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Electoral Rules and the Emergence of New Issue Dimensions*

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Abstract

Different electoral rules provide different incentives for parties competing for votes to adopt emerging issues. As a result, new societal issues will be integrated at different speeds into the political arena, and ultimately, into policy. In order to study this question formally, I propose an extension of the standard spatial model of political competition that allows for issue adoption and more generally, issue prioritizing at the platform level. The paper then compares the outcome of party competition under proportional and plurality rule. Entry is allowed and incumbent parties act as Stackelberg leaders vis-à-vis potential entrants. The analysis highlights the interaction between entry barriers and the type of emerging issue in determining when and how a new issue will be introduced. The theory explains both internal (that is, without entry by a new party) realignments of party systems along new dimensions and entry as part of the process of political realignment.

Keywords: Comparison of electoral rules, new issue, electoral competition, entry, realignment, party system change.

JEL codes: D72, D78.

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1 Introduction

Suppose that ethnic identity is becoming more important, or that people's concern for the environment is rising. When and how should we expect these new issues to enter the political arena? And how does the answer to these questions depend on the way the competition among parties is organized? These questions are important because parties in modern democracies act as intermediaries between the population, whose preferences the electoral system seeks to aggregate, and policy.

In this paper, I provide a first analysis of how electoral rules mediate the changes in the underlying political space, and as a result, affect the dynamics of party systems (How many parties are present? What are the views they represent? And how do these elements change?). Obviously, electoral rules vary in many respects. My analysis focuses on the role of entry barriers and compares the outcome of political competition under the plurality rule and the proportional rule.

The model seeks to capture the following features of political environments. First, there is some level of strategic positioning by parties around issues and cleavages. In this paper, I make the simplest assumption consistent with this view, namely that parties maximize their chances of electoral success. Second, historical accounts of the dynamics of party systems include cases where new cleavages were absorbed by existing parties and cases where entry took place. This suggests that a comprehensive theory of party system change needs to include entry. Third, parties tend to prioritize the issues on their platforms and single-issue parties have been credited for introducing new issues.

In practice, the model has much of the flavor of a standard spatial model. There are two issues, an "old" and a "new", and they differ in terms of saliency in the electorate. A new issue is said to emerge when its saliency increases and becomes significant. Incumbent parties have a first-mover advantage. At any point, they take the prominence of the new issue as given and choose their electoral platforms, acting as Stackelberg leaders vis-à-vis potential entrants.[1] The analysis distinguishes between partisan and non partisan issues. Partisan issues correspond to issues on which parties can take opposite stances. Only one stance can be taken on non partisan issues. Voters are assumed to be sincere and parties are purely electoralist.

The novel (and key) feature of the model is the idea that parties prioritize the issues on their platforms. In addition to bringing a greater level of realism, this assumption provides a natural way to model the question of issue adoption: there may be issues that people care about, yet these

[1] This is in the spirit of Palfrey (1984) and Weber (1992 and 1998).

may not be "politically mobilized" in the sense that no party finds it advantageous to devote any attention to them on a platform. It turns out that a very straightforward extension of the standard model is enough to account for this added feature of political competition. The assumption is introduced and further motivated in the next secion.

In the model, there are two basic mechanisms that lead parties to respond to the changing electoral tastes. First, the competition among established parties forces them to choose popular platforms. Second, the threat of entry by a new party constrains them to differentiate their platforms enough and it induces them to prefer platforms that are robust to entry. *A priori*, it is unclear whether entry should speed up or slow down issue adoption. Indeed, entry can bite from both ends: from a party's conservative side if it adopts the new issue too fast or from the new issue dimension if the party does not adopt it fast enough. In the paper, these two mechanisms are called inside and outside competition respectively.

The difference between inside and outside competition is well-illustrated in sections 4 and 5 which deal with an emerging non partisan issue. Under the plurality (PL) rule, effective entry barriers are very high. Therefore, inside competition is the dominant force affecting issue adoption. As the new issue becomes more salient, established parties add it to their platforms. Entry never takes place and the party system evolves progressively to integrate the new dimension. By contrast, outside competition is more important under the proportional (PR) rule and it significantly affects equilibrium behavior. First, entry can no longer be avoided. Second, the entrant will be the one introducing the new issue. Third, entry deterrence can delay adoption in the PR system relative to the PL system. *In other words, though the PR system has formally a lower threshold of representation, the political system might be less permeable to new issues.* The reason is that, when the new issue is salient enough for parties in the PL system to adopt it but not salient enough to dominate political competition, entry on parties' conservative sides remains very attractive if the established parties decided to adopt the new issue. As a result, they prefer to postpone its adoption.

In section 6, I consider an emerging partisan issue. Because partisan issues correspond to a political cleavage in the classic sense, they allow for a greater scope for political differentiation (parties can take different stances on the new issue). This means that established parties can better coordinate to avoid entry. As such, political differentiation reinforces the role of entry costs. For the PL system this is unimportant since outside competition did not affect equilibrium behavior much. However, I find that parties in the PR system might now be able to absorb the emerging cleavage internally, that is, without entry.

Summarizing, the theory suggests that both entry cost and the scope for political differentiation are important in determining whether party systems are likely to absorb a new issue internally or whether entry will take place. In particular, both scenarios are possible under the PR rule.

Literature

The question of issue diversity and new issue adoption has ramifications in several strands of the literature. First, there is a rich literature in political science on realignment and party system change, starting from the contributions of Lipset and Rokkan (1967), Butler and Stokes (1969), Burnham (1970) and Sundquist (1973). This literature is empirical in focus and has emphasized detailed country studies as the basis for theory building and comparative analysis. An important contribution of this literature is the identification of the factors that influence political realignment and the generation of possible scenarios for such realignments.[2] The approach taken is this paper is complementary, in the sense that it focuses on only one factor (entry barriers) but uses the full force of equilibrium analysis in selecting among the likely scenarios.

Second, the literature on new / third parties has emphasized the role that such parties play in introducing new issues (see e.g. Rosenstone, Behr and Lazarus, 1984). In this paper, entry takes on a similar role, though to some extent for a different reason. This is further discussed in section 7.

In terms of formal theory, there appear to be fewer contributions. Of course, at the most basic level, all (spatial) models that are solved using a distribution of voter preferences can answer the question of how parties' positions on *existing* issues change when voter preferences change or when the electoral rule changes. However, because these models assume that parties take position on all issues at all times, they do not allow for new issue adoption.[3] In that vein, Cox (1987 and 1990) studies the effect of electoral rules on the dispersion of candidates over the unit line, a property which can be interpreted as providing representation for minorities (when compared to the standard median voter result). See also Myerson (1993).

Outside of the spatial theory of political competition, Hug (1994 and 1996) analyzes the effect of entry costs on the introduction of new parties. Hug models the electoral competition as the interaction between an interest group and a single established party. The interest group asks the incumbent to integrate the new issue on its agenda. If the demand is rejected, it can decide to form a party. An essential feature of Hug's model is that parties are not equally informed about the likely electoral support for the new issue. In my model, asymmetric information does not play

[2] Of particular interest is the model presented in Sundquist (1973) which emphasizes the role of a new "cross-cutting" issue in any political realignment.

[3] The literature on ambiguity (Shepsle, 1972) does not assume that parties take precise position on all dimensions, but unfortunately this modelling approach is not useful here (see also, Aragones and Postlewaite, 1999).

any role. All parties - incumbents and the potential entrant - are equally informed about voter preferences.

2 Issue prioritizing

Any model of political competition must define what distinguishes parties and how voters evaluate these characteristics in making their choices. In this section, I introduce and motivate the key assumption of the model: the fact that parties prioritize the issues on their platforms.

Empirically, there is ample evidence that parties do indeed emphasize some issues more than others (see, e.g, Robertson, 1976, Laver and Hunt, 1992, and Budge, 1992). Allowing for issue prioritizing at the platform level also seems to be an essential part of any model of issue adoption: An issue can exist (in the sense that voters care about it) without being politically mobilized. Such an issue is absent from party platforms or, which is equivalent, given zero priority on these platforms. Likewise, an issue is politically activated when a party places some priority on it in its platform.

Another advantage is that it makes it possible to distinguish between generalist and focused, single-issue parties. A generalist party gives more or less equal weights to most issues voters care about. Single-issue parties devote most of their energy to pursuing a specific issue. In many countries, single-issue parties have been credited for introducing new issues. It is therefore important to be able to include them in a theory of party system change.

Practically, a straightforward extension of the standard model can account for this additional feature of electoral competition. Suppose there are I issues voters care about. A party's platform is a vector $(a_i, e_i)_{i=1}^{I}$ where a_i stands for the party's policy position on the ith issue and e_i stands for the amount of effort (or priority) this party intends to allocate to the issue. (Without loss of generality, we can normalize $\sum\limits_i e_i = 1$).

A voter with characteristics θ evaluates platform $(a_i, e_i)_{i=1}^{I}$ according to:

$$\sum_i f(e_i) v_i(a_i, \theta) \tag{1}$$

where $v_i(a_i, \theta)$ (≥ 0) is voter θ's utility from policy a_i on issue i and $f(.)$ is a function that translates effort into output levels (legislative activity, probability of success, ...). Reasonable assumptions on f include: $f \geq 0$, $f(0) = 0$ (the issue is inexistent at the political level when parties do not devote any effort to it), $f' \geq 0$ (the more effort a party exerts on an issue the more likely it is to be effective at carrying out its policy promise) and $f'' \leq 0$ (decreasing returns to scale).

This formulation of party platforms and voters' assessment of them is the key assumption of the model. It will determine the trade-off that parties face when deciding whether or not to adopt a new issue and, more generally, when deciding on their portfolio of issues. The next sections use a simplified version of this model with only two issue dimensions and discrete platform choices to investigate the question of new issue adoption and how it depends on the electoral rule.

3 The model

There are two established parties. In the first stage, these parties select simultaneously a platform, taking into account the possibility of entry by a third party in the second stage. Platform choices are definitive and voters vote sincerely for their most preferred alternative.

Under plurality rule, the winner is the party which attracts the most votes and parties maximize their chances of winning. Under proportional rule, seats are allocated proportionally to the share of votes and parties maximize their voter support.[4]

Plaform choice

We will be analyzing a stylized version of the model introduced in section 2. There are two issues. The first issue can be thought of as the traditional left-right cleavage. The second issue is new. A party can choose to deal with one or both issues on its electoral platform and, if it chooses to deal with both issues, it gives them equal weight. In other words, $e_i \in \{0, \frac{1}{2}, 1\}$. On each issue, there are at most two different stances that parties can take: L (for left) or R (for right) on the first issue and U (for up) or D (for down) on the second issue. This assumption can be seen as an extreme simplification of a continuous policy space, or as the actual representation of the discreteness of the possible positions on that issue.[5] A single issue platform is described by its position on that issue, a generalist platform is described by the pair of letters that stand for its position on each issue. (for instance, L stands for a single issue leftwing platform, LU stands for a platform that takes position L on the first issue and U on the second issue).

The following distinction will be useful:

Definition: A partisan issue is an issue on which parties can take opposite stances. A non partisan issue is an issue that is non controversial in the sense that its reverse cannot be integrated onto a platform.

I will assume that the first issue is partisan but that the new issue may be partisan or non partisan. In the latter case, this means that U (say) is the only available policy position.

[4] For simplicity, I ignore the post-election coalition formation game. With coalition formation, parties' objective may differ from vote maximization. I take vote maximization as an approximation.

[5] For instance, abortion is often seen as allowing for only two policy positions.

The distinction between partisan and non partisan issue has two motivations: one is substantive, the other expositional. At the substantive level, there is evidence that some issues have a distinct non-conflictual character: People may vary in terms of their intensity of preferences for them but there is no disagreement about the desirable course of action.[6] Many post-materialistic issues (the ecology, education, consumer rights, ...) seem to fit in this category and it is important to allow for them in the analysis. Of course, this raises the question of whether the non-partisan outcome should be the result of an ex-ante restriction on the strategy space (like here) or the consequence of electoral competition in face of voters unanimously preferring one side of the issue. This is a valid point. An easy way to answer this concern is to assume that voters attach zero utility to the policy D. Hence, any platform including position D is a dominated platform choice (it does not pay to allocate any effort to it) and will never be selected at equilibrium. This assumption is equivalent to an ex-ante restriction of the policy choice to U.

A more subtle point however is the idea that different issues do indeed offer different possibilities for policy differentiation. This can be due to the nature of the issue it-self, or simply because policy nuances on the issue, though real, are difficult to convey to the electorate. This brings us to the expositional motivation for the distinction between partisan and non partisan issues. This distinction is the coarsest possible to capture the scope for policy differentiation on an issue and it will be useful to carry out the analysis into two steps: First, to look at an emerging non partisan issue to focus on the trade-off that parties face when they decide whether to add a new issue, and then to consider how a greater scope for differentiation reduces the extent of platform "cannibalization" and thereby increases the possibilities for entry deterrence.

Voter preferences

There is a continuum of voters whose preferences are summarized by their coordinates (x, y) in $[0, 1] \times [0, 1]$. Voters evaluate platforms using the expression (1) with weights $f(0) = 0$, $f(\frac{1}{2}) = \delta \in (\frac{1}{2}, 1)$ and $f(1) = 1$, and $v_1(L; x, y) = 1 - x$, $v_1(R; x, y) = x$, $v_2(U; x, y) = sy$ and $v_2(D; x, y) = s(1 - y)$[7] where $s \geq 0$ is a saliency parameter for the second issue: When $s = 0$, voters do not care about the new issue. When $s = 1$, both issues are equally salient. Let $u_{xy}(Z)$ be voter (x, y)'s benefit from platform Z. These assumptions imply among others that $u_{xy}(L) = 1 - x$ and $u_{xy}(LU) = \delta(1 - x) + \delta sy$.

[6] Stokes (1963) was the first to raise this point. The example he provided is corruption. In a factor analysis of British party platforms, Robertson (1976) finds that "support for open government and democratic procedures" is an essentially non polar issue.

[7] Hence, in the standard spatial model, L and R can be seen as the extreme positions on the first issue and a voter's preference depends (negatively) on the distance between his most preferred position on that issue and L (or R).

Finally, I assume that the distribution of voters' characteristics is uniform over $[0,1] \times [0,1]$ and that this is common knowledge. In particular, this means that a voter's views on the second issue are completely independent from his views on the first issue. In some cases, this is of course a strong assumption but it will serve as a first approximation.[8]

Remark: When $s = 0$, there is no interest in the new issue so U, LU and RU are dominated platform choices. In that case, the restriction of platform choice to $\{L, R\}$ can be motivated ex-post by the results of Palfrey (1984) and the generalization of Weber (1992 and 1998), which show that parties competing in a one-dimensional space will tend to *differentiate* their platforms when entry is a possibility (non convergence result). More on this in section 5.

Equilibrium

This is formally a two-stage game so the relevant equilibrium concept is the subgame perfect Nash equilibrium. Because incumbents have a first mover advantage, we can suppose that they will be able to coordinate on the best equilibrium from their perspective. So, to simplify proofs, I will focus on the Pareto optimal (from incumbents' perspective) pure strategy subgame perfect Nash equilibria (PSNE) of the game. I assume that the entrant mixes with equal probabilities when he is indifferent between several platforms. When the entrant is indifferent between entering or not, he does not enter.

To complete the description of the game, we need to define the level of the entry costs. I assume that entry is profitable only if the entrant can secure 25% of the seats. This is a high entry threshold but it will facilitate the comparison between electoral rules. Indeed, with an entry cost of 25%, the political outcome under both rules is identical when $s = 0$: One party chooses platform L and another chooses platform R. No further entry takes place. This justifies the fact that we are looking at two incumbents. Extensions are discussed in section 5.

4 An emerging non partisan issue

In this section, I characterize the outcome of political competition under the PR and PL rule when the emerging issue is non partisan. For future reference, I first consider how two incumbent parties adjust to the changing electoral landscape when there is no threat of entry. Proposition 1 applies to both the PL and PR rule.

Proposition 1: Two-party equilibrium in the absence of entry

At equilibrium, parties choose platforms $(L, R), (L, L)$ or (R, R) as long as $s \leq \frac{1-\delta}{\delta}$, they adopt

[8] The other extreme is to assume perfect correlation

generalist platforms (LU, RU), (LU, LU) *or* (RU, RU) *for* $\frac{1-\delta}{\delta} \leq s \leq \frac{\delta}{1-\delta}$ *and platforms* (U, U) *for* $s \geq \frac{\delta}{1-\delta}$.

The results in proposition 1 are intuitive. Inside competition alone forces parties to be responsive to electoral tastes and the adoption threshold for the new issue depends as expected on the cost of dilution when moving from a single issue platform to a generalist platform. When the new issue is not very salient ($s < \frac{1-\delta}{\delta}$), parties prefer to ignore it rather than dilute their effort. When the new issue is dominant ($s > \frac{\delta}{1-\delta}$), parties prefer to ignore the old cleavage. For intermediate salience levels, both issues have similar importance in the public and parties present generalist platforms.

Proposition 1 characterizes the equilibrium for all values of s for the sake of completeness. However, note that $\frac{\delta}{1-\delta}$ is greater than 1, so the new issue dimension is dominant for $s > \frac{\delta}{1-\delta}$, a position obviously more akin to an established issue than an emerging issue. In the remainder of this section, I will focus on equilibrium behavior for $s \leq 1$.

The proof comes down to comparing the payoffs to different choices. From (1), we can derive the loci of voters who are indifferent between two platforms. This yields:

$$\begin{aligned}
y|_{L,LU} &= \frac{1-\delta}{s\delta} - \frac{1-\delta}{s\delta}x \\
y|_{L,RU} &= \frac{1}{s\delta} - \frac{1+\delta}{s\delta}x \\
y|_{L,U} &= \frac{1}{s} - \frac{x}{s} \\
y|_{R,LU} &= -\frac{1}{s} + \frac{1+\delta}{s\delta}x \\
y|_{R,RU} &= \frac{1-\delta}{s\delta}x \\
y|_{R,U} &= \frac{x}{s} \\
x|_{L,R} &= x|_{LU,RU} = \frac{1}{2} \\
y|_{LU,U} &= \frac{\delta}{s(1-\delta)} - \frac{\delta}{s(1-\delta)}x
\end{aligned}$$

For $s < \frac{1-\delta}{\delta}$, these are represented in figure 1.

[insert figure 1 here]

Proof: Let votes$(X)|_Y$ be the share of votes that a platform X party gets when competing against platform Y. First note that as long as $s < \frac{1-\delta}{\delta}$, $s < 1$ and therefore votes$(U)|_{LU}$ and votes$(U)|_L$ are both less than 50 % (see figure 1). Moreover, the intersection of the indifference loci between L, R,

RU and LU (point A in figure 1) lies in the upper half of the square. Therefore, the best response to R is L or R (yielding a 50 % chance of winning under the PL rule and 50 % of the seats under the PR rule), the best response to RU is R or L under the PL rule (which guarantees winning) and R under the PR rule, and the best response to U is any of the four other possibilities under the PL rule and LU or RU under the PR rule. (The other best responses are derived by symmetry). As a result, (R, R), (L, L) and (R, L) are the only pure strategy Nash equilibria for $s < \frac{1-\delta}{\delta}$.

For $\frac{1-\delta}{\delta} < s < \frac{\delta}{1-\delta}$, A lies in the lower half of the square (see figure 2). LU and RU now dominate L and R respectively. In addition, $\text{votes}(U)|_{LU}$ and $\text{votes}(U)|_{RU} < 0.5$ as long as $s < \frac{\delta}{1-\delta}$. Therefore, U cannot be part of an equilibrium. This leaves (RU, RU), (LU, LU) and (LU, RU) as the only PSNE for $\frac{1-\delta}{\delta} < s < \frac{\delta}{1-\delta}$.

[insert figure 2 here]

When $s > \frac{\delta}{1-\delta}$, L and R remain dominated by LU and RU respectively and $\text{votes}(U)|_{RU} > 0.5$. Hence, the best response to LU and RU is U. Finally, the best response to U is U. Therefore, the unique equilibrium for $s > \frac{\delta}{1-\delta}$ is (U, U). ■

We now allow for entry. In the first stage, the two incumbent parties choose their platforms simultaneously, taking into account the potential subsequent entry by a third party. In the second stage, entry takes place if it is profitable.

Under the PL rule, entry barriers are very high since entry is profitable only if the entrant can secure more votes than any of the incumbents (or tie with one of them) – in all cases, this corresponds to an effective entry cost higher than 33% of the votes. Therefore, we can expect the equilibrium behavior under the PL rule to bear much in common with the no entry case. Proposition 2 shows that this is indeed the case:

Proposition 2: Equilibrium under plurality rule and entry

1. For $s < \frac{1-\delta}{\delta}$, (L, R) *is the unique Pareto optimal PSNE platform choice for incumbents.*
2. For $\frac{1-\delta}{\delta} < s \leq 1$, (LU, RU) *is the unique Pareto optimal PSNE platform choice for incumbents. At* $s = \frac{1-\delta}{\delta}$, *both platform pairs are equilibria. Entry never takes place.*

The proof illustrates the mechanism at play. In practice, the threat of entry eliminates some of the best responses of the no entry game (for example, L as a best response to L). However, *one* best response always subsists and it constitutes the equilibrium of the game when entry is allowed. As a consequence, the adoption threshold for the new issue under the PL rule corresponds to the threshold identified in proposition 1: in both cases, the new issue appears on electoral platforms for

$s > \frac{1-\delta}{\delta}$. In other words, the competition among incumbents - inside competition - is what drives the incentives for adopting the emerging issue.

Proof: The proof proceeds in 3 steps. First, we show that entry always takes place later in front of (R, LU) than in front of (R, L), and that this happens for $s > \widehat{s} > \frac{1-\delta}{\delta}$. Steps 1 and 2 then prove that (R, L) and (RU, LU) are the unique equilibria over their respective intervals.

Step 1: Entry in front of (R, LU) takes place later than in front of (R, L)

Entry in front of (R, LU) is only possible at LU. Indeed, the most an entrant at R can get is when $s = 0$, in which case it shares $\frac{1}{1+\delta} < \frac{2}{3}$ of the votes with the R incumbent and loses for sure. Similarly, an entry at U would get less than 25% of the votes (it is easily checked graphically that U gets 25% of the votes in front of (R, L) when $s = 1,$ and the (R, LU) configuration is worse for U). Finally, because of a higher level of platform cannibalization, an entry at RU collects less votes than the LU incumbent and an entry at L collects less votes than the R incumbent.

Now, entry at LU in front of (LU, R) is profitable when $\text{votes}(LU)|_R > \frac{2}{3}$, that is (using figure 2) $\frac{(2+s)\delta}{2(1+\delta)} > \frac{2}{3}$[9] or

$$s > \widehat{s} = \frac{4-2\delta}{3\delta} > \frac{1-\delta}{\delta} \tag{2}$$

Claim: Entry is profitable at LU in front of (L, R) for all $s \geq \widehat{s}$.

Proof: It is readily checked that

$$\text{votes}(L)|_{LU,R} < \text{votes}(R)|_{LU,L} \tag{3}$$

so R is the alternative to beat. At $\widehat{s}$,

$$\text{votes}(R)|_{LU} = \frac{1}{3} > \text{votes}(R)|_{LU,L} \tag{4}$$

Putting (3) and (4) together, we conclude that $\text{votes}(LU)|_{L,R} = 1$ - $\text{votes}(R)|_{LU,L}$ - $\text{votes}(L)|_{LU,R} > \frac{1}{3} > \max\{\text{votes}(R)|_{LU,L}, \text{votes}(L)|_{LU,R}\}$ so entry at LU is profitable. QED

Step 2: (R, L) is the unique PSNE for $s < \frac{1-\delta}{\delta}$

From proposition 1, both R and L are best responses to L. However, (L, L) will trigger entry. By contrast, (R, L) is robust to entry on this interval. Indeed, it is easy to see from figure 1 that $\text{votes}(U)|_{L,R}$, $\text{votes}(LU)|_{L,R}$ and $\text{votes}(RU)|_{L,R}$ are all less than the triangular area above A. In turn, this area represents less than 25 % of the votes, so at least one of the incumbents must be getting a $\frac{1-0.25}{2} > 0.25$ share of the votes. As a result, R becomes the unique best response to L when entry is allowed and (R, L) is a PSNE.

[9] This formula is valid as long as $\frac{(1+s)\delta}{1+\delta} < 1$ which can be checked ex-post for the relevant value of $\widehat{s}$ in (2).

To prove uniqueness, we need to rule out (RU, LU) and (U, LU) as potential equilibria (configurations where both incumbents choose the same platform can be ruled out by Pareto optimality). Step 1 together with proposition 1 implies that R remains a best response to LU (ensuring winning for the R incumbent) so (RU, LU) and (U, LU) cannot be equilibria.

Step 3: (LU, RU) is the unique Pareto optimal PSNE on $\frac{1-\delta}{\delta} < s \leq 1$

From proposition 1, both LU and RU are best responses to RU on this interval. However (RU, RU) triggers entry. By constrast, (LU, RU) is robust to entry. Indeed, the most L can get in front of (LU, RU) is when $s = \frac{1-\delta}{\delta}$, and it is easily checked graphically that $\text{votes}(L)|_{LU,RU} < \text{votes}(RU)|_{LU,L}$ (the same reasoning holds for an entry at R). Similarly, entry at U in front of (LU, RU) collects less than 25% of the votes for $s < \frac{\delta}{1-\delta}$ so it can be ruled out too over $[\frac{1-\delta}{\delta}, 1]$. Hence, (LU, RU) is a PSNE over this interval.

To prove uniqueness, we need to rule out (R, L) and (U, L) as potential equilibria. Step 1 together with proposition 1 implies that as long as (R, L) does not trigger entry, (RU, L) does not trigger entry either and therefore RU remains a best response to L. (U, L) can be ruled out by Pareto optimality (entry is always profitable in front of (U, L) over this interval. In particular, an entrant at R would secure a 50% chance of winning). ■

Under the PR rule, entry barriers are lower and the threat of entry is now more effective at influencing equilibrium behavior by incumbents above and beyond simply selecting one of the Nash equilibria of the no entry game. First, it changes the adoption threshold for the new issue. Second, entry necessarily takes place for some values of s. Formally, we have:

Proposition 3: Equilibrium under proportional rule and entry

Let $s^* = \min\{\frac{1-3\delta+\sqrt{9+2\delta-7\delta^2}}{4\delta}, 1\}$ *and* $s^{**} = \frac{2-\delta-\delta^2}{\delta(1+\delta)}$. Then $\frac{1-\delta}{\delta} < s^* < s^{**}$ *and at the unique Pareto optimal equilibrium, parties choose* (L, R) *for* $s \leq s^*$ *and (if* $s^{**} < 1$*)* (LU, RU) *on* $[s^{**}, 1]$*. Over these intervals, entry is deterred. Entry cannot be avoided on* (s^*, s^{**})*. The exact pattern of entry accommodation depends on the parameters* (δ, s)*. Close to* s^**, the incumbents keep their mainstream platforms* (L, R) *and entry takes place at* LU *,* RU *or (for very low values of* δ*) at* U*.*

The proof proceeds in 3 steps. First, we determine for each possible platform choice by incumbents whether and where entry takes place. In practice only two platform pairs are robust to entry for some values of s, namely (R, L) for $s \leq s^*$ and (RU, LU) for $s \geq s^{**}$. This is illustrated in figure 3. (For $s > s^*$, entry in front of (R, L) takes place at LU or U depending on which strategy is best for the entrant). Second, we prove that over these intervals, these platform choices constitute the only Pareto optimal PSNE. Third, over (s^*, s^{**}), entry cannot be avoided and we determine the optimal pattern of entry accommodation close to s^*. Details can be found in the appendix.

[insert figure 3 here]

Two features of the equilibrium are noteworthy. First, notice that the incumbents keep their mainstream platforms for some values of s greater than $\frac{1-\delta}{\delta}$ for which they would have preferred to switch to generalist platforms if entry had not been a concern (proposition 1). The reason is that entry is profitable at L or R in front of (LU, RU) on $[\frac{1-\delta}{\delta}, s^{**}]$ whereas it can be deterred by (R, L) up to s^*.

Second, entry cannot be avoided for $s \in (s^*, s^{**})$ and proposition 3 suggests that close to s^*, the incumbents keep their mainstream platforms when they optimally accommodate entry. In other words, *entry by a third party represents the dominant medium for the introduction of the new issue into the political arena*. Green parties across continental Europe provide an excellent illustration of a new (and largely non partisan) societal demand being introduced by a new party. The next section elaborates further on these points.

5 Entry deterrence as a conservative force

Comparing proposition 2 with proposition 3, we get:

Corollary 1: *The entry of the new issue into the political arena takes place earlier under the PL rule than under the PR rule.*

Corollary 1 follows directly from propositions 2 and 3. Under the PL rule, the radical issue is introduced at $s = \frac{1-\delta}{\delta}$ when both incumbent parties add it to their existing mainstream platforms. Under the PR rule, the incumbents postpone their adoption of the new issue in order to deter entry on their more conservative sides. As a result, the new issue enters the political arena only after $s^* > \frac{1-\delta}{\delta}$.

This result highlights the dual role that entry costs play. Given a platform choice by incumbents, entry costs affect the entry decision by third parties: the lower the cost, the more likely the entry. At the same time, entry costs also alter the incumbents' platforms choice *ex-ante*.

This second effect generates a non-monotonic relationship between the speed of issue adoption and the level of the entry cost. Obviously, with zero entry costs, any new issue will automatically trigger entry by a third party and it will therefore get represented in the political arena as soon as it is formulated. At the other extreme, when barriers to entry are very high (like under the PL rule), the threat of entry does not affect the behavior of incumbents. They simply balance the benefits and costs in terms of votes of adopting the new issue, and they adopt it when the benefits of doing so surpass the costs. Entry is effectively blockaded. By contrast, entry deterrence does play a role for intermediate levels of the entry cost and corollary 1 suggests that, when this is the case, it will *postpone* the adoption of the radical issue.

To investigate more formally the effect of the entry cost on the adoption threshold, we need to relax our assumption of a 25% entry cost. However, we immediately run into a difficulty. Indeed, for an entry cost corresponding to less than 25% of the seats, our focus on two incumbent parties at $s = 0$ is no longer justified under the PR rule.

There are two ways to reestablish internal consistency for lower entry costs. One is to assume that when faced with an incumbent and a new party competing on an identical platform, voters will only cast votes for the established party. The result in proposition 3 is easily extended for this case. Let c be the entry cost (in terms of vote share) and define $s^*(c)$ and $s^{**}(c)$ as the entry thresholds in front of (R, L) and (RU, LU) respectively. From proposition 3, we know that for $c = 0.25$, $\frac{1-\delta}{\delta} < s^*(c) < s^{**}(c)$. Moreover, $s^*(c)$ is increasing in c (entry in front of (R, L) becomes more difficult as c increases) and $s^{**}(c)$ is decreasing in c. As long as $s^*(c) < s^{**}(c)$, entry cannot be avoided. It takes place at $s^*(c)$ in front of (R, L) and the entrant enters on a generalist or single issue U platform (when $s^*(c) > \frac{1-\delta}{\delta}$, entry is deterred on $[\frac{1-\delta}{\delta}, s^*(c)]$). For c big enough, $s^{**}(c) < s^*(c)$, entry can be avoided for all values of s and the established parties are those who introduce the new issue as part of a generalist platform. This takes place at $\max\{\frac{1-\delta}{\delta}, s^{**}(c)\}$ (for $s < \frac{1-\delta}{\delta}$, the incumbents prefer to keep their mainstream platforms since it does not trigger entry. For $s < s^{**}(c)$, entry takes place in front of (RU, LU) whereas it could be deterred by choosing (R, L)). Summarizing, the adoption threshold becomes

$$\min\{s^*(c), \max\{\frac{1-\delta}{\delta}, s^{**}(c)\}\} \tag{5}$$

Since $s^*(c)$ is increasing in c and $s^{**}(c)$ is decreasing in c, the adoption threshold depends non-monotonically on the entry cost. It is first increasing in c then decreasing.

A second way to reestablish internal consistency for lower entry costs is to allow for more platform locations on the mainstream dimension in order to capture the fact that with lower entry costs and a PR system, more than two parties are likely to compete for votes even when the political space is unidimensional.[10] Using the insights from Palfrey (1984) and Weber (1992 and 1998), we could define the original platform choice for $s = 0$ as the set $X(c) = \{x_i(c) : i = 1, ..., I\} \in [0, 1]$ where I is the minimum number of platforms such that $\{x_1(c), ..., x_I(c)\}$ corresponds to the equilibrium platform choice by I incumbent parties when $s = 0$ and no further entry is profitable given an

[10] Notice that this approach makes sense if we consider that the earlier restriction to two positions was a simplification of a more complex reality.

entry cost of c.[11] For $s > 0$, parties can keep a mainstream platform, add the new issue onto their platforms (that is, select $x_i^U(c)$) or choose a single issue platform on the new dimension, U. With these assumptions and voter preferences adapted from section 2, we are back to the familiar mechanic of the previous section. Let s_i^T be the transition threshold of party i between a mainstream platform and a generalist platform absent the threat of entry. A similar pattern of equilibrium behavior emerges: (1) keeping a mainstream platform a little bit longer than s_i^T is a profitable entry deterring strategy, and (2) entry is inevitable and the entrant is the one who introduces the new issue. Moreover, entry now takes place earlier for lower entry costs.

Either way we go, we obtain:

Corollary 2: *There is a non-monotonic relationship between the adoption threshold for the new issue and the level of the entry cost. Moreover, when entry is deterred, it postpones the adoption of the new issue relative to the no entry game.*

Summarizing the discussion so far, we have found that: (1) lower costs of entry do not necessarily mean that a political system is more permeable to new ideas – entry deterrence can delay adoption, and (2) when entry takes place, the new party is the one who introduces the new issue.

In fact, these two results are closely related. They are both driven by the platform dilution embodied in (1), and more specifically, by the fact that voters who care more about a specific issue are also those most affected when a party adds a new issue to its platform. As a consequence,

$$\begin{aligned}&\text{When party 1 is indifferent between platforms } L \text{ and } LU \\ &\text{in front of } R, \text{ votes}(L)|_{LU,R} \text{ is greater than votes}(LU)|_{L,R}\end{aligned} \tag{6}$$

Intuitively, on the left-wing turf, the single issue left-wing platform will attract more votes than a generalist platform. What (6) implies is that at $s = \frac{1-\delta}{\delta}$, entry in front of (RU, LU) is easier than in front of (R, L) (and *a fortiori* for $s < \frac{1-\delta}{\delta}$). As a result, there might be cases for which entry at $s = \frac{1-\delta}{\delta}$ is profitable in front of (RU, LU) but not in front of (R, L). Entry deterrence on a conservative platform configuration (R, L) is then profitable for some $s > \frac{1-\delta}{\delta}$, i.e. entry deterrence delays adoption. Another consequence is that as the new issue becomes more salient, parties prefer to accomodate entry by keeping their mainstream platforms (since their expected vote share from keeping (R, L) will be greater than if they switched to (RU, LU)). In other words, the new party is the one who introduces the new issue.

[11] Weber (1998) guarantees that for all $c < 0.5$ there exists a unique I and a unique entry deterring equilibrium configuration $\{x_1(c), ..., x_I(c)\}$. For example, given a uniform distribution of voters over the unit line, $I = 3$ and $X(c) = \{c, \frac{1}{2}, 1 - c\}$ for $c \in [\frac{1}{6}, \frac{1}{4})$.

The condition in (6) is the critical property of the model that biases platform choices against the adoption of new issues. In particular, the qualitative predictions of the model hold for more general preferences such as:

$$v_2(U; x, y) = \alpha_1 + \alpha_2 s + \alpha_3 sy \tag{7}$$

for α_1, α_2 and $\alpha_3 \in \mathbb{R}$, $\alpha_3 \geq 0$ and $\alpha_1 + \alpha_2 s \geq 0$ (which imposes that the minimum utility that voters can derive from the emerging issue U be positive). The parameter α_1 can be seen as a scaling factor between the utility derived from the mainstream issue and that derived from the new issue. α_2 and α_3 allow for a richer pattern of issue emergence. When $\alpha_2 > 0$ and $\alpha_3 = 0$, the new issue is completely non-controversial: all voters agree on the importance of U and U becomes more salient as s increases. When $\alpha_2 = 0$ and $\alpha_3 > 0$, some voters care more than others about U and opinions become (in a welfare kind of way) more differentiated as s increases. Ethnic issues enter this category. Arguably the ecology combines both elements: people have generally been more receptive to the environmental cause over the past 30 years, though at the same time, opinions are also more differentiated on this issue.

To see that entry deterrence will slow down issue adoption under (7), we simply need to check that (6) is satisfied. Deriving the indifference lines between L and LU and R and LU respectively, we get:

$$\begin{aligned} y|_{L,LU} &= -\frac{\alpha_1 + \alpha_2 s}{\alpha_3 s} + \frac{(1-\delta)}{\delta \alpha_3 s} - \frac{(1-\delta)}{\delta \alpha_3 s} x \\ y|_{R,LU} &= -\frac{1 + \alpha_1 + \alpha_2 s}{\alpha_3 s} + \frac{(1+\delta)}{\delta \alpha_3 s} x \end{aligned}$$

The slope of the LLU indifference locus is negative while that of the RLU locus is positive. When $\text{votes}(LU)|_R = \text{votes}(L)|_R$, both indifference loci go through the center of the square. Since $\alpha_1 + \alpha_2 s \geq 0$, we can check graphically that $\text{votes}(L)|_{R,LU} > \frac{1}{4}$ and $\text{votes}(LU)|_{R,L} < \frac{1}{4}$. Therefore (6) must hold.

6 An emerging partisan issue

In this section, I turn to the case of an emerging partisan issue, that is, a new cleavage in society. Now both U and D are possible positions on the second dimension. Together with the two stances along the old cleavage, this yields 4 possible single issue platforms and 4 possible generalist platforms. Figure 4 illustrates various indifference loci for $\frac{1-\delta}{2\delta} < s < \frac{1-\delta}{\delta}$. As before, we first derive the equilibrium behavior absent the threat of entry.

[insert figure 4 here]

Proposition 4: Two-party equilibrium under the PL and PR rule

At equilibrium, parties choose platforms $(L, L), (R, R)$ *or* (L, R) *for* $s \leq \frac{1-\delta}{\delta}$, *platforms* (X, Y) *with* $X, Y \in \{LU, LD, RU, RD\}$ *for* $\frac{1-\delta}{\delta} \leq s \leq \frac{\delta}{1-\delta}$ *and platforms* $(U, U), (D, D)$ *or* (U, D) *for* $s \geq \frac{\delta}{1-\delta}$.

The proof of proposition 4 follows the lines of that of proposition 1 and is omitted. With two full dimensions, *relative salience* is all that matters for electoral support. Therefore the equilibrium is symmetric around $s = 1$. When $s < 1$, the old cleavage dominates the new one. When $s > 1$, it is the new cleavage that becomes dominant. As before, the transition thresholds depend on the cost of platform dilution. In particular, proposition 4 implies that for $s < \frac{1-\delta}{\delta}$, the old cleavage drives party alignment (the new issue is not salient enough to be represented in the political arena) whereas for $s > \frac{\delta}{1-\delta}$, the new issue is the one driving party alignment.

When we allow for entry, issue adoption patterns vary with the electoral system. For the PL system, we have:

Proposition 5: Equilibrium under plurality rule and entry

The set of Pareto optimal PSNE of the game consists of:

1. For all δ,

a. (R, L) *for* $s \leq \frac{1-\delta}{\delta}$,

b. (LU, RD) *and* (LD, RU) *for* $s \in [\frac{1-\delta}{\delta}, \frac{\delta}{1-\delta}]$; *and*

c. (U, D) *for* $s \geq \frac{\delta}{1-\delta}$.

In addition,

2. (RU, LU) *and* (RD, LD) *are equilibria for* $s \in [\frac{1-\delta}{\delta}, \min\{1, \frac{9\delta}{4(2-\delta)}\}]$ *and* (LU, LD) *and* (RU, RD) *are equilibria for* $s \in [\max\{1, \frac{4(2-\delta)}{9\delta}\}, \frac{\delta}{1-\delta}]$ *when these intervals are well-defined, and*

3. *Let* $s_2(\delta) = \min\{\frac{1+3\delta-\delta^2-\sqrt{1+6\delta+4\delta^2-15\delta^3-14\delta^4}}{3\delta^2}, \frac{4(2-\delta)}{9\delta}\}$ *and* $s_1(\delta) = \min\{\frac{1-\delta}{2\delta} + \frac{\sqrt{6(1-\delta^2)}}{4\delta}, \frac{9}{8}\}$. *Then,* $s_2(\delta) > \frac{1-\delta}{\delta}$ *and* (R, L) *is an equilibrium on* $(s_2(\delta), s_1(\delta)]$ *and* (U, D) *is an equilibrium on* $[\frac{1}{s_1(\delta)}, \frac{1}{s_2(\delta)})$ *when these intervals are well-defined.*

The proof is left for the appendix. Two types of equilbria emerge under the PL rule. First, some of the equilibria with entry correspond to those of the no entry game (points 1 and 2 in proposition 5). The intuition is identical to the one behind the equivalent result for the non partisan case. Under the PL rule, the threat of entry eliminates some of the "unreasonable" best responses that sustain the equilibria of the no entry game, *but it does not eliminate all of them.* Indeed, entry barriers are so high that if parties differentiate their platforms enough, entry is avoided. As a result, the first set of equilibria form a subset of the set of equilibria of no entry game. They describe a progressive pattern of party system adjustment to the emergence of the new cleavage. Transition thresholds are driven by the competition among incumbents and correspond therefore to those identified in proposition 4.

However, new equilibria also appear when we allow for entry. Point 3 of proposition 5 identifies intervals beyond $s = \frac{1-\delta}{\delta}$ where (R, L) can be an equilibrium. One interval is strictly included in $(\frac{1-\delta}{\delta}, \frac{\delta}{1-\delta})$ and arises for intermediate values of δ. Another takes place around $s = \frac{\delta}{1-\delta}$ and is possible only for very low values of δ ($\delta < 0.54$). Over these intervals, entry in front of (R, L) is not profitable. The reason why these equilibria emerge when the new issue is partisan whereas they did not appear when the new issue was non partisan is that (LU, R) (LU was the best response to R when the issue was non partisan) now triggers entry (at LD or D).[12]

Turning to the PR rule, we have:

Proposition 6: Equilibrium under proportional rule and entry
Let $s^* = \min\{\frac{1-3\delta+\sqrt{9+2\delta-7\delta^2}}{4\delta}, 1\}$ *and* $s^{**} = \frac{-1-3\delta+2\delta^2+\sqrt{9-2\delta-7\delta^2+4\delta^3}}{2\delta(1-\delta)}$. *Then* $\frac{1-\delta}{\delta} < s^* < s^{**}$ *and*
1. (L, R) *is the unique Pareto optimal PSNE for* $s \leq s^*$,
2. (U, D) *is the unique Pareto optimal PSNE for* $s \geq \frac{1}{s^*}$, *and*
3. When $s^{**} < 1$, (RU, LD) *and* (RD, LU) *are the only Pareto optimal PSNE over* $[s^{**}, \frac{1}{s^{**}}]$.
Over these intervals, entry is deterred. Entry is accommodated elsewhere.

[insert figure 5]

The proof is very similar to that of proposition 3 and is left for the appendix. The key difference between a non partisan issue and a partisan issue is the scope for platform differentiation that each offers. When the new issue is non partisan, incumbents have little scope for deterring entry *and* integrate the new issue onto their platforms at the same time. By contrast, when the new issue is partisan, the incumbents can integrate it while continuing to cover much of the political space (for example, by coordinating on (LU, RD) or (LD, RU)). This is illustrated in proposition 6. In practice, s^* and s^{**} are very close to one another (see figure 5) so entry is effectively avoided for most values of s while the party configuration moves from (R, L) to (RU, LD) (or (RD, LU)) and from (RU, LD) to (U, D). In other words, the party system under the PR rule is able to *adjust internally* to the changing electoral landscape: no entry takes place and existing parties subsist, though with a different electoral platform.

[12] These new equilibria could be ruled out by imposing $\delta > 0.54$ and that best responses be robust to small trembles of the opponent among his preferred alternatives (for example, when (R, L) is an equilibrium for $\delta > 0.54$, party 1 is actually indifferent between RU and R. But if he mixed over these alternatives, party 2 would prefer to choose LD).

For the rest, the equilibrium pattern described in proposition 6 is very similar to that of proposition 3. As in the case of the non partisan issue, entry deterrence under the PR system delays the adoption of the new issue relative to the no entry case ($s^* > \frac{1-\delta}{\delta}$). However, the transition from generalist platforms to single issue platforms along the new dimension also takes place earlier under the PR system relative to the benchmark of proposition 4 ($\frac{1}{s^*} < \frac{\delta}{1-\delta}$). This illustrates the fact that entry deterrence - combined with platform dilution in the presence of multiple issues - not only slows down the adoption of new issues but more fundamentally puts *a limit on issue proliferation.*

7 Discussion

How responsive are political institutions to the preferences of citizens? The analysis in this paper has focused on electoral competition as a factor of change. We have identified two specific channels. First, competition among incumbents (inside competition) forces parties to be responsive to voter preferences. Second, actual or potential entry (outside competition) also affects issue adoption.

The results suggest that the balance between these two forces depends on the entry barriers to the political system and the nature of the emerging issue. Lower entry barriers favors outside competition. A greater scope for political differentiation on the new issue favors inside competition. When inside competition drives party realignment, we can expect the political system to adjust internally (that is, without entry) to the changing electorate. The political players remain the same – but identity of labels does not mean identity of contents. When outside competition is more important, entry will necessarily take place.

Internal adjustment

Models like this one are bound to provide a very partial view of real phenomena. However, it is interesting to relate the results to the observation of Lipset and Rokkan (1967) that, *by cutting alliances across issues*, parties in Western Europe have largely been able to adjust (internally) to the new cleavages, at least for the period before 1920. This is exactly the strategy that the model suggests. For a more recent example, consider the repositioning in the 1950s of the Belgian "Liberals" as the relative salience of religion versus the economic dimension was declining. The party left the anticlerical pole of the Religion vs. State cleavage to position itself on the (conservative side of the) economic cleavage (Mabille, 1986). As the relative salience of these two cleavages changed, no new party appeared on these dimensions so the existing party system *absorbed internally* the changing electoral landscape.

The role of entry

In this paper, entry only plays a role in the PR system and when it occurs, it constitutes an integral part of the realignment process: the entrant is expected to stay and be part of the resulting party system.

In the PL system, entry is not an equilibrium phenomenon. Yet, there are many examples of entry in PL systems, from the rise of the regionalist parties in the UK in the second half of this century to the many short-lived "third parties" in US history.[13] Various explanations have been put forward.

One reason invoked for the UK is that the national parties were constrained to adopt a single platform at the national level and that their absence from the decentralization / regionalist debate is what led to the development of nationalist parties in Scotland and Wales (Rasmusen, 1991). Analysts of American political history suggest at least two other explanations. In particular, Burnham (1970) argues that in all major realignments, the entry of a new party on a new issue has led to the adoption of its themes by one of the main parties, to a party system realignment and the subsequent disappearance of the entrant. In other words, entry here is ephemeral. One interpretation is that it helps established parties to overcome organizational inertia due to some internal conflict about the position to adopt on the new issue (see also Sundquist, 1973). A second interpretation is that entry helps established parties realize the extent of public support for the new issue.

All of these explanations suggest a much richer role for entry in the dynamics of party systems than what is allowed in the current simple model, and they provide as many directions for extending the model.

8 Concluding remarks

This paper has started with the question of how electoral rules mediate the changes in the electorate and in particular, the emergence of new issues. To answer this question, I have proposed an extension of the standard spatial model to allow for an additional element of political differentiation: issue prioritizing. I believe that this extension captures an important feature of party competition as emphasized in the substantive literature but so far neglected in the formal literature. More down to earth, allowing for issue prioritizing at the platform level provided a natural way to talk about an issue as being important for voters, yet not present in the political arena.

The implementation of the model has relied on several very specific assumptions, in particular in

[13] One could also add how the Republican party appeared on the electoral scene in the mid-nineteenth century - on a new platform - and completely replaced one of the incumbents. For an entertaining account of this episode, see e.g. Riker (1986).

terms of the possible platform choices, the distribution of preferences and entry costs. My feeling is that the thrust of the message is robust to the relaxation of these assumptions and that it is mainly driven by the dilution effect embodied in the issue prioritizing at the platform level: (1) Lower entry costs do not imply that new issues are adopted faster when established parties have a first mover advantage, (2) Greater scopes for differentation increase entry barriers and (3) When entry takes place, the entrant is the party which introduces the new issue.

The analysis leaves several open questions for research. An obvious one is the study of the properties of a full-fledged version of the issue prioritizing model, both at the abstract level and in the context of applications to new questions.

Second, one of the implications of the model is that entry in the PL system cannot be an equilibrium phenomenon and that instead the new issue will be adopted by the existing parties. This contradicts the historical evidence for the U.S. and the U.K. My interpretation is that the model falls short of explaining this evidence because it takes parties as unitary organizations with the objective of maximizing their chance of electoral success. Historical accounts very often stress the internal divisions within parties when new cleavages emerge. Therefore, allowing parties to have more subtle objective functions appears to be a very promising route.[14]

Finally, and from a broader perspective, it is important to note that political institutions provide many channels through which new issues can be integrated into the policy agenda. This paper has focused on one: electoral competition. Referendums, the activity of lobbies and popular protests are other channels. In addition, parties are made of politicians and party discipline tends to vary across electoral systems. In the US for example, individual candidates maintain a large degree of autonomy regarding which issues they defend. In parliamentary systems, voting along party lines is much more common. In turn, this is also likely to affect the diversity of issues represented at the political level. To what extent some of these features are complements or substitutes and how correlated they are in practice is an open question.

[14] The informational route is another possibility, see Castanheira (2000) for a model of the informational role of "small parties."

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9 Appendix

Proof of proposition 3: The proof proceeds in three steps. First, we determine for each possible platform pair, the pattern of entry by a third party, as well as its consequence for the vote share of incumbents. In practice, only (L, R) and (LU, RU) are robust to entry for some values of the parameter s. Second, we prove that over these intervals, these platform pairs constitute the only Pareto optimal PSNE. Finally, we look at the pattern of optimal entry accommodation.

Step 1: Windows of entry and of entry deterrence

There are 6 generic platform pairs to consider: (L, R), (LU, RU), (L, U), (LU, U), (LU, L) and (LU, R). (platform pairs where both incumbents choose the same platform always elicit entry and can never be an equilibrium).

Step 1.1: Entry in front of (L, R)

It is easy to check graphically that entry at LU or RU is not profitable for $s < \frac{1-\delta}{\delta}$. For $s > \frac{1-\delta}{\delta}$, we find that (using figure 2):

$$\begin{aligned} \text{votes}(LU)|_{L,R} &= \frac{1}{4}[2 - \frac{3}{2}\frac{1-\delta}{s\delta}] + \frac{1}{2}[\frac{(1+s)\delta}{1+\delta} - \frac{1}{2}][1 - \frac{1-\delta}{2s\delta}] \\ &= \frac{2\delta^2(1+s)^2 - (1+\delta)}{4\delta(1+\delta)s} \end{aligned}$$

Solving for when this expression is equal to $\frac{1}{4}$, we get $s = \frac{1-3\delta+\sqrt{9+2\delta-7\delta^2}}{4\delta}$.[15] Turning to U, we find that $\text{votes}(U)|_{L,R} \geq \frac{1}{4}$ as soon as $s \geq 1$. Putting these two elements together, we conclude that entry in front of L and R is profitable for:

$$s > s^* = \min\{\frac{1-3\delta+\sqrt{9+2\delta-7\delta^2}}{4\delta}, 1\} > \frac{1-\delta}{\delta}$$

Step 1.2: Entry in front of (LU, RU)

We first consider entry by a conservative platform, say L. At $s = \frac{1-\delta}{\delta}$, entry is profitable. Indeed, the L, LU indifference locus cuts the political space into two equal areas and an L platform therefore collects at least $\frac{3}{8}th$ of the votes. For $s > \frac{1-\delta}{\delta}$, we can use figure 2 to compute:

$$\begin{aligned} \text{votes}(L)|_{LU,RU} &= \frac{3-3\delta}{8\delta s} + \frac{1}{2}(\frac{1}{1+\delta} - \frac{1}{2})(\frac{1-\delta}{2\delta s}) \\ &= \frac{2-\delta-\delta^2}{4\delta(1+\delta)s} \end{aligned}$$

[15] This expression is less than $\frac{1}{\delta}$ for all δ, hence $\frac{\delta(1+s)}{(1+\delta)} < 1$ so the formula used to derive $\text{votes}(LU)|_{L,R}$ is the correct one.

Hence,

$$\text{votes}(L)|_{LU,RU} > \frac{1}{4} \text{ for } s < \frac{2-\delta-\delta^2}{\delta(1+\delta)}$$

Turning to U, we have

$$\text{votes}(U)|_{LU,RU} > \frac{1}{4} \text{ for } s > \frac{\delta}{1-\delta} > 1$$

so entry is never profitable at U for the relevant values of s. Let $s^{**} = \frac{2-\delta-\delta^2}{\delta(1+\delta)}$. Checking numerically, we find that $s^{**} < 1$ for $\delta > 0.61803$, and that $s^* < s^{**}$ everywhere.

Step 1.3: Entry in front of (L, U)

Entry at L is dominated by an entry at R (in both cases, the entrant has the same vote share as the L incumbent but in the second case the U incumbent gets less). By step 1.1, an entry at R yields at least $\frac{3}{8}th$ of the votes. Entries are LU or RU are also possible if they yield a higher vote share for the entrant.

Claim: The U incumbent gets less than 31.25% of the votes.
Proof: By step 1.1, an entry at R leaves less than 25% of the votes to the U incumbent. An entry at RU is even worse for the U incumbent. Now, if entry takes place at LU (because the entrant would get more than $\frac{3}{8}th$ of the votes this way), the U incumbent gets less than 50% of the remaining (since for $s \leq 1$, L gets more votes in front of LU than U[16]), that is 31.25%. QED

Step 1.4: Entry in front of (LU, U)

Entry takes place for all s. On $s < \frac{1-\delta}{\delta}$, it takes place at L (yielding more than 50% of the votes for the entrant). For $s \geq \frac{1-\delta}{\delta}$, step 1.2 implies that an entry at RU secures at least $\frac{3}{8}th$ of the votes. Entry at L and R are also possible.

Step 1.5: Entry in front of (LU, L).

Entry is again unavoidable. For $s < \frac{1-\delta}{\delta}$, an entry at R gets more than $\frac{3}{8}th$ of the votes (step 1.1) and it dominates entries at RU (by geometry), at L (LU gets more in this case and the entrant has to share 50-50 the remaining votes) and at LU. So entries at R or U are the only possible. For $s > \frac{1-\delta}{\delta}$, we can check graphically that RU gets more than $\frac{3}{8}th$ of the votes and it dominates entries at R, L and LU (by similar arguments), so entries at RU or U are the only possible.

Claim 1: The LU incumbent gets less than 25% of the votes for $s \leq s^*$.
Proof: First, note that $\text{votes}(LU)|_{RU,L} < \text{votes}(LU)|_{R,L}$, hence step 1.1 implies a vote share lower than 25% for the LU incumbent if entry occurs at R or RU. When entry takes place at U, it must yield more than $\frac{3}{8}th$ of the votes to the entrant. Using the fact that $\text{votes}(U)|_{LU,L} \leq \text{votes}(L)|_{LU,U}$ (footnote 22), this leaves the LU incumbent with less than 25% of the votes again. QED

[16] Indeed, for $s < \frac{1-\delta}{\delta}$, $\text{votes}(L)|_{LU,U} > 50\%$. For $s \geq \frac{1-\delta}{\delta}$, $\text{votes}(L)|_{LU,U} = \text{votes}(L)|_{LU} = \frac{1-\delta}{2\delta s}$ and $\text{votes}(U)|_{LU,L} = \text{votes}(U)|_{LU} = \frac{s(1-\delta)}{2\delta}$.

Claim 2: On $[s^{**}, 1]$, the L incumbent gets strictly less than 50% of the votes.

Proof: When entry takes place at RU, this follows from step 1.2. When entry takes place at U, $\text{votes}(L)|_{LU,U} = \text{votes}(L)|_{LU} < 0.5$ since $s^{**} > \frac{1-\delta}{\delta}$. QED

Step 1.6: Entry in front of (LU, R)

Entry is always profitable. For $s \leq \frac{1-\delta}{\delta}$, we can check graphically that $\text{votes}(L)|_{LU,R} > \frac{3}{8}$. For $s > \frac{1-\delta}{\delta}$, entry at LU is profitable.

Claim 1: For $s \leq s^*$, the R incumbent gets more than $\frac{3}{8}th$ of the votes and the LU incumbent gets less than $\frac{3}{8}$.

Proof: First note that entry at U is never profitable as long as $s \leq 1$. For $s \leq \frac{1-\delta}{\delta}$, an entry at LU or RU is not profitable either. By contrast, entry is profitable at both L and R. Now,

$$\begin{aligned} \text{votes}(L)|_{R,LU} &\geq \frac{3}{8} > \frac{1}{3} \\ \text{votes}(R)|_{R,LU} &< \frac{1}{2}\frac{1}{1+\delta} < \frac{1}{3} \end{aligned}$$

so entry takes places at L.

When $s \in (\frac{1-\delta}{\delta}, s^*]$, we can rule out entry at R and RU : a R platform would get less than 25% of the votes. As for RU, $\text{votes}(RU)|_{LU,R} < \text{votes}(RU)|_{L,R} \leq 0.25$ for all $s \leq s^*$ by step 1.1. Entry at LU or L is profitable.

Entry at L is the worst case scenario for the party at R. However, even then, R gets more than $\frac{3}{8}th$ of the votes (this follows from step 1.1 and the fact that LU takes more votes from L than from R). Since the entrant gets more than 25% of the votes, this leaves the LU incumbent with less than $\frac{3}{8}th$ of the votes. QED

Claim 2: When $s^{**} < 1$, the R incumbent gets strictly less than 50% of the votes on $s \geq s^{**}$.

Proof: Again, we need to determine where entry takes place. Entry at R and U can easily be ruled out (not profitable). Similarly, $\text{votes}(L)|_{LU,R} < \text{votes}(L)|_{LU,RU} \leq 25\%$ by step 1.2. So entry is only feasible at LU and RU. When entry takes place at RU, the R incumbent gets less than 25% of the votes by step 1.2. When entry takes place at LU, the R incumbent gets less than 50% of the votes since $s^{**} > \frac{1-\delta}{\delta}$. QED

Step 2: Entry deterring equilibria

Claim 1: On $s \leq s^*$, (L, R) is the unique Pareto optimal Nash equilibrium.

Proof: At (L, R), the L incumbent gets 50% of the votes. The claims in steps 1.3, 1.5 and 1.6 imply that deviations yield less than $\frac{3}{8}th$ of the votes and are therefore not profitable (a deviation to R triggers entry at L if $s < \frac{1-\delta}{\delta}$ and RU if $s > \frac{1-\delta}{\delta}$). This means that (L, R) is an equilibrium.

To show that it is unique, we need to rule out (RU, LU) and (U, LU) as potential equilibria. Step 1.6. implies that R in front of LU yields at least $\frac{3}{8}th$ of the votes. This is more than what RU

could yield (using steps 1.2). To rule out (U, LU) as an equilibrium, first note that U gets less than 50% of the votes *prior* to entry. By step 1.4, we need to consider 3 entry scenarios, securing at least $\frac{3}{8}th$ of the votes to the entrant. If entry takes place at RUor R, then U gets less than 25% of the votes (step 1.1) and R is a profitable deviation. Now suppose that entry takes place at L *and* U gets more than $\frac{3}{8}th$ of the votes. Then LU must be getting less than 25% of the votes and we can use Pareto optimality to rule out (U, LU). QED

Claim 2: When $s^{**} < 1$, (LU, RU) is the unique Pareto optimal Nash equilibrium for $s \in [s^{**}, 1]$.
Proof: At (LU, RU), the incumbents each get 50% of the votes. Steps 1.5 and 1.6. imply that a deviation to L or R is not profitable. A deviation to RU triggers entry at LU and yields 25% of the votes. Finally, $\text{votes}(U)|_{RU} < 0.5$ for $s < \frac{\delta}{1-\delta}$ even before entry.
To show that it is unique, we need to rule out (R, L) and (U, L) as potential equilibria. Entry is profitable in front of (R, L) (step 1.1). In front of (U, L) entry takes place at LU, RU or R (step 1.3) and both incumbents get less than 50% of the votes. We can then use Pareto optimality to rule out these potential equilibria. QED

Step 3: Patterns of entry accommodation

Claim: Close to s^*, incumbents optimally accommodate entry by choosing (L, R).
Proof: For $s > s^*$, step 1.1. implies that entry takes place in front of (L, R), yielding an expected vote share to incumbents of (arbitrarily) less than $\frac{3}{8}$. By claim 1 of step 2, any deviation from (L, R) remains unprofitable for s arbitrarily close to s^*. ■

Proof of proposition 5: The proof proceeds in 6 steps. Steps 1 and 2 determine the windows of entry in front (L, R) and (LU, R), an information that will be useful to compute best responses. Steps 3 to 5 check that some of the equilibria identified in proposition 4 remain equilibria when entry is allowed. Step 6 identifies the new equilibria that emerge because entry is allowed. At various points, the proof appeals to the symmetry of equilibrium with respect to $s = 1$.

Step 1: Entry in front of (L, R) takes place for $s > s_1(\delta) = \min\{\frac{1-\delta}{2\delta} + \frac{\sqrt{6(1-\delta^2)}}{4\delta}, \frac{9}{8}\} > \frac{1-\delta}{\delta}$.
Using the symmetry of the original configuration, we only need to check for two kinds of generic entry: U and LU.
We first consider an entry at U. For $s > 1$, we find that $\text{votes}(U)|_{R,L} = \frac{4s-3}{4s} > \frac{1}{3}$ for $s > \frac{9}{8}$.
Entry at LU is impossible for $s = \frac{1-\delta}{\delta}$ (this is seen graphically). For $s > \frac{1-\delta}{\delta}$, R is the alternative to beat and we can check graphically that $\text{votes}(LU)|_{R,L} > \text{votes}(R)|_{LU,L}$ iff $\text{votes}(L)|_{R,LU} = \frac{3(1-\delta)}{8\delta s}$ is smaller than twice the triangular area to the upper right of the RL indifference locus, that is[17]

$$\frac{3(1-\delta)}{8\delta s} < \frac{2(2\delta s - 1 + \delta)^2}{8\delta(1+\delta)s} \qquad \text{i.e.}$$

[17] The formula used assumes that $s < \frac{1}{\delta}$, which can be checked expost.

$$s \quad > \quad \frac{1-\delta}{2\delta} + \frac{\sqrt{6(1-\delta^2)}}{4\delta}$$

Putting these two elements together, we conclude that entry in front of (R, L) takes place for $s > s_1(\delta) = \min\{\frac{1-\delta}{2\delta} + \frac{\sqrt{6(1-\delta^2)}}{4\delta}, \frac{9}{8}\}$. In both cases, it leads to a zero payoff to incumbents.

Step 2: Entry in front of (LU, R) takes place for $s > s_2(\delta) = \min\{\frac{4(2-\delta)}{9\delta}, \frac{1+3\delta-\delta^2-\sqrt{1+6\delta+4\delta^2-15\delta^3-14\delta^4}}{3\delta^2}\}$.

Claim 1: We only need to consider entry at LD, D or RD.

Proof: Entry at L is impossible (R would get more votes). Whenever entry at LU is profitable so is an entry at LD. Hence we can focus without loss of generality on LD. The maximum R gets is when $s = 0$ but even then it gets $\frac{1}{1+\delta} < \frac{2}{3}$ of the votes so we can rule it out. RU would get less votes than LU. Entry at RD is profitable (and yields $\frac{1}{2}$) for $s > \frac{1}{\delta}$. Whenever entry at U is profitable, so is entry at D. Hence we can ignore U. QED

Claim 2: Entry at LD is profitable for $s > \frac{4(2-\delta)}{9\delta} > \frac{1-\delta}{\delta}$.

Proof: To see this, we determine the indifference loci

$$\begin{aligned} y|_{LU,R} &= -\frac{1}{s} + \frac{(1+\delta)}{\delta s}x \\ y|_{LD,R} &= \frac{1+s}{s} - \frac{(1+\delta)}{\delta s}x \end{aligned}$$

With this, we get votes$(R)|_{LU,LD} = \frac{4-3\delta s}{4(1+\delta)} < \frac{1}{3}$ for $s > \frac{4(2-\delta)}{9\delta}$. QED
(It can be checked that $\frac{4(2-\delta)}{9\delta} < \frac{1}{\delta}$ so we can ignore entry at RD).

Claim 3: Entry at D is profitable for $s > \frac{1}{(1+3\delta-\delta^2)}\left(3\delta^2 + \delta\sqrt{6\delta^2+3+9\delta}\right)$.

Proof: The relevant indifference loci are described by

$$\begin{aligned} y|_{LU,D} &= \frac{s-\delta}{(1+\delta)s} + \frac{\delta}{(1+\delta)s}x \\ y|_{LU,R} &= -\frac{1}{s} + \frac{(1+\delta)}{\delta s}x \\ y|_{D,R} &= 1 - \frac{x}{s} \end{aligned}$$

These intersect at $x = \frac{\delta(1+s)}{1+2\delta}$ and $y = \frac{s-\delta+\delta s}{1+2\delta}$. For $s \in (\delta, \frac{1}{\delta})$, they are represented in figure 6. Note that the two triangles making up LU's votes are of equal surface, that is, relative to the (R, D) configuration, LU takes as many votes from D as from R. Given that votes$(D)|_R < 0.5$ for all $s < 1$, votes$(D)|_{R,LU} <$ votes$(R)|_{D,LU}$ for all $s < 1$ and entry at D is impossible. Also this means that for $s > 1$, LU is the alternative for D to beat. Using figure 6,[18]

$$\text{votes}(LU)|_{R,D} \quad = \quad \frac{\delta^2(1+s)^2}{(1+\delta)(1+2\delta)s}$$

[18] These expressions are derived for $s \in [1, \frac{1}{\delta}]$. Given claim 1 this is the relevant range.

$$\text{votes}(D)|_{R,LU} = \frac{2s-1}{2s} - \frac{\delta^2(1+s)^2}{2(1+\delta)(1+2\delta)s}$$

Solving for when these expressions are equal we get that $\text{votes}(D)|_{R,LU} > \text{votes}(LU)|_{R,D}$ for

$$s > \frac{1+3\delta-\delta^2-\sqrt{1+6\delta+4\delta^2-15\delta^3-14\delta^4}}{3\delta^2}$$

This completes the proof. QED

[insert figure 6 here]

Step 3: (L,R) is a PSNE for $s \le \frac{1-\delta}{\delta}$.

This is a direct consequence of step 1 and proposition 4.

Step 4: (LU,RD) and (LD,RU) are PSNE for $s \in [\frac{1-\delta}{\delta}, \frac{\delta}{1-\delta}]$.

From proposition 4, we simply need to check that entry is not profitable. Given the symmetry of the original platform configuration, there are two kinds of entry to check for: on a generalist platform or at L. We can check graphically that an entry on a generalist platform would get only 25% of the votes so it can be ruled out. At $s = \frac{1-\delta}{\delta}$, entry at L is unprofitable (this is seen graphically). Hence, it is impossible for all $s \ge \frac{1-\delta}{\delta}$. By symmetry, entry at U or D is impossible for all $s \le \frac{\delta}{1-\delta}$.

Step 5: (LU,RU) is a PSNE for $s \in [\frac{1-\delta}{\delta}, \min\{1, \frac{9\delta}{4(2-\delta)}\}]$.

Given proposition 4, we simply need to check that entry is not profitable over this interval. Using the symmetry of the initial configuration leaves us with 4 generic entries: L, LD, U and D. At $s = \frac{1-\delta}{\delta}$, we can check graphically that entry at L is not profitable (RU gets more votes) so we can rule it out for all $s \ge \frac{1-\delta}{\delta}$. Entry at LD is only profitable for $s > 1$. Whenever entry at U is profitable so is an entry at D. Finally, using claim 2 of step 2 together with symmetry, we find that entry at D is profitable for $s > \frac{9\delta}{4(2-\delta)}$. Solving for when $\frac{9\delta}{4(2-\delta)} = \frac{1-\delta}{\delta}$, we get $\delta = 0.54356$. We conclude that for $\delta > 0.54356$, (LU,RU) is an equilibrium over $[\frac{1-\delta}{\delta}, \min\{1, \frac{9\delta}{4(2-\delta)}\}]$.

By symmetry, (LU,LD) is a PSNE for $s \in [\max\{1, \frac{4(2-\delta)}{9\delta}], \frac{\delta}{1-\delta}]$.

Step 6: Other equilibria.

On $s < \frac{1-\delta}{\delta}$, R is the unique best response to L. By step 2, entry in front of (LU,R) does not take place over this interval so R remains a best response to LU. This rules out any potential equilibrium with a generalist platform. It remains to consider (U,D).

On $(\frac{1-\delta}{\delta}, \frac{\delta}{1-\delta})$, step 4 implies that the only potential other equilibria are (R,L), (U,L) and (U,D). (U,L) necessarily triggers entry and so it can be ruled out by Pareto optimality. There remain (R,L) and (U,D).

To determine under what conditions (R, L) is a Pareto equilibrium for $s > \frac{1-\delta}{\delta}$, we appeal to steps 1 and 2 above. Step 1 implies that (R, L) is robust to entry for $s \leq s_1(\delta)$, yielding a payoff of $\frac{1}{2}$ to each incumbent. By deviating to U or D player 1 gets at most $\frac{1}{2}$ (entry is always profitable). A deviation to L, LU or LD also triggers entry and yields a zero payoff to the deviator. Finally, we use step 2 to rule out a profitable deviation on RU or RD for $s > s_2(\delta) > \frac{1-\delta}{\delta}$. In other words, (R, L) can be sustained as a Pareto optimal PSNE for $s \in (s_2(\delta), s_1(\delta)]$ when this interval is well defined.

Investigating, we identify two such intervals: (1) for very low values of δ (< 0.5465), $s_1(\delta) = \frac{9}{8} > s_2(\delta)$ and entry in front of (LU, R) takes place at D and (2) for intermediate values of δ, $s_1(\delta) > s_2(\delta)$ again, $(s_2(\delta), s_1(\delta)] \subset (\frac{1-\delta}{\delta}, \frac{\delta}{1-\delta})$ and entry takes place at LD in front of (LU, R).

The intervals over which (U, D) is an equilibrium are derived by symmetry. ■

Proof of proposition 6: The proof proceeds in two steps. First, we determine for each possible platform pair, the pattern of entry by a third party, as well as its consequence for the vote share of incumbents. In practice, only (L, R) and (U, D) are robust to entry for some values of the parameter s, and for δ big enough, there is also an interval of s around 1 where (LD, RU) and (LU, RD) deter entry. Second, we prove that over these intervals these platform pairs constitute the only Pareto optimal PSNE.

Step 1: Windows of entry and of entry deterrence

Given the symmetry of the political space, there are 6 generic platform pairs to check for: (L, R), (L, LU), (L, RU), (L, U), (LD, LU) and (LD, RU).

Step 1.1: Entry in front of (L, R)

There are two (generic) types of entry in front of (L, R) : LU and U. Appealing to step 1.1 of the proof of proposition 3, we find that entry takes place in front of (L, R) for all

$$s > s^* = \min\{\frac{1 - 3\delta + \sqrt{9 + 2\delta - 7\delta^2}}{4\delta}, 1\} > \frac{1-\delta}{\delta}$$

By symmetry, entry in front of (U, D) takes place for all $s < \frac{1}{s^*}$.

Step 1.2: Entry in front of (L, LU)

It is easy to check graphically that either R or RU (or both) get more than $\frac{3}{8}th$ of the votes in front of (L, LU) so entry takes place for all values of s. By symmetry, no platform pair consisting of a single issue platform and an adjacent generalist platform is robust to entry for any value of s.

Claim: The LU incumbent gets less than 50% of the votes for $s \leq 1$.

Proof: When $s < \frac{1-\delta}{\delta}$, LU gets less than 50% of the votes in front of L *absent entry*. Therefore, it also gets less than 50% of the votes when entry takes place. To determine how much the LU

incumbent gets when $s > \frac{1-\delta}{\delta}$, we first need to find out where entry takes place in front (L, LU). We can easily rule out entries at L (not profitable), LU (dominated by an entry at LD - in both cases, the entrant has the same vote share as the LU incumbent, but in the second case, the L incumbent gets less) and R (dominated by an entry at RU). There remain the entries at LD, RU, RD, U and D. When entry takes place at LD, RU and RD, it is easily checked that the LU incumbent gets less than 50% of the votes (strictly in the two last cases[19]). Indeed, without L, the entrant and the LU incumbent would share the votes equally, and L takes votes from both platforms. Entry takes place at U if it yields more votes than the alternative entry strategies, that is, at least $\frac{3}{8}th$ of the votes. Since $s \leq 1$, $\text{votes}(L)|_{LU,U} \geq \text{votes}(U)|_{LU,L}$, so the LU incumbent must be getting strictly less than 25% of the votes. Finally, to determine the impact of an entry at D on the vote share of the LU incumbent, notice that the L incumbent is better off than when entry takes place at LD, and by assumption the entrant gets more votes than if he had entered at LD. As a result, the LU must be worse off. QED

Step 1.3: Entry in front of (L, RU).

Entry is always profitable. For example, we can check graphically that R collects more than $\frac{3}{8}th$ of the votes on $s < \frac{1-\delta}{\delta}$, and on $s > \frac{1-\delta}{\delta}$, RD gets more than 25% of the votes.

Claim: The RU incumbent gets strictly less than 50% of the votes for $s \leq 1$.

Proof: When $s \leq \frac{1-\delta}{\delta}$, RU gets less than 50% of the votes in front of L absent entry. Therefore, it also gets less than 50% of the votes when entry takes place. For $s > \frac{1-\delta}{\delta}$, we can rule out entries at L (not profitable), U (never profitable for $s \leq 1$ - by step 1.6 of proposition 3) and RU (dominated by an entry at RD). Entries at LU, LD, RD, R and D remain to be considered. When entry takes place at LU, LD or RD, the RU incumbent gets less than 50% of the votes. Indeed, without L, the entrant and the RU incumbent would share the votes equally, and L takes votes from both platforms. If entry is profitable at R, RU gets again less than 50% since L must be getting more votes than R. Finally, to determine the impact of an entry at D, notice that the L incumbent is better off than when entry takes place at LD, and by assumption the entrant gets more votes than if he had entered at LD. As a result, the RU incumbent must be worse off. QED

Step 1.4: Entry in front of (L, U)

Entry at R gets more than $\frac{3}{8}th$ of the votes for $s < 1$ and entry at D gets more than $\frac{3}{8}th$ of the votes for $s > 1$ so entry cannot be deterred.

Step 1.5: Entry in front of (LD, LU)

[19] When $s \geq \frac{1-\delta}{2\delta}$, an entry at LD in front of (L, LD) yields a 50% vote share for the entrant (and the LD incumbent). The L incumbent gets no vote.

It is easy to check graphically that an entry at RU or RD collects more than 25% of the votes so entry cannot be avoided. Given the symmetry of the incumbents' positions and the assumption that when the entrant is indifferent, he randomizes with equal probability, this means that the incumbents get each less than $\frac{3}{8}th$ of the votes.

Step 1.6: Entry in front of (LD, RU)

An entry on a mixed platform gets only 25% of the votes and so can be ruled out. It remains to check when entry at L (or R) is profitable (Entry at U or D is symmetric).

At $s = \frac{1-\delta}{\delta}$, votes$(L)|_{LD,RU} > \frac{1}{4}$. For $s \in (\frac{1-\delta}{\delta}, \frac{1}{\delta})$,

$$\text{votes}(L)|_{LD,RU} = \frac{1-\delta}{2\delta s} - \frac{\delta(1+s)^2(1-\delta)}{4(1+\delta)s}$$

Solving for when this expression is equal to $\frac{1}{4}$, we get that entry at L is profitable for all

$$s < s^{**} = \frac{-1 - 3\delta + 2\delta^2 + \sqrt{9 - 2\delta - 7\delta^2 + 4\delta^3}}{2\delta(1-\delta)}$$

It can be checked that $s^* < s^{**}$ everywhere (figure 5 in the text) and that $s^{**} < 1$ for $\delta > 0.55$. By symmetry, entry at U or D is profitable for all $s > \frac{1}{s^{**}}$. In all cases, the incumbents get less than $\frac{3}{8}th$ of the votes in expectation.

Figure 7 summarizes these results. For $s < s^*$, (R, L) is the only platform pair that deters entry. For $s > \frac{1}{s^*}$, (U, D) is the only platform pair that deters entry. When $s^{**} < 1$, (LU, RD) and its symmetric configuration (LD, RU) deter entry on $(s^{**}, \frac{1}{s^{**}})$.

[insert figure 7 here]

Step 2: Entry deterring equilibria

Claim 1: On $s \leq s^*$, (L, R) is the unique Pareto optimal Nash equilibrium.

Proof: At (L, R), the L incumbent gets 50% of the votes. Deviation is not profitable (using step 1.2, 1.3 and that fact that U and D get less than 50% of the votes *absent entry* in front of R).

To prove uniqueness, we need to rule out equilibria (1) where both incumbents choose a generalist platform, (2) (U, D), and (3) (U, LU) and (D, LU). In the first case, entry always takes place (steps 1.5 and 1.6) and given the symmetry of the platform choice both incumbents end up with a vote share lower than 50%. A similar argument applies for (U, D). In both cases, these equilibria are ruled out by Pareto optimality. To rule out the equilibria in (3) we appeal again to pareto optimality (party 1 always gets less than 50% of the votes, even prior to entry, entry is inevitable and leads to a vote share for party 2 lower than 50%). QED

Claim 2: On $(s^{**}, \frac{1}{s^{**}})$, (LU, RD) and (LD, RU) are the only Pareto optimal Nash equilibria.

Proof: Consider a party's vote share if its competitor chooses RD. It gets 50% by selecting LU. Since $\frac{1-\delta}{\delta} < s^{**} < \frac{1}{s^{**}} < \frac{\delta}{1-\delta}$, all other alternative platforms yield at most a 50% vote share *absent entry* (proposition 4). Now, steps 1.2, 1.3, 1.5 and 1.6 imply that entry is unavoidable so the deviator is strictly worse off. Hence (LU, RD) and (LD, RU) are equilibria.

To prove that they are the only equilibria, we need to rule out (R, L), (U, L) and (U, D) as potential equilibria. Steps 1.1 and 1.4 imply that entry cannot be avoided. Pareto optimality does the rest. QED

Claim 3: On $s > \frac{1}{s^{*}}$, (U, D) is the unique Pareto optimal Nash equilibrium.

Proof: This follows from claim 1 and symmetry. ■

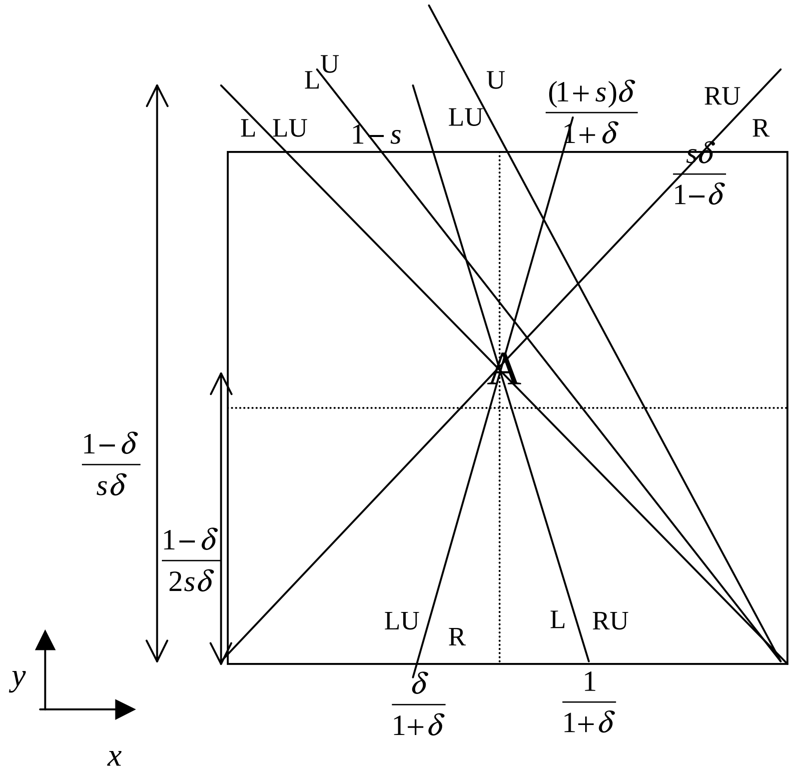

Figure 1: Indifference lines for $\frac{1-\delta}{2\delta} < s < \frac{1-\delta}{\delta}$ (the first inequality is necessary to get A inside the square).

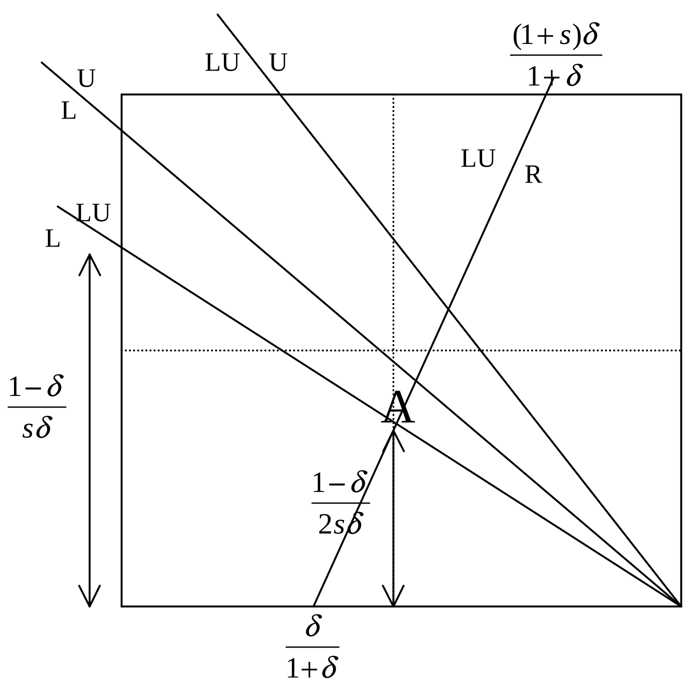

Figure 2: Indifference lines for $\frac{1-\delta}{\delta} < s < \frac{1}{\delta}$

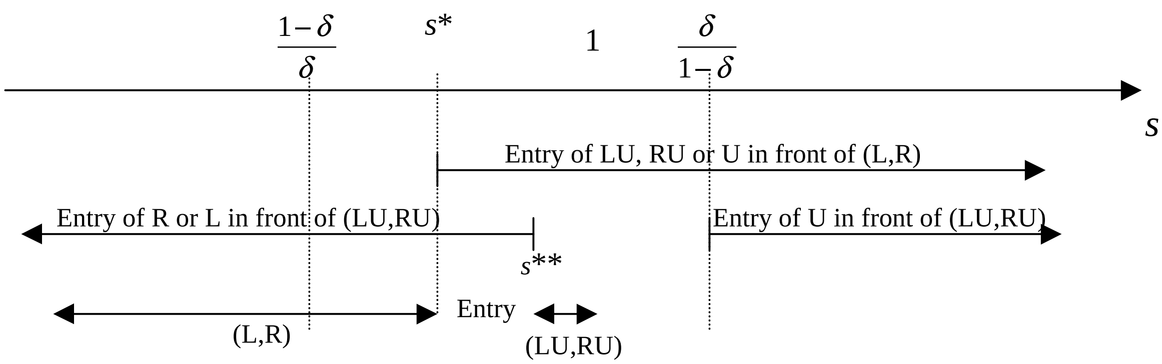

Figure 3: Equilibrium under proportional rule and entry

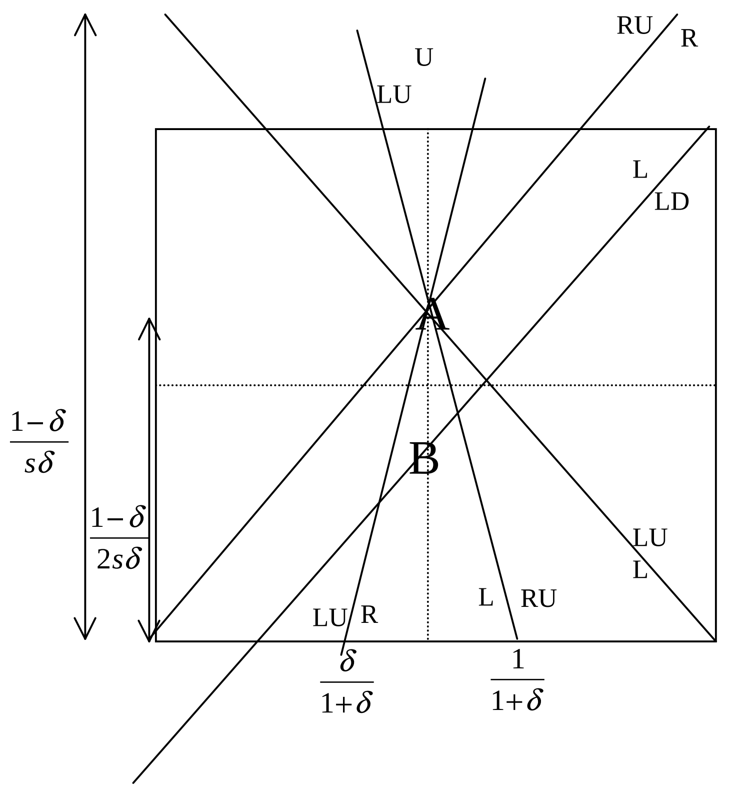

Figure 4: $\frac{1-\delta}{2\delta} < s < \frac{1-\delta}{\delta}$

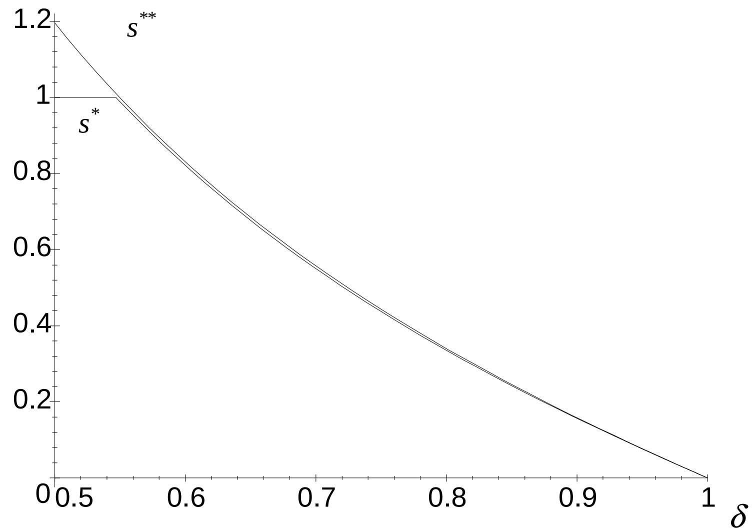

Figure 5

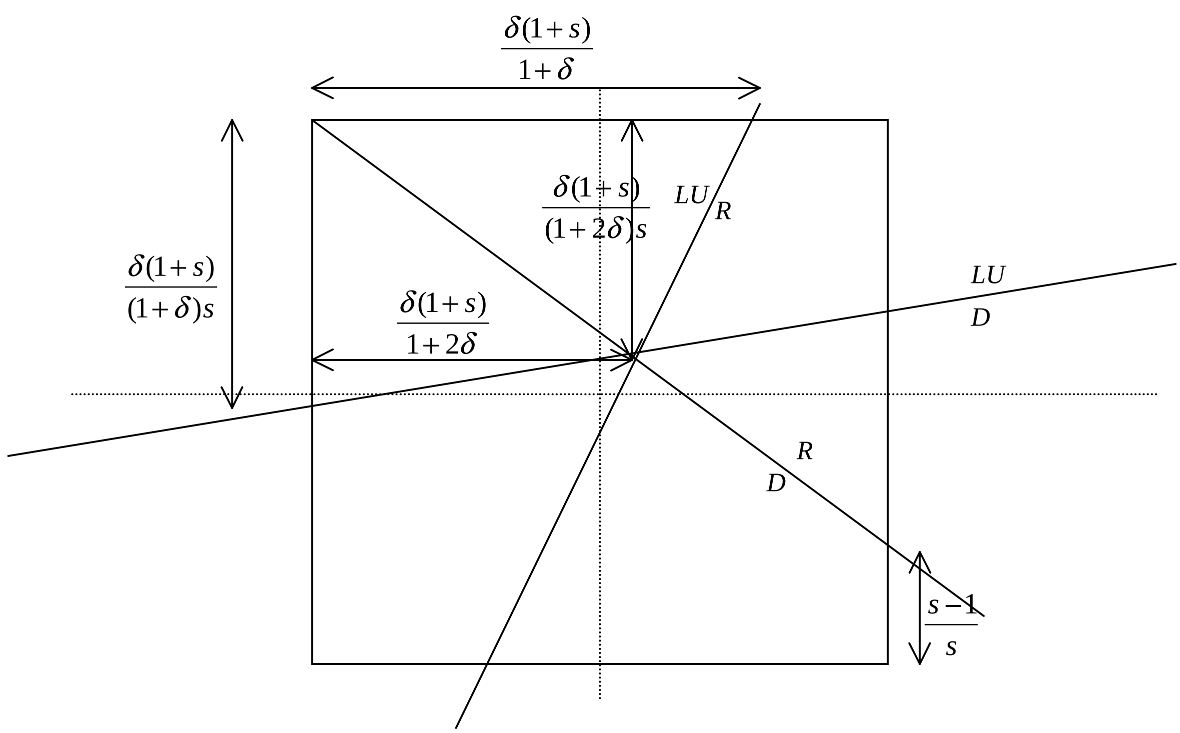

Figure 6

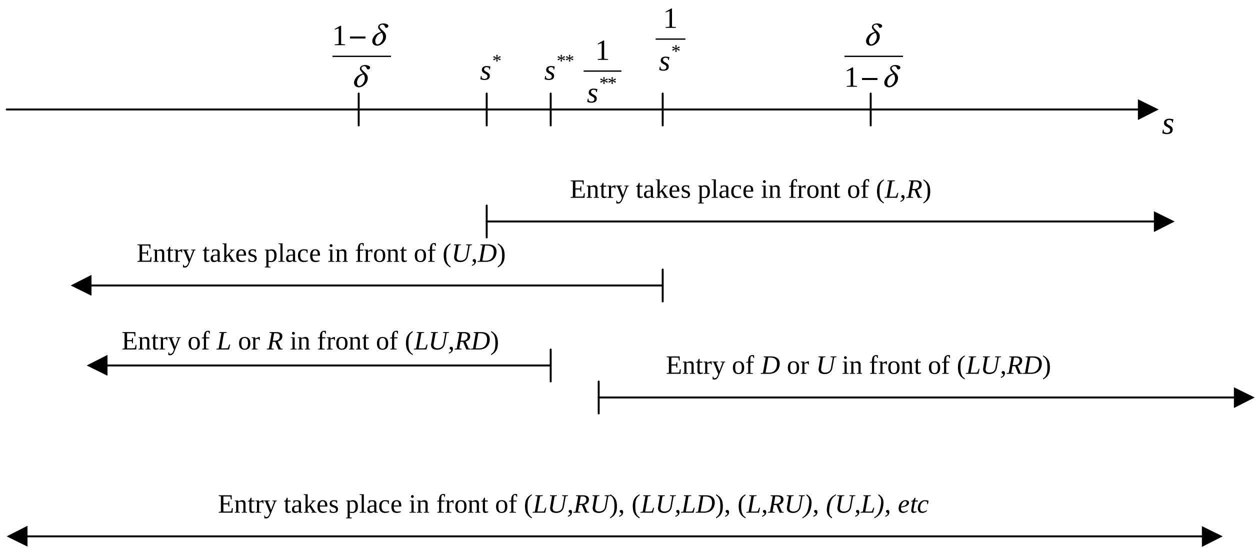

Figure 7: Entry and entry deterrence under the PR rule