4-1-1984

Optimal Taxation of Capital Income in Economies with Identical Private and Social Discount Rates

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COWLES FOUNDATION DISCUSSION PAPER NO. 699

OPTIMAL TAXATION OF CAPITAL INCOME IN ECONOMIES

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April 1984

ABSTRACT

The optimal capital income tax is analyzed in the framework of inter-
temporal efficient taxation. The relation between the zero tax in the long-
run and the equality between private and social discount rates is emphasized.
The properties of the dynamic second best path described for a specific
example (convergence to a steady state and values of the capital income tax
in the transition). The case where wealth is a specific utility argument
is also considered.

I. Introduction

The non taxation of interest income has often been advocated
for two reasons. Such a policy would stimulate savings and reduce inter-
temporal welfare distortions. Both arguments are examined by Feldstein
(1978a, b), who shows that they cannot be supported unambiguously in models
with individual intertemporal optimization and exogenous factor prices.

The ambiguous conclusion of the welfare analysis is to be expected
from the standard results of optimal taxation. Pestieau (1974) shows that
in a simple general equilibrium of overlapping generations, the optimal ratio
between the interest tax and the wage is determined in the steady state,
by the Ramsey rules (Diamond and Mirrlees, 1971).[1] This ratio depends on
the properties of the lifetime utility function. An important and well
known feature of the life-cycle model is that the rate of return of capital
in the steady state is not in general, equal to the social discount rate.
This equality between the market rate and the social discount rate
may be generated in the long-run steady state, by an operative bequest
motive.

Auerbach (1979), considers among other issues, the second best prob-
lem of the interest tax in an overlapping generation model with population
growth. The welfare criterion is the life-time utility in the steady state.
With an operational bequest motive, the long-run supply of savings is in-
finite, and the rate of return is equal to the social discount rate (which
under the steady state welfare criterion, is equal to the population growth
rate).

A purpose of this paper is to show that this result is very general
and in particular, is valid even if the private discount rate is endogenous
(and the supply of private savings is not infinitely elastic at a given

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value of the rate of return. When the social discount rate is different
from the population growth rate, the method of comparative statics between
steady states is not an appropriate approach. It is replaced here by a
criterion which measures welfare over the entire dynamic path. The fiscal
instruments are the interest rate and other distortionary taxes (typically
the wage tax). The government deficit (or surplus), may be used as fiscal
instrument or may be constrained to be equal to zero. The main result is
that when the second-best dynamic path converges to a steady state where
the private and the social discount rates are identical, the tax rate on
interest income is equal to zero in the long-run steady state.

The terms private and social discount rates have a precise meaning
only in specific frameworks. Such contexts are provided by the models
considered in the paper. However, in order to see that the argument may
apply beyond the framework of these specific models, it may be useful to
give first a heuristic presentation.

Consider an economy with one produced good which can be used for
consumption or as capital input. Represent by λ_t the current value of
the <u>social</u> marginal value of capital in the economy in period t , and
μ_t the current <u>social</u> marginal value of the public debt. An increment
of the debt does not change the real wealth of the economy. It only in-
creases the obligation of the government to the private sector which has
to be repaid by a distortionary tax. The absolute value of μ is there-
fore equal to the marginal efficiency cost of taxation. Its sign is negative.

Following an intuitive argument similar to that of Bradford (1975),
there is a relation between the values of λ_t , λ_{t+1} and μ_{t+1} :

$$\lambda_t = \delta_t (\lambda_{t+1}(1 + r_{t+1}) - \mu_{t+1}(r_{t+1} - \overline{r}_{t+1})) \; ,$$

where r_t is the rate of return gross of tax, to capital in period t , $\bar{r}_t$ is the rate of return net of tax, and δ_t is the <u>social</u> discount factor between periods t and $t+1$. This relation has a simple interpretation: an exogenous marginal increment of capital in period t increases the quantity of available goods at time $t+1$ by the amount $1+r_{t+1}$, which has a social marginal value λ_{t+1} . In addition there is an increase of tax revenues equal to $r_{t+1} - \bar{r}_{t+1}$, which enables the government to reduce the level of the debt by the same amount. The reduction of the deadweight loss is equal to $-\mu_{t+1}(r_{t+1} - \bar{r}_{t+1})$. The sum of these two effects in period $t+1$ is discounted back by the discount factor δ_t , and determines the marginal value λ_t .

In the steady state, all variables are constant and,

$$\lambda = \delta(\lambda(1+r) - \mu(r-\bar{r})) \ .$$

The optimization of intertemporal resources by the private sector implies that in the steady state,

$$1 = \delta_p(1+\bar{r}) \ , \quad \text{where} \quad \delta_p \text{ is the private discount rate,}$$

When the private and the social discount rates are identical, these two expressions can be combined such that

$$(\lambda-\mu)(r-\bar{r}) = 0 \ .$$

Since $\lambda-\mu$ is positive, the net rate of return is equal to the gross rate, and the interest tax rate is equal to zero.

This intuitive argument is formalized in the next section with a dynamic model of an infinitely lived individual. The utility function is of the type introduced by Koopmans. Some extensions with more general

assumptions are also discussed.

Since the zero interest tax is a long-term (steady state) result, the convergence of the second best dynamic path towards a steady state in the long-run is an essential part of the argument. The standard method in dynamic models of second-best is to assume this convergence and to analyze the fiscal policies in the steady state.[2] In this study the stability of the steady state is analyzed explicitly for a specific class of utility functions. This discussion is presented in Section III and has two by-products.

First the proof of the (local) stability of the steady state suggests a computation method of the dynamic path. Second, on the dynamic path, the interest tax is equal to zero not only asymptotically but for all instants t after some data τ . Before this date the interest tax is bounded by an institutional constraint. This situation raises obviously the issue of time consistency. This problem is more the rule than the exception in dynamic taxation and it raises issues beyond the scope of the present paper.[3]

The equality between the private and the social discount rate is violated when wealth is an argument in the utility function of individuals. This case is analyzed in Section IV. The technical discussion relies on the methods presented in Section III. However, its main argument can be read directly after Section II. All technical parts can be omitted at first reading.

The concluding remarks are presented in the last section.

II. The Optimal Capital Income Tax in the Long-Run

A. The Model

The private sector is represented by a single individual who is infinitely lived. His utility function has the form proposed by Koopmans (1960),[4] and is defined as follows:

$$(1) \qquad J(_tX) = U(x_t, J(_{t+1}X)) \ , \quad t \geq 1 \ ,$$

with the notation:

> x_t is a vector of consumption of produced goods and leisure at time t,
>
> $_tX$ is the program $(x_t, \ldots, x_{t+k}, \ldots)$.

The function U is called the utility aggregator. The most restrictive property which is implied by the above functional form may be the limited non-complementarity between periods.[5] This axiom will be partially relaxed below.

Without loss of generality one can assume that there is a unique produced good, and that the vector x_t represent the levels of consumption of produced goods c_t, labor supply ℓ_t, and government consumption g_t, respectively:

$$(2) \qquad x_t = (c_t, \ell_t, g_t) \ .$$

The unique good can be used for consumption or as capital input in the production technology which is represented by the neoclassical function with constant returns to scale:

$$(3) \qquad y = f(k,\ell) \ .$$

The level of output and capital are equal to y and k , respectively.

The variation of the capital stock between periods is equal to the difference between output and consumption:

(4) $\qquad k_{t+1} = k_t + f(k_t, \ell_t) - c_t - g_t$.

The government finances expenditures with linear taxes on the incomes of capital and labor. An equivalent assumption is that the fiscal instruments are the factor prices net of taxes, of capital and labor in each period, $\overline{r}_t$ and $\overline{w}_t$, respectively. There is no restriction on the variations of the tax rates between periods. However, for a reason which will be clear below, the net rate of return $\overline{r}$ is constrained, by assumption to be nonnegative:[6]

(5) $\qquad \overline{r}_t \geq 0$ for all t .

The government may also match any imbalance between expenditures and revenues by issuing a debt which is perfectly substitutable with capital for the private individual (there is no uncertainty). If a surplus is generated the government redeems the debt or buys capital. The variation of the debt b_t is given by

(6) $\qquad b_{t+1} = (1 + \overline{r}_t)b_t + \overline{r}_t k_t + \overline{w}_t \ell_t - f(k_t, \ell_t) - g_t$

(b_t is negative if the government has a positive wealth and owns capital).

Balanced budget policies can be analyzed in the same framework, with the additional restriction $b_t \equiv 0$, for all t . In the remaining part of this section, one does not need to specify whether the budget is

balanced in every period since this constraint does not affect the main result presented below.

B. The Second Best Problem

In the first period, the government announces the program of tax rates (or net factor prices), and expenditures. The representative agent is endowed with <u>perfect</u> <u>foresight</u> and behaves <u>competitively</u>: although the factor returns are endogenous to the decisions of savings and labor supply, they are considered as unaffected by the agent's actions. For a given path of net factor prices $(\bar{r}_t, \bar{w}_t)_{t \geq 1}$, the representative agent chooses a programme which maximizes the utility function (1), under the budget constraint. This programme satisfies the first order conditions about the intra-temporal and inter-temporal choices, respectively,

$$(7) \qquad \bar{w}_t \frac{\partial U(c_t, \ell_t, g_t, J_{t+1})}{\partial c_t} + \frac{\partial U}{\partial \ell_t}(c_t, \ell_t, g_t, J_{t+1}) = 0$$

$$(8) \quad \frac{\partial U}{\partial c_t}(c_t, \ell_t, g_t, J_{t+1}) - (1+\bar{r}_{t+1})\frac{\partial U}{\partial J_{t+1}}(c_t, \ell_t, g_t, J_{t+1})\frac{\partial U}{\partial c_{t+1}}(c_{t+1}, \ell_{t+1}, g_{t+1}, J_{t+2}) \quad ,$$

where J_t is equal to $J(_t X)$. The levels of J_t and J_{t+1} are related by the equation:

$$(9) \qquad J_t - U(c_t, \ell_t, g_t, J_{t+1}) = 0 .$$

As in the standard atemporal efficient tax problem with one person, the objective of the government is to choose values of the fiscal instruments such that the representative individual's utility is maximized. The constraints are imposed by the private sector's optimizing behavior and by the budget of the government (i.e., the value of b_t discounted at time

1 with the net rate of return, tends to zero when t tends to infinity, or the budget is balanced in all periods).

The second-best problem can be formulated by the maximization of the utility function $U(_1X)$ subject to the above constraints (4)-(9). The endogenous variables are the paths of c_t, ℓ_t, g_t, J_t, $\bar{r}_t$, $\bar{w}_t$, k_t and b_t for $t \geq 1$. Although the government does not directly control the levels of c_t and ℓ_t, the framework is of second-best because the government chooses a program which satisfies the optimization restrictions (7)-(9) of the private agent. When the dynamic path converges to a steady state, the budget constraints of the economy and of the government are satisfied (because of (4) and (5)). Therefore, the budget constraint of the private agent is also satisfied.

It is worth recalling that even in the first-best the dynamic path which maximizes a utility function satisfying Koopman's axioms, does not always converge to a steady state (see Iwai, 1973). However, in this paper the convergence of the second-best program to a steady state will always be postulated. This property is discussed more specifically for a special case in the next section.

Theorem 1. When the represented individual's utility function has the form (1), and the second-best dynamic path converges to a steady state, the tax rate on capital income is equal to zero in this steady state.

Proof. Represent by $\tilde{\lambda}_t$, $\tilde{\mu}_t$ the Lagrange multipliers associated to the constraints (4) and (6) respectively. The Hamiltonian is equal to

$$(10) \quad H = U(_1X) + \sum_{t \geq 1} \tilde{\lambda}_t (-k_{t+1} + k_t + f(k_t, \ell_t))$$

$$+ \sum_{t \geq 1} \tilde{\mu}_t (-b_{t+1} + (1 + \bar{r}_t)b_t + \bar{r}_t k_t + \bar{w}_t \ell_t - f(k_t, \ell_t) - g_t) + A ,$$

where the terms in A do not include k_t as arguments.

We introduce the <u>current value</u> multipliers which are denoted without a tilde:

$$(11) \qquad \tilde{z}_t = \left\{ \prod_{k=1}^{t-1} \frac{\partial U}{\partial J}(x_k, J_{k+1}) \right\} z_t ,$$

where z represents one of the multipliers and x_k represents the vector (c_k, ℓ_k, g_k) .

Differentiating H in (10), the first order condition with respect to k_{t+1} implies that:

$$(12) \qquad \lambda_t = \frac{\partial U}{\partial J}(x_t, J_{t+1})[\lambda_{t+1}(1+r_{t+1}) + \mu_{t+1}(\bar{r}_{t+1} - r_{t+1})] ,$$

where r_t is equal to the marginal product of capital. Denote by δ_t the "discount rate" $\frac{\partial U}{\partial J}(x_t, J_{t+1})$. In the steady state all endogenous variables in (12) are constant over time. Omitting the time subscript,

$$(13) \qquad \lambda = \delta(\lambda(1+r) + \mu(\bar{r}-r)) .$$

In the steady state the intertemporal first-order condition (8) of the private agent, takes the form:

$$(14) \qquad 1 = \delta(1+\bar{r}) , \text{ with the } \underline{same} \text{ value for } \delta \text{ as in (13)}.$$

By combination of (13) and (14),

$$(15) \qquad (\lambda-\mu)(r-\bar{r}) = 0 .$$

One can check readily that $\lambda-\mu$ is the social cost of the following experiment: the government raises an additional unit of revenues (through

indirect taxation, hence the term -μ), in order to purchase a unit of

produced goods from the private agents and destroys it. The term λ-μ is

positive, and $\bar{r}$ is equal to r .   Q.E.D.

C. Applications and Remarks

The central argument in the previous result is the equality between

the social and the private discount rate in (13) and (14) respectively.[7]

This equality implies in the steady state, the zero interest tax result.

The equality between the social and private discount rate is clearly

valid for a larger class than the utility functions introduced by Koopmans.

An important restriction of this functional forms is the separability be-

tween periods (implied by the axiom of limited non-complementarity), which

rules out the formation of habits: the marginal rate of substitutions

between consumption levels in the future is independent of past consumption.

But this axiom can be partially relaxed without altering the above result.

Under the assumption that the consumption in a given period t , does not

affect the marginal rates of substitutions beyond period t+k (with k

fixed), the interest tax is still equal to zero, in the steady state.

Consider for example, the following utility function

$$(16) \qquad J = \sum_{t \geq 1} \beta^t u(x_t, x_{t+1}, g_t) \; ,$$

where x_t represents the vector of consumption and labor supply at time

t . One can easily show that in the steady state, the social and the pri-

vate rates of discount are identical.

The previous discussion has been conducted in a model of a one good

economy, for the sake of simplicity. The result can easily be extended to an

economy with many consumption goods.[8] If ad valorem taxes are available, their

rates are, in general, different from zero under an optimal policy. Assume, for example, that there are two consumption goods and that the function given in (1) takes the

$$(17) \qquad J_t = U(u(c_{t1}, c_{t2}, \ell_t, g_t), J_{t+1})$$

where c_{t1} and c_{t2} represent the levels of the consumption of the two goods at time t. Assume also that the set of fiscal instruments is the wage tax, the tax on the rate of return, and an ad valorem tax on c_2. In the steady state the interest tax rate is nil, and the tax rate on c_2 is, in general, different from zero.

The functional form (17) is particularly interesting because it can be interpreted as the utility function of a family where each individual lives two periods and has an operative bequest motive.[9] In this context, the values of c_{t1} and c_{t2} represent the levels of consumption in the two periods. The combination of the tax on c_2 and of a general tax on the rate of return is equivalent to the combination of a tax on the rate of return for savings consumed during the life-cycle and another tax on bequests. Theorem 1 implies that there should be no inheritance tax in the long-run. However, the intra-generational or internal discount rate (to use a terminology of Diamond, 1973), is in general, different from the inter-generational or external discount rate. If a specific tax can be implemented on the interest income of savings used for life-cycle consumption, its rate is in general different from zero.[10]

It is reasonable from the institutional point of view, to consider the case where no distinction can be made between the savings for life-cycle consumption, and those accumulated for bequest. Theorem 1 implies that the zero tax on bequest overrides the efficient tax or subsidy on

savings for life-cycle consumption. <u>If there is a unique tax rate on
the incomes of savings for life-cycle consumption and bequest, this rate
is equal to zero in the long-run</u>.

One should emphasize that the zero interest tax result does not de-
pend on an infinitely elastic supply of savings. In the present framework,
the discount rate is endogenous and depends on the levels of consumption
and labor supply. In the steady state this rate is equal to the rate of
return. Therefore, the long-run interest elasticity of supply of capital
of the private agent is, in general, not infinite.

The essential property of the equality between the private and the
social rates of discount is violated if the level of wealth enters the
utility function (1) as a separate argument and the private sector puts
a greater premium on holding wealth than the public sector. Then, the
sign of the tax on capital income in the long-run is in general, ambiguous.
This is shown for an example which for technical reasons, is presented
after the next section.

Although the external private and the social discount rates are
always equal for a utility function (1), the zero tax result is in general
not valid when the economy is not in a steady state. It turns out however,
that for a specific class of utility functions the optimal interest tax
is also equal to zero at least for all periods sufficiently distant in
the future. We now turn to this specific class. The more restrictive
framework will also enable us to solve the problem of the existence of a
stable steady state.

III. The Dynamic Path--An Example

In this section the full dynamic path is analyzed for a specific class of utility functions. This context is somewhat restrictive. However it generates a remarkable programme for the interest tax. Also it facilitates the proof of the existence of a dynamic path which converges in the long-run, to a steady state.

For reasons of algebraic simplicity, the time is continuous. The utility function of the private agent is assumed to be of the form:[11]

$$(18) \qquad U = \int_0^\infty e^{-\rho t}\left[\frac{1}{1-\sigma}c_t^{1-\sigma} + L(1 - \ell_t)\right]dt \ ,$$

where ρ and σ are fixed parameters, and L is a concave function ($L'(0)$ is infinite).

For simplicity the level of government expenditures is considered as given and is not an argument in the utility function.[12] The expenditures program $(g_t)_{t \geq 0}$ is assumed to converge to a constant level when time goes to infinity.[13] The fiscal instruments are the tax rates on the rate of return and the wage rate. In addition, the government can issue a debt perfectly substitutable with capital (or save capital). The level of the public debt at time zero is taken as given.

A jump of the level of the debt (of the government to the private agent), at time zero, would be equivalent to a lump-sum transfer between the private agent and the government and is ruled out in this second-best framework.

As in the previous section, an additional restriction is imposed on the admissible fiscal policies: the rate of return on assets, net of taxes, $\bar{r}$, is constrained to be non negative (another lower bound different from

zero, could also be considered). The meaning of this constraint should now be clarified. At the beginning of time, the capital stock is in fixed supply and the constraint $\bar{r} \geq 0$ is obviously binding. Otherwise the efficient policy would be to apply an infinite and negative net rate of return during an infinitesimal interval of time. Asymptotically, this policy is equivalent to a lump-sum taxation, and is obviously efficient.

In general one can expect from economic intuition that the constraint $\bar{r} \geq 0$ is binding during some interval of time starting at zero until time t_1. After time t_1 the net rate of return should converge to the gross rate if the dynamic path converges to a steady state (according to Theorem 1). This description of the dynamic path applies for any utility function of the Koopmans type. The special case of the additivity separable iso-elastic function (18) is remarkable. The transition from the regime of a high interest tax to that of zero tax is instantaneous.

Theorem 2. Assume that the utility function of the private agent is of the form (18) (additively separable and iso-elastic in consumption). If the fiscal policy is efficient, there is a time τ such that for $t < \tau$, the constraint $\bar{r}_t \geq 0$ is binding, and for $t > \tau$, capital income is untaxed $(\bar{r}_t = r_t)$.

Proof. In the framework of continuous time, the expressions which determine the variations of the capital stock and the public debt are equal to:

$$(19) \qquad \dot{k} = f(k,\ell) - c - g \, ,$$

$$(20) \qquad \dot{b} = \bar{r}b + \bar{r}k + \bar{w}\ell - f(k,\ell) - g \, .$$

Call $u(c,\ell)$ the current utility function:

$$u(c,\ell) = \frac{1}{1-\sigma}c^{1-\sigma} + L(1-\ell) \, .$$

Following a standard approach, the private individual's programme imposes the following restrictions on the second-best policy:

$$(21) \qquad u'_c = q$$

$$(22) \qquad u'_\ell = -q\overline{w}$$

$$(23) \qquad \dot{q} = q(\rho-\overline{r}) \ .$$

The variable q represents the private marginal utility of assets. Using (21) and (22), c and ℓ can be substituted in u as functions of $\overline{w}$ and q to generate the function $u(c(\overline{w},q), \ell(\overline{w},q)) = v(\overline{w},q)$.

The current value Hamiltonian of the second-best optimization problem is then equal to:

$$(24) \quad H = v(\overline{w},q) + \xi q(\rho-\overline{r}) + \lambda(f(k,\ell) - c - g) + \mu(\overline{r}b + \overline{r}k + \overline{w}\ell - f(k,\ell) + g) + \nu\overline{r} \ .$$

The efficient policy satisfies the first order conditions:

$$(25) \qquad \dot{\xi} = \rho\xi - \frac{\partial H}{\partial q}$$

$$(26) \qquad \dot{\lambda} = \rho\lambda - \frac{\partial H}{\partial k}$$

$$(27) \qquad \dot{\mu} = \mu(\rho-\overline{r})$$

$$(28) \qquad \frac{\partial H}{\partial \overline{w}} = 0$$

$$(29) \qquad \frac{\partial H}{\partial \overline{r}} = -\xi q + \mu a + \nu = 0 \ , \quad \text{if} \quad \nu > 0 \ , \quad \text{then} \quad \overline{r} = 0 \ ,$$

a is the level of private assets: $a = k+b$.

At time zero, there is no constraint on the level of q , and the associated costate variable ξ is equal to zero. One verifies that the

value of ν is positive (it is equal to $-\mu a$, and the marginal tax excess-burden $-\mu$ is positive), and that the constraint $r \geq 0$ is binding.

Differentiating (29) and using (25), (23), (27), and the equation of private saving $\dot{a} = (\bar{r}-n)a + \bar{w}\ell - c$, the time variation of ν is equal to:

$$(30) \qquad \dot{\nu} = \rho\nu + G ,$$

with[14]
$$G = -q\nu'_q - \lambda(wq\ell'_q - qc'_q) + \mu(w-\bar{w})q\ell'_q + \mu(c - \bar{w}\ell) .$$

Because of the additive separability of u , the labor supply ℓ , depends only on the product $q\bar{w}$, and $\bar{w}\ell'_{\bar{w}} = q\ell'_q$. Using this relation and the condition $H'_{\bar{w}} = 0$, the expression of G can be rewritten:

$$(31) \qquad G = \frac{c}{\sigma}(q - \lambda + \sigma\mu) .$$

Therefore the equation (30) is equivalent to

$$(32) \qquad \dot{\nu} = \rho\nu + \frac{c}{\sigma}J , \quad \text{with} \quad J = q-\lambda+\sigma\mu .$$

From (23), (26) and (27), the time derivative of J is equal to

$$(33) \qquad \dot{J} = J(\rho-\bar{r}) + (\lambda-\mu)(r-\bar{r}) .$$

On an interval of time where the constraint $\bar{r} \geq 0$ is not binding, $\nu \equiv 0$. Therefore the relation (32) implies that $J \equiv 0$, and in (33) $r-\bar{r}$ is equal to zero (because $\lambda-\mu$ is positive).

The constraint $\bar{r} \geq 0$ cannot be binding forever (the marginal utility of private consumption q , would grow to infinity since $q = q(\rho-\bar{r})$, which is absurd). Call τ the lowest value of t for which the constraint

$\bar{r} \geq 0$ is not binding. To conclude the proof, it remains to show <u>ad absurdum</u> that $\bar{r} = r$ for $t > \tau$.

Assume now that there is an interval (t_1, t_2) such that on this interval, $\bar{r} \geq 0$ is binding, and that t_1 is the lowest value of t <u>greater than</u> τ , for this to occur. At time t_1 , $v = 0$ and $\dot{v} > 0$. In the relation (32), this implies that J is strictly positive, which is impossible since J is a continuous function of time and before time t_1 , J is equal to zero.   Q.E.D.

The previous result shows that there are two regimes for the interest tax. The policy is either to tax as much as possible or not at all. The tax has two effects. It is a tax on existing capital but it also introduces intertemporal distortions in saving. The lump-sum effect of the capital tax overrides the savings distortions for relatively small values of t $(t < \tau)$. In the second regime, the savings distortions become the predominant factor. The constraint $\bar{r} \geq 0$ is no longer binding. In fact there is no interest tax and the government generates revenues only from taxing wage income (or other commodities in a multi-good economy).

The duration of the first regime depends on the excess-burden of these other taxes. Typically, when the value of the excess-burden increases, the duration of the first regime τ , increases: the government extends the "tax recovery" period, accumulates wealth (or decreases the public debt), in order to lower the total discounted value of revenues generated in the second regime.[15]

The marginal value of the excess-burden is equal to the social value of the level of the public debt when the latter is optimized, i.e., to the shadow price μ . It is simpler to express this value in terms of private consumption, by the ratio μ/q . The comparison of the inter-temporal

first-order conditions of the private sector (21), and of the government (25), shows that they are identical, and that the ratio μ/q is constant over time:

(34) $\frac{\mu}{q} = \phi$ is constant over time.

This relation is simply an application of the principle according to which the government debt is used to smooth out variation of the excess-burden between periods.[16]

According to economic intuition, the value of ϕ should depend on the initial level of public debt. If the government owns initially a stock of capital equal to the present value of future expenditures, there is no need for (distortionary) taxation, and the marginal excess-burden ϕ is equal to zero. In the standard case, the initial level of public wealth is smaller than the value of expenditures. Taxation is necessary and the value of ϕ is negative (an increment of initial public debt generates a loss of social welfare). This relation between the values of marginal excess-burden ϕ and the public debt is made more explicit in the following results. For technical reasons, one has to make a distinction between the two regimes described above. Consider first the second regime which occurs after time τ .

Theorem 3. Let the marginal value of the excess-burden ϕ and of the capital stock at time τ , k_τ (where τ is defined in Theorem 2) be given. If the level of k_τ is sufficiently close to the steady state value of the capital stock, there is a unique value of the debt at time τ , b_τ , such that the dynamic path of second-best converges to the steady state, when t increases from τ to infinity.

The Theorem is proven in the Appendix by determining the signs of the eigenvalues of the dynamic system linearized at the steady state.

To complete the solution of the second-best problem, consider now the first regime in the time interval $(0,\tau)$. Assume first that the values of ϕ , k_τ and τ are given (and that the condition in the Theorem 3 applied). There exists a unique set of values $(q_\tau , k_\tau , b_\tau , \xi_\tau , \lambda_\tau)$ such that the dynamic path converges to a steady state after time τ . The integration of the system of dynamic equations (23), (19), (20), (25), (26), with the condition (28), back from time to time zero, determines the initial values (q_0 , k_0 , b_0 , ξ_0 , λ_0),[17] which depend on the given values of ϕ , k_τ , and τ . The three constraints on the fixed values of k_0 , b_0 and ξ_0 [18] provide the additional equations which determine the values of ϕ , k_τ and τ .[19] This completes the determination of the dynamic path.

It may be interesting at this point to have a rough estimate of the duration of the regime with interest taxation. Assume for tractability, that the gross rate of return r is equal to the discount rate ρ ,[20] and that the value of σ is equal to one. The value of τ is equal to[21]

$$(35) \qquad \tau = \frac{1}{\rho} \log \left[1 + \sqrt{-2\phi \, \frac{\rho a_0}{c_\tau}} \right] ,$$

where a_0 is the level of private assets at time zero, and c_τ is the level of consumption at time τ . If $r = \rho$, the value of c_τ is the same as in the steady state. Taking a ratio $(\rho a_0/c_1)$ equal to 1/4, ρ = 4% (per year), and $\phi = -1/4$, the value of τ , is approximated by

$$(36) \qquad \tau \sim \frac{1}{\rho} \sqrt{-2\phi \left(\frac{\rho a_0}{c_1} \right)} = 8.8 \text{ years.}$$

Of course this computation is only illustrative and the values of ϕ ,
c_τ and τ should be determined simultaneously. It indicates, however,
that the length of the period with capital income taxation at the 100 per-
cent rate can be non-negligible.

IV. The Wealth Effect

The fundamental result of the paper does not hold when there is a specific wealth argument in the utility function of the private sector.[22] To see this, consider an extension of the additively separable utility function given in (18), and assume that it now takes the form:

$$(18a) \quad U = \int_0^\infty e^{-\rho t} \left[\frac{1}{1-\sigma} c_t^{1-\sigma} + L(1 - \ell_t) + \frac{A}{1-\gamma} a_t^{1-\gamma} \right] dt \ ,$$

where A is a constant. Also, the government budget is balanced at each instant and there is no public debt. For simplicity, government expenditures have a constant level through time.

Theorem 4: Assume that the utility function of the private agent is of the form (18a), and that the fiscal instruments are the tax rates on capital and labor income. Then, in the long-run steady state, the sign of the tax rate on capital income is identical to the sign of the difference $\gamma-\sigma$. This tax rate is equal to zero when γ is equal to σ .

The proof is presented in Appendix 3.[23]

This result is in agreement with the standard intuition of the optimal taxation. Consider the levels of assets a_t in the utility function (18a) as separate goods.[24] When the value of γ increases with respect to σ , the demands for these goods become relatively less elastic with respect to the demands for consumption at all dates, and these "goods" should be taxed more heavily.

V. Conclusion

The main result of this paper is the non taxation of capital income in the long-run. This result follows from the equality between the inter-temporal marginal rates of substitution for consumption in the private and the public sector, respectively. This property is satisfied by most of the standard models where representative families have a utility function which is separable between the programs of different generations (these families do not necessarily have the same endowments). Also wealth does not produce utility per se, but is valued only because it is a claim on future resources.

The property of separability is somewhat restrictive, but it does not have to be additive, or it can apply only between generations suffi-ciently distant from each other. Indeed, the problem of the existence of a long-run steady state for a general form of utility functions with an infinite horizon is at the present time, an open question even in the first best situation.

The result presented here obviously does not contradict the standard views on second best taxation in the life cycle framework, which depend on the properties of the life-cycle utility function, when the number of fiscal instruments is sufficient. But if there is a uniform tax on capital income and agents have an operational bequest motive, the issue of long-run effi-ciency between generations prevails over the efficiency problem in the life-cycle and the (long-run) optimal interest tax is nil.

In general, the interest tax is equal to zero only asymptotically, in the long-run. However for a specific class of utility functions, the result applies for all instants after a specific date (which depends on all the parameters of the problem), although the economy is not in a steady

state. Before this date, the optimal tax rate is only limited by insti-

tutional constraints. As most second best dynamic policies, this fiscal

program is time-inconsistent, an issue which is beyond this

study.[25]

The zero tax result is invalid if individuals enjoy wealth per se.

Whether this effect induces a tax or a subsidy depends as usual on the

properties of the utility function. In a simple example we have seen that

standard intuition is correct, i.e., wealth is taxed only if the demand

for wealth per se is relatively inelastic with respect to that of consump-

tion of produced goods.

FOOTNOTES

[1]A crucial assumption in Pestieau (1974) is that the public debt (with the same return as capital), is a fiscal instrument. For a further analysis with the case of balanced budget, see Atkinson and Sandmo (1980).

[2]See, for example, Turnovsky and Brock (1980).

[3]In other terms it is assumed here that the government is commited to·a fiscal policy once it is announced.

[4]See also Koopmans et al. (1964).

[5]See Koopmans (1965). The other important property which is implied by (1) is the stationarity of the utility function. It seems less restrictive in the present context since the main emphasis is on steady state results.

[6]One could also assume $r \geq M$, with an arbitrarily fixed value for M .

[7]The two rates of discount are always identical for the functional form given in (1). They are identical only in the steady state for the functional form in (16) below.

[8]In the same way with many capital goods, there is no capital income tax in the steady state.

[9]Note that the bequest motive arises because the utility level J_{t+1} is an argument of U . If the level of bequest is an argument (as in Atkinson, 1971), the result does not hold.

[10]In the special framework of two-period lives with the consumption of the old at time zero taken as given, and no constraint of the public debt, the optimal tax rates are determined by the Ramsey rules (see Pestieau, 1974, or Atkinson and Sandmo, 1980).

[11]Without loss of generality, the population is constant.

[12]As another possible simplification, one can assume that government expenditures enter the function U as additively separable terms.

[13]This convergence could be proven when g_t is an argument of U , and the government optimizes the level of expenditures.

[14] A derivative as v'_q represents the partial derivative of v with respect to q .

[15] This is shown explicitly in an example given below (expression (36)).

[16] This principle has been analyzed by Barro (1979) in the framework where the tax excess-burden is at each instant, an _ad_ _hoc_ quadratic loss function which depends on the revenues raised at the same instant.

[17] The value of μ is equal to $q\phi$.

[18] The value of the shadow price ξ_0 is equal to zero since there is no constraint on the value of q at time zero (the equation (23) is only a constraint on the _variation_ of q).

[19] This argument relying on the equality between the numbers of unknowns and equations may not convince the scrupulous reader. In the special case of fixed exogenous factor prices and $\sigma = 1$, and can indeed show rigorously (the proof is available from the author), that there is a unique path converging to the steady state (the stability result is global in this case).

[20] This assumption is approximately true when the economy is near the steady state. When r is greater than ρ the result given below overestimates the value of τ .

[21] See Appendix 2 .

[22] For an analysis of first best policies in this context, see Kurz (1968). The uniqueness and stability problems raised by Kurz do not arise for the example which is considered here.

[23] The proof also shows that the result is still valid when the values of σ and γ are not fixed but depend on the levels of c and a , respectively.

[24] Of course this is an imaginary experiment, since assets produce utility both as direct arguments in the function (18a), and as claims to future consumption.

[25] The time-consistency problem is not solved if one assume that by institutional constraints, there is no interest tax before instant τ , where τ is chosen sufficiently large.

APPENDIX

1. Proof of Theorem 3

The stability analysis is simplified when the level of labor ℓ, is substituted to the net wage rate $\bar{w}$, as a control variable. The net wage rate is defined implicitly as a function of ℓ and q, by the first order condition of the private sector:

$$(37) \qquad L'(1-\ell) = q\bar{w} .$$

By concavity of L, $\bar{w}'_\ell$ is positive. As shown in the text, when $\mu = \phi q$, ϕ is independent of time, and $\lambda = q + \sigma\mu$ (see the proof of Theorem 2).

The Hamiltonian is now equal to

$$(38) \qquad H = u(c,\ell) + \xi q(\rho-\bar{r}) + \lambda(f(k,\ell) - c - g)$$

$$+ \mu(\bar{r}b + (\bar{w}-w)\ell + (\bar{r}-r)k - g) ,$$

where $\bar{r}$ is equal to r, c is a function of q, and $\bar{w}$ is a function of ℓ and q (given in (37)). The first order conditions imply that:

$$\begin{cases} \dot{k} = f(k,\ell) - c - g \\[2mm] \dot{q} = q(\rho-r) \\[2mm] \dot{b} = rb + (\bar{w}-w)\ell - g \\[2mm] \dfrac{\partial H}{\partial \ell} = 0 . \end{cases}$$

The last equation defines ℓ implicitly as a function of the state variables k, q and b, and of the constant μ. By differentiation of H through ℓ, one finds the equation:

$$(39) \qquad q(w-\bar{w}) + \mu(\bar{w} - (1-\sigma)w) + \mu\ell\frac{\bar{w}}{1-\ell} = 0 .$$

This equation can be rewritten

$$(40) \qquad (1-\ell)\left[\frac{w}{\overline{w}}(1 - \phi(1-\sigma)) - 1\right] + \phi = 0 \ ,$$

Since w is a function of k and ℓ (in the production technology), and $\overline{w}$ is a function of q and ℓ in (37), the equation (40) is of the form:

$$(41) \qquad B(\ell,k,q) = 0 \ .$$

This equation defines implicitly the value of ℓ as a function of the state variables k and q on the dynamic path. The partial derivatives of B are equal to:[1]

$$(42) \qquad \begin{aligned} B'_\ell &= \frac{\phi}{1-\sigma} + [1 - \ell - \phi]\left[-\frac{1}{\overline{w}\ell'_{\overline{w}}} + \frac{w'_\ell}{w}\right] \\[2mm] B'_k &= [1 - \ell - \phi]\frac{w'_k}{w} \\[2mm] B'_q &= [1 - \ell - \phi]\frac{\ell'_q}{\overline{w}\ell'_{\overline{w}}} \end{aligned}$$

The second order condition $\partial B/\partial \ell < 0$ is assumed to be satisfied.

By linearization near the steady state (where the values of variables are denoted by an asterisk), the dynamic equations become

$$(43) \qquad \begin{pmatrix} \dot{k} \\ \dot{q} \\ \dot{b} \end{pmatrix} = M \begin{pmatrix} k - k^* \\ q - q^* \\ b - b^* \end{pmatrix}.$$

The matrix M has the form

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[1] In these expressions, w'_ℓ is the partial derivative of w with respect to ℓ, keeping k constant. $\overline{w}'_\ell$ is the partial derivative of $\overline{w}$ with respect to ℓ keeping q constant, where $\overline{w}(\ell,q)$ is defined implicitly in (37), and $\ell'_{\overline{w}}$ is the inverse of $\overline{w}'_\ell$.

$$(44) \qquad M = \begin{pmatrix} r + w\ell_k' & w\ell_q' - c_q' & 0 \\ -qr_k' - qr_\ell'\ell_k' & -qr_\ell'\ell_q' & 0 \\ \cdot & \cdot & \rho \end{pmatrix}$$

The partial derivatives of ℓ with respect to k and q are defined

implicitly in the system (42): $\ell_k' = -B_k'/B_\ell'$, and $\ell_q' = -B_q'/B_\ell'$.

A dot represents a non-zero element. ρ is a positive eigenvalue of the

matrix. To prove that the two other eigenvalues have opposite signs, it

is sufficient to show that the first determinant of order two Δ , is negative:

$$(45) \qquad \Delta = -(r + w\ell_k')qr_\ell'\ell_q' + q(r_k' + r_\ell'\ell_k')(w\ell_q' - c_q')$$

Using the relations $qc_q' = -c/\sigma$, $c = rk + w\ell$ in the steady state,

$r_\ell' = -(k/\ell)r_k'$ (by the constant return to scale assumption), after some

manipulations,

$$(46) \qquad \Delta = \frac{cr_k'}{\sigma}\left[1 + \frac{\sigma q}{\ell}\ell_q' - \frac{k\ell_k'}{\ell}\right] .$$

Substitute ℓ_q' and ℓ_k' by the ratios $-(B_q'/B_\ell')$ and $-(B_k'/B_\ell')$ of the

expressions in (42) and use $w' = (k/\ell)w_k'$, to find

$$(47) \qquad \Delta = -\frac{cr_k'}{\sigma B_\ell'}\left[\frac{-\phi}{1-\sigma} + \frac{1 - \ell - \phi}{\frac{w\ell'}{w}}\left(1 + \sigma q\frac{\ell_q'}{\ell}\right)\right] .$$

Since $\phi < 0$, $\frac{\ell'}{w} > 0$, $\ell_q' > 0$, $r_k' < 0$ and $B_\ell' < 0$, the sign of the

determinant Δ is negative.

The matrix M of the linearized system has two positive eigenvalues,

and one negative eigenvalue. Therefore when the value of the constant

variable ϕ is given, for each value of k near its steady level, there

are unique values of q and b such that the dynamic system converges

to the steady state.

2. The Length of the Period of Capital Income Taxation

For tractability, the marginal productivities of capital and labor are fixed: $r = \rho$ (for the existence of a steady state), and $w = 1$ (by normalization). Consider the utility function given in (18), with $\sigma = 1$ The first order condition (26) implies that:

$$(48) \qquad \dot{\lambda} = \rho\lambda - r\lambda - \mu(r-\bar{r}) \ .$$

Using (27), in the interval $(0,\tau)$,

$$(49) \qquad \dot{\lambda} - \dot{\mu} = (\rho-r)(\lambda-\mu) \ .$$

Since $r = \rho$ (this is where the assumption is used), $\lambda-\mu$ is _constant_ over time. But according to (32), $\nu_t \equiv J_t \equiv 0$ for $t \geq \tau$, and therefore $\lambda_\tau - \mu_\tau = q_\tau$. This implies that

$$(50) \qquad \lambda_t - \mu_t = \lambda_\tau - \mu_\tau = q_\tau \ , \quad \text{for } 0 \leq t \leq \tau \ .$$

In the interval $(0,\tau)$, $\bar{r} = 0$, the relation (23) reduces to $\dot{q} = \rho q$, and

$$(51) \qquad q_t = q_\tau e^{\rho(t-\tau)} \ .$$

Substituting $\lambda_t - \mu_t$ and q_t in (33), the expression (32) is equivalent to:

$$(52) \qquad \dot{\nu}_t = \rho\nu_t + 1 - e^{\rho(\tau-t)} \ , \quad \text{for } 0 \leq t \leq \tau \ .$$

Since $\nu_0 = -\mu a_0$ in (29) ($\xi_0 = 0$) , and $\nu_\tau = 0$, the value of τ is found by integration of (52), and is given by the formula (35) in the text.

3. Proof of Theorem 4

The private agent's intertemporal first order condition now takes the form:

(53) $\qquad \dot{q} = q(\rho - \bar{r}) - Ak^{-\gamma}$.

The private wealth is equal to the capital stock because there is no public debt. As in the proof of Theorem 2 in the text (equation (24)), the Hamiltonian can be written:

(54) $\quad H = v(\bar{w}, q) + \frac{1}{1-\gamma} k^{1-\gamma} + \xi(q(\rho - \bar{r}) - Ak^{-\gamma}) + \lambda(f(k,\ell) - c - g) + \mu(\bar{r}k + \bar{w} - f(k,\ell) + g) + v\bar{r}$

The equation $\partial H / \partial k = \rho \lambda - \dot{\lambda}$ becomes, in the steady state:

(55) $\quad Ak^{-\gamma} + A\gamma\xi k^{-\gamma-1} + (\lambda - \mu)(r - \bar{r}) + \lambda(\bar{r} - \rho) = 0$.

Since $\partial H / \partial \bar{r} = 0$ and the constraint $\bar{r} \geq 0$ is not binding,

(56) $\quad \xi = \frac{\mu k}{q}$.

Replacing the value of ξ in (55) and using (53) with $\dot{q} = 0$, the equation (55) is equivalent to:

(57) $\quad (\lambda - \mu)(\bar{r} - r) = \frac{Ak^{-\gamma}}{q}(q + \gamma\mu - \lambda)$

To determine the sign of the right hand side of this equation, consider the first order conditions of the second best program (in the steady state) which are equivalent to $\partial H / \partial q = \rho q$ and $\partial H / \partial \bar{w} = 0$, respectively:

(58) $\quad qv'_q + \frac{\mu k}{q} Ak^{-\gamma} + \lambda(wq\ell'_q - qc'_q) - \mu(\bar{w} - w)q\ell'_q - \mu\rho k = 0$

(where (56) and (53) are used for substitution in the second and the last terms), and

(59) $\quad \overline{w}v'_{\frac{1}{w}} + \lambda w\overline{w}\ell'_{\frac{1}{w}} + \mu\overline{w}\ell - \mu(\overline{w}-w)\overline{w}\ell'_{\frac{1}{w}} = 0$.

Because of the separability of U between c and ℓ , $\overline{w}\ell'_{\frac{1}{w}} = q\ell'_q$. The difference between (58) and (59) is then equivalent to:

(60) $\quad qv'_q - \overline{w}v'_{\frac{1}{w}} + \frac{\mu k}{q}Ak^{-\gamma} - \lambda qc'_q - \mu\overline{w} - \mu\overline{r}k + \mu(\overline{r}-\rho)k = 0$.

Using again (53) with $\dot{q} = 0$ to substitute in the last term, and the equality $c = \overline{r}k + \overline{w}\ell$, to find:

(61) $\quad qv'_q - \overline{w}v'_{\frac{1}{w}} - \lambda qc'_q - \mu c = 0$.

But $v'_q = \frac{\partial u}{\partial c}c'_q + \frac{\partial u}{\partial \ell}\ell'_q = qc'_q - q\overline{w}\ell'_q$, and in the same way, $v'_{\frac{1}{w}} = -q\overline{w}\ell'_{\frac{1}{w}}$. Substituting in (61), with $q\ell'_q = \overline{w}\ell'_{\frac{1}{w}}$, and $c'_q = -\sigma\frac{c}{q}$,

(62) $\quad q = \lambda - \sigma\mu$.

The expression (57) can now be rewritten:

(63) $\quad \overline{r} - r \equiv \frac{-\mu Ak^{-\gamma}}{q(\lambda-\mu)}(\sigma-\gamma)$.

The sign of μ is negative, and the result in Theorem 4 follows.   Q.E.D.

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