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Abstract

We investigate firms' incentives for cost reduction in the first price sealed bid auction, a format largely used for procurement. A central feature of the model is that we allow firms to be heterogeneous. Though private value first price auctions are not games with monotonic best responses, we find that for comparative statics purposes they behave like these games. In particular, firms will tend to underinvest in cost reduction because they anticipate fiercer head-on competition. Using the second price auction as a benchmark, we also find that the first price auction will elicit less investment from market participants. Moreover, both auction formats tend to favor investment by the current market leader and are therefore likely to reinforce asymmetries among market participants.

Keywords: Asymmetric auctions, endogenous distributions, investment incentives.
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1. Introduction

Consider the following procurement situation. Several firms are competing for a government contract through a sealed bid procedure. These firms are not necessarily equally competitive a priori, and, indeed, in many procurement situations, some firms do have a clear cost advantage and are aware of these intrinsic differences among them (for systematic evidence see for instance Carnaghan and Bracewell-Milnes, 1993, Bajari, 1998 and 1999, Porter and Zona, 1999).

In this paper, we are interested in understanding how the rules for the auction are likely to affect the dynamics of competition and market structure in such a procurement setting. More specifically, we investigate the incentives for cost reduction under the first and the second price auctions. That is, we step back and, rather than focus on the existence of asymmetries in procurement auctions, we ask what the incentives are for firms to improve their competitive situations relative to their rivals.[1]

A standard framework to study this question is to start with a two-stage game, where firms invest in cost reduction in the first stage and compete through a procurement auction in the second stage. As Fudenberg and Tirole (1984) and Bulow, Geanakoplos and Klemperer (1985) have shown, it then turns out that the nature of strategic interactions in the second-stage game (i.e. whether best responses are increasing or decreasing) is a key element of the analysis.

Unfortunately, this approach does not work for the first price auction. Indeed, though the complete information analog of the procurement auction (a Bertrand game) has increasing best response schedules, best responses in the space of strategies for the first price auction are non-monotonic. To overcome this difficulty, we impose the condition of equilibrium and compare equilibria directly.

Our approach is based on the characterization of equilibrium behavior in first price auctions as the solution to a system of differential equations (Maskin and Riley, 1996, Lebrun, 1999). Our thought experiment is the following: Consider an initial configuration of bidders for a procurement contract. Suppose that one of them has the opportunity to upgrade his technology in the sense of generating a "better" ex-ante distribution of costs. What are his incentives to do so if this investment is observed by his competitors?

[1] Efficiency investments are important in several markets, though the best example is probably the defense industry. In his study of defense procurement, Lichtenberg (1988) finds evidence that competitive procurement stimulates considerable private R&D investment.

In section 3, we show that the investor's opponents will collectively bid more aggressively after the upgrade than before (propositions 1 and 2). Therefore, any given bid by the investor has a lower chance to win the market after the upgrade. In terms of investment incentives, this means that bidders in the first price auction will tend to underinvest prior to the procurement stage according to simple static efficiency arguments (holding competitors' strategies fixed). Put differently, firms will invest less in the case of observable (overt) investment than in the case of covert investment.

In section 4, we investigate how the first and second price auctions compare when it comes to pre-auction investment. We find that the first price auction will induce less investment than the second price auction (proposition 3), and that this effect is related to the properties of the first price auction identified in propositions 1 and 2. Moreover, we show that the second price auction yields the socially optimal level of investment incentives (proposition 4).

Section 5 is more exploratory in nature. There, we investigate how our results could translate in a fully dynamic setting. In particular, an important question for procurement authorities is whether given market institutions tend to reinforce existing asymmetries among market participants. Our examples suggest that both auction rules tend to favor a pattern of increasing dominance by the current market leader.

Our research contributes to three strands of the literature: the recent literature on asymmetric first price auctions, the work on investment incentives in symmetric auctions and the broader field of market dynamics in oligopoly settings.

Existence and uniqueness of the equilibrium in the independent private value first price auction have been proved under increasingly general assumptions by Lebrun (1996) and Maskin and Riley (1996 and 1999a). Maskin and Riley (1999b) and Li and Riley (1999) provide more precise characterizations of the equilibrium when a stochastic dominance relationship exists among bidders. The results in section 3 are closest to Lebrun (1998). Our first two propositions extend his comparative statics result in several directions. First, we do not restrict bidders to have a common support for their distributions of costs. Second, we allow for risk aversion and endogenous quantity. Finally, and more importantly, our results apply to $N > 2$ bidders.

There is a series of papers that study investment incentives in first price auctions for the case of ex-ante symmetric firms and simultaneous investment (the key reference here is Tan (1992)). A finding of this literature is that the firms invest the same amount and therefore remain symmetric ex-post. A "Revenue Equivalence" kind of result is also shown to hold whereby the first and second price auctions elicit the same amount of investment by market participants and therefore yield the same expected revenue for the procurement authority (Tan, 1992). By contrast, we find that the first price auction generates less investment than the second price auction when firms are allowed to be heterogeneous and investment is sequential.[2] Finally, section 3 also suggests that the fact that this literature has focused on the Nash equilibrium of the investment game (instead of imposing subgame perfection) is crucial in obtaining the result that ex-ante symmetric firms automatically invest the same amount and remain symmetric ex-post.

Finally, this paper relates to the literature on strategic investment and market dynamics in industrial organization. This literature has by and large confined itself to complete information settings. We offer a first investigation of the question of market dynamics under incomplete information. There are two ways to see the contributions of our paper to this literature. First, allowing for uncertainty about the costs of the competitors provides some robustness check for the predictions of models with complete information. Our results suggest that, for comparative statics purposes, the first price auction behaves very much like a game with strategic complementarities. Therefore, we expect much of the insight and intuition gained in pricing games under complete information to transpose to the first price auction.[3] For instance, the results in section 5 suggest a pattern of increasing market dominance, which is a common finding in that literature.

Second, auction environments have the advantage that they are well-defined for the modeler as well as for the participants. By changing the rules of the auction, we can start investigating the dynamic effect of specific market designs. This is important because, as our results in sections 4 and 5 suggest, market institutions

[2] The case of simultaneous investment for heterogenous firms remains an open question.

[3] Different approaches have been used to investigate the dynamics of competition and market structure. Some authors have modeled competition as patent races or sequences of patent races (see, for instance, Reinganum, 1985, Vickers, 1986, Grossman and Shapiro, 1987 or Tirole, 1989 for a summary). Another popular way to model the dynamics of competition is learning-by-doing (see, for instance, Dasgupta and Stiglitz, 1988). Athey and Schmutzler (1999) have recently provided a unifying framework to study market dynamics in oligopoly models. Notice that the kind of cost reduction considered in this paper corresponds to the "non drastic" innovation in that literature.

affect market structure and in turn, this feeds back into the effectiveness and efficiency of these institutions.

2. The model

In this section, we present the model and characterize its equilibrium. There is a single buyer (e.g. a government agency) in charge of procuring a given good or service. As in Hansen (1988), we allow quantities to be endogenous. Let $D(b)$ be the buyer's demand at price b. We make the following standard assumptions on demand:[4]

Assumption 1: $D(b) \geq 0, D'(b) \leq 0$ and increasing price elasticity $\frac{d}{db}[\frac{D'(b)b}{D(b)}] \leq 0$.

$N \geq 2$ firms take part in a first-price, sealed-bid auction for the procurement contract. That is, the contract is awarded to the firm offering to provide the good or service at the lowest price, and the winner is paid the per unit price she bid. Ties are resolved by a random draw among the lowest bidders.

Firms' constant marginal costs have support on $[\underline{c}_i, \overline{c}_i]$, where $0 \leq \underline{c}_i < \overline{c}_i$. They are independently distributed according to the twice continuously differentiable cumulative distribution function $F_i(.)$, with a density bounded away from zero on its support. These distributions are assumed to be common knowledge. They can be interpreted as representing the technology available to firms. Notice that we do not restrict bidders to have cost levels distributed on a common support. Firm i's profit when its cost is c_i and it makes a bid b is given by:

$$\pi_i(b, c_i) = \begin{cases} V_i((b - c_i)D(b)) & \text{if it wins} \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

Assumption 2: For all i, $V_i(0) = 0$, $V_i' > 0$ and $V_i'' \leq 0$.

Lemma 1: *Under assumptions 1 and 2, $\pi_i(b, c)$ is strictly log-supermodular in (b, c), i.e. $\frac{\partial}{\partial c}[\frac{\frac{\partial \pi_i}{\partial p}}{\pi_i}] > 0$ over the domain where $\pi_i > 0$.*

Proof. We first claim that, at any equilibrium, $D(b) + (b - c)D'(b) > 0$ for all b such that b is bid by some firm i. $D(b) + (b - c)D'(b) = 0$ corresponds to the first order condition of the monopolist facing demand $D(b)$. It trades

[4] These guarantee that the complete information monopolist problem is quasiconcave (see, e.g. Caplin and Nalebuff, 1991, proposition 11).

off the marginal benefit of increasing prices with the marginal cost of lost trade. In a procurement setting, increasing prices has an additional cost: the potential loss of the whole market. Therefore, $D(b)+(b-c)D'(b)$ must be strictly positive at any bid b submitted in equilibrium by some firm i.

Together with assumptions 1 and 2, this implies that:

$$\begin{aligned}
\frac{\partial}{\partial c}\left[\frac{\frac{\partial \pi_i}{\partial b}}{\pi_i}\right] &= \frac{1}{\pi_i}\frac{\partial^2 \pi_i}{\partial b \partial c} - \frac{1}{\pi_i^2}\frac{\partial \pi_i}{\partial b}\frac{\partial \pi_i}{\partial c} \\
&= \frac{1}{\pi_i}\underbrace{\{-V_i''[D(b)+(b-c)D'(b)]D(b) - V_i' D'(b)\}}_{\text{positive}} \\
&\quad + \frac{1}{\pi_i^2}\underbrace{(V_i')^2 D(b)[D(b)+(b-c)D'(b)]}_{\text{strictly positive}} \\
&> 0 \blacksquare
\end{aligned}$$

The recent results about existence and uniqueness of equilibrium in the first price auction form the basis for our analysis (see, for instance, Maskin and Riley, 1996 and 1999a). An equilibrium in this auction is described by an $N-$tuple of bidding functions $b_i : [\underline{c}_i, \overline{c}_i] \rightarrow \mathbb{R}_+$, $i = 1, ..., N$. For our purposes, it is convenient to look at the inverse bidding functions. We denote them by $\phi_i : \mathbb{R}_+ \rightarrow [\underline{c}_i, \overline{c}_i]$, $i = 1, ..., N$.

Maskin and Riley (1996 and 1999a) have shown that there exists a unique equilibrium in this environment.[5] The corresponding equilibrium inverse bidding functions $\phi_i(.)$ have support on $[l_i, u]$, $i = 1, ..., N$, and solve the system of differential equations

$$\sum_{j \neq i} \frac{F_j'(\phi_j(b))\phi_j'(b)}{1 - F_j(\phi_j(b))} = \frac{\frac{\partial}{\partial b}\pi_i(b, \phi_i(b))}{\pi_i(b, \phi_i(b))} \qquad i = 1, ..., N \tag{2.2}$$

with boundary conditions $F_i(\phi_i(l_i)) = 0$, and with u, the maximum equilibrium winning bid, determined uniquely by the following lemma:

[5] If one bidder's support is very far to the left of all the other bidders' supports, then the equilibrium is degenerate. We shall ignore this case.

For the $N > 2$ case, Maskin and Riley (1996) require an additional condition on the payoff functions to ensure uniqueness. It is satisfied if all bidders are risk neutral or if they have the same CARA or CRRA utility function.

Lemma 2 : **Upper bound of the support of the equilibrium distribution of winning bids** (adapted from Maskin and Riley, 1996): *Suppose that the distributions* $(F_1, ..., F_N)$ *are ordered so that* $\bar{c}_1 \leq \bar{c}_2 \leq ... \leq \bar{c}_{N-1} \leq \bar{c}_N$. *Then, if* $\bar{c}_1 = \bar{c}_2 = \bar{c}$, *then* $u = \bar{c}$. *Otherwise,* u *solves*

$$\min\{\arg\max_b \pi_1(b, \bar{c}_1) \prod_{i \neq 1} (1 - F_i(b))\} \in (\bar{c}_1, \bar{c}_2) \tag{2.3}$$

If $u < \bar{c}_i$ for some i, we can consider that, for any realization of cost $c_i > u$, firm i bids its own cost (and never wins) or stays out of the auction.

Notice that the lower bounds of the supports of equilibrium bids are endogenously determined by the boundary condition of (2.2). In general, those minimum bids need not be common to all firms but $\min\{l_i\}$ must be common to at least two of them. Minimum equilibrium bids depend on the lower bounds of the support of costs, $\underline{c}_i$, and it can be shown that $l_i \leq l_j$ iff $\underline{c}_i \leq \underline{c}_j$. Finally, it can also be shown that the equilibrium inverse bidding functions are strictly increasing and twice differentiable on their support. For further details on the structure of the equilibrium, we refer the interested reader to Maskin and Riley (1996).

To see why $\phi_i(.)$, $i = 1, ..., N$ are indeed equilibrium inverse bidding functions, it suffices to realize that equations (2.2) are the first-order conditions of the firms' pseudo-concave maximization problem. That is, firm i with cost level c_i will choose its bid by solving the problem

$$\max_b \pi_i(b, c_i) \underbrace{\prod_{j \neq i} (1 - F_j(\phi_j(b))}_{\text{probability of winning}}$$

Noting that, at the optimal value of b, we have $c_i = \phi_i(b)$ for all i, equations (2.2) follow.

We want to understand how the equilibrium in the procurement auction is affected by changes in the distribution of cost levels for one firm. For that purpose, we need to define a proper notion of "better" distribution of costs. The following definition provides such a partial ordering:

Definition 1: Consider two cumulative distribution functions F and $\widetilde{F}$ with bounded support. We shall say that $\widetilde{F} \succ F$ if, for all c, c' such that $c' > c$,

$$\frac{1 - \widetilde{F}(c')}{1 - \widetilde{F}(c)} < \frac{1 - F(c')}{1 - F(c)} \tag{2.4}$$

whenever these expressions are well defined.

The requirement in (2.4) is one of conditional stochastic dominance. It means that, conditioning on any minimum level of costs, it is always more likely for F to yield a higher cost level than it is for $\widetilde{F}$. It can be shown that this condition implies that there is a relation of first-order stochastic dominance between the distributions: $F(c) < \widetilde{F}(c)$ for all c on the interior of their common support. Note that, given our differentiability assumption, (2.4) can be rewritten as

$$\frac{d}{dc}\left(\frac{1-\widetilde{F}(c)}{1-F(c)}\right) < 0$$

or, in terms of hazard rates,

$$\frac{\widetilde{F}'(c)}{1-\widetilde{F}(c)} > \frac{F'(c)}{1-F(c)} \tag{2.5}$$

for all c where these expressions are well-defined.

Definition 1 (or its variant for the standard auction) has become quite common in the asymmetric first price auction literature (see Lebrun, 1998, Maskin and Riley, 1999b, or Li and Riley, 1999, for instance). In practice, it is a little bit stronger than needed and a weak inequality in (2.5) would do for our purpose. However, it would also lengthen the proofs without adding any new insight, hence our decision to keep the stronger version. Comparing (2.5) with (2.2), it should also be clear that this is a natural way to order distributions for the first price auction.

In what follows, whenever there is a shift in firm i's distribution of cost levels from F_i to $\widetilde{F}_i \succ F_i$, we will refer to such a shift as an *upgrade*, and to firm i as the *upgrader* . Examples of distributional upgrades that satisfy definition 1 include: (1) Additional random draws from the same distribution ($\widetilde{F}(c) = 1-(1-F(c))^x$ for $x > 1$),[6] which is a common way of modeling R&D; (2) Shifts

[6] Note that $\widetilde{F}(c)$ does not satisfy the assumption of strictly positive density made above since $\widetilde{F}'(c) = x(1-F(c))^{x-1}F'(c) = 0$ for $c = \overline{c}$, the upper bound of the F distribution.
This assumption is required for the uniqueness result of Maskin and Riley (1996). However, it is not crucial. Maskin and Riley need this condition to prove that the slope of the inverse bid functions at u is bounded. This can be shown to hold here too as long as $F'(c) > 0$ for all c. To make things simple, suppose that there are 2 firms and firm 1 has distribution

of distributions to the left (i.e. $\widetilde{F}(c) = F(c+a)$ for $a > 0$) for distributions with a strictly increasing monotone hazard rate.[7] This can be a convenient way to model a locational investment when transportation costs are important, and (3) Distributional contractions with a fixed lower end of the support ($\widetilde{F}(c) = \theta F(c)$ for $\theta > 1$ and $c \in (\underline{c}, F^{-1}(1/\theta))$, which could represent the shift to a more reliable technology. Distributional stretches with a fixed upper end of the support $(1 - \widetilde{F}(c)) = \theta[1 - F(c)]$ for $\theta < 1$ and c in the support of F) satisfy the weaker requirement of first order stochastic dominance and weakly higher hazard rate.

3. Comparing equilibria

To determine firms' investment incentives when investment is observable, we need to understand how a distributional upgrade by one firm affects the resulting equilibrium in the procurement auction. More precisely, starting from an initial configuration of firms $(F_1, ..., F_N)$, suppose that firm j has the opportunity to upgrade its distribution of costs to $\widetilde{F}_j \succ F_j$. How does the equilibrium in this new auction $(\widetilde{F}_j, F_{-j})$ compare with that of the initial one, (F_j, F_{-j})?

Referring back to (2.2), it is easy to see that such an investment by firm j shifts its opponents' best response schedules upwards (remember, by lemma 1, the right hand side of (2.2) is increasing in ϕ_i), that is, they now react more aggressively to any given bidding behavior of firm j. If auctions were games with increasing best responses, this would be the end of the story. Indeed, the "commitment" of j's opponents to bidding more aggressively together with increasing best responses would result in more aggressive bidding behavior by all participants in the "post-upgrade" equilibrium (see, e.g., Milgrom and Roberts, 1994 who generalize the earlier analyses by Fudenberg and Tirole, 1984 and Bulow, Geanakoplos

$F_1(c) = 1 - (1 - F(c))^x$. We claim that $\phi_2'(\overline{b}) = 1 + \frac{1}{x}$. To see this, consider firm 2's FOC:

$$\frac{1}{b - \phi_2(b)} = \frac{F_1'(\phi_1(b))\phi_1'(b)}{F_1(\phi_1(b))}$$

Solving for ϕ_1' and using the definition of F_1, we get: $\lim_{b\to\overline{b}} \phi_1'(b) = \lim_{b\to\overline{b}} \frac{(1-F(\phi_1(b)))}{xF'(\phi_1(b))(b-\phi_2(b))}$ $= \lim \frac{-F'(\phi_1)\phi_1'}{xF''(\phi_1)(b-\phi_2)+x(1-\phi_2')F'(\phi_1)} = \frac{-\phi_1'}{x(1-\phi_2')}$ (using l'Hopital's rule). Solving for ϕ_2', we get $\phi_2'(\overline{b}) = 1 + \frac{1}{x}$ as claimed.

[7] Distributions that satisfy this condition include the uniform, the normal, the logistic, the extreme value, the exponential and the χ^2 distributions, as well as the Weibull, γ and β distributions for some parameter values (see Bagnoli and Bergstrom, 1989).

and Klemperer, 1985). Unfortunately, first price auctions are not games with monotonic best responses as the following example illustrates.

Example 1: Consider the following auction environment. Two risk neutral firms bid for a single object. Firms' costs are distributed independently and uniformly over the interval $[0, 1]$. This is a symmetric first price auction and it is easy to check that the equilibrium bidding functions are $b_i(c_i) = \frac{1+c_i}{2}$ for $i = 1, 2$ (this means that $\phi(b) = 2b - 1$). Now suppose that firm 1 suddenly bids more aggressively: $\widehat{b}_1(c_1) = \sqrt{c_1} < \frac{1+c_1}{2}$ (this corresponds to an inverse bidding function of $\widehat{\phi}_1(b) = b^2$). Firm 2's best response solves $\max_b$ $(b - c_2)(1 - b^2)$. Let $\widehat{\phi}_2(b)$ be the inverse bid function that corresponds to this optimization problem. $\widehat{\phi}_2(b) = \frac{3b^2-1}{2b}$ and has support on $[1/\sqrt{3}, 1]$. The interesting element here is that though firm 1 has become *more aggressive*, $\widehat{\phi}_1(b) > \phi(b)$, firm 2's best response to $\widehat{\phi}_1, \widehat{\phi}_2$, is *less aggressive* than his best response to ϕ.[8] Examples where firm 2 would respond to a more aggressive behavior of firm 1 by being more aggressive can similarly be generated.

Example 1 runs a bit counter our intuition about the nature of competition in the private value first price auction. However, on a second thought, it should not be so surprising. Indeed, firm 2's maximization problem is identical to that of the monopolist who faces demand $D(b) = (1 - F_1(\phi_1(b))$ and has constant marginal cost c_2. Firm 2 in example 1 is then analogous to the monopolist who might respond to a decrease in demand by raising prices. Put differently, in the same way as a monopolist cares about the elasticity and not the level of the demand he is facing, bidders in the first price auction care about the hazard rate of their opponents' highest bid (see (2.2)).

Nevertheless, example 1 is problematic because it rules out the kind of comparative statics exercise based on the slope of best responses. The alternative approach that we take here is to impose the condition of equilibrium, and *compare the equilibria* (prior and after the upgrade) *directly.* In this section, we want to show that the kind of comparative statics that did not hold for best responses (more aggressive response to more aggressive behavior) does hold at equilibrium.

Consider the two configurations (F_j, F_{-j}) and $(\widetilde{F}_j, F_{-j})$ with $\widetilde{F}_j \succ F_j$. Denote their respective equilibria by (ϕ_j, ϕ_{-j}) and $(\widetilde{\phi}_j, \widetilde{\phi}_{-j})$. Let l and u (respectively $\widetilde{l}$

[8] Indeed, $\widehat{\phi}_2(b) = \frac{3b^2-1}{2b}$ is less than $\phi(b) = 2b - 1$ iff $3b^2 - 1 < 4b^2 - 2b$ or $b^2 - 2b + 1 > 0$.

and $\widetilde{u}$) be the lower and upper bounds of the equilibrium bids under (F_j, F_{-j}) (respectively $(\widetilde{F}_j, F_{-j})$). For later use, we also define $p_i(b) = F_i(\phi_i(b))$ i.e. the probability that firm i submits a bid lower than b in configuration (F_j, F_{-j}). $\widetilde{p}_i(b)$ is similarly defined.

In the asymmetric first price auction, we cannot in general solve for the equilibrium explicitly. Therefore, we need to resort to firms' FOCs in order to compare equilibria. Though the actual proofs tend to be lengthy, the gist of the argument is actually quite simple. We want to show that j's opponents will tend to bid more aggressively if j makes the investment. In other words, we want to rank the p_i functions defined above (indeed, if $\widetilde{p}_i(b) > p_i(b)$ for all b, then we can conclude that firm i bids more aggressively after the investment). With $N = 2$ and a slight abuse of notation, (2.2) becomes:

$$\frac{p_i'(b)}{1 - p_i(b)} = \frac{\frac{\partial}{\partial b}\pi_j(b, \phi_j)}{\pi_j(b, \phi_j)} \qquad i \neq j \tag{3.1}$$

where the term on the right-hand side is increasing in ϕ_j (from lemma 1). To prove that $\widetilde{p}_2(b) > p_2(b)$ everywhere, we show that this holds on an interval close to the minimum bid and then we use the FOCs to rule out a crossing later on. For example, suppose that at some point $\widehat{b}$, $p_2(\widehat{b}) = \widetilde{p}_2(\widehat{b})$ and $p_2'(\widehat{b}) > \widetilde{p}_2'(\widehat{b})$, that is, p_2 is crossing $\widetilde{p}_2$ from below at $\widehat{b}$. Then, firm 1's FOC (3.1) implies that $\phi_1(\widehat{b}) > \widetilde{\phi}_1(\widehat{b})$. In other words, using firms' FOCs, we are able to deduct from what is happening to one given firm's behavior across configurations, what is happening to the other firm's behavior. The rest of the argument usually makes use of the relationship between F_j and $\widetilde{F}_j$ to get a contradiction.

For $N > 2$ firms, the equations in (2.2) can be rewritten as:

$$\sum_{j \neq i} \frac{p_j'(b)}{1 - p_j(b)} = \frac{\frac{\partial}{\partial b}\pi_i(b, \phi_i)}{\pi_i(b, \phi_i)} \tag{3.2}$$

Solving for $\frac{p_j'(b)}{1-p_j(b)}$ yields:

$$(n-1)\frac{p_j'(b)}{1 - p_j(b)} = \sum_{i \neq j} \frac{\frac{\partial}{\partial b}\pi_i(b, \phi_i)}{\pi_i(b, \phi_i)} - (n-2)\frac{\frac{\partial}{\partial b}\pi_j(b, \phi_j)}{\pi_j(b, \phi_j)} \tag{3.3}$$

where n is the number of firms who bid down to b.[9] Since the only property of $\frac{\frac{\partial}{\partial b}\pi_i}{\pi_i}$ that is used in the proofs is the fact that it is increasing in ϕ_i, for simplicity we will often write the equivalent of (3.3) for the single object risk neutral case:

$$(n-1)\frac{p_j'(b)}{1-p_j(b)} = \sum_{i\neq j} \frac{1}{b-\phi_i(b)} - (n-2)\frac{1}{b-\phi_j(b)} \tag{3.4}$$

It should be clear that any result proved for this case also holds for the more general case (allowing for risk aversion and endogenous demand).

Our argument proceeds in 3 steps. First, we show that the upper bound to the equilibrium bids must be non increasing i.e. $\widetilde{u} \leq u$ (lemma 3). Second, we show that the lower bound to equilibrium bids is strictly decreasing, $\widetilde{l} < l$ (lemma 5). Finally, we show that, for 2 firms, bidding in the new configuration is more aggressive (in the sense of first order stochastic dominance), $\widetilde{p}_j(b) > p_j(b)$ (proposition 1). For more than two firms and with some additional conditions on the technologies available (the F functions), we prove that, for any b, the probability that the upgrader wins the market with b is lower after the upgrade than in the original configuration (proposition 2).

Lemma 3: *Let $u(F_1, ..., F_N)$ be the upper bound of the equilibrium bids in configuration $(F_1, ..., F_N)$. $u(F_1, ..., F_N)$ is weakly decreasing in its arguments. That is, if $\widetilde{F}_j \succ F_j$, then $u(\widetilde{F}_j, F_{-j}) \leq u(F_j, F_{-j})$.*

Proof. Let $u = u(F_i, F_{-i})$ and $\widetilde{u} = u(\widetilde{F}_i, F_{-i})$, and assume without loss of generality that $\overline{c}_1 \leq \overline{c}_2 \leq ... \leq \overline{c}_N$. Let $\widetilde{\overline{c}}_i$ be the maximum cost under $\widetilde{F}_i$ ($\widetilde{\overline{c}}_i \leq \overline{c}_i$).

If $\overline{c}_1 = \overline{c}_2$, then $\widetilde{u} \leq u$ follows trivially from lemma 2. If $\overline{c}_1 < \overline{c}_2$, lemma 2 implies that u solves:

$$\min\{\arg\max_b \pi_1(b, \overline{c}_1) \prod_{i\neq 1} (1 - F_i(b))\}$$

In particular, u satisfies the FOC:

$$\frac{\frac{\partial}{\partial b}\pi_1(b, \overline{c}_1)}{\pi_1(b, \overline{c}_1)} = \sum_{i\neq 1} \frac{F_i'(b)}{1 - F_i(b)} \tag{3.5}$$

[9] Remember from the discussion in the previous section that firms need not share the same minimum equilibrium bid.

When $b \downarrow \overline{c}_1$, the expression in the left-hand side (LHS) goes to infinity while the expression in the right-hand side (RHS) remains bounded. Therefore, at the solution (remember, u is the smallest value that solves the FOC), the LHS crosses the RHS from above. In other words, the slope of the LHS is always less than that of the RHS.

If firm 1 is the upgrader, there are two possibilities. Either $\widetilde{\overline{c}}_1 = \overline{c}_1$, in which case $\widetilde{u} = u$, or $\widetilde{\overline{c}}_1 < \overline{c}_1$. In that case, the LHS of (3.5) decreases (using lemma 1) and $\widetilde{u} < u$ follows.

If $\widetilde{F}_i \succ F_i$ for $i \neq 1$, the RHS of (3.5) increases for all $b \in (\overline{c}_1, \overline{c}_2)$ when firm i upgrades its distribution, and the lowest solution to (3.5) falls: $\widetilde{u} < u$ follows. ■

Lemma 4 is central to the rest of the argument:

Lemma 4: *It cannot be that, at some point, bidding is less aggressive for all bidders after the investment than before. More precisely, it cannot be that, at any point $\widehat{b} \in [\max\{l, \widetilde{l}\}, \widetilde{u})$, $\widetilde{\phi}_i(\widehat{b}) \leq \phi_i(\widehat{b})$ for all i for whom both functions are defined, including a non-upgrading firm.*

Proof. The proof proceeds in two steps. First, we show that $\widetilde{\phi}_i(\widehat{b}) \leq \phi_i(\widehat{b})$ for all i for whom both functions are defined, implies that $\widetilde{\phi}_i(b) \leq \phi_i(b)$ for all i and for all $b > \widehat{b}$ (with a strict inequality close to $\widetilde{u}$). Second, we show that this leads to a contradiction with the fact that $\widetilde{u} \leq u$ (lemma 3).

Step 1: Suppose that for all i for which both functions are defined,

$$\widetilde{\phi}_i(\widehat{b}) \leq \phi_i(\widehat{b}) \tag{3.6}$$

Towards a contradiction, imagine that this condition is violated at a later point, b^* where:

$$b^* = \inf_{b > \widehat{b}} \{b \text{ s.t. } \widetilde{\phi}_j > \phi_j \text{ for some } j\} \tag{3.7}$$

The expression in (3.7) accounts for two distinct possibilities. (3.6) can be violated if the inverse bid functions cross for one of the bidders in this group (in that case, $\widetilde{\phi}_j(b^*) = \phi_j(b^*)$ and $\widetilde{\phi}_j'(b^*) > \phi_j'(b^*)$). Alternatively, it is possible that $l_k > \widehat{b}$ for some bidder k, $l_k > \widetilde{l}_k$ and $\widetilde{\phi}_k(l_k) > \phi_k(l_k) = \underline{c}_k = b^*$). (The proof proceeds by assuming that the first scenario holds but the argument is easily adapted for the second possibility.)

At b^*, there are potentially three groups of firms bidding:

1. The firms that bid down to b^* under both configurations, (F_j, F_{-j}) and $(\widetilde{F}_j, F_{-j})$. We index them by i.
2. Firms that bid down to b^* only under (F_j, F_{-j}). We index them by q.
3. Firms that bid down to b^* only under $(\widetilde{F}_j, F_{-j})$. We index them by r.

Since $\widetilde{\phi}_j(b^*) = \phi_j(b^*)$, firm j's FOC yields:

$$\frac{1}{b^* - \widetilde{\phi}_j(b^*)} = \sum \frac{\widetilde{p}'_r}{1-\widetilde{p}_r} + \sum_{i \neq j} \frac{\widetilde{p}'_i}{1-\widetilde{p}_i} = \sum \frac{p'_q}{1-p_q} + \sum_{i \neq j} \frac{p'_i}{1-p_i} \tag{3.8}$$

(notice that the firms in the second group only appear on the right-hand side since they are active opponents only in the configuration (F_j, F_{-j}) and similarly for the firms in the third group).

Because for all the other firms in group 1, $\widetilde{\phi}_k \leq \phi_k$ at b^*, we also have:

$$\sum \frac{\widetilde{p}'_r}{1-\widetilde{p}_r} + \sum_{i \neq k} \frac{\widetilde{p}'_i}{1-\widetilde{p}_i} \leq \sum \frac{p'_q}{1-p_q} + \sum_{i \neq k} \frac{p'_i}{1-p_i} \tag{3.9}$$

Now, $\widetilde{\phi}'_j(b^*) > \phi'_j(b^*)$ implies that $\frac{\widetilde{p}'_j}{1-\widetilde{p}_j} > \frac{p'_j}{1-p_j}$. Moreover, comparing (3.8) with (3.9), we find that, for all the firms in group 1:

$$0 < \frac{\widetilde{p}'_j}{1-\widetilde{p}_j} - \frac{p'_j}{1-p_j} \leq \frac{\widetilde{p}'_i}{1-\widetilde{p}_i} - \frac{p'_i}{1-p_i} \tag{3.10}$$

Therefore, going back to (3.8) we conclude that there must be some firms in group 2. Let p be one of them. We have,

$$\sum \frac{\widetilde{p}'_r}{1-\widetilde{p}_r} + \sum_{i \neq j} \frac{\widetilde{p}'_i}{1-\widetilde{p}_i} > \sum_{q \neq p} \frac{p'_q}{1-p_q} + \sum_{i \neq j} \frac{p'_i}{1-p_i}$$

So, adding $\frac{\widetilde{p}'_j}{1-\widetilde{p}_j} > \frac{p'_j}{1-p_j}$

$$\frac{1}{b^* - \underline{\widetilde{c}}_p} > \sum \frac{\widetilde{p}'_r}{1-\widetilde{p}_r} + \sum \frac{\widetilde{p}'_i}{1-\widetilde{p}_i} > \sum_{q \neq p} \frac{p'_q}{1-p_q} + \sum \frac{p'_i}{1-p_i} = \frac{1}{b^* - \phi_p(b^*)} > \frac{1}{b^* - \underline{c}_p}$$

where the first inequality follows from the assumption that firm p does not bid down to b^* under $(\widetilde{F}_j, F_{-j})$, the equality corresponds to firm p's FOC and the last inequality comes from the fact that inverse bid functions are increasing. This last expression implies that $\widetilde{\underline{c}}_p > \underline{c}_p$, a contradiction. We conclude that $\widetilde{\phi}_i(b) \leq \phi_i(b)$ for all i and for all $b > \widehat{b}$.

To prove the stronger claim that $\widetilde{\phi}_i(b) < \phi_i(b)$ for all i and for all b in a neighborhood of $\widetilde{u}$, we need a few more steps since we also have to rule out $\widetilde{\phi}'_j(b^*) = \phi'_j(b^*)$. In that case, the first inequality in (3.10) is weak if j is not the upgrader. To get a strong second inequality for at least one firm, we need that, for some i in the first group, $\widetilde{\phi}_i < \phi_i$ at b^* or that the upgrader belongs to the first group (in which case, even if $\widetilde{\phi}_i = \phi_i$, $\frac{\widetilde{p}'_i}{1-\widetilde{p}_i} > \frac{p'_i}{1-p_i}$ by definition 1). If none of these conditions is satisfied, $\widetilde{\phi}_i = \phi_i$ for all i and for all $b > b^*$ until we meet a firm in category 2 or 3 (and we know that this must happen at some point before $\widetilde{u}$ since the upgrader does not belong to group 1). At that point, we can argue as before (there must be a firm in group 2) and get a contradiction.

Step 2: Consider one of the non-upgrading firm in group 1, say k. Step 1 implies that $\widetilde{\phi}_k(\widetilde{u}) < \phi_k(\widetilde{u})$. However, $\widetilde{\phi}_k(\widetilde{u}) = \min\{\overline{c}_k, \widetilde{u}\}$ (lemma 2) and $\phi_k(\widetilde{u}) \leq \min\{\overline{c}_k, \widetilde{u}\}$ (lemmas 2 and 3) implying $\widetilde{\phi}_k(\widetilde{u}) \geq \phi_k(\widetilde{u})$. A contradiction. ■

Lemma 5: *$l(F_1,..., F_N)$ is strictly decreasing in its arguments. That is, if $\widetilde{F}_j \succ F_j$, then $l(\widetilde{F}_j, F_{-j}) < l(F_j, F_{-j})$.*

Proof. Towards a contradiction, suppose that $\widetilde{l} \geq l$. Then, for all i where both functions are defined, $\widetilde{\phi}_i(\widetilde{l}) \leq \phi_i(\widetilde{l})$ ($\widetilde{\phi}_i(\widetilde{l}) = \underline{c}_i$ or $\widetilde{\underline{c}}_i$ for the upgrader). We claim that there is at least a non-upgrading firm bidding down to $\widetilde{l}$ under both configurations. To see this, remember that there are at least two firms bidding to the lower bound l and $\widetilde{l}$. Moreover, we know from section 2 that, in any configuration, $l_i \leq l_j$ iff $\underline{c}_i \leq \underline{c}_j$. Given that there is a non-upgrading firm that has the first or second lowest minimum cost under both configurations, it must be bidding under both configurations down to $\widetilde{l}$. We can then apply lemma 4 to get a contradiction. ■

We are now able to prove the main result of this section. We start with the simplest case: two firms. Then, we know from our discussion in section 2 that, at equilibrium, both firms bid on a common support $[l, u]$ (and $[\widetilde{l}, \widetilde{u}]$ after the investment). Moreover, lemma 5 implies that $\widetilde{l} < l$ so $\widetilde{p}_j(b) > p_j(b)$, $j = 1, 2$, for b close to l.

Proposition 1: Let $N = 2$. Then $\widetilde{p}_j(b) > p_j(b)$ *for all* j *and for all* b *in the interior of their common support.*

Proof. Let 1 be the upgrader. From lemma 5, $\widetilde{p}_j > p_j$ close to l. In addition, as long as $\widetilde{p}_2(b) > p_2(b)$, $\widetilde{\phi}_2(b) > \phi_2(b)$ and so (using firm 2's FOC) $\frac{\widetilde{p}_1'}{1-\widetilde{p}_1} > \frac{p_1'}{1-p_1}$. Therefore, starting from the left, $\widetilde{p}_1 > p_1$ as long as $\widetilde{p}_2 > p_2$.

Now, towards a contradiction, suppose that $\widetilde{p}_2$ and p_2 intersect first at $b_1 < \widetilde{u}$:

$$\phi_2(b_1) = \widetilde{\phi}_2(b_1) \tag{3.11}$$

In addition, we have $\frac{p_2'(b_1)}{1-p_2(b_1)} > \frac{\widetilde{p}_2'(b_1)}{1-\widetilde{p}_2(b_1)}$ so (using firm 1's FOC)

$$\phi_1(b_1) > \widetilde{\phi}_1(b_1) \tag{3.12}$$

By lemma 4, (3.11) and (3.12) together are impossible. ■

Comparative statics results on firms' aggregate bidding behavior (the p_i functions) are all we need to answer questions about investment incentives in the procurement auction. It is nevertheless useful to remark that proposition 1 implies that, for the non upgrading firm, bidding is also more aggressive for *every cost realization.* Indeed, for firm 2, $\widetilde{p}_2(b) = F_2(\widetilde{\phi}_2(b)) > p_2(b) = F_2(\phi_2(b))$, hence $\widetilde{\phi}_2(b) > \phi_2(b)$. This does not necessarily hold for the upgrader.

When we move to $N > 2$ firms, the system of differential equations that describes the equilibrium puts much less structure on the solution. Intuitively, fixing the bidding behavior of one firm places contraints on the *aggregate* bidding behavior of its opponents (since what matters for the firm is its probability of winning). However, it leaves much room for maneuver concerning their *individual* bidding behavior. To illustrate, suppose that there are three firms and let us try to argue as in the proof of proposition 1. We can easily claim again that $\widetilde{p}_1 > p_1$

as long as $\widetilde{p}_2 > p_2$ and $\widetilde{p}_3 > p_3$.[10] Now suppose that the first p's and $\widetilde{p}$'s cross at b_1 for firm 2. We have $\phi_2(b_1) = \widetilde{\phi}_2(b_1)$ and $\widetilde{\phi}_3(b_1) > \phi_3(b_1)$ and, since $\frac{p_2'}{1-p_2} > \frac{\widetilde{p}_2'}{1-\widetilde{p}_2}$ must hold, $\widetilde{\phi}_1(b_1) < \phi_1(b_1)$. This is a perfectly possible situation and we cannot rule it out (remember that by FOSD $\widetilde{\phi}_1 < \phi_1$ is compatible with $\widetilde{p}_1 > p_1$). In a nutshell, the more firms, the more degrees of freedom we have to allow for various patterns in the way the solutions behave.

To get analytical results, we need to impose further conditions. First, we assume that firms have the same utility functions, V_i for all i. Second, we impose that the distributions of costs are partially ordered according to definition 1 for some firms. This is useful because it can be shown that if $F_i \succ F_j$ then $\phi_i < \phi_j$ and $p_i > p_j$ at equilibrium (see, e.g., Maskin and Riley, 1999b and for a generalization to $N > 2$ bidders, Li and Riley, 1999). Intuitively, firm i, which has a more efficient technology, can afford to take a higher profit margin $b - \phi_i(b)$ at equilibrium. This is because, when trading off between a lower probability of winning and a higher price-cost margin, it takes into account the fact that its opponent is unlikely to have low costs. How strong is this condition? Probably stronger than needed for the claim to hold. On the other hand, it seems that firms participating in an auction have a good idea of their relative cost advantages and, in that case, definition 1 seems appropriate.

An important consequence of our assumption of common payoff functions is that firms that have the same technology will bid identically at equilibrium.[11] So, in particular, this means that we can already "stretch" the interpretation of proposition 1. Suppose there are N firms with firms 2 to N sharing the same payoff function and the same technology F. Firm 1 is the upgrader. Then the claim of proposition 1 also applies to describe the relationship between the equilibria prior and after the investment.

In proposition 2, we use these additional assumptions to prove a similar claim for any number of partially ordered technologies. Proposition 2 covers any patterns of "catching-up" by the investing firm, whereby the investor is either a

[10] Indeed, $\widetilde{p}_2 > p_2$ and $\widetilde{p}_3 > p_3$ imply that $\widetilde{\phi}_2 > \phi_2$ and $\widetilde{\phi}_3 > \phi_3$. $\widetilde{p}_1 = p_1$ implies by FOSD that $\widetilde{\phi}_1 < \phi_1$. Therefore, using (3.4), we have:

$$2\frac{\widetilde{p}_1'}{1-\widetilde{p}_1} = \frac{1}{b-\widetilde{\phi}_2} + \frac{1}{b-\widetilde{\phi}_3} - \frac{1}{b-\widetilde{\phi}_1} > 2\frac{p_1'}{1-p_1}$$

[11] Lebrun (1998) was the first to point out and use this property of equilibrium.

"laggard" or an "average" bidder prior to the investment and it joins the average or becomes a leader after the investment.[12]

Proposition 2: *Let $\widetilde{F} \succeq F$ stands for $\widetilde{F} \succ F$ according to definition 1 or $\widetilde{F} = F$. Suppose there are N firms with $\widetilde{F}_1 \succeq F_j \succeq F_1$ for all $j \neq 1$ with a least one of these inequalities strict. Let firm 1 be the upgrader. Define $W_1(b) = \prod_{j \neq 1} (1 - p_j(b))$ i.e. $W_1(b)$ is the probability that firm 1 wins with a bid of b* in the original configuration. *Define $\widetilde{W}_1(b)$ similarly. Then $\widetilde{W}_1(b) < W_1(b)$ on the interior of their common support.*

Proof. For future reference, notice that bidders' FOCs (3.2) can be rewritten as:

$$-\frac{W_j'(b)}{W_j(b)} = \frac{1}{b - \phi_j(b)}$$

Because the technologies are partially ranked, equilibrium inverse bid functions can also be ordered. We have:

$$\begin{aligned} \widetilde{\phi}_1 &\leq \widetilde{\phi}_j \\ \phi_j &\leq \phi_1 \end{aligned} \tag{3.13}$$

for all $j \neq 1$ (with some strict inequalities).

We first assume that the supports of winning bids are common to all firms in the post-upgrade configuration. Then, by lemma 5 ($\widetilde{l} < l$), $\widetilde{p}_j > p_j$ for all j, $\widetilde{W}_1 < W_1$, and $\widetilde{\phi}_j > \phi_j$ for all $j \neq 1$ close to l.

Claim 1: Starting from the left (i.e. from l onwards), the first $\widetilde{p}$ and p to cross cannot be for the upgrader. Moreover, at that first crossing, it must be that $\widetilde{\phi}_1 < \phi_1$.

Proof: Since $\widetilde{p}_1$ would need to cross p_1 from above, we must have

$$\frac{\widetilde{p}_1'}{1 - \widetilde{p}_1} < \frac{p_1'}{1 - p_1} \tag{3.14}$$

[12] These also form the relevant cases when one wants to talk about market turnover and leadership changes. When the investment is made by a leader or a laggard who retain their positions, a similar claim seems to hold though we were not able to prove it analytically (see claim 4 of the proof where the partial ordering is needed).

At the same time, because $\widetilde{\phi}_j \geq \phi_j$ for all $j \neq 1$ and $\widetilde{\phi}_1 < \phi_1$ (since $\widetilde{F}_1 > F_1$), we have

$$
\begin{aligned}
(N-1)\frac{\widetilde{p}_1'}{1-\widetilde{p}_1} &= \sum_{j\neq 1}\frac{1}{b-\widetilde{\phi}_j} - (N-2)\frac{1}{b-\widetilde{\phi}_1} \\
&> \sum_{j\neq 1}\frac{1}{b-\phi_j} - (N-2)\frac{1}{b-\phi_1} = (N-1)\frac{p_1'}{1-p_1}
\end{aligned}
$$

a contradiction with (3.14). Now, suppose that the first $\widetilde{p}$ and p to cross from the left are for bidder $j \neq 1$. Because, by hypothesis, $\widetilde{\phi}_k \geq \phi_k$ for all $k \neq j, 1,$ we need $\widetilde{\phi}_1 < \phi_1$ to get $\frac{\widetilde{p}_k'}{1-\widetilde{p}_k} < \frac{p_k'}{1-p_k}$ (and this is possible since, by FOSD, $\widetilde{\phi}_1 < \phi_1$ is compatible with $\widetilde{p}_1 > p_1$).

Claim 2: $\widetilde{W}_1(b) < W_1(b)$ close to u.

Proof: When $\widetilde{u} < u$, this is straightforward. If $\widetilde{u} = u$, suppose towards a contradiction that $\widetilde{W}_1 \geq W_1$ close to $\widetilde{u} = u$. In the appendix, we prove (lemma 6) that this implies that $\frac{\widetilde{W}_1'}{\widetilde{W}_1} \leq \frac{W_1'}{W_1}$ close to u, and using firm 1's FOC, $\widetilde{\phi}_1 \geq \phi_1$ close to u. Using (3.13), this implies that $\widetilde{\phi}_j \geq \phi_j$ for $j \neq 1$ (some of them strict) and $\widetilde{W}_1 < W_1$ close to u. A contradiction.

Claim 3: $\widetilde{\phi}_1 = \phi_1$ implies that $\widetilde{\phi}_j \geq \phi_j$ for all j (some strict)

Proof: This follows directly from (3.13).

Claim 4: $\widetilde{W}_1 < W_1$ for all b.

Proof: First note that if for all $j \neq 1, \widetilde{\phi}_j > \phi_j$ on their common support, then $\widetilde{p}_j > p_j$ for all j and $\widetilde{W}_1 < W_1$ follows directly. So suppose firm 2 is the first from the left (say, at b_1) for whom $\widetilde{\phi}_2 = \phi_2$. We have $\widetilde{W}_1 < W_1$ for all $b \leq b_1$, and by claim 1, $\widetilde{\phi}_1(b_1) < \phi_1(b_1)$.

Case 1: $\widetilde{\phi}_1(b) < \phi_1(b)$ for all $b > b_1$.

Using bidder 1's FOC, this implies

$$
-\frac{\widetilde{W}_1'}{\widetilde{W}_1} = \frac{1}{b-\widetilde{\phi}_1} < \frac{1}{b-\phi_1} = -\frac{W_1'}{W_1} \tag{3.15}
$$

for all $b > b_1$. Then using claim 2, we conclude that $\widetilde{W}_1 < W_1$ for all b (since a crossing in $(b_1, \widetilde{u})$ would require $\frac{\widetilde{W}_1'}{\widetilde{W}_1} < \frac{W_1'}{W_1}$ in contradiction with (3.15)).

Case 2: There exists $b_2 > b_1$ such that $\widetilde{\phi}_1(b_2) = \phi_1(b_2)$ (b_2 is the first one from b_1).

Using claim 3, we have $\widetilde{W_1}(b_2) < W_1(b_2)$ and arguing as in case 1, we have $\widetilde{W_1} < W_1$ for all $b < b_2$. The scenario from b_2 on is identical (the first p and $\widetilde{p}$ cannot be for the upgrader and at that point we must have $\widetilde{\phi}_1 < \phi_1, ...$) and we can replicate the argument.

In the appendix, we extend the proof for the case where the lower bound $\widetilde{l}$ is not common to all firms. ■

Propositions 1 and 2 allow us to answer our initial question concerning the incentives of firms to upgrade their distributions. When firm i upgrades its distribution, it needs to take two effects on its ex-ante expected payoff into account. First, a *direct* effect through an improvement in the ex-ante distribution of its costs (holding its opponents' strategies fixed) and, second, an *indirect* or *strategic* effect through its opponents' adjustments to the new configuration. Propositions 1 and 2 tell us that, under the new configuration $(\widetilde{F}_i, F_{-i})$, firm i's opponents will bid, collectively, more aggressively. This means that the strategic effect is *negative* for distributional upgrades in the first price procurement auction.

At this point, it might be useful to remember that the investments we are considering shift the best response schedule of the investor's opponents upwards. In other words, holding the bidding strategy of the investor fixed, his opponents prefer to bid more aggressively after the investment than before (refer to (2.2) if needed). We can then interpret our results as indicating that, for comparative statics purposes, the first price auction behaves as a standard game with increasing best response schedules. Firms will tend to invest less in case of overt investments than in case of covert investments.

How strong is the strategic effect? It can be quite strong as example 2 illustrates. There, an inefficient firm is better off avoiding a cost reducing investment, *even if it came at no cost*!

Example 2: Consider the following initial configuration for firms 1 and 2: F_1 is uniform over [0,10] and F_2 is uniform over [0,5]. Suppose that firm 1 has the possibility to upgrade its distribution to $\widetilde{F}_1 = F_2$. Denote by $\Pi_i(F, \widehat{F})$ firm i's ex-ante payoff when firm 1's distribution is F and firm 2's distribution is $\widehat{F}$. A numerical solution to the first-price auction yields: $\Pi_1(F_1, F_2) = 0.90445$, $\Pi_2(F_1, F_2) = 1.93245$, $\Pi_1(F_2, F_2) = \Pi_2(F_2, F_2) = 0.83333$. The change in its distribution leaves firm 1 worse off.

Proposition 1 and 2 also shed light on the results derived when firms are ex-ante symmetric and investment is simultaneous. Typically, this literature has focused on the Nash equilibrium of the investment game and found a unique symmetric equilibrium (see e.g. Tan (1992)). Our results and example 2 in particular suggest that this equilibrium might not be subgame perfect when investment is observable (and so bidders can react to deviations at the investment stage). Indeed, suppose that the symmetric Nash equilibrium of the investment game is given by $(x^*, ..., x^*)$ (where x represents firms' investment level, indexed according to definition 1) and consider firm 1's incentive to deviate. Since $(x^*, ..., x^*)$ is a Nash equilibrium, firm 1 has no incentive to deviate *when its opponents' behavior* (including at the procurement stage) *is held fixed*. Because of the negative strategic effect identified in propositions 1 and 2, investing more than x^* would not be profitable either if investment is observable. However, investing *less* than x^* might be a profitable deviation: by choosing a lower investment level, firm 1 induces its opponents to bid less aggressively in the second stage, and this, together with the cost saving involved, might overcome the effect of the lower probability of winning. In other words, our results suggest that the focus on the symmetric equilibrium of the simultaneous investment game can only be justified if investment is not observable. Otherwise, the simultaneous investment game among symmetric firms might admit asymmetric (subgame perfect) equilibria.

4. Investment Incentives in Procurement Auctions

In this section, we turn to our original question of investment incentives and compare the first price auction (FPA) and the second price auction (SPA). Comparing the two auction formats is interesting in two respects. First, both are commonly used auction rules (remember that in our setting the SPA is equivalent to the English auction). Second, the SPA provides an excellent benchmark to analyze the properties of the FPA because the strategic effect identified in the previous section for the FPA is absent for the SPA. Indeed, bidding one's own cost is a dominant strategy in the SPA, irrespective of the distributions of one's opponents. Therefore bidding behavior at the procurement stage is unaffected by firms' investment in the first stage. To allow for a comparison between the two auction formats (using the Revenue Equivalence Theorem), we return in this section to the standard assumptions of risk neutrality and a single indivisible object. Moreover, it is convenient to restrict the analysis to two firms.

To provide intuition for our next result, consider the following example. Suppose that firms have originally the same distribution of costs and consider an incremental investment by firm 1. Firm 1's change in payoff can be decomposed into two terms: a direct effect (holding its opponents' behavior fixed) and a strategic effect. By the Revenue Equivalence Theorem, the direct effect for an incremental change is the same under the FPA and the SPA. Moreover, propositions 1 and 2 suggest that the strategic effect is negative for the FPA. Because there is no strategic effect for the SPA, this implies that firm 1 will invest less under the FPA than under the SPA.

Proposition 3 shows that this intuition extends to situations where firms are not ex-ante symmetric and investment is not necessarily incremental: Firms will tend to invest less when the FPA format is used because they anticipate the more aggressive behavior of their opponents. As in proposition 2, proposition 3 requires some level of leadership change for the analytical proof to go through, though again we expect this result to hold generally.

Proposition 3: *Let $N = 2$. The FPA provides less incentives than the SPA for investments that involve a change of leadership. Formally, let firm 1 be the upgrader. Then, investment incentives are lower in the FPA than in the SPA for an investment such that $\widetilde{F}_1 \succeq F_2 \succeq F_1$ (with at least one strict inequality).*

Proof. Let $H_2 \succ H_1$. The statement of proposition 3 can be decomposed into two parts: (1) the incentive to catch up, i.e. to move from (H_1, H_2) to (H_2, H_2); and (2) the incentives to overtake i.e. to move from (H_1, H_1) to (H_2, H_1). We need to show that both are weaker under the FPA.

Proposition 3 is then an almost direct consequence of Maskin and Riley (1999b)'s proposition 2.6. There, they show that the inefficient firm prefers the FPA format to the SPA auction format. Let $\Pi_1^{FPA}(H, \widehat{H})$ be the ex-ante expected profit of firm 1 when its cost distribution is H and firm 2's cost distribution is $\widehat{H}$. $\Pi_1^{SPA}(H, \widehat{H})$ is the equivalent for the SPA auction. Proposition 2.6 of Maskin and Riley implies that $\Pi_1^{FPA}(H_1, H_2) > \Pi_1^{SPA}(H_1, H_2)$ and $\Pi_2^{FPA}(H_1, H_2) < \Pi_2^{SPA}(H_1, H_2)$. Therefore:

$$\Pi_1^{FPA}(H_2, H_2) - \Pi_1^{FPA}(H_1, H_2) < \Pi_1^{FPA}(H_2, H_2) - \Pi_1^{SPA}(H_1, H_2) = \Pi_1^{SPA}(H_2, H_2) - \Pi_1^{SPA}(H_1, H_2)$$

where the equality of the second and third terms follows from the Revenue Equivalence theorem. To prove (2), we proceed similarly:

$$\Pi_1^{FPA}(H_2, H_1) - \Pi_1^{FPA}(H_1, H_1) < \Pi_1^{SPA}(H_2, H_1) - \Pi_1^{FPA}(H_1, H_1) =$$
$$\Pi_1^{SPA}(H_2, H_1) - \Pi_1^{SPA}(H_1, H_1) \blacksquare$$

The result in proposition 3 could overturn the revenue ranking found by Maskin and Riley (1999) for exogenous and fixed distributions of valuations. Though neither auction format is generally better in their environment, they find that the FPA performs better for plausible asymmetries. Proposition 3 suggests that allowing for endogenous distributions could change this result.[13]

Our next proposition confirms the qualities of the SPA: Not only does it induce more investment (which is good news for the auctioneer), it also induces the *socially optimal level* of investment. The efficiency of the SPA (i.e. the fact that the firm with the lowest cost always wins the contract) is what drives this result. Intuitively, consider firm 1's investment decision. In the SPA, firm 1 wins if and only if it has the lowest cost. Investment makes this occurrence more likely. Specifically, an investment changes firm 1's payoff in two circumstances: (1) Firm 1 wins the auction in both cases but it now wins it with a lower cost. Then, its gain equals the difference between the two costs, and this corresponds to the change in social welfare (because the social cost of the contract has been decreased by that amount), and (2) firm 1 wins after the investment but not before. Its payoff then equals the difference between its cost and the second lowest cost, and this corresponds again to the change in social welfare. Formally,

Proposition 4: *The second price auction provides the socially optimal level of investment incentives.*

Proof. Let (F_1, F_2) be the initial configuration, with support on $[\underline{c}_1, \overline{c}_1]$ and $[\underline{c}_2, \overline{c}_2]$. Suppose firm 1 is considering an investment to $\widetilde{F}_1 \succ F_1$. We just need to prove that private and social incentives are aligned under the SPA.

Claim 1: $\Pi_1^{SPA}(F_1, F_2) = \int_{min\{\underline{c}_1,\underline{c}_2\}}^{max\{\overline{c}_1,\overline{c}_2\}} F_1(c)(1 - F_2(c))dc$

Proof: Suppose $\overline{c}_1 \geq \overline{c}_2$. By definition,

$$\Pi_1^{SPA}(F_1, F_2) \;\; = \;\; \int_{\underline{c}_1}^{\overline{c}_2} dF_1(c) \int_{c}^{\overline{c}_2} (x - c) dF_2(x)$$

[13] A similar reversal of revenue ranking has been found by Persico (2000) for the case of common values and unobservable investment.

$$
\begin{aligned}
&= F_1(c)\int_c^{\overline{c}_2}(x-c)dF_2(x)\Big|_{\underline{c}_1}^{\overline{c}_2} + \int_{\underline{c}_1}^{\overline{c}_2} F_1(c)(1-F_2(c))dc \\
&\quad \text{(integration by parts)} \\
&= \int_{min\{\underline{c}_1,\underline{c}_2\}}^{max\{\overline{c}_1,\overline{c}_2\}} F_1(c)(1-F_2(c))dc
\end{aligned}
$$

The proof when $\overline{c}_1 < \overline{c}_2$ is analogous.

Given claim 1, the private return to investment is equal to $\Pi_1^{SPA}(\widetilde{F}_1, F_2) - \Pi_1^{SPA}(F_1, F_2)$

$$
= \int_{min\{\widetilde{\underline{c}}_1,\underline{c}_2\}}^{max\{\widetilde{\overline{c}}_1,\overline{c}_2\}} \widetilde{F}_1(c)(1-F_2(c))dc - \int_{min\{\underline{c}_1,\underline{c}_2\}}^{max\{\overline{c}_1,\overline{c}_2\}} F_1(c)(1-F_2(c))dc \quad (4.1)
$$

In a procurement setting, the lower the minimum of the two cost realizations, the higher the social surplus. Hence, a measure of social surplus is the negative of the expected value of the second order statistics. Let $S(c)$ be the distribution of the second order statistics.

$$
\begin{aligned}
S(c) &= F_2(c)(1-F_1(c)) + F_1(c)(1-F_2(c)) + F_1(c)F_2(c) \\
&= F_1(c) + F_2(c) - F_1(c)F_2(c)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
SS(F_1, F_2) &= -\int_{min\{\underline{c}_1,\underline{c}_2\}}^{min\{\overline{c}_1,\overline{c}_2\}} cdS(c) \\
&= -min\{\overline{c}_1, \overline{c}_2\} + \int_{min\{\underline{c}_1,\underline{c}_2\}}^{min\{\overline{c}_1,\overline{c}_2\}} S(c)dc \\
&= -max\{\overline{c}_1, \overline{c}_2\} + \int_{min\{\underline{c}_1,\underline{c}_2\}}^{max\{\overline{c}_1,\overline{c}_2\}} S(c)dc
\end{aligned}
$$

and the social return to investment, $SS(\widetilde{F}_1, F_2) - SS(F_1, F_2)$

$$
\begin{aligned}
&= \int_{min\{\widetilde{\underline{c}}_1,\underline{c}_2\}}^{max\{\widetilde{\overline{c}}_1,\overline{c}_2\}} \widetilde{F}_1(c)(1-F_2(c))dc - \int_{min\{\underline{c}_1,\underline{c}_2\}}^{max\{\overline{c}_1,\overline{c}_2\}} F_1(c)(1-F_2(c))dc \\
&\quad -max\{\widetilde{\overline{c}}_1, \overline{c}_2\} + max\{\overline{c}_1, \overline{c}_2\}
\end{aligned}
$$

$$
\begin{aligned}
&+\int_{min\{\underline{\widetilde{c}}_1,\underline{c}_2\}}^{max\{\overline{\widetilde{c}}_1,\overline{c}_2\}} F_2(c)dc - \int_{min\{\underline{c}_1,\underline{c}_2\}}^{max\{\overline{c}_1,\overline{c}_2\}} F_2(c)dc \\
= &\int_{min\{\underline{\widetilde{c}}_1,\underline{c}_2\}}^{max\{\overline{\widetilde{c}}_1,\overline{c}_2\}} \widetilde{F}_1(c)(1-F_2(c))dc - \int_{min\{\underline{c}_1,\underline{c}_2\}}^{max\{\overline{c}_1,\overline{c}_2\}} F_1(c)(1-F_2(c))dc
\end{aligned}
\tag{4.2}
$$

The claim follows by comparing (4.1) and (4.2) ■

Proposition 4 is actually a very general result. The proof straightforwardly extends to $N > 2$. With one qualification, it also easily extends to situations where firms invest simultaneously: when investment is simultaneous, the socially optimal outcome is always an equilibrium but not necessarily the only one.[14] To see this, denote firm i's investment choice $x_i \in X_i$. Let $SS(x_1, ..., x_N)$ be the social welfare resulting from firms' investment decisions $(x_1, ..., x_N)$. Suppose that $(x_1^*, ..., x_N^*)$ maximizes this expression. This will also be a subgame perfect equilibrium of the simultaneous investment game. Indeed, fix $x_{-1}^* = (x_2^*, ..., x_N^*)$ and consider firm 1's incentive to deviate. This is similar to considering the one-firm investment incentive of our model. So, applying the same logic as in the proof of proposition 4, we find that private and social incentives are again perfectly aligned. $(x_1^*, ..., x_N^*)$ is a subgame perfect equilibrium investment choice when investment is simultaneous.

5. Market turnover and the dynamics of competition

An important question in industrial organization is whether asymmetries between firms tend to increase or decrease over time. Maintaining a healthy degree of competition is also a concern for procurement authorities.[15] The analysis of a full-blown dynamic model is of course outside the scope of this paper. However, we can already use the insights from our previous results to explore the likely dynamics of competition under both auction formats.

First, note that the results of section 3 transpose to a fully dynamic multi-stage setting. Indeed, what matters for our comparative statics result to go through is the log-supermodularity of the payoff function (lemma 1), which can be shown to

[14] Stegeman (1996) provides a proof of proposition 4 for the case of simultaneous investment.

[15] Repeated procurement auctions are also an active area for empirical research (see among others, the work of Bajari (1998 and 1999) and Jofre-Bonet and Pesendorfer (1999)).

hold for the value function of a dynamic version of the model. Therefore, at any point, the FPA will generate less investment than the SPA.

However, from a revenue perspective, the fact that the SPA elicits more investment does not in it-self imply that it should be favored by the procurement authority. Indeed, market asymmetries among bidders reduce competition and hurt revenue (Cantillon, 1999). Therefore, the lower level of investment in the FPA might not be such a bad news for the procurement authority *if,* in the long run*,* the FPA were likely to favor a more symmetric market structure than the SPA.

To illustrate, let us consider the following simple thought experiment. Today, two firms, 1 and 2, bid for a contract for which, a priori, firm 1 has a competitive advantage i.e. $F_1 \succ F_2$. Next period, a similar contract is put for tender and because of project synergies, winning the contract today is helpful (say, in the form of shifting the distribution of costs for the second period auction). Costs are independently drawn in each period.

In this setting, winning today corresponds to an "investment" in our framework and firms will need to account for this extra advantage of winning when bidding for the first period contract.[16] The underinvestment result from the previous section suggests that this advantage will be valued less if the FPA is used rather than the SPA. However, it is unclear which firm, of the laggard or the leader, will value the investment more. If the leader values it more, then he will be more likely to win the contract today and therefore confirm his competitive advantage tomorrow. By contrast, if the laggard values it more, then this can help him win more often today (because he will discount his current bid more) and this could possibly reverse his current market disadvantage, leading to a more symmetric market structure tomorrow.

In the SPA, we can use the fact that private and social investment incentives are perfectly aligned (proposition 4) to compare the incentives of the leader and the laggard. Recently, Athey and Schmutzler (1999) have found sufficient conditions for leaders in oligopoly markets to invest more than followers (an outcome they term "weak increasing dominance"). In our setting, these conditions reduce to: (1) The return to investment must be decreasing in the initial position of one's opponents (i.e. investments are strategic substitutes); and (2) The higher one's

[16] Given that only one firm wins the contract at a time, firms' decisions correspond to our one-firm investment set-up. The difference with our basic model is that the identity of the investor is now endogenously determined.

initial position, the higher the return on investment. The social return to an investment is clearly decreasing in the initial position of the investor's opponents so condition (1) holds straightforwardly for the SPA. Moreover, it is easily checked that condition (2) holds for all the examples of distributional upgrades given in section 2.[17] This suggests that leaders in a SPA environment will have at least as high an inventive as laggards to invest. Therefore, the SPA is likely to increase current market asymmetries.

Could the FPA favor a more symmetric market structure? Unfortunately, there is some reason to suspect that the negative strategic effect identified in section 3 is greater for investments by laggards than by leaders. Intuitively, when a laggard invests, he closes the gap with his competitors and this intensifies competition. By contrast, when a leader invests, he does not need to change his behavior much and he is therefore able to keep a greater part of the benefit of his investment.

In this section we investigate this question numerically.[18] In the examples that follow, we keep the distribution of costs for firm 2 fixed, and order the potential distributions of costs for firm 1 by the parameter a with $F_1^a \succ F_1^{a+1}$ (a lower value for a means a more efficient cost distribution). Let $\Pi_1(F, \widehat{F})$ be the ex-ante expected profit for firm 1 when its costs are distributed according to F and firm 2's costs are distributed according to $\widehat{F}$. Define $\Delta = \Pi_1(F_1^a, F_2) - \Pi_1(F_1^{a+1}, F_2)$. Δ is the ex-ante expected increase in firm 1's profit from moving to F_1^{a+1} to F_1^a.

The tables report the values of $\Pi_1(F_1^a, F_2)$ and Δ.[19] They all indicate that, holding the position of firm 2 fixed, the higher the initial position of firm 1, the greater its incentives to invest further. An appropriate rescaling of the numbers in the table allows us to conclude that, of the two firms in stage 1, the current leader values the investment more.[20]

[17] Except for the case of additional random draws for which the social returns to an investment depend on the *aggregate* investment only (from a social perspective, you don't care which bidder gets an extra draw). In this setting then, leaders and laggards have equal incentives to invest.

[18] To answer this question analytically, we would need to compare the investment incentives for firms with different initial competitive situations. However, the analysis in section 3 only pinned down the sign of the strategic effect, not its size, and given the lack of explicit solution to the equilibrium, we were not able to study this question analytically.

[19] Our numerical simulations are based on Li and Riley's Bidcomp2 program, extended to compute bidders' ex-ante expected payoffs. We refer to their paper for technical details about the program.

[20] Alternatively, we could appeal to Athey and Schmutzler (1999)'s sufficient conditions. Condition (1) is intuitive and the tables illustrate condition (2).

Distributional contraction (fixed lower end): F_2 is uniform on $[0,5]$ and F_1^a is uniform on $[0,a]$. Table 1 presents the payoffs that result from the numerical solution to each of the auction for a between 10 and 1. The row in bold type refers to the symmetric configuration.

a	$\Pi_1(F_1^a, F_2)$	Δ
10	0.90445	
9	0.87073	-0.03372
8	0.84941	-0.02132
7	0.83392	-0.01549
6	0.82701	-0.00691
5	**0.83333**	**0.00632**
4	0.85391	0.02058
3	0.91935	0.06544
2	1.00454	0.08519
1	1.12233	0.11779

Table 1

As reflected in the last column, the strategic effect outweighs the direct effect for high levels of a. Catching up leaves firm 1 worse off.[21] However, as soon as firm 1 becomes the most efficient firm, further distributional upgrades always result in an increase in its payoffs.

Distributional stretch (fixed upper end): F_2 is uniform on $[5,10]$ and F_1^a is

[21] A particular feature of the kind of upgrade considered here is that when firm 1 upgrades, the length of the support of its distribution is reduced and so is the "privateness" of its cost realization.

uniform on $[a, 10]$. The corresponding values are presented in Table 2.

a	$U_1(F_1^a, F_2)$	Δ
9	0.04191	
8	0.15958	0.11767
7	0.33874	0.17916
6	0.56666	0.22792
5	**0.83333**	**0.26667**
4	1.13113	0.29780
3	1.45416	0.32303
2	1.79797	0.34381
1	2.15986	0.36189
0	2.53459	0.37473

Table 2

In this case, the strategic effect is not significant enough to outweigh the direct effect on firm 1's payoffs from any starting distribution. However, the last column exhibits once more a pattern of increasing incentive in the investor's original position.

Distributional shift: F_1^a is uniform on $[a-4, a+4]$, while F_2 is uniform on

[12, 20]. The corresponding values are presented in Table 3.

a	$\Pi_1(F_1^a, F_2)$	Δ
24	0.10255	
23	0.14931	0.04676
22	0.21325	0.06392
21	0.29937	0.08614
20	0.41182	0.11245
19	0.56437	0.15255
18	0.75644	0.19207
17	0.96974	0.21330
16	**1.24499**	**0.27516**
15	1.57223	0.32724
14	1.97229	0.40006
13	2.39907	0.42678
12	2.87819	0.47912
11	3.42465	0.54646
10	4.02154	0.59689
9	4.66895	0.64741
8	5.36238	0.69343

Table 3

Finally, table 4 presents the numerical results of a shift to the left of a truncated normal distribution. Specifically, F_1^a has mean a, whereas F_2 has mean 10. The standard deviation for both distributions is 1 and they are truncated three standard deviations away from the mean.

a	$\Pi_1(F_1^a, F_2)$	Δ
17	0.01813	
16	0.02587	0.00774
15	0.03705	0.01118
14	0.05701	0.01996
13	0.09742	0.04041
12	0.17411	0.07669
11	0.31627	0.14216
10	**0.55835**	**0.24208**
9	0.92902	0.37067
8	1.43440	0.50538
7	2.05971	0.62531
6	2.77621	0.71650
5	3.57305	0.79684
4	4.41764	0.84459
3	5.28299	0.86535

Table 4

To sum up then, all these numerical results support the conjecture that leaders have greater incentives to upgrade than laggards do.[22] Therefore they suggest that the dynamics of competition under the FPA will similarly be biased in favor of the current leader. As a result, we do not expect a fuller dynamic model to provide more support for the FPA.

6. Concluding remarks

Asymmetries among bidders are widespread in procurement situations. They are also a source of concern for procurement authorities. However, our understanding

[22] Of course, these results are largely indicative. Moreover, the numbers in the table refer to the *gross* benefits from the investment. If the origin of the change in firms' competitive position arise from synergies with other auctions, then of course the "investment" comes in for free. Otherwise, a definite conclusion would require us to model the investment technology as well. If costs increase with the initial competitive position of firms, this could of course provide a countervailing force.

of these market situations has been largely limited to date by the lack of explicit solutions for the equilibrium in the asymmetric first price auction.

In this paper, we have provided comparative statics results for a class of investments in cost reduction in the first price auction. In section 3, we showed that, after the investment, the investor's opponents bid collectively more aggressively. In the terminology of industrial organization, this means that investments in the first price auction have a negative strategic effect. In turn, we found in section 4 that this effect leads to lower investment levels in the first price auction than in the second price auction. Finally, the results in section 5 suggested that market leaders tend to invest more than laggards. It is tempting to interpret the low level of competition and of turnover in many procurement markets in light of these results.

At a purely theoretical level, our results contribute to the current efforts by various researchers to characterize and describe the equilibrium in the asymmetric first price auction. Our analysis deals with more than two bidders and provides a systematic treatment of potentially different bidding supports.

Finally, it is interesting to stress the fact that most of auction theory and, in particular, the comparison between market rules, take the distributions of private information as exogenously given. In practice, this should not be the case as market institutions are likely to affect the incentives for entry and investment. In this paper, we have offered a first comparison between the first price auction and the second price auction when firms are not necessarily symmetric ex-ante, the distributions of costs are endogenous and investment is observable. Our analysis has highlighted two attractive features of the second price auction: (1) it generates higher investment levels than the commonly used first price auction and (2) these investment levels are socially efficient. These results suggest that in markets where investment prior to the auction is deemed important or where there exist positive synergies between auctions, the second price auction is likely to be better at fostering a healthy level of competition.

In any case, further research is needed. We have assumed that only one firm has the opportunity to invest at any single point in time. It would be interesting to investigate whether our underinvestment result continues to hold and asymmetries are also self-reinforcing under the first price auction when investment is simultaneous (we conjecture that this is the case).[23] Another important open

[23] Whether the sequential or the simultaneous investment assumption is more relevant depends

question for the first price auction is the source of bidders' profits, since these are ultimately driving incentives. In the meantime, economists will need to rely on numerical methods for gaining understanding of the basic forces at play in asymmetric first price auctions (and they have already successfully done so: see Marshall et al., 1994, Athey, 1997, and Li and Riley, 1999).

of course on the economic environment under study. An important area of application for the sequential investment case is when the auction is repeated and there is some linkages (through capacity, learning, ...) between them.

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7. Appendix

Lemma 6: Behavior of $W_1(b)$ close to $u = \widetilde{u}$.
Let $W_1(b) = \prod_{j \neq 1} (1 - p_j(b))$, i.e. $W_1(b)$ is the probability that bidder 1 bid wins the auction with a bid of b in the original configuration. Define $\widetilde{W_1}(b)$ similarly. With these notations, firm 1's FOC can be rewritten as:

$$-\frac{W_1'(b)}{W_1(b)} = \frac{1}{b - \phi_1(b)} \text{ and } -\frac{\widetilde{W}_1'(b)}{\widetilde{W_1}(b)} = \frac{1}{b - \widetilde{\phi}_1(b)} \tag{A.1}$$

In the procurement first price auction, the upper bound to the equilibrium winning bids, u, is a singularity point for at least one of the differential equations that characterize the equilibrium (since for at least one firm, $p_j(u) = 1$ and so the left-hand side of (A.1) is undetermined at u). We want to pin down the relationship between W_1 and $\widetilde{W_1}$ when $u = \widetilde{u}$.

Lemma 6: *Let* 1 *be the upgrader and suppose that* $\widetilde{u} = u$. *Then*
(a) if $\frac{W_1'(b)}{W_1(b)} > \frac{\widetilde{W}_1'(b)}{\widetilde{W_1}(b)}$ *for all* $b \in (u - \delta, u)$ *for some* δ *positive, then* $\widetilde{W_1}(b) > W_1(b)$ *in* $(u - \delta, u)$,
(b) if $\widetilde{W_1}(b) > W_1(b)$ *in some neighborhood of* u, *then* $\exists\ \delta > 0$ *such that* $\frac{W_1'(b)}{W_1(b)} > \frac{\widetilde{W}_1'(b)}{\widetilde{W_1}(b)}$ *for all* $b \in (u - \delta, u)$,
and the same claims hold by inverting the roles of W_1 *and* $\widetilde{W_1}$.

Proof of lemma 6: (a) Towards a contradiction, suppose that there exists $\widehat{b} \in (u - \delta, u)$ such that $\widetilde{W_1}(\widehat{b}) < W_1(\widehat{b})$ (this is without loss of generality since if $\widetilde{W_1}(\widehat{b}) = W_1(\widehat{b})$, then $\widetilde{W_1}(\widehat{b} + \varepsilon) < W_1(\widehat{b} + \varepsilon)$ for ε small enough) . $\frac{\widetilde{W_1}(b)}{\widetilde{W_1}(b)} < \frac{W_1(b)}{W_1(b)}$ for all $b \in (u - \delta, u)$ implies that $\frac{d}{db}\left(\frac{\widetilde{W_1}(b)}{W_1(b)}\right) < 0$ on the same interval. Then,

$$\int_{\widehat{b}}^{u} \frac{d}{db}\left(\frac{\widetilde{W_1}(b)}{W_1(b)}\right) db = \lim_{b \to u} \frac{\widetilde{W_1}(b)}{W_1(b)} - \frac{\widetilde{W_1}(\widehat{b})}{W_1(\widehat{b})} < 0.$$

That is,

$$\lim_{b \to u} \frac{\widetilde{W_1}(b)}{W_1(b)} < \frac{\widetilde{W_1}(\widehat{b})}{W_1(\widehat{b})} < 1. \tag{A.2}$$

From lemma 3, we know that $u = \widetilde{u}$ only in two cases: (i) if there are at least two firms j such that $u = \overline{c}_j$ and $\overline{c}_1, \widetilde{\overline{c}}_1 \geq u$; (ii) if $\widetilde{\overline{c}}_1 = \overline{c}_1 < \min\limits_{j \neq i} \{\overline{c}_j\}$. The configuration where all firms have the same maximum cost is included in the first case. Bajari (1998) and, for the more general form of the profit function, Maskin and Riley (1996) have found expressions for the first derivative of inverse bidding functions at u. They satisfy $\phi_j'(u) = -\frac{N}{N-1} \frac{\frac{\partial}{\partial b}\pi_j(u,u)}{\frac{\partial}{\partial c}\pi_j(u,u)} < \infty$. We refer to these papers for a proof of this result. Case (i) also includes the possibility that two firms share the same maximum cost equal to u but some other firms have a strictly higher maximum cost. We shall argue that this possibility is a knife-edge case (we can rule it out by imposing the condition that, when some maximum costs differ, then they should all differ) and ignore it.[24] With this in mind, we need to consider two cases:

(i) All firms have the same maximum cost, u. Using L'Hôpital's rule and the fact that $\phi_j'(u) = \widetilde{\phi}_j'(u) < \infty$, we get that:

$$\lim_{b \to u} \frac{\widetilde{W}_1(b)}{W_1(b)} = 1,$$

and this contradicts (7.1).

(ii) $\overline{c}_1 = \widetilde{\overline{c}}_1 < \overline{c}_j$ for all $j \neq i$. In such a case,

$$\lim_{b \to u} \frac{\widetilde{W}_1(b)}{W_1(b)} = \frac{\prod\limits_{j \neq i} [1 - F_j(u)]}{\prod\limits_{j \neq i} [1 - F_j(u)]} = 1,$$

since $F_j(u) < 1$ for all $j \neq i$, and we get a contradiction again with (7.1). The same proof can be used reversing the roles of $\widetilde{W}_1(b)$ and $W_1(b)$.

(b) Towards a contradiction, suppose that for all b in a neighborhood of u, $\frac{W_1'(b)}{W_1(b)} \leq \frac{\widetilde{W}_1'(b)}{\widetilde{W}_1(b)}$. Then, applying the argument of part (a), we get that $\widetilde{W}_1(b) \leq W_1(b)$ in that neighborhood. This contradicts the hypothesis. ■

Proof of proposition 2 when the support of bids need not be common: What changes when the supports of bids are not common to all firms is that we can no longer start on the premise that $\widetilde{p}_j > p_j$ for all $j \neq 1$ close to l since it

[24] A continuity argument would suffice as well.

could be that for some j, $l \leq l_j < \widetilde{l}_j$ (by contrast, claims 2 and 3 in the main body of the proof continue to hold). To account for this, we simply need to redefine the first "relevant crossing", b_1 as used in claim 4.

$$b_1 = \min_{b \geq l} \{b \text{ s.t. } \widetilde{\phi}_j(b) = \phi_j(b) \text{ for } j \neq 1;\ l_j \text{ when } l_j < \widetilde{l}_j \text{ for } j \neq 1\} \tag{A.3}$$

With this definition, it is easy to check that $\widetilde{W}_1(b) < W_1(b)$ for all $b \leq b_1$. Moreover, we have:

Claim 5: $\widetilde{\phi}_1(b_1) < \phi_1(b_1)$

Proof: First, notice that firm 1 must be bidding down to $\widetilde{l}$ after the investment since, by assumption, $\widetilde{F}_1 \succeq F_j$ for all $j \neq 1$ (and remember from our discussion in section 2 that $l_i \leq l_j$ iff $\underline{c}_i \leq \underline{c}_j$). The rest of the proof adopts the same technique as lemma 4. Towards a contradiction, suppose that $\widetilde{\phi}_1(b_1) \geq \phi_1(b_1)$. At b_1, there are potentially three types of firms bidding:

1. The firms that bid down to b_1 under both configurations, (F_j, F_{-j}) and $(\widetilde{F}_j, F_{-j})$. We index them by i.
2. Firms that bid down to b_1 only under (F_j, F_{-j}). We index them by q.
3. Firms that bid down to b_1 only under $(\widetilde{F}_j, F_{-j})$. We index them by r.

Suppose that b_1 is such that $\widetilde{\phi}_j(b_1) = \phi_j(b_1)$ for $j \neq 1$. Using firm j's FOC, we have that, at b_1,

$$\frac{1}{b_1 - \phi_j(b_1)} = \sum_{i \neq j} \frac{\widetilde{p}_i'}{1 - \widetilde{p}_i} + \sum \frac{\widetilde{p}_r'}{1 - \widetilde{p}_r} = \sum \frac{p_q'}{1 - p_q} + \sum_{i \neq j} \frac{p_i'}{1 - p_i} \tag{A.4}$$

and, for all other firms in category 1, $\widetilde{\phi}_k \geq \phi_k$, so

$$\sum_{i \neq k} \frac{\widetilde{p}_i'}{1 - \widetilde{p}_i} + \sum \frac{\widetilde{p}_r'}{1 - \widetilde{p}_r} \geq \sum \frac{p_q'}{1 - p_q} + \sum_{i \neq k} \frac{p_i'}{1 - p_i} \tag{A.5}$$

Now, because at b_1 $\widetilde{\phi}_j$ is crossing ϕ_j from above, we have $\frac{\widetilde{p}_j'}{1 - \widetilde{p}_j} < \frac{p_j'}{1 - p_j}$. Comparing (A.4) and (A.5), we find that for all $k \neq j$ in category 1,

$$\frac{\widetilde{p}_k'}{1 - \widetilde{p}_k} - \frac{p_k'}{1 - p_k} \leq \frac{\widetilde{p}_j'}{1 - \widetilde{p}_j} - \frac{p_j'}{1 - p_j} < 0 \tag{A.6}$$

Comparing (A.6) with (A.4) and (A.5) again, we conclude that there must exist a firm in category 3 at b_1. Let's call it s. Using (A.6) and (A.4), we have

$$\frac{1}{b_1 - \underline{c}_s} \leq \frac{1}{b_1 - \widetilde{\phi}_s(b_1)} = \sum \frac{\widetilde{p}'_i}{1-\widetilde{p}_i} + \sum_{r \neq s} \frac{\widetilde{p}'_r}{1-\widetilde{p}_r} < \sum \frac{p'_q}{1-p_q} + \sum \frac{p'_i}{1-p_i}$$

But this contradicts the fact that firm s does not bid down to b_1 under (F_j, F_{-j}). We now turn to the case where b_1 corresponds l_j when $l_j < \widetilde{l}_j$ for $j \neq 1$. Then, at b_1, we have $0 = \frac{\widetilde{p}'_j}{1-\widetilde{p}_j} < \frac{p'_j}{1-p_j}$. We also have that $\underline{c}_j = \widetilde{\phi}_j(b_1) < \phi_j(b_1)$. Using bidder j's FOC this means that:

$$\sum_{i \neq j} \frac{\widetilde{p}'_i}{1-\widetilde{p}_i} + \sum \frac{\widetilde{p}'_r}{1-\widetilde{p}_r} < \sum \frac{p'_q}{1-p_q} + \sum_{i \neq j} \frac{p'_i}{1-p_i}$$

Now, for all other bidders in category 1, $\widetilde{\phi}_k \geq \phi_k$, so (A.5) holds for them. Arguing as before, we find that (A.6) must hold, that there must be a firm in category 3, and we get the same contradiction as above.

From claim 5, we can continue the argument along the same lines as claim 4 in the main body of the proof. ■