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Information and Timing in Repeated Partnerships

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In a repeated partnership game with imperfect monitoring, we distinguish among the effects of (1) shortening the period over which actions are held fixed, (2) increasing the frequency with which accumulated information is reported, and (3) reducing the amount of discounting of payoffs between successive periods. While reducing the amount of discounting generally improves incentives for cooperation, the other two changes can have the reverse effect. When the game is specified in the customary way with information reported at the end of each period of fixed action, the net effect of shortening the period length can be to destroy all incentives for cooperation, reversing the usual conclusion associated with the Folk Theorem for repeated games. Moreover, when interest rates are low, reducing the frequency of information reporting can greatly enhance the efficiency of equilibrium.

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1. Introduction

In many economic settings the efficiency of explicit and implicit contractual arrangements is limited by the presence of imperfect monitoring: some agents cannot observe perfectly the actions of others. Often the economic problem of interest involves the indefinite repetition of some strategic situation such as a partnership problem (see, for example, Fudenberg and Maskin [1986b], Radner [1986], and Radner, Myerson and Maskin [1986]), an oligopoly (Green and Porter [1984], Porter [1983], and Abreu, Pearce and Stacchetti [1986]) or a principal-agent problem (Fudenberg, Holmstrom and Milgrom [1986], Fudenberg and Maskin [1986a], Holmstrom and Milgrom [1987], Radner [1981, 1983], Rogerson [1985], Rubinstein [1979], and Spear and Srivastava [1987]). This paper uses repeated partnership games with moral hazard to study a number of questions arising from interactions among information flows, interest rates and the frequency with which choices are made and revised. The results correct some natural misconceptions regarding the roles of action frequency and informational delays in determining the limits of intertemporal cooperation. They also reveal the potency of a certain sort of information suppression as a tool for enhancing the efficiency of repeated partnerships.

Cooperation in supergames depends upon players' responding aggressively to indications that not all participants are honoring the implicit agreement. When imperfect information about the players' actions arrives periodically, improving the information always strictly expands the equilibrium value set (Kandori [1988]) and hence the possibilities for cooperation. There is some presumption, then, that possibilities of cooperation are also enhanced when signals related to players' behavior are observed without delay, and when players can respond quickly to new information. While this is exactly what happens under perfect monitoring, we show that the presumption is entirely misleading for games with imperfect monitoring. First, reducing the frequency with which actions are taken may allow much greater efficiency in equilibrium. Further, in some cases all cooperation collapses when players are able to move very frequently. These results have significant implications for the interpretation of limit theorems as the discount factor δ approaches 1. Letting δ approach unity is often understood as a study not of extraordinarily patient people, but rather of players who can adjust their actions very quickly. (Think for a moment about the easier case with perfect monitoring--all players' actions are publicly observed. Allowing agents to act more frequently creates a new supergame identical to the original one in all respects except that, because the periods are shorter, future payoffs are discounted (relative to current payoffs) less than before. Thus, asking what happens when players can act very quickly is equivalent to asking what happens when δ approaches 1.) While this point of view seems to provide a compelling motivation for limit theorems in δ, it is inappropriate under imperfect monitoring for informational reasons: it implicitly assumes that as players become able to change their actions more frequently, there is a corresponding increase in the rate of arrival of signals in the economy. A more natural exercise would be one in which the underlying informational structure of the game is held fixed, while players are allowed to react to new information more quickly. Section 2 presents a simple model that allows action frequency and the information arrival process to be varied independently. Then, as noted above, increasing action frequency is not the same as changing the interest rate r and does not generally enhance the possibilities for cooperation.

The second part of the paper concerns the effects of delaying the release of signals to the players. Although such delay would appear to involve a deterioration in the flow of information, it often improves the possibilities for cooperation, especially when δ is not too low. Indeed, when δ is very close to 1, long delays in information release typically achieve nearly perfect efficiency, in the first-best sense. This stands in stark contrast to the asymptotic inefficiency results of Radner, Myerson and Maskin [1986] and Fudenberg and Maskin [1986b]. When there is only infrequent release of accumulated signals (related to players' behavior) it is possible to use a "single punishment" to deter a multitude of different potential deviations. We use the term "global deterrent" to describe such a punishment. Section 3 explores these issues involving the timing of the release of information in a simple model; the benefits of temporary information suppression in more general games with patient players are established with a "folk theorem" in Section 4. Section 5 concludes.

2. Varying Action Frequency with Continuous Observation

The Model and Some Basic Results

We wish to consider repeated partnership games with imperfect monitoring in a formulation which makes it possible to vary the length of time for which players' actions are held constant while keeping fixed the information arrival process. To avoid an arbitrary choice of fundamental time unit, we adopt a specification in which signals arrive stochastically in continuous time. This allows us in particular to analyze the behavior of the model as the players' reaction time goes to zero. The latter in turn permits comparison with the usual asymptotic exercise of letting δ tend to 1, and has the additional advantage of being analytically tractable.

The model developed below is possibly the simplest one in which the effects we wish to highlight appear in a natural and clear way.

The Stage Game $G(t,r)$

We consider a simultaneous, symmetric stage game which is a stochastic, n-player version of a "prisoners' dilemma." Each player has available two actions, labelled c for "cooperate" and d for "defect." For example, c might involve a high level of care or effort, and d could represent "shirking." The imperfect monitoring takes the following form: publicly observed signals arise according to a simple Poisson process whose arrival rate $\gamma(\underset{\sim}{x})$ is a function of the profile $\underset{\sim}{x} = (x_1, \ldots, x_n)$ of actions chosen. The arrival rates when no one deviates, and when any single player deviates, respectively, are denoted by $\lambda > 0$ and $\mu > 0$. The relative size of λ and μ depends upon the interpretation of the signal, which could be a desirable event (in which case assume $\lambda > \mu$) such as the sale of a product or a research breakthrough, or a failure $(\lambda < \mu)$ such as a defective product or an industrial accident. The stage game is parametrized by its length $t > 0$, and by $r > 0$, the rate at which payoffs are discounted. Player i's payoff in the stage game is the expected value of his realized payoffs. The latter depend on his own action x_i, which contributes an instantaneous, or "flow" payoff at rate

$f(x_i)$, and on the realizations of the signal during the time interval. The arrival of a signal causes a discrete loss (in the case of "bad news": $\lambda < \mu$) or gain which is denoted $\ell(x_i)$. If k signals arrive at times t_j , $j = 1, \ldots, k$, $0 \le t_j \le t$, player i's realized payoff is $f(x_i)(\int_0^t e^{-rs}ds) + \ell(x_i) \sum_{j=1}^{k} e^{-rt_j}$, where $f : (c,d) \to R$ and $\ell : (c,d) \to R$ are independent of i . Hence, expected payoffs for i in the stage game, expressed as a flow, are:

$$u_i(\underset{\sim}{x}) = f(x_i) + \gamma(\underset{\sim}{x})\ell(x_i) .$$

To summarize, player i's (expected) payoff depends on his own action, and also on the other players' actions, to the extent that these influence $\gamma(\underset{\sim}{x})$. The payoff structure resembles that of a prisoners' dilemma insofar as:

(1) (d, ..., d) is a Nash equilibrium of the stage game. Furthermore, it yields players their individually rational payoff, which we normalize to 0.

(2) A player's expected flow payoff when all players cooperate is strictly positive, and is denoted Π .

(3) The <u>increase</u> or "gain" in a deviating player's expected flow payoff when he alone defects from (c, ..., c) is strictly positive, and is denoted g .

No further restrictions on payoffs are needed, but because we investigate symmetric equilibria in what follows, a natural assumption is that for all profiles $\underset{\sim}{x}$, $\sum_i u_i(\underset{\sim}{x}) \le \sum_i u_i(c, \ldots, c)$. The probability of k signal arrivals in the stage game of length t , given the arrival rate γ , is $p(k|t,\gamma) = e^{-\gamma t}[(\gamma t)^k)/k!]$, that is, the distribution of k is Poisson with mean γt .

<u>The Repeated Game</u> $G^{\infty}(t,r)$

For any $t > 0$, consider an infinite horizon game in which the stage game $G(t,r)$ is repeated indefinitely. In order to allow for public randomization following any history of play, we assume that at the beginning of every stage $h = 1, 2, \ldots,$ an independent draw ω_h from the uniform distribution on $[0,1]$ is publicly observed. Player i can condition his $(s+1)^{th}$ stage action on his own past actions (which remain unobservable to others), on k_h (the number of signal arrivals in stage h)[1] for all $h \leq s$, and on ω_h for all $h \leq s+1$. Thus a pure strategy σ_i for player i in $G^{\infty}(t,r)$ is a sequence of measurable functions $\{\sigma_i(s)\}_{s=1}^{\infty}$, where $\sigma_i(1) : [0,1] \to \{c,d\}$ and for $s \geq 1$, $\sigma_i(s+1) : [\{c,d\} \times N]^s \times [0,1]^{s+1} \to \{c,d\}$, and N denotes the set of non-negative integers. A profile σ of pure strategies induces a distribution of action profiles (and hence an expected flow payoff for each player) in each stage. The flow payoffs are discounted continuously at the rate $r > 0$; multiplying these discounted sums by r yields the average payoffs corresponding to σ . A profile σ is <u>symmetric</u> if for all i and j and any $s \in N$, $\sigma_i(s) = \sigma_j(s)$. For any symmetric profile σ , let $v(\sigma)$ denote the average payoff to each player, given σ . Here the parameters t and r that identify the supergame have been suppressed.

We are interested in the maximal average payoff $\bar{v}(t,r)$ that can be

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[1]Notice that players' strategies depend only on the number of signal arrivals in stage h , and not on the arrival times. This entails no loss of generality (given that we allow for public randomization) since the number of signals is a sufficient statistic for the Poisson parameter.

achieved in any <u>pure strategy symmetric "sequential equilibrium"</u>[2] (hereafter, S.S.E.) of $G^{\infty}(t,r)$. Again suppressing t and r, denote the S.S.E. average value set by $V = \{v(\sigma) | \sigma \text{ is an S.S.E.}\}$. The following definitions are analogous to those introduced in Abreu, Pearce and Stacchetti [1986], hereafter APS. Define $g^*(b;a) = u_1(b, a, \ldots, a) - u_1(a, \ldots, a)$. For any set $W \subseteq R$, a pair $(a;w) \in \{c,d\} \times [coW]^N$ is <u>admissible with respect to W</u> if $(e^{rt} - 1)g^*(b;a) \leq \sum_k [p(k|t, \gamma(a, \ldots, a)) - p(k|t, \gamma(b, a, \ldots, a))]w_k$ for $b \in \{c,d\}$. Let $E(a;w) = (1 - e^{-rt})u_1(a, \ldots, a) + e^{-rt}\sum_k p(k|t, \gamma(a, \ldots, a))w_k$ and $B(W) = co\{E(a;w) | (a;w) \text{ is admissible w.r.t. } W\}$. The three results below, whose proofs we omit, are straightforward adaptations of theorems in APS; they simplify our analysis considerably.

(R1) Self-generation: $[W \subseteq B(W) \text{ and } W \text{ bounded}] \Rightarrow [W \subseteq V]$.

(R2) Factorization: $V = B(V)$

(R3) Compactness and Convexity: V is compact and convex.

We now use the specific structure of the model to develop a result (Proposition 1) that essentially gives algebraic expression to (R1) and (R2). The presence of the ω_s's allows for public randomizations over the continuation paths to be followed; without loss of generality, then, restrict attention to continuation payoffs with (average) values $\bar{v}$ or 0,

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[2]The definition of sequential equilibrium (Kreps and Wilson 1988) does not cover the case encountered here of an infinity of information sets. However, the belief system associated with a profile can be computed here using Bayes' Rule; we require that the profile and the associated beliefs satisfy sequential rationality. We abuse notation by calling a profile (rather than an assessment) a sequential equilibrium. The qualifier "symmetric" simply indicates that the strategy profile is symmetric.

the best and worst elements of V, respectively. Players are willing to choose c rather than d only if defecting increases the probability of receiving a continuation value of 0 by an amount Δ sufficient to wipe out the one-period gain $[(1 - e^{-rt})/r]g$ from defecting. (Recall that $g = g^*(d;c)$.) For an arbitrary "probability wedge" $\Delta > 0$, the linear program below determines the subset of the one-stage signal space $[0,1] \times N$ which, when used as the punishment region, most efficiently creates the required wedge. The subset is described by the sequence $\{\alpha_k\}$, where α_k is the probability of punishing when exactly k signals are observed.

$$\text{LP:}\quad P(t,\Delta) = \min_{\{\alpha_k\}} \sum_k \alpha_k p(k|t,\lambda)$$

$$\text{subject to}\quad \sum_k \alpha_k[p(k|t,\mu) - p(k|t,\lambda)] = \Delta$$

$$\text{and}\quad 0 \le \alpha_k \le 1 \;,\quad k = 0,\ 1,\ 2,\ \ldots\ .$$

Let $m(t,\Delta) = \Delta/P(t,\Delta)$. Notice that $m(t,\Delta)$ is a (transformed) likelihood ratio, being the ratio of the increased probability of triggering a punishment when someone defects, to the probability when no one defects. It measures the efficiency with which the punishment region distinguishes statistically between cooperation and defection. It may be verified that:

(P1) if $\lambda \neq \mu$ there exists $\bar{\Delta}(t) > 0$ such that $m(t,\Delta)$ is well-defined for all $\Delta \in (0, \bar{\Delta}(t))$. As a function of Δ , m is continuous and non-increasing.

Furthermore, on this interval if $\mu > \lambda$, the unique solution $\{\alpha_k(t,\Delta)\}$ to LP is characterized by a cutoff value $k^*(\Delta)$ such that $\alpha_k(t,\Delta) = 0$

for all $k < k^*(\Delta)$ and $\alpha_k(t,\Delta) = 1$ for all $k > k^*(\Delta)$. Conversely, if $\lambda > \mu$, $\alpha_k(t,\Delta) = 1$ for all $k < k^*(\Delta)$ and $\alpha_k(t,\Delta) = 0$ for all $k > k^*(\Delta)$.

Proposition 1 provides a clean characterization of $\bar{v}(t,r)$, the maximal S.S.E. payoff, and is basic to the analysis of this section.

<u>Proposition 1</u>. Consider the equations

$$g(e^{rt} - 1) = v\Delta \qquad \text{Incentive Equation} \tag{1}$$

$$v = \Pi - \frac{g}{m(t,\Delta)} \qquad \text{Value Equation} \tag{2}$$

and define $V^*(t,r) = \{v | v = 0 \text{ or } \exists\Delta > 0 \text{ s.t. } (v,\Delta) \text{ satisfies (1) and (2)}\}$. Then $\bar{v}(t,r) = \max V^*(t,r)$.

<u>Proof</u>: Fix $r > 0$. For simplicity we suppress the dependence of functions on r in what follows. Consider $\bar{v}(t) = \max V(t)$, which by (R3) is well-defined. We first show that there exists $\tilde{v} \in V^*(t)$ such that $\tilde{v} \geq \bar{v}(t)$. This is obviously true if $\bar{v}(t) = 0$. Now suppose $\bar{v}(t) > 0$. By (R2), $V(t) = B(V(t))$. Hence there exists $(a;w)$ admissible with respect to $V(t)$ such that

$$\bar{v}(t) = \begin{cases} (1 - e^{-rt})\Pi + e^{-rt}\sum_k p(k|t,\lambda)w_k & \text{if } a = c \\ e^{-rt}\sum_k p(k|t, \gamma(d, \ldots, d))w_k & \text{if } a = d \end{cases} \tag{2'}$$

Since $w_k \in \text{co } V(t)$ if $a = d$, $\bar{v}(t) \leq e^{-rt}\bar{v}(t)$, a contradiction. Thus $a = c$. Let $\alpha_k \in [0,1]$ be defined by $\alpha_k \cdot 0 + (1 - \alpha_k)\bar{v}(t) = w_k$. Admissibility of $(a;w)$ now implies

$$g(e^{rt} - 1) \le (Q-P)\bar{v}(t) \ , \tag{1'}$$

where $P = \sum_k \alpha_k p(k|t,\lambda)$ and $Q = \sum_k \alpha_k p(k|t,\mu)$. (1') and (2') imply that

$$\bar{v}(t) \le \Pi - \frac{g}{(Q-P)/P} \le \Pi - \frac{g}{m(t,\bar{\Delta})} \ , \tag{2"}$$

where $\bar{\Delta} = Q-P$, and by definition $m(t,\bar{\Delta}) \ge (Q-P)/P$. Substituting (2") into (1'), we obtain

$$g(e^{rt} - 1) \le \left[\Pi - \frac{g}{m(t,\bar{\Delta})}\right]\cdot\bar{\Delta} \ . \tag{1"}$$

If this holds as an equality, $\bar{v}(t) \in V^*(t)$, and we may set $\tilde{v} = \bar{v}(t)$. If not, define

$$f(x) = \left[\Pi - \frac{g}{m(t,\ \bar{\Delta}-x)}\right]\cdot(\bar{\Delta}-x) - g(e^{rt} - 1) \ .$$

Since $m(t,\cdot)$ is continuous in Δ , f is continuous. Also $f(0) > 0$ and for some $\varepsilon \in (0,\bar{\Delta})$, $f(\bar{\Delta}-\varepsilon) < 0$. Hence there exists $\tilde{x} > 0$ such that $f(\tilde{x}) = 0$. Then since $m(t,\cdot)$ is non-increasing,

$$\tilde{v} = \left[\Pi - \frac{g}{m(t,\ \bar{\Delta}-\tilde{x})}\right] \ge \bar{v}(t) \ , \quad \text{and} \quad \tilde{v} \in V^*(t) \ .$$

To complete the proof we show that $V^*(t) \subseteq V(t)$. Together with the preceding argument this implies $\bar{v}(t) = \max V^*(t)$. Let (v,Δ) satisfy (1) and (2), and let $\{\alpha_k\}$ now be defined to solve LP for Δ , t . That is, $m(t,\Delta) = (Q-P)/P$ where P and Q are as defined earlier. We argue that $\{0,v\}$ is a self-generating set and therefore, by (R1), $v \in V(t)$.

Clearly $0 \in B(\{0,v\})$. Consider the pair $(c;w)$ where for each k , $w_k = \alpha_k \cdot 0 + (1 - \alpha_k) \cdot v$. Then

$$g(e^{rt} - 1) = (Q-P)v \ . \tag{3}$$

It remains only to show that $E(c;w) = \Pi - [g/m(t,\Delta)]$. But this follows directly from $E(c;w) = \Pi(1 - e^{-rt}) + e^{-rt}(1-P)v$, (3), (2) and the identity $m(t,\Delta) = (Q-P)/P$. Q.E.D.

The "likelihood ratio" $\bar{m}(t,r)$ associated with the best S.S.E. is

$$\bar{m}(t,r) = \max\{m(t,\Delta) \mid \exists v \text{ for which } (v,\Delta) \text{ satisfies (1) and (2)}\} \ .$$

From Proposition 1 we then have the following equation which summarizes the analysis thus far:

$$\bar{v}(t,r) = \Pi - \frac{g}{\bar{m}(t,r)} \ .$$

In words, cooperative behavior yields flow payoffs Π , but also provides the opportunity of gains from cheating, at the flow rate g . Deterring the latter involves losses proportional to g , and inversely proportional to the efficiency of the best test available.

Proposition 1 can be extended to much more general moral hazard models, by reformulating LP to include additional constraints, but that will not concern us here. We proceed to study in turn the cases of "bad news" and "good news," focussing in particular on the behavior of $\bar{v}(t,r)$ in a neighborhood of $t = 0$, and $r = 0$ respectively.

Case A: "Bad News"

In this case the signal represents occurrences of a "failure" which is more likely when someone defects than when all cooperate: $\mu > \lambda$. The key result here is that for a wide range of parameter values (including some fixed r) the function $\bar{v}(t,r)$ has an interior maximum at $t^* > 0$. This contradicts the common presumption that possibilities for collusion improve monotonically as players are able to act more quickly. Lemma 2 offers necessary and sufficient conditions for the possibility of cooperation in a neighborhood of $t = 0$.

By Proposition 1, $\bar{v}(t,r)$ and $\Delta = g(e^{rt} - 1)/\bar{v}(t,r)$ satisfy equations (1) and (2) above. Let $\{\bar{\alpha}_k(t,r)\}$ be the unique solution of the LP for Δ so defined. Hence $\bar{\alpha}_k(t,r)$ is the probability of punishing the occurrence of k signals (by playing $(d, \ldots, d)$ forever) in the best S.S.E. of $G^\infty(t,r)$. Define $m_0 = (\mu-\lambda)/\lambda$.

Lemma 2 establishes that for t very small, it is possible to sustain some cooperation if and only if $\left[\frac{1}{r}\left(\Pi - \frac{g}{m}\right)(\mu-\lambda)t\right] > gt$. The right side evidently approximates the (gross) gain from defecting for an interval of length t ; the left side is the corresponding loss, the expression in square brackets representing the permanent loss of $\bar{v}$ (see Prop. 1) if a single signal arrives, and $(\mu-\lambda)t$ being the increased probability of this event.

Lemma 2. If $\frac{1}{r}(\Pi - g/m_0)(\mu-\lambda) > g$, there exists $T > 0$ such that $\bar{v}(t,r) > 0$ for all $t \in (0,T]$, and $\lim_{t\to 0} \bar{v}(t,r)$ and $\lim_{t\to 0} \bar{\alpha}_1(t,r)$ are well defined. Conversely, if $\frac{1}{r}(\Pi - g/m_0)(\mu-\lambda) \le g$, there exists $t > 0$ such that $\bar{v}(t,r) = 0$ for all $t \in (0,T]$.

Proposition 4 below observes that for any $t > 0$, perfect efficiency is approximated when players are extremely patient. Comparing Propositions 3 and 4, one notices that letting players move quickly has completely different consequences from making them very patient. The latter leads to asymptotic efficiency; the former decidedly does not. Notice also the contrast between Proposition 4 and the asymptotic inefficiency results of Radner, Myerson and Maskin [1986] and Fudenberg and Maskin [1986b]; in their models, there are a finite number of possible observations in a period, and hence the likelihood ratios of the tests they could use are bounded above. In our Poisson stage game, however, there are rare multiple occurrences (k very large) having arbitrarily large likelihood ratios; this means that there is no limit to the efficiency of the implicit tests that can be used in equilibrium, provided that $\bar{v}(t,r)/r$ is large enough.

Proposition 3: For any $r > 0$,

$$\lim_{t\to 0} v(t,r) = \begin{cases} \Pi - \frac{g}{m_0} & \text{if } \frac{1}{r}\left(\Pi - \frac{g}{m_0}\right)(\mu-\lambda) > g \\ 0 & \text{otherwise.} \end{cases}$$

Proposition 4: For any $t > 0$, $\lim_{r\to 0} \bar{v}(t,r) = \Pi$.

Proof of Lemma 2 and Proposition 3: As in the earlier proof, for simplicity we suppress the dependence of functions on r . Consider a sequence $\{\alpha_k(t)\} \subseteq R$, and define $Q = \sum_k \alpha_k(t)p(k|t,\mu)$, $P = \sum_k \alpha_k(t)p(k|t,\lambda)$ and $\Delta = Q-P$. We will say that $\{\alpha_k(t)\}$ satisfies (*) if:

(a) $\{\alpha_k(t)\}$ solves LP for (t,Δ) .

(b) $v = \Pi - g\frac{P}{\Delta} > 0$

(c) $g(e^{rt} - 1) = v\Delta$.

Note that (a) implies $m(t,\Delta) = \Delta/P$ where Δ , P are as defined above. Hence by Proposition 1. $\bar{v}(t) > 0$ if and only if there exists $\{\alpha_k(t)\}$ satisfying (*). Substituting for v in requirement (c) above yields

$$g(e^{rt} - 1) = \Pi\Delta - gP .$$

Expanding and dividing by t , we have

$$g\left(r + \frac{r^2 t}{2} + \dots\right) = \Pi\left[\frac{\alpha_0(t)}{t}(e^{-\mu t} - e^{-\lambda t}) + \alpha_1(t)(e^{-\mu t}\mu - e^{-\lambda t}\lambda) + \alpha_2(t)\left(e^{-\mu t}\frac{\mu^2 t}{2} - e^{-\lambda t}\frac{\lambda^2 t}{2}\right) + \dots\right]$$
$$- g\left[\frac{\alpha_0(t)}{t}e^{-\lambda t} + \alpha_1(t)e^{-\lambda t}\lambda + \alpha_2(t)e^{-\lambda t}\frac{\lambda^2 t}{2} + \dots\right] \tag{4}$$

For t sufficiently small,

$$e^{-\mu t} < e^{-\lambda t} \quad , \quad \text{and}$$

$$\frac{e^{-\mu t}(\mu t)^{k+1}}{e^{-\lambda t}(\lambda t)^{k+1}} > \frac{e^{-\mu t}(\mu t)^k}{e^{-\lambda t}(\lambda t)^k} > 1 \ , \quad k = 1,\ 2,\ \dots \ .$$

Together with (4) this implies that there exists $T' > 0$ such that for any $t \le T'$, $\{\alpha_k(t)\}$ satisfies (*) if and only if

(a') $\alpha_0(t) = 0$, $\alpha_1(t) \in (0,1]$, $\alpha_k(t) = 1$, $k = 2, 3, \dots$

(b') as before, and

(c') $\alpha_1(t)$ satisfies:

$$g\left[r + \frac{r^2t}{2} + \ldots\right] = \Pi\left[\alpha_1(t)(e^{-\mu t}\mu - e^{-\lambda t}\lambda) + \left(e^{-\mu t}\frac{\mu^2 t}{2} - e^{-\lambda t}\frac{\lambda^2 t}{2}\right) + \ldots\right]$$
$$- g\left[\alpha_1(t)e^{-\lambda t}\lambda + e^{-\lambda t}\frac{\lambda^2 t}{2} + \ldots\right] . \tag{5}$$

In what follows we assume $t \le T'$, $\alpha_0(t) = 0$, $\alpha_k(t) = 1$, $k = 2, 3, \ldots$, and that $\alpha_1(t)$ is given by (5). These requirements define $\{\alpha_k(t)\}$ uniquely. Hence if $\bar{v}(t) > 0$, then $\bar{\alpha}_k(t) = \alpha_k(t)$ and $\bar{v}(t) = \Pi - \frac{g}{\Delta}P$. It is clear from (5) that $\alpha_1(0) = \lim_{t\to 0} \alpha_1(t)$ is well-defined. Taking limits,

$$gr = (\Pi(\mu-\lambda) - g\lambda)\alpha_1(0) .$$

Thus

$$\alpha_1(0) \in (0,1) \Leftrightarrow \frac{1}{r}\left(\Pi - \frac{g}{m_0}\right)(\mu-\lambda) > g .$$

When the latter condition is satisfied,

$$\lim_{t\to 0}\left(\Pi - \frac{g}{\Delta/P}\right) = \Pi - \frac{g}{(\mu-\lambda)}\lambda > \Pi - g\frac{(\lambda+r)}{(\mu-\lambda)} > 0 ,$$

and there exists $T \in (0,T']$ such that for <u>all</u> $t \in (0,T)$, $\{\alpha_k(t)\}$ satisfies (*), or equivalently, $\bar{v}(t) > 0$. Moreover $\bar{v}(0) = \lim \bar{v}(t) = (\Pi - g/m_0)$. Conversely, if $\frac{1}{r}(\Pi - g/m_0)(\mu-\lambda) < g$, there exists $T > 0$ such that for <u>all</u> $t \le T$, $\alpha_1(t) > 1$. Since this violates (a'), $\bar{v}(t) = 0$ for all $t \le T$. Finally we deal with the case $\alpha_1(0) = 1$, which corresponds to $\frac{1}{r}(\Pi - g/m_0)(\mu-\lambda) = g$. Differentiating (5), evaluating (right-hand) derivatives at $t = 0$, and substituting $\alpha_1(0) = 1$, we have

$$g\frac{r^2}{2} = \alpha_1'\Pi(\mu-\lambda) - \frac{1}{2}\Pi(\mu^2 - \lambda^2) - \alpha_1' g\lambda + \frac{1}{2}g\lambda^2 ,$$

which simplifies to

$$gr^2 + \lambda((\Pi m_0 - g)\lambda + \Pi m_0\mu) = 2\alpha_1'\lambda(\Pi m_0 - g) .$$

As noted above, $(\Pi - g/m_0) > 0$, therefore $\alpha_1' > 0$. Hence there exists $T > 0$ such that $\alpha_1(t) > 1$ for all $t \le T$, and thus $\bar{v}(t) = 0$. Q.E.D.

Proof of Proposition 4: Fix $t > 0$. Let

$$D(r) = \left\{\Delta > 0 \,\middle|\, g(e^{rt} - 1) = \left(\Pi - \frac{g}{m(t,\Delta)}\right)\Delta\right\} \subseteq [0,1] .$$

As noted earlier, $m(t,\cdot)$ is continuous in Δ. Hence $D(r)$ is compact. Also, there exists $\bar{r} > 0$ such that $D(r)$ is non-empty for all $r \in (0,\bar{r})$. Let r lie in this range and define $\Delta(r) = \min D(r)$. Clearly $\Delta(r)$ is strictly increasing in r. By Proposition 1, $\bar{v}(t,r) = \Pi - [g/m(t, \Delta(r))]$. Since $m(t,\cdot)$ is strictly decreasing in Δ, $\bar{v}(t,r)$ is strictly decreasing in r. Thus $\lim_{r\to 0} (\Pi - g/m(t,\Delta(r))) > 0$, and hence $\Delta(r) \to 0$ as $r \to 0$. Finally note that $\lim_{\Delta\to 0} m(t,\Delta) = \infty$. Q.E.D.

Let $\bar{\alpha}_1(0,r) = \lim_{t\to 0} \bar{\alpha}_1(t,r)$ be the limiting probability of punishment given the occurrence of a single signal in the first period of the best S.S.E. From the proof of Lemma 2 and Proposition 3, $\bar{\alpha}_1(0,r) = rg/[\Pi(\mu-\lambda) - \lambda g]$. Hence $\bar{\alpha}_1(0,r)$ tends to be "small" if there is little discounting, the rate of arrival of signals is high, or the gains from cheating are small. We next show that whenever $\bar{v}(0,r) > 0$

(cooperation is possible near the limit) and $\bar{\alpha}_1(0,r) < 1/2$, the right-hand derivative $\partial v(t,r)/\partial t$ at $t = 0$ is strictly positive. Consequently, for this parameter range, <u>increasing speeds of reaction can inhibit cooperation</u>: there is some strictly positive t^* at which profits in the most collusive S.S.E. are maximized.

<u>Proposition 5</u>: Suppose that $\bar{v}(0,r) > 0$. Then at $t = 0$,

$$\frac{\partial \bar{v}(t,r)}{\partial t} > 0 \quad \text{if} \quad \bar{\alpha}_1(0,r) < \frac{1}{2}$$

$$\frac{\partial \bar{v}(t,r)}{\partial t} < 0 \quad \text{if} \quad \bar{\alpha}_1(0,t) > \frac{1}{2} .$$

<u>Proof</u>: As usual, we suppress dependence on r. From Lemma 2 and its proof it follows that there exists $T > 0$ such that for $t \in (0,T]$,

$\bar{v}(t) > 0$ and

$$\bar{m}(t) = \frac{\bar{\alpha}_1(t)e^{-\mu t}\mu t + e^{-\mu t}\left[\frac{\mu^2 t^2}{2!} + \ldots\right]}{\bar{\alpha}_1(t)e^{-\lambda t}\lambda t + e^{-\lambda t}\left[\frac{\lambda^2 t^2}{2!} + \ldots\right]} - 1 ,$$

where $\bar{v}(t) = \Pi - (g/\bar{m}(t))$, and $\bar{\alpha}_1(t)$ is defined by (5). Since $\bar{\alpha}_1(t)$ is differentiable at $t = 0$, so is $\bar{m}(t)$. Dividing the numerator and denominator of the expression for $\bar{m}(t)$ by t and differentiating, we obtain

$$\left.\frac{d\bar{m}}{dt}\right|_{t=0} = -\frac{\mu m_0}{\bar{\alpha}_1(0)}\left(\alpha_1(0) - \frac{1}{2}\right) .$$

Clearly $\left.\frac{d\bar{v}}{dt}\right|_{t=0}$ and $\left.\frac{d\bar{m}}{dt}\right|_{t=0}$ have the same signs. Q.E.D.

Case B: "Good News"

The signals are now taken to represent successes; $\lambda > \mu$. This case differs from the "bad news" model in that cooperation cannot necessarily be guaranteed simply by considering very patient players. Proposition 6 shows that a necessary condition for the emergence of cooperation, regardless of the rate of interest, is that the likelihood ratio corresponding to the event "no successes" is sufficiently large. Punishing this event affords the most efficient test possible. The associated likelihood ratio is $M(t) = (e^{-\mu t}/e^{-\lambda t}) - 1$

Proposition 6: If $M(t) \le g/\Pi$, $\bar{v}(t,r) = 0$ for all $r > 0$.

Proof: Since $\lambda > \mu$,

$$\frac{e^{-\mu t}}{e^{-\lambda t}} > \frac{e^{-\mu t}}{e^{-\lambda t}} \frac{(\mu t)^k/k!}{(\lambda t)^k/k!} , \quad k = 1, 2, \ldots .$$

Hence $m(t,\Delta) \le M(t)$ for all $\Delta \in [0, \bar{\Delta}(t)]$. Therefore if $\Pi < g/M(t)$, there exists no $(v,\Delta) > 0$ which satisfies (1) and (2) of Proposition 1.

Q.E.D.

Proposition 7 provides a partial converse.

Proposition 7: If $M(t) > g/\Pi$, $\lim_{r \to 0} \bar{v}(t,r) = \Pi - [g/M(t)]$.

Proof: It is clear from LP that $m(t,\Delta) = M(t)$ for $0 < \Delta \le e^{-\mu t} - e^{-\lambda t}$. Also there exists $\bar{r} > 0$ such that $g(e^{rt} - 1) \le (\Pi - [g/M(t)])(e^{-\mu t} - e^{-\lambda t})$ for all $r \in (0,\bar{r}]$. Thus for $r \le \bar{r}$, $r = \Pi - [g/M(t)]$ and $\Delta = g(e^{rt} - 1)/(\Pi - [g/M(t)]) > 0$ satisfy (1) and (2) of Proposition 1, and hence $\bar{v}(t,r) \ge \Pi - (g/M(t))$. Since

$m(t,\Delta) \le M(t)$ for all $\Delta > 0$, $\Pi - [g/M(t)]$ is also an upper bound on $\bar{v}(t,r)$, and we are done. Q.E.D.

We see that if parameter values are not too unfavorable, the limit results for this case as r approaches 0 conform to one's expectations given the analysis in Radner, Myerson and Maskin [1986] and Fudenberg and Maskin [1986b]. The contrast with the limit as t tends to 0, however, is even more vivid than in the "bad news" case: regardless of parameter values, if players can move extremely quickly, there is no possibility of cooperation:

Proposition 8: There exists $T > 0$ such that $\bar{v}(t,r) = 0$ for all $t \le T$ and all $r > 0$.

Proof: Clearly there exists $T > 0$ such that $M(t) \le g/\Pi$ for all $t \le T$. The result now follows from Lemma 5. Q.E.D.

3. Monitoring Delays with Fixed Frequency of Actions

This section explores the rather potent effects of delaying the release of signals while action frequency is held constant. No longer needing to subdivide time arbitrarily finely, we now revert to the standard model in which there is a fundamental unit of time, called a period, during which actions cannot be changed. As before, each player in this symmetric model has available in any period s two actions, c or d . The payoff-relevant random variable θ_s (corresponding to the Poisson variable in Section 2) now takes on only two values: 1 ("success") or 0 ("failure"). Players' actions $\underset{\sim}{x}_s = (x_s^1, \ldots, x_s^n)$ in period s affect the probabilities $p(\theta_s|\underset{\sim}{x}_s)$, $\theta_s = 0, 1$. Player i's payoff $u^i(\underset{\sim}{x}_s)$

in period s is the expected value of a realized reward that depends on x_s^i directly and on θ_s. The payoff structure is analogous to that of a prisoners' dilemma, and conditions (1) to (3) of Section 2 are retained, Π and g now representing expected payoffs per period rather than flow rates of payoffs in continuous time. Let $\lambda \in (0,1)$ and $\mu \in (0,1)$ be the probabilities of failure when all players cooperate and, respectively, when a single player deviates from cooperation. We assume $\mu > \lambda$.

A stage game of length t, where t is a positive integer, is comprised of t of the period games just described, but with the following information structure: players remain ignorant of signal realizations within the stage game until the end of the t^{th} period, when they are informed of the realizations within the stage (and hence their own realized payoffs for the stage). Thus, in a particular period, player i's action is a function of his own past actions and realizations of θ_h for all past periods h excluding those in the current stage. It is convenient to convexify the S.S.E. value set of the supergame $G^\infty(t,\delta)$ with stage game of length t, by including at the beginning of each stage a publicly observed random drawing from the uniform distribution on $[0,1]$, on which all subsequent choices by players can be conditioned.

Some additional notation is needed for a formal definition of the supergame strategies. For positive integers s and t, let $\eta(s,t)$ and $\rho(s,t)$ be defined by:

$$s = \eta(s,t) + \rho(s,t)\ , \quad \text{where } \eta(s,t) \text{ is a non-negative integral multiple of } t \text{ and } \rho(s,t) < t\ .$$

In other words, $\rho(s,t) = s \bmod t$. Also, define the integer of (s,t)

by $I(s,t) = \eta(s,t)/t$. Then in period $s+1$ of $G^{\infty}(t,\delta)$, player i has observed s past actions of his own, $\eta(s,t)$ realizations of the payoff-relevant signal, and $I(s,t)+1$ realizations of the public randomizing device. A supergame strategy σ_i for player i is a sequence $\{\sigma_i(s)\}_{s=1}^{\infty}$ where $\sigma_i(1) : [0,1] \to \{c,d\}$, and for $s = 1, 2, \ldots$,

$$\sigma_i(s+1) : \{c,d\}^s \times \{0,1\}^{\eta(s,t)} \times [0,1]^{I(s+1,t)+1} \to \{c,d\} .$$

Employing the same argument as that given in APS, we may without loss of generality restrict attention to strategies for each player i that are not functions of actions taken by i before the current stage:

$$\sigma_i(s+1) : \{c,d\}^{\rho(s,t)} \times \{0,1\}^{\eta(s,t)} \times [0,1]^{I(s+1,t)+1} \to \{c,d\} .$$

Period t payoffs accrue at the end of period t , and are discounted to the beginning of period 1. We denote by $V(t,\delta)$ the set of S.S.E. (average) values in $G^{\infty}(t,\delta)$.

A stream of action profiles, one for each period of a stage, is denoted $(\underset{\sim}{x}_k)_{k=1}^{t}$, and $\theta = (\theta_1, \ldots, \theta_t) \in \Theta = \{0,1\}^t$ is a possible vector of signal realizations in a stage. The probability of θ , given that play in a stage is $(\underset{\sim}{x}_k)_{k=1}^{t}$, is

$$q\left(\theta \,\middle|\, (\underset{\sim}{x}_k)_{k=1}^{t}\right) = \underset{k=1}{\overset{t}{\times}} p(\theta_k | \underset{\sim}{x}_k) .$$

That is, the signals are identically, independently distributed across periods of a stage. For $a, b \in \{c,d\}^t$ and e_n the unit vector with n coordinates, let $\Delta(\theta|b,a) = q(\theta|(a_k \cdot e_n)_{k=1}^{t}) - q(\theta|(b_k,\ a_k \cdot e_{n-1})_{k=1}^{t})$.

This is the amount by which the probability of θ falls when a single player defects from a by playing $b = (b_1, \ldots, b_t)$, while all others continue to play a. As before, for $x, y \in \{c,d\}$, $g^*(y;x) = u_1(y, x, \ldots, x) - u_1(x, \ldots, x)$ denotes a player's one-period gain from switching from x to y, when all others are playing x.

We now state the appropriate definition of admissibility for this model, which will allow us to invoke (R1), (R2) and (R3) of Section 2 for $G^\infty(t,\delta)$, reading $V(t,\delta)$ for V.

<u>Definition</u>: For any set $W \subseteq \mathbf{R}$, a pair $(a;w) \in \{c,d\}^t \times (\text{co}W)^{|\Theta|}$ is <u>admissible with respect to</u> W if for all $b \in \{c,d\}^t$,

$$\sum_{k=1}^{t} \delta^k g^*(b_k;\ a_k) \le \frac{\delta}{1-\delta}\ \delta^t \sum_{\theta\in\Theta} \Delta(\theta|b,a)w(\theta) \ .$$

Also, let $E(a;w) = \frac{1-\delta}{\delta} \sum_{k=1}^{t} \delta^k u_1(a_k \cdot e_n) + \delta^t \sum_{\theta\in\Theta} q(\theta|(a_k \cdot e_n)_{k=1}^{t})w(\theta)$

and $\qquad B(W) = \text{co}\{E(a;w)|(a;w) \text{ is admissible w.r.t. } W\}$.

$\qquad W$ is self-generating if W is bounded and $W \subseteq B(W)$.

The arguments in APS can be adapted to the current model to establish (R1), (R2) and (R3) (see Section 2). Because $V(t,\delta)$ is compact, $\bar{v}(t,\delta) = \max V(t,\delta)$ is well-defined.

Let $m = (\mu-\lambda)/\lambda$ and recall that $g = g^*(d,c)$ and that $u_1(c, \ldots, c) = \Pi$. We assume that $\Pi > g/m$. Lemma 9 provides an upper bound on S.S.E. payoffs in $G^\infty(t,\delta)$.

<u>Lemma 9</u>: Suppose $\bar{v}(t,\delta) > 0$. Then

$$\bar{v}(t,\delta) \le \Pi - \left(\frac{\delta}{\delta + \dots + \delta^t}\right)\frac{g}{m} \qquad \text{for all} \quad \delta \in (0,1) \ .$$

<u>Proof</u>: By (R2), $W = [0, \bar{v}(t,\delta)]$ is a self-generating set. Let $(a;w)$ be admissible with respect to W , and $E(a;w) = \bar{v}(t,\delta)$. Let α_θ satisfy $(1 - \alpha_\theta)\bar{v} = w(\theta)$, where for convenience we write $\bar{v} = \bar{v}(t,\delta)$. Since $\bar{v} > 0$ there is a smallest integer $\ell \le t$ such that $a_\ell = c$. Denote by $q(\theta)$ the probability of θ when in all periods $k = 1, \dots, t$, all players use a_k . Then

$$\bar{v} = E(a;w) \le \Pi(\delta^\ell + \dots + \delta^t)\frac{(1-\delta)}{\delta} + \delta^t(1-P)\bar{v}$$

where $P = \sum_{\theta\in\Theta} \alpha_\theta q(\theta)$. This implies

$$\Pi(\delta^\ell + \dots + \delta^t) - \bar{v}(\delta + \dots + \delta^t) \ge \frac{\delta}{1-\delta}\,\delta^t P\bar{v} \ . \tag{6}$$

$$\text{Let} \quad q^*(\theta) = \begin{cases} \frac{\mu}{\lambda}q(\theta) & \text{if} \quad \theta_\ell = 0 \\ \left(\frac{1-\mu}{1-\lambda}\right)q(\theta) & \text{if} \quad \theta_\ell = 1 \ . \end{cases}$$

$q^*(\theta)$ is the probability of θ when in all periods k , a_k is used by all players, with the exception that a single player deviates to d in period ℓ . Let $Q = \sum_{\theta\in\Theta} \alpha_\theta q^*(\theta)$. By the definition of admissibility, $\delta^\ell g \le \delta^t \sum_\theta \Delta(\theta|b,a)w(a)\frac{\delta}{1-\delta}$, where $b_\ell = d$ and $b_k = a_k$, $k \ne \ell$. The above may be manipulated to yield

$$\delta^{\ell} g \leq \delta^{t}(Q-P)\bar{v} \frac{\delta}{1-\delta} . \tag{7}$$

It is easy to check that

$$\frac{Q-P}{P} \leq \frac{\mu-\lambda}{\lambda} = m . \tag{8}$$

Combine (6)-(8) to obtain:

$$\Pi(\delta^{\ell} + \ldots + \delta^{t}) - \bar{v}(\delta + \ldots + \delta^{t}) \geq \delta^{\ell} \frac{g}{m} , \quad \text{that is}$$

$$\bar{v}(\delta + \ldots + \delta^{t}) \leq (\delta^{\ell} + \ldots + \delta^{t})\left[\Pi - \left(\frac{\delta}{\delta + \ldots + \delta^{t*}}\right)\frac{g}{m}\right]$$

where $t* = t-\ell+1$. Since

$$\frac{\delta}{\delta + \ldots + \delta^{t*}} \geq \frac{\delta}{\delta + \ldots + \delta^{t}} \quad \text{and} \quad \frac{\delta^{\ell} + \ldots + \delta^{t}}{\delta + \ldots + \delta^{t}} \leq 1 ,$$

the proof is complete. Q.E.D.

Proposition 10 states that for sufficiently high δ , the bound on S.S.E. payoffs established in Lemma 9 is actually achieved by $\bar{v}(t,\delta)$. The corollaries and discussion that follow the proposition underline the significance of the result.

<u>Proposition 10</u>: For all $t \in \mathbf{N}$ there exists $\underline{\delta}(t) \in (0,1)$ such that

$$\bar{v}(t,\delta) = \Pi - \left(\frac{\delta}{\delta + \ldots + \delta^{t}}\right)\frac{g}{m} \quad \text{for all} \quad \delta \geq \underline{\delta}(t) .$$

Proof: Let $\underline{\delta}(t) \in (0,1)$ solve

$$g(1-\delta) = m(\lambda\delta)^t\left[\Pi - \frac{\delta}{\delta + \ldots + \delta^t}\frac{g}{m}\right] .$$

Note that the right hand side increases strictly monotonically to $m\lambda^t\left[\Pi - \frac{1}{t}\frac{g}{m}\right]$ as $\delta \to 1$. The latter is strictly positive by the assumption that $\Pi - (g/m) > 0$; $\underline{\delta}(t)$ is therefore well-defined. Let

$$\bar{v} = \Pi - \left[\frac{\delta}{\delta + \ldots + \delta^t}\right]\frac{g}{m} . \tag{9}$$

We will show that for $\delta \geq \underline{\delta}(t)$, $W = \{0,\bar{v}\}$ is a self-generating set, and hence, by (R1), $\bar{v} \in V(t,\delta)$. As usual, $0 \in B(\{0,\bar{v}\})$. Let $\underline{\theta} = (0, \ldots, 0)$. To obtain $\bar{v}$, consider $(a;w)$ such that $a_k = c$, $k = 1, \ldots, t$, and $w(\theta) = \bar{v}$ for all $\theta \neq \underline{\theta}$. Let $w(\underline{\theta})$ be defined by:

$$g\delta = \frac{\delta}{1-\delta}\delta^t(\mu-\lambda)\lambda^{t-1}(\bar{v} - w(\underline{\theta})) . \tag{10}$$

For $\delta \geq \underline{\delta}(t)$, $w(\underline{\theta}) \geq 0$. Hence $(a;w)$ is admissible with respect to W if for all $K \subseteq \{1, \ldots, t\}$, $\sum_{k=1}^{t} \delta^k g(b_k, a_k) \leq \frac{\delta}{1-\delta}\delta^t \sum_{\theta} \Delta(\theta|b,a)w(\theta)$, where

$$b_k = \begin{cases} d & \text{if } k \in K \\ a_k & \text{otherwise.} \end{cases}$$

This reduces to

$$\sum_{k\in K} \delta^k g \leq (\mu^\ell\lambda^{t-\ell} - \lambda^t)\frac{\delta}{1-\delta}\delta^t(\bar{v} - w(\underline{\theta})) \tag{11}$$

where $\ell = |K|$. Clearly $g \sum_{k \in K} \delta^k \le g \sum_{k=1}^{\ell} \delta^k$. Also, since $\mu > \lambda$, and $\delta \in (0,1)$, (10) implies

$$g\delta^k \le (\mu-\lambda)\mu^{k-1}\lambda^{t-k}L = (\mu^k\lambda^{t-k} - \mu^{k-1}\lambda^{t-(k-1)})L$$

where $L = \frac{\delta}{1-\delta}\delta^t(\bar{v} - w(\underline{\theta}))$. Hence

$$g \sum_{k=1}^{\ell} \delta^k \le \left(\sum_{k=1}^{\ell} \mu^k\lambda^{t-k} - \sum_{h=0}^{\ell-1} \mu^h\lambda^{t-h} \right) L = (\mu^\ell\lambda^{t-\ell} - \lambda^t)L .$$

Thus (10) implies (11), and $(a;w)$ is admissible with respect to W . Finally,

$$E(a;w) = \Pi(\delta + \ldots + \delta^t)\frac{(1-\delta)}{\delta} + \delta^t\bar{v} - \delta^t\lambda^t(\bar{v} - w(\underline{\theta})) .$$

By (10), $\delta^t\lambda^t(\bar{v} - w(\underline{\theta})) = \frac{g}{m}(1-\delta)$. Hence $E(a;w) - \bar{v} = \Pi(1 - \delta^t) - \frac{g}{m}(1-\delta) - (1 - \delta^t)\bar{v}$, where we use $(1-\delta)/(1 - \delta^t) = \delta/(\delta + \ldots + \delta^t)$. It follows from (9) that the right hand side equals 0. Hence $E(a;w) = \bar{v}$, and $\{0,\bar{v}\}$ is a self-generating set. By Lemma 9 we are done. Q.E.D.

It seems plausible, on first considering the problem, that informational delays would make collusion more difficult to sustain, because deviating is more attractive. For example, if $t = 10$, a player can cheat in period 1 and not face any negative consequence (in expected terms) until period 11. Moreover, he can now cheat not once, but ten times in a favorable environment: others' actions are independent of his own in the first 10 periods. Admittedly one could do a "joint test," at the end of period 10, of the hypothesis that the players cooperated in all periods. But they still must be deterred, as in the "no delay" case, from cheating in any single period; what point could there be in confounding 10 different incentive problems, when there is no statistical interdependence to exploit? An immediate implication of Proposition 10 is that there _are_ advantages to delay: when δ exceeds $\underline{\delta}(t)$, maximal S.S.E. payoffs in $G^{\infty}(t,\delta)$ strictly exceed those in $G^{\infty}(k,\delta)$ for any $k = 1,\ldots, t-1$.

Proposition 11: For any fixed $\delta \geq \underline{\delta}(t)$,

$$\bar{v}(t,\delta) > \bar{v}(k,\delta) \quad \text{for all} \quad k = 1, \ldots, t-1 .$$

To understand the benefit from delay, consider the problem of designing a "pseudo-equilibrium" in which it is necessary only to deter cheating in the _first_ period of each stage (all other potential deviations may be ignored). Proposition 1 can be adapted to show that the best pseudo-equilibrium payoff is $\Pi - [(1-\delta)/(1-\delta^{t})]\cdot(g/m)$. But this is exactly what Proposition 10 establishes the true equilibrium payoff $\bar{v}(t,\delta)$ to be. Thus, _with monitoring delays it is "as if" one needed to deter only the first potential defection_. In fact, this is how the optimal scheme constructed in proof works. The implicit future reward offered at the end of t periods is always $\bar{v}(t,\delta)$, _unless_ there were failures in _all_ t periods. The punishment in the latter case must certainly be severe enough to deter a player from cheating in period 1, and is an efficient way of providing that deterrent. But the punishment treats the various periods symmetrically, so cheating in period 2, for example, is equally dangerous, whereas the gain is realized one period later than that from a period 1 cheat. So having deterred the first deviation, one has strictly deterred all other single deviations. Multiple deviations are even less attractive: if $k > 1$ and a player has cheated $k-1$ times, cheating a k^{th} time increases the probability of t failures more than does a single deviation.

A second corollary to Proposition 10 notes that if players are sufficiently patient, choosing t large admits equilibria with payoffs approximately the first-best.

Proposition 12: For all $\varepsilon > 0$ there exist $\delta_\varepsilon \in (0,1)$ and T such that

$$\bar{v}(T,\delta) \geq \Pi - \varepsilon \quad \text{for all} \quad \delta \geq \delta_\varepsilon .$$

We shall contrast this below with the well-known asymptotic ineffi- ciency results for repeated partnership games. Before doing so we remark that it is erroneous to explain the Corollary above by saying that when one "saves up" signals over many periods, the test eventually performed involves a very high likelihood ratio, and hence is very efficient. This would be appropriate if the players' only options were to cheat in all periods, or not at all. On the contrary, single deviations must be discouraged, and the associated likelihood ratio is not extremely high. The efficiency is explained instead by a feature of the "global deterrent" used: punishing the single event "failure in all periods" is effective in deterring every possible pattern of cheating, but "costs" no more than deterring a single deviation in period 1.

Setting $t = 1$ in Lemma 9 gives a final corollary which asserts that regardless of δ , without temporary suppression of signals the efficiency loss caused by imperfect monitoring is at least g/m .

Corollary: For all $\delta \in (0,1)$,

$$\bar{v}(1,\delta) \leq \Pi - \frac{g}{m} .$$

This recalls the asymptotic inefficiency results of Radner, Myerson and Maskin [1986] and Fudenberg and Maskin [1986b], which apply to models such as that of this section, but with $t = 1$. This point can be summarized by saying that if there is an upper bound on the likelihood ratios of potential "punishment regions" in the signal space of the period game, and if carrying out a punishment has an efficiency cost,[3] then there is an inescapable efficiency wedge which neither repetition nor patience serves to remove. We were surprised to find that this source of inefficiency could be avoided simply by impeding the flow of information. While it is plausible that in practice, delaying the release of information may be less costly than improving the quality of the signal, for example, how easily this can be accomplished will depend on the institutional details of a particular partnership.

4. A Folk Theorem for Information Delays

The benefits of delaying the release of information, and the phenomenon of global deterrents, are not features special to the example studied in Section 3. The arguments used there can be generalized in a straightforward manner. We emphasize this by proving a "folk theorem" which

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[3]Important work by Radner and Williams [1987], Matsushima [1987] and Fudenberg and Levine [1988] indicates that inefficiencies from imperfect monitoring need not be substantial if the players affect the signal differently from one another.

extends Proposition 12 to general symmetric games. As the ideas are similar to those already encountered, we proceed rapidly, leaving details to the reader.

We stick as far as possible to the notation of the previous section, redefining terms only when their natural extensions are ambiguous. The period game is still symmetric, differing from that of Section 3 in that players now choose from an arbitrary finite action set $A = \{a^1, \ldots, a^H\}$ and signal realizations lie in some finite set $\{\theta^1, \ldots, \theta^L\}$. Denote by $p^j_{hl} = p(\theta^\ell | (a^h, a^j \cdot e_{n-1}))$ the probability of the signal θ^ℓ arising when $n-1$ players use action a^j and a single player uses action a^h. Let $p^j_{h\cdot} = (p^j_{h1}, \ldots, p^j_{hL})$. We assume that:

(1) The period game has at least one symmetric Nash equilibrium. The expected payoff of the worst of these is normalized to zero.

(2) For all $j = 1, \ldots, H$, $p^j_{j\cdot} \notin co\{p^j_{i\cdot} | i \neq j\}$.

Condition (2) is a statistical distinguishability assumption. It requires that the probability distribution over signals when all players play action a^j is distinguishable from the probability distribution associated with any single-player mixed strategy deviation from this symmetric profile. When Condition (2) fails, there is some mixed strategy deviation that can never be detected by any statistical test; if such a deviation is profitable, it cannot be deterred. The condition is in the spirit of, though weaker than, the "full-rank condition" of Fudenberg and Maskin [4].

The definition of a stage game of length t, and in particular, the information structure assumed, are as in the previous section. Let $U^* = co\{u_1(a^h \cdot e_n) | h = 1, \ldots, H\} \cap R_{++}$. We show that any $u \in U^*$ may be approximated arbitrarily closely as a payoff of the repeated stage game by taking the stage game to be long enough, and the discount factor close enough to one. This result generalizes Proposition 12.

Proposition 13: For all $u \in U^*$ and $\varepsilon > 0$ there exist $T \in N$ and $\underline{\delta} \in (0,1)$ such that for all $\delta \geq \underline{\delta}$,

$$v \geq u - \varepsilon \quad \text{for some} \quad v \in V(T,\delta) .$$

Proof: Assume for the moment that $u = u(a^j \cdot e_n)$ for some $a^j \in A$. Since by assumption $p^j_{j\cdot} \notin co\{p^j_{i\cdot} | i \neq j\}$, $p^j_{j\cdot}$ can be separated by a hyperplane: there exists $\beta^j \in R^L$ such that $\beta^j \cdot p^j_{j\cdot} < \beta^j \cdot p^j_{i\cdot}$ for all $i \neq j$. Furthermore since $p^j_{h\cdot} \in \Delta^{L-1}$, the L-1 dimensional simplex, for all h, we may w.l.o.g. take $\beta^j \in \Delta^{L-1}$. Let $P^j = \beta^j \cdot p^j_{j\cdot}$ and $Q^j_i = \beta^j \cdot p^j_{i\cdot}$. Let $i(j) \in \arg\max_{i \neq j} g^*(a^i, a^j) + (Q^j_i - P^j)/P^j$, and $z_j = g(a^{i(j)}; a^j) + (Q^j_{i(j)} - P^j)/P^j$. We prove the result by showing that for all $t \in N$ for which $v = (u - [\delta/(\delta + \ldots + \delta^t)]z_j) > 0$, there exists $\underline{\delta}^j(t)$ such that $v \in V(t,\delta)$ for all $\delta \geq \underline{\delta}^j(t)$. Let $\underline{\delta}^j(t)$ solve

$$\delta(g^*(a^{i(j)}; a^j)) = \delta^t(Q^j_{i(j)} - P^j)(P^j)^{t-1} \frac{\delta}{1-\delta} v .$$

We argue that for $\delta \geq \underline{\delta}^j(t)$, $W = \{0,v\}$ is a self-generating set. As usual, it suffices to show that $v \in B(\{0,v\})$. Let $e \in [0,1]$ solve:

$$\delta(g(a^{i(j)}, a^j)) = \delta^t(Q^j_{i(j)} - P^j)(P^j)^{t-1} \frac{\delta}{1-\delta} ve . \qquad (12)$$

(We have implicitly assumed $z_j > 0$. The case $z_j = 0$ is trivial.) Let $\ell(\theta_k)$ be the index of the signal in period k. Consider $(a; w^j)$, where $a_k = a^j$, $k = 1, \ldots, t$ and $w^j(\theta_1, \ldots, \theta_t) = v(1 - e \times_{k=1}^{t} \beta^j_{\ell(\theta_k)})$.

It will be convenient to think of $w^j(\theta)$ as the expected payoff of the lottery which yields 0 with probability $e \prod_{k=1}^{t} \beta^j_{\ell(\theta_k)}$ and v otherwise. First observe that:

$$E(a;w) = u(\delta + \ldots + \delta^t)\frac{(1-\delta)}{\delta} + \delta^t\left[1 - e(P^j)^t\right]v \ , \tag{13}$$

where we have substituted:

$$e \sum_{\theta\in\Theta} \prod_{k=1}^{t} \beta^j_{\ell(\theta_k)} P^j_{j\ell(\theta_k)} = e \prod_{k=1}^{t} \sum_{\ell=1}^{L} \beta^j_{\ell} P^j_{j\ell} = e(P^j)^t \ ,$$

the probability of obtaining a zero payoff at the end of the stage, given w^j and given that players play a^j in every period. From (12) and the definition of z_j, $\delta^t e(P^j)^t v = (1-\delta)z_j$. Hence (13) may be rewritten

$$E(a;w) - v = u(1 - \delta^t) - (1-\delta)z_j - (1 - \delta^t)v \ .$$

The definition of v implies that the right hand side is zero. Hence $E(a;w) = v$.

To establish that $(a;w)$ is admissible, we therefore need to show:

$$\sum_{k=1}^{t} \delta^k g^*(b_k;\ a_k) \le \frac{\delta}{1-\delta}\, \delta^t \sum_{\theta} \Delta(\theta|b,a)w^j(\theta)$$

for all $b \in A^t$. Let i_k be defined by $b_k = a^{i_k}$. The above inequality reduces to:

$$\sum_{k=1}^{t} \delta^k g^*(a^{i_k};\ a^j) \le \delta^t\left(\prod_{k=1}^{t} Q^j_{i_k} - \prod_{k=1}^{t} P^j\right)\frac{\delta}{1-\delta}\, ve \ . \tag{14}$$

Since $Q_i^j \geq P^j$, (12) and the definition of $i(j)$ imply

$$\delta^k g^*(a^{i_k};\ a^j) \leq \delta^t (Q_{i_k}^j - P^j)\left(\mathop{\times}_{s=1}^{k-1} Q_{i_s}^j\right)(P^j)^{t-k} \frac{\delta}{1-\delta}\, ve .$$

Hence

$$\sum_{k=1}^{t} \delta^k g(a^{i_k},\ a^j) \leq \delta^t \left[\sum_{k=1}^{t}\left(\mathop{\times}_{s=1}^{k} Q_{i_s}^j\right)(P^j)^{t-k} - \sum_{k=1}^{t}\left(\mathop{\times}_{s=1}^{k-1} Q_{i_s}^j\right)(P^j)^{t-(k-1)}\right]\frac{\delta}{1-\delta} ve$$

which implies (14), as required. This completes the argument for the case $u = u(a^j \cdot e_n)$. Now suppose $u = \sum_{j=1}^{J} \tau_j u(a^j \cdot e_n)$ where $(\tau_1, \ldots, \tau_J) \in \Delta^{J-1}$. Now we simply use the admissible pair $(a;\ w^j)$ where $a_k = a^j$, $k = 1, \ldots, t$, with probability τ_j and we obtain

$$v = \Sigma\, \tau_j \left[u(a^j) - \frac{\delta}{\delta + \ldots + \delta^t} z_j\right] = u - \frac{\delta}{\delta + \ldots + \delta^t} \Sigma\, \tau_j z_j .$$

Q.E.D.

5. Conclusion

Extrapolation from repeated games with perfect monitoring might lead one to expect that in repeated partnerships with moral hazard, collusive possibilities deteriorate when players can change their actions less frequently, or when information is temporarily suppressed. The partnership models explored in this paper allow the period length (over which actions are constant) and the time for which signals are suppressed to be varied independently; the results reveal that the analogy to the perfect monitoring case is treacherous.

Two effects must be considered in studying how increases in period length or the interval of information suppression affect the ability of partners to sustain cooperative behavior. The first effect – and the only important one for games with perfect monitoring – is that increases in either of these variables cause punishments to be delayed, and therefore (in a world with discounting) less likely to deter deviations from the equilibrium. This effect favors quick observation and short periods of action. The second effect is that multiplying the period length or the duration of information suppression reduces the set of strategies – and in particular the set of deviation strategies – available to a player. This effect may favor delayed observation and long periods of action. In our formulation, long periods of action make it easier to distinguish statistically between random outcomes arising from desired cooperative behavior and the systematic deviations associated with cheating. Depending on the balance of the two effects, this improved information may allow a punishment region with a higher likelihood ratio to be used, which decreases the amount of waste in equilibrium.

Lengthening the duration of signal suppression also reduces inefficiencies from imperfect monitoring (if the players are fairly patient), but not by changing the likelihood ratios of the punishment region. What we showed in Section 3 was that if the players are sufficiently patient, one of the efficient ways of deterring a deviation during the first action period in an information interval is also sufficient to deter <u>any</u> pattern of cheating in that interval. As the information interval grows longer, more deterrence is achieved for the one low price. Even in rather general symmetric partnership games, such "global deterrents" allow the first-best payoffs to be approximated when the discount factor is near unity.

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