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Abstract

The swiftness and devastating impact of recent financial crises have taken many market participants by surprise, and pose challenges for economists seeking a theory of the onset of a crisis. We propose such a theory based on two features. The actions of diverse economic actors which undermine the currency are mutually reinforcing, while the fragmented nature of the media create small disparities in their information. In such circumstances, the beliefs of market participants can be tracked in the same way as the economic fundamentals, and an attack is triggered when the economic fundamentals deteriorate sufficiently to fall below the minimum level of market confidence necessary to support the currency. We give a characterization of such a minimum level of confidence.

JEL Classification numbers: F31, D82.

Keywords: Currency crisis, common knowledge.

1. Introduction

The swiftness and devastating effect of recent financial crises pose considerable challenges for economists seeking an explanation of the onset of a crisis. It is easy to give a narrative of the sequence of events leading up to the crisis with the benefit of hindsight. However this falls short of an explanation, since it begs the question of why the crisis occurred at that particular moment in time. More importantly, it does not explain the *absence* of a crisis in apparently similar countries, or in the same country at different moments in history. The challenge is all the more acute in the light of evidence that the onset of the Asian financial crisis of 1997 was largely unanticipated by market participants, as well as by the international agencies. Radelet and Sachs (1998) note that credit risk spreads for borrowers in the region increased only after the crisis was in full swing, and the credit rating agencies were largely reacting to events rather than acting in advance. Nor was there much indication from international agencies or the country analysts of normally canny investment banks that a crisis of such magnitude was brewing.

Given the difficulties in coming up with a rigorous theory, it is tempting to fall back on unexplained shifts of sentiment on the part of fickle investors, or the unexplained onset of panic among creditors as an explanation of crisis. As a formal counterpart to such an approach, multiple equilibrium models of currency attacks have gained acceptance among many commentators, and such acceptance owes a great deal to the difficulty in predicting the exact timing of currency attacks, as well as the observation that they are triggered without any apparent change in the underlying fundamentals of the economy. Such models incorporate the self-fulfilling nature of the belief in an imminent speculative attack. If speculators and exposed borrowers believe that a currency will come under attack, their actions in anticipation of this precipitate the crisis itself, while if they believe that a currency is not in danger of imminent attack, their inaction spares the currency from attack, thereby vindicating their initial beliefs. Thus, the onset of a currency attack is explained in terms of a shift from one equilibrium to another.

A large and growing literature has emerged developing this theme and formalizing the intuition[1]. Obstfeld's work (1986, 1994, 1995) has been influential in this regard, and has served to draw a line between multiple equilibrium models of currency attacks and the earlier generation of theories which rely on a secular deterioration of fundamentals (such as Krugman (1979) and Flood and Garber (1984a, b)), and whose argument builds on insights from models of price-fixing in

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[1]An excellent bibliography on the Asian financial crisis is maintained by Nouriel Roubini on http://www.stern.nyu.edu/~nroubini/asia/AsiaHomepage.html. Corsetti, Pesenti and Roubini (1998) is a recent survey of the debate, as is Corbett and Vines (1998). Edison, Luangaram and Miller (1998) offer a perspective on the Thai financial crisis based on the role of assets as collateral.

exhaustible goods markets (Salant and Henderson (1978)).

However, the multiple equilibrium approach is open to the charge that it does not fully explain a currency attack, since the shift in beliefs which leads to the shift from one equilibrium to another is left unexplained. In short, there is an indeterminacy in the theory. The beliefs of the economic actors are seen as being autonomous from the economic fundamentals, and liable to unexplained coordinated shifts. Such a view not only runs counter to our theoretical scruples against indeterminacy, but more importantly, runs counter to our intuition that bad fundamentals are somehow "more likely" to trigger a crisis. Indeed, a growing empirical literature has examined the relationship between the incidence of currency attacks and the underlying economic fundamentals[2]. A satisfactory theory of the onset crisis must explain the shift in beliefs which trigger the attack.

In this paper, we attempt to construct such a theory of the onset of currency attacks. The theory builds on two main features.

- The actions of diverse economic actors which exacerbate a currency crisis are mutually reinforcing. For instance, a hedge fund will find it profitable to attack a currency if it can rely on borrowers with unhedged dollar liabilities to scramble to cover their positions, and thereby exacerbate the crisis. Conversely, the borrower will find it more attractive to hedge if the currency is under attack from speculators.

- Market participants have access to a large mass of information concerning the economic fundamentals, and hence are often well informed of the underlying state of the economy. However, perhaps because of the sheer volume of information, there are small disparities in the information at the disposal of each economic actor.

The first of these features is standard in multiple equilibrium accounts, and we adopt this basic starting point. Our innovation comes with the second feature. When there are small disparities in the information of the market participants, the indeterminacy of beliefs inherent in the multiple equilibrium story is largely removed. Instead, it is possible to track the shifts in beliefs as we track the shifts in the economic fundamentals. This is so, since uncertainty about others' beliefs now takes on a critical role, and such uncertainty often dictates a particular course of action as being the uniquely optimal one. Even vanishingly small differences in information suffice to generate such uncertainty about others' beliefs. When we consider the sheer quantity of information available to market participants - the news wire services, in-house research, leaks from official sources, as well as the

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[2]The Roubini bibliography cited earlier contains a comprehensive list. A recent paper with suggestive results is Kumar, Moorthy and Perraudin (1998).

press and broadcasters, exact uniformity of information is the last thing we can expect.

Indeed, the fragmentation of the media in modern times has generated the paradoxical situation in which ever greater quantities of information is generated and disseminated, but comes at the expense of the shared knowledge of its recipients. Apart from totalitarian regimes in which there is a singe source of information (or perhaps in the heyday of the BBC Home Service), the receipt of information is rarely accompanied by the knowledge that everyone else is also receiving precisely this information at that time. Even among financial markets, the foreign exchange market is especially fragmented. Its market microstructure is characterized by the decentralized nature of the trade necessitated by round-the-clock trading, and the geographical spread which goes with it.

At its most basic, a speculative attack is a resolution of a coordination problem among the diverse interested parties - both foreign and domestic. Small disparities of information determine the outcome of such coordination problems.

In earlier work (Morris and Shin (1998)), we have illustrated this point in the context of a simple static model where speculators observe accurate, but idiosyncratic signals concerning the fundamentals, and they hold uniform prior beliefs about the fundamentals. In this case, the multiplicity of equilibrium is completely eliminated, and a unique outcome emerges in equilibrium.

Here, we develop this line of inquiry further, and clarify the role of differential information in currency attacks. The static framework and the strong distributional assumptions in our earlier work did not allow us to distinguish between the issue of the uniqueness of equilibrium from the more general issue of how the set of equilibria of the imperfect information game is affected by the departures from common knowledge. Although changes in the equilibrium set to shifts in the nature of differential information is to be expected, uniqueness of equilibrium requires additional pieces of the jigsaw to be in place. Also, the static nature of the model in our earlier work detracted from the goal of serving as a theory of the *onset* of currency attacks. Any such theory must take into account the evolution of the fundamentals over time, and incorporate differential information in this dynamic context.

In what follows, the fundamentals evolve according a Brownian motion process, and market participants monitor the fundamentals accurately, but with small differences in their information. We demonstrate the existence of an accompanying stochastic process - called the "hurdle process" - whereby, as long as the fundamentals lie above the realization of the hurdle process, there is no currency attack. However, as soon as the fundamental process falls below the hurdle process, an attack follows inevitably. The imagery is intended to be suggestive. As long as fundamentals can negotiate the hurdle, there is no attack. However, as soon as it

4

"trips over" the hurdle, an attack is triggered. This hurdle process also has the feature that it moves in the opposite direction to the fundamentals. Thus, when fundamentals deteriorate, the hurdle shifts upwards, making it more difficult to clear the hurdle.

We readily acknowledge that such a model is still too rudimentary to yield detailed policy implications. However, it makes a small step in the direction of giving us the framework in which such questions can be addressed *within* the theory, rather than appealing to forces outside it.

2. Elements of a Theory

Defending a currency peg in adverse circumstances entails large costs for the government or monetary authorities. The costs bear many depressingly familiar symptoms - collapsing asset values, rising bankruptcies, the loss of foreign exchange reserves, high interest rates and the resulting reduction in demand leading to increases in unemployment and slower growth. Whatever the perceived benefits of maintaining a currency peg, and whatever their official pronouncements, all monetary authorities have a pain threshold at which the costs of defending the peg outweighs the benefits of doing so. Understanding the source and the severity of this pain is a key to understanding the onset of currency attacks.

Facing the monetary authority is an array of diverse private sector actors, both domestic and foreign, whose interests are affected by the actions of the other members of this group, and by the actions of the monetary authority. The main actors are domestic corporations, domestic banks and their depositors, foreign creditor banks, and outright speculators - whether in the form of hedge funds or the proprietary trading desks of the international financial houses. Two features stand out, and deserve emphasis.

- Each actor faces a choice between actions which exacerbate the pain of maintaining the peg and actions which are more benign.

- The more prevalent are the actions which increase the pain of holding the peg, the greater is the incentive for an individual actor to adopt the action which increases the pain. In other words, the actions which tend to undermine the currency peg are mutually reinforcing.

For domestic corporations with unhedged foreign currency liabilities, they can either attempt to hedge their positions or not. The action to hedge their exposure - of selling Baht to buy dollars in forward contracts, for example, is identical in its mechanics (if not in its intention) to the action of a hedge fund which takes a net short position in Baht. For domestic banks and finance houses which have

5

facilitated such dollar loans to local firms, they can either attempt to hedge their dollar exposure on their balance sheets or not. Again, the former action is identical in its consequence to a hedge fund short-selling Baht. As a greater proportion of these actors adopt the action of selling the domestic currency, the greater is the pain to the monetary authorities, and hence the greater is the likelihood of abandonment of the peg. This increases the attractiveness of selling Baht. In this sense, the actions which undermine the currency peg are mutually reinforcing. They are "strategic complements", in the sense used in game theory.

Indeed, the strategic effects run deeper. As domestic firms with dollar liabilities experience difficulties in servicing their debt, the banks which have facilitated such dollar loans attempt to cover their foreign currency losses and improve their balance sheet by a contraction of credit. This in turn is accompanied by a rise in interest rates, fall in profit and a further increase in corporate distress. For foreign creditor banks with short-term exposure, this is normally a cue to cut off credit lines, or to refuse to roll over short term debt. Even for firms with no foreign currency exposure, the general contraction of credit increases corporate distress. Such deterioration in the domestic economic environment exacerbates the pain of maintaining the peg, thereby serving to reinforce the actions which tend to undermine it. To make matters worse still, the belated hedging activity by banks is usually accompanied by a run on their deposits, as depositors scramble to withdraw their money.

The following table contains a (somewhat simplistic) taxonomy of the various actors and their actions which undermine the peg. The feature to be emphasized is the increased pain of maintaining the peg in the face of widespread adoption of such actions, and hence the *mutually reinforcing* nature of the action which undermines the peg. The greater is the prevalence of such actions, the more attractive such actions become to the individual actor.

Actor	Action(s) undermining peg
Speculators	Short sell Baht
Domestic firms	Sell Baht for hedging purposes
Domestic banks	$\begin{cases} \text{Sell Baht for hedging purposes} \\ \text{Reduce credit to domestic firms} \end{cases}$
Foreign banks	Refuse to roll over debt
Depositors	Withdraw deposits

To be sure, the actual *motives* behind these actions are as diverse as the actors themselves. A currency speculator rubbing his hands and looking on in glee as his target country descends into economic chaos has very different motives from a desperate owner of a firm in that country trying frantically to salvage what he can, or a depositor queuing to salvage her meagre life savings. However, whatever

the motives underlying these actions, they are similar in their consequences. They all lead to greater pains of holding to the peg, and hence hasten its demise.

For the purposes of the formal development of the theory, we will abstract from the diverse motives of the private sector actors, and simply treat everyone as being a potential "speculator" against the currency. Hence, in what follows, the label of "speculator" should be taken to apply to the array of economic actors discussed above.

We summarize by θ the overall perception of the monetary authorities concerning the robustness of the underlying economy, and by implication, the ease with which the monetary authorities can withstand speculative selling of the currency. When θ is low, the economy is in bad shape and the costs of defending the currency peg is high. When θ is high, the reverse is true, and the cost of defending the peg is low. When θ is sufficiently low, the monetary authorities abandon the peg irrespective of the actions of the speculators. Conversely, when θ is sufficiently high, the government maintains the peg irrespective of the actions of speculators. However, the cost to defending the peg depends on the extent to which the peg comes under concerted attack by speculators. For intermediate values of θ, the cost of maintaining the peg is pivotal in the government's decision on whether to abandon the peg.

Let $a(\theta)$ be the degree of ferocity of the attack on the currency which is just sufficient to induce the monetary authorities to abandon the peg, as measured by the proportion of speculators who sell the currency (we assume that each speculator has the binary choice of whether to attack the currency, or not to do so). In other words, if proportion $a(\theta)$ or greater attack the currency at state θ, the monetary authorities abandon the peg, while if the proportion attacking the currency is less than $a(\theta)$, the government maintains the peg. We further assume that

- There is $\underline{\theta}$ such that $a(\theta) = 0$ for $\theta \leq \underline{\theta}$

- There is $\bar{\theta}$ such that $a(\theta)$ is undefined for $\theta > \bar{\theta}$

- $a(\theta)$ is strictly increasing in θ when $0 < a(\theta) < 1$, and there is a bound b on the slope of $a(\cdot)$, so that $0 < b \leq a'(\theta)$.

2.1. Evolution of θ

Time is discrete, and advances in increments of $\Delta > 0$. The value of θ at time t is denoted by $\theta(t)$. Conditional on $\theta(t)$, the value of θ at time $t + \Delta$ is distributed normally with mean $\theta(t)$, and variance Δ. Such a feature would result if observations of θ are snapshots of a process which evolved according to the Brownian

7

motion process

$$d\theta = z\sqrt{\Delta}dt, \tag{2.1}$$

where z is the standard normal random variable.

The monetary authorities observe θ perfectly (after all, θ is the *perception* of the monetary authorities). However, other parties do not observe θ perfectly. In particular, the speculators are able to observe θ only after a delay of Δ. Thus, at time $t + \Delta$, they observe $\theta(t)$.

However, although the speculators do not observe the current value of θ, they do have a noisy signal of the current θ. Speculator i observes at time t the random variable

$$x_i(t) = \theta(t) + \eta_i \tag{2.2}$$

where η_i is a normal random variable with mean zero, and variance $\varepsilon\Delta$, where ε is a small positive number. So, the variance of the noise term is ε times the one-period ahead variance of θ itself. Furthermore, each η_i is independent of θ, and of η_j for all $j \neq i$.

To summarize, at time t, the information at the disposal of the monetary authorities and the speculators are as follows.

- Monetary authorities: $\{\theta(t)\}$

- Speculator i: $\{\theta(t - \Delta), x_i(t)\}$.

2.2. Payoffs

In each period, the speculators decide whether to attack the currency or not based on their information. There is a cost of attacking the currency, given by a constant $c > 0$. As well as the transaction costs associated with attacking a currency, c incorporates any differences in the interest rates between the target currency and the dollar. For a speculator who borrows the target currency and sells it for dollars, the higher interest cost of the borrowing can sometimes be substantial. Our model does not address the determination of this cost. We assume it to be a known parameter.

The monetary authorities observe the aggregate short-selling of the speculators and maintains the peg at θ if and only if the proportion of speculators who attack the currency does not exceed the threshold level $a(\theta)$. When the currency peg is removed, the currency depreciates by a known amount $D > 0$, and remains at this lower level forever. We normalize payoffs and assume from now on that $D = 1$. We thus have the following matrix of payoffs to a particular speculator.

	Peg maintained	Peg abandoned
Attack	$-c$	$1 - c$
Refrain	0	0

$$\tag{2.3}$$

If θ were common knowledge among the speculators, there is the familiar multiplicity of equilibria in the range $(\underline{\theta}, \bar{\theta})$. If speculators believe that the peg will be maintained, they refrain from attacking the currency, which leads to the peg being maintained. If, however, they believe that the peg will be abandoned, they attack the currency, leading to its downfall.

However, common knowledge of fundamentals would be an inappropriate assumption in the context of financial markets, as we shall argue below.

2.3. Joint Distributions

In thinking about the joint distributions generated by our model, recall that if (X, Y) has a bivariate normal distribution, then the conditional distribution of X given $Y = y$ is normal with mean

$$\mu_X + (\rho \sigma_X / \sigma_Y)(y - \mu_Y)$$

and variance

$$\sigma_X^2 \left(1 - \rho^2\right)$$

where μ denotes the mean of the subscripted random variable, σ^2 denotes its variance and ρ is the correlation coefficient between X and Y.

In our case, we will be interested in the one-step ahead covariances conditional on θ at time $t - \Delta$. From our assumptions,

$$
\begin{aligned}
\text{Cov}\left(x_i\left(t\right), x_j\left(t\right) | \theta\left(t - \Delta\right)\right) &= \text{Cov}\left(x_i\left(t\right), \theta\left(t\right) | \theta\left(t - \Delta\right)\right) \\
&= \text{Var}\left(\theta\left(t\right) | \theta\left(t - \Delta\right)\right) \\
&= \Delta
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
\text{Var}\left(x_i\left(t\right) | \theta\left(t - \Delta\right)\right) &= \text{Var}\left(\theta\left(t\right) | \theta\left(t - \Delta\right)\right) + \text{Var}\left(\eta_i\right) \\
&= \Delta\left(1 + \varepsilon\right)
\end{aligned}
$$

When no confusion is possible, we will economize on notation, and drop the time argument. Unless otherwise stated, all covariances are conditional on the realization of θ in the previous period. Hence, we write $\text{Cov}(x_i, x_j)$ and $\text{Var}(x_i)$ for the expressions above.

The one-step ahead correlation coefficients between x_i and x_j and between x_i and θ are given by

$$\rho\left(x_i, x_j\right) = \frac{\text{Cov}\left(x_i, x_j\right)}{\sqrt{\text{Var}\left(x_i\right)\text{Var}\left(x_j\right)}}$$

$$= \frac{\text{Var}\left(\theta\right)}{\text{Var}\left(\theta\right) + \text{Var}\left(\eta_i\right)}$$

$$= \frac{1}{1+\varepsilon}$$

and

$$\rho\left(x_i, \theta\right) = \frac{\text{Cov}\left(x_i, \theta\right)}{\sqrt{\text{Var}\left(x_i\right)\text{Var}\left(\theta\right)}}$$

$$= \frac{\text{Var}\left(\theta\right)}{\sqrt{\left[\text{Var}\left(\theta\right) + \text{Var}\left(\eta_i\right)\right]\text{Var}\left(\theta\right)}}$$

$$= \frac{\Delta}{\sqrt{\Delta(1+\varepsilon)\Delta}}$$

$$= \frac{1}{\sqrt{1+\varepsilon}}.$$

In both cases, the correlation is high when ε is small, and the speculators have good information concerning θ and the signals of others. In terms of the inference problem, the distributions of interest are, first, the conditional distribution of $\theta\left(t\right)$ on $x_i\left(t\right)$ and the previous realization of θ denoted by

$$f\left(\theta|x_i, \theta_{-\Delta}\right) \tag{2.4}$$

and the conditional distribution of $x_i\left(t\right)$ on $\theta\left(t\right)$, denoted by $f\left(x_i|\theta\right)$. The former summarizes the beliefs of speculator i concerning the fundamentals, while the latter gives the distribution of signals for a given state of fundamentals. We know that

$$f\left(x_i|\theta\right) = \eta_i \tag{2.5}$$

so that it is normal with mean zero and variance $\varepsilon\Delta$. As for $f\left(\theta|x_i, \theta_{-\Delta}\right)$, normality of the underlying random variables implies that $f\left(\theta|x_i, \theta_{-\Delta}\right)$ is also normal whose mean is given by

$$E\left(\theta\right) + \frac{\text{Cov}\left(x_i, \theta\right)}{\text{Var}\left(x_i\right)}\left(x_i - E\left(x_i\right)\right)$$

$$= \theta_{-\Delta} + \frac{\text{Var}\left(\theta\right)}{\text{Var}\left(\theta\right) + \text{Var}\left(x_i\right)}\left(x_i - \theta_{-\Delta}\right)$$

$$= \left(\frac{\varepsilon}{1+\varepsilon}\right)\theta_{-\Delta} + \left(\frac{1}{1+\varepsilon}\right)x_i$$

In other words, when trader i observes signal x_i, he forms his beliefs on the current value of θ by taking a convex combination of his current signal x_i and the previous

10

realization of θ. As the signal gets more accurate (i.e. as ε becomes small), the trader puts more weight on his signal, and less on the prior realization. The variance of $f(\theta|x_i)$ is given by $\text{Var}(\theta)(1-\rho^2)$, or

$$\frac{\varepsilon\Delta}{1+\varepsilon}.$$

To summarize,

- $f(x_i|\theta)$ is normal with mean zero and variance $\varepsilon\Delta$.

- $f(\theta|x_i, \theta_{-\Delta})$ is normal with mean $\left(\frac{\varepsilon}{1+\varepsilon}\right)\theta_{-\Delta} + \left(\frac{1}{1+\varepsilon}\right)x_i$, and variance $\frac{\varepsilon\Delta}{1+\varepsilon}$.

- The correlation between x_i and x_j is $1/(1+\varepsilon)$.

2.4. Failure of Common Belief

Although the signals of the speculators are highly correlated when ε is small, there is a qualitative difference between the case when ε is small but positive and when ε is precisely zero for the degree of common knowledge. In the former, there is common knowledge of the fundamentals, but in the latter even *approximate* common knowledge fails, as we shall demonstrate.

The fact that an individual believes some feature of the economy to be this or that way is as much a description of the world as any statement about the fundamentals of the economy. For event E, we can associate those states of the world at which some group of individuals hold certain beliefs concerning E. Define the operator $B_q(.)$ as:

$$B_q(E) \equiv \left\{ \theta \,\middle|\, \begin{array}{l} \text{proportion } q \text{ or higher of speculators} \\ \text{believe } E \text{ with probability } q \text{ or higher} \end{array} \right\}.$$

When θ belongs to $B_q(E)$, proportion q or higher of speculators believe event E with probability q or higher at θ. Consider the event $E = [\underline{\theta}, \infty)$ - i.e. the event that the fundamentals are consistent with the peg. When $\varepsilon = 0$, we have $B_1(E) = E$, and hence

$$E = B_1(E) = B_1(B_1(E)) = B_1(B_1(B_1(E))) = \cdots$$

for any number of iterations of the operator $B_1(.)$, so that whenever fundamentals are consistent with the peg (i.e. when $\theta \in E$), everyone believes this with probability 1, everyone believes that everyone believes it, everyone believes that everyone believes that everyone believes it, and so on, without bound.

Contrast this with the case when ε is small, but positive. Consider when at least 90% or speculators believe E with probability at least 0.9. See figure 1.

[Figure 1 here]

The top graph illustrates the density $f(\theta|x, \theta_{-\Delta})$ - the posterior density over θ given the information $(x, \theta_{-\Delta})$. In order for a speculator to place belief 0.9 or greater on the event E, the signal x must be at least as high as x_*. The next graph illustrates the density of the signals generated by the noise. In order for 90% of speculators to receive a signal greater than x_*, the value of θ must be at least θ_*. Thus, the event in which at least 90% of speculators believe E with probability at least 0.9 is given by the interval $[\theta_*, \infty)$. In other words,

$$B_{0.9}(E) = [\theta_*, \infty) \subsetneq E.$$

Indeed, for $q > 1/2$, we have

$$\theta \notin B_q(\cdots(B_q(B_q(E)))\cdots)$$

for some finite number of iterations. In other words, when ε is small but positive, even *approximate* common knowledge of fundamentals fails.

We should think of common belief not in terms of the mental gymnastics of higher order beliefs, but in terms of the "transparency" of the situation. When two individuals are seated across the same table in a well-lit room, we can reasonably claim that there is common knowledge of this fact, given its transparency to both individuals. The fundamentals, in this case, satisfy a fixed point property in that this situation obtains if and only if both individuals know that this is so[3].

Away from such special circumstances, common knowledge, and even approximate common knowledge is very rarely in place in the real world. For financial markets, common knowledge of fundamentals is a singularly inappropriate assumption. In what follows, therefore, the results which stand in contrast to the benchmark case should be attributed to the failure of common knowledge. We build on the work of game theorists who have investigated the effects of higher order uncertainty (Rubinstein (1989), Monderer and Samet (1989), Carlsson and Van Damme (1993a, b), Morris, Rob and Shin (1995), and Kajii and Morris (1997)). Morris and Shin (1997) is a survey of some of the main results to date.

3. Incomplete Information Game

At date t, if the currency peg is still intact, each speculator decides whether or not to attack the currency based on his information. A strategy for speculator i

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[3]This fixed point characterization of common knowledge was first given formal treatment by Aumann (1976), and emphasized in Barwise (1988), Shin (1993) and others. Monderer and Samet (1989) discuss the analogus fixed point charaterization of common p-belief.

12

is a function

$$(x, \theta_{-\Delta}) \mapsto \{\text{Attack, Refrain}\}. \tag{3.1}$$

In principle, a speculator can choose an action conditional on the whole history of θ, but since history reveals no more information than the previous realization, we restrict attention to Markov strategies of the above form.

The monetary authority observes θ and the proportion s of speculators who attack. It abandons the peg if and only if

$$s \geq a(\theta).$$

If the peg is abandoned, it is never reinstated. In effect, the game ends when the peg is abandoned. The payoffs of the game are given by (2.3).

We can now state the main result of the paper. We shall do it in terms of the following pair of theorems[4].

Theorem 1. For ε sufficiently small, there is a stochastic process $\{h(t)\}$ such that the currency peg is maintained as long as $\theta > h$, but the peg is abandoned as soon as $\theta \leq h$.

Theorem 2. $h(t) \geq h(t - \Delta)$ if and only if $\theta(t - \Delta) \leq \theta(t - 2\Delta)$.

The first theorem states that when market participants have sufficiently accurate information concerning the fundamentals, we can construct a stochastic process - an accompanying "hurdle process" - such that the onset of a speculative attack can be characterized in terms of the fundamentals "tripping over" the hurdle.

The second theorem states that the hurdle moves in the opposite direction to the fundamentals. So, when economic fundamentals deteriorate, the hurdle becomes higher than before. Conversely, when fundamentals improve, the hurdle falls further away. This implies that when the fundamentals deteriorate, the prospect of a currency attack increases *more than proportionately* to the deterioration of the fundamentals. The following figure illustrates a typical time path of the two stochastic processes. $\theta(t)$ and $h(t)$ move in opposite directions until they cross at some point, at which time the peg is abandoned.

[Figure 1a here]

The model and conclusions presented here may be contrasted with our earlier work, Morris and Shin (1998). The economic model in this paper is more

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[4]We are grateful for Jonathan Thomas for suggesting this particular formulation of our results.

reduced form; but it captures the same essential features. Our earlier work made distributional assumptions that guaranteed uniqueness of equilibrium; the normal processes assumed in this paper do not guarantee uniqueness, but uniqueness is guaranteed for sufficiently small noise, and, as we discuss below, this noise does not need to be too small. Finally, by explicitly modelling a dynamic process, we are able to see how the hurdle changes through time. Previous period fundamentals influence the threshold at which an attack occurs because they influence speculators' beliefs about other speculators' beliefs.

3.1. Overview of the argument

Before giving the detailed proofs for theorems 1 and 2, we give an outline of the shape of our argument. The first step in our analysis is to identify bounds on equilibrium actions. For any given profile of strategies by the speculators, denote by

$$\pi\left(x, \theta_{-\Delta}\right) \tag{3.2}$$

the proportion of traders who attack the currency given $(x, \theta_{-\Delta})$

Define the set $\mathcal{E}$ as the set of all π which may arise in an equilibrium of the game. In other words, $\pi \in \mathcal{E}$ if and only if there is some equilibrium in which the proportion of speculators who attack given $(x, \theta_{-\Delta})$ is given by $\pi\left(x, \theta_{-\Delta}\right)$. Define

$$\underline{x}\left(\theta_{-\Delta}\right) = \inf\left\{x \middle| \pi\left(x, \theta_{-\Delta}\right) < 1 \quad \text{and} \quad \pi \in \mathcal{E}\right\} \tag{3.3}$$

$$\bar{x}\left(\theta_{-\Delta}\right) = \sup\left\{x \middle| \pi\left(x, \theta_{-\Delta}\right) > 0 \quad \text{and} \quad \pi \in \mathcal{E}\right\}. \tag{3.4}$$

Thus, $\underline{x}\left(\theta_{-\Delta}\right)$ is the greatest lower bound on the signal at which at least some of the traders do *not* attack the currency. Thus, if $x < \underline{x}\left(\theta_{-\Delta}\right)$, we can be sure that every speculator attacks given $(x, \theta_{-\Delta})$ in *every* equilibrium, while if $x > \bar{x}\left(\theta_{-\Delta}\right)$, every speculator refrains given $(x, \theta_{-\Delta})$ in every equilibrium.

We will show that the bounds $\underline{x}\left(\theta_{-\Delta}\right)$ and $\bar{x}\left(\theta_{-\Delta}\right)$ can be identified by constructing a continuous function $U\left(x, \theta_{-\Delta}\right)$ which has four features.

- $U \to 1 - c$ as $x \to -\infty$, and $U \to -c$ as $x \to \infty$

- $\underline{x}\left(\theta_{-\Delta}\right) = \min\left\{x \middle| U\left(x, \theta_{-\Delta}\right) = 0\right\}$

- $\bar{x}\left(\theta_{-\Delta}\right) = \max\left\{x \middle| U\left(x, \theta_{-\Delta}\right) = 0\right\}$

- $U\left(x, \theta_{-\Delta}\right)$ is decreasing in $\theta_{-\Delta}$.

The function U is positive for small values of x, and is negative for large values. It is continuous, and so must cut the horizontal axis at least once. The smallest value of x at which U cuts the horizontal axis is shown to be $\underline{x}\left(\theta_{-\Delta}\right)$, while the

largest value at which U cuts the horizontal axis is shown to be $\bar{x}(\theta_{-\Delta})$. Thus, once this function has been identified, the characterization of equilibrium actions can be reduced to the simple task of checking where it cuts the horizontal axis.

Moreover, this function is also shown to have the feature that, for sufficiently small ε,

$$\frac{\partial U}{\partial x} < 0, \tag{3.5}$$

so that U cuts the horizontal axis precisely once. This implies that $\underline{x}(\theta_{-\Delta}) = \bar{x}(\theta_{-\Delta})$, so that we can tie down equilibrium actions precisely. For sufficiently small ε, there is a unique θ^* for each $\theta_{-\Delta}$ such that, in *any* equilibrium, the government abandons the currency peg if and only if

$$\theta \leq \theta^* \tag{3.6}$$

The hurdle process $\{h(t)\}$ can then be constructed by defining the realization of $h(.)$ at time t to be the value of θ^* associated with the realization $\theta(t - \Delta)$. Theorem 1 follows from this definition and (3.6).

We now present the proofs of theorems 1 and 2, as well as illustrating the argument with a number of simulations.

4. The Argument

4.1. Conditional Expected Payoff

Denote by $s(\theta, \pi)$ the proportion of speculators who end up attacking the currency when the state of fundamentals is θ, given the aggregate selling strategy π. It is given by

$$s(\theta, \pi) \equiv \int_{-\infty}^{\infty} \pi(x, \theta_{-\Delta}) f(x|\theta) \, dx. \tag{4.1}$$

Denote by $A(\theta_{-\Delta}, \pi)$ the event in which the monetary authority abandons the currency peg when the speculators' aggregate short selling is π. In other words,

$$A(\theta_{-\Delta}, \pi) = \{\theta | s(\theta, \pi) \geq a(\theta)\}. \tag{4.2}$$

When the government abandons the peg, there is a devaluation in the currency of $D = 1$. Since a speculator does not observe θ directly, the optimal decision rests on the expected payoff from attacking the currency conditional on the signal x received. We denote by $u(x, \theta_{-\Delta}, \pi)$ the expected payoff from attacking the currency conditional on $(x, \theta_{-\Delta})$ when the aggregate short selling is given by π. Then,

$$u(x, \theta_{-\Delta}, \pi) = \int_{A(\theta_{-\Delta}, \pi)} f(\theta | x, \theta_{-\Delta}) \, d\theta - c \tag{4.3}$$

15

4.2. Defining $U(x, \theta_{-\Delta})$

The function $U(x, \theta_{-\Delta})$ is defined to be the expected payoff conditional on $(x, \theta_{-\Delta})$ when speculators follow the strategy of attacking the currency if the realization of the signal is x or lower. In other words,

$$
\begin{aligned}
U(x, \theta_{-\Delta}) &\equiv u(x, \theta_{-\Delta}, I_x) \\
&= \int_{A(\theta_{-\Delta}, I_x)} f(\theta|x)\, d\theta - c
\end{aligned}
\tag{4.4}
$$

where $I_x(y)$ is the indicator function which takes the value 1 when $y \leq x$, and takes value 0 when $y > x$. That is

$$
I_x(y) = \begin{cases} 1 & \text{if } y \leq x \\ 0 & \text{if } y > x \end{cases}
\tag{4.5}
$$

In order to express U more succinctly, we characterize the event $A(\theta_{-\Delta}, I_x)$ - i.e. the event in which the peg is abandoned when the speculators' aggregate sales of the currency at given by I_x.

The distribution of x given θ is normal with mean θ and standard deviation $\sqrt{\varepsilon\Delta}$. Denoting by $\Phi(k, \mu, \sigma)$ the cumulative normal distribution at k when the mean is μ and the standard deviation is σ, we have

$$
\begin{aligned}
s(\theta, I_x) &= \Phi\left(x, \theta, \sqrt{\varepsilon\Delta}\right) \\
&= 1 - \Phi\left(\theta, x, \sqrt{\varepsilon\Delta}\right)
\end{aligned}
$$

So, $A(I_x, \theta_{-\Delta}) = (-\infty, \psi(x)]$ where $\psi(x)$ is the unique θ which solves

$$
1 - \Phi\left(\theta, x, \sqrt{\varepsilon\Delta}\right) = a(\theta).
\tag{4.6}
$$

The solution is unique, since a is increasing, while $1 - \Phi\left(\theta, x, \sqrt{\varepsilon\Delta}\right)$ is decreasing. The following figure illustrates $\psi(x)$

[Figure 2 here]

Hence,

$$
\begin{aligned}
U(x, \theta_{-\Delta}) &= \int_{-\infty}^{\psi(x)} f(\theta|x, \theta_{-\Delta})\, d\theta - c \\
&= \Phi\left(\psi(x), \frac{x + \varepsilon\theta_{-\Delta}}{1 + \varepsilon}, \sqrt{\frac{\varepsilon\Delta}{1 + \varepsilon}}\right) - c
\end{aligned}
\tag{4.7}
$$

We note the following properties of this function.

16

- $\underline{\theta} < \psi(x) < \bar{\theta}$

- U is positive for small x (tends to $1 - c$)

- U negative for large x (tends to $-c$)

- U is continuous in x

We can conclude therefore, that U is positive for small values of x, negative for large values of x, and that it crosses the horizontal axis at least once. Consider the smallest and largest values of x for which $U = 0$. We can prove:

Lemma 1.

$$
\begin{aligned}
\underline{x}(\theta_{-\Delta}) &= \min\{x|U(x, \theta_{-\Delta}) = 0\} \\
\bar{x}(\theta_{-\Delta}) &= \max\{x|U(x, \theta_{-\Delta}) = 0\}
\end{aligned}
$$

In our argument for this result,we will need the following preliminary result.

Lemma 2. If $\pi \geq \pi'$, then $u(x, \theta_{-\Delta}, \pi) \geq u(x, \theta_{-\Delta}, \pi')$

Lemma 2 states that the payoff to attacking the currency is higher when the attack on the currency is stronger. In other words, speculators' decisions to attack are strategic complements. To prove Lemma 2, note that if $\pi(x, \theta_{-\Delta}) \geq \pi'(x, \theta_{-\Delta})$, we have $s(\theta, \pi) \geq s(\theta, \pi')$ for every θ, so that

$$A(\pi, \theta_{-\Delta}) \supseteq A(\pi', \theta_{-\Delta}).$$

In other words, the event in which the currency peg is abandoned is strictly larger under π. Thus,

$$
\begin{aligned}
u(x, \theta_{-\Delta}, \pi) &= \int_{A(\theta_{-\Delta}, \pi)} f(\theta|x)\, d\theta - c \\
&\geq \int_{A(\theta_{-\Delta}, \pi')} f(\theta|x)\, d\theta - c \\
&= u(x, \theta_{-\Delta}, \pi')
\end{aligned}
$$

To prove lemma 1, note that

$$
\begin{aligned}
\underline{x}(\theta_{-\Delta}) &\leq \inf\{x|0 < \pi(x, \theta_{-\Delta}) < 1 \text{ and } \pi \in \mathcal{E}\} \\
&\leq \sup\{x|0 < \pi(x, \theta_{-\Delta}) < 1 \text{ and } \pi \in \mathcal{E}\} \\
&\leq \bar{x}(\theta_{-\Delta})
\end{aligned}
$$

17

Now, if $\pi < 1$, then some speculators do not attack. This is consistent with equilibrium only if the payoff from not attacking is at least as high as attacking. By continuity, the same is true at $\underline{x}(\theta_{-\Delta})$. Hence,

$$u(\underline{x}(\theta_{-\Delta}), \theta_{-\Delta}, \pi) \leq 0. \tag{4.8}$$

By strategic complementarity of actions, (lemma 2),

$$U(\underline{x}(\theta_{-\Delta}), \theta_{-\Delta}) \leq u(\underline{x}(\theta_{-\Delta}), \theta_{-\Delta}, \pi) \leq 0 \tag{4.9}$$

implying that

$$\min\{x | U(x, \theta_{-\Delta}) = 0\} \leq \underline{x}(\theta_{-\Delta}) \tag{4.10}$$

Meanwhile, we can construct a symmetric equilibrium in switching strategies at $\min\{x | U(x, \theta_{-\Delta}) = 0\}$. In other words, there is an equilibrium in which every speculator attacks if and only if

$$x \leq \min\{x | U(x, \theta_{-\Delta}) = 0\}$$

To see this, suppose that every speculator follows this strategy. Then, by construction, the speculator who receives the marginal message $\min\{x | U(x, \theta_{-\Delta}) = 0\}$ is indifferent between attacking the currency and not. But from (4.7) the expected payoff from attacking the currency is decreasing in the message x. Thus, any speculator who receives a message greater than the marginal one prefers to refrain, while a speculator who receives a message lower than the marginal one prefers to attack. Thus, we have an equilibrium. The fact that there is a symmetric equilibrium in switching strategies at $\min\{x | U(x, \theta_{-\Delta}) = 0\}$ implies

$$\min\{x | U(x, \theta_{-\Delta}) = 0\} \geq \underline{x}(\theta_{-\Delta}), \tag{4.11}$$

so that together with (4.10),

$$\min\{x | U(x, \theta_{-\Delta}) = 0\} = \underline{x}(\theta_{-\Delta}) \tag{4.12}$$

There is an analogous argument for

$$\max\{x | U(x, \theta_{-\Delta}) = 0\} = \bar{x}(\theta_{-\Delta}) \tag{4.13}$$

This completes the proof of lemma 1.

The shape of the U function determines the equilibrium set, and when ε is small, U is a monotonic function of x.

Lemma 3. $\partial U / \partial x < 0$ if ε is sufficiently small.

Proof. From

$$s\left(\psi\left(x\right), I_x\right) = \Phi\left(x, \psi\left(x\right), \sqrt{\varepsilon\Delta}\right),$$

and denoting by Φ_n the partial derivative of Φ with respect to its nth argument, total differentiation with respect to x yields

$$\Phi_1 + \Phi_2 \psi'\left(x\right) = a'\left(\psi\left(x\right)\right) \psi'\left(x\right)$$

Rearranging,

$$\psi'\left(x\right) = \frac{\Phi_1}{a'\left(\psi\left(x\right)\right) - \Phi_2}$$

However, Φ_1 is the value of the normal density at x, so that $\Phi_1 = \phi\left(x, \psi\left(x\right), \sqrt{\varepsilon\Delta}\right)$, where ϕ is the density corresponding to Φ. The partial derivative Φ_2 is the negative of Φ_1. Thus,

$$\psi'\left(x\right) = \frac{\phi\left(x, \psi\left(x\right), \sqrt{\varepsilon\Delta}\right)}{a'\left(\psi\left(x\right)\right) + \phi\left(x, \psi\left(x\right), \sqrt{\varepsilon\Delta}\right)} \tag{4.14}$$

Consider $\Phi\left(k, \mu, \sigma\right)$. If both k and μ are differentiable functions of x while the variance is constant, then $\Phi\left(k, \mu, \sigma\right)$ is decreasing in x if and only if $k' < \mu'$. Hence,

$$\partial U/\partial x < 0 \iff \psi'\left(x\right) < \frac{1}{1+\varepsilon} \tag{4.15}$$

We know that

$$\begin{aligned}
\psi'\left(x\right) &= \frac{\phi\left(x, \psi\left(x\right), \varepsilon\Delta\right)}{a'\left(\psi\left(x\right)\right) + \phi\left(x, \psi\left(x\right), \varepsilon\Delta\right)} \\
&= \frac{1}{\frac{a'}{\phi} + 1}.
\end{aligned}$$

Thus, $\psi'\left(x\right) < \frac{1}{1+\varepsilon}$ if and only if $a'/\phi > \varepsilon$, or

$$a' > \varepsilon\phi \tag{4.16}$$

Any normal density attains its maximum value at its mean and this is $\frac{1}{\sigma\sqrt{2\pi}}$, where σ is its standard deviation and π is the number pi (not to be confused with the use we have made of it so far). In our case, $\sigma = \sqrt{\varepsilon\Delta}$. Thus,

$$\varepsilon\phi \le \sqrt{\frac{\varepsilon}{2\Delta\pi}} \tag{4.17}$$

19

Hence $\varepsilon\phi \to 0$ as $\varepsilon \to 0$. Thus, for sufficiently small ε the inequality (4.16) holds. This is sufficient for U to be decreasing in x. This proves lemma 3.

The following diagram illustrates the point.

[Figure 3]

The horizontal axis measures x, the vertical axis measures θ. For any given realization of θ in the previous period, the mean of the conditional distribution $f(\theta|x, \theta_{-\Delta})$ is a linear function of x, with slope $1/(1+\varepsilon)$. The smaller is ε, the greater is the weight placed on the noisy signal and smaller is the weight placed on the previous realization of θ. The whole distribution $f(\theta|x, \theta_{-\Delta})$ is depicted above. The function $\psi(\cdot)$ maps x into the value of θ at which the government switches from maintaining the peg to abandoning the peg. $\psi(x)$ takes values in the open interval $(\underline{\theta}, \bar{\theta})$, and is increasing.

From the diagram, we can see that $U(x, \theta_{-\Delta}) + c$ is given by the area under $f(\theta|x, \theta_{-\Delta})$ to the left of the point $\psi(x)$. This is what (4.7) says. Moreover, we can see that the question of whether $U(x, \theta_{-\Delta})$ is decreasing or not depends on the "race" between the mean of $f(\theta|x, \theta_{-\Delta})$ and the point $\psi(x)$ as x increases. If the mean is increasing faster than the point $\psi(x)$, then we indeed have decreasing U. In general, this cannot be guaranteed. However, we saw above that when ε is sufficiently small, this can be guaranteed.

4.3. Example

We illustrate the effect of shifts in ε by means of a numerical example. Let $\Delta = 1$, $\underline{\theta} = 0$, $\bar{\theta} = 1$, and $\theta_{-\Delta} = 0.5$. We plot the function $\psi(x)$ and the posterior mean $(x + \varepsilon\theta_{-\Delta})/(1+\varepsilon)$ for a variety of values of ε. In the following diagrams, the horizontal axis measures x, while the vertical axis measures θ. The posterior mean is the straight line, while $\psi(x)$ is the curve.

Case 1: $\varepsilon = 0.2$

Case 2: $\varepsilon = 1$

Case 3: $\varepsilon = 5$

Case 4: $\varepsilon = 15$

As is clear from these plots, even for moderately large values of ε, the slope of the posterior mean is steeper than the slope of ψ, so that U is monotonic. Only when ε is very large (certainly larger than 5) do we have the possibility of ψ being steeper than the posterior mean.

These simulations suggest that the possible multiple equilibria resulting from a non-monotonic U function may not be important in practice.

From lemmas 1 and 3, Theorem 1 follows from the following argument. For sufficiently small ε, we have a unique point $x^* (\theta_{-\Delta})$ at which $U (x, \theta_{-\Delta})$ cuts the horizontal axis. Hence, in every equilibrium, every speculator attacks given $(x, \theta_{-\Delta})$ if and only if $x \leq x^* (\theta_{-\Delta})$. Then, consider the value of θ given by $\psi (x^* (\theta_{-\Delta}))$, where ψ is the function defined in (4.6). The *hurdle process* $\{h (t)\}$ is defined to be the stochastic process such that

$$h (t) \equiv \psi (x^* (\theta (t - \Delta))). \tag{4.18}$$

21

To prove Theorem 2, note from (4.7) that $U(x, \theta_{-\Delta})$ is decreasing in $\theta_{-\Delta}$ since an increase in $\theta_{-\Delta}$ induces a rightward shift in the posterior density $f(\cdot|x, \theta_{-\Delta})$. This, in turn implies a lower value of $x^*(\theta_{-\Delta})$, and hence a lower value of $\psi(x^*(\theta_{-\Delta}))$. This completes the proof.

5. Limiting case

A case of particular interest to us is the limiting case when the noise ε tends to zero. This serves as a benchmark in several respects. Since we envisage ε as being very small, the limit gives us an indication of the likely shape of the various quantities we have been working with, in particular the function U. Indeed, the U function has a particularly simple characterization in terms of the pain threshold function $a(.)$.

Theorem 3. $\lim_{\varepsilon \to 0} \partial U / \partial x = -a'$.

Since U lies between $1 - c$ and $-c$, Theorem 3 determines the U function uniquely as the "upside-down" version of the function $a(.)$, where its level is fixed to lie between $1 - c$ and c. To prove this result, let us use the shorthand of $\phi = \phi\left(x, \psi(x), \sqrt{\varepsilon\Delta}\right)$, and $\hat{\phi} = \phi\left(\psi(x), \frac{x+\varepsilon\theta_{-\Delta}}{1+\varepsilon}, \sqrt{\frac{\varepsilon\Delta}{1+\varepsilon}}\right)$, then we have

$$
\begin{aligned}
\frac{\partial U}{\partial x} &= \hat{\phi}\left[\frac{\phi}{a' + \phi} - \frac{1}{1+\varepsilon}\right] \\
&= \left[\frac{\hat{\phi}}{(1+\varepsilon)(a'+\phi)}\right](\varepsilon\phi - a').
\end{aligned}
$$

In the interval $(\underline{\theta}, \bar{\theta})$, these densities become degenerate as ε becomes small, so that the expression in square brackets tends to 1, while $\varepsilon\phi \to 0$. Hence,

$$
\frac{\partial U}{\partial x} \to -a'. \tag{5.1}
$$

This is a very appealing result, in that it gives a simple characterization of the U function in terms of the fundamentals of the problem. The shape of the U function in the limit is the mirror image of the "pain threshold" function a. This puts the focus squarely on the factors which determine the a function.

For instance, if the interval $(\underline{\theta}, \bar{\theta})$ is wide, then slope of a is shallow, and a small increase in cost has large impact on the cutoff θ^*. In terms of the economic interpretation, a wide interval $(\underline{\theta}, \bar{\theta})$ translates into the statement that speculators' actions are more influential/decisive in dictating the exchange rate. A variety of

22

factors will influence such decisiveness. The size of country relative to pool of hot money will certainly be a factor, as well as the composition of financial flows and the maturity structure of debt. A shallow $a(.)$ function can also be seen reflecting the strength of the mutually reinforcing nature of the actions undermining the peg.

6. Concluding Remarks

Whilst the theory advanced in this paper is too rudimentary to serve as a tool for assessing practical policy alternatives, it does set out the considerations which could guide our thinking. We regard the contribution here very much as a conceptual one. We believe that our approach provides a handle on the evolution of beliefs which trigger the change of sentiment, which in turn precipitates the attack. In this sense, we propose our theory as one of the *onset* of currency attacks. Developments of this framework may shed further light on the problem.

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