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Entrepreneurial Abilities and Liabilities in a Model of Self-Selection

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1. Introduction

According to Schumpeter the function of the entrepreneur is "to reform or revolutionize the pattern of production by exploiting an untried technological possibility."[1] For such a task a superior ability in organizing production processes with predictable outcomes[2] is not sufficient. The main quality of the entrepreneur is to deal with unforeseen events. Because of this uncertainty, the entrepreneurial function has often been associated with risk taking. Frank Knight in his classical study,has emphasized that entrepreneurs and employees can be characterized by different degrees of risk-aversion (using a modern term). This idea has been recently formalized by Kihlstrom and Laffont [1979] in a general equilibrium model of occupational choice. The more risk-averse individuals choose to be employees and receive a fixed wage; the less risk-averse

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[1] Schumpeter [1941, Chapter 12, p. 132].

[2] This term is not restricted to deterministic outcomes; no entrepreneurial talent is necessary to choose between projects described by well determined probability distributions of outcomes.

become entrepreneurs and receive all the (uncertain) profits. The equilibrium is inefficient (in a first best sense). However, there is *a priori*, no reason why capital markets could not insure entrepreneurs and support an efficient equilibrium.

The non insurable risk which has often been considered as the most typical aspect of the entrepreneurial function, may be only a byproduct of two entrepreneurial qualities emphasized by Schumpeter.[3] Entrepreneurs have to produce some effort in "getting things done;" this is related to the problem of moral hazard.

Also, the ability to deal with unforeseen events is difficult to evaluate *ex ante* by outsiders who may not share the same "vision." This situation of asymmetric information seems essential in the entrepreneurial function. In this context the choice of liability form (limited or unlimited) acts as a signal[4] of entrepreneurial abilities to financial institutions which finance production undertakings.

Some entrepreneurial risk prevents individuals of low skill to compete with more able individuals for capital funds and may be socially desirable. Indeed the welfare value of the limited liability institution is *a priori* ambiguous. We address these issues in a simple model of occupational choice.[5]

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[3]The best short summary of the economists' view on the entrepreneurial function may be found in Schumpeter's *History of Economic Analysis*, by checking the references to "entrepreneur."

[4]Since the seminal paper of Akerlof [1970], markets with asymmetric information and the role of signals have been analyzed in various contexts by numerous authors. See for example Spence [1974], Rothschild and Stiglitz [1976], Ross [1977], Jaynes [1978], Wilson [1978, 1979] and Shavell [1980].

[5]Kanbur [1979] analyzes the problem of income distribution in a model of occupational choice with uncertainty.

The model is presented in the next section. The equilibrium with no limited liability is analyzed in Section 3. Equilibria with both liability forms and conditions of existence are considered in Section 4. The institution of limited liability is evaluated from a welfare point of view in Section 5. The results are summarized in the last section.

2. The Model

We consider a one good economy with three types of agents, employees, entrepreneurs, and financial institutions. The economy is divided in two production sectors; in the first sector, production is risk-free, and is a function of the capital and labor inputs. In the second sector we find a large number of small firms; each firm is headed by one entrepreneur. The output of this firm is uncertain, and depends on the input of capital,[6] and on the skill of its entrepreneur. The size of the first (risk-free) sector is assumed to be sufficiently large so that the wage rate and the risk-free rate of return on capital are fixed.

For simplicity, we make the following assumptions: all individuals have the same level of wealth a ; each individual is characterized by the level of his entrepreneurial ability s . Individuals are distributed according to the distribution function $F(s)$. To simplify the exposition we will make the following assumption about F .[7]

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[6]This could be generalized to the case where inputs of capital and labor are required (assuming that entrepreneurs do not default on the wage bill): inputs depend on factor prices. The only factor price which will vary across entrepreneurs is the price of capital, and the utility of an entrepreneur can be written as a function of this price. An example with labor input is given in Section 5.

[7]We could also have considered discrete distribution. The difference between the highest and the lowest skill level will matter for the type of equilibrium.

(A) F has a density function $g(s)$ which is strictly positive for $0 \leq s \leq 1$.

Each individual lives one period. In the beginning of the period he chooses to be an entrepreneur or an employee. The occupations are described as follows:

An entrepreneur invests an amount k in a production process. Two outcomes are possible: if the production process is successful, the output is a function of the capital invested $f(k)$, where $f' > 0$, $f'' < 0$, $f'(k) \to \infty$ if $k \to 0$. If the process is unsuccessful, it is a total loss, and output is equal to zero. The probability of success is equal to the entrepreneur's skill level s.

An entrepreneur can invest his own wealth in his own firm or lend it to financial intermediaries which will pay him the risk-free total rate of return R (principal and interest). He can also borrow from financial institutions at the rate r (which also refers to principal and interest). Loans are made only for investment purposes and there is no rationing.[8] Lenders do not know the skill level of an entrepreneur and can only observe his liability form.[9] The interest rates on loans to firms with limited liability (type L) and unlimited liability (type N) will be called r_L and r_N, respectively. The utility level of an entrepreneur is represented by the von Neumann-Morgenstern utility function:

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[8]Financial institutions can monitor if a loan is used for the purchase of machinery. Entrepreneurs cannot borrow large cash amounts under fictitious corporate names, to buy castles in Europe. For an analysis of the rationing problem, see Stiglitz and Weiss [1981].

[9]When financial institutions can observe the entrepreneur's complete personal portfolio, an equilibrium with full information may exist (an equilibrium of this type has been presented by Ross (1977)). The cost of monitoring may prevent financial intermediaries to collect this information _ex ante_. If the probability of business failure is relatively small, this information may be easier to acquire _ex post_ on the small number of bankrupt firm. This argument is reinforced by the fact that entrepreneurs with unlimited liability do not invest outside of their own firm (as it will be shown below). Financial intermediaries need then only to verify ex post that this investment is equal to zero.

$$(1) \qquad U = su(y_1) + (1-s)u(y_0) \ ,$$

where $u'' \leq 0$, (U is defined for $x \geq 0$), and y_1 and y_0 represent levels of wealth when the production outcome is successful or unsuccessful. These levels depend on the capital invested in the firm, the amounts lent to and borrowed from financial institutions, and on the type of liability chosen by the entrepreneur prior to the investment decision.

For a successful production outcome, $y_1 = f(b+i) - rb + R(a-i)$, where i represents the amount of personal wealth invested by an entrepreneur in his own firm, and b the amount borrowed from financial markets.

When the production outcome is unsuccessful, the income depends on the choice of liability by the entrepreneur. With unlimited liability, the entrepreneur is liable up to his total income: $y_0 = \mathrm{Max}(0, -rb + R(a-i))$. If he chooses limited liability, he is liable only up to the income generated by the firm. Should his venture fail, he keeps the interest and principal of his other financial investments: $y_0 = R(a-i)$.

The other two types of agents can be described very simply: employees have a risk-free activity. Their fixed income is equal to $w + Ra$ and they have the same utility function u as the entrepreneurs. Individuals choose the type of activity (entrepreneur or employee), which provides the highest level of utility.

Financial institutions are risk-neutral (independent risks are fully diversified), and do not make a profit. They pay their depositors the risk-free rate R , and lend these funds to entrepreneurs of types L and N (if they exist) at the rates r_L and r_N , respectively, with the information structure described above. The rates r_L and r_N are computed such that the expected return on a loan (principal and interest, or a fraction of it) is equal to the risk-free rate R .

Finally, in order to ensure that entrepreneurs and financial institutions trade, it is assumed that an individual's wealth is not too large, and that entrepreneurs need to borrow if the production process is worthwhile. More specifically, the level of personal wealth a, satisfies the condition

(B) $$f(a) < w + Ra \; .$$

Under this condition the income of an entrepreneur who does not borrow is always smaller than the income of an employee; such an individual does not exist because $u'' \leq 0$.

Also, the function f is assumed to satisfy the following condition

(C) $$M = f(k) - Rk + Ra > w + Ra \quad \text{with} \quad f'(k) = R$$

This condition has a very simple interpretation: assume that the individual with the highest skill $s = 1$ is identified by lenders. Since he does not face any risk, he can borrow at the risk free rate R. When his investment is optimal, his net income is sufficient to induce him to choose to be an entrepreneur. This assumption is minimal: if it is not satisfied, no enterprise exists in the equilibrium, under any informational structure (and is obviously not socially desirable). The value M will be on upperbound for for all incomes in the equilibrium.

3. Equilibrium without Limited Liability

In order to analyze the effect of limited liability on the allocation of resources, we first have to consider how individuals choose their activity when they are all liable up to their total wealth for production losses.

If an entrepreneur borrows from financial intermediaries, he does not at the same time lend to them because such an investment provides a rate of return R only after all borrowings are repaid (at a rate of return r greater than R). By assumption (B), all entrepreneurs borrow a positive amount b , therefore their income is equal to zero in the case of unsuccessful production; since no income can be claimed by lenders in this situation, the loan is a total loss. From the assumptions made in the previous section about the behavior of financial institutions, we can deduce that the equilibrium interest rate on loans to firms is equal to:

$$r_N = \frac{R}{\bar{s}_N} , \tag{2}$$

where $\bar{s}_N$ is the mean level of skills of active entrepreneurs (who are all of type N). The utility of an entrepreneur of ability s is then equal to:

$$U = su(f(b+a) - r_N b) + (1-s)u(0) . \tag{3}$$

The optimal amount of borrowing b_N , is defined by the relation $f'(b+a) = r_N$. All entrepreneurs borrow the same amount and have the same income if successful. The utility of entrepreneurs is an increasing linear function of the skill s . It is immediate that an equilibrium, if it exists, is characterized by the values of the lowest skill level of active entrepreneurs, s_N , the interest rate r_N and the level of borrowing b_N , which are determined by the following equations:

$$u(w+Ra) = s_N u(f(b_N+a) - r_N b_N) + (1-s_N)u(0) \tag{4}$$

$$r_N = \frac{R}{\bar{s}_N}, \quad \text{where} \quad \bar{s}_N = \frac{\int_{s_N}^{1} s\,dF(s)}{\int_{s_N}^{1} dF(s)} \tag{5}$$

$$f'(b_N+a) = r_N . \tag{6}$$

The first of these equations determines for given b_N and r_N, the skill level of the marginal entrepreneur s_N. Of course b_N and r_N depend themselves on s_N by (5) and (6). Substituting b_N and r_N by these functions in (4), the RHS can be considered as a function of s_N, $\phi(s_N)$. It is a trivial exercise to show that this function is monotonically strictly increasing. Also $\phi(0) = u(0) < u(w+Ra)$, and $\phi(1)$ is equal to $u(f(b+a) - R(a+b) + Ra)$, with $f'(b+a) = R$. By assumption (B), $\phi(1) > u(w+Ra)$. We have proven the following proposition:

<u>Proposition 1</u>. When limited liability is not allowed, under conditions (A), (B), (C), there exists a unique equilibrium level of skill s_N $(0 < s_N < 1)$, which separates the entrepreneurs (with $s > s_N$) from the employees (with $s < s_N$). The equilibrium is characterized by the values of s_N, r_N and b_N which are determined by equations (4), (5) and (6).

4. Equilibrium with Limited Liability

Assume now that entrepreneurs can choose between the two types of liability form. We first analyze the behavior of individuals, then the properties of the equilibrium if it exists, and finally the conditions of existence of an equilibrium.

The income generated by a firm of an unsuccessful entrepreneur is equal to zero, and the loan is a total loss for the lender. Therefore, as for firms of type N, the equilibrium rate of interest (including principal) on loans to entrepreneurs of type L is equal to

$$r_L = \frac{R}{\bar{s}_L}, \tag{7}$$

where $\bar{s}_L$ is the mean level of skill of L-entrepreneurs. The expected utility of an L-entrepreneur of skill s is equal to

$$U = su[f(b+i) - r_L b + R(a-i)] + (1-s)u(R(a-i)) . \tag{8}$$

He maximizes this utility with respect to b and i $(0 \leq i \leq a)$. Using (7), the first order condition can be written as follows:

$$f'(b+i) = r_L = \frac{R}{\bar{s}_L} \tag{9}$$

$$\begin{cases} su'_{1L}(s)\left(\frac{1}{\bar{s}_L} - 1\right) - (1-s)u'_{0L}(s) = 0 \quad \text{if} \quad 0 < i < a \\ i = \begin{matrix} 0 \\ a \end{matrix} \quad \text{if} \quad su'_{1L}(s)\left(\frac{1}{\bar{s}_L} - 1\right) - (1-s)u'_{0L}(s) \begin{matrix} < 0 \\ > 0 \end{matrix} \end{cases} \tag{10}$$

$$u'_{1L}(s) = u'(y_{1L}(s)) \quad \text{and} \quad u'_{0L}(s) = u'(y_{0L}(s)) .$$

The terms $y_{1L}(s)$ and $y_{0L}(s)$ are the incomes of the entrepreneur with skill s and type L when successful and unsuccessful, respectively. They depend on s because the level of personal investment may be positive (which is not the case for N-firms), and increases with s. In this case $y_{0L}(s)$ decreases with s, and $y_{1L}(s)$ is an increasing function.

For a given value of r_L, the utility level of an L-entrepreneur can be written as a function of his skill s : $v_L(s, r_L)$. This function is represented by the curve AD in Figure 1.[10] By construction the graph of v_L is the envelope of the segment joining the utility levels $u(y_{0L}(s))$ and $u(y_{1L}(s))$ on the two vertical axes, respectively. It is a trivial exercise to show that when $s = 0$, $i = 0$, $y_{0L}(0) = Ra$, and that when $s = 1$, $i = a$ and $y_{0L}(1) = 0$. Also $y_{1L}(s)$ is an increasing function of s (it increases strictly only when s in the interval (s_B, s_C), where i increases from 0 to a).

We have seen in the previous section, that entrepreneurs with unlimited liability (of type N), invest all the same amount and have the same income when successful, y_{0N}. Their utility level is a function of skill s and of r_N, $v_N(s, r_N)$, and it is represented in Figure 1 by the linear segment OE. If both types L and N are to exist, the segment OE must intersect the curve AD. Since CD is on the segment OD, the point E must be above D. Therefore there exists a threshold level of skill s_N such that entrepreneurs with skill s greater than s_N choose the type N, and individuals of skill s smaller than s_N choose to be of type L or to be employees.

From the geometry of Figure 1, we also deduce that if firms of type L exist, there is a level s_L such that individuals with skill smaller

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[10] The function $v_L(s, r_L)$ can be determined for all s $(0 \leq s \leq 1)$ even if not all individuals choose to be L-entrepreneurs.

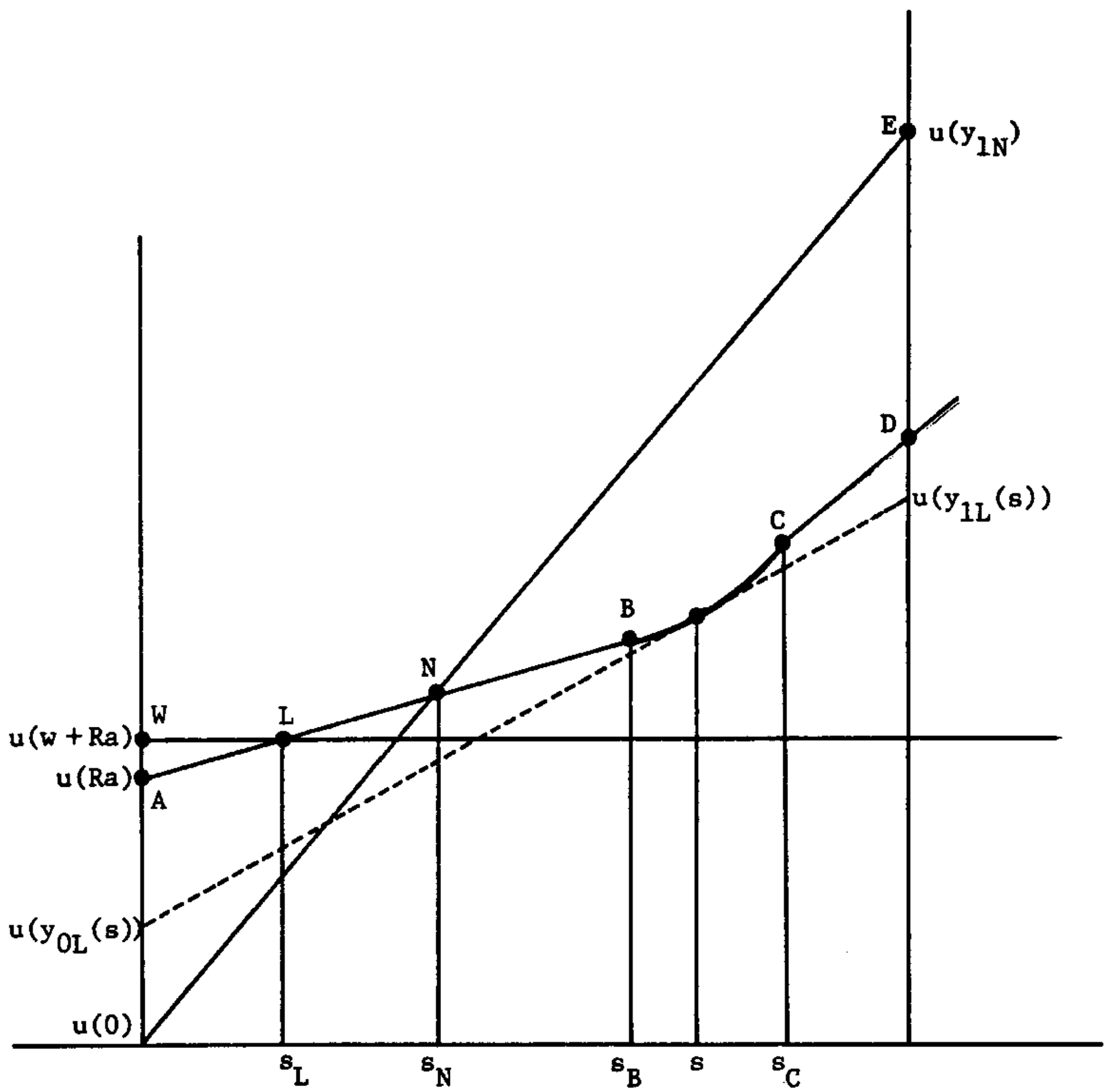

FIGURE 1

The equilibrium with two types of liability forms (greater) than s_L choose to become employees (entrepreneurs). This description of an equilibrium with two types of liability forms is summarized in the following proposition.

Proposition 2. If firms with limited and unlimited liabilities exist in the equilibrium, there exist two numbers s_L and s_N such that individuals of ability greater than s_N choose to be entrepreneurs of type N , individuals of ability between s_L and s_N choose the type L , and the others are employees.

Ceteris paribus, all entrepreneurs would prefer to choose limited liability in order to shift at least a fraction of the risk of production to the financial institutions which are risk-neutral. However, in the situation of asymmetric information, the entrepreneurs of higher skill prefer to use the unlimited liability form as a signal of their lower probability of failure. In this way, they gain better terms for their borrowings. The "price" of this signal is the penalty in case of business failure (it is of course lower for the highly skilled individuals). This insurance aspect of limited liability is only valued by risk-averse individuals. If they are risk-neutral, no entrepreneur chooses limited liability because it would only reveal a lower level of skill. An equilibrium with two types of liability may arise only if individuals are sufficiently risk-averse. From this discussion we expect that if the degree of risk-aversion is small, type L does not arise. On the other hand if individuals are sufficiently risk-averse for low incomes, the penalty of business failure is high and both types L and N are found in the equilibrium. These intuitive properties are stated more explicitly in the following proposition (the proof is given in the appendix).

Proposition 3. For a distribution of skills and a technology which satisfy conditions (A), (B) and (C), there exists an occupational equilibrium with employees and entrepreneurs.

There is a positive number such that if $|u''| < \varepsilon$ on $[0,M]$, the equilibrium is unique and all entrepreneurs have unlimited liability.

Assume that the utility function $u(x)$ is fixed for $x \geq Ra$, there is a number η such that if $u(0) < \eta$ both types of liability are found in the equilibrium, which may not be unique.

The equilibrium may not be unique because of the externalities between individuals of different skills. Assume that the values (s_L^1, s_N^1) and (s_L^2, s_N^2) define two equilibria with $s_L^1 < s_L^2$. In this case, the interest rates on loans to L-entrepreneurs in the second equilibrium r_L^2 is smaller than r_L^1 (in order to attract more employees to the L-occupation). This implies that the mean skill of L-entrepreneurs is higher, and that $s_N^2 > s_N^1$.

A risk averse entrepreneur is willing to pay the "price" of unlimited liability only if this allows him to borrow at a significantly lower rate. When the difference between the highest and the lowest level of skill is small, there is no such incentive, and no N-entrepreneur exists. To see this, we relax the assumption (A), and we assume that all skills are contained in the interval $[\alpha,\beta]$, $(\alpha \geq 0$, $\beta \leq 1)$. We then have the following proposition:[11]

Proposition 4. Assume that an occupational equilibrium exists for a given utility function and a technology which satisfies the assumptions (B) and (C). There exists a positive number γ such that if $\alpha-\beta < \gamma$, all entrepreneurs have a limited liability.

[11]The proof is given in the appendix, with a sketch of a construction of an equilibrium.

To some extent, the previous discussion about the equilibrium liability types can be summarized by the following table (where the dispersion of skill is defined by the difference between the highest and the lowest levels of skills):[12]

		Risk Aversion	
		Low	High*
Dispersion of Abilities	Low	?	Type L
	High	Type N	Types L and N

*The precise meaning of "High" or "Low" risk-aversion is relative to the skill dispersion (and vice-versa); it is made more precise in the sufficiency conditions of Propositions 3 and 4. For example, types L and N exist if the range of skills includes 1, and if $|u''(x)|$ is sufficiently large for <u>some</u> x, $0 < x \leq Ra$.

In the occupational equilibrium with one of the types L and N, or both of them, inefficient externalities arise between individuals of different skills. More precisely, an equilibrium is always inefficient in a first best sense: for example, if marginal L-entrepreneurs with lowest skill could be identified, they could be displaced to the pool of employees. Assuming their number to be small with respect to the labor force, they would not affect the welfare of other employees. The mean ability of L-entrepreneurs would be raised, and therefore, the welfare of <u>all</u> entrepreneurs (also of N-entrepreneurs if they exist). The same can be said about the marginal N-entrepreneurs (in this case also the welfare of both types is raised). Of course this observability condition is not

[12] An equilibrium without N-entrepreneurs exists only if <u>all</u> skill levels are contained in an interval sufficiently small. For the existence of an N-entrepreneur, it is sufficient to assume the existence of only <u>one</u> individual with skill $s = 1$.

met in our framework, and the concept of first best inefficiency has only a speculative purpose.

However, the equilibrium may also not be optimal in a framework where the planner has no more information than financial institutions. Consider the combination of a fixed fee for a liability charter, and a reward to successful L-entrepreneurs. Because the differential between the reward and the fee is higher for individuals at the margin between L and N than for the marginal L-entrepreneurs with lowest skill, this combination can induce a movement of individuals from type N to L, and from type L to the pool of employees. It is possible that there is a combination of fee and reward which improves the welfare of all individuals, and generates a surplus of revenues at the same time. In this sense, the equilibrium may not be a Pareto optimum under institutional constraints.

Although the possibility of a Pareto improvement under institutional constraints seems appealing, the determination of such a policy is subject to strong informational requirements about the model itself (technology, utility). Because these conditions are unlikely to be met, we now address a more fundamental question, i.e. the social value of the institution - of limited liability.

5. Private and Social Value of Limited Liability

5.1. Partial Equilibrium

Under the assumptions (A), (B), and (C), when no entrepreneur is allowed to take limited liability the occupational equilibrium exists and is unique (Section 3). Assume now that limited liability "charters" are given without charge to applicants. The initial equilibrium may be replaced by a new equilibrium with entrepreneurs of types L and N.[13]

The charters have a private value for some entrepreneurs who are better off with a higher income in case of business failure, even though their income is lower when the production outcome is successful (because of a higher borrowing rate). The behavior of individuals who change their occupation, or their liability type will also affect other individuals; limited liability charters have a positive value for some individuals, but this does not imply that they are desirable from a social point of view (in a sense which remains to be defined). We consider first the case where the relative size of the entrepreneurial sector is small with respect to the rest of the economy; the (risk-free) interest and wage rate are fixed.

Proposition 5. When the wage rate and the risk-free rate of return are fixed, the introduction of limited liability charters is Pareto optimal.

We prove the proposition in the only nontrivial case which is described as follows:[14] In the equilibrium without limited liability, the borrowing rate is r_N^* , and the lowest skill level of active entrepreneurs s^* , is defined by:

$$(11) \qquad u(w+Ra) = v_N(s^*, r_N^*) ,$$

where $v_N(s^*, r_N^*)$ is the "indirect" utility of an N-entrepreneur.

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[13]Proposition 3 gives a sufficiency condition for the existence of such an equilibrium. The stability of the "old" equilibrium with N-entrepreneurs after the introduction of limited liability charters, depends on the signalling function of limited liability out of equilibrium. One may consider the "worst" case where at the initial position of an N-equilibrium, with lowest skill s^* , any entrepreneur who chooses limited liability is attributed a skill level equal to s^* by financial institutions. In this case it is possible that the equilibrium with entrepreneurs of type N only, is stable after the introduction of limited liability charters, even though another (stable) equilibrium exists with entrepreneurs of types L and N .

[14]No particular assumption (like (A), (B) or (C)) is necessary for Proposition 5 except for the existence of equilibrium.

Introduce now limited liability charters; a new equilibrium arises with entrepreneurs of types L and N, with minimum skills s_L and s_N, respectively $(s_L < s_N)$.[15] We must have $s_N > s^*$; otherwise $r_N \geq r_N^*$, and we have an unfeasible chain of inequalities:

$$u(w+Ra) = v_L(s_L, r_L) < v_L(s_N, r_L) = v_N(s_N, r_N) \tag{12}$$

$$\leq v_N(s^*, r_N) \leq v_N(s^*, r_N^*) = u(w+Ra)$$

When $s_N > s^*$, $r_N < r^*$ and the utility of N-entrepreneurs is greater in the new equilibrium. It is strictly greater if entrepreneurs borrow. The utility of individuals who choose to be entrepreneurs of type L is higher than if they had chosen to be employees or N-entrepreneurs in the second equilibrium, and *a fortiori*, higher than in their first equilibrium occupation. Also, one can show with a geometrical argument in Figure 1 that $s_L < s^*$. If $s_L \leq s \leq s^*$, $v_L(s, r_L) > u(w+Ra)$: the utility level of new entrants in the entrepreneurial occupation is strictly increased. If the institution of limited liability affects the choice of occupations, the new equilibrium is strictly Pareto superior.

5.2. General Equilibrium

The assumption of partial equilibrium is a good approximation when the sector of the economy affected by the introduction of limited liability charters has a small size. When this sector is large with respect to the rest of the economy, the reallocation of resources may alter the wage rate and the risk-free rate of return on capital.

In this case the introduction of limited liability contracts could

[15] If the new equilibrium is not unique, one can use the same method of proof to show that the equilibria can be ranked by the Pareto criterion with the same order as the values of s_N. See also the previous footnote.

have an adverse effect for a large number of people, and be undesirable from a social point of view. This effect is illustrated by the following example which introduces labor in the production technology.

The total capital stock is equal to K . The labor force is equal to L . Each worker provides one unit of labor. In addition, there are $2n$ individuals who can act as entrepreneurs (these people can also act as employees). In order to neglect small variations of the labor force, we assume that the number $2n$ of potential employers is relatively small with respect to the total population. This class is divided equally among people with an ability $s = \frac{1}{4}$, and people with an ability $s = \frac{3}{4}$. Each entrepreneur runs one firm which is successful with a probability s .

When it is successful, its output is given by the neoclassical production function:

$f(k,\ell)$, where k and ℓ represent the amounts of capital and labor used in the production process.

We take $f(k, \ell) = k^{\alpha}\ell^{\beta}$, where $\alpha+\beta < 1$; $\gamma = 1 - \alpha - \beta$ is equal to the share of entrepreneurial input.

The alternative outcome (with probability $1-s$) is an output equal to zero.

The total capital invested in the production process is equal to the sum of the real capital and of the wage bill; with the notation of the previous section,

$$k + w\ell = b + i .$$

Labor is allocated competitively between firms.[16] For a given

[16] In this two-period process, the wage bill is paid before the outcome is known; therefore labor bears none of the production uncertainty.

firm, the values of ℓ , k , b , i and the liability form are chosen by the entrepreneur maximizing his utility function.

He follows the efficiency conditions:

$$\frac{\partial f}{\partial k} = r \ , \quad \frac{\partial f}{\partial \ell} = wr \ ,$$

where r represents his borrowing interest rate. Using the definition of f , we find:

$$w = \frac{\frac{\partial f}{\partial \ell}}{\frac{\partial f}{\partial k}} = \frac{\beta}{\alpha}\frac{k}{\ell} = \frac{\beta}{\alpha}\frac{K}{L} \ , \quad \text{since } w \text{ is the same for all firms.}$$

The wage rate is determined by the aggregate capital labor ratio,

Assume that when limited liability is allowed, all individuals with a positive ability chose to be L-entrepreneurs.[17] The mean ability is equal to 1/2, in the equilibrium, the capital stock is equal to $k_1 = K/2n$ for all firms, and the risk-free rate is equal to:

$$R_1 = \frac{1}{k_1^{\gamma}} \frac{\alpha}{2}\left(\frac{L}{K}\right)^{\beta} .$$

Assume now that when limited ability is not allowed, the individuals with skill 1/3 choose to become employees. In the new equilibrium, the risk-free rate R_2 can be expressed as a function of R_1 :

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[17] The following numerical example supports the argument: $\alpha = 1/2$, $\beta = 1/4$, $n = 1$, $L = 40$, $a = 0.5$ for all individuals (which implies that $K = 21$); the utility function is piecewise linear with $u(1) = 1$ and $u'(x) = 50$ for $x \leq 0.1$, $u'(x) = 1$ for $0.1 \leq x \leq 3$, $u'(x) = 0.1$ for $x \geq 3$. In the first equilibrium (with type $\bar{L}$), $\bar{i} = 0$ for all entrepreneurs.

$$R_2 = \frac{3}{2}\,\frac{1}{2^{\gamma}}R_1$$

When the share of entrepreneurial input in production γ, is not too large, R_2 is greater than R_1.

The abolition of limited liability is beneficial to the capitalists. When the number n is small with respect to L, the decrease of the wage rate is small. If wealth is uniformly distributed, the income of employees (equal to $w + Ra$) is greater when limited liability is not allowed.[18] The utility levels of individuals with skills 1/3 and 2/3 is lower without limited liability. However it should be noted that the former can be considered as wasting the capital of society, and the latter have a utility level which is higher than the utility of employees (because of their entrepreneurial skills).[19]

The model considered here is a one period model. It can easily be embedded in a dynamic framework where individuals live one period (financial institutions do not know the entrepreneurial abilities of new born individuals), and capitalists have an infinitely elastic supply of savings in the long-run (through an operative bequest motive *à la* Barro). The risk-free interest is fixed in the long-run, and the capital stock is endogenous. For proper parametric values, the suppression of limited liability increases the risk-free return in the short-run and stimulates savings; in the long run the capital stock and the wage rate increase (for the numerical example the wage rate is more than doubled in the long-run). Overall the policy may improve the welfare of both capitalists and workers.

[18]For the example described in footnote 17, $R_1 = 0.163$, $R_2 = 0.207$, $w_1 + R_1a = 0.344$, $w_2 + R_2a = 0.359$. The utility levels of entrepreneurs in the first and the second equilibrium (indexed by L and N, respectively), are functions of the entrepreneurial ability: $U_L(1/4) = 0.366$, $U_L(3/4) = 1.32$, $U_N(3/4) = 0.99$.

[19]For the numerical case, the level of expected social utility (additive) in the equilibrium without limited liability is equal to 16.0, and is greater than in the first equilibrium (where it is equal to 15.45).

6. Conclusion

This paper has analysed the institution of limited liability in a simple model of occupational choice with asymmetric information between entrepreneurs and capitalists. Limited liability performs two functions: the partial insurance enables entrepreneurs to reduce their risk, and may induce some individuals to exert their entrepreneurial skill when otherwise they would choose to be employees. Also some individuals may waiver this insurance possibility and signal their greater skill to capitalists. The limited liability institution reduces the (negative) externality that the less skilled convey on the more skilled, and improves the working of the entrepreneurial function in situations of asymmetric information. In the absence of general equilibrium response, other opportunities for individuals are unaffected, and the institution of limited liability is Pareto optimal. These general equilibrium effects cannot be ignored when we consider substitutions between occupations which affect a large sector of the economy. In this case, the welfare properties of limited liability are ambiguous. Entrepreneurs may still enjoy the insurance and the signalling aspect of the institution. However, the number of highskill entrepreneurs taking limited liability may be too large from a socialpoint of view. These entrepreneurs, by conveying a positive externality, induce an excessive number of low skilled individuals to undertake production projects.

In the absence of limited liability, low skilled individuals cannot bear the high penalty of failure (more likely for them), and capital is employed only by individuals with higher skills. This induces a higher risk-free rate of return on capital and may stimulate capital accumulation, the wage rate, and total output in the long-run.

Of course, the present paper provides only a partial view on the limited liability institution; for example, one could consider the twin problem of moral hazard mentioned in the introduction, or more general types of probability distributions for the production outcomes,[20] different degrees of risk aversion, etc. The complexity of these problems arises from the variety of situations.

The approach followed here may also have some application to the study of the corporate tax. One of the important attributes of a corporate charter is the property of limited liability.[21] Traditionally, studies of the corporate tax have assumed the existence of a corporate sector, with a corporate production function.[22] The corporate tax affects the relative inputs of the corporate and the noncorporate sector. This approach seems to be valuable for some important sectors where production has to be organized under the corporate form (it is difficult to imagine General Motors in a noncorporate form).

However, in some sectors where entrepreneurial talent becomes important in deciding success or failure, two production processes, identical from a technical point of view can be undertaken under different legal structures.[23] This paper analyzes one of the aspects of the substitution between two legal forms of production. Despite the gloomy predictions of Schumpeter,[24] the entrepreneurial sector seems still very active, and seems to require further theoretical and empirical investigations.

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[20]Stiglitz and Weiss [1980] have shown that financial intermediaries may use credit rationing to discriminate against risky projects.

[21]For discussion of the properties of the corporate form, see King [1977].

[22]See, for example, Shoven (1976).

[23]For an introduction to some aspects of the taxation on small firms, see Clark (1977).

[24]"The romance of earlier commercial adventure is rapidly wearing away, because so many more things can be strictly calculated that had of old to be visualized in a flash of genius" (Schumpeter (1941), p. 132).

APPENDIX

Proof of Proposition 3

Define by $h(r)$ the function:

$$h(r) = \max_k (f(k) - rk) \ , \quad h'(r) < 0 \ .$$

Using (5) and the description of the behavior of N-entrepreneurs in Section 3, we can express the utility of the N-entrepreneur with lowest ability s_N as a function of s_N :

$$\phi(s_N) = s_N u\left[h\left(\frac{R}{\bar{s}_N}\right) + \frac{Ra}{\bar{s}_N}\right] + (1 - s_N)u(0) \ ,$$

where for a given density of skills f , $\bar{s}_N$ is the mean skill on the interval $(s_N, 1)$, and is a function of s_N . Because $d\bar{s}_N/ds_N > 0$ and $h' < 0$, the function ϕ is increasing; its graph represented in Figure 2. Its extreme values are:

$$\phi(0) = u(0) \ ; \quad \phi(1) = u(h(R) + Ra) = u(M) \ .$$

In the same way, the utility of an L-entrepreneur of skill s (assuming existence), is a function of the lowest and the highest skills of L-entrepreneurs, s_L and s_N respectively:

$$\psi(s,\ s_L,\ s_N) = su\left[h\left(\frac{R}{\bar{s}_L}\right) + \frac{R}{\bar{s}_L}i + R(a-i)\right] + (1-s)u(R(a-i)) \ ,$$

where the value of personal investment i is optimal $(0 \leq i \leq a)$, and $\bar{s}_L$ depends on s_L and s_N . The partial derivative of ψ with respect to s_L is equal to:

$$\frac{\partial\psi}{\partial s_L} = s u_1' b \frac{R}{\bar{s}_L^2} \frac{d\bar{s}_L}{ds_L} ,$$

where b is the amount borrowed by the entrepreneur ($b > 0$ by assumption (B)). Since $d\bar{s}_L/ds_L > 0$,

(a-1) $$\frac{\partial\psi}{\partial s_L} > 0 .$$

In the same way,

(a-2) $$\frac{\partial\psi}{\partial s_N} > 0 , \quad \text{and obviously} \quad \frac{\partial\psi}{\partial s} > 0 .$$

An occupational equilibrium can be defined with the function ϕ and ψ . If only type N exists, s_N is defined by:

(a-3) $$u(w + Ra) = \phi(s_N) .$$

If both types L and N exist, s_L and s_N are defined by:

(a-4) $$u(w + Ra) = \psi(s_L, s_L, s_N)$$

(a-5) $$\psi(s_N, s_L, s_N) = \phi(s_N) .$$

The locus of points with coordinates s_L and s_N which satisfy these last two relations is represented in Figure 3 by the curves AD and FB respectively. To establish the geometric properties of the figure, we need to analyze more explicitly the function ψ .

Define the following functions:

$$\chi(s_N) = \psi(s_N, s_N, s_N)$$

$$\begin{aligned}\xi(s_L, s_N) &= \chi(s_N) \quad \text{if} \quad s_L \geq s_N \\ &= \psi(s_N, s_L, s_N) \quad \text{if} \quad s_L \leq s_N\end{aligned}$$

By (a-1) , $\xi(s_L, s_N) \leq \chi(s_N)$, and we have:

$$\xi(s_L, 0) = u(Ra) \text{ , and } \xi(s_L, 1) = u(z_1,(s_L)) \text{ , with}$$

$$z_1(s_L) = h\left(\frac{R}{\bar{s}_L(s_L, 1)}\right) + \frac{R}{\bar{s}_L(s_L, 1)}i + R(a-i) ,$$

and i is optimal $(0 \leq i \leq a)$. When $s_L < 1$, $\bar{s}_L(s_L, 1) < 1$ because of assumption (A), and

$$z_1(s_L) \leq h\left(\frac{R}{\bar{s}_L(s_L, 1)}\right) + \frac{R}{\bar{s}_L(s_L, 1)}a < h(R) + Ra .$$

Also, for a fixed s_L , by (a-1) and (a-2), $\xi(s_L, s_N)$ is an increasing function of s_L . It is represented in Figure 2. The important properties of the functions ϕ and ξ are expressed by the following relations which are valid for all s_L :

$$\phi(0) = u(0) < u(Ra) = \xi(s_L, 0)$$

$$\phi(1) = u(h(R) + Ra) > u(z_L(L)) = \xi(s_L, 1) .$$

Therefore, the graphs of the functions ϕ and ξ intersect at least once (multiple intersections cannot be excluded because ϕ and ξ are both increasing), for a given s_L, define by G the point of intersection with the highest value for s_N , $s_N(G)$. At this point, the slope of ϕ is smaller than the slope of ϕ ; by (a-2), when s_L increases, the graph of ξ shifts upwards, the value of $s_N(G)$ increases, and the point G tends to the point B at the intersection of the graph of ϕ and

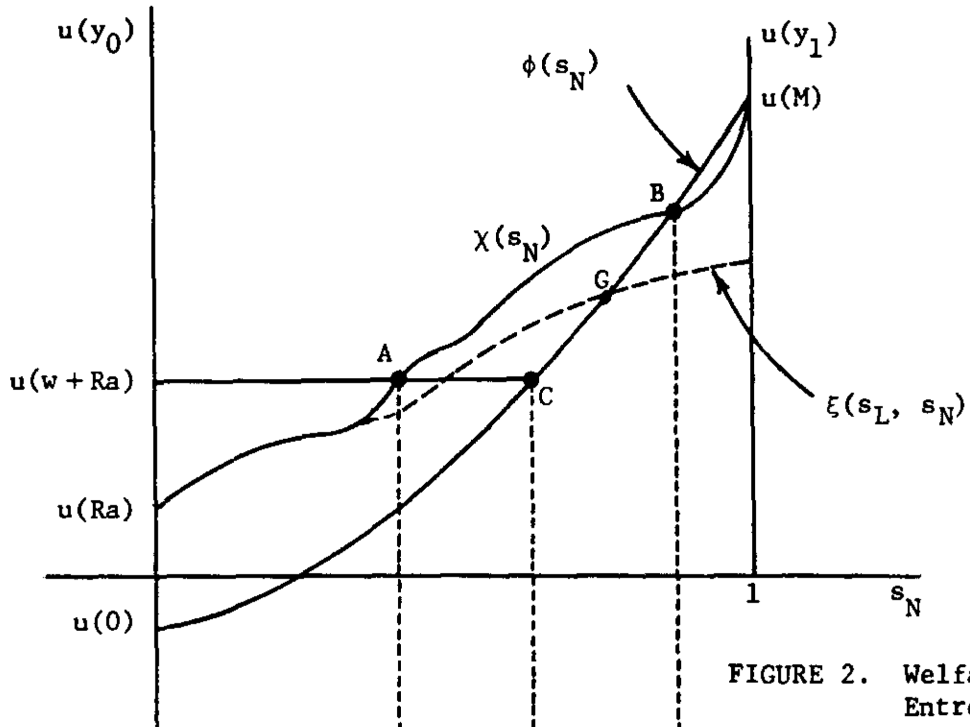

FIGURE 2. Welfare of Marginal Entrepreneurs in the Equilibrium

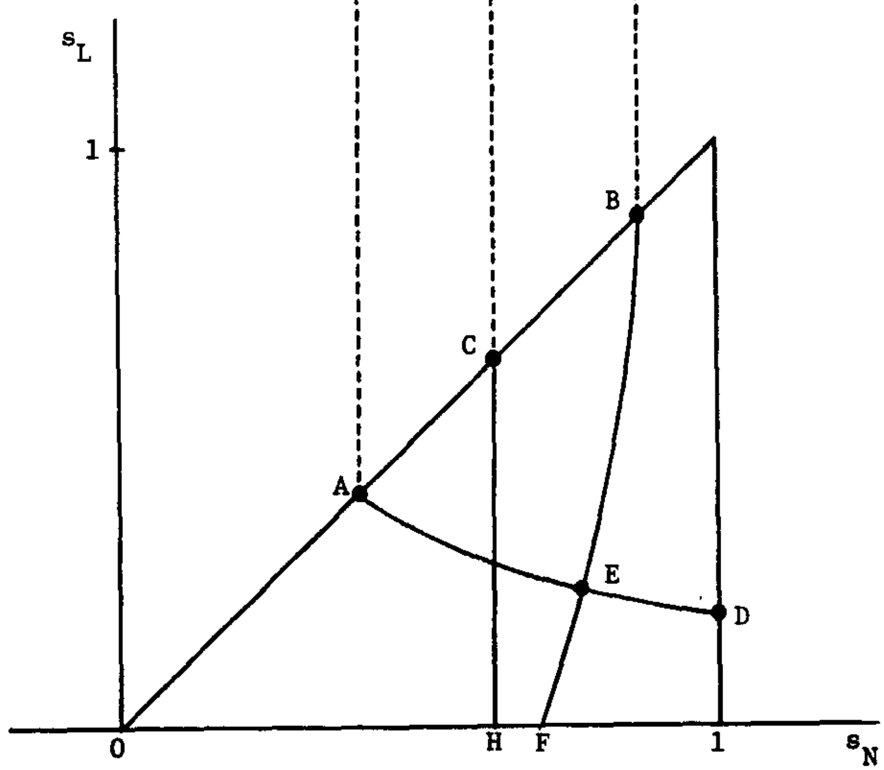

FIGURE 3. Marginal Abilities in the Equilibrium

the graph of the function χ . (The function χ measures the utility level of an L-entrepreneur with a skill perfectly monitored by lenders.) The value of $s_N(B)$ may be equal to one, or smaller than one. For all values of s_L $(0 \leq s_L \leq s_N)$, there exists at least one value of s_N such that the equilibrium between types L and N (relation (a-5)), is attained. The largest value is an increasing function of s_L and is represented in Figure 4 by the curve FB .

The equilibrium condition (a-4) between employees and L-entrepreneurs is represented on Figure 3 by the curve AD . The point A is on the 45° line $(s_L = s_N)$. At this point, the value of s_N is equal to the solution of the equation:

$$u(w+Ra) = \chi(s_N) \ .$$

This solution exists and is unique (see Figure 2).
Because of (a-1) and (a-2) the curve AD on Figure 3 has a downward slope, the value of s_L at the point D $(s_N = 1)$, is the solution of

$$u(w+Ra) = \psi(s_L, s_L, 1) \ .$$

Since $\psi(0,0,1) = u(Ra)$, $\psi(1,1,1) = u(h(R)+Ra) > u(w+Ra)$ (by assumption (C)),and $\psi(s_L, s_L, 1)$ is monotonically increasing, the solution of this equation is unique, strictly positive and strictly smaller than one.

Finally the equilibrium condition (a-3) (between employees and N-entrepreneurs), is represented in Figure 3 by the segment HC . The point C on Figure 3 corresponds also to the point C on Figure 2.

An equilibrium with entrepreneurs of types L and N is represented on Figure 3 by the point E . Its existence depends on the relative positive of the points A and B on the 45° line.

Assume that individuals are risk neutral, and $u(x) = x$. From the optimal conditions (9) and (10), the functions ϕ and ψ take the special forms:

$$\phi(s_N) = s_N\left(f(b_1 + a) - \frac{R}{\bar{s}_N}b_1\right)$$

$$\chi(s_N) = s_N\left(f(b_2 + a) - \frac{R}{s_N}b_2\right) ,$$

where b_1 and b_2 are defined by $f'(b_1 + a) = R/\bar{s}_N$, and $f'(b_2 + a) = R/s_N$ respectively.

Since $\bar{s}_N > s_N$ (by assumption (A)), and all active entrepreneurs borrow (by assumption (B)), the utility of an individual with skill s_L is greater with unlimited liability than with limited liability because of a smaller interest rate: $\phi(s_N) > \chi(s_N)$. This case is represented in Figures 4 and 5. The curves AD and BF do not intersect, and there is only one equilibrium with unlimited liability represented by the point C . (By construction of BF all other possible combinations of points (s_L, s_N) which satisfy (a-5) are to the left of BF .)

By a simple continuity argument, there is a positive number ε such that if $|u''| < \varepsilon$ on the range of income values $[0,M]$ (with $M = h(R) + Ra$), the point B is on the left of the point A , and no entrepreneur chooses the limited liability.

Assume that the function $u(x)$ is given for $x \geq Ra$. When $u(0) \to -\infty$, $\phi(s_N) \to -\infty$ if $s_N < 1$. Since $\phi(1) = h(R) + Ra$ is independent of $u(0)$, there is a number η such that if $u(0) < \eta$, the point B is to the right of the point A in Figure 2. (The point A is independent of $u(0)$.) This proves the second part of Proposition 3.

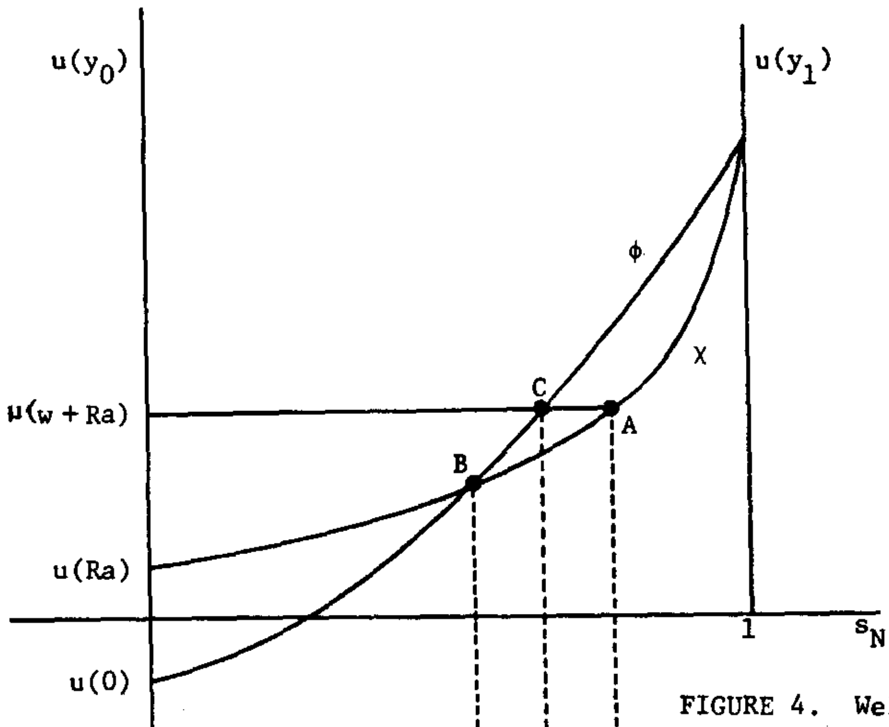

FIGURE 4. Welfare of Marginal Entrepreneurs

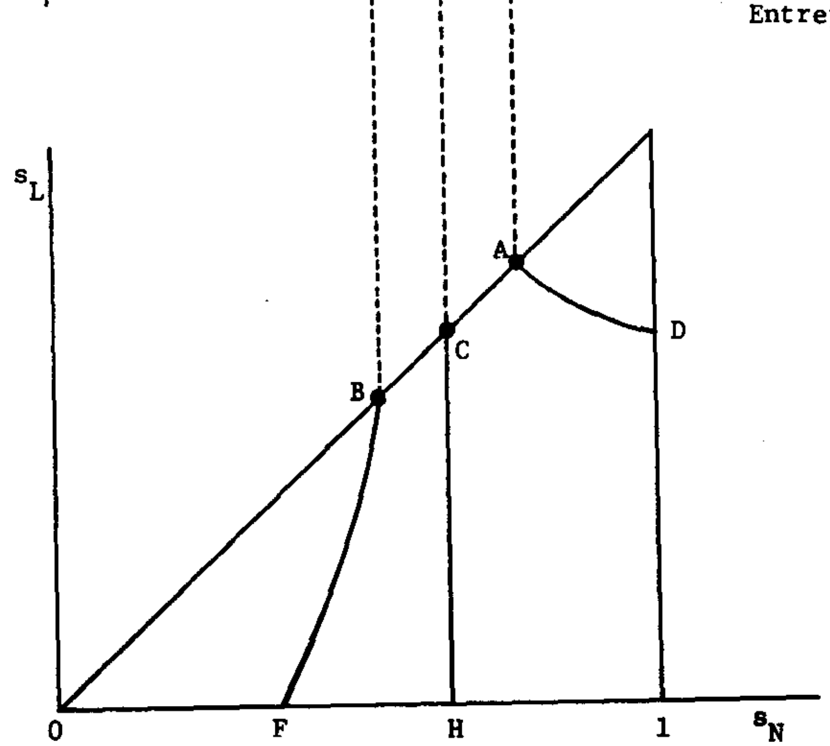

FIGURE 5. Marginal Abilities

The equilibrium may not be unique because for a given value of s_L , there may be multiple solutions of the equation (a-5) to which correspond more than one curve BF on Figure 3.

Proof of Proposition 4

Assume that $\alpha \leq s \leq \beta$, and consider an individual with skill β . If he chooses to be an N-entrepreneur, his utility is equal to:

$$v_N = \beta u\left(h\left(\frac{R}{\bar{s}_N}\right) + \frac{Ra}{\bar{s}_N}\right) + (1-\beta)u(0)$$

where $h(r) = f(b) - rb$ $(f'(b) = r)$ as in the proof of Proposition 3 $(h' < 0)$.

Assume now that if he chooses the type L , his skill will be estimated by financial institutions to be at the lowest possible level α . It is a straightforward exercise to show that in this situation, he would invest none of his assets in his own firm, and his utility would be equal to

$$v_L = \beta u\left(h\left(\frac{R}{\alpha}\right) + Ra\right) + (1-\beta)u(R\alpha) .$$

Using the concavity of the utility function u , we have the following inequality:

$$v_N \leq \beta u(y_1) + \beta u_1'\left(\frac{R}{\bar{s}_N}a - Ra\right) + (1-\beta)u(0)$$

$$\text{with } u_1' = u'(y_1) = u'\left(h\left(\frac{R}{\bar{s}_N}\right) + \frac{R}{\bar{s}_N}a\right).$$

In the same way,

$$v_L \geq \beta u(y_2) + (1-\beta)u(0) + (1-\beta)u_0'R$$

$$\text{with} \quad y_2 = h\left(\frac{R}{\alpha}\right) + Ra \ , \quad \text{and} \quad u_0' = u'(Ra) \ .$$

Combining these two inequalities, we find:

$$\text{(a-6)} \qquad v_N \leq v_L + \beta(u(y_1) - u(y_2)) + Ra\left[\beta u_1'\left(\frac{1}{\bar{s}_N} - 1\right) - (1- \ )u_0'\right]$$

When $|\alpha-\beta| \to 0$, $|\bar{s}_N - \alpha| \to 0$, therefore $|y_1 - y_2| \to 0$, and $|u(y_1) - u(y_2)| \to 0$. The term in the last parentheses in (a-6) tends to $u_1' - u_0'$. At the limit, $y_1 > w + Ra > Ra$. (Otherwise no individual is an entrepreneur.) Therefore there exists a number γ such that if $\beta-\alpha < \gamma$, $v_N < v_L$: The entrepreneur with highest skill β chooses the type N . _A fortiori_, the same is true for all entrepreneurs.

A geometrical representation of the equilibrium can be given by the intersection of the curve AD and the vertical line $(s_N = \beta)$ in a figure similar to Figure 3. The construction of an equilibrium is a simple exercise: for a given w , the point A is independent of the skill distribution and can define α . The value of β can be chosen such that the above inequalities are satisfied.

REFERENCES

Akerlof, G. "The Market for 'Lemons': Qualitative Uncertainty and the Market Mechanism," Quarterly Journal of Economics, August 1970, pp. 629-649.

Clark, R. C., "The Morphogenesis of Subchapter C: An Essay in Statutory Evolution and Reform," The Yale Law Journal, 1977. pp. 90-162.

Jaynes, Gerald D., "Equilibria in Monopolistically Competitive Insurance Markets, Journal of Economic Theory, December 1978, pp. 394-422.

Kihlstrom, Richard F. and Jean-Jacques Laffont, "A General Equilibrium Entrepreneurial Theory of Firm Formation Based on Risk-Aversion," Journal of Political Economy, August 1979, pp. 719-748.

King, Mervyn A., Public Policy and the Corporation, Chapman and Hall, London, 1977.

Knight, Frank H., Risk, Uncertainty and Profit. New York: Harper & Row, 1965.

Kanbur, S. M., "On Risk Taking and the Personal Distribution of Income," Journal of Political Economy, August 1979, pp. 769-797.

________., "Risk Taking and Taxation: An Alternative Perspective," Journal of Public Economics, April 1971, pp. 163-184.

Ross, Stephen, "The Determination of Financial Structure: The Incentive-Signalling Approach," The Bell Journal of Economics, Spring 1977, pp. 33-40.

Rothschild, Michael and Joseph E. Stiglitz, "Equilibrium in Competitive Insurance Markets: The Economics of Markets with Imperfect Information," Quarterly Journal of Economics, November 1976.

Schumpeter, Joseph A., Capitalism, Socialism and Democracy. New York: Harper & Row, 1942.

__________, History of Economic Analysis. New York: Oxford University Press, 1954.

Shoven, John B., "The Incidence and Efficiency Effects of Taxes on Income from Capital," The Journal of Political Economy, December 1976, pp. 1261-1283.

Shavell, Stephen, "Accidents, Liability and Insurance," Mimeo, Harvard University, February 1979.

Spence, Michael A., Market Signalling: Information Transfer in Hiring and Related Processes. Cambridge: Harvard University Press, 1974.

Stiglitz, Joseph F. and Andrew Weiss, "Credit Rationing in Markets with Imperfect Information, American Economic Review, June 1981, pp. 393-410.

Wilson, Charles, "The Welfare Benefits of Price Maintenance in Markets with Adverse Selection," Mimeo, University of Wisconsin, Madison, August 1978.

__________, "The Nature of Equilibrium in Markets with Adverse Selection," The Bell Journal of Economics, Spring 1980, 108-130.