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On the Internationalization of Portfolios

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by

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ON THE INTERNATIONALIZATION OF PORTFOLIOS*

Portfolio theory has been an important component of open-economy macroeconomic models. In those models, it is essential to distinguish among several categories of assets, both foreign and domestic, and to specify their demands and supplies. This framework has become increasingly relevant. Movements of capital across regional and national boundaries, and across currencies, have exploded in volume, thanks to the dismantling of currency and exchange controls and other financial regulations and to revolutionary economies in technologies of communication and transactions. The globalization of financial markets was stimulated by the floating-exchange-rate regime established in 1973.

Figures 1 and 2 show the trends of internationalization of, respectively, American owned wealth and foreign ownership of American assets. The latter has been growing especially rapidly in the last ten to twelve years, a period when the U.S. net wealth position vis-a-vis the rest of the world deteriorated sharply and turned negative. Nevertheless U.S. direct investments abroad and holdings of foreign equities grew sharply during the recent cyclical recovery. Evidently U.S. current account deficits were financed mainly in credit and bond markets. These Figures say that internationalization in both directions remains quite modest. Americans as private households and firms have put only 4 percent of their net worth in foreign assets. Foreigners' gross claims on Americans amount to about 7

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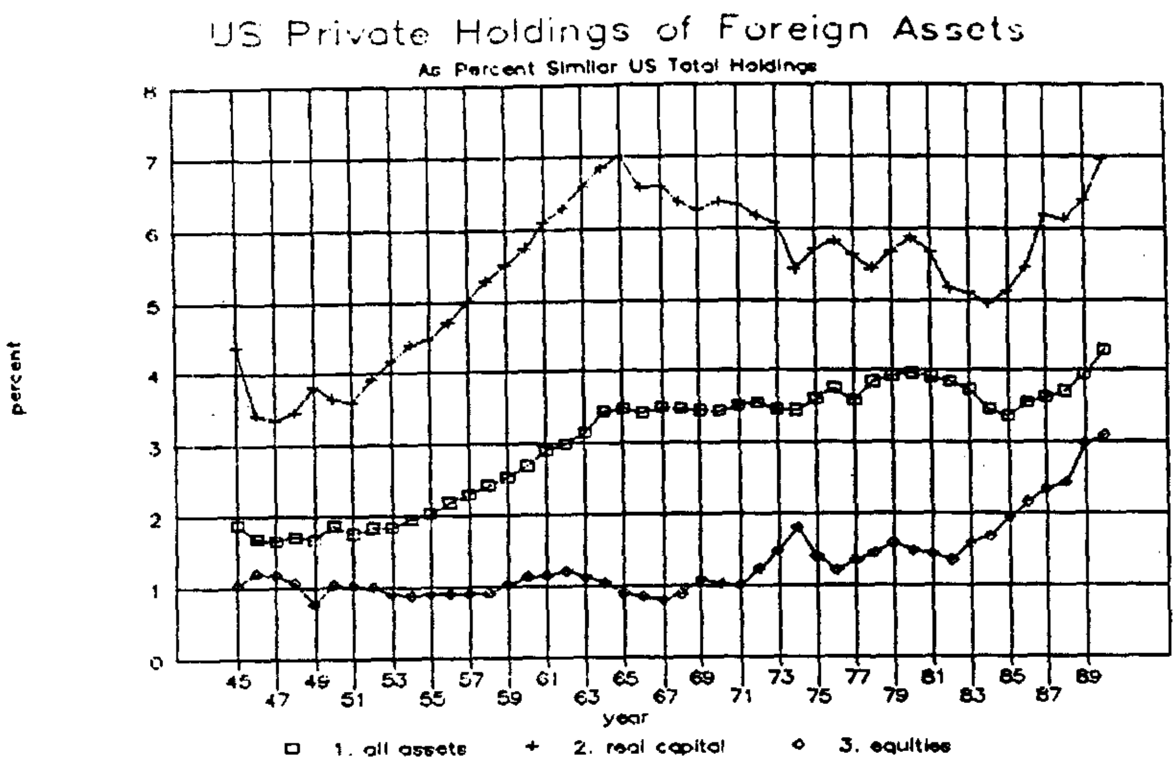
*In this expository paper we have adapted material from previous articles, in particular Brainard and Tobin (1968), Brainard and Dolbear (1971), and Tobin (1982 and 1990), as well as articles specifically cited.

FIGURE 1



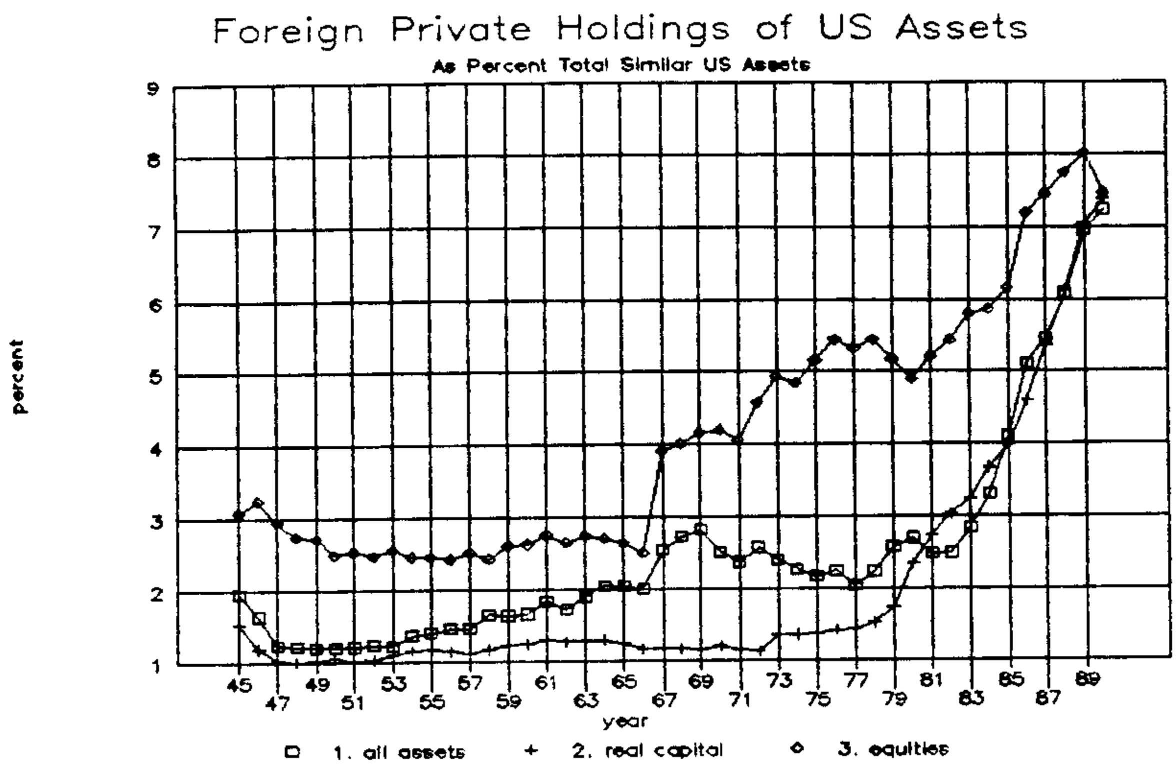
Source: Board of Governors of the Federal Reserve System, Balance Sheets for the U.S. Economy 1945-90, C.9 Balance Sheets Flow of Funds, March 1991.

Ratio 1: US Private Holdings of Foreign Assets / US Private Net Worth
Numerator: Table for Net Foreign Assets (NFA) pp. 7-12:
Line 2 Foreign assets owned by US residents
- Line 3 US official fgn. exchange & net IMF position
- Line 14 US government loans
- Line 18 US equity in IBRD etc.
- Line 19 US government deposits
Denominator: Table for National Net Worth (NNW) pp. 1-6:
Line 19 Private net worth, consolidated

Ratio 2: US Direct Investment Abroad / US Business Capital
Numerator: NNW Line 9 US dir. invest. abroad
Denominator: NNW Line 4 Nonres. plant and equipment
+ Line 5 Inventories

Ratio 3: US Private Holdings of Foreign Equities / US Nonfinancial Equities
Numerator: NFA Line 5 Foreign corporate equities
Denominator: Table for Nonfinancial Corporate Business (NCB) pp. 31-36.
Line 45 Market value of equities

FIGURE 2



Source: Federal Reserve Flow of Funds Balance Sheets March 91. See Figure 1.

Ratio 1 Foreign Privately Owned US Assets / US Private Net Worth
Numerator: NFA Line 22 US assets owned by foreigners
Denominator: NNW 19 Private net worth consol.

Ratio 2 Foreign direct investment in US / US business capital
Numerator: NFA Line 38 Direct investment in US
Denominator: NNW Line 4 Nonres. plant and equip.+ Line 5 Inventories

Ratio 3 Foreign holdings of US equities / US nonfinancial equities
Numerator: NFA Line 27 US corporate equities (owned by foreigners)
Denominator: NCB Line 45 Market value of equities

TABLE 1. FOREIGN ASSETS AND DEBTS 1987

1A. billions of dollars

	Assets	Debts	Net Assets	GNP
United States	1168	1536	-368	4527
Japan	1072	831	241	2369
Germany	584	436	148	1126
United Kingdom	1316	1149	167	667

1B. Relative to GNP (percent)

	Assets	Debts	Net Assets
United States	25.8	33.9	-8.1
Japan	45.3	35.1	10.2
Germany	51.9	38.7	13.1
United Kingdom	197.3	172.3	25.0

Source: Assets and Debts figures fom Kuroyanagi, Hamada, and Sakurai, 1989, Table 6. GNP figures from Statistical Abstract of the United States 1990, Table 1446, p. 840.

TABLE 2. DIRECT FOREIGN INVESTMENT STOCK 1986

2A. billions of dollars

	From	To	Net Position
United States	260	220	40
Japan	58	7	51
Germany	49	31	18
United Kingdom	142	69	73

2B. relative to GNP (percent)

	From	To	Net Position
United States	5.74	4.86	0.88
Japan	2.45	0.30	2.15
Germany	4.35	2.75	1.60
United Kingdom	21.29	10.34	10.94

Source. Kuroyanagi et al (1989), Table 12. GNP as for Table 1.

percent of private net worth. However, projections of recent trends by Hamada and Iwata (1989) suggest that Japan and Germany will own one third of the United States in 2010. Presumably these trends themselves are setting in motion adjustments that will slow them down and even reverse them.

Kuroyanagi, Hamada, and Sakurai (1989) are in process of building a world asset and debt matrix from scattered sources. It is a difficult task. Some of their preliminary estimates are reported in Tables 1a and 2a. We have tried to put their numbers in perspective by scaling them to GNP. Stocks of wealth and capital are 4 to 6 times GNP, so the numbers in Tables 1b and 2b would be much smaller if scaled by those stocks. The Tables decisively establish the primacy of the United Kingdom as an international lender, borrower, and investor. After all, the City and Whitehall both have been at it for a long time. The rest of us have a long way to go before our portfolios become as international as the British.

Exchange Rates and Portfolio Theory

The 1973 regime shift accentuated the interest of macroeconomists in explaining the determination of exchange rates. "The exchange rate is an asset price," a fashionable aphorism fifteen years ago, symbolized the centrality of asset markets, and thus of portfolio theory, in open-economy macroeconomics. There is truth in the aphorism, but only half truth. The exchange rate is, of course, a factor, but only a factor, in the price of an asset in any currency other than the one in which it is denominated. That is not the only macroeconomic role of the exchange rate. It is also a factor in the external prices of goods and services.

The characteristic feature of portfolio theory in its macroeconomic applications is the assumption that assets are imperfect substitutes, that their relative prices and returns vary systematically as their relative quantities are varied by policies or other events. In contrast, models strongly influenced by finance theory often reduce the effective number of distinct assets by assuming perfect substitutability. Those models are convenient for rational expectations methodology. However, the imperatives of empirical relevance and econometrics work in the direction of multi-asset models.

As usual in macroeconomics, a distinction can be made between short-run and long-run applications of portfolio theory. Short-run theoretical and empirical models concern cyclical fluctuations and demand management policies for counter-cyclical stabilization. In long-run trends of capital accumulation, growth, and terms of trade, real fundamentals can be expected to dominate. We shall discuss both.

Asset markets in short-run open-economy macroeconomics

Mundell-Fleming models, the open-economy extensions of IS/LM, are still the tools of first resort. They apply to single "small" economies, to which the rest of the world is exogenous. They are still useful, but some shortcomings of IS/LM are underscored and magnified by the extension. The IS/LM short-cut assumption that all assets other than money are perfect substitutes for each other is bad enough for a closed economy. It is worse when the asset menu is expanded to embrace foreign assets. The rudimentary asset menu needs to be amended by recognizing foreign assets as distinct items on the menu. But often this is not explicitly done, even when a "BP" curve is added in r - Y space, purporting to show the locus of (r, Y) at which external payments in and out are balanced. That locus cannot be other than horizontal unless foreign and domestic assets are imperfect substitutes. And if they are, then the stocks of those assets, not just their flows, are relevant and must be tracked.

Besides overdoing the assumption of perfect substitutability among assets other than money, the IS/LM model does not deal with asset stock accumulations. Although the solution of the model generally implies that capital and other assets are begin accumulated, the effects of the accumulations are not explicitly modeled. The excuse is that the flows, during the period to which the model is relevant, are too small to alter the stocks significantly. The excuse is especially implausible for a country's stock of net foreign assets, which could be close to zero, small relative to current account flows within a year.

Over the years we, Brainard and Tobin, have proposed the appropriate multi-asset stock adjustment framework. See Brainard and Tobin (1968) and Tobin (1982). Let us remind you of that framework now. The strategy is to specify for each asset on the menu of portfolio choice the desired end-of-period stock, at the asset's current-period price. This stock demand is a function of the vector of the rates of return for one period ahead (inclusive of capital gains or losses expected) on all the assets on the menu. Stock demand is also a function of national income, the commodity price level, and perhaps other nonfinancial variables. The list of variables is in principle the same for all the assets; every individual's portfolio demands are, in effect, a single decision. Stock demands will in general differ from the values, at the current period asset price, of the quantities of the asset carried over from the previous period. Flow demands for the period are taken to be fractions of these discrepancies, plus a growth factor. Equation (1) illustrates the specification.

(1) $\alpha(A_i(t) - q_i(t)a_i(t-1)) + gq_i(t)a_i(t-1) = S_i(t)$

Here α is the stock adjustment fraction; A_i is the function describing desired end-of-period value of holdings of asset i; q_i is the price of asset i, usually related negatively to its rate of return; $a_i(t-1)$ is the quantity, not value, of the asset carried over from the previous period; g is the growth rate. The left hand side of (1) is the flow demand in period t. It is equated to the flow supply $S_i(t)$.

For equities and direct investments in business capital, the S_i are the increments in capital stocks, evaluated at current replacement costs or commodity prices. For government obligations, base money and interest-bearing debt, the S_i are the changes in outstanding supplies resulting from central bank and Treasury transactions with the public. These equal in total the government deficit. For claims on foreign countries, the S_i are supplies made available by the current account surplus. New asset supplies may contain exogenous elements, but they are generally not wholly exogenous. Domestic capital investments and government deficits are in part policy decisions and in part functions of other variables. The current account surplus or deficit, related to domestic and foreign economic activity and prices, and to exchange rates, is largely endogenous.

The summation of equations like (1) is the "IS" equation for the economy. The left-hand sides add up to private saving, and the right hand sides to its absorption in net domestic investment, government deficit, and current account surplus. If there are n assets on the menu, the n independent asset demand-supply equations determine n variables. These include the rates of return, but at least one of them, the rate on nominal base money, is exogenously fixed, and others may be also. That is why the system can determine one or more other variables, notably Net National Product or the commodity price level.

Standard Mundell-Fleming one-period multipliers for fiscal and monetary policies and internal and external shocks can be qualitatively confirmed in a multi-asset model, on several assumptions. The most important are: (a) Assets are gross substitutes for each other; in particular, an increase in wealth due to a rise in the return on any asset does not raise the demand for holdings of any other asset. (b) The private sector holds non-negative quantities of all assets, foreign and domestic (this does not exclude that the country is a net debtor internationally). (c) Expectations of future returns, inclusive of capital gains, on assets are not extrapolative, but are neutral or regressive. This means, in particular, that appreciation of the currency today does not lead portfolio managers to expect the currency to appreciate further tomorrow. (d) Real net exports are increased by depreciation of the currency and reduced by appreciation. See (Tobin and Macedo, 1980) and (Tobin, 1982).

However, the multi-asset approach does lead to different conclusions on some important issue. Our model does not, except in extreme cases, justify the proposition that fiscal policy is impotent in a floating-exchange-rate regime or that monetary policy is impotent in a fixed-rate regime. Nor does it validate the extravagant claim that a floating rate insulates an economy from external demand shocks.

Models with more than one country endogenous.

The "small country" assumption that the rest of the world is exogenous is scarcely applicable to the relationships among the Group of Seven economies, particularly the three economic superpowers, North America, the European Community, and Japan. In principle a 2-country or n-country model can be constructed from building blocks like those of the one-country model. We give an example for the two countries, United States and Japan.

Assets in both countries are available to wealth owners and portfolio managers in both countries. (Some assets may be held in negative amounts). The notation is tedious, because the identity of the asset, its country of origin, and the residence of holders of it must all be described.) Amounts of American assets are designated by a_i , amounts of Nipponese assets by n_j . Holdings by Americans are denoted by superscript A , those by Japanese by superscript N. Thus $a_i = a_i^A + a_i^N$ and $n_j = n_j^A + n_j^N$. The units of assets are par values for nominally dominated securities and commodity units for equities or real properties. The economies' steady-state growth trend rates are g_a and g_n .

The prices of assets are q_{ai} and q_{nj} , in dollars or yen per asset unit. These prices are related, normally inversely, to rates of return r_{ai} and r_{nj} . In each country there is one asset, base money, whose price is identically 1 in local currency, and whose nominal return is zero or some other constant. The exchange rate is e dollars per yen.

The values of desired asset holdings in currency of the country of issue are, for Americans A_i^A and N_j^A, for Japanese A_i^N and N_j^N. In the currency of the holder they are A_i^A and eN_j^A, $(1/e)A_i^N$ and N_j^N . These latter are functions of the vectors of rates of return (r_{ai}, r_{nj}), of prices and expected inflation rates, of Net National Product (current and in principle future too) in the investors' home country, and of expected change in the exchange rate.

Here are the flow demands for the various assets, all expressed in dollars:

$$(2)\quad fA_i^A(t) = \alpha(A_i^A(t) - q_{ai}(t)a_i^A(t-1)) + g_a q_{ai}(t)a_i^A(t-1)$$

$$fA_i^N(t) = \beta e(t)([(1/e(t))A_i^N(t)] - (q_{ai}(t)/e(t))a_i^N(t-1)) + g_n e(t)(q_{ai}(t)/e(t))a_i^N(t-1)$$

$$fN_j^A(t) = \alpha([e(t)N_j^A(t-1)] - e(t)q_{nj}(t)n_j^A(t-1)) + g_a e(t)q_{nj}(t)n_j^A(t-1)$$

$$fN_j^N(t) = \beta e(t)(N_j^N - q_{nj}(t)n_j^N(t-1)) + g_n e(t)q_{nj}(t)n_j^N(t-1)$$

In each case, the first term is the stock adjustment, and the second term is the flow required to make the holding increase at the rate of the economy's growth. The speeds of adjustment, α and β are multiplied by the excess of desired holdings over the value of previous holdings at the prices of the current period.

The flow demands for each asset -- $(f_{ai}^A + f_{ai}^N)$ and $(f_{nj}^A + f_{nj}^N)/e$ in local currencies -- are to be equated to the new supplies.

Total wealth stocks and total private saving in both economies are implicit in the above equations. They are just the sums of stocks and flows for the wealth owners of the country. In steady states, with stock adjustments zero, asset supplies and holdings and total wealth would be growing at rates g_a and g_n .

There are two more supply-demand equations than there are endogenous asset prices (or corresponding rates of return) to be determined. That is because the local prices of currencies and their nominal returns are fixed.

And there is one more equation, the payments balance equation. It is natural to think of the equality of net capital flow to current account balance as determining the exchange rate, just as in the one-country framework sketched above. This is the equation (3), with the net capital inflow to the United States in dollars on the left hand side and on the right hand side the American current account deficit. This deficit has two parts, the trade deficit and the deficit in investment incomes. Here p and Y with superscripts are local price levels and Net National Products.

$$(3) \quad \Sigma_i fA_i^N - \Sigma_j fN_j^A = X^N(ep^N/p^A, Y^A) - eX^A(p^A/ep^N, Y^N) + \Sigma_i r_{ai} q_{ai} a_i^N - e\Sigma_j r_{nj} q_{nj} n_j^A$$

What variables do the two extra equations determine? In some contexts these would be the two Y's, or the two price levels. If aggregate supply equations -- "AS curves" -- were added for each economy, all four of these variables could be determined. Alternatively, two instruments of monetary policy, local interest rates or monetary bases, could be the free variables. The two price levels would be taken as predetermined in the short run and the two Y's would be those chosen by the monetary authorities. There are many other possibilities.

Models of this kind are designed to exploit imperfect substitutabilities among assets, to analyze the effects of variations in relative asset supplies, and to explain variations in differentials among asset rates of return. For these purposes the crucial parameters are the partial derivatives of desired asset stocks with respect to rates of return. It is intuitively natural to expect own partials to be positive, at least non-negative. Furthermore, the gross substitutes assumption is convenient and plausible, though not mandated by rationality. High absolute values of cross partials indicate high substitutability between assets, low values the reverse. Common vulnerabilities to certain risks make for high substitutability. Assets whose returns, including capital gains or losses, do not share the same risks, or move in opposite directions in response to the same shocks, will be poor substitutes. We give an example below, where we argue that equities and real properties in two countries could be good portfolio substitutes for each other but poor substitutes for nominal assets in either currency.

Table 3 illustrates an accounting scheme for equations (2) and (3). In the matrix, the columns represent sectors of the two economies, America on the left and Nippon on the right. Only four sectors of each economy are distinguished here. The rows represent assets, American at the top, Nipponese at the bottom. Seven asset categories are shown for each country. A plus sign + in a cell indicates that the sector holds positive quantities of the asset, a minus sign - that the sector is a debtor in this category of asset. The entries in a column, for both local and foreign assets, describe the sector's portfolio; they sum to the sector's net worth. The entries in a row show how the positive and negative holdings of a given asset are distributed among sectors in the two countries. The row sums, in the final column, are zero for all the assets that are obligations of one or more of the sectors. They are positive only for real property, capital, in the two economies. The sum of the values of the two capital stocks are equal to the sum of the sector net worths. Of course, one country -- in present circumstances, Japan -- may have a net worth greater than its own capital stock by the amount of its net claims on the other country.

Table 3 is a schematic balance sheet, in which the entries are stocks.

An analogous schema would describe the flows in any period, as in the model above. The illustration is for a two-country world, but the same format could be used for n countries. Obviously we are a long way from having data for such spread sheets. Kuroyanagi, Hamada, and Sakurai (1989) have made a pioneering start.

In a two-country model, it is not possible to get definitive qualitative results like those reported above for a one-country open economy. This is true *a fortiori* for n countries. However, in a two-country floating-rate model, the presumption is that expansionary fiscal policies and other positive "IS" shocks in one country will raise real outputs and/or prices in both countries. On the other hand. expansionary monetary policies and other positive "LM" shocks in one country will raise real outputs and/or prices in that country but lower them elsewhere. (Tobin and Macedo, 1980)

Home asset preferences.

International financial and capital markets determine simultaneously the prices of the various assets and currencies, as well as domestic commodity prices and/or incomes. Asset values will depend on their relative supplies and on the asset preferences of wealth owners. A common assumption is that wealth owners display home currency preference, both on average and on the margin. That is, Americans hold most of their wealth in dollar assets and, at given rates of return and exchange rates, will place most of a given increment of wealth in dollars. They will require a premium over local rates of return to invest abroad.

As asset supplies and total wealth of the several countries evolve over time, so will their trade balances and current account deficits or surpluses. These depend on prices, exchange rates, interest rates, and domestic absorptions. Current account deficits equal capital inflows. The identity says nothing about causation, which can run either way. Monetary and financial shocks may lead to international asset transfers whose counterparts are current account imbalances; exchange rates may change in the process.

On the other hand, shocks of taste and technology that alter the competitiveness of nations in trade may lead to current account imbalances and shifts of wealth, which set in train adjustments of prices, interest rates, and exchange rates along with international asset transfers. These adjustments are required because wealth owners differ in preferences. If those in a surplus country prefer local assets, their currency will become more valuable.

Home currency preference is by no means an implication of portfolio theory. The risks of nominal assets are those of commodity price fluctuation, interest rate variation, and default. Economies with different currencies, geographies, and central governments will be more independent of each other in these dimensions than, for example, the states of a central union.

Exchange risk may work either way. Here it is necessary to consider the correlations of deviations of exchange rates from expected trends with the other risks of nominal assets. If a foreign currency appreciates when home inflation increases and depreciates when foreign inflation is relatively high, foreign currency assets are a hedge. If the foreign currency depreciates when domestic interest rates move up, foreign securities are not a good hedge against capital losses on home bonds.

In the past, home asset preferences have been due less to risk-return calculus than to other factors: legal restrictions, transactions costs, and information gaps. These obstacles have all been diminishing. Institutional and individual portfolios are slowly becoming more international.

Equities and direct investments in real properties in foreign countries present opportunities and risks quite different from those of currency-denominated assets. Some of the geographical, political, legal, and informational reasons for home preference apply to both, to be sure. But one consideration that leads to home preference among currency-denominated assets, the avoidance of exchange risk in consuming the income or principal, applies with considerably less force.

Equities, and the real capital assets to which they are claims, are not entitlements to specified amounts of any currencies. An extreme view is that goods are goods, capital goods are capital goods, factories are factories, wherever they are located. Earnings of multi-national companies come from worldwide sales in numerous currencies. Neither the earnings nor the value of the shares in any currency need be particularly correlated with the price of the currency of the country where the company is domiciled. Indeed if a company is leveraged by debt in its home currency, owners of its equity are going short in that currency.

Consider Japanese direct investments in the United States, say in particular the acquisition of facilities for producing internationally traded goods like automobiles. In the first instance Japanese investors use dollars they bought from Japanese exporters to buy or build a factory, instead of purchasing U.S. Treasury bonds or other nominal dollar assets. This substitution of American equity for bonds may be only transitory.

Japanese investors' demand for nominal dollar assets should not decline by the full amount of their direct investments in the U.S. The reason is that an automobile factory in Ohio may well be a closer substitute for factories, properties, and equities in Japan and elsewhere than it is for future dollars _qua_ dollars.

The international car market can be supplied from Tennessee or from Tokyo. The long-run real returns from owning a plant in Tennessee are not very dependent on the dollar/yen exchange rate and not very vulnerable to the factors that might generate losses to Japanese holders of American bonds. U.S. inflation, for example, would raise the dollar earnings from operating the plant at the same time as it depreciated the dollar against the yen. Direct investment of this kind in the U.S. is a portfolio reallocation vis-a-vis Japanese plants, real properties, and equities more than vis-a-vis dollars per se. Indeed scattering production for the world market over various locations may reduce risk by diversifying risks due to national or regional productivity shocks.

When Japanese investments overseas are in real assets productive of non-traded goods, like office space in Rockefeller Center, they are not so obviously substitutes for similar real assets in Japan. But neither are they particular substitutes for U.S. bonds or other U.S. nominal assets. However, the principal location and legal and tax domicile of a business do entangle its earnings with the domestic and exchange value of the currency. Equity in a country's businesses may be a closer substitute for home than for foreign debt securities.

Allowing for risks to consumption.

A basic contribution to portfolio theory has been recognition that what really concerns an individual saver and investor is the return on his total "portfolio", which includes nontradable assets (and possibly debts) as well as marketable assets. Indeed human capital, the value of labor, is the major component of wealth of most of us for most of our lives. The possibilities of selling it or borrowing against it are strictly limited. The returns on it are uncertain. They are subject to their own idiosyncratic shocks, but also to many of the same shocks that affect the returns on tradable assets. It is this insight that led to "consumption-betas" in asset pricing models.[2]

Assuming, as we have right along, that individuals are predominantly risk averse, they are willing to give up some mean expectation of portfolio return to reduce their risk. Applying this assumption to their total portfolios, inclusive of nontradable human capital, implies that they will give up some return on their market portfolios or accept some additional risk on them, in order to diminish their overall risk.

Considerations of this kind suggest that a worker should not invest his retirement savings in the shares of the company that employs him. And that a farmer should not invest in agricultural real estate subject to the same meteorological shocks as his own farm.

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2. An early paper with this perspective, distinguishing between tradable and nontradable assets and emphasizing social risk, is Brainard and Dolbear (1971). Golub (1990) has recently studied empirically opportunities for international diversification between U.S. and Japan. On the general use of "consumption betas," stressing covariances with consumption rather than with market, the basic references are Merton (1973) and Breeden (1979).

The same considerations may well suggest the reverse of home asset preference, on the ground that domestic macro-economic shocks affect capital and labor incomes in the same directions. If foreign assets are indeed useful hedges against major shocks to domestic wage incomes, then the small amount of international diversification observed is an even greater puzzle than it appears when only tradable-asset portfolios are considered. The puzzle leads to a forecast, that global diversification has a long way to go.

Appendix A gives a simple example of the opportunities afforded by adding foreign assets to a portfolio. Using mean-variance analysis, risk-return frontiers are computed for two asset menus: (1) two tradable domestic assets, nominal bonds and equities, (2) the first menu augmented by two foreign assets, nominal bonds and equities. These frontiers show the minimum risk to the portfolio return for a sequence of required mean portfolio returns. It is shown that the addition of foreign assets to the menu improves the frontier.

Calculations are also made for the risks of a more inclusive "portfolio" inclusive of human capital yielding wages, a nontradable asset that the typical agent can neither buy nor sell. Taking into account this non-discretionary asset along with the menu of market assets whose holdings are discretionary, makes significant difference. When shocks to wages are correlated with those of other assets, the composition of efficient market portfolios will be quite different if the objective is to minimize total variance for given mean return, rather than to minimize portfolio variance alone.

In the illustrative simulations of Appendix A the several asset returns are affected in different ways and degrees by several shocks, domestic, foreign, and common: shocks to productivity, inflation, nominal interest rates, world oil prices, and the nominal exchange rate. In the example, the availability of the foreign assets improves the frontier substantially, especially when account is taken of the covariances of wages with portfolio asset returns. The calculations also illustrate how different the efficient tradable portfolios are when efficiency is defined to be minimization of total risk rather than just risk on the portfolio of tradable assets.

The illustration does not give a full general equilibrium solution. It shows there is "room for a deal." The home side would want a deal, and since foreign worker-investors are in a symmetrical position, they would want a deal too. Of course, the deal would not be as good as the partial calculations suggest, because the reallocations of assets will change their rates of return.

Gains from international asset trades in the long run.

Our discussion so far concerned relatively short horizons, when shocks to real and nominal aggregate demand and shocks to nominal interest rates are major sources of risks to domestic asset holdings. Likewise, exchange rate fluctuations induced by such shocks at home and abroad are major sources of risks in holding claims on foreigners. Over longer horizons demand-side fluctuations in economic activity and nominal shocks to financial markets are less important sources of risk. Changes in levels and rates of growth of productivity, directly affecting the real value of a country's endowment of labor, land, and capital are the basic domestic sources of risk. And shocks to the terms of trade a country faces in international commodity markets are likely to be the dominant external sources of risk. Although many investors appear to focus on short run fluctuations in financial markets, over an individual's lifetime fundamental long-term risks are probably more important. For nations, this is _a fortiori_ true.

Here we examine the long-run gains from international trade in assets, an inquiry similar to the theory of gains from commodity trade. (If our model seems applicable to asset trade between individuals or households as much as between nations or regions, that is of course not surprising. So does the theory of trade in commodities, including the law of comparative advantage.) We investigate the importance of the basic risks identified above, and the potential of international diversification to reduce them and thereby to improve welfare. Our illustrative model is simple and abstract; nonetheless it is not easy to analyze, and we use numerical simulations.

We consider a two-country competitive exchange economy.[3] The two countries are mirror images of each other. There are two goods, one produced by each country. They are not perfect substitutes in consumption, and they are traded between the countries in a spot market in which the terms of trade are competitively determined. The representative agent in each country consumes both commodities but has a preference for home goods. In the simulations reported in Appendix B, the demand functions are contrived to imply a 20 percent share of imports when the price ratio between the two goods is unity. Agents have infinite horizons but discount the future. Preferences are intertemporally additive and display constant relative risk

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[3] The model is in effect a two-country version of the model used by Mehra and Prescott (1985) to examine the equity risk premium.

aversion. The demands in each period are derived from a CES "felicity"[4] function, with constant relative risk aversion for commodity bundles with any fixed composition of home and foreign goods. The felicity and utility functions for the two agents have the same elasticity of substitution, risk aversion and time preference.

In this economy an asset is simply a claim on current and future endowments. Each country is exogenously endowed each period with some quantity of its perishable consumption good. Over time the endowment of each country follows a Markov process, wandering around a mean level of one. The two processes are independent of each other. Specifically, in the simulations of Appendix B, a country with an endowment e_t in period t has either the same endowment, s percent more, or s percent less in period t+1, with specified probabilities for the three changes of state. These probabilities are not independent of the state. Movements of endowments away from their central value, 1, are limited by a floor and a ceiling. As the endowment gets close to its floor, the probability of a downward move declines, and that of staying put correspondingly increases. At the floor the probability of further decline is zero. The adjustment of probabilities near the ceiling is symmetric. The probability of a regressive movement is always the same. Although this process gives a stationary distribution of endowments, in the neighborhood of 1.0 it is approximately a random walk. Since the probabilities of movement up and down are symmetrical, no upward

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[4] It is convenient to call the utility generated by consumption in a single period "felicity" and to use the term "utility" to describe the properly weighted sums of period-by-period felicities over the consumer's horizon.

drift in endowments occurs.[5]

Since the purpose of this exercise is to investigate the importance of international portfolio diversification, we calculate equilibrium terms of trade, asset prices, rates of return and risk premia for two asset menus. In the first, capital market autarky, residents of a country can hold claims only on their own country's endowments. They have no opportunity to diversify or to hedge on future changes in the terms of trade. Second, agents are allowed to hold claims for pro-rata shares of the foreign country's endowment in every future period. The optimal two-asset portfolio for each country's agents is computed, and the effect of diversification on rates of return and on welfare are calculated.[6]

Tables B-1a through B-1e report, for all one-period states of the world (of which there are a finite number), the commodity price ratio, country 1's consumption of the two commodities, marginal utilities of the home goods, and country 1's "felicity." In addition, the dependence of a country 1 agent's expected utility on the shares she owns of the future endowments in the two countries is calculated. Given the stochastic process assumed and the assumption that portfolios are unchanging over time, this expected

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[5] In our simulations, as explained in Appendix B, periods correspond to five years, and the steps s are 10%, approximately 2% at annual rates. Locally the implied endowment growth process is essentially the same as the two state Markov process used by Mehra and Prescott (1985). They find that annual steps of 3.6% with a drift of 1.8% and a small amount of serial correlation mimics U.S. data on real per-capita consumption for the period 1889-1978. As they note, allowing for serial correlation has only a minimal impact on the results since there is only small (negative) serial correlation in observed consumption growth. Here, however, a time discount rate, rather than declining marginal utility due to trend growth, is the reason for a positive risk-free rate of interest.

[6] We can, and shall in future, compute equilibria with a full set of Arrow-Debreu securities, i.e. one for each potential state of the world in each time period.

utility is uniquely determined by the current state of the two countries' endowments. In order to indicate the magnitude of the gains from diversification, we also report the certainty equivalent, i.e. the hypothetical sure permanent endowment which has the same utility as the actual uncertain claims. The simulations are made for a variety of assumptions about the elasticity of substitution between the two goods, varying from the near perfect substitutability to near total inelasticity.

Each table reports the outcomes under capital-market autarky and for various amounts of diversification (including the optimally diversified portfolio, which maximizes utility). Our discussion will focus mainly on the gains from diversification to economies initially at the expected value of endowments (1, 1). To economize on computation each "period" is taken to be 5 years but the results are reported on an annualized basis.

The near-perfect substitutes case shown in Table B-1a is the simplest to analyze. As expected, terms of trade are almost constant. Under autarky the marginal utilities of the two countries are essentially independent, but vary a great deal with the own countries' endowments. Endowment risk is important; the certainty equivalent of the uncertain future faced by an agent holding only domestic claims is .927, a 7.3% loss when the world is in state (1, 1). In the high substitutability case, the two representative agents have very similar preferences. Therefore, when trade between them in endowments is allowed, the optimal portfolio is essentially half domestic and half foreign claims. This cuts the risk discount from the certainty equivalent roughly in half, from 7.3% to 3.1%. Actually, much of this gain (2.2%) can be achieved with a portfolio of which only a quarter is invested in foreign claims.

Diversification changes the <u>ex post</u> distribution of consumption between countries. Just how it changes them depends on the elasticity of substitution between the two consumption goods. If the elasticity exceeds one, favorable productivity draws in one country give a larger share of world income to foreigners. Since foreigners spend a smaller share of their income on the good, its relative price falls more than under autarky. Even in the near-perfect substitutes case, this effect of diversification on the variance of relative prices can be seen. The variance of relative prices increases, but only very slightly, with increased diversification. These effects are reversed when the elasticity is smaller than one.

The more elastic the demands, the smaller are the variations of the terms of trade. A favorable endowment draw does result in an offsetting deterioration in the terms of trade, but the offset is incomplete so long as the elasticity is greater than one. As can be seen in Tables B-1b and B-1c, this induces a correlation between the (home-good) marginal utilities in the two countries. Good outcomes in country 2 not only lower the marginal utility of good 2 in country 2, but also lower the marginal utility of good 1 in country 1.

Changes in the terms of trade accomplish some of the sharing of risks that could be achieved by capital markets. For a substitution elasticity of 4, for example, the certainty equivalence of (1, 1) under capital market autarky is .932. The optimal portfolio, still approximately 1/2-1/2, has a certainty equivalent of .965, a gain of 3.3% as compared to 4.2% when the elasticity is 20.

The possibility that endogenous adjustments in the terms of trade mitigate the gains from international diversification is shown most clearly in case the substitution elasticity is one (Table B-1c). In this case terms of trade effects exactly offset the endowment changes. Owning foreign assets does not provide insurance against changes in the terms of trade; when home country endowment is low, the value of claims on foreign goods is also low. As a consequence, expected utility and certainty equivalent are insensitive to the share of wealth invested abroad.

With elasticities below 1 increases in the supply of a commodity decrease its share in world income. Hence under autarky positive endowment shocks make home agents worse off and improve the welfare of foreigners. This redistribution could in principle be quite dramatic. As shown in Table B-1, with an elasticity as low as 1/4, autarky is disastrous -- the certainty equivalent for endowment state (1,1) is less than .04. (Agricultural countries and regions are familiar with this kind of disaster.) As can be seen in the table, the adjustments in the terms of trade generate a negative correlation of marginal utilities, suggesting substantial gains from even modest international diversification. Moving from a 0% claim on foreign assets to 5% raises the certainty equivalent of (1,1) from .0439 to .515. The optimal portfolio is very close to a hedging portfolio, a 20% portfolio share for foreign assets, corresponding to the assumed normal 20% share of imports. (If only the farmers owned shares in the industries of their customers...)

The computed competitive equilibria provide values, for each single-period state of the two countries, of pro rata claims on each country's future endowments in terms of the country's good in the current period. We refer to such claims as equities, although they are unlike real-world equities in encompassing all claims on the country's resources. This model lumps profits, interest, and wages. Each equilibrium also determines the terms of trade, the price of country 2's good in terms of country 1's good. Thus equity prices in home goods can be translated into foreign goods. Equilibrium in any period means that world consumption of each good in that period is equal to the endowment of that good in the state of nature realized in that period. It also means that, at prevailing goods and asset prices, the demands of the agents for both country's equities add up to the supplies, i.e. to the entire future endowments of the two economies.

Holding equity from one period to the next yields a share of next period's endowment plus the value of the same claim in the state realized in the next period. Because the state of the world next period is uncertain, the return to holding equity from one period to the next is uncertain. Economists always like to compare the expected one-period return on equity to a one-period risk-free rate of return, calling the difference between them the risk premium. This can be done here, but it is important to be clear about the concept of risk-free rate.

If an economy contains numerous agents with identical preferences and resources, there is of course no reason for trade among them. In autarky, for example, all agents would consume pro rata shares of their country's endowment and hold pro rata equity claims on its future. There would be no function for assets with different risk characteristics, in particular no reason for separate markets for risk-free real bills or bonds.

Nevertheless, from the state prices implicit in the competitive equilibria described above can be derived prices of any assets with specified state distributions of returns. Consider a promise to deliver one unit of commodity 1 next period, whatever state of nature is realized. This promise will have a price in terms of commodity 1 today, and from that price can be determined the risk-free rate of return. In a certain world or a world of risk-neutral agents, and in the absence of growth, this would simply be the rate of time preference, 4 percent in our simulations. But in an uncertain world risk averse investors may be willing to buy a risk-free contract at a different rate, lower or higher than their time preference rate.

With a "constant relative risk aversion (CRRA)" utility function, or with any other function with a positive third derivative, increases in risk actually decrease all one-period interest rates, including the risk-free rate. The reason is that the expected marginal utility of an uncertain consumption next period is greater than the marginal utility of the expected value, which would be delivered by the hypothetical risk-free contract. Thus an increase in basic risk inclines agents to shift consumption from the current period to the next period. No such shift is possible -- goods are not storable and the stream of endowments is exogenous. To make agents content not to shift, interest rates have to be lower. In our simulations this effect is substantial, causing variations in the risk-free rate of more than one percentage point. The same mechanism makes the return on equity vary negatively with risk. The risk premium varies much less.

Table B-2 reports the expected values and distributional characteristics of these two rates of return and of the risk premium for both economies, and tells how they are affected by widening the menu of assets to permit international diversification.

As already noted, when substitutability between the two goods is nearly perfect, variations in terms of trade are relatively inconsequential. Under capital market autarky, an agent's risk is almost entirely a reflection of the substantial volatility in the home country's endowments, The risk premium is 1.7 percent. Even though the response of terms of trade to relative supplies is small, it does result in a slight positive correlations in the two countries' rates of returns. That is, a good harvest in one country improves welfare in the other.

To indicate the incentives for internationalization of capital markets -- the room there is for a deal between agents in the two autarchic markets -- we calculate the premium country 1 agents would be willing to pay to acquire the equity of country 2, as a percentage of the value of that equity. This premium is substantial, 10 percent when the economies are in state (1,1).

The composition of the optimal international portfolios held when diversification is allowed depends on the elasticity of substitution between the two consumption goods. In the near-perfect-substitutes case, agents will hold roughly equal quantities of each equity. This diversification cuts the variance of returns and risk premia by about a half. With the same claims being held in both countries, the inter-country correlations of incomes and equity returns are high -- nearly 0.6 in the case of equity returns.

When the elasticity of substitution is unity, changes in terms of trade suffice to accomplish efficient allocation of risks. There is little merit in opening up international asset markets. With them or without them, asset returns in the two countries are highly correlated.

With low substitution elasticity autarky entails large risks. With an elasticity as low as 0.25, fluctuations in equity prices are almost boundless. Even with 5 percent of portfolios in foreign equity, variations in asset prices are four times what they would be with unitary elasticity, and risk premia are correspondingly high. Positive home endowment shocks lead to losses of home income and gains in foreign income; thus equity returns in the two economies show strong negative correlation. Another indication of the potential gains from unconstrained asset trade is that residents of country 1 would be willing to pay nearly 80 times the prevailing foreign price for shares of country 2.

In this case the optimal portfolio is hedged, in the sense that portfolio shares mimic consumption shares, for example in our simulations 20 percent foreign. Given this degree of diversification, equity risks and risk premia fall dramatically to levels comparable to those of the case of unitary elasticity. The correlations between the two incomes become strongly positive instead of strongly negative.

Problems in empirical estimation.

Current asset prices depend on expectations, and on expectations of expectations. Portfolio choices involve comparisons of returns on many assets and thus involve expectations and joint probability distributions of large dimension. The market prices that emerge today reflect choices by many diverse investors and depend on their estimates of the price vectors that similar market processes will generate in the future.

Unfortunately the relevant expectations are largely unobservable. Rational expectations theory confines price-determining expectations to those that will in fact be realized, at least actuarially. Those expectations are supposed to be anchored in a future equilibrium on which the views of investors converge. But econometricians, and presumably market participants too, have great difficulty in identifying such an equilibrium and much more in estimating it.

Our standard textbook theory of exchange rates says that a country's exchange rate is on a (singular) path towards an equilibrium rate. Its rate of change along that path is in the meanwhile making up for differences in national interest rates. A macroeconomic intervention or event or news item can affect the current exchange rate by tilting the path (for example, if the interest differential is changed) or by altering the expected future equilibrium rate (for example, a change in expected fiscal policy). But that equilibrium is hard to pin down. Purchasing power parity and zero current account balance are sometimes advanced as candidates for defining equilibrium, but they are treacherous. As for "ppp," the real exchange rate is generally not independent of the trade balance but reflects the imperfect substitutability of imports and domestically produced goods. As for current account balance, there is no reason to expect it in any practical long run.

Moreover, the <u>nominal</u> exchange rate is not foreseeable on the basis of the real economic forces involved in projections of trade balances and terms of trade. We would have to model and estimate the monetary and fiscal policies of the several countries, their political and social tolerances of inflation and unemployment, the evolution of their price- and wage-setting institutions and of their financial systems, and many other phenomena relating to nominal variables and their interactions with real economic outcomes.

Observed flows of capital can reflect either or both of two conceptually distinct phenomena, reallocations of portfolio stock and allocations of new wealth in established patterns. Stock reallocations occur in response to changes in current and expected asset yields and in perceived risks. These are one-shot effects. Because of transactions costs and other frictions, they do not take place instantaneously in response to news. They take time, but as they are completed and portfolios are reshaped they die out. In the absence of further changes of yields and risks, i.e. without new news, these flows between markets and across frontiers and currencies, become zero. Accretions to wealth, on the other hand, are a continuing source of capital flows.

For example, suppose U.S. assets are about 10 percent of fully adjusted Japanese portfolios. In stable circumstances we should expect 10 percent of the regular annual additions to these portfolios to flow to U.S. markets. Japan's wealth is estimated to be six times its GNP. Suppose that steady state growth for both Japanese wealth and Japanese GNP is in nominal terms 9 percent per year. Thus the steady flow into U.S. assets would be .10x.09x6xJGNP, about $16 billion. In the recent decade, of course, Japanese holdings of U.S. assets have grown sharply in relation both to Japan's wealth and to U.S. wealth. One may infer that a stock adjustment has been occurring, and is still going on. Obviously it must taper off sooner or later. But when?

The lags in portfolio adjustment make the econometrician's life difficult, because he or she cannot assume that observations reflect desired holdings. The long process of internationalization means that observations during the process are not good guides to investors' behavior once the reallocation is completed. The "bootstrap" method of calculating asset mean returns, variances, and covariances from market histories -- the same market histories the method is supposed to explain -- is particularly likely to be misleading. A preferable approach may be to use "fundamental" data only. (Tobin and Brainard 1977, Brainard Shoven and Weiss 1980, Brainard Shapiro and Shoven 1990, Tobin 1988) But, as already acknowledged above, it is difficult to identify the "fundamentals" of nominal variables.

APPENDIX A

The illustrative simulation involves two countries and four portfolio assets, U bonds and U equities in country U, J bonds and J equities in country J. Equities are pure, unlevered; leverage can be provided by negative holdings of bonds. All the assets yield stochastic real returns. Using mean-variance analysis, we compute risk-return loci for a representative investor in U. That is, for various levels of expected real portfolio return we find that portfolio with the minimum variance. In the Figures, the minimum standard deviations of portfolio return are plotted against the sequence of assumed levels of expected return. The locus for an asset menu confined to the two local U assets is compared with the locus for a 4-asset menu, to show the potential gains from international diversification.

In some calculations, a fifth risky asset, human capital yielding stochastic real wages, is added. Unlike the four portfolio assets, this asset cannot be bought or sold. The mean-variance calculation is then to find the portfolio of market assets that minimizes the total variance, taking into account also the nonmarket asset, for each given expectation of total real return. The relative sizes of the wealth invested in the market portfolio and human wealth are invariant, in proportions 1:y. Variation in total expected real return comes solely from variation in portfolio return.

Portfolio variance is:

(A1) $\sigma_R^2 = \Sigma_{ij}\sigma_{ij}x_i x_j$,

where i and j run from 1 to n, the number of assets, σ_{ij} are variances and covariances, and x_i and x_j are the shares of the assets in the portfolio.

This is to be minimized subject to two constraints:

(A2) $\Sigma_i x_i r_i = R$,

for various values of R, the portfolio return. The r_i are the expected values of real returns on the assets i.

(A3) $\Sigma^i{}_{xi} = 1$

Total variance, taking account of stochastic deviations from expectations for the non-market asset, wages w, as well as for portfolio assets, is:

(A4) $\sigma_T^2 = \sigma_R^2 + y^2\sigma_w^2 + 2\Sigma_i y x_i \sigma_{wi}$

This variance can be calculated when σ_R^2 is being minimized. Or, as suggested above, σ_T^2 itself can be minimized.

These minimizations are solutions of n + 2 linear equations, the n first order conditions and the two constraints, giving the n x_i and the shadow prices of the two constraints. To compute the overall minimum variance portfolio, the first constraint (A2) is omitted. A rational investor will not be interested in any portfolio with a mean return smaller than that available at overall minimum risk.

The calculations reported here allow negative values of x_i, borrowing or short-selling. It would be possible, of course, to impose constraints on negative positions.

Table A-1 lists the ten shocks here considered, their assumed standard deviations, and the participations of the five assets in those shocks. The two countries U and J are taken to be virtual mirrors of one another. The major assumptions in the assets/shocks matrix are the following:[7]

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[7]. We have been guided to some extent by the empirical correlations reported by Golub (1989).

TABLE A-1. SHOCKS AND THEIR EFFECTS ON VARIABILITY OF ASSET RETURNS

ASSETS:	U bonds	U equity	J bonds	J equity	U wages	Std devs
SHOCKS						
U prod		1.5		-0.5	0.8	0.03
J prod		-0.5		1.5	0.2	0.015
World int.	-1	-1	-1	-1		0.02
U int	-1	-1				0.02
J int			-1	-1		0.02
World infl	-1		-1			0.025
U infl	-0.8					0.015
Jinfl	-0.2		-1			0.015
Oil	-0.5	-0.5		-1	-0.2	0.05
Exchge rate			1			0.03

IMPLIED VARIANCES AND COVARIANCES OF ASSET RETURNS

	U bonds	U equity	J bonds	J equity	U wages
U bonds	0.002203	0.001425	0.00107	0.00165	0.00025
U equity	0.001425	0.003506	0.0004	0.000806	0.001307
J bonds	0.00107	0.0004	0.00255	0.0008	0
J equity	0.00165	0.000806	0.0008	0.004031	0.000207
U real wages	0.00025	0.001307	0	0.000207	0.000685
Mean returns	0.02	0.06	0.02	0.06	

Human capital (wages) assumed 5 times as large as discretionary portfolio. Variance/covariance matrix above not adjusted for this assumption.

TABLE A-2. RISKS AND RETURNS: COMPARING PORTFOLIO AND TOTAL RISKS FOR TWO- AND FOUR-ASSET MENUS.

Required Return	no req	0.030	0.040	0.050	0.060	0.070	0.080	0.090	0.100
Std Devs Return (min var)									
2 asset ptfolios									
min var R									
stddev R	0.002	0.045	0.046	0.051	0.059	0.069	0.079	0.091	0.103
stdev total	0.026	0.026	0.027	0.029	0.031	0.032	0.034	0.036	0.038
min total var									
stddev R	0.012	0.045	0.046	0.051	0.059	0.069	0.079	0.091	0.103
stdev total	0.020	0.026	0.027	0.029	0.031	0.032	0.034	0.036	0.038
4 asset ptfolios									
min var R									
stddev R	0.038	0.039	0.038	0.040	0.044	0.049	0.056	0.063	0.071
stdev total	0.025	0.025	0.025	0.026	0.027	0.028	0.029	0.030	0.031
min total var									
stddev R	0.110	0.084	0.084	0.085	0.087	0.089	0.093	0.098	0.103
stdev total	0.018	0.021	0.022	0.023	0.024	0.025	0.026	0.027	0.028
Ptfolio Shares:									
2 asset ptfolios									
min var R									
U bonds	0.728	0.750	0.500	0.250	0.000	-.25	-.50	-.75	-1.00
U equities	0.272	0.250	0.500	0.750	1.000	1.250	1.500	1.750	2.000
Portfolio R	0.031	0.030	0.040	0.050	0.060	0.070	0.080	0.090	0.100
min total var									
U bonds	2.577	0.750	0.500	0.250	0.000	-.250	-.500	-.750	-1.00
U equities	-1.577	0.250	0.500	0.750	1.000	1.250	1.500	1.750	2.000
Portfolio R	-0.043	0.030	0.040	0.050	0.060	0.070	0.080	0.090	0.100
4 asset ptfolios									
min var R									
U bonds	0.178	0.332	0.100	-.131	-.362	-.594	-.825	-1.06	-1.29
U equity	0.260	0.177	0.303	0.429	0.555	0.681	0.807	0.933	1.059
J bonds	0.406	0.418	0.400	0.381	0.362	0.344	0.325	0.307	0.288
J equity	0.155	0.073	0.197	0.321	0.445	0.569	0.693	0.817	0.941
Portfolio R	0.037	0.030	0.040	0.050	0.060	0.070	0.080	0.090	0.100
min total var									
U bonds	1.514	-.085	-.316	-.548	-.780	-1.011	-1.242	-1.473	-1.705
U equity	-1.611	-.739	-.613	-.487	-.361	-.235	-.109	0.017	0.143
J bonds	0.964	0.835	0.817	0.798	0.780	0.761	0.742	0.724	0.705
J equity	0.132	0.989	1.113	1.237	1.361	1.485	1.609	1.733	1.857
Portfolio R	-0.039	0.030	0.040	0.050	0.060	0.070	0.080	0.090	0.100

--Home productivity shocks hit home equities and wages positively. They have smaller negative effects on foreign equities. Foreign productivity gains do benefit real wages, because they make foreign imports cheaper.

--The two countries share worldwide interest rate shocks, and are in addition vulnerable to idiosyncratic local interest rate movements. Interest rates, as the discount rates for capitalizing future payments, affect both bonds and equities.

--Likewise, the two countries share worldwide inflation shocks, and in addition are affected by local inflation shocks. Real returns on U bonds are affected negatively by J inflation, because worker-consumer-investors do consume imports from J.

--An oil shock is taken to be tougher on J equities than on U equities, but to have no effect on J inflation and thus none on J bonds.

--Finally, an increase in the nominal value of the yen, arising from speculative causes and not from any of the other listed shocks, is taken to increase the real payoffs of J bonds to U investors.

The variance/covariance matrix for the five assets, as derived from the shocks/assets matrix, is also displayed in Table A-1. In the final line of the table are shown the assumed mean returns for the four portfolio assets. The mean return on the fifth asset is irrelevant to the portfolio choices, although the variance of wages and their covariance with the market asset returns are very relevant.

Table A-2 reports the solutions, for both 2-asset and 4-asset portfolios and for both minimands, "var R" and "total var." Here "stdev total" is $\sigma_T/6$, where σ_T comes from (A4). The first column gives the calculated minimum risk portfolio, along with the portfolio R it implies.

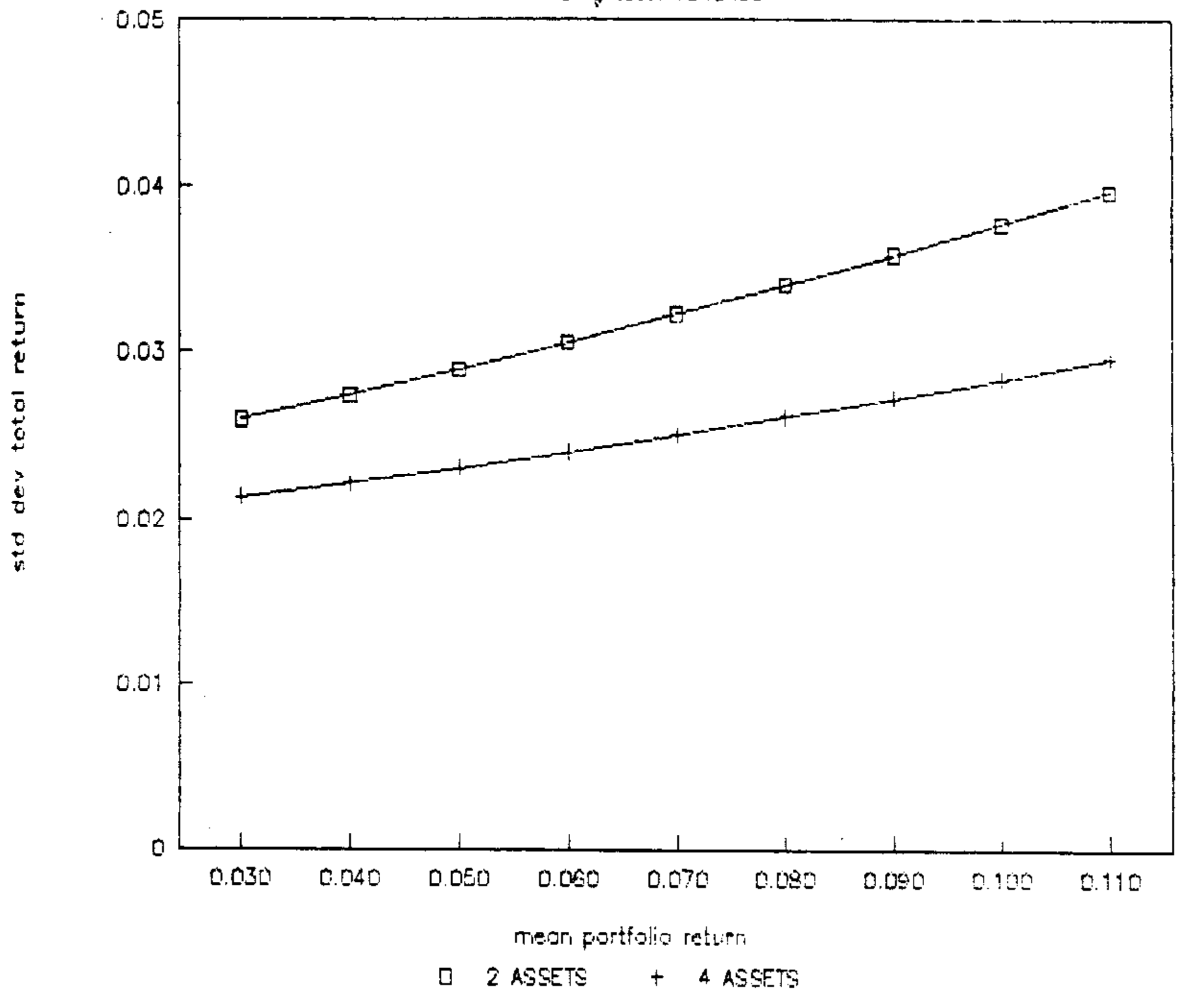
The remaining columns are solutions subject to the constraint that Portfolio R be the Required Return of the first row.

Figures A-1 and A-2 show graphically that adding the two foreign assets to the menu improves the risk-return frontier. As Table A-2 shows, the 4-asset portfolios that accomplish this improvement depend importantly on which variance is being minimized. In both cases, large positive holding of J bonds and J equities, partly financed by borrowing the U bond market, are efficient. This is especially true when it is total variance that is being minimized, and selling U equity short is also risk-reducing. This reflects the assumption that domestic productivity shocks have large positive effects on both the marginal productivity of capital and the wages of labor. J assets do not share this risk.

Figure A-3 makes the point that the choice of the minimand makes a big difference for the variance of portfolio return R. See the upper two loci in the Figure. A high variance of R is efficient if the portfolios that produce it take advantage of covariances to reduce total variance. The bottom two panels of Table A-2 show radical differences in portfolio composition when the covariances of asset returns with wages are taken into account. Adding J assets to the menu provides these opportunities. The composition of the two-asset portfolios is, in this example, independent of the minimand. However, the differences in the 2-asset minimum variance portfolios (first column) indicates that risk-shy investors worried about the variance of total return might stay out of equities or even sell them short.

Fig A-1 Total Risk-Return Frontiers

Minimizing total variance



Total Fig. A-2 Risk-Return Frontiers

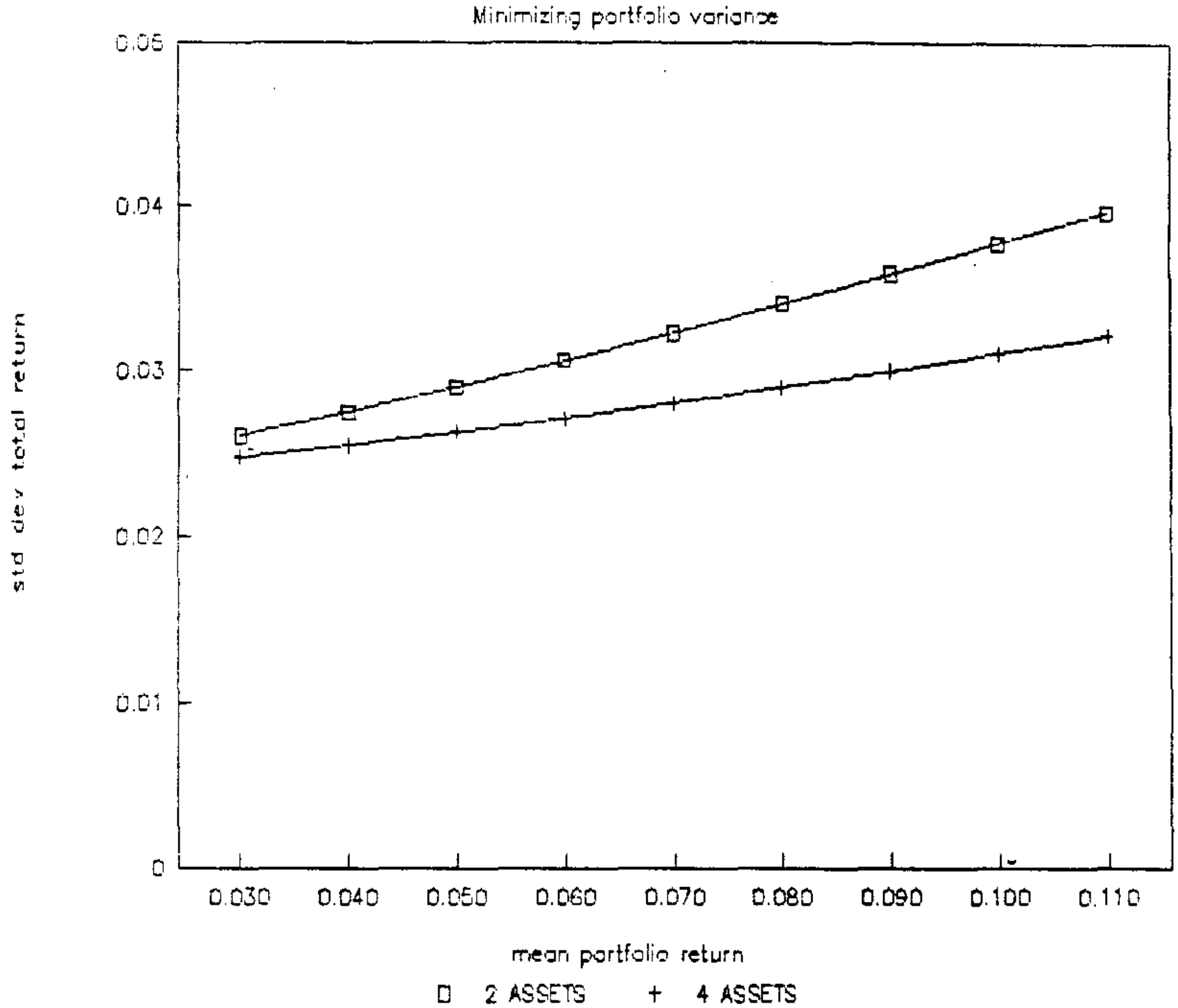
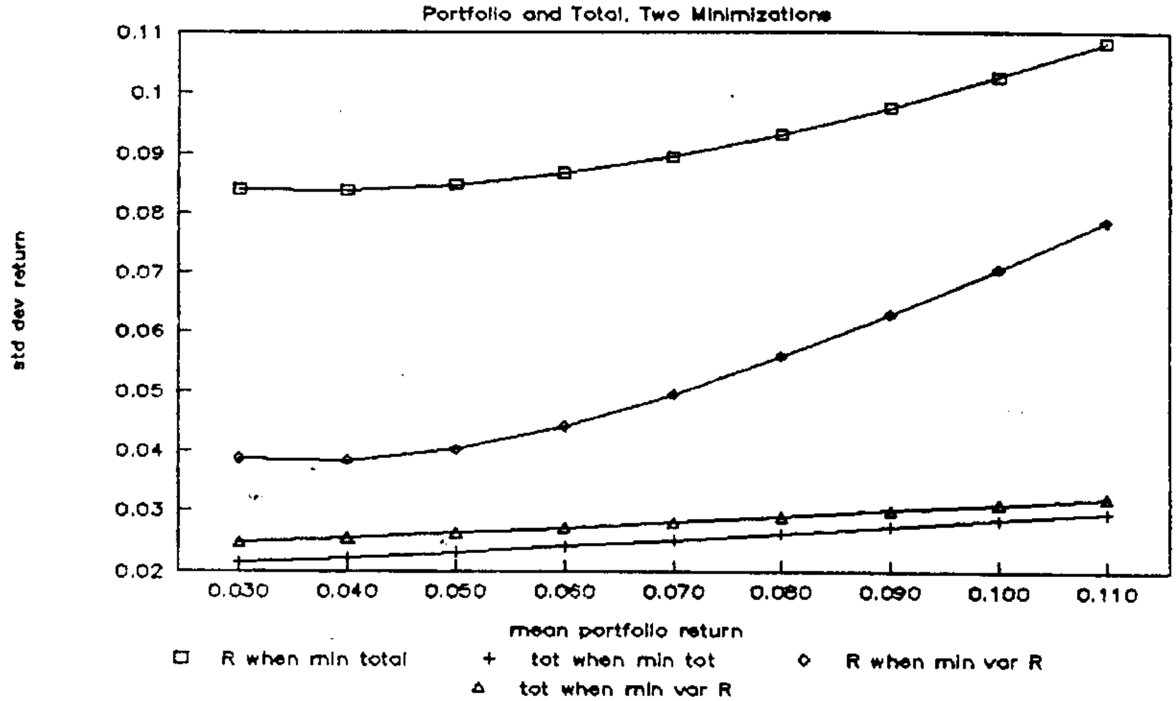


Fig. A–3 Comparing Risk–Return Loci



APPENDIX B

The model described in the text is a two-good two-country exchange economy. Each country is represented by a risk-averse agent with intertemporally additive preferences. The single-period felicity function displays constant relative risk aversion (CRRA). Each agent ($i = 1,2$) maximizes:

(1) $U_i = \Sigma\beta^t u_i(C_{1i},C_{2i})$,

where C_1 and C_2 are consumptions of goods 1 and 2, originating in countries 1 and 2. β is a time discount factor, and u_i is given by a CRRA-CES function:

(2) $u_i = (1/(1-\theta))(\alpha_i C_{1i}{}^{\gamma}+(1-\alpha_i)C_{2i}{}^{\gamma})^{(1-\theta)/\gamma}$

In this felicity function θ is the degree of relative risk aversion, the α_i are the CES "share" parameters for the two countries, and γ determines the constant elasticity of substitution $\varepsilon = 1/(1-\gamma)$.

The two representative agents are assumed to have the same β and γ. In the simulations the periods are meant to correspond to five years, but to promote understanding our tables of simulation results report the parameters of the model on an annualized basis. Thus the β of .2 used in the calculations is approximately equivalent to a discount rate of 4 percent per year. Simulations were run for substitution elasticities ε of 1/4, 1/2, 2, 4, and 20. Agents have home good preferences, mirror images of each other. In the simulations α is always chosen so that for a unitary ratio between the prices of the two goods the share of imports is 0.2.

Each country is exogenously endowed each (five-year) period with only one good, which is perishable. Each endowment follows a Markov process. The two processes are independent. A country with an endowment e_t in period t has either the same endowment, 10 percent more, or 10 percent less in period t+1, except when its endowment is far from its central value of 1. In particular, for $1 < e_t < e_{max}$:

(3) $e_{t+1} = 1.1e_t$ with prob p_{out}

$e_{t+1} = 0.9e_t$ prob p_{in}

$e_{t+1} = e_t$ prob $1-p_{in}-p_{out}$

The probabilities are reversed for endowments below 1. When the endowment is 1.0 it grows or shrinks with equal probability $(p_{out} + p_{in})/2$. p_{in} is set, equal to 0.5 in the simulations. In order to bound the range of possible endowments, to 11 for each country, the probability of moving out is reduced from p_{in} to 2/3, 1/3, and 0 times p_{in} in the three outermost states. The expected endowment is 1, the minimum is .62, and the maximum is 1.61. This process gives a stationary distribution of endowments, but in the neighborhood of 1.0 it is approximately a random walk.

The world economy can be in any of 11x11=121 states in any period. Competitive equilibria are calculated for each of these states by the following sequential procedure: First, the state-dependent terms of trade are calculated, using the condition that the current goods markets clear. These state prices depend, of course, on the incomes of the two country's agents, and hence on their postfolios as well as their endowments. The implied state-dependent consumptions of the two agents give state-dependent marginal utilities $(\lambda_1(s), \lambda_2(s))$ for the home goods. These are shadow prices on the home endowments. The first order conditions correspondinag to the fact that in equilibrium agents are indifferent between an extra unit of current consumption and an uncertain claim on future consumption make it possible to compute the value of an asset with state-dependent future returns. The valuation of country 1's future endowment, for example, is obtained by solving the following recursive equation:

(4) $V1(h) = \beta\Sigma\phi(h,j)(e1(j)+V1(j))\lambda1(j)/\lambda1(h)$,

where V1(k) is the value of country 1's endowment in state k, $\phi(h,j)$ is the transition matrix giving the probability of the world's moving from state h to state j, e1(j) is country 1's endowment in state j.

Table B.1a Outcomes for Internationally Diversified Portfolios

Own consumption share= .8				elasticity=20			relative risk aversion=6.0		
share in home economy=1.0									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1.00	0.66	0.17	95.00	95.00	-15.7	-71.6	0.805
0.83	1.00	0.99	0.64	0.18	94.38	30.58	-15.6	-71.2	0.806
0.83	1.21	0.99	0.63	0.20	93.74	9.77	-15.5	-70.8	0.807
1.00	0.83	1.01	0.82	0.18	30.46	93.86	-6.1	-35.6	0.926
1.00	1.00	1.00	0.80	0.20	30.27	30.27	-6.1	-35.3	0.927
1.00	1.21	0.99	0.78	0.22	30.07	9.74	-6.0	-35.2	0.928
1.21	0.83	1.01	1.02	0.19	9.76	93.17	-2.4	-16.5	1.080
1.21	1.00	1.01	0.99	0.21	9.71	29.91	-2.4	-16.4	1.082
1.21	1.21	1.00	0.97	0.24	9.65	9.65	-2.3	-16.3	1.082
share in home economy=.9									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1.00	0.66	0.17	95.00	95.00	-15.7	-67.6	0.814
0.83	1.00	0.99	0.65	0.19	83.66	33.80	-14.1	-61.4	0.830
0.83	1.21	0.98	0.64	0.22	72.07	11.82	-12.4	-54.9	0.849
1.00	0.83	1.01	0.81	0.17	33.73	83.20	-6.6	-35.5	0.927
1.00	1.00	1.00	0.80	0.20	30.27	30.27	-6.1	-32.5	0.943
1.00	1.21	0.99	0.79	0.23	26.66	10.77	-5.4	-29.5	0.961
1.21	0.83	1.02	0.99	0.18	11.78	71.70	-2.8	-17.4	1.069
1.21	1.00	1.01	0.98	0.20	10.75	26.51	-2.6	-16.1	1.085
1.21	1.21	1.00	0.97	0.24	9.65	9.65	-2.3	-14.9	1.103
share in home economy=.75									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1.00	0.66	0.17	95.00	95.00	-15.7	-63.8	0.824
0.83	1.00	0.99	0.67	0.20	70.26	39.25	-12.2	-51.3	0.860
0.83	1.21	0.98	0.67	0.25	49.94	15.72	-9.2	-40.1	0.904
1.00	0.83	1.01	0.79	0.16	39.38	70.48	-7.6	-36.4	0.922
1.00	1.00	1.00	0.80	0.20	30.27	30.27	-6.1	-29.9	0.959
1.00	1.21	0.99	0.81	0.24	22.39	12.50	-4.7	-24.0	1.002
1.21	0.83	1.02	0.95	0.16	15.76	50.12	-3.5	-19.5	1.044
1.21	1.00	1.01	0.96	0.20	12.55	22.46	-2.9	-16.4	1.081
1.21	1.21	1.00	0.97	0.24	9.65	9.65	-2.3	-13.6	1.123
share in home economy=.6									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1.00	0.66	0.17	95.00	95.00	-15.7	-61.9	0.829
0.83	1.00	0.99	0.68	0.22	59.60	45.78	-10.6	-44.9	0.884
0.83	1.21	0.97	0.69	0.29	35.72	21.15	-6.9	-31.2	0.951
1.00	0.83	1.01	0.78	0.15	46.00	59.84	-8.6	-38.5	0.911
1.00	1.00	1.00	0.80	0.20	30.27	30.27	-6.1	-28.7	0.967
1.00	1.21	0.99	0.82	0.26	18.99	14.59	-4.1	-20.6	1.033
1.21	0.83	1.03	0.92	0.14	21.22	35.79	-4.5	-22.7	1.013
1.21	1.00	1.01	0.94	0.19	14.66	19.07	-3.3	-17.4	1.068
1.21	1.21	1.00	0.97	0.24	9.65	9.65	-2.3	-12.9	1.134
share in home economy=.5									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1.00	0.66	0.17	95.00	95.00	-15.7	-61.5	0.830
0.83	1.00	0.99	0.68	0.23	53.77	50.79	-9.8	-41.8	0.896
0.83	1.21	0.97	0.70	0.31	28.97	25.88	-5.8	-27.1	0.978
1.00	0.83	1.01	0.77	0.15	50.92	53.87	-9.4	-40.6	0.902
1.00	1.00	1.00	0.80	0.20	30.27	30.27	-6.1	-28.4	0.969
1.00	1.21	0.99	0.83	0.27	17.13	16.18	-3.8	-19.0	1.049
1.21	0.83	1.03	0.90	0.13	26.01	29.03	-5.4	-25.5	0.990
1.21	1.00	1.01	0.93	0.18	16.23	17.16	-3.6	-18.4	1.056
1.21	1.21	1.00	0.97	0.24	9.65	9.65	-2.3	-12.8	1.136
end1,2 = endowments in country 1,2									
price2/1 = price of country 2's good in terms of country 1's good									
C11,C21 = consumption of goods 1 and 2 by country 1									
MU1,2 = marginal utility of good 1 in country 1, good 2 in country 2									
u1 = the undiscounted felicity from consumption (C11,C21)									
EU1 = present discounted value of expected utility for country 1									
Cert equiv = the riskless consumption stream with the same utility as the uncertain claims on future endoments									

Table B.1b Outcomes for Internationally Diversified Portfolios

own consumption share=.8				elasticity=4			relative risk aversion=6.0		
share of home economy=1.0									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1.00	0.66	0.17	76.49	76.49	-12.64	-56.69	0.808
0.83	1.00	0.97	0.65	0.19	73.74	25.33	-12.19	-55.00	0.813
0.83	1.21	0.93	0.63	0.21	71.09	8.30	-11.75	-53.21	0.818
1.00	0.83	1.04	0.82	0.18	25.24	73.20	-5.05	-28.65	0.926
1.00	1.00	1.00	0.80	0.20	24.37	24.37	-4.87	-27.75	0.932
1.00	1.21	0.97	0.78	0.23	23.50	8.07	-4.70	-27.00	0.937
1.21	0.83	1.07	1.01	0.19	8.28	70.53	-2.00	-13.51	1.076
1.21	1.00	1.04	0.99	0.21	8.04	23.32	-1.95	-13.12	1.082
1.21	1.21	1.00	0.97	0.24	7.77	7.77	-1.88	-12.80	1.088
share of home economy=.75									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1.00	0.66	0.17	76.49	76.49	-12.64	-51.72	0.823
0.83	1.00	0.95	0.67	0.20	57.93	31.09	-9.94	-42.17	0.857
0.83	1.21	0.90	0.67	0.25	42.69	12.35	-7.63	-33.58	0.897
1.00	0.83	1.05	0.79	0.16	31.26	58.21	-6.04	-29.36	0.921
1.00	1.00	1.00	0.80	0.20	24.37	24.37	-4.87	-24.32	0.957
1.00	1.21	0.95	0.81	0.24	18.46	9.91	-3.83	-19.82	0.997
1.21	0.83	1.10	0.96	0.16	12.42	42.89	-2.82	-15.68	1.044
1.21	1.00	1.05	0.96	0.20	9.96	18.55	-2.33	-13.25	1.080
1.21	1.21	1.00	0.97	0.24	7.77	7.77	-1.88	-11.04	1.120
share of home economy=.6									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1.00	0.66	0.17	76.49	76.49	-12.64	-50.50	0.827
0.83	1.00	0.94	0.67	0.21	51.50	34.82	-9.00	-38.08	0.875
0.83	1.21	0.89	0.68	0.27	33.63	15.36	-6.23	-27.85	0.931
1.00	0.83	1.06	0.78	0.16	35.05	51.70	-6.66	-30.56	0.914
1.00	1.00	1.00	0.80	0.20	24.37	24.37	-4.87	-23.51	0.963
1.00	1.21	0.94	0.82	0.26	16.41	11.09	-3.47	-17.64	1.020
1.21	0.83	1.12	0.93	0.15	15.47	33.75	-3.39	-17.51	1.022
1.21	1.00	1.06	0.95	0.19	11.17	16.47	-2.57	-13.80	1.072
1.21	1.21	1.00	0.97	0.24	7.77	7.77	-1.88	-10.63	1.129
share of home economy=.55									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1.00	0.66	0.17	76.49	76.49	-12.64	-50.26	0.827
0.83	1.00	0.94	0.68	0.22	49.75	36.06	-8.74	-37.06	0.879
0.83	1.21	0.88	0.69	0.28	31.38	16.45	-5.88	-26.43	0.941
1.00	0.83	1.06	0.78	0.15	36.29	49.89	-6.85	-31.03	0.911
1.00	1.00	1.00	0.80	0.20	24.37	24.37	-4.87	-23.35	0.965
1.00	1.21	0.94	0.82	0.26	15.85	11.49	-3.37	-17.10	1.027
1.21	0.83	1.13	0.92	0.14	16.57	31.44	-3.60	-18.18	1.014
1.21	1.00	1.06	0.95	0.19	11.56	15.90	-2.64	-14.03	1.068
1.21	1.21	1.00	0.97	0.24	7.77	7.77	-1.88	-10.55	1.131
share of home economy=.50									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1.00	0.66	0.17	76.49	76.49	-12.64	-50.10	0.828
0.83	1.00	0.94	0.68	0.22	48.07	37.30	-8.49	-36.14	0.884
0.83	1.21	0.88	0.69	0.29	29.33	17.59	-5.55	-25.17	0.950
1.00	0.83	1.06	0.78	0.15	37.59	48.21	-7.06	-31.56	0.908
1.00	1.00	1.00	0.80	0.20	24.37	24.37	-4.87	-23.24	0.965
1.00	1.21	0.94	0.82	0.26	15.32	11.88	-3.27	-16.62	1.032
1.21	0.83	1.14	0.92	0.14	17.74	29.38	-3.81	-18.90	1.006
1.21	1.00	1.06	0.94	0.18	11.98	15.36	-2.72	-14.29	1.064
1.21	1.21	1.00	0.97	0.24	7.77	7.77	-1.88	-10.50	1.132

Table B.1c Outcomes for Internationally Diversified Portfolios (e=1)

Own consumption share=.8				elasticity=1			relative risk aversion=6.0		
share in home economy=1.0									
end1	end2	price2/1	C11	C21	MU1	MU2	u1	Exp U1	Cert equiv
0.83	0.83	1	0.66	0.17	38.31	38.31	-6.33	-26.61	0.818
0.83	1	0.83	0.66	0.2	31.66	14.77	-5.23	-22.73	0.845
0.83	1.21	0.68	0.66	0.24	26.17	5.69	-4.33	-19.3	0.873
1	0.83	1.21	0.8	0.17	14.77	31.66	-2.95	-14.82	0.92
1	1	1	0.8	0.2	12.21	12.21	-2.44	-12.67	0.949
1	1.21	0.83	0.8	0.24	10.09	4.71	-2.02	-10.83	0.979
1.21	0.83	1.46	0.97	0.17	5.69	26.17	-1.38	-7.83	1.045
1.21	1	1.21	0.97	0.2	4.71	10.09	-1.14	-6.73	1.077
1.21	1.21	1	0.97	0.24	3.89	3.89	-0.94	-5.78	1.111
share in home economy=.75									
end1	end2	price2/1	C11	C21	MU1	MU2	u1	Exp U1	Cert equiv
0.83	0.83	1	0.66	0.17	38.31	38.31	-6.33	-26.61	0.818
0.83	1	0.83	0.66	0.2	31.66	14.77	-5.23	-22.73	0.845
0.83	1.21	0.68	0.66	0.24	26.17	5.69	-4.33	-19.3	0.873
1	0.83	1.21	0.8	0.17	14.77	31.66	-2.95	-14.82	0.92
1	1	1	0.8	0.2	12.21	12.21	-2.44	-12.67	0.949
1	1.21	0.83	0.8	0.24	10.09	4.71	-2.02	-10.83	0.979
1.21	0.83	1.46	0.97	0.17	5.69	26.17	-1.38	-7.83	1.045
1.21	1	1.21	0.97	0.2	4.71	10.09	-1.14	-6.73	1.077
1.21	1.21	1	0.97	0.24	3.89	3.89	-0.94	-5.78	1.111
share in home economy=.50									
end1	end2	price2/1	C11	C21	MU1	MU2	u1	Exp U1	Cert equiv
0.83	0.83	1	0.66	0.17	38.31	38.31	-6.33	-26.61	0.818
0.83	1	0.83	0.66	0.2	31.66	14.77	-5.23	-22.73	0.845
0.83	1.21	0.68	0.66	0.24	26.17	5.69	-4.33	-19.3	0.873
1	0.83	1.21	0.8	0.17	14.77	31.66	-2.95	-14.82	0.92
1	1	1	0.8	0.2	12.21	12.21	-2.44	-12.67	0.949
1	1.21	0.83	0.8	0.24	10.09	4.71	-2.02	-10.83	0.979
1.21	0.83	1.46	0.97	0.17	5.69	26.17	-1.38	-7.83	1.045
1.21	1	1.21	0.97	0.2	4.71	10.09	-1.14	-6.73	1.077
1.21	1.21	1	0.97	0.24	3.89	3.89	-0.94	-5.78	1.111

Table B.1d Outcomes for Internationally Diversified Portfolios (e=.5)

consumption share=.8				elasticity=.50			relative risk aversion =6		
share of home economy=1.00									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1	0.66	0.17	21.59	21.59	-3.57	-21.2	0.763
0.83	1	0.4	0.71	0.28	10	20.78	-1.65	-11.1	0.87
0.83	1.21	0.17	0.75	0.46	6.13	27.62	-1.01	-6.28	0.974
1	0.83	2.45	0.72	0.11	20.09	10.25	-4.02	-27.4	0.725
1	1	1	0.8	0.2	6.88	6.88	-1.38	-12	0.855
1	1.21	0.4	0.86	0.34	3.19	6.62	-0.64	-5.57	0.997
1.21	0.83	5.65	0.76	0.08	24.96	6.34	-6.04	-39.3	0.675
1.21	1	2.45	0.87	0.14	6.4	3.27	-1.55	-15.3	0.815
1.21	1.21	1	0.97	0.24	2.19	2.19	-0.53	-5.89	0.986
share of home economy=.90									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1	0.66	0.17	21.59	21.59	-3.57	-14.9	0.819
0.83	1	0.61	0.67	0.22	16.2	9.56	-2.61	-11.6	0.861
0.83	1.21	0.37	0.68	0.28	12.7	4.48	-2	-9.27	0.901
1	0.83	1.62	0.78	0.15	9.58	16.34	-1.98	-9.38	0.899
1	1	1	0.8	0.2	6.88	6.88	-1.38	-7.1	0.95
1	1.21	0.61	0.81	0.26	5.16	3.05	-1.01	-5.51	1
1.21	0.83	2.63	0.93	0.14	4.47	12.83	-1.17	-5.88	0.987
1.21	1	1.62	0.95	0.19	3.05	5.2	-0.76	-4.32	1.05
1.21	1.21	1	0.97	0.24	2.19	2.19	-0.53	-3.24	1.112
share of home economy=.85									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1	0.66	0.17	21.59	21.59	-3.57	-15.1	0.817
0.83	1	0.66	0.67	0.21	17.68	8.61	-2.83	-12.6	0.848
0.83	1.21	0.43	0.67	0.26	14.9	3.57	-2.32	-10.6	0.877
1	0.83	1.52	0.79	0.16	8.65	17.79	-1.8	-8.82	0.91
1	1	1	0.8	0.2	6.88	6.88	-1.38	-7.2	0.948
1	1.21	0.66	0.81	0.25	5.63	2.74	-1.09	-5.99	0.983
1.21	0.83	2.31	0.95	0.16	3.58	14.99	-0.94	-5.04	1.018
1.21	1	1.52	0.96	0.19	2.76	5.67	-0.69	-4.03	1.064
1.21	1.21	1	0.97	0.24	2.19	2.19	-0.53	-3.29	1.108
share of home economy=.8									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1	0.66	0.17	21.59	21.59	-3.57	-15.3	0.815
0.83	1	0.68	0.66	0.2	18.74	8.11	-2.99	-13.3	0.838
0.83	1.21	0.47	0.66	0.24	16.61	3.13	-2.57	-11.7	0.86
1	0.83	1.46	0.8	0.17	8.11	18.74	-1.69	-8.57	0.915
1	1	1	0.8	0.2	6.88	6.88	-1.38	-7.33	0.944
1	1.21	0.68	0.8	0.24	5.97	2.58	-1.15	-6.39	0.971
1.21	0.83	2.14	0.97	0.17	3.13	16.61	-0.83	-4.65	1.034
1.21	1	1.46	0.97	0.2	2.58	5.97	-0.65	-3.92	1.07
1.21	1.21	1	0.97	0.24	2.19	2.19	-0.53	-3.36	1.104
share of home economy=.75									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1	0.66	0.17	21.59	21.59	-3.57	-15.5	0.813
0.83	1	0.7	0.66	0.2	19.57	7.78	-3.11	-13.9	0.831
0.83	1.21	0.49	0.65	0.23	17.99	2.86	-2.77	-12.6	0.848
1	0.83	1.43	0.8	0.17	7.73	19.48	-1.62	-8.43	0.918
1	1	1	0.8	0.2	6.88	6.88	-1.38	-7.46	0.941
1	1.21	0.7	0.8	0.24	6.24	2.48	-1.2	-6.72	0.961
1.21	0.83	2.04	0.98	0.17	2.84	17.94	-0.75	-4.41	1.045
1.21	1	1.43	0.97	0.2	2.46	6.21	-0.62	-3.85	1.074
1.21	1.21	1	0.97	0.24	2.19	2.19	-0.53	-3.42	1.099

Table B 1e Outcomes for Internationally Diversified Portfolios (e=.25)

consumption share= .8				elasticity =.25		relative risk aversion= 6			
share of home economy =1.00									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1	0.66	0.17	13.8	13.8	-2.28	-5E+07	0.037
0.83	1	0	0.82	0.96	3.17	3E+09	-0.52	-2E+07	0.046
0.83	1.21	0	0.83	1.19	3.15	5E+10	-0.52	-6E+06	0.057
1	0.83	193.42	0.07	0	4E+07	3.24	-9E+06	-1E+08	0.031
1	1	1	0.8	0.2	4.4	4.4	-0.88	-4E+07	0.039
1	1.21	0	1	1.16	1.01	9E+08	-0.2	-1E+07	0.05
1.21	0.83	308.32	0.06	0	1E+08	3.21	-3E+07	-2E+08	0.027
1.21	1	193.42	0.09	0.01	1E+07	1.03	-3E+06	-7E+07	0.034
1.21	1.21	1	0.97	0.24	1.4	1.4	-0.34	-2E+07	0.044
share of home economy= .95									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1	0.66	0.17	13.8	13.8	-2.28	-165.24	0.463
0.83	1	0.03	0.77	0.48	4.69	138.6	-0.74	-55.14	0.577
0.83	1.21	0.01	0.78	0.62	4.45	97.75	-0.7	-19.69	0.709
1	0.83	29.73	0.52	0.06	129.41	4.8	-56.48	-279.21	0.417
1	1	1	0.8	0.2	4.4	4.4	-0.88	-97.35	0.515
1	1.21	0.03	0.94	0.58	1.49	44.16	-0.28	-29.86	0.652
1.21	0.83	68.72	0.57	0.05	103.02	4.54	-82.34	-373.13	0.393
1.21	1	29.73	0.63	0.07	41.23	1.53	-21.78	-149.83	0.472
1.21	1.21	1	0.97	0.24	1.4	1.4	-0.34	-48.12	0.593
share of home economy= .90									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	0.83	1	0.66	0.17	13.8	13.8	-2.28	-14.54	0.753
0.83	1	0.15	0.72	0.29	7.7	12.16	-1.17	-8.11	0.846
0.83	1.21	0.05	0.73	0.39	6.62	7.54	-0.99	-5.57	0.912
1	0.83	6.17	0.71	0.11	11.97	7.84	-3.38	-16.36	0.735
1	1	1	0.8	0.2	4.4	4.4	-0.88	-7.7	0.855
1	1.21	0.15	0.87	0.35	2.45	3.87	-0.45	-4.01	0.974
1.21	0.83	18.43	0.81	0.1	7.67	6.74	-4.01	-18.12	0.721
1.21	1	6.17	0.86	0.14	3.81	2.5	-1.3	-8.37	0.841
1.21	1.21	1	0.97	0.24	1.4	1.4	-0.34	-3.65	0.993
share of home economy= .85									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	1	0.34	0.68	0.22	10.98	6	-1.65	-7.97	0.849
0.83	1.21	0.13	0.69	0.29	9.62	2.71	-1.4	-6.71	0.879
1	0.83	2.85	0.78	0.15	6.04	11.07	-1.45	-7.1	0.869
1	1	1	0.8	0.2	4.4	4.4	-0.88	-5	0.932
1	1.21	0.34	0.82	0.27	3.5	1.91	-0.64	-3.85	0.982
1.21	0.83	7.41	0.92	0.14	2.76	9.72	-1.08	-5.28	0.922
1.21	1	2.85	0.94	0.18	1.92	3.53	-0.56	-3.37	1.008
share of home economy= .80									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	1	0.47	0.66	0.2	12.99	4.84	-1.96	-9.01	0.829
0.83	1.21	0.22	0.66	0.24	12.52	1.78	-1.79	-8.28	0.843
1	0.83	2.14	0.8	0.17	4.84	12.99	-1.12	-5.81	0.904
1	1	1	0.8	0.2	4.4	4.4	-0.88	-4.93	0.935
1	1.21	0.47	0.8	0.24	4.14	1.54	-0.76	-4.39	0.957
1.21	0.83	4.59	0.97	0.17	1.78	12.52	-0.61	-3.4	1.007
1.21	1	2.14	0.97	0.2	1.54	4.14	-0.43	-2.7	1.055
share of home economy= .75									
end1	end2	price2/1	C11	C21	M U 1	M U 2	u1	Exp U1	Cert equiv
0.83	1	0.54	0.65	0.19	14.23	4.4	-2.15	-9.94	0.812
0.83	1.21	0.29	0.64	0.22	14.82	1.46	-2.1	-9.71	0.816
1	0.83	1.86	0.81	0.17	4.35	14.19	-0.99	-5.48	0.915
1	1	1	0.8	0.2	4.4	4.4	-0.88	-5.1	0.928
1	1.21	0.54	0.79	0.23	4.54	1.4	-0.83	-4.9	0.936
1.21	0.83	3.49	0.99	0.18	1.43	14.81	-0.47	-2.85	1.043
1.21	1	1.86	0.98	0.21	1.39	4.52	-0.38	-2.54	1.068

Table B 2 Returns and Risk Premia

elasticity=20										
	share in home economy=1.00									
end1	end2	SDev1	SDev2	SDev2p	corr	corrp	Rrf1	Rend1	Risk Prem	Prem2/1
0.83	0.83	0.050	0.050	0.049	0.004	0.025	0.014	0.036	0.022	0.13
0.83	1.00	0.050	0.036	0.036	0.007	0.021	0.014	0.036	0.022	0.10
0.83	1.21	0.050	0.041	0.041	0.006	0.019	0.014	0.036	0.022	0.11
1.00	0.83	0.036	0.050	0.050	0.007	0.018	0.014	0.031	0.017	0.13
1.00	1.00	0.036	0.036	0.035	0.000	0.029	0.014	0.031	0.017	0.10
	share in home economy=.50									
end1	end2	SDev1	SDev2	SDev2p	corr	corrp	Rrf1	Rend1	Risk Prem	Prem2/1
0.83	0.83	0.033	0.033	0.033	0.774	0.800	0.031	0.042	0.011	0.00
0.83	1.00	0.029	0.029	0.028	0.682	0.736	0.031	0.040	0.009	0.00
0.83	1.21	0.031	0.034	0.033	0.759	0.798	0.031	0.041	0.010	0.00
1.00	0.83	0.029	0.029	0.029	0.682	0.729	0.031	0.041	0.010	0.01
1.00	1.00	0.026	0.026	0.025	0.536	0.580	0.031	0.039	0.008	0.00
elasticity=1										
	share in home economy=1									
end1	end2	SDev1	SDev2	SDev2p	corr	corrp	Rrf1	Rend1	Risk Prem	Prem2/1
0.83	0.83	0.042	0.042	0.033	0.215	0.799	0.022	0.039	0.017	0.03
0.83	1.00	0.043	0.031	0.027	0.210	0.934	0.022	0.039	0.017	0.01
0.83	1.21	0.043	0.036	0.030	0.250	0.906	0.022	0.039	0.016	0.02
1.00	0.83	0.031	0.043	0.032	0.210	0.795	0.022	0.035	0.013	0.03
1.00	1.00	0.032	0.032	0.025	0.151	0.913	0.022	0.035	0.013	0.02
	share in home economy=.5									
end1	end2	SDev1	SDev2	SDev2p	corr	corrp	Rrf1	Rend1	Risk Prem	Prem2/1
0.83	0.83	0.042	0.042	0.033	0.215	0.799	0.022	0.039	0.017	0.03
0.83	1.00	0.043	0.031	0.027	0.210	0.934	0.022	0.039	0.017	0.01
0.83	1.21	0.043	0.036	0.030	0.250	0.906	0.022	0.039	0.016	0.02
1.00	0.83	0.031	0.043	0.032	0.210	0.795	0.022	0.035	0.013	0.03
1.00	1.00	0.032	0.032	0.025	0.151	0.913	0.022	0.035	0.013	0.02
elasticity=.25										
	share in home economy=.95									
end1	end2	SDev1	SDev2	SDev2p	corr	corrp	Rrf1	Rend1	Risk Prem	Prem2/1
0.83	0.83	0.142	0.142	0.563	-0.037	-0.758	-0.239	-0.086	0.152	83.87
0.83	1.00	0.095	0.222	0.964	0.538	0.578	-0.025	-0.027	-0.002	112.04
0.83	1.21	0.135	0.035	0.138	0.080	0.999	0.014	0.042	0.028	0.08
1.00	0.83	0.222	0.095	0.202	0.538	-0.949	0.056	0.175	0.119	-0.32
1.00	1.00	0.139	0.139	0.553	-0.003	-0.755	-0.239	-0.090	0.149	78.18
	share in home economy=.82									
end1	end2	SDev1	SDev2	SDev2p	corr	corrp	Rrf1	Rend1	Risk Prem	Prem2/1
0.83	0.83	0.045	0.045	0.102	0.215	0.853	0.018	0.035	0.018	0.00
0.83	1.00	0.047	0.031	0.112	0.279	0.836	0.016	0.036	0.019	0.00
0.83	1.21	0.048	0.034	0.111	0.335	0.859	0.015	0.036	0.021	0.00
1.00	0.83	0.031	0.047	0.096	0.279	0.780	0.020	0.032	0.013	0.00
1.00	1.00	0.032	0.032	0.106	0.215	0.792	0.018	0.031	0.014	0.00

end1,2 = endowments countries 1 and 2

Sdev1,2 = standard deviations of returns on endowments, countries 1 and 2

Sdev2p = standard deviation of return on endowment of country 2, valued in country 1's good

corr, corrp = correlation of returns on endowment, valued in home goods, valued in good 1

Rrf1 = the risk free rate of return on commodity 1 in country 1

Rend1= expected rate of return on country 1's endowment

Risk Prem = difference between Rend1 and Rrf1

Prem2/1 = excess of what country 1 agents are willing to pay for claims on country 2 and their value in country 2

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