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Output Supply, Employment, and Intra-Industry Wage Dispersion

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Introduction

A striking feature of the world that is not usually reflected in economic models is the payment of non-identical wages to workers with virtually identical qualifications and duties. Suggested causes of this phenomenon have included disequilibrium, incomplete information, and institutional features such as partial unionization. In this paper we construct a simple model of a labor market which exhibits wage dispersion in the absence of these possible causes. Specifically, our model exhibits equilibrium wage dispersion even though all firms are identical, all workers are identical, and all agents possess complete information about prevailing market opportunities.

Our model considers a single industry, with firms hiring homogeneous workers to produce a common homogeneous output. Each firm considers itself so small relative to the industry that it anticipates that any change in its own wage and employment policy will have no significant effect upon aggregate supply and will result in all other firms doing exactly what they were doing before the change.

An industry equilibrium (Nash equilibrium) satisfies three conditions. First, each firm, taking as given the wage and employment policies of all other firms, chooses its own wage and employment levels to maximize profit. Second, workers make optimal employment decisions; which means that they change jobs to fill any vacancies at a higher wage than their current wage. Finally, the prevailing output price equates the industry's supply with its Marshallian demand.

A critical assumption of the paper is the occurrence of firm specific hiring and training costs each time a new worker is hired. A firm paying a low wage will incur such costs more frequently than a firm paying a higher wage, since workers will tend to quit the low wage firm to fill vacancies occurring in higher wage firms, therefore a firm may be indifferent between offering any one of two or more wages because the increase in the wage bill caused by paying a higher wage is just compensated by the decrease in hiring and training costs.

The model has a variety of equilibria; ranging from equilibria with any finite number of wages, including a single wage, to the equilibrium with a continuum of wages on a connected interval of the real number line. The connected continuum equilibrium is particularly interesting because of the simple closed form distribution of wages by workers which we derive endogenously from the model, and because all other equilibria can be characterized in terms of that equilibrium.

Two interesting features of some of the equilibria are unemployment and underemployment. Unemployment or underemployment in other lines of work occur whenever the maximum wage offered in the industry is higher than the Marshallian wage. This subject and its implications for Keynesian wage rigidity is discussed at length for an economy without wage dispersion, in Dybvig and Jaynes [1980].

Before proceeding with the model we now discuss the relationship of this paper to previous work on wage dispersion.

In recent years economists, spurred by a seminal paper of George Stigler, have attempted to give explicit theoretical recognition to the common empirical observation that homogeneous commodities and services often trade at different prices. Since it is generally recognized that such a situation is perfectly consistent with disequilibrium much effort has been devoted to explaining price-wage dispersion as an equilibrium phenomenon in the presence of optimizing agents. A common intuition prevailing in this work has been that price-wage dispersion over homogeneous commodities is essentially a manifestation of imperfect information about trading opportunities. It has, however, become increasingly clear that the construction of an equilibrium model with wage dispersion is not an easy project.[1] Under the usual characterization of imperfect information in labor markets, in which the searcher knows the distribution of wages but not the specific location of any wage, it has been found that each firm pays the monopsony wage in the only equilibrium consistent with optimal agent behavior. Two attempts to resolve this collapse to a single wage have been to impose nonoptimal behavior upon searchers and to assume that searchers have imperfect information about the distribution as well as location of wages.[2] Motivation for this paper was generated by the belief that some of the more salient properties of labor markets, such as wage dispersion and unem-

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[1] See M. Rothschild [1973] for a discussion of these problems.

[2] Two such examples are provided by L. Wilde [1978] and M. Rothschild [1974]. A third approach in a paper by J. E. Stiglitz [1976] is similar to our model and will be commented on below.

ployment, are not fundamentally due to problems of imperfect information about wage opportunities, but to more basic phenomena inherent to the technology of production (e.g., training costs). Indeed, the above mentioned difficulties with constructing an equilibrium model of wage dispersion based upon imperfect information alone seem to strengthen this belief. But even more importantly, George Stigler has pointed out that products whose prices figure prominently in searcher's budgets will induce intensive search making significant personal gains from search by individuals very unlikely. In the case of labor markets in particular, the abundance of want-ads, employment agencies, and telephones, indicates that specific information about wage opportunities is available rather cheaply.

These observations and the positive results reported in this paper seem in our opinion to indicate that although imperfect information has been shown to have important implications for many economic problems; lack of information about wage opportunities may not be that significant. As a first maxim we argue that imperfect (asymmetric) information will be of fundamental importance only if some agents privately possessing the information hope to gain by keeping the information imperfect. This is certainly the case in markets where adverse selection and moral hazard with nonhomogeneous or imperfectly identifiable commodities are traded.

As Greenwald [1979] has illustrated, a "lemons" problem may arise if current employers have better information about workers' abilities than prospective employers. Also, as Spence [1973] and others have shown, if agents have superior information about their own characteristics there is potential for signalling phenomena. Whether these informational problems are more or less important than the phenomenon we are describing is an empirical question, the answer to which probably varies across industries.[3]

The plan of this paper is as follows. Section 1 sets forth our model. Section 2 describes firm behavior and its consequences for the equilibrium wage distribution in the interval case. Section 3 derives Marshallian supply and employment correspondences for the interval case, and Section 4 characterizes the general class of equilibrium wage distributions and shows that the results of Section 3 are valid more generally. Section 5 concludes the paper.

1. The Model

All active and potentially active firms (we assume free entry) are capable of producing a homogeneous good using inputs capital and labor with the concave, constant returns production function $Q[K,L]$. Letting $k = K/L$ we denote the intensive form of this production function by $q(k) = Q[K,1]$ with,

$$q' > 0 \ , \quad q'' < 0 \ .$$

In addition to the wage paid, each new worker costs the hiring firm T dollars in firm specific hiring and training costs. These costs are assumed to occur instantly and are identical across firms. Job vacancies occur at firms through deaths and voluntary resignations. The death rate is exogenous and is taken to be δ percent of the labor force per unit time. The endogenous resignation rate $r(w)$ is the

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[3]See Bruce Greenwald [1979], A. M. Spence [1973] and J. E. Stiglitz [1975].

fraction of workers leaving firms paying w to accept jobs at higher paying firms, per unit time. We shall write as if the death rate is certain although we are really thinking of uncertain death with firms maximizing expected profits. The vacancy rate of a firm paying wage w is therefore

$$V(w) = \delta + r(w) . \quad (1)$$

We define p and ρ to be the price of output and rental rate of capital. The profit of a firm is

$$\pi(k,p,\rho,w) = [pq(k) - (w + TV(w)) - \rho k]L . \quad (2)$$

The rental rate of capital is assumed to be determined in a competitive market outside the industry and will be considered a fixed parameter throughout the paper.

The vacancy rate of a firm offering a wage w is determined in equilibrium by a simple rule characterizing optimal agent behavior, and the assignment of workers to jobs. We assume that the total pool of workers available for employment is constant and equal to N. Each identical worker has the same reservation money wage w_m at which he will just be willing to accept employment in the industry. Under the assumptions of complete information and zero transaction costs each worker will apply to every firm paying w_m or more; accepting his first offer and quitting his current position whenever a higher wage is offered. When there is a job opening in a firm we assume that each worker in the pool of applicants who would accept the firm's wage has an equal chance of getting the offer.

If there is a finite set of wages offered in equilibrium with

N_w workers working at wage w, the vacancy rate $V(\hat{w})$ for a firm paying $\hat{w}$ will be,

$$V(\hat{w}) = \delta + r(\hat{w}) \tag{3}$$

$$= \delta + \sum_{w>\hat{w}} \frac{N_w \cdot V(w)}{U + \sum_{w\leq\hat{w}} N_w} .$$

Where U is the number of workers which are unemployed or underemployed outside the industry at the reservation wage w_m. If there is a continuum of wages with the distribution of workers by wages $F(w)$, so that $NF(w)$ workers work in the industry at wages w or less, or are unemployed or underemployed in another industry at w_m, we have;

$$V(\hat{w}) = \delta + \int_{\hat{w}}^{w_M} \frac{NV(w)F'(w)dw}{NF(w)} . \tag{4}$$

Here we have assumed that $F(\cdot)$ has a density F', except perhaps at $\hat{w}$. The numerator in the integrand represents the number of vacancies per unit time in the interval $[w, w+dw]$, and the denominator represents the number of agents willing to fill those vacancies. The ratio gives the probability that any worker at a wage w or less will receive an offer. Our model is most easily studied in the case which has an interval of wages offered from the alternative wage w_m to some maximum wage w_M (we have referred to this as the interval case). Therefore, this case will be the immediate focus of our study of equilibrium wage dispersion. An interesting feature of the model is the existence of many equilibria. Since every equilibrium can be constructed from the interval case, we will not be losing any generality by restricting our immediate attention to this case.

2. Firm Behavior and the Equilibrium Wage Distribution

A firm chooses labor, capital, and its offered wage to maximize profits, given prices and the distribution of wages. This leads to first order conditions

(5) $$\frac{\partial \pi}{\partial L} = 0 \quad \text{or} \quad \pi = 0$$

(6) $$\frac{\partial \pi}{\partial k} = 0 \quad \text{or} \quad q'(k) = \frac{\rho}{p}$$

(7) $$\frac{\partial \pi}{\partial w} = 0 \quad \text{or} \quad -v'(w)T = 1 \ .$$

The first two conditions are standard. The third expresses the fact that at an optimal wage the marginal decrease in training costs due to a wage increase should just offset the increase in the wage bill. Figures 1 and 2 depict these conditions.[4] By the concavity assumptions on $q(\cdot)$ the first two conditions are necessary and sufficient. The last is only necessary. Suppose that all the wages between w_m and w_M are paid, that is the support of $F(\cdot)$ is $[w_m, w_M]$. To verify that the firm cannot earn higher profits by offering a wage not in $[w_m, w_M]$, first recognize that no wage below w_m will attract labor, so no firm would choose such a wage. Alternatively, no firm will offer a wage above w_M, because $TV(w) = \delta T$ for all $w \geq w_M$ and any higher wage gives a greater wage bill with no fewer quits than at w_M, and therefore smaller profits.

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[4] In Figure 1 the capital intensity k^* is chosen to satisfy equation (6). At this optimal intensity $pq(k) - \rho k = w + T\delta$ by equation (5). Note that k^* is invariant to the wage offer. In Figure 2 the vacancy rate is plotted under the assumption that an interval $[w_m, w_M]$ of wages is offered. $V(w)$ is determined by equation (7) and the boundary condition at w_M. Below w_m $V(w)$ is effectively infinite.

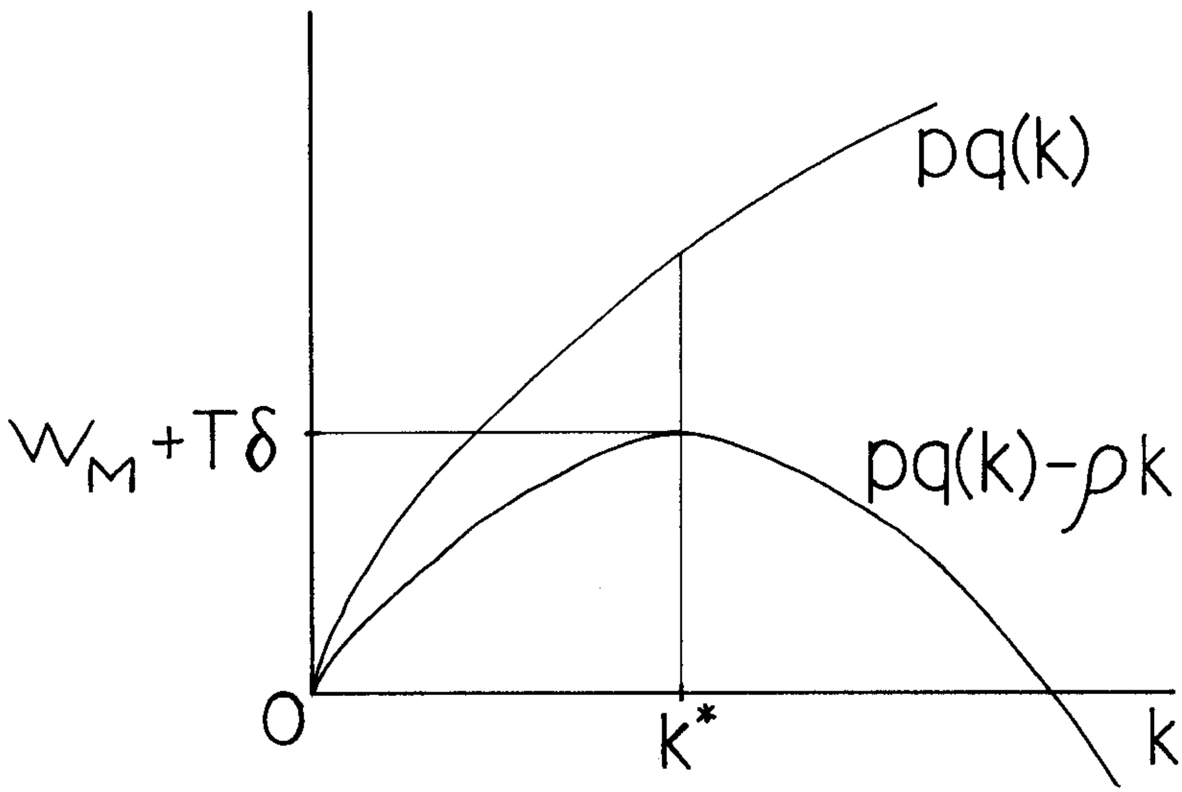

FIGURE 1

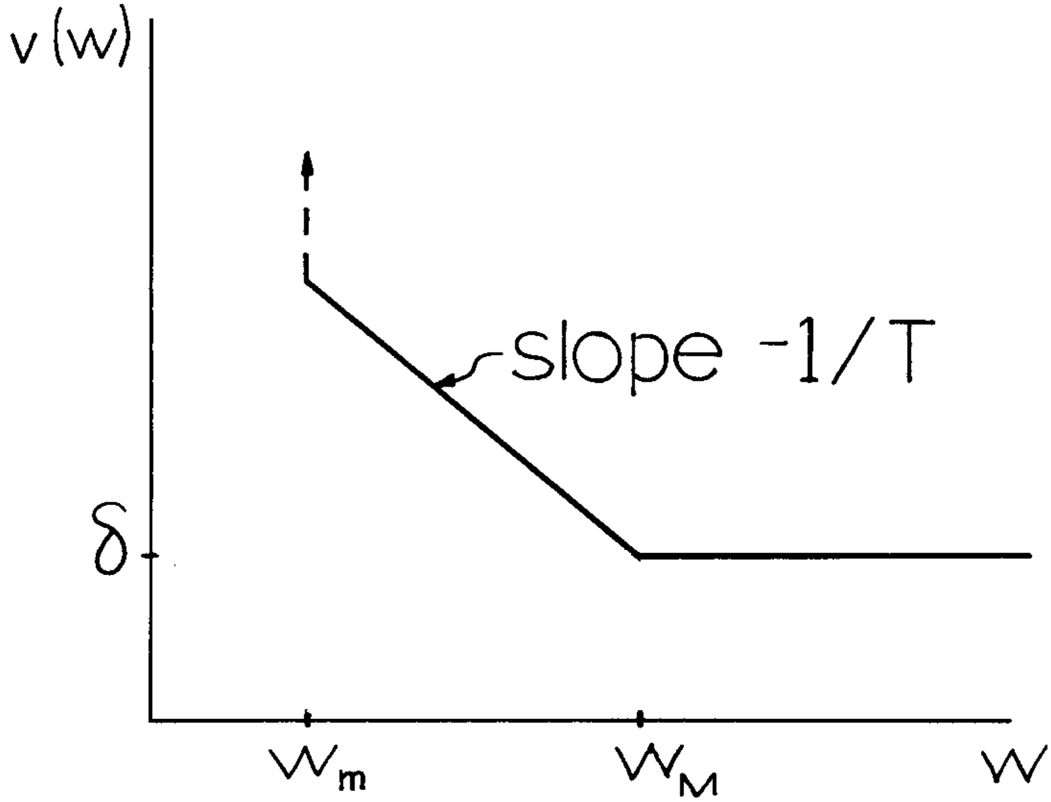

FIGURE 2

By using equations (4) and (7) we can determine the functional form of the vacancy rate $V(w)$ from the distribution $F(\cdot)$ of wages by workers. We assume $F(w) = \int F'(w)$ for some density $F'(w)$ for $w > w_m$. In fact this must be true if all wages between w_m and w_M are offered in equilibrium, as is shown in footnote 5 (there may, however, be a mass point of workers at w_m).

$$V(\hat{w}) = \delta + r(w)$$

(8)

$$= \delta + \int_{\hat{w}}^{w_M} \frac{V(w)F'(w)}{F(w)} dw ,$$

therefore,

$$V'(\hat{w}) = \frac{-V(\hat{w})F'(\hat{w})}{F(\hat{w})}$$

and

$$-\frac{V'(\hat{w})}{V(\hat{w})} = \frac{F'(\hat{w})}{F(\hat{w})} .$$

Integrating, we have

$$\log V(\hat{w}) + \log F(\hat{w}) = c_1 = \log c_2$$

for some c_2. This means

$$V(\hat{w}) = \frac{c_2}{F(\hat{w})}$$

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[5] Suppose to the contrary that there is a mass point of workers earning $\bar{w}$ for $w_m < \bar{w} \leq w_M$. Then for ε sufficiently small, firms prefer offering $\bar{w}$ to offering $\bar{w} - \varepsilon > w_m$ because of the discrete jump in the quite rate at $\bar{w}$.

where the constant of integration c_2 is found to equal δ by setting $\hat{w} = w_M$. We have

$$(9) \qquad V(w) = \frac{\delta}{F(w)}$$

Using equation (7) allows an explicit determination of $V(w)$ and $F(w)$.

$$V'(w) = -1/T ,$$

so

$$V(w) = -w/T + c_3$$

where c_3 is a constant of integration. Using the fact that $V(w_M) = \delta$ we have

$$(10) \qquad V(w) = \delta + \frac{w_M - w}{T} .$$

Then finally we have from (9) and (10)

$$(11) \qquad \begin{aligned} F(w) &= \frac{\delta T}{w_M - w + \delta T} , & w_m \le w \le w_M \\ &= 1 , & w > w_M \\ &= \mu , & w < w_m \end{aligned}$$

where μ is the unemployment rate $\mu = U/N$ and represents the number of qualified workers not working in the industry. All we can say about μ at this point is that $\mu \in [0, \delta T/(w_M - w_m + \delta T)]$. For $\mu = 0$, this distribution is plotted in Figure 3.

We have just proven:

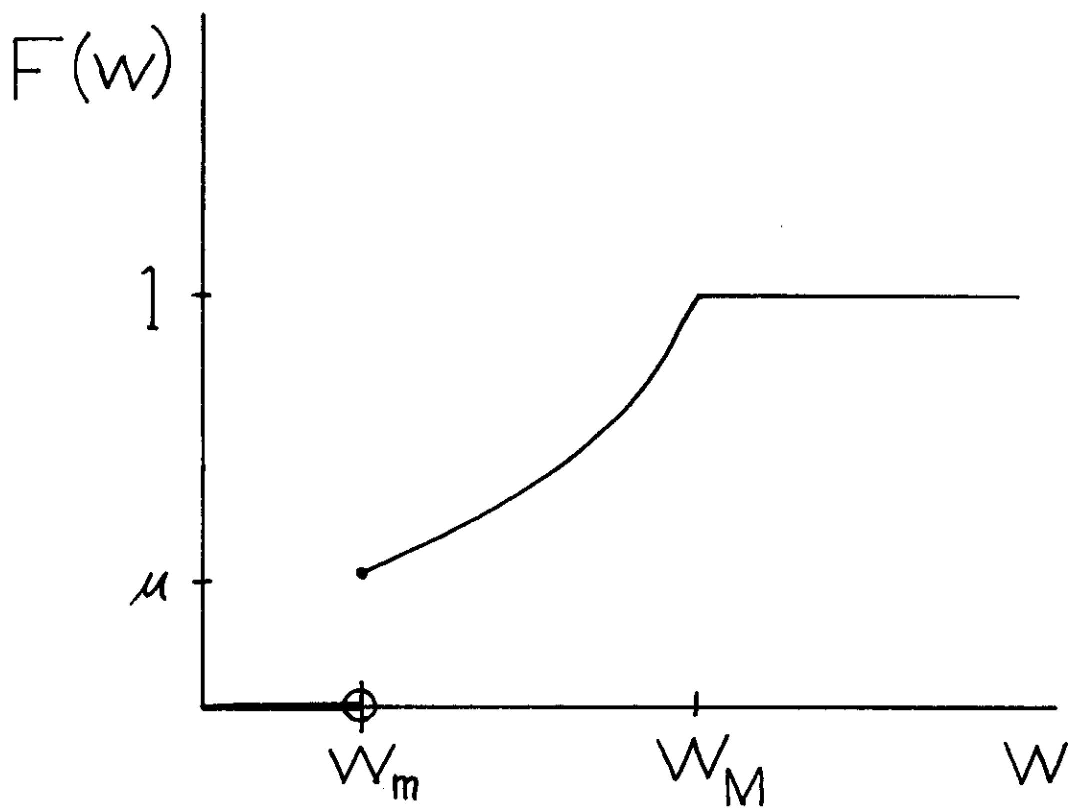

FIGURE 3

Proposition 1. In an equilibrium with an interval of wages the distribution of workers must be given by:

$$F(w) = \frac{\delta T}{w_M - w + \delta T} \quad , \quad w_m \leq w \leq w_M$$

$$= 1 \quad , \quad w \geq w_M$$

$$\equiv \mu \in [0, \delta T/(w_M - w_m + \delta T)] \quad , \quad w < w_m .$$

3. Industry Supply and Employment

Denote the level of employment in the industry by E. By the results of the last section we have in the interval case,

(12) $$E = N(1-\mu) \in [N(w_M - w_m)/(\delta T + w_M - w_m), N] .$$

Since $\pi = 0$ in equilibrium we have,

(13) $$w_M = pq(k) - \rho k - \delta T$$

which depends on p directly and through k by equation (6). (Recall that the rental rate ρ is exogenous.)

We have from these last two equations and equation (6), the relationship between employment and the money price of output. For each output price there is a range of employment levels, where the lower bound E_{min} depends on p and the upper bound is N. It is easy to verify that the lower bound increases in p:

$$\frac{dE_{min}}{dP} = N\left[\frac{\delta T}{(\delta T + w_M - w_m)^2}\right]\frac{dw_M}{dP} > 0$$

because from equations (6) and (13), using $k = q'^{-1}[\rho/p]$,
$w_M = pq(q'^{-1}(\rho/p)) - \rho q'^{-1}(\rho/p) - \delta T$.

$$\frac{dw_M}{dP} = q + (pq' - \rho)(q'^{-1})'(-\rho/p^2) = q > 0 \ .$$

The reason that the lower bound increases in p is that the price increase increases the separation between w_m and w_M , allowing more "room" for workers in the industry. (Also when $w_M >> w_m$, it is very easy for firms to enter at w_m when there is significant unemployment.)

The industry's supply of output is just the intensity of output times employment:

$$Q = q(k)E \ .$$

Since E lies in an interval for each p and q(k) is a function of p (recalling that $k = q'^{-1}(\rho/p))$, Q can take on any value in an interval for each p . Because $\partial q/\partial P > 0$
and because the lower bound for E is increasing in p and the upper bound constant, both of the endpoints of the interval in which Q must lie increase as p increases.

It should be apparent that this description of output quantities as a function of price is the Marshallian supply correspondence for this market. Since this supply correspondence is well-behaved (upper hemi-continuous, convex-valued, non-decreasing and zero at zero price), equilibrium will exist with any well-behaved demand function or correspondence (continuous or upper hemi-continuous and convex-valued, non-increasing, asymptotically zero for sufficiently high price).

In Figures 4 and 5 we have graphed the industry employment and

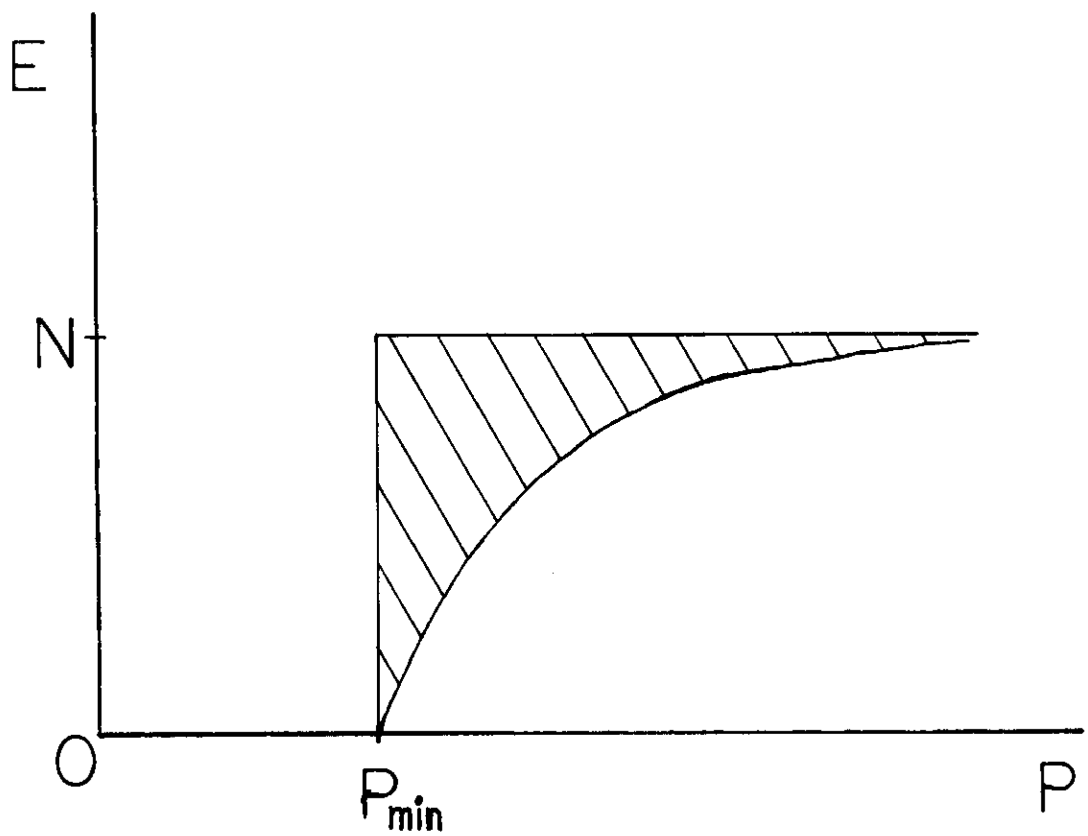

FIGURE 4

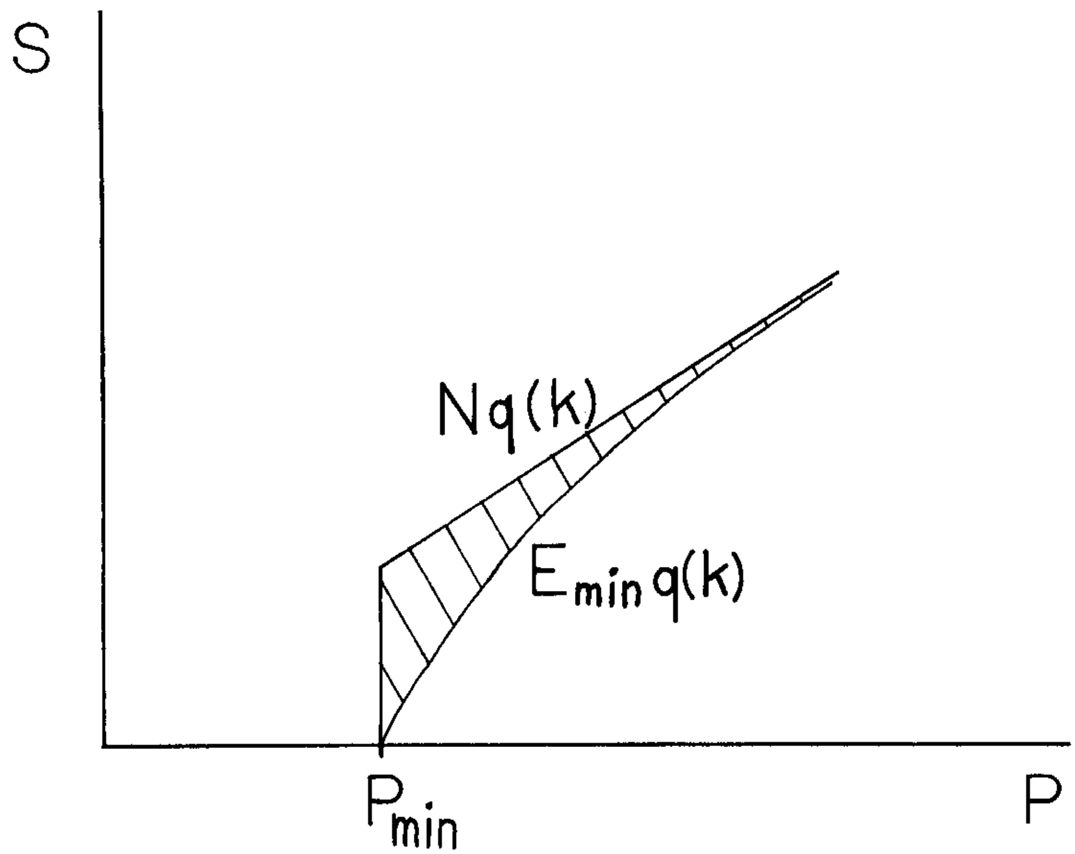

FIGURE 5

supply correspondences for the specific case where $q(k) = 2k^{1/2}$. The achievable levels of employment have been shown to depend upon the output price level. The price p_{min} is the output price at which a firm paying wage w_m and experiencing the minimum vacancy rate δ just breaks even. So p_{min} is the industries shutdown point of elementary textbook fame. For each price level above p_{min} , an interval of employment levels is possible. Analogously, Figure 5 depicts the fact that industry output also depends upon the price level. At a given price, maximal output corresponds to full employment. Equilibrium occurs at the intersection of the supply correspondence and the industry demand curve (not shown). The linear <u>upper</u> boundary of the supply correspondence results from the particular choice of $q(k) = 2k^{1/2}$.[6]

4a. Characterizing Equilibrium

In Sections 1 through 3, we have characterized the equilibria in which an interval $[w_m, w_M]$ of wages is observed in the market. Now we will derive the general class of equilibria from this case. We will find that the equilibrium employment, maximum wage, price, quantity of output, and capital intensity will always correspond to those of some interval equilibrium. Therefore, our calculations of industry supply and employment in Section 3 are valid more generally. What is different is the degree to which training costs are incurred when workers

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[6] $q'(k) = k^{-1/2}$ and from the first order condition $pq'(k) = \rho$ we derive $q(k) = 2(p/\rho)$. $S(p)_{max} = q(k) \cdot N = (2p/\rho)N$. One can also derive that $S(p)_{min} = (2p/\rho)N(1 - [\delta T/(p^2 - \rho w_m)/\rho])$.

change jobs. Since the retraining serves no socially productive purpose, all retraining is inefficient. Some retraining occurs whenever more than a single wage is offered, so the only efficient equilibria have a single wage. Unemployment is also inefficient (unless $w_m = w_M$), so we know that only full employment single wage equilibria are efficient. (In fact, these equilibria are efficient and correspond to the Marshallian equilibria.)

In constructing all equilibria from interval equilibria, agents who were offered wages in an interval are all given instead the maximum wage in the interval. The firms for which they work still have the same wage bill (including training costs) and output, so the new wage distribution is still in equilibrium. However, the workers are better off, with their increase in welfare exactly financed by the decrease in training costs. In general, a countably infinite number of such operations may be required to generate the desired equilibrium. However, we can do this formally in a single step through the following result:

Proposition 2. $F(\cdot)$ is an equilibrium wage distribution with corresponding attributes μ, w_M, p, Q, and k if and only if there is an equilibrium $\hat{F}(\cdot)$, with an interval $[w_m, w_M]$[7] of wages offered, having the same attributes just listed, and satisfying $F(w) = \hat{F}(w)$ on the support of $F(\cdot)$ and $F(w) \leq \hat{F}(w)$ outside the support of $F(\cdot)$. (Recall that the support of $F(\cdot)$ is the set of wages actually offered when wages have the distribution function $F(\cdot)$).

We give a sketch of the formal proof of this Proposition. To see

[7] We must include the degenerate case for which $w_m = w_M$.

that any $F(\cdot)$ is related to some $\hat{F}(\cdot)$ in this way, we need only note that all equilibrium conditions (including supply = demand) still hold. An important condition to note is that $F(w) = \hat{F}(w)$ for offered wages implies that the profits are constant at zero for these wages and that $F(w) \leq \hat{F}(w)$ elsewhere implies higher training costs and negative profits.

To see that every equilibrium distribution $F(\cdot)$ has this property we need only note that it must be related to the unique interval distribution having the same output price, unemployment rate, and maximum wage. The requirements that $F(w) = \hat{F}(w)$ for w offered (in $F(\cdot)$) and $F(w) \leq \hat{F}(w)$ are implied by two facts. First, since $F(\cdot)$ is an equilibrium, profits are zero for offered wages and non-positive for other wages. Second, since $\hat{F}(\cdot)$ has all wages in $[w_m, w_M]$ offered, profits are zero on this whole interval. The requirements on $F(\cdot)$ (for $[w_m, w_M]$) follow immediately from these two facts and the definition of profits. That $F(\cdot)$ and $\hat{F}(\cdot)$ agree elsewhere follows from our choice which required equal unemployment rates and equal maximum wage.

4b. Some Examples of Equilibria

Proposition 2 demonstrates the existence of equilibria with any finite number of wages as well as equilibria with a continuum of wages offered on a set of disconnected intervals each contained in $[w_m, w_M]$. Examples of this are depicted in Figures 6 and 7. In Figure 6 is shown a discrete wages equilibrium with six wages. The solid lines represent the discrete equilibrium distribution, while the dashed curve is the corresponding continuum equilibrium of Proposition 1. A more general

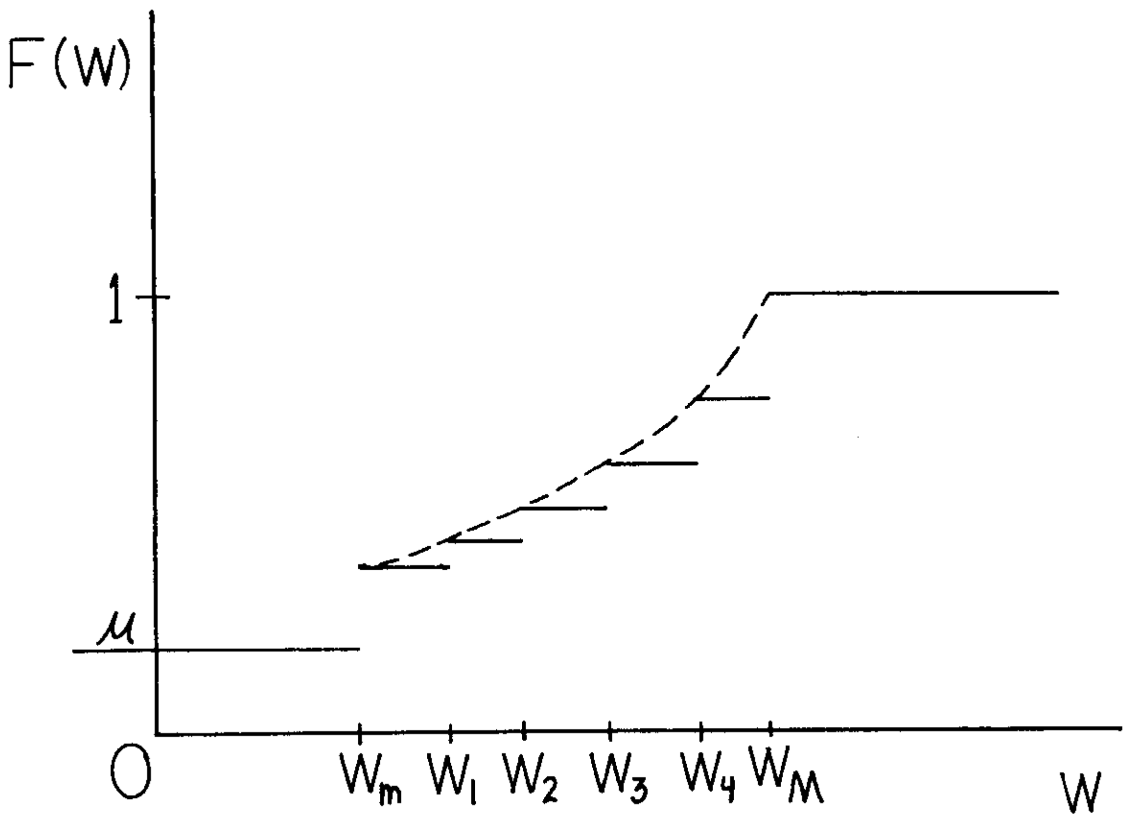

FIGURE 6

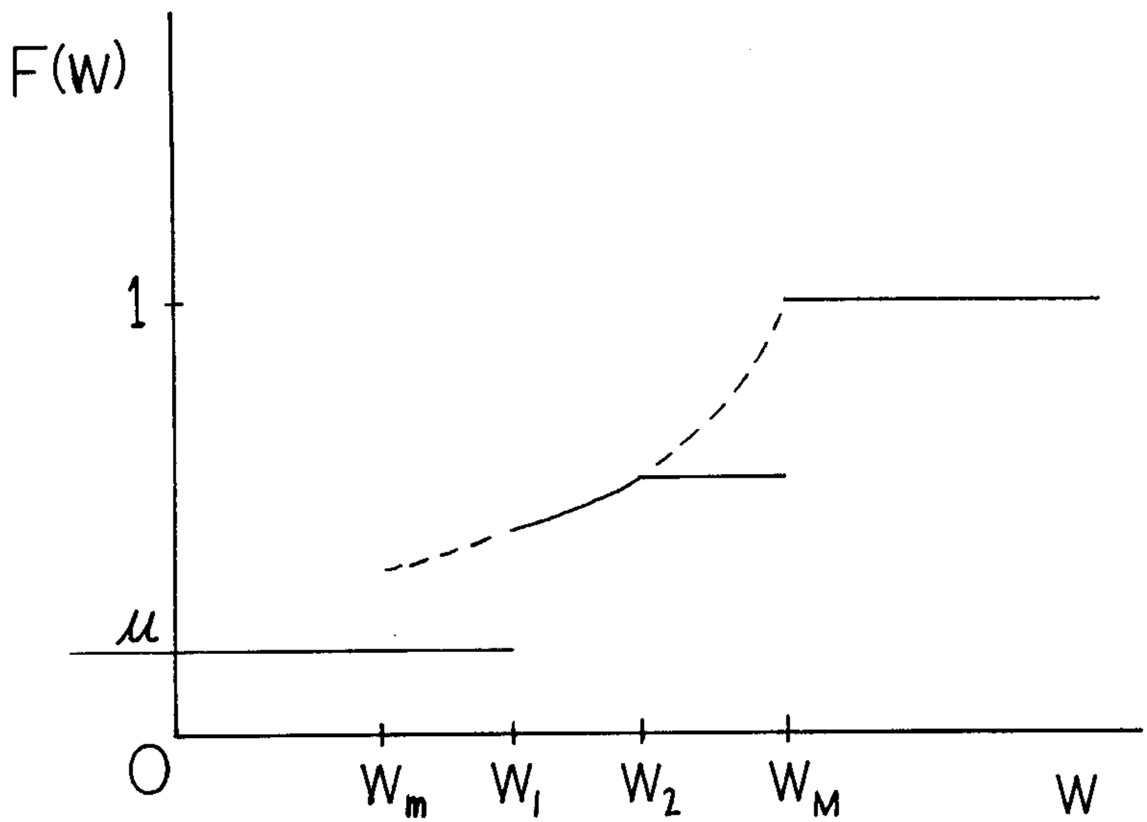

FIGURE 7

case is shown in Figure 7 where the distribution of wages by workers is again the solid lines. In this equilibrium the offered wages are the interval of wages $[w_1, w_2]$ and the wage w_M. This example illustrates the corollary to Proposition 2 that in any equilibrium, the maximum wage w_M of the corresponding continuum single interval equilibrium must be offered, while the minimum wage w_m need not be.

The existence of an infinite number of equilibria is likely to be unsettling to some economists. All of the equilibria with more than one wage are Pareto inferior to the single wage equilibrium. In the multiple wage equilibria all workers at low wage firms could be made better off with no loss to firms. It might be thought that since all the multiple wage equilibria are Pareto inferior to the corresponding single wage equilibrium (at w_M) some kind of competitive force might lead to the elimination of the dominated equilibria as in the labor market signalling-screening models. In this labor market there are no such forces because unlike screening equilibria each of the industry equilibria here is a Nash-Cournot Equilibrium.[8]

5. Conclusions

The main result of this paper, an equilibrium distribution of wages with homogeneous labor, identical firms, and perfect information seems contrary to intuition. In fact the absence of imperfect information is necessary to support the result. If we were to introduce imperfect information into the model in the usual way by assuming that all workers

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[8] This inefficiency property of Nash Equilibria is similar to the Nash equilibria in insurance markets with adverse selection analyzed in Jaynes [1978].

knew the distribution of workers (jobs) by wages, but not the location of any particular job it is well known that optimal agent behavior would result in the collapse of the wage distribution to a single wage between w_m and w_M . Imperfect information endows either sellers or buyers with monopoly or monopsony power!

In an interesting model that is similar to ours J. E. Stiglitz [1976] prevents this problem by assuming that the continual arrival of new entrants into the labor market causes enough ignorance about the distribution of wages to prevent the collapse to monopsony. The basic structure of his model is however, similar enough to ours that a reader familiar with it should be able to see that the Stiglitz model would also support equilibrium wage distributions if workers were endowed with perfect information. The essential elements of both models are the existence of hiring-training costs for employees and the fact that firms unaided by a centralized Walrasian auctioneer must make their own wage policies. Given these two conditions what we have shown is that the standard competitive equilibrium is not the only Nash-Cournot Equilibrium consistent with atomistic competition in the labor market. Therefore, if firm hiring-training costs are not negligible, the common hypothesis of the existence of a Walrasian auctioneer to facilitate the work of the "invisible hand" in our competitive models of the labor market is not as innocuous an assumption as has been believed.

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