs from earlier studies, because we include smaller and private bankruptcies.

We found 63 cases with data on both APR violations and the number of secured and unsecured creditors. Loans with both secured and unsecured components are separated and their components are assigned into the correct categories. The earliest case in our sample was filed in July 1995, the most recent in July 2000. The average firm size is $17.372 million, and the average value of liabilities is $34.488 million. In particular, we obtained information on the number of secured and unsecured creditors, and on APR violations, if any. In our average firm, there are 3.5 secured creditors, and 174 unsecured creditors. APR is violated 3 times against creditors in favor of equityholders, and 9 times against secured creditors in favor of unsecured creditors.

Unsecured creditors are consolidated into one creditor committee. Secured creditors have to find their own mechanisms to coordinate their efforts. Define the "secured creditors outcome" as what was granted to secured creditors minus the value of the secured claim or total assets (whichever is smaller), divided by the size of the secured claim. (Although quoting outcome in percent already offers some size adjustment, our regressions also control for total assets.) A White heteroskedasticity-adjusted regression among our 63 observations yields

Secured Creditors Outcome		=	0.11	$-0.01 \cdot N^S$	$+ 0.39 \cdot (N^U/1000)$	$+ \gamma_1 \cdot \text{TA} + \gamma_2 \cdot \text{TA}^2$	$+\epsilon$
(p-value):			(0.012)	(0.025)	(0.010)	(not significant)	
Mean:	0.14		0.03	3.50	0.173		
Stddev:	(0.32)		(0.19)	(9.91)	(0.483)		

with an adjusted R^2 of 6.0%.[5] The negative coefficient on N^S indicates that secured creditors receive a worse outcome the higher the number of secure creditors. The positive coefficient on $(N^U/1000)$ indicates that the higher the number of unsecured creditors (divided by 1,000), the better the outcome to secured creditors. However, unsecured creditors's legal expenses are subsidized by the firm, and they have a coordinating committee, indicating caution in interpreting the number of creditors as a proxy for dischord. Nevertheless, the two relationships indicate to us that concentration is a positive factor in the competition between secured and unsecured creditors. Put differently, creditor dispersion is not helpful. (Total assets [TA] and total assets squared [TA^2] are included as control variables, but are not statistically significant.) A similar regression to explain the outcome for unsecured

[5] We also tried a non-linear transformation, in which we estimated $\lambda = 0.5$ on a $1/(1 + N^\lambda)$ specification with 0.00 statistical significance. (The test of whether concentration is a positive or negative influence is now that $\lambda > 0$.) Although the result is more significant and again in favor of our theory, because the results are more difficult to explain, we report the simple linear regression instead.

creditors shows no significant variables.[6]

We also estimated a logit regression for violation of APR from secured creditors *in favor of* unsecured creditors. Again, concentration indicates a stronger negotiation ability. In our data set, which also included many smaller non-public firms (many of which were probably simply liquidated), there were only three APR violations in favor of equity. Thus, we cannot reliably report how creditor concentration influenced negotiating ability relative to equity. Finally, we wish to reemphasize that evidence does not reject Bolton and Scharfstein (1996) in other situations.

III The Cost of A Given Number of Creditors

We begin with a simple "creditor concentration" model. Our primary intent is to derive the in-equilibrium costs/waste of claims collection as a function of the number of creditors.

A The Assumptions

Insert Table 1 here
[Table of Symbols]

Table 1 lists the symbols used in our paper. In stage 1 of the game, the entrepreneur owns in-place assets worth V_{Old}. To adopt a project that provides 0 with probability π and V_{New} with probability $(1-\pi)$, the entrepreneur must raise external financing I ($\Rightarrow I > V_{\text{Old}}$). We also assume that the project is intrinsically worthwhile, i.e., $(1-\pi)V_{\text{New}} > I$. This financing can be in the form of debt raised from an (endogenously determined) number of creditors, N.[7]

[6] Unsecured creditors are coordinated through a committee, and thus may act more like a single creditor. In addition, their legal activities are subsidized by the court, allowing them to mount an effective campaign more easily.

[7] The model in this section finds a first-best solution. Thus, we could permit the firm to raise capital via equity, and it would not improve the firm's fate. When we introduce benefits to a limited number of creditors in the next section, the firm voluntarily avoids the first-best solution, and thus would avoid raising equity, too. Thus, omitting raising equity is without loss of generality.

If the project later were to fail, the firm still owns its project in place, V_{Old}. Although creditors "should" receive what the absolute priority rule (APR) promises them, collection costs (such as courts, lawyers, and "legal maneuvers") will allow management to reduce creditors' claims in financial distress by up to X. The fact that financial distress is not free or ex-ante completely contracted away (Schwartz and Watson (2000)), and that part of the function of lawyers is to influence courts and obtain rents is reasonably realistic (Cooter and Rubinfeld (1989), Glaeser and Shleifer (2001)). However, the specific details of conflict are extremely complex, and thus our paper relies on a flexible, parameterized "black box."[8] To "fight" for X, both creditors and management can devote effort. The exact allocation of X to management (equity) is determined by a *contest success function*. For example, equity may grab fraction α_e of the contested amount X if equity spends L_e on debt collection avoidance/influence-seeking and debt spends L_d on debt collection/influence-seeking. We specify in

$$\alpha_e(L_e, L_d) = \frac{L_e^\lambda}{L_d^\lambda + L_e^\lambda} = \frac{1}{1 + ({}^{L_d}/{}_{L_e})^\lambda} = \alpha({}^{L_d}/{}_{L_e}) \qquad . \tag{1}$$

λ (≥ 0) is a parameter which measures the relative effect of disproportionate spending, e is a mnemonic for equity, d for debt, and $\alpha(x) \equiv {}^1/{}_{(1 + x^\lambda)}$. Our context success function is a *ratio* function, as discussed in Hirshleifer (1989). The fraction not allocated to equity, $\alpha_d = 1 - \alpha_e$, goes to the creditors:

$$\alpha_d(L_e, L_d) = 1 - \alpha_e(L_e, L_d) = \frac{L_d^\lambda}{L_d^\lambda + L_e^\lambda} = \frac{1}{1 + ({}^{L_e}/{}_{L_d})^\lambda} = \alpha({}^{L_e}/{}_{L_d}) \qquad . \tag{2}$$

When $\alpha({}^{L_e}/{}_{L_d}) = 1$, the absolute priority rule prevails. When $\alpha({}^{L_e}/{}_{L_d}) < 1$, APR is violated in favor of equity. α can be considered as a probability of holding onto APR, or as a fraction of the disputed amount X that is allotted to debt in financial distress, or both. The combination of a parametrized X with a contest success

[8] Similar simplifying functional forms about underlying values and monitoring, as well as similar assumptions about an inability to write complete contracts are often made in the monitoring literature.

function can cover a wide range of possible allocation scenarios.

Because legal effort goes beyond pure litigation, and extends to such out-of-court activities as settlement negotiations and "fact finding," we adopt the term "lobbying" instead of "litigating" in this paper. Both equity and creditors are assumed to pay for their own lobbying expenses.[9] One unit of lobbying (collection) costs c_d for creditors, c_e for equity. As is the law, creditors in the same class must be treated equally.[10] Neither management nor creditors can commit not to act opportunistically in case of financial distress. Capital markets are perfectly competitive, the firm is acting strategically. All participants are risk-neutral optimizers, and there is no asymmetric information in the financial distress game. (Any asymmetric information is assumed to be fully captured by the known contest success function. We will introduce a signaling component later in the paper.)

B The Financial Distress Game

B.1 The Creditors' Problems

First consider the problem of a single among N creditor if the firm enters financial distress. Under full APR, he receives V_{Old}/N, because $V_{\text{New}} = 0$ and this creditor has first claim to the remaining firm's assets, which are assumed to be insufficient to cover the required investment. Under maximum violation, he receives $V_{\text{Old}}/N - X/N$. He benefits from both his own lobbying, denoted l_d, and the lobbying of other

[9]The insights of this paper are largely unaffected if the firm reimburses creditors and management for their legal costs (as in Chapter 11). This arrangement defacto subsidizes the legal efforts of lower-priority claimants from higher-quality claimants. However, the algebra becomes substantially more complex. See also Welch (1997).

[10]If one were to allow creditors to compete with one another to collect from a limited amount of funds, and management would pay off the loudest claimants in the same class but leave other claimants dry, free-riding of creditors on one another would be mitigated. However, there would then be a conflict game among creditors, and perhaps even a "run" (Diamond and Dybvig (1983)) on the firm's assets. We focus on our simpler model only.

creditors, denoted l_o. (Recall that $\alpha\left[{}^{L_e}/_{(l_d+l_o)}\right] = \alpha_d$ is the fraction of X accruing to *creditors*.) Thus, one single creditor maximizes with respect to l_d

$$\alpha\left[{}^{L_e}/_{(l_d+l_o)}\right]\cdot\left(\frac{V_{\text{Old}}}{N}\right)+\left(1-\alpha\{{}^{L_e}/_{(l_d+l_o)}]\right\}\cdot\left(\frac{V_{\text{Old}}}{N}-\frac{X}{N}\right)-c_d l_d$$

$$=\frac{V_{\text{Old}}-\left\{1-\alpha\left[{}^{L_e}/_{(l_d+l_o)}\right]\right\}X}{N}-c_d l_d \qquad . \tag{3}$$

This creditor's first-order condition is

$$-\frac{L_e X\alpha'\left[{}^{L_e}/_{(l_d^\star+l_o)}\right]}{N(l_d^\star+l_o)^2}=c_d \qquad . \tag{4}$$

Note that all creditors are equal. Thus, a minimal equilibrium condition is that $l_o^\star=(N-1)l_d^\star$ and aggregate creditor collection effort is $L_d^\star=N\cdot l_d^\star$.

B.2 The Management's Problem

Unlike creditors, management does not suffer from a free-riding problem. Under APR, management receives 0. The entrepreneur maximizes with respect to L_e in financial distress (i.e., $V_{\text{Old}}-I+(1-\pi)V_{\text{New}}$ are sunk costs, and we are only investigating the bankruptcy payoffs, which occurs with probability π):

$$\alpha\left({}^{L_e}/_{L_d}\right)\cdot 0+\left[1-\alpha\left({}^{L_e}/_{L_d}\right)\right]\cdot X-c_e L_e \tag{5}$$

Her first-order condition is

$$-\frac{X\alpha'\left[L_e^\star/L_d\right]}{L_d}=c_e \qquad . \tag{6}$$

B.3 The Joint Solution

Solving the two first order conditions, we find that the in-distress equilibrium choices are

$$L_e^\star = -\frac{\alpha' c_d N X}{c_e^2} \qquad l_d^\star = -\frac{\alpha' X + c_e l_o}{c_e} \tag{7}$$

Using $L_d^\star = N \cdot l_d^\star$, it follows that

$$N = \frac{c_e L_e^\star}{c_d L_d^\star} \quad . \tag{8}$$

Thus, if there is one creditor and the fighting costs are equal, both debt and equity will fight equally hard for X. However, if there are two creditors, management will spend twice as much as debt on lobbying effort in equilibrium. This is due to the free-riding among creditors in spending money on claims collection.

Equation 8 further allows us to replace the endogenous choice variables in the contest success function *in equilibrium* with known parameters. Define

$$K \equiv N\left(\frac{c_d}{c_e}\right) \quad . \tag{9}$$

so $L_e^\star / L_d^\star = K$. (If we assume that creditors are not intrinsically better at fighting than management, i.e., if $c_d \geq c_e$, then $K \geq 1$.) Consequently

$$\alpha\left[L_e^\star/(l_d^\star + l_o^\star)\right] = \alpha\left[L_e^\star/L_d^\star\right] = \alpha(K) \quad . \tag{10}$$

where $\alpha(K)$ is the allocation fraction favoring of creditors. (A higher K means a lower α, which makes creditors happier.)

We shall now return from arbitrary ratio functions back to the specific ratio function in eq. 1. Substituting eq. 10 back into the first-order conditions eqs. 6 and 4, we can eliminate α to find that

$$L_e^\star = \left[\frac{\lambda K^\lambda}{c_e(1+K^\lambda)^2}\right] \cdot X \tag{11}$$

and

$$L_d^\star = \left[\frac{\lambda K^{\lambda-1}}{c_e(1+K^\lambda)^2}\right] \cdot X \qquad . \tag{12}$$

In equilibrium, deadweight waste W is

$$W^\star(N) \equiv c_d L_d^\star + c_e L_e^\star = \left[\frac{\lambda(N+1)K^\lambda}{N(1+K^\lambda)^2}\right] \cdot X \qquad . \tag{13}$$

Asymptotically, the waste in this conflict game is smaller when there are significant asymmetries in strength between the debt and equity contestants, i.e., when K (which itself embeds N!) and λ are large. Here, creditors are weakest when their number is high. Thus, a very large number of creditors can drive in-equilibrium conflict costs to zero. This also implies that it is sufficient if c_d is not much less than c_e, i.e., as long as creditors are not intrinsically better fighting for the potentially reallocable component X than management and thus $K \geq 1$, waste is lower when N increases above 1.[11] This enhances asymmetry. Although the exact derivative of waste with respect to N depends on the ratio between c_d and c_e (see subsection C), asymptotically, as $N \to \infty$, waste $W^\star(N) \to 0$.

As in all models of competitive credit provision, the entrepreneur internalizes these ex-post waste costs in equilibrium. Thus, without any other considerations which could induce the entrepreneur into restricting the number of creditors, having as many creditors as possible maximizes the entrepreneur's firm value.[12]

[11]Remember that $K \equiv N^{c_d}/_{c_e}$. Not too surprisingly, the comparative statics discussed below can reverse *locally* if $c_d << c_e$, i.e., if creditors were far more efficient at producing a unit of legal influence than management. Doubling the number of creditors in such situations can drive $c_d \cdot N$ close to c_e.

[12]We are ignoring the side condition management that the entrepreneur may have to sell more than 100% of the firm to raise the necessary credit.

C The Ex-Ante Price of Debt

To obtain credit of I, which is assumed necessary to finance the project, an entrepreneur has to offer debt face value FV that satisfies

$$I = \pi \cdot [\alpha^\star V_{\text{Old}} + (1-\alpha^\star)(V_{\text{Old}} - X) - c_d L_d^\star] + (1-\pi) \cdot FV^\star \quad , \tag{14}$$

where $\alpha^\star \equiv \alpha(L_e^\star / L_d^\star) = {}^1/[1+K^\lambda] = {}^1/[1+(Nc_d/c_e)^\lambda]$ is the in-equilibrium fraction of X that creditors expect to receive and $L_d^\star$ is given in eq. 12. The first term is the expected payoff to creditors in bankruptcy, the second term is the promised payoff to creditors outside of bankruptcy. In bankruptcy, the claimants can recover V_{Old}, the assets in place (because the value of the new project V_{New} is worthless), net of their in-equilibrium reduction due to managerial ex-post opportunism and net of their own fighting costs. We also assume that $FV^\star \le V_{\text{Old}} + V_{\text{New}}$, so that the firm is able to pay off the debt in the non-bankrupt state.[13]

Solving for $FV^\star$, the in-equilibrium solution for the face value of debt, is

$$FV^\star = \frac{I + \pi \left\{ K^\lambda X \left[\frac{1}{1+K^\lambda} + \frac{\lambda}{N(1+K^\lambda)^2} \right] - V_{\text{Old}} \right\}}{1-\pi} \quad . \tag{15}$$

Note that, given our formulation, if X is very large and $\alpha^\star \to 0$, the term "creditor" is almost a misnomer. In financial distress, such creditors would not receive very much at all. They would effectively be more of a residual claimant than equity itself.

D The Ex-Ante Entrepreneur's Problem

The Entrepreneur is maximizing equity's value (E) today, i.e.

$$E \equiv \pi \cdot [\alpha^\star \cdot 0 + (1-\alpha^\star) \cdot X - c_e L_e^\star] + (1-\pi) \cdot (V_{\text{Old}} + V_{\text{New}} - FV^\star) \tag{16}$$

[13] It is sufficient if $\lambda < \overline{\lambda}$, where $\overline{\lambda}$ is defined by $K^{\overline{\lambda}}(\overline{\lambda} - 1) < 1$. $\overline{\lambda}$ is usually a high number, so this constraint is rarely an issue.

with respect to the number of creditors. A quick check shows that in financial distress, $E + I = V_{\text{Old}} - W^{\star}(N)$; if the project is successful, $E + I = V_{\text{Old}} + V_{\text{New}}$.

The first-order condition of E with respect to N is a long algebraic expression, but it is easier to derive the sign of the comparative statics from the insight that entrepreneurs internalize all waste in a competitive capital market, i.e., from eq. 13 :

$$E^{\star} = V_{\text{Old}} + [-I + (1-\pi) \cdot V_{\text{New}} - \pi \cdot W^{\star}(N)] \quad . \tag{17}$$

As $N \to \infty$, $E^{\star}$ converges to the first-best $V_{\text{Old}} - I + (1-\pi)V_{\text{New}}$. The interesting comparative statics are

$$\frac{\partial E^{\star}}{\partial X} = -\pi \left[\frac{\lambda K^{\lambda}(N+1)}{(1+K^{\lambda})^2 N} \right] < 0 \quad , \tag{18}$$

$$\frac{\partial E^{\star}}{\partial K} = \pi \left[\frac{\lambda^2 K^{\lambda-1}(K^{\lambda}-1)(N+1)}{(1+K^{\lambda})^3} \right] \left(\frac{X}{N} \right) > 0 \quad , \tag{19}$$

$$\frac{\partial E^{\star}}{\partial N} = \frac{\partial E^{\star}}{\partial K} \cdot \frac{\partial K}{\partial N} > 0 \quad , \tag{20}$$

$$\frac{\partial E^{\star}}{\partial ({}^{c_d}/_{c_e})} = \frac{\partial E^{\star}}{\partial K} \cdot \frac{\partial K}{\partial ({}^{c_d}/_{c_e})} = \frac{\partial E^{\star}}{\partial K} \cdot N > 0 \quad , \tag{21}$$

$$\frac{\partial E^{\star}}{\partial \lambda} = \pi \left[\frac{K^{\lambda}(N+1)\left[\lambda(K^{\lambda}-1)\log(K) - (K^{\lambda}+1)\right]}{(1+K^{\lambda})^3} \right] \left(\frac{X}{N} \right) \quad . \tag{22}$$

These equations state that the entrepreneur is better off if when $c_d >> c_e$, X is small, and N and λ are large. There is no clear comparative static with respect to the decisiveness parameter λ: for small values of K and λ, it can be negative. As either K or λ becomes large, the derivative of $E^{\star}$ with respect to λ is positive.

IV Creditor Concentration and Financial Distress Conflict In a Capital Structure Model: A Signaling Model

Almost all theories of capital structure center around the effects of an increase in the expected costs of bankruptcy (probability of and waste in) when the firm takes on additional debt. Our model is no exception. It merely identifies the deadweight costs of bankruptcy as the waste of socially inefficient claims collection, and it relates this specific cost of debt to the number of creditors.

To obtain an equilibrium in which some firms are willing to incur these financial distress costs in equilibrium, there must also be some advantages to the otherwise disadvantageous debt choice to a finite number of creditors. We now discuss four different mechanisms: signaling (in some detail) in this section; and agency, optimal termination/continuation, and marketing costs in the following section.

A A Revised Model

In the prior model, there was no drawback to the use of multiple creditors. Creditors were maximally expropriated in financial distress, but compensated ex-ante for being ex-post expropriated.

Now, consider the presence of two different kinds of firms: good, high-quality (G) firms with a lower probability of bankruptcy (π_G), and bad, low-quality (B) firms with a higher probability of bankruptcy (π_B).

B Signaling With The Number of Creditors

Signaling works if there is a differentially higher cost for low-quality firms to send the signal. To deter imitation, high-quality firms therefore want to minimize corporate payoffs if they enter financial distress. These payoffs are lower if [a] litigation waste upon financial distress is higher and [b] entrepreneur's relative (post-litigation) share of the firm is lower. Having fewer creditors accomplishes both objectives. Thus, signaling through creditor concentration is a relatively efficient separation mechanism.

We have set up the problem intentionally so that the signaling equilibrium is easy to construct. Because signaling equilibria are well understood, we shall be casual on formal equilibrium definitions, and just focus on the pareto-dominant signaling equilibrium. For the sake of brevity, we shall also treat integer constraints on the number of creditors rather casually.

In a separating equilibrium, the low-quality entrepreneur prefers revelation to imitation. Revelation provides the low-quality entrepreneur with her full-information first-best proceeds of

$$V_{\text{Old}} - I + (1 - \pi_B) V_{\text{New}} \qquad . \tag{23}$$

To achieve this, the entrepreneur would offer highly dispersed (public) debt. Imitation would provide a potentially cheating entrepreneur with

$$\pi_B \cdot [\alpha^\star \cdot 0 + (1 - \alpha^\star) \cdot X - c_e L_e^\star] + (1 - \pi_B) \cdot (V_{\text{New}} + V_{\text{Old}} - \mathsf{FV}_G) \qquad , \tag{24}$$

where the FV_G indicates that an out-of-equilibrium imitating low-quality firm can receive the high-quality firms' price of credit (based on the good firm's distress probability π_G, not the imitator's true distress probability π_B). FV_G is given in eq. 15. A reasonable signaling equilibrium emerges in which the difference in profits between a cheating and a truthful low-quality firm, i.e., the gain from imitation (GFI), are

$$\mathsf{GFI} \equiv \pi_B \cdot [(1 - \alpha^\star) \cdot X - c_e L_e^\star] + (1 - \pi_B) \cdot (V_{\text{New}} + V_{\text{Old}} - \mathsf{FV}_G) - [V_{\text{Old}} - I + (1 - \pi_B) V_{\text{New}}]$$

$$= \overbrace{\left[\left(1 - \frac{\lambda}{1 + K^\lambda}\right) \pi_B - \pi_G \left(1 + \frac{1}{N}\right) \left(\frac{1 - \pi_B}{1 - \pi_G}\right) \right]}^{(-) \text{ in equilibrium}} \cdot \overbrace{\left(\frac{K^\lambda}{1 + K^\lambda}\right) X + \left(\frac{\pi_B - \pi_G}{1 - \pi_G}\right) (I - V_{\text{Old}})}^{(+)}$$

$$= \overbrace{\left(\frac{\pi_B - \pi_G}{1-\pi_G}\right)(I - V_{\text{Old}})}^{(+)} + \frac{K^\lambda X}{1+K^\lambda} \overbrace{\left[\pi_B - \pi_G\left(\frac{1-\pi_B}{1-\pi_G}\right) - \left(\frac{\lambda}{1+K^\lambda}\right)\left(\pi_B + \frac{\pi_G}{N}\cdot\frac{1-\pi_B}{1-\pi_G}\right)\right]}^{\text{must be } (-) \text{ in equilibrium}} \tag{25}$$

is just below zero. ${}^{\partial\mathsf{GFI}}/_{\partial N}$ is a complex expression (because K embeds N). However, we do know that larger numbers of creditors are preferred when there is no signaling, and the low-quality firm's outcome does not depend on N if it confesses its identity. (The optimal N for revealing bad firms is infinity.)

$$\frac{\partial\mathsf{GFI}}{\partial N} > 0 \tag{26}$$

Thus, a potential low-quality imitator has less to gain from imitation when there are fewer creditors. For the signaling equilibrium to have a solution in which the low-quality firm is indifferent between imitating and not imitating, the right part of eq. 25 must be negative.

C Comparative Statics

For the most part, the comparative statics are messy, but straightforward. The task is easy, because we have constructed the model so that the number of creditors only matters in the cost of sending the signal (N), not in the benefits of the signal. The comparative statics are determined by the incentive compatibility constraint to prevent low-quality firms' imitation. Appealing again to our side condition that $c_d \geq c_e$, the sign of the implicit differentiated $({}^{\partial N^\star}/_{\partial .})$ is the opposite to the sign of $({}^{\partial\mathsf{GFI}}/_{\partial .})$. (Using the implicit function theorem, ${}^{\partial N^\star}/_{\partial .} = -{}^{\partial\mathsf{GFI}}/_{\partial .}/{}^{\partial\mathsf{GFI}}/_{\partial N}$. Consequently, $sign({}^{\partial N^\star}/_{\partial .}) = -sign({}^{\partial\mathsf{GFI}}/_{\partial .})$.)

New Opportunities Value V_{New}: V_{New} is irrelevant.

$$\frac{\partial\mathsf{GFI}}{\partial V_{\text{New}}} = 0 \quad \Rightarrow \quad \frac{\partial N^\star}{\partial V_{\text{New}}} = 0 \tag{27}$$

__**Preexisting Firm Value V_{Old}**__**:** If V_{Old} is high relative to I, high-quality firms need to sell little debt and prefer to simply wait instead. Thus, imitation is relatively less attractive, and N can be larger. Formally,

$$\frac{\partial \mathsf{GFI}}{\partial V_{\text{Old}}} = \frac{\pi_G - \pi_B}{1 - \pi_G} < 0 \qquad \Rightarrow \qquad \frac{\partial N^\star}{\partial V_{\text{Old}}} > 0 \tag{28}$$

Thus, $N^\star$ increases when firms have more assets in place (V_{Old}).

__**New Opportunities Cost I**__**:** If V_{Old} is high relative to I, high-quality firms need to sell little debt and prefer to simply wait instead. Thus, imitation is relatively less attractive, and N can be larger. Formally,

$$\frac{\partial \mathsf{GFI}}{\partial I} = \frac{\pi_B - \pi_G}{1 - \pi_G} > 0 \qquad \Rightarrow \qquad \frac{\partial N^\star}{\partial I} < 0 \tag{29}$$

$N^\star$ decreases when firms have to raise more money to take the project (I).

__**Disputable Amount X**__**:** The signaling schedule requires a fixed cost to potential imitators. If X is large, the fixed imitation prevention cost can be achieved with more creditors. Formally, X appears only as a factor in the term that must be negative so that $\mathsf{GFI}^\star = 0$. Consequently

$$\frac{\partial \mathsf{GFI}}{\partial X} = \left[\left(1 - \frac{\lambda}{1 + K^\lambda}\right) \pi_B - \pi_G \left(1 + \frac{1}{N}\right) \left(\frac{1 - \pi_B}{1 - \pi_G}\right) \right] \cdot \left(\frac{K^\lambda}{1 + K^\lambda}\right) < 0$$

$$\Rightarrow \qquad \frac{\partial N^\star}{\partial X} > 0 \tag{30}$$

Firm Bankruptcy Probabilities π_G and π_B: If good firms never go bankrupt, $\pi_G = 0$, good firms know they must never bear ex-post negotiations, so the lowest number of creditors imposes the highest cost on bad-quality firms. Alternatively, when π_G is close to π_B, bad firms have little incentive to imitate (there is little to be gained), and thus N can be large.

Formally, if the difference between firm types tends to zero, we know that GFI could not be positive (N here can be finite):

$$\pi_B = \pi_G = \pi \Rightarrow \mathsf{GFI} = -\left(\frac{K^\lambda X}{1+K^\lambda}\right)\left(\frac{\lambda}{1+K^\lambda}\right)\left(\frac{N+1}{N}\right)\pi \leq 0$$

Therefore, for $\mathsf{GFI}^\star = 0$, for $\pi_B > \pi_G$, it must be that the gains to imitation decrease in the probability of bankruptcy for the good firm ($^{\partial \mathsf{GFI}}/_{\partial \pi_G} < 0$), and increase in the probability that the bad firm goes bankrupt ($^{\partial \mathsf{GFI}}/_{\partial \pi_B} > 0$). After all, GFI is a monotonic function of both π_B and π_G for $0 \leq \pi_B < 1$, $0 \leq \pi_G < 1$. Consequently

$$\Rightarrow \qquad \frac{\partial N^\star}{\partial \pi_G} > 0, \qquad \frac{\partial N^\star}{\partial \pi_B} > 0 \qquad . \tag{31}$$

Effective Entrepreneurial Advantage K: If resistance is futile (too costly for creditors), then high-quality firms find it more difficult to obtain enough creditor resistance to induce the low-quality firm not to follow. Thus, to signal, a high-quality firm needs to choose even fewer creditors when $^{c_d}/_{c_e}$ and thus K is large.

Formally, the partial derivative $^{\partial \mathsf{GFI}}/_{\partial K}$ is positive. First note that because $K \geq 1$ ($c_d \geq c_e$ and $N \geq 1$), then $K^\lambda \geq 1$, $\forall \lambda \geq 0$. Therefore

$$\begin{aligned}\frac{\partial \mathsf{GFI}}{\partial K} \;=\; & \left[\pi_B - \pi_G\left(\frac{1-\pi_B}{1-\pi_G}\right) - \left(\frac{\lambda}{1+K^\lambda}\right)\left(\pi_B + \frac{\pi_G}{N}\cdot\frac{1-\pi_B}{1-\pi_G}\right)\right]\left[\frac{\partial\left(\frac{K^\lambda X}{1+K^\lambda}\right)}{\partial K}\right] \\ & + \left(\frac{K^\lambda X}{1+K^\lambda}\right)\left(\pi_B - \frac{\pi_G}{N}\cdot\frac{1-\pi_B}{1-\pi_G}\right)\left[\frac{\lambda K^{\lambda-1}}{(1+K^\lambda)^2}\right]\end{aligned} \tag{32}$$

And $\partial\left[(K^\lambda X)/(1+K^\lambda)\right]/\partial K = \left[\lambda K^{\lambda-1}/(1+K^\lambda)^2\right] X$. Therefore

$$\begin{aligned}
\frac{\partial \text{GFI}}{\partial K} &= \left[\pi_B - \pi_G\left(\frac{1-\pi_B}{1-\pi_G}\right) - \left(\frac{\lambda}{1+K^\lambda}\right)\left(\pi_B + \frac{\pi_G}{N}\cdot\frac{1-\pi_B}{1-\pi_G}\right)\right]\left[\frac{\lambda K^{\lambda-1}}{(1+K^\lambda)^2}\right] X \\
&\quad + \left(\frac{K^\lambda X}{1+K^\lambda}\right)\left[\frac{\lambda K^{\lambda-1}}{(1+K^\lambda)^2}\right]\left[\pi_B + \frac{\pi_G}{N}\cdot\left(\frac{1-\pi_B}{1-\pi_G}\right)\right] \\
&= \left[\frac{\lambda K^{\lambda-1}}{(1+K^\lambda)^2}\right]\left[\pi_B - \pi_G\left(\frac{1-\pi_B}{1-\pi_G}\right) + \frac{\lambda(K^\lambda - 1)}{1+K^\lambda}\left(\pi_B + \frac{\pi_G}{N}\cdot\frac{1-\pi_B}{1-\pi_G}\right)\right] X > 0 \quad (33)
\end{aligned}$$

because $\pi_B > \pi_G\left[(1-\pi_B)/(1-\pi_G)\right] \geq \pi_G/N \cdot (1-\pi_B)/(1-\pi_G)$, and $K^\lambda \geq 1$. Thus it follows that

$$\frac{\partial N^\star}{\partial K} < 0 \quad \Rightarrow \quad \frac{\partial N^\star}{\partial c_e} > 0 \quad \frac{\partial N^\star}{\partial c_d} < 0 \quad . \tag{34}$$

Fighting Decisiveness λ: The sign with respect to λ is ambiguous, as it was in the no-signaling comparative static eq. 22.

These comparative statics should be unsurprising to connoisseurs of signaling models. They are determined by the self-punishing mechanisms necessary to deter low-quality imitation.

D Signaling By Debt Pricing And Debtor Concentration

When separation by choice of creditors is insufficient, entrepreneurs may have to underprice their debt, i.e., pay a relatively high interest rate. Interestingly, this has a direct implication: Even though the *required* yields on highly concentrated bank debt can be lower than those on dispersed public debts (to allow for banks superior ability to defend their APR), banks earn excess rents (positive expected returns) from their loans made. This is not to purchase bank services, but to assure separation.[14]

[14] Necessarily, we would expect competitive banks to compete these rents away (e.g., through higher fixed costs).

Proposition 1 *When firms can use either yields or creditor concentration for signaling, two choices emerge in equilibrium:*

1. *The firm offers fairly priced debt to a creditor base, concentrated or unconcentrated.*
2. *The firm offers good-deal debt to a single concentrated creditor (bank debt).*

In particular, the firm will not offer good-deal debt to public creditors.

The proof is in the appendix. The intuition is that signaling with creditor concentration is the more efficient signal: it inflicts pain when the firm goes bankrupt, which is more likely to happen to a low-quality firm. When the signal is exhausted, i.e. $N = 1$, which we interpret as bank debt, a high-quality firm then must pay a higher price for credit to separate. High bank debt interest rates do not arise from credit-rationing or poor quality or the purchase of monitoring services, but instead from high-quality, high uncertainty, and the need to separate from other firms! Naturally, in real life, banks probably both monitor and permit signaling.

E A Numerical Illustration of The Signaling Model

For easy checking, Table 1 contains some numerical values that help gathering intuition. We use as parameters $X = \$80$, $I = \$100$, $V_{\text{Old}} = 80$, $V_{\text{New}} = \$250$, $c_d = c_e = \$1$, $\lambda = 1.5$, $\pi_B = 0.5$, and $\pi_G = 0.4$. Note that, under full APR violation (which happens with probability $1 - \alpha^\star$), creditors are fully expropriated by equityholders.

In the non-signaling case, suppose there is only one type of firms, that is $\pi = 0.5$. The optimal solution is for the firm to set $N = \infty \Rightarrow \alpha^\star = 0$. The face value of the debt equals $\mathsf{FV} = \$200$, and $L_d = L_e = 0$. Profits to equityholders are $E^\star = \pi X + (1 - \pi)(V_{\text{Old}} + V_{\text{New}} - \mathsf{FV}) = \$105$, which equals the full information value $V_{\text{Old}} + (1 - \pi)V_{\text{New}} - I$ (after paying off creditors).

Suppose this was the bad firm ($\pi_B = 0.5$), and there is a good firm in the market, with $\pi_G = 0.4$ now. Its full information value would be $130 (see Table 1). The high-quality firm prefers to have a lower number of creditors. Its optimal $N^\star$ solving eq. 25 is $N^\star = 2.26$ ($\simeq 2$). This is costly, because if the high-quality firm goes bankrupt, $L_d = \$9.31$, $L_e = \$21.06$, and $\alpha^\star = 0.22$. However, because the firm borrows from a fewer number of creditors, $FV = \$160.76$ (creditors know now that they will recover more in the bankrupt state). Consequently, the high-quality firm's equity value is $E^\star = \$117.84$. The difference between this amount, and the full information value of the good firm, \$12.15, is the signaling cost. Note that, by imitating the good firm, the bad firm would be worth

$$\pi_B\left[(1-\alpha^\star)X - c_e L_e\right] + (1-\pi_B)\left[V_{\text{Old}} + V_{\text{New}} - FV^\star\right] = \$105 \qquad ,$$

exactly its full information value. In equilibrium, there are no incentives for the bad firm to imitate the good firm. There is no need for the good firm to signal with the debt yield r, because we know, from Proposition 1, that signaling with creditor concentration alone is preferred.

Suppose instead that the good firm is in fact very good, and $\pi_G = 0.2$. Thus, its full information value is \$180. As in the previous example, the optimal $N^\star$ solves eq. (25), and $N^\star = 0.188$. This is impossible, so the firm must set $N^\star = 1$. There would still be gains from imitation for the bad firm, because by imitating the good firm with $N^\star = 1$, the bad firm pays $\mathsf{FV} = \$122.5$, $\alpha^\star = 0.5$, $L_e = L_d = 30$, and $\mathsf{GFI} = \$3.75$. The good firm needs to additionally increase the debt yield to $r^\star = 6\%$. Now $\mathsf{FV}^\star = \$130$, with $\alpha^\star = 0.5$, $L_e = L_d = 30$ (with one creditor and one entrepreneur, the legal expenses for both are equal if their cost is also equal). The higher face value restrain the bad firm from imitation, since imitation yields

$$0.5\,(\$40 - \$30) + 0.5\,(\$80 + \$250 - \$130) = \$5 + \$100 = \$105 \qquad ,$$

exactly the full information value of the bad firm. For the good firm, however, separation yields

$$0.2\,(\$40 - \$30) + 0.7\,(\$80 + \$250 - \$130) = \$2 + \$140 = \$142 \qquad .$$

This is still lower than its full information value \$180. The cost of signalling has therefore increased to \$38.

Insert Figure 1 here
[In-Signaling-Equilibrium Regions]

Figure 1 shows the two regions for which it is optimal to signal with either N only, or with N and the debt yield r. For $\pi_B > 0.6$, at least the bad firm's (and possibly also the good firm's) project has a negative NPV, so a signaling equilibrium makes no sense. The upward sloping curve solves π_G as a function of π_B in (25), where $N^\star = 1$. $N^\star$ becomes larger as π_B and π_G become closer. When $r^\star > 0$, the debt yield decreases as both probabilities of default become closer.

Insert Figure 1 here
[In-Signaling-Equilibrium Promised Yields]

Figure 2 plots the promised rate of return ($\frac{FV}{I} - 1$) to creditors of the good firm for different levels of creditor concentration. For any value of N to be optimal, we let $\pi_B = 0.5$, and allow π_G to vary. As $N \to \infty$, the face value of the debt tends to \$200, and therefore the promised rate of return tends to \$200/\$100 − 1 = 100%. In the figure, the *expected* yield $r^\star$ would be zero for $N^\star > 1$. For $N^\star = 1$, the yield can range from 0% to 45%.

V Alternative Mechanisms

A An Agency Model Alternative

It would be similarly easy to embed ex-post financial lobbying costs into an agency/monitoring model instead of a signaling model. In such a model, although a single creditor can fight better and thus cause more waste in financial distress than multiple creditors, he would also be assumed to monitor management better. As before, the costs of fewer creditors is more waste in bankruptcy. The benefits of fewer creditors would depend on the functional form determining

$$\overset{(-)}{\frac{\partial V}{\partial N}} = \overset{(+)}{\frac{\partial V}{\partial M}} \cdot \overset{(-)}{\frac{\partial M}{\partial N}} \tag{35}$$

where V is firm value, M is the amount and quality of monitoring, and N is the number of creditors. The in-equilibrium tradeoff would be that fewer creditors would monitor better (increasing firm value and/or reducing the probability of distress), but also cost more in financial distress.

Depending on the functional specification of agency benefits, an interior solution could emerge in which firms more in need of monitoring would be more willing to live with fewer creditors. The comparative statics are straightforward:

1. $N^\star$ decreases (facilitating more monitoring) when the value to monitoring ($^{\partial V}/_{\partial M}$) is high.

2. $N^\star$ increases (facilitating less monitoring) when it is difficult to influence the aggregate monitoring by choosing the number of creditors ($^{\partial M}/_{\partial N}$).

As is typical, many of the comparative statics of signaling models are different from those of agency models—but with both theories relying on variables that are difficult to observe (or which by definition *must* be unobservable), it is often difficult to empirically distinguish between the two.

B An Optimal Continuation Alternative

In Bolton and Scharfstein (1996), the costs of more creditors is mutual free-riding when it comes to the ex-ante efficiency of the ex-post optimal continuation choice (terminate if termination is optimal; continue if continuation is optimal). The fighting tradeoff considered in our paper could also be embedded in such a model. If a single creditor were able to wrestle the firm from management in financial distress, he could internalize the continuation choice. Thus, fewer creditors would mean more claims collection waste, but better continuation choice. The comparative statics would depend on the creditors' ability to internalize the continuation choice. (Empirical evidence to the continuation/termination choice can be found in Kahl (2001) and Andrade and Kaplan (1998).)

In contrast to Bolton and Scharfstein (1996), however, fewer creditors obtain more, not fewer resources in financial distress in our model. Consequently, the tradeoffs are different: creditors intrinsically prefer coalescing, management is ambivalent. In Bolton and Scharfstein (1996), creditors intrinsically prefer dispersing, and management has to try to prevent this to preserve firm value.

C Marketing Costs

It requires little explanation to point out that finding multiple creditors may, *under some circumstances*, require more effort than finding a single creditor. Multiple creditors may require more shoeleather and road shows than a single creditor or bank. A single creditor may also find it easier to conduct the normal due diligence than many dispersed creditors. To raise the same amount of funding might thus be costlier through multiple creditors. If this assumption about costs of finding creditors is correct, the model predicts that entrepreneurs trade off the marketing costs of more creditors against the waste costs of fewer creditors.

VI Implications

The intent of our approach is not to stress the monitoring, signaling, continuation or transaction cost aspects. These have already been treated in other literature and thus are probably familiar to the academic reader. Instead, our intent is to stress the consequences of the assumption that spending money on lobbying/lawyers can better one's position in financial distress.

Insert Table 2 here
[Comparison With Bolton and Scharfstein (1996) Model of Creditor Concentration.]

The main intuition and empirical implication of our approach are summarized in Table 2: Dispersed creditors are at a relative disadvantage in financial distress. Coordination and free-riding costs among creditors allow the firm to escape some of their contractual obligations. Put succinctly, concentrated and/or coordinated creditors spend more on lobbying and representation than dispersed creditors. Thus, they receive a better settlement than unconcentrated debt would. To respond to concentrated debt, management also spends more on lobbying in equilibrium. The more creditors, the more management spends *relative* to the aggregate creditor legal expenses, but the less management spends in absolute terms. Aggregate waste is lower when creditors are dispersed.

Our model also predicts that there are incentives for creditors to concentrate *ex-post*. Concentration enhances creditors' bargaining power. It is true that this implications can be generated within other contexts. For example, in Bolton and Scharfstein (1996), it could be that creditors could choose to concentrate in order to avoid inefficient liquidation. But, under other parameters, this theory could suggest that creditors could choose to diffuse in order to avoid inefficient continuation. In this sense, our own implication is more robust.

Holding the amount of debt constant, firms which choose a low creditor concentration *ex-ante* do so in order to later have an ability to expropriate them *ex-post*. Thus, small creditors demand a promised premium for offering credit to such firms. Bank debt requires less of a promised premium. However, in the signaling variant of our model, banks also may earn positive rents, which is required for the high-quality firm to separate.

The implications of our model are relatively sharp, and different from those in which diffuseness helps creditors (Bolton and Scharfstein (1996)). As already mentioned in the introduction, we believe that small credit (such as trade credit) and debt that is uncoordinated by a strong bond trustee are the prime applications for the model.

A Existing Empirical Evidence

Naturally, there are no tests for our theory, yet. Indeed, with the exception of our pilot study above, we failed to find evidence relating APR violations to creditor concentration. But we did find evidence relating the duration of the workout period to creditor concentration: Helwege (1999) analyzes junk bond defaults in the 1980s. Her abstract summarizes the findings relevant to us: "bondholder holdouts are not a significant problem, as firms with proportionately more bonds have shorter default spells...bargaining problems arising from contingent liabilities, lawsuits, and size delay the process, although multiple bond classes do not. Neither information problems nor firm value appear to matter." Of course, time holdout is not necessarily a good measure of settlement, although it is indicative of the presence of shareholder holdout issues. The lack of time holdout is thus not necessarily a smoking gun, but it is hint that creditor holdout issues are perhaps not too important.

Brunner and Krahnen (2001) find that creditor pools increase the probability of workout success, and that coordination costs are higher when there are more creditors. However, they do not determine if such pools improve or worsen the settlements obtained by creditors.

Hotchkiss and Mooradian (1997) find that vulture investors became more prominent in the 1980's and 1990's. These vulture investors serve many roles, and not all of them are proactive. Still, the paper suggests that vultures can enhance not only their own claims (both for their class and for themselves), but also the firm's overall value, by actively pressuring management. The very fact that active vultures purchase large blocks in financial distress, often from dispersed claimants, seems to indicate at least that the claimant's loss of bargaining power may not be drastic and/or outweighed by the creditors' gains from "undispersing" themselves.

There is some disagreement as to the extent of the direct costs of financial distress. The order of magnitude of direct court-filed fees are about 2-4% of the value of assets (20% of the market value of equity), depending on whether one includes costs of failed workouts and exchange offers, etc. Lubben (2000) reviews the evidence and provides some new evidence. In a sample including mostly mid-size companies, debtor's expenses for attorneys tend to be about $680,000 (mean), $300,000 (median). Adding accountants and investment bankers roughly doubles these figures. Creditor Committees spend about $230,000 (mean), $70,000 (median). Accountants and investment bankers add only about 50 percent more. (If we use these figures to calibrate our model, this implies an effective creditor diffuseness of about $N = 3$.) However, in "ten [of 22] cases in the sample the United States Trustee was unable to appoint creditors' committee, most often because of lack of interest among unsecured creditors" [p.530]. The article also points out cases in which businesses misjudged the difficulties of complying with code requirements, and thus were denied reimbursements for their claims; and lack of understanding of and frustration with the Bankruptcy Code by businessmen.

Although we believe that financial contracts can be and often are written in a way to mitigate legal costs, there are instances in which priority changes unexpectedly. In such cases, one can get an indication of the (usually) out-of-equilibrium costs of litigation. Anderson (1987, p.442) describes the Manville asbestos experience, in which customers unexpectedly received priority over creditors in bankruptcy

> An Institute for Civil Justice–Rand Corp study estimates that for every dollar paid to injured claimants, nearly two dollars are spent on litigation expenses. More specifically, of the total amount paid by producers and insurers, 37 percent was received by plaintiffs, 26 percent by plaintiffs' attorneys, and 37 percent was spent by producers and insurers on defense costs.

Even though our model has situations in which (high-quality) issuers like higher litigation costs in order to deter low-quality issuers, a calibration of our model indicates that we would not expect to see such legal costs in equilibrium. Indeed, for the most part, our model predicts relatively moderate expenses and only some APR violations. The Manville experience is supportive of our argument only insofar as it indicates that out-of-equilibrium legal costs can be quite significant, and that observed legal costs may be small *by intent*, i.e., by choice of the mechanisms considered in our and other papers.

In sum, although there is little evidence that directly relates creditor (claimant) concentration and coordination to the ultimate settlement (except that presented above in our paper), there is good evidence that creditors often coalesce in financial distress. Although coalition-forming could be (partially) to avoid a "creditor run on the firm" and to enhance firm value, it is also possible that this coalition-forming serves to increase (not decrease) the relative bargaining strength of creditors.

VII Conclusion

Our paper has reexamined the question posed in the Bolton and Scharfstein (1996) JPE paper regarding the optimal number of creditors. We have taken an alternative approach to offer an intuition and a set of implications that differ from those of our predecessor. Thus, empiricists can easily test Bolton and Scharfstein (1996) against our own theory: For one, our mutual alternative is not just an unspecified straw man. For two, creditor concentration is a relatively easily empirically accessible variable. Our pilot study has found preliminary evidence that creditor concentration helps rather than hurts.

Although banks are undoubtedly unique among many dimensions, the fact that bank credit is typically very concentrated is among its more unusual features. A variant of our model offered the specific implication that even though promised yields on bank debt may be higher or lower than comparable public debt, bank debt may offer (single) banks excess rents.

Again, even among public creditors, concentration and coordination measures are relatively easy to obtain. We thus hope that our theory will be put to further empirical work—and preferably by third parties which are less likely to be suspected of trying to find evidence in support of their own model.

References

Anderson, D. R., 1987, "Financing Asbestos Claims: Coverage Issues, Manville's Bankruptcy and the Claims Facility," *Journal of Risk and Insurance*, 54(3), 429–451.

Andrade, G., and S. N. Kaplan, 1998, "How Costly is Financial (Not Economic) Distress? Evidence from Highly Leveraged Transactions that Became Distressed," *The Journal of Finance*, 53(5), 1443–1493.

Bergloef, E., G. Roland, and E.-L. von Thadden, 2000, "An Incomplete Contracts Approach to Bankruptcy," working paper, CEPR.

Bernardo, A., E. Talley, and I. Welch, 2000, "A Theory of Legal Presumptions," *Journal of Law, Economics, and Organization*, 16(1), 1–49.

Bernardo, A. E., and E. L. Talley, 1996, "Investment Policy And Exit-Exchange Offers Within Financially Distressed Firms," *The Journal of Finance*, 51(3), 871–888.

Biais, B., and C. Gollier, 1997, "Trade Credit and Credit Rationing," *Review of Financial Studies*, 10(4), 903–937.

Bolton, P., and D. S. Scharfstein, 1996, "Optimal Debt Structure And The Number Of Creditors," *Journal of Political Economy*, 104(1), 1–25.

Brunner, A., and J. P. Krahnen, 2001, "Corporate debt restructuring: Evidence on Lender Coordination in Financial Distress," working paper, Humboldt Universität and Goethe-Universität of Frankfurt.

Cooter, R. D., and D. L. Rubinfeld, 1989, "Economic Analysis of Legal Disputes and Their Resolution," *Journal of Economic Literature*, 27, 1067–1097.

Diamond, D. W., and P. H. Dybvig, 1983, "Bank Runs, Deposit Insurance, And Liquidity," *Journal of Political Economy*, 91(3), 401–419.

Gertner, R., and D. Scharfstein, 1991, "A Theory of Workouts and The Effect of Reorganization Law," *The Journal of Finance*, 46, 1189–1222.

Glaeser, E. L., and A. Shleifer, 2001, "The Rise of the Regulatory State," working paper 8650, NBER.

Grossman, S. J., and O. D. Hart, 1980, "Takeover Bids, The Free-Rider Problem, And The Theory Of The Corporation," *Bell Journal of Economics*, 11(1), 42–64.

Hege, U., 1997, "Multi-party debt renegotiation and the maturity and seniority of large creditors," working paper, University of Tilburg.

Helwege, J., 1999, "How Long do Junk Bonds Spend in Default," *The Journal of Finance*, 54(1).

Hirshleifer, J., 1978, "Competition, Cooperation, And Conflict In Economics And Biology," *American Economic Review*, 68(2), 238–243.

——— , 1989, "Conflict And Rent-Seeking Success Functions: Ratio Vs. Difference Models Of Relative Success," *Public Choice*, 63(2), 101–112.

——— , 1991, "The Technology Of Conflict As An Economic Activity," *American Economic Review*, 81(2), 130–134.

Hotchkiss, E. S., and R. M. Mooradian, 1997, "Vulture Investors and the market for control of distressed firms," *Journal of Financial Economics*, 43(3), 401–432.

Kahl, M., 2001, "Financial Distress as a selection mechanism: Performance and Survival from the Onset to the Resolution of Financial Distress," working paper, University of California, Los Angeles.

Lubben, S. J., 2000, "The Direct Costs of Corporate Reorganization: An Empirical Examination of Professional Fees in Large Chapter 11 Cases," *American Bankruptcy Law Journal*, 74.

Morris, S., and H. S. Shin, 1998, "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *American Economic Review*, 88(3), 587–597.

——— , 1999, "Coordination Risk and the Price of Debt," working paper, Yale University.

Obstfeld, M., 1996, "Models of Currency Crisis with Self-Fulfilling Currency Attacks," *European Economic Review*, 40, 1037–1047.

Petersen, M. A., and R. G. Rajan, 1997, "Trade Credit: Theories and Evidence," *Review of Financial Studies*, 10(3), 661–691.

Preece, D., and D. J. Mullineaux, 1996, "Monitoring, Loan Renegotiability, and Firm Value: The Role of Lending Syndicates," *Journal of Banking and Finance*, 20, 577–593.

Rajan, R. G., 1992, "Insiders and Outsiders: The Choice between Informed and Arm's Length Debt," *The Journal of Finance*, 47, 1367–1400.

Rajan, R. G., and L. Zingales, 1998, "Power In A Theory Of The Firm," *Quarterly Journal of Economics*, 108(2), 387–432.

Repullo, R., and J. Suarez, 1998, "Monitoring, Liquidation, and Security Design," *Review of Financial Studies*, 10(4), 1203–1236.

Rubinstein, A., 1982, "Perfect Equilibrium In A Bargaining Model," *Econometrica*, 50(1), 97–110.

Schwartz, A., and J. Watson, 2000, "Economic And Legal Aspects Of Costly Recontracting," working paper, Yale Law School.

Welch, I., 1997, "Why Is Bank Debt Senior? A Theory Of Asymmetry And Claim Priority Based On Influence Costs," *Review of Financial Studies*, 10(4), 1203–1236.

Zingales, L., 1995, "Insiders Ownership and the Decision to Go Public," *Review of Economic Studies*, 62(212), 425–448.

A Proof of Proposition 1: Signaling With Debt Pricing and Concentration.

This appendix proves that the firm prefers to use only the number of creditors for signaling, if possible, and uses interest rate signaling only after bumping against the $N = 1$ limit. We need to modify eq. 14 to accommodate non-zero debt yields:

$$I \cdot (1 + r) = \pi_G \left[\alpha^\star V_{\text{Old}} + (1 - \alpha^\star)(V_{\text{Old}} - X) - c_d L_d\right] + (1 - \pi_G)\mathsf{FV}^{\mathsf{NY}} \quad , \tag{36}$$

where the superscript NY on FV reflects the fact that the good firm uses both N and the debt yield as signals. Hence

$$\mathsf{FV}^{\mathsf{NY}} = \frac{I(1 + r) - \pi_G \left[V_{\text{Old}} - (1 - \alpha^\star)X - c_d L_d\right]}{1 - \pi_G} \quad . \tag{37}$$

Separation will occur as long as $\mathsf{GFI}^{\mathsf{NY}} = 0$, where GFI is, from eq. 25:

$$\mathsf{GFI}^{\mathsf{NY}} = \pi_B \left[(1 - \alpha^\star)X - c_e L_e\right] + (1 - \pi_B)(V_{\text{New}} + V_{\text{Old}} - \mathsf{FV}^{\mathsf{NY}}) - \left[V_{\text{Old}} - I + (1 - \pi_B)V_{\text{New}}\right] = 0 \tag{38}$$

Substituting $\mathsf{FV}^{\mathsf{NY}}$ with his value:

$$\begin{aligned}
\mathsf{GFI}^{\mathsf{NY}} &= \pi_B \left[(1 - \alpha^\star)X - c_e L_e\right] \\
&\quad + (1 - \pi_B)\left\{V_{\text{New}} + V_{\text{Old}} - \frac{I(1 + r) - \pi_G \left[V_{\text{Old}} - (1 - \alpha^\star)X - c_d L_d\right]}{1 - \pi_G}\right\} \\
&\quad - \left[V_{\text{Old}} - I + (1 - \pi_B)V_{\text{New}}\right] \\
&= \left[X(1 - \alpha^\star) + I - V_{\text{Old}}\right](\pi_B - \pi_G) - (1 - \pi_B) r I \\
&\quad - \left[\pi_B(1 - \pi_G)c_e L_e + (1 - \pi_B)\pi_G c_d L_d\right] \\
&= 0
\end{aligned} \tag{39}$$

Setting this expression to zero defines the signaling equilibrium $(N^\star, r^\star)$. Solving for $r^\star$ as a function of $N^\star$:

$$r^\star = \frac{\left[X(1 - \alpha^\star) + I - V_{\text{Old}}\right](\pi_B - \pi_G) - \left[\pi_B(1 - \pi_G)c_e L_e^\star + (1 - \pi_B)\pi_G c_d L_d^\star\right]}{(1 - \pi_B)I} \tag{40}$$

$r^\star$ depends on $N^\star$ through $\alpha^\star$, $L_d^\star$, and $L_e^\star$. Substitute the value of $r^\star$ into $\mathsf{FV}^{\mathsf{NY}}$:

$$\mathsf{FV}^{\mathsf{NY}} = \frac{I}{1 - \pi_B} + \pi_B(1 - \alpha^\star)X - \left(\frac{\pi_B}{1 - \pi_B}\right)(V_{\text{Old}} + c_e L_e^\star) \tag{41}$$

Finally, substitute $\mathsf{FV}^{\mathsf{NY}}$ into the expression for E (from equation (16?)):

$$\begin{aligned} E^{\mathsf{NY}} &= \pi_G\left[(1-\alpha^\star)X - c_e L_e^\star\right] + (1-\pi_G)(V_{\text{Old}} + V_{\text{New}} - \mathsf{FV}^{\mathsf{NY}}) \\ &= \left(\frac{\pi_B - \pi_G}{1-\pi_B}\right)\left[c_e L_e^\star - X(1-\alpha^\star)\right] + (V_{\text{Old}} - I)\left(\frac{1-\pi_G}{1-\pi_B}\right) + (1-\pi_G)V_{\text{New}} \end{aligned} \tag{42}$$

In terms of entrepreneurial proceeds, the equilibrium $(\hat{N}, r = 0)$ dominates the equilibrium $(N^\star, r^\star \neq 0)$ defined in eq. 39. This is because E^{NY} increases with $\alpha^\star$, but is indendent of r. Thus, any equilibrium with both signals is dominated by an equilibrium of the type $(N, r = 0)$, as long the latter is feasible (i.e., does not run into the $N = 1$ constraint).[15]

When $N^\star = 1$**:** We now consider when N alone is not sufficient for the firms to separate (i.e., even with $N^\star = 1$). We now show that the firm needs to additionally increase the debt yield to induce separation. To characterize this equilibrium, let us define

$$\alpha^1 = \frac{1}{1 + \left(\frac{c_d}{c_e}\right)^\lambda} \quad , \tag{43}$$

that is, the value of α when $N = 1$. In this case, the entrepreneur offers debt with face value such that:

$$I(1+r) = \pi_G\left[\alpha^1 V_{\text{Old}} + (1-\alpha^1)(V_{\text{Old}} - X) - c_d L_d^1\right] + (1-\pi_G)\mathsf{FV}^{\star\star} \quad . \tag{44}$$

Because $N = 1$,

$$L_d^1 = \frac{\lambda\left(\frac{c_d}{c_e}\right)^{\lambda-1}}{c_e\left[1 + \left(\frac{c_d}{c_e}\right)^\lambda\right]^2} X \quad . \tag{45}$$

Solving for $\mathsf{FV}^{\star\star}$:

$$\mathsf{FV}^{\star\star} = \frac{I(1+r^{\star\star}) - \pi_G\left[\alpha^1 V_{\text{Old}} + (1-\alpha^1)(V_{\text{Old}} - X) - c_d L_d^1\right]}{1-\pi_G} \tag{46}$$

Therefore separation will occur as long as the bad firms find the gains from separation equal to zero.

$$GFI^{\star\star} = \pi_B\left[(1-\alpha^1)X - c_e L_e^1\right] + (1-\pi_B)(V_{\text{New}} + V_{\text{Old}} - \mathsf{FV}^{\star\star}) - \left[V_{\text{Old}} - I + (1-\pi_B)V_{\text{New}}\right] = 0 \tag{47}$$

[15] The single-crossing property also assures us that the high-quality firm prefers to adhere to the equilibrium over pretending that it is a low-quality firm.

The last two equations define $r^{\star\star}$ as a function of the parameters in the model, together with the condition that:

$$GFI = \pi_B\left[(1-\alpha^1)X - c_e L_e^1\right] + (1-\pi_B)(V_{\text{New}} + V_{\text{Old}} - \mathsf{FV}_{r=0,N=1}) - [V_{\text{Old}} - I + (1-\pi_B)V_{\text{New}}] > 0 \quad (48)$$

This equation states that $N = 1$ is insufficient to separate (profits from imitation are greater than zero). That is, separation with N only is not enough, even for $N = 1$.

It is also the case that signaling with $N = 1$ and $r^{\star\star}$ is preferred to signaling with r alone: From eq. 42, $E(1, r^{\star\star}) > E(\infty, r)$, where $E(\infty, r)$ is the value of equity when the firm optimally signals with r alone.

Table 1. Table of Symbols

Symbol	Explanation	Example
V_{Old}	Value of Assets in Place	$V_{\text{Old}} = \$80$
V_{New}	Value of New (Extra) Project in Non-Distress (Zero in distress).	$V_{\text{New}} = \$250$
V	$V_{\text{New}} + V_{\text{Old}}$	$\rightarrow V = \$330$
X	Amount that can be lobbied for in financial distress	$X = \$80$
I	Cost of Extra Project. $V_{\text{Old}} + V_{\text{New}} > I > V_{\text{Old}}$	$I = \$100$
π	Probability of Distress, generic	$\pi = 40\%$
π_G	(Signaling Game:) Probability of Distress for Good Firm.	$\pi_G = 40\%$
π_B	(Signaling Game:) Probability of Distress for Bad Firm.	$\pi_B = 50\%$
λ	Conflict Decisiveness Parameter in α.	$\lambda = 1.5$
c_e	unit cost of lobbying for equity.	$c_e = \$1$
c_d	unit cost of lobbying for debt.	$c_d = \$1$
	Solutions in Signaling Model for High-Quality Firm	
L_e	Lobbying Effort by Equity (Management, Entrepreneur) for X.	$\rightarrow L_e^\star = 21.067$
L_d	Aggregate lobbying effort by all creditors for X.	$\rightarrow L_d^\star = 9.313$
l_d	Lobbying effort by a single creditors for X.	$\rightarrow l_d^\star = 4.117$
l_o	Lobbying effort by other creditors for X.	$\rightarrow l_o^\star = 5.196$
α	Contest Success Function, allocation of X between equity and debt,	$\rightarrow \alpha = 0.227$
$\equiv \alpha(L_e, L_d)$	depending on exerted lobbying effort.	$\alpha(21.067, 9.313)$
N	Number of Creditors (Endogenous Choice Variable).	$\rightarrow N^\star = 2.26$
E	Entrepreneurial Profit	$\rightarrow E^\star(N = 2.26) = 117.85$
FV	Debt Face Value	$\rightarrow \text{FV}^\star(N = 2.26) = 160.76$
GFI	Gains from Imitation in Signaling Model	$\rightarrow GFI^\star = 0$
	(Full Information Value: \$130 $\Rightarrow$ Cost of Signaling: \$12.152	
	Side Conditions	
$(1-\pi)V_{\text{New}} \geq I$	The project is worthwhile.	
$X \leq V_{\text{Old}}$	Only a part of the firm value can be lobbied for.	
$N \in I$	There are no negative or fractional creditors.	
$c_d \geq c_e$	Management's lobbying costs are no higher than creditors'.	
$0 < \lambda < \overline{\lambda}$	Facilitates sensible contest success function.	
	where $\overline{\lambda}$ is defined by $K^{\overline{\lambda}}(\overline{\lambda} - 1) < 1$.	

Figure 1. In-Signaling-Equilibrium Regions

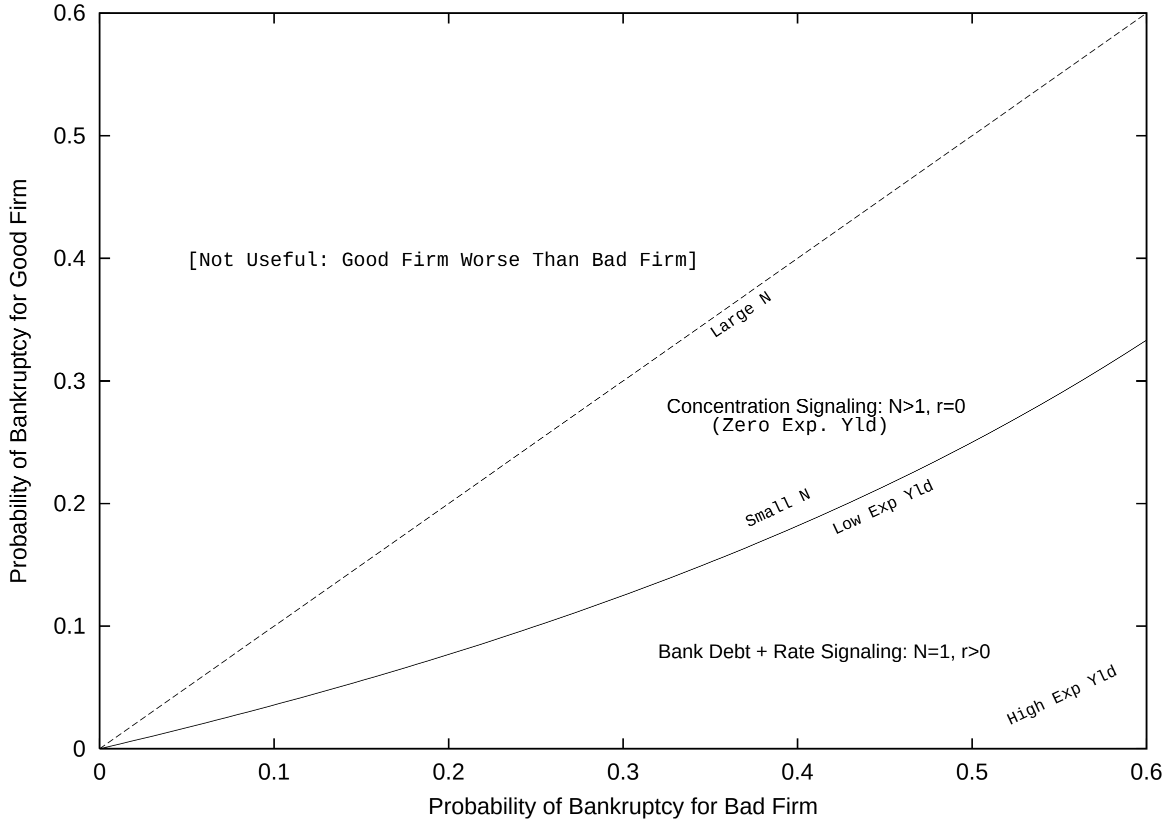

Figure 1 plots the regions in which signaling by creditor concentration alone signals creditor quality and in which signaling requires not only the ultimate concentration ($N = 1$, i.e., bankdebt), but also an expected interest above zero. The parameter values for this figure are as in our numerical examples: $V_{\text{Old}} = \$80, V_{\text{New}} = \$250, X = \$80, I = \$100, c_e = c_d = \$1$, and $\lambda = 1.5$. A positive interest rate is required when $\pi_g < {}^{\pi_b(\lambda-3)}/_{2\lambda\pi_b-\lambda-3}$. If $(1-\pi_B)\cdot V_{\text{New}} < I$, i.e., when $\pi_B > 0.6$, the new project is not a positive NPV project.

Figure 2. Signaling Equilibrium Promised Yields

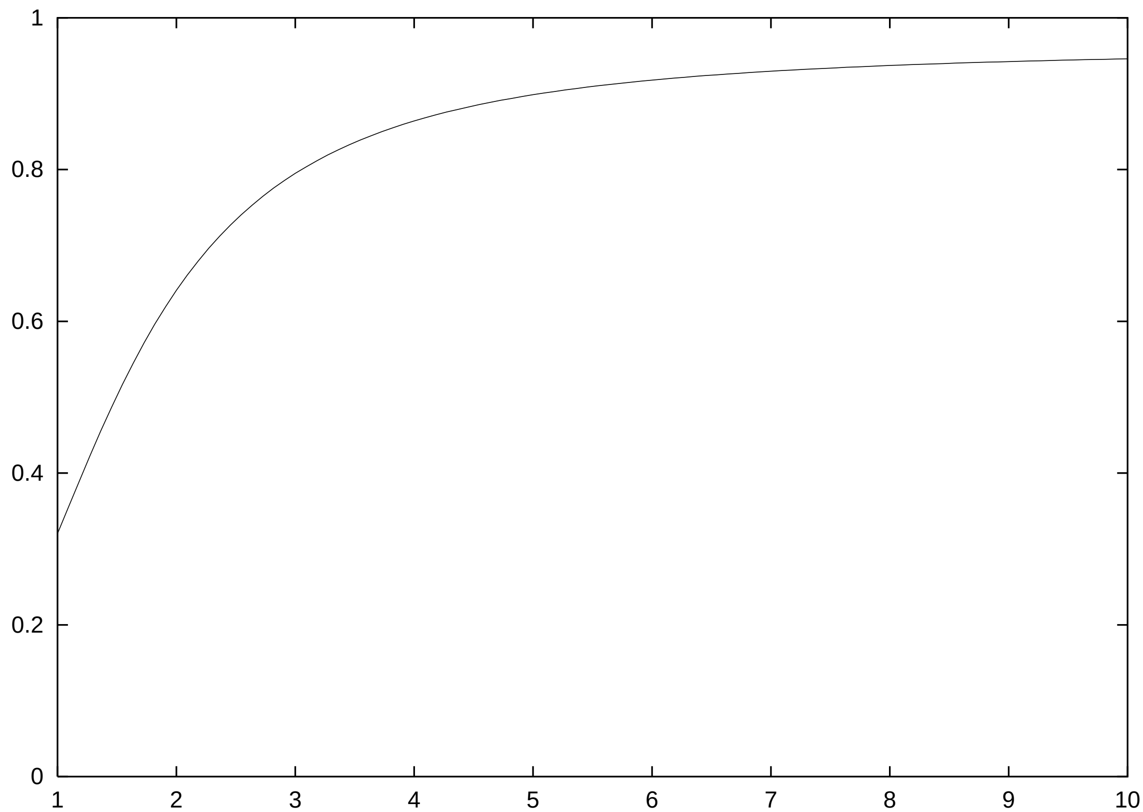

Figure 2 plots the *promised* rate of returns as a function of the optimal number of creditors. (To obtain different optimal number of creditors, we vary π_G. [Changing π_B would have the same effect.]) When $N > 1$, this is simply the yield required to offer creditors a zero expected rate of return. (For numerical convenience and to keep in-text computations easy to repeat, we are working with numbers that produce unrealistically high promised yields.) The expected yield is always zero, except when $N = 1$. Not plotted: When $N = 1$, i.e., (house) bank debt, the *expected* yield can range anywhere from 0 to 45%. (The promised yield would thus be higher.)

Table 2. Comparison With Bolton and Scharfstein (1996) Model of Number of Creditors

Relationship	BS 1996	BW 2001
Low Concentration (Public Debt)	more APR: Creditor-Friendly Settlement Firm Unfriendly Settlement	less APR: Creditor-Unfriendly Settlement Firm Friendly Settlement
High Concentration (Bank Debt)	less APR: Creditor-Unfriendly Settlement Firm Friendly Settlement	more APR: Creditor-Friendly Settlement Firm Unfriendly Settlement
Concentration vs. Corporate Termination	Less Frequent	Undef
Concentration Vs. Promised Interest Rate	High or Undef	Low
Bank Debt Vs. Expected Interest Rate	Zero	Zero (or Positive)
Concentration Vs. Holdout (Time)	Negative	Undef
Concentration Vs. Creditor Lobbying Expenses	Undef	Higher
Concentration Vs. Lobbying Expenses of Firm	Undef	Higher
Concentration Vs. Total Lobbying Expenses	Undef	higher
Concentration Vs. Inefficient Outcome	Ambiguous	Higher (except with add-on)
Concentration Incentives for Creditors Ex-Post	Negative or Ambiguous	Positive
Lawyer Expenses	Uncover Value	Seek Rents

Note: Public Debt is assumed equivalent to highly dispersed debt. Bank Debt is assumed equivalent to highly concentrated debt.