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On Disequilibrium Economic Dynamics. Part IV. The Theory of Long-Run Phillips Curve

Katsuhito Iwai

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

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by

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1. Introduction

It is the purpose of the present paper to demonstrate that the economy with downward rigidity of money wage will never lose its Keynesian features no matter how long it is run. Rather paradoxically, however, Keynes himself seems to have entertained an opposite view in "The General Theory of Employment, Interest and Money," when he wrote--

> Our criticism of the accepted [neo-]classical theory of economics has consisted not so much in finding logical flaws in its analysis as in pointing out that its tacit assumptions are seldom or never satisfied, with the result that it cannot solve the economic problems of the actual world. But if our central controls suceed in establishing an aggregate volume of output corresponding to full employment as nearly as is practicable, the [neo-]classical theory comes into its own again from this point onwards.[1]

This remark has become the foundation of the "neoclassical synthesis" and we need little textual evidence to assert that many contemporary economists more or less share the same view.[2] Even Axel Leijonhufvud who has recently

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*This is Part IV of a series of papers on disequilibrium economic dynamics. This paper is, however, self-contained. Research described in this paper was supported by grants from the National Science Foundation and the Ford Foundation.

[1] Keynes [11], p. 378.

[2] See, for example, Samuelson [18] for the textbook account of the neoclassical synthesis.

given us a forceful argument that the Keynesian notion of underemployment equilibrium should be interpreted as a state of "persistent disequilibrium" is not exempt from this.[3]

Keynes meant by the [neo-]classical theory a set of doctrines according to which all the "real" variables, such as employment of resources, production and consumption of commodities and their relative prices, are determined by equilibrium between supply and demand and all the "nominal" variables are left to be determined by the quantity theory of money. The neoclassical description of the economy can therefore be characterized by the following two interdependent propositions--(i) that there is no such thing as "involuntary" unemployment and (ii) that money is neutral in the sense that all the "real" variables are determined independently of the nominal quantity of money.

Recent controversy over the theoretical interpretation of the "Phillips curve"--the empirical finding of a negative relation between the rate of change in money wage and the rate of unemployment--has directly centered around the validity of these neoclassical propositions in the long-run. The proponents of the so-called "natural rate theory of unemployment," led by Milton Friedman and Edmund Phelps, deny the existence of the Phillips curve relation in the long-run.[4] They argue that there is only one rate of unemployment which is consistent with an equilibrium in which anticipations about the rate of inflation are not disappointed by the actual rate of inflation. This equilibrium rate of unemployment is called the

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[3] Leijonhufvud [12].

[4] Friedman [3] and Phelps [16]. See also Lucas [13, 14] and Sargent [19] for the recent development of the natural rate theory of unemployment.

"natural rate of unemployment" and its level is considered to be invariant under changes in the time-pattern of inflation rate. In the short-run, there is always a "trade-off" between unemployment and unanticipated inflation. But, in the long-run if a given rate of inflation comes to be correctly anticipated, the rate of unemployment will return to the constant natural rate; hence, there is no permanent trade-off between unemployment and inflation. The Phillips curve, thus argue the natural rate theorists, is vertical in the long-run at the level equal to the constant natural rate. In spite of its theoretical sophistication, the natural rate theory of unemployment has only elaborated the two neoclassical propositions stated previously. It has left the neoclassical picture of the economy essentially intact. In fact, Milton Friedman characterized the natural rate of unemployment as "the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is imbedded in them the actual structural characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labor availabilities, the cost of mobility, and so on."[5]

In the present paper, we shall show that even if the central authority succeeded in establishing a level of aggregate demand in balance with that aggregate supply and even if no anticipations were ever disappointed by actual market outcomes the achieved state of the economy--a long-run equilibrium--would have nothing in common with the picture painted by the neoclassical equilibrium theory. First, we shall establish that in the economy with wage rigidity the "involuntary" unemployment would persist

[5] Friedman [3], p. 8.

above the constant natural rate level even in the long-run equilibrium. There is no state of the economy where there exists no involuntary unemployment. Second, we shall demonstrate that in the economy with downward rigidity of money wage the long-run equilibrium rate of involuntary unemployment is negatively correlated with the steady-state rate of wage inflation. Or, to put this proposition differently, we shall prove that money is not neutral even in the long-run, nor does the long-run Phillips curve become vertical. The economy we happen to live in can never approach the beautiful and optimum neoclassical world "even in the economist's never never land of the long run."

2. The Structure of the Labor Market

Let us consider a closed economy whose total labor market consists of m business firms as employers and a much larger number of workers as potential employees. We shall distinguish firms by indexes $i = 1, 2, \ldots, m$. To simplify the analysis we assume that all workers are homogeneous and that they can be regarded as a completely variable factor in production.[6] However, firms as employers are assumed to be differentiated from each other to the eyes of workers; either because their geographical locations are different; or because they offer different working conditions and non-pecuniary returns to workers; or simply because they can exploit the imperfect information of workers about the distribution of wage offers among them. This means that we can regard the whole labor market as being composed of numerous interdependent labor markets, each controlled by a single firm. We assume that the only means available to each firm in the short-run in regulating the supply of labor-services to itself is the level of its own

[6] This assumption can be somewhat relaxed. See Hall [4].

money wage relative to the other firms' money wages. Furthermore, we also assume that the number of firms m is very large, so that each firm determines its own money wage level without taking account of possible repercussions on the future labor supply schedules through other firms' responses.

In short we picture the labor market of our Keynesian economy as a world of numerous monopsonists competing with each other for a given aggregate level of labor supply.

Let us start our analysis by reviewing the micro-dynamic theory of an individual firm, developed in Part III of this series of papers.[7] The reader familiar with Part III may skip the next section and directly proceed to section 4.

3. A Keynesian Model of Money Wage Adjustment

Consider the entrepreneur of the i^{th} firm who must decide the level of money wage at the beginning of period, say, t. Denote by w^*_{it} the logarithmic level of money wage that would uniquely maximize the i^{th} firm's short-run expected profit in period t, were there not any money wage adjustment costs. We shall call w^*_{it} the logarithmic level of the "optimal" money wage, for short. (Warning: in the following we shall measure all the variables by logarithmic scale!) This "optimal" money wage is determined by such factors as the entrepreneur's expectations of the current as well as future states of product and labor markets, the fixed productive factors endowed in his firm, the technology available to him, and so on. But its detailed specification does not concern us in

[7] Iwai [10].

the present section.[8]

Let w_{it} denote the logarithmic level of money wage actually quoted by the entrepreneur of the i^{th} firm in period t. If there were no costs involved in the money wage adjustment activity, his optimal money wage adjustment policy would be simply to equate w_{it} with w^*_{it} in every period. However, the introduction of money wage adjustment costs would prevent the entrepreneur from indulging in a quiet life.

Rather than attempting to deduce the fully optimal money wage adjustment policy, however, we assumed in Part III of this series that the entrepreneur chooses the best policy out of a restricted class of adjustment rules which can be specified as follows:

$$(1)\qquad \begin{cases} w_{it} = w_{i,t-1} & \text{when } \lambda_{2i} < w^*_{it} + \lambda_{0i} - w_{i,t-1} < \lambda_{1i}\,, \\ w_{it} = w^*_{it} + \lambda_{i0} & \text{when } w^*_{it} + \lambda_{0i} - w_{i,t-1} \leq \lambda_{2i} \text{ or } w^*_{it} + \lambda_{0i} - w_{i,t-1} \geq \lambda_{1i}\,, \end{cases}$$

where λ_{0i} ($\gtreqless 0$), λ_{1i} (> 0) and λ_{2i} (< 0) are constant parameters whose values must be chosen by the entrepreneur.[9] Defining

$$(2)\qquad x_{it} - \lambda_{0i} \equiv w^*_{it} - w_{it}\,,$$

$$(3)\qquad \xi_{it} \equiv w^*_{it} - w^*_{i,t-1}\,,$$

we can tranforms (1) into the following rule:

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[8] See Iwai [7,8] or section 5, for the determination of the "optimal" money wage in a highly parametrized model of the firm.

[9] Under the conditions that a money wage adjustment incurs only a fixed transactions cost and that the future motion of w^*_{it} can be characterized by the Bernouilli trial random walk model it is likely that the form of money wage adjustment rule given by (1) is the "optimal" one. See Vial [23] for the proof of this proposition in the case of continuous-time cash management problem.

$$(2)\quad \begin{cases} x_{it} = x_{i,t-1} + \xi_{it} & \text{when } \lambda_{2i} < x_{i,t-1} + \xi_{it} < \lambda_{1i}\,, \\ x_{it} = 0 & \text{when } x_{i,t-1} + \xi_{it} \leq \lambda_{2i} \text{ or } x_{i,t-1} + \xi_{it} \geq \lambda_{1i}\,. \end{cases}$$

Then, the parameter λ_{1i} is the "ceiling threshold," the parameter λ_{2i} the "floor threshold," and the eliminated parameter λ_{0i} is the "returning point," of the specified money wage adjustment rule. It should be also noted that $x_{it} - \lambda_{0i}$, defined by (2), can be regarded as the i^{th} entrepreneur's "subjective or ex ante measure of disequilibrium" in his own labor market. If we further assume that the entrepreneur believes that the sequence of the rates of change of the "optimal" money wage, $\{\xi_{it}\}$, are mutually independent random variables drawn from a common subjective probability distribution:[10]

$$(5)\qquad {}^{i}\hat{F}(\xi) \equiv {}^{i}\hat{\Pr}\{\xi_{it} \leq \xi\}\,,$$

whose mean is equal to:

$$(6)\qquad {}^{i}\hat{\mu} \equiv \int_{-\infty}^{\infty} \xi \cdot d\,{}^{i}\hat{F}(\xi)\,,$$

then the sequence of the random variables $\{x_{it}\}$, given by (4), constitutes (subjectively to him) a "random walk model with two return barriers at λ_{1i} and λ_{2i}."

Suppose that at the beginning of period say, zero, the i^{th} entrepreneur predicts the future position of the random variable x_{it} on the basis of the information available to him at that time, represented by

[10] We denote by ${}^{i}\hat{\Pr}\{z\}$ the i^{th} entrepreneur's subjective probability of an event z.

x_{i0} . He can summarize this prediction by the subjective "transition probability":

$$(7) \qquad {}^{i}\hat{\Pi}_{t}(x|x_{i0}) \equiv {}^{i}\hat{Pr}\{x_{it} \leq x|x_{i0}\} \ ;$$

where ${}^{i}\hat{Pr}\{z_{t}|{}^{i}\theta\}$ denotes the i^{th} entrepreneur's subjective expectation of z_{t} conditional upon his information ${}^{i}\theta$. As time goes on, he can expect that the level of money wage will be adjusted over and over again and that the motion of the random variable x_{it} will approach a "stochastic steady-state." In the Mathematical Appendix of Part III,[11] we were able to prove

Proposition 1 (Steady-State Theorem). If (i) both λ_{1i} and λ_{2i} are finite, or (ii) λ_{1i} is finite, $\lambda_{2i} = -\infty$ and $0 < {}^{i}\hat{\mu} < \infty$, or (ii) λ_{2i} is finite, $\lambda_{1i} = +\infty$ and $-\infty < {}^{i}\hat{\mu} < 0$, then as $t \to \infty$ the sequence of the random variables $\{x_{it}\}$ converges to a steady-state random variable $\underset{\sim}{x}_{i}$ in the sense that ${}^{i}\hat{\Pi}_{t}(x|x_{i0})$ converges to a steady-state distribution ${}^{i}\hat{\underset{\sim}{\Pi}}(x)$, independently of the initial condition x_{0i} ; that is, we have

$$(8) \qquad \lim_{t\to\infty} {}^{i}\hat{\Pi}_{t}(x|x_{i0}) = {}^{i}\hat{\underset{\sim}{\Pi}}(x) \equiv {}^{i}\hat{Pr}\{\underset{\sim}{x}_{i} \leq x\} \ .$$

This steady-state distribution is determined by the two threshold parameters λ_{1i} and λ_{2i} and by the subjective probability distribution ${}^{i}\hat{F}(\xi)$. We can interpret ${}^{i}\hat{\underset{\sim}{\Pi}}(x)$ either as the description of the stochastic motion of the random variable x_{it} in the "stochastic steady-state" or as the "long-run average" description of the motion of x_{it} starting from an arbitrary initial condition.

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[11] Iwai [10], Theorem 1.

It is clear that the movement of the rate of change of the actual money wage, $\Delta w_{it} \equiv w_{it} - w_{i,t-1}$, is inherently discrete. There is an upward jump by the rate equal to $w^*_{it} + \lambda_{0i} - w_{i,t-1} = x_{i,t-1} + \xi_{it}$ when $x_{i,t-1} + \xi_{it} \geq \lambda_{1i}$ in period t, and a downward jump by the rate equal to $-(x_{i,t-1} + \xi_{it})$ when $x_{i,t-1} + \xi_{it} \leq \lambda_{2i}$ in period t; otherwise the adjustment of the level of money wage is postponed in the future. However, the analysis of its short-run as well as long-run average behaviors would shed a useful light on our understanding of the nature of the entrepreneur's money wage adjustment activity. It is in fact easy to calculate the subjective expected rate of money wage change in period t, predicted on the basis of the initial condition x_{i0}:[12]

$$(9) \qquad {}^{i}\hat{\mathcal{E}}(\Delta w_{it} | x_{i0}) = {}^{i}\hat{\mathcal{E}}\{x_{i,t-1} + \xi_{it} | x_{i,t-1} + \xi_{it} \geq \lambda_{1i} \text{ or } x_{i,t-1} + \xi_{it} \geq \lambda_{2i};\ x_{i0}$$

$$= {}^{i}\hat{\mu} - \int_{{}^{i}\hat{\lambda}^*_2}^{{}^{i}\hat{\lambda}^*_1} \{z \cdot d\,{}^{i}\hat{F}(z-y) - y\} \cdot d\,{}^{i}\hat{\Pi}_{t-1}(y | x_{i0}) \ ;$$

where ${}^{i}\hat{\mathcal{E}}(z_t | {}^{i}\theta)$ denotes the i^{th} entrepreneur's subjective expectation of z_t conditional upon his information ${}^{i}\theta$. Under certain reasonable specifications of ${}^{i}\hat{F}(\xi)$, the expected rate of money wage change can be shown to be an increasing function of the initial condition x_{i0} (or the ex ante measure of labor market disequilibrium in period zero $x_{i0} - \lambda_{0i}$). Thus, we can regard (9) as the i^{th} entrepreneur's "perceived law of supply and demand" in period t based upon his information available in period zero. However, in the long-run, this perceived law of supply and demand will evaporate! That is, as time goes on, the influence of the initial

[12] Iwai [10], eq. (10).

condition will gradually fade away; and in the limit the expected rate of change in the actual money wage will become equal to ${}^{i}\hat{\mu}$, the constant expected rate of change in the "optimal" money wage.[13] We thus have

Proposition 2. Under one of the conditions stated in Proposition 1, we have

$$(10) \qquad {}^{i}\hat{\mathcal{E}}(\underset{\sim}{\Delta w}_i) \equiv \lim_{t\to\infty} {}^{i}\hat{\mathcal{E}}(\Delta w_{it} | x_{i0}) = {}^{i}\hat{\mu} ,$$

independently of the initial condition x_{i0}.

The entrepreneur of the i^{th} firm must choose the optimal values of the thre parameters, λ_{0i}, λ_{1i} and λ_{2i}. He is faced with a trade-off between the opportunity cost associated with the disequilibrium in his labor market and the direct cost associated with the money wage adjustment activity itself. We assumed in Part III that when there is no money wage adjustment cost the i^{th} firm's short-run expected profit in period t can be specified as ${}^{i}\hat{\rho}(w^{*}_{it} - w_{it}) \cdot \exp(\hat{v}_{it})$; where $\exp(\hat{v}_{it})$ represents the "trend level" of the i^{th} firm's expected profit which is beyond the control of the entrepreneur's short-term money wage adjustment policy, and ${}^{i}\hat{\rho}(w^{*}_{it} - w_{it}) \equiv {}^{i}\hat{\rho}(x_{it} - \lambda_{0i})$ is the "trend-free expected profit function" which depends only upon his ex ante measure of labor market disequilibrium.[14] Now, since w^{*}_{it} is defined as the logarithmic level of money

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[13] Iwai [10], eq. (12).

[14] If we substitute eq. (34) of section 5 into eq. (48) of Iwai [8], we can obtain a short-run expected profit function of this form. But a variety of models can generate the same expected profit function.

wage that would uniquely maximize the short-run expected profit were there not money wage adjustment costs, ${}^{i}\hat{\rho}'(0) = 0$ and ${}^{i}\hat{\rho}''(0) < 0$, if ${}^{i}\hat{\rho}(\cdot)$ is twice-differentiable. Then, the "trend-free opportunity cost" associated with the ex ante labor market disequilibrium (or the "trend-free disequilibrium cost," for short) can be unambiguously defined as the difference between the maximum attainable value of the trend-free short-run expected profit ${}^{i}\hat{\rho}(0)$ and the actual trend-free short-run expected profit ${}^{i}\hat{\rho}(w^{*}_{it} - w_{it}) \equiv {}^{i}\hat{\rho}(x_{it} - \lambda_{0i})$. We can approximate this quadratically as

$$(11) \qquad {}^{i}\hat{\rho}(0) - {}^{i}\hat{\rho}(x_{it} - \lambda_{0i}) \cong - \frac{{}^{i}\hat{\rho}''(0)}{2} \cdot (x_{it} - \lambda_{0i})^{2} .$$

Against this disequilibrium cost, the entrepreneur must weigh the direct cost of money wage adjustment. We assumed in Part III that if he raises the level of money wage in period t he incurs a lump-sum adjustment cost, $c_{1i} \cdot \exp(\hat{v}_{it})$; and if he cuts it in period t he incurs a different lump-sum cost, $c_{2i} \cdot \exp(\hat{v}_{it})$. The trend-free cost parameters, c_{1i} and c_{2i}, are all assumed to be invariant over time.

The "total loss" for the firm in period t is nothing but the sum of the disequilibrium cost and the lump-sum adjustment cost. We supposed in Part III that the entrepreneur chooses the values of λ_{0i}, λ_{1i} and λ_{2i} so as to minimize the "long-run average of the trend-free total loss per unit period."[15] He has a very long planning horizon and is concerned only with the trend-free loss in the determination of the optimal money wage adjustment policy.

Minimizing the long-run average trend-free loss with respect to

[15] See eq. (17) of Iwai [10] for the explicit formula.

λ_{0i}, we established

Proposition 3. The optimal value of λ_{0i}, denoted by $\hat{\lambda}^*_{0i}$ is equal to the long-run average of the random variable x_{it}, i.e., we have

$$\hat{\lambda}^*_{0i} = {}^{i}\hat{\mathcal{E}}(\underset{\sim}{x}) \ . \tag{12}$$

It then follows from the definition of x_{it} given by (2) that if the entrepreneur has chosen the value of λ_{0i} optimally the expected value of his ex ante measure of labor market disequilibrium, $w^*_{it} - w_{it} \equiv x_{it} - \hat{\lambda}^*_{0i}$, will vanish in the long-run. Thus, we can restate the above proposition as:

Proposition 3'. As $t \to \infty$, we have

$$ {}^{i}\hat{\mathcal{E}}(w^*_{it} - w_{it} | x_{i0}) = {}^{i}\hat{\mathcal{E}}(x_{it} - \hat{\lambda}^*_{0i} | x_{i0}) \to 0 \ , \tag{13}$$

independently of the initial condition x_{i0}.

Note that the validity of this proposition hinges upon the quadratic approximation of the disequilibrium cost function (11).

The optimal values of λ_{0i}, λ_{1i} and λ_{2i}, respectively denoted by $\hat{\lambda}^*_{0i}$, $\hat{\lambda}^*_{1i}$ and $\hat{\lambda}^*_{2i}$, are, in general, functions of the basic subjective parameters of the model such as $-{}^{i}\hat{\rho}''(0)$, c_{1i}, c_{2i} and ${}^{i}\hat{F}(\xi)$. However, in order to shed more light on the nature of the optimal money wage adjustment policy we must turn to a special case which allows us to calculate $\hat{\lambda}^*_{0i}$, $\hat{\lambda}^*_{1i}$ and $\hat{\lambda}^*_{2i}$ explicitly.

In the Mathematical Appendix of Part III we were able to calculate the steady-state probability distribution ${}^{i}\hat{\underset{\sim}{\Pi}}(\underset{\sim}{x})$ explicitly in the special case where the entrepreneur's subjective probability distribution of the rate of change of the "optimal" money wage ${}^{i}\hat{F}(\xi)$, is the simple Ber-

nouilli trial distribution given by

$$(14) \qquad {}^{i}\hat{\Pr}\{\xi_{it} = s_i\} = q_i \ , \quad {}^{i}\hat{\Pr}\{\xi_{it} = -s_i\} = 1 - q_i \ ;$$

where $s_i > 0$ is a step size and $0 < q_i < 1$ is the probability of a positive jump.[16] (In this case, the expected value ${}^{i}\hat{\mu}$ of the rate of change of the "optimal" money wage is equal to $(2q_i - 1)\cdot s_i$ and its variance ${}^{i}\hat{\mathrm{Var}}(\xi_t)$ is equal to $s_i^2 - {}^{i}\hat{\mu}^2$.) When we further assumed that the money wage is "absolutely rigid downwards" in the sense that $c_{2i} = +\infty$, we could compute the optimal parameter values, $\hat{\lambda}^*_{0i}$, $\hat{\lambda}^*_{1i}$ and $\hat{\lambda}^*_{2i}$ explicitly.[17]

Proposition 3' maintains that if the entrepreneur of the i^{th} firm chooses the value of the return parameter λ_{0i} optimally he can predict tha the ex ante measure of labor market disequilibrium, $x_{it} - \hat{\lambda}^*_{0i}$, will be averaged out to zero in the long-run. However, this by no means implies that the subjective disequilibrium in his labor market will disappear in the long-run. On the contrary, after a sufficiently long time his money wage adjustment activity is expected to approach only a "stochastic steady-state" inwhich his ex ante measure of labor market disequilibrium fluctuates stochastically between the state of positive disequilibrium and the state of negative disequilibrium. As long as the lump-sum money wage adjustment costs are not negligible, its "variance" will never shrink to zero. In Part III under the special assumptions stated in the above paragraph we were able to give an explicit characterization of the variance of the subjective measure of the i^{th} firm's labor market disequilibrium in the stochastic steady-state.

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[16] See eq. (29) of Iwai [10].

[17] See eq. (35) and eq. (36) of Iwai [10].

Proposition 4. If (i) ${}^{i}\hat{F}(\xi)$ is the Bernouilli trial distribution, given by (14), and if (ii) the money wage level is absolutely rigid downwards in the sense that $c_{2i} = +\infty$, the steady-state variance of the ex ante measure of labor market disequilibrium can be expressed as

$$(15) \qquad {}^{i}\hat{\mathrm{Var}}(x_i - \hat{\lambda}^{*}_{0i}) = {}^{i}\hat{\mathrm{Var}}(x_i)$$

$$= \frac{s_i^2}{4}\left(\frac{s_i^2}{{}^{i}\hat{\mu}^2} - 1\right) \qquad \text{when } 0 < {}^{i}\hat{\mu} \leq \frac{-s_i^3 \cdot {}^{i}\hat{\rho}''(0)}{12c_{1i}} ;$$

$$= 48^{-1/3}\left(\frac{2\,{}^{i}\hat{\mu}c_{1i}}{-{}^{i}\hat{\rho}''(0)}\right)^{2/3} - \frac{1}{3}s_i^2 + \frac{1}{4}\frac{s_i^2}{{}^{i}\hat{\mu}} \text{ when } \frac{-s_i^3 \cdot {}^{i}\hat{\rho}''(0)}{12c_{1i}} \leq {}^{i}\hat{\mu} \leq s ;$$

the steady-state variance becomes infinite when ${}^{i}\hat{\mu} \leq 0$.

It should be emphasized here that the steady-state variance is a decreasing function of the expected rate of change in the "optimal" money wage ${}^{i}\hat{\mu}$ at least for a relevant range of the values of the latter.

4. Stochastic Macro-Equilibrium

Let us now embark upon a macroeconomic exploration of our Keynesian economy.

In order to highlight the essential feature of our theory, let us assume in the following that all the firms in our Keynesian economy are symmetric in the sense that they are identically 'structured' from the behavioral standpoint. Therefore, we shall erase the index i from all the subjective parameters in their behavioral equations and write them simply as c_1, c_2, $\hat{\rho}(\cdot)$, $\hat{F}(\cdot)$, $\hat{\mu}$, $\hat{\lambda}^{*}_{0}$, $\hat{\lambda}^{*}_{1}$, $\hat{\lambda}^{*}_{2}$ and so on. However, this does not mean that all the firms actually behave identically.

On the contrary, their actual behaviors are quite heterogeneous both cross-sectionally and intertemporally. Workers' relative preferences for a particular firm's working condition as well as buyers' relative preference for a particular firm's product are randomly changing over time; introductions of new production processes, new marketing techniques and new recruiting methods are never uniform among firms; and accumulations of both tangible and non-tangible capitals which are based upon different long-term expectations are following different patterns from firm to firm. As a result, each of the entrepreneurs are gathering quite different market experiences and forming quite different expectations even about such variables as aggregate demand, aggregate labor supply, aggregate price and aggregate money wage that may affect all the firms' product demand and labor supply schedules uniformly. Since the "optimal" money wage w^*_{it} depends upon the entrepreneur's expectations about these relevant random variables, the motions of the actual money wage w_{it}, which is occasionally adjusted by the entrepreneur in order to keep up with the motions of w^*_{it}, will follow a non-uniform stochastic path from firm to firm. Let us make this idea more formal.

We have measured the extent of subjective or ex ante disequilibrium in the i^{th} firm's labor market by $x_{it} - \hat{\lambda}^*_0 \equiv w^*_{it} - w_{it}$ in period t. Starting from a given initial condition $x_{i0} - \hat{\lambda}^*_0$ or x_{i0}, the i^{th} entrepreneur adjusts the level of money wage or equivalently the level of the random variable x_{it}, according to the following adjustment rule:

$$(16) \qquad \begin{cases} x_{it} = x_{i,t-1} + \xi_{it} & \text{if } \hat{\lambda}^*_2 < x_{i,t-1} + \xi_{it} < \hat{\lambda}^*_1 , \\ x_{it} = 0 & \text{if either } x_{i,t-1} + \xi_{it} \leq \hat{\lambda}^*_2 \text{ or } x_{i,t-1} + \xi_{it} \geq \hat{\lambda}^*_1 . \end{cases}$$

We shall now assume that the actual sequence of the rates of change of the "optimal" money wage $\{\xi_{it}\}$ is drawn from an objective probability distribution $F(\xi)$ which is uniform across firms and independent over time; that is, we put[18]

$$(17) \qquad F(\xi) \equiv \Pr\{\xi_{it} \leq \xi\}, \quad \text{for all } t \text{ and } i,$$

whose (objective) mean μ is given by

$$(18) \qquad \mu \equiv \int \xi \cdot dF(\xi) .$$

In other words, we shall assume that the objective sequence of the "optimal" money wages of each firm will follow the multiplicative random walk path with the common average growth rate μ.[19] It then follows that the sequence of the actual values of the random variables $\{x_{it}\}$ constitutes an objective random walk model with two return barriers at $\hat{\lambda}_1^*$ and $\hat{\lambda}_2^*$, generated by the objective probability distribution $F(\xi)$. Obviously, this objective random walk model is mathematically equivalent to the subjective random walk model that was extensively investigated in the Mathematical Appendix of Part III of this series and briefly reviewed in Section 3 of the present article.

Crucial for our subsequent analysis is an observation that the ex ante measure of labor market disequilibrium in period t, $x_{it} - \hat{\lambda}_0^*$,

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[18] We denote by $\Pr\{z\}$ the objective probability of an event z. By "objective" probability we mean the probability that is consistent with all the information provided by the model as a whole, including the parameter values of the model itself. This does not necessarily imply that we believe in the "frequency" interpretation of probability.

[19] Therefore, we implicitly assume that the underlying "objective" stochastic structure itself is of the random walk type.

contains all the necessary information for our (objective) prediction of the movements of the i^{th} firm's ex ante labor market disequilibrium from the t^{th} period on, $\{x_{i,t'} - \hat{\lambda}_0^*\}$ for all $t' > t$, as long as the same money wage adjustment rule is kept in the entire future. In other words, the individual labor market in this economy possesses a "Markov" information structure. It then follows from the assumption of symmetricity among firms that if two firms happen to have the same ex ante measure of labor market disequilibrium in period t, our prediction of the future movements of these two firms' ex ante disequilibrium are identical on the basis of any information available at the beginning of period t. In period t, therefore, we can treat them simply identical (except for the scales of their money wage levels). In consequence, it is more convenient for our analysis to distinguish firms by the ex ante disequilibrium, $x_t - \hat{\lambda}_0^*$, or simply by the position of the random variable, x_t, in each period, rather than differentiating them by the fixed index i. Note that the new level x attached to a particular firm will vary from period to period, as the entrepreneur will gather new information and revise his expectations over time.

Denote by $\Pi_t(x|x_0)$ the objective transition probability distribution of the position of the variable index x_t in period t from the initial position x_0 in period zero; that is, we put[20]

$$\Pi_t(x|x_0) \equiv \Pr\{x_t \leq x|x_0\} . \tag{19}$$

It should be borne in mind that by the assumption of symmetricity $\Pi_t(x|x_0)$ is uniform for all firms.

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[20] We denote by $\Pr\{z|\theta\}$ the objective probability of z conditional upon θ.

Let us, then, suppose that all the firms in the economy happen to start from the same initial index x_0 in period zero. (This assumption is adopted solely for the simplicity of exposition; we could dispense with it entirely.) From period zero, each of the entrepreneurs adjusts the level of money wage according to the same adjustment rule (16) but based upon different market experiences unfolding over time. Since $\Pi_t(x|x_0)$ represents the probability that the position of the variable index x_t of an individual firm is not higher than x in period t, the (objective) expected number of firms whose positions of x_t are not higher than x must be equal to $m \cdot \Pi_t(x|x_0)$, where m is the number of the firms in the economy. Furthermore, if m is sufficiently large, the "law of large number" implies that we can regard $\Pi_t(x|x_0)$ as approximating the actual proportion of firms whose x_t's are not higher than x in period t; that is, we have

(20) $$\Pi_t(x|x_0) \cong \{\text{The actual proportion of firms whose } x_t\text{'s are not higher than } x | \text{All firms started from } x_0 \text{ in period zero}\}.$$

Accordingly, $\Pi_t(x|x_0)$ can be interpreted as the "cross-sectional distribution" of the variable indexes among firms in period t. Since $x_t - \hat{\lambda}_0^*$ is the ex ante measure of labor market disequilibrium of an individual firm, $\Pi_t(x|x_0)$ completely characterized how ex ante labor market disequilibrium are dispersed among firms in period t. We can hardly overemphasize the importance of this macroeconomic interpretation of $\Pi_t(x|x_0)$.

After a sufficiently long time, according to Proposition 1 (or Steady-State Theorem in Part III of this series), the motion of the variable index

x_t will settle down to a "steady-state random index" $\underset{\sim}{x}$ in the sense that $\Pi_t(x|x_0)$ will converge to a "steady-state distribution" $\underset{\sim}{\Pi}(x)$, independently of the initial index x_0; that is, as $t \to \infty$, we have

$$(22) \qquad \Pi_t(x|x_0) \to \underset{\sim}{\Pi}(x) \equiv \Pr\{\underset{\sim}{x} \leq x\} .$$

From the microeconomic viewpoint, this objective steady-state distribution can be interpreted as the representation of the motion of the steady-state random index $\underset{\sim}{x}$ in the "stochastic steady-state" of an individual firm. By the Mean Ergodic Theorem,[21] it can be also interpreted as the "long-run average" representation of the intertemporal movements of the variable index x_t of an individual firm which starts from an arbitrary initial index x_0; or to put it in a different way, we can regard $\underset{\sim}{\Pi}(x)$ as representing the average proportion of periods during which the positions of x_t are not higher than x within a very long period of time.

On the other hand, from the "macroeconomic" viewpoint, this objective steady-state distribution is nothing but the cross-sectional representation of a "stochastic macro-equilibrium" of the labor markets as a whole --an equilibrium which is maintained by offsetting motions of a large number of firms perpetually thrown out of equilibrium by incessant stochastic disturbances of labor supplies, product demands, capital stocks, technical knowledge, etc.[22] As long as the basic parameters (c_1 , c_2 ,

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[21] See, for example, Billingsely [1] for the precise discussion on the mean ergodic theorem.

[22] It is Tobin [21] who first introduced the notion of stochastic macro-equilibrium into macroeconomics. Similar equilibrium notions also appear in Champernowne [2] as an equilibrium distribution of income sizes and in Simon and Bonini [20] as an equilibrium size distribution of firms.

$\hat{\rho}(\cdot)$, $\hat{F}(\cdot)$ and $\hat{\mu}$) remain the same, the stochastic macro-equilibrium once attained would exhibit no tendency to change. However, it is by no means a state of "tranquility." In fact, the labor market of an individual firm never shows any tendency towards equilibrium; its ex ante measure of labor market disequilibrium is fluctuating stochastically between the state of positive and negative disequilibria. As a matter of fact, the prediction of the position of the variable index $x_{t+\tau}$ in period $t+\tau$ of the firm whose variable index in period t is x_t is still given by the transition probability distribution $\Pi_\tau(x|x_t)$ even in this stochastic macro-equilibrium; and the steady-state distribution $\underset{\sim}{\Pi}(x)$ has little to do with the short-run movement of an individual firm's variable index x_t . That the labor markets as a whole are in a stochastic macro-equilibrium means only that the diverse movements of labor market disequilibria among firms delicately balance with each other and keep reproducing the same cross-sectional distribution from period to period.

The state of stochastic macro-equilibrium is a macroscopic equilibrium of microscopic disequilibria. Its characteristic feature lies in the contrast between the regular and homogeneous aggregative behavior and the volatile and heterogeneous individual behaviors. "The myth of macroeconomics," according to James Tobin, "is that relations among aggregates are enlarged analogues of relations among corresponding variables for individual households, firms, industries, markets. The myth is a harmless and useful simplification in many contexts, but sometimes it misses the essense of the phenomenon."[23] Indeed, the myth has been the root of the persistent failure to explain the persistence of involuntary unemployment in the conventional economic theories.

[23] Tobin [21], p. 9.

5. A Further Specification of the Model of the Firm

More light can be thrown on the nature of stochastic macro-equilibrium of labor markets if a further specification of their structure is provided. In what follows we shall present the model of labor markets adopted in Part I of this series,[24] but we could obtain qualitatively the same results under various alternative specifications.

Let us assume that the firm whose variable index equals x in period t is faced with the constant elasticity labor supply schedule:

$$(22) \qquad n^{s}_{xt} = \epsilon \cdot w_{xt} + \beta_{xt} \ ;$$

where n^{s}_{xt} is the logarithmic level of the labor supply to this firm in period t ; $\epsilon > 0$ is the constant labor supply elasticity, which is by the assumption of symmetricity uniform across firms; β_{xt} summarizes all the factors which are beyond the control of the firm but which affect the amount of labor supply to this firm in period t . To simplify our exposition, we shall suppose in the following that the entrepreneur of the firm has a correct estimate of the value of ϵ . On the other hand, since the entrepreneur can have only incomplete information about factors (other than w_{xt}) which affect n^{s}_{xt} when he decides the level of money wage at the beginning of period t ,[25] he must regard β_{xt} as a random variable and attach a subjective probability distribution to it. Denote by ${}^{x}_{t}\hat{\beta}_{xt}$ the subjective expectation of the random variable β_{xt} by the entrepreneur of the firm with the variable index x conditional upon his information

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[24] Iwai [8].

[25] See Section 2 of Iwai [8] for the more extensive discussion of this point.

${}^{x}\theta_t$ available at the beginning of period t ; that is, we put

$$(23) \qquad {}^{x}_{t}\hat{\beta}_{xt} \equiv {}^{x}\hat{\mathcal{E}}(\beta_{xt}|{}^{x}\theta_t) \ .$$

We then assume that he believes that his own expectation error of β_{xt}, given by $\beta_{xt} - {}^{x}_{t}\hat{\beta}_{xt}$, is a time-independent random variable drawn from a common (subjective) probability distribution ${}^{x}\hat{\Phi}(\cdot)$; that is, we assume

$$(24) \qquad {}^{x}\hat{\Pr}\{\beta_{xt} - {}^{x}_{t}\hat{\beta}_{xt} \leq z|{}^{x}\theta_t\} \equiv \hat{\Phi}(z) \ ,$$

independently of t . (To be consistent with our symmetricity assumption we have also assumed that $\hat{\Phi}(z)$ is uniform across all firms.)

The specification of the demand for labor is more complicated, for it depends, among others, upon the productivity of the firm's fixed productive factors and upon the entrepreneur's subjective expectation as to the state of product demand in the future period when he is able to sell the output from the production process he starts in period t . In Part I of this series we represented the former by the index k_{xt} and the latter by ${}^{x}_{t}\hat{\alpha}_{x,t+\tau} \equiv {}^{x}\hat{\mathcal{E}}(\alpha_{x,t+\tau}|{}^{x}\theta_t)$; where $\alpha_{x,t+\tau}$ is the random variable summarizing all the factors, other than its own product price, that affect the amount of demand for his product in period $t+\tau$; and finally τ (≥ 0) denotes the gestation period of a unit production process. Then, it was shown that under certain assumptions the labor demand of the firm x , whose logarithmic level is denoted by n^{d}_{xt} , can be expressed by the following log-linear equation:

$$(25) \qquad n^{d}_{xt} = \frac{1}{\eta(1-\gamma)+\gamma}\{\sigma_0 + (\eta-1)k_{xt} + {}^{x}_{t}\hat{\alpha}_{x,t+\tau} - \eta w_{xt}\} \ ;$$

where σ_0 is a constant, η (> 1) is the constant price-elasticity of product demand and γ (> 0) is the constant output-elasticity of labor input.[26]

It is clear that the actual labor employment, whose logarithmic level is denoted by n_{xt}, is determined by the "short-side" of the current labor market; that is, we have

$$n_{xt} = \min[n^s_{xt},\ n^d_{xt}] \ . \tag{26}$$

We can then approximate the "rate" of excess labor supply and the "rate" of involuntary unemployment respectively by

$$h_{xt} \equiv n^s_{xt} - n^d_{xt} \ , \tag{27}$$

$$u_{xt} \equiv \max[h_{xt},\ 0] \ . \tag{28}$$

We can also express the subjective expectations of h_{xt} and u_{xt} respectively as follows

$$\begin{aligned} {}^x_t\hat{h}_{xt} &\equiv {}^x\hat{\mathcal{E}}(h_{xt}|{}^x\theta_t) = {}^x\hat{\mathcal{E}}(n^s_{xt}|{}^x\theta_t) - n^d_{xt} \\ &= \frac{\eta+\gamma\epsilon+\eta\epsilon(1-\gamma)}{\eta(1-\gamma)+\gamma} w_{xt} - \frac{1}{\eta(1-\gamma)+\gamma}[\sigma_0 + (\eta-1)k_{xt} + {}^x_t\hat{\alpha}_{x,t+\tau} - \{\eta(1-\gamma)+\gamma\}{}^x_t\hat{\beta}_{xt}] \end{aligned} \tag{29}$$

$$\begin{aligned} {}^x_t\hat{u}_{xt} &\equiv {}^x\hat{\mathcal{E}}(u_{xt}|{}^x\theta_t) = {}^x\hat{\mathcal{E}}\{\max[h_{xt},\ 0]|{}^x\theta_t\} \\ &= {}^x\hat{\mathcal{E}}\{\max[{}^x_t\hat{h}_{xt} + (h_{xt} - {}^x_t\hat{h}_{xt}),\ 0]|{}^x\theta_t\} \\ &= {}^x\hat{\mathcal{E}}\{\max[{}^x_t\hat{h}_{xt} + (\beta_{xt} - {}^x_t\hat{\beta}_{xt}),\ 0]|{}^x\theta_x\} \\ &= \int_{-{}^x_t\hat{h}_{xt}}^{\infty} z \cdot d{}^x\hat{\Phi}(z) + {}^x_t\hat{h}_{xt}\{1 - \hat{\Phi}(-{}^x_t\hat{h}_{xt})\} \ . \end{aligned} \tag{30}$$

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[26] See eq. (35) of Iwai [8].

It is easy to show that ${}^{x}_{t}\hat{u}_{xt}$ is an increasing and convex function of ${}^{x}_{t}\hat{h}_{xt}$. It is also bounded below by zero.

One of the major results in Part I of this series is that, if there is no money wage adjustment cost, the "optimal" money wage that would maximize the short-run expected profit from the unit production process starting from period t can be characterized by the extremely simple condition that ${}^{x}_{t}\hat{h}_{xt}$ be equal to a constant $\hat{h}^*$; that is,

$$(31) \qquad {}^{x}_{t}\hat{h}_{xt} = \hat{h}^* \ .$$

The constant $\hat{h}^*$ is called the firm's "normal rate of excess labor supply"; its value is determined solely by ϵ, η, γ and $\hat{\Phi}(\cdot)$.[27] Substituting (30) into (31), we can alternatively express the optimal condition as the equality between ${}^{x}_{t}\hat{u}_{xt}$ and another constant $\hat{u}^*$ which is called the firm's "normal rate of involuntary unemployment"; that is, we have

$$(32) \qquad {}^{x}_{t}\hat{u}_{xt} = \hat{u}^* \equiv \int_{-\hat{h}^*}^{\infty} z \cdot d\hat{\Phi}(z) + \hat{h}^*\{1 - \hat{\Phi}(-\hat{h}^*)\} \ .$$

Again, the value of $\hat{u}^*$ is determined solely by ϵ, η, γ and $\hat{\Phi}(\cdot)$. The explicit formula of the "optimal" money wage can be obtained by substituting (29) into (31) and solving it with respect to w_{xt}. We have

$$(33) \qquad w^*_{xt} = \frac{1}{\eta+\gamma\epsilon+\eta\epsilon(1-\gamma)}[\sigma_0 + (\eta-1)k_{xt} + {}^{x}_{t}\hat{\alpha}_{x,t+\tau} - \{\eta(1-\gamma)+\gamma\}({}^{x}_{t}\hat{\beta}_{xt} - \hat{h}^*)] \ .$$

The "optimal" money wage in period t is therefore determined by the

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[27] See eq. (50) of Iwai [8] for the equation determining $\hat{h}^*$.

index of productivity of fixed factor, by the entrepreneur's subjective expectation about the state of product demand in the future period and by his subjective expectation about the state of labor supply in the current period. That it is governed by the entrepreneur's expectations is of the utmost importance for our disequilibrium economic dynamics.

If the entrepreneur incurs certain adjustment costs whenever he changes the level of money wage, it no longer pays for him to adjust w_{xt} in every period. w_{xt} tends to diverge from w^*_{xt} in most of the periods. Their difference, $w^*_{xt} - w_{xt}$, which is by definition equal to $x_t - \hat{\lambda}^*_0$, have been interpreted as the ex ante measure of labor market disequilibrium in period t. Now, since the "optimal" money wage is characterized as the level of money wage that would equate ${}^{x}_{t}\hat{h}_{xt}$ with $\hat{h}^*$, we can immediately derive the following relation from equation (29):

$$(34) \qquad x_t - \hat{\lambda}^*_0 \equiv w^*_{xt} - w_{xt} = - \frac{\eta(1-\gamma)+\gamma}{\eta+\gamma\epsilon+\eta\epsilon(1-\gamma)}({}^{x}_{t}\hat{h}_{xt} - \hat{h}^*) \ .$$

In words, the ex ante measure of disequilibrium in the x^{th} firm's labor market is negatively proportional to the rate of deviation of the entrepreneur's subjective expected rate of excess labor supply from its normal rate. Here, we have obtained a perfect justification for our interpretation of $x_t - \hat{\lambda}^*_0$ as the ex ante measure of labor market disequilibrium.

6. Long-Run Macro-Equilibrium

In order to focus upon long-run macro-economic problems, in particular, upon the problem of whether involuntary unemployment can persist even in the long-run, the present article will consider only the state of the economy in which all the entrepreneurs' expectations are simultaneously

"self-fulfilling" (or "rational" in the sense of J. Muth).[28] We say that the entrepreneur of a firm has "self-fulfilling expectations" if his subjective probability distributions of all the random variables relevant to his decisions happen to coincide with their objective probability distributions conditional upon the same information set available to him. The entrepreneur with self-fulfilling expectations has nothing to learn from his observations in markets and feels no need to revise his "economic theory" or his "econometric model" or his "view of the working of the economy," on the basis of which his subjective expectations are formed. Everything relevant to his decisions is on average foreseen by him and nothing in his observations will on average disappoint him. This notion of self-fulfilling expectations is a generalization of the classical notion of perfect foresight into the world of uncertainty.

Note that for the microdynamic model of an individual firm presented in Section 3 and Section 5 we can specify the conditions for self-fulfilling expectations by the entrepreneur of the firm x as follows.

$$(35) \qquad {}^{x}_{t}\hat{\beta}_{xt} \equiv {}^{x}\hat{E}(\beta_{xt}|{}^{x}\theta_{t}) = E(\beta_{xt}|{}^{x}\theta_{t}) \; ;$$

$$(36) \qquad {}^{x}_{t}\hat{\alpha}_{x,t+\tau} \equiv {}^{x}\hat{E}(\alpha_{x,t+\tau}|{}^{x}\theta_{t}) = E(\alpha_{x,t+\tau}|{}^{x}\theta_{t}) \; ;$$

$$(37) \qquad \hat{\Phi}(z) \equiv {}^{x}\hat{Pr}\{\beta_{xt} - {}^{x}_{t}\hat{\beta}_{xt} \leq z|{}^{x}\theta_{t}\}$$

$$= Pr\{\beta_{xt} - E(\beta_{xt}|{}^{x}\theta_{t}) \leq z|{}^{x}\theta_{t}\} \equiv \Phi(z)$$

where $\Phi(x)$ is assumed to be time-independent and uniform across all firms; and finally,

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[28] See Muth [15]. Our definition of self-fulfilling expectations is in accordance with that of Lucas [13].

(38) $\hat{F}(\xi) = F(\xi)$,

where $\hat{F}(\xi)$ is defined by (5) and $F(\xi)$ by (17).

If the entrepreneur of the firm x happens to have self-fulfilling expectations, then the normal rate of excess labor supply $\hat{h}^*$, the normal rate of involuntary unemployment $\hat{u}^*$ and the trend-free expected profit function $\hat{\rho}(\cdot)$ all become determined by the objective elasticity values η , ϵ and γ and by the objective probability distributions of expectation errors $\Phi(\cdot)$. We shall denote the "objective" values of $\hat{h}^*$ and $\hat{u}^*$ by h^* and u^* and the "objective" functional form of $\hat{\rho}(\cdot)$ by $\rho(\cdot)$. We shall then call h^* and u^* the "natural rate" of excess labor supply and the "natural rate" of involuntary unemployment. By the same token, the threshold parameters, $\hat{\lambda}_0^*$, $\hat{\lambda}_1^*$ and $\hat{\lambda}_2^*$, of the optimal money wage adjustment rule become determined by the objective parameters such as c_1 , c_2 , $\rho(\cdot)$ and $F(\cdot)$. We shall denote their "objective" values by λ_0^* , λ_1^* and λ_2^* . Finally, the cross-sectional distribution $\Pi_t(x|x_0)$ and its steady-state distribution $\underset{\sim}{\Pi}(x)$ which have been determined by the two thresholds $\hat{\lambda}_1^*$ and $\hat{\lambda}_2^*$ are now determined by the their objective values λ_1^* and λ_2^* . We shall denote their "objective" functional forms by $\Pi_t^0(x|x_0)$ and $\underset{\sim}{\Pi}^0(x)$, respectively.

The main proposition established in Part II of this series is, however, that all the entrepreneurs can have self-fulfilling expectations simultaneously only if what we call the modern "Say's condition" happens to hold; by Say's condition we mean a statistical balance between the aggregate product demand and the aggregate product supply.[29] If Say's condition is disturbed, we showed in Part II, a majority of entrepreneur's expectations are bound to be falsified by their own interdependent but uncoordinated

[29] See Iwai [9]. The definition of the "modern Say's condition" is given by eq. (35) in it.

price and wage adjustment activities. Since this fundamental proposition was deduced independently of the assumption about the costly nature of money wage adjustment activity, it holds true even in our dynamic economy with money wage adjustment costs. Therefore, when we stated in the second paragraph of this section in a seemingly innocuous manner that we would consider only the state of the economy in which all the entrepreneurs' expectations are simultaneously self-fulfilling, we had implicitly presupposed the fulfillment of the Say's condition! It goes without saying that in the "monetary economy" in which some markets for futures goods are missing and hence certain durable assets, including money, function as stores of value, the Say's condition is in constant danger of being upset by unforeseen shocks from both inside and outside of the economy. Hence the so-called Say's law that supply creates its own demand loses its general validity in the monetary economy. Therefore, in order to justify our implicit presupposition about Say's condition we must assume that the government consciously "leans against the wind" and successfully keeps the balance between the aggregate demand and supply through compensationary fiscal, monetary and other policy instruments.[30]

The economy is then said to be in "a state of long-run equilibrium" if (i) Say's condition is maintained by the conscious stabilization policy of the government and (ii) all entrepreneurs' expectations are simultaneously self-fulfilling. Note that the second condition is contingent upon the first condition. If, in addition to the above two conditions, the labor markets as a whole satisfy the condition for stochastic macro-equilibrium,

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[30] Of course, one can imagine an economy in which automatic stabilizing forces are so strong that the economy can be hardly away from long-run equilibrium. See Iwai [9] for a discussion on the stability of long-run equilibrium. See also Tobin [22].

we say that the economy is in a "state of long-run macro-equilibrium." The present paper will investigate only the nature of the long-run macro-equilibrium of our Keynesian economy. The analysis of its behaviors outside of the long-run macro-equilibrium will be left for other opportunities.

7. Aggregate Structure of Long-Run Macro-Equilibrium

We are now in a position of analyzing the "aggregate" structure of the long-run macro-equilibrium of our Keynesian economy. Recall that the steady-state distribution $\underset{\sim}{\Pi}^0(x)$ can be interpreted from the microscopic viewpoint as the long-run average representation of the motions of the variable index x_t of an individual firm and from the macroscopic viewpoint as the cross-sectional distribution of variable indexes x_t's among firms in a stochastic macro-equilibrium. This implies that we can easily approximate the economy-wide geometrical averages of relevant variables by employing $\underset{\sim}{\Pi}^0(x)$ as follows

$$(39)\qquad \underset{\sim}{W}_t \cong \int_{\lambda_2^*}^{\lambda_1^*} w_{xt} \cdot d\underset{\sim}{\Pi}^0(x) \ , \quad \underset{\sim}{W}^*_t \cong \int_{\lambda_2^*}^{\lambda_1^*} w^*_{xt} \cdot d\underset{\sim}{\Pi}^0(x)$$

$$\underset{\sim}{N}_t \cong \int_{\lambda_2^*}^{\lambda_1^*} n_{xt} \cdot d\underset{\sim}{\Pi}^0(x) \ , \quad \underset{\sim}{N}^d_t \cong \int_{\lambda_2^*}^{\lambda_1^*} n^d_{xt} \cdot d\underset{\sim}{\Pi}^0(x)$$

$$\underset{\sim}{N}^s_t \cong \int_{\lambda_2^*}^{\lambda_1^*} n^s_{xt} \cdot d\underset{\sim}{\Pi}^0(x) \ , \quad \underset{\sim}{H}_t \cong \int_{\lambda_2^*}^{\lambda_1^*} h_{xt} \cdot d\underset{\sim}{\Pi}^0(x) \ ,$$

$$\underset{\sim}{U}_t \cong \int_{\lambda_2^*}^{\lambda_1^*} u_{xt} \cdot d\underset{\sim}{\Pi}^0(x) \ , \quad \underset{\sim}{\hat{H}}_t \cong \int_{\lambda_2^*}^{\lambda_1^*} {}^{x}\hat{h}_{xt} \cdot d\underset{\sim}{\Pi}^0(x)$$

$$\underset{\sim}{\hat{U}}_t \cong \int_{\lambda_2^*}^{\lambda_1^*} {}^{x}\hat{u}_{xt} \cdot d\underset{\sim}{\Pi}^0(x) \ ;$$

where $\underset{\sim}{W}_t$, for instance, denotes the economy-wide average of w_{xt} in the state of long-run macro-equilibrium.

Now we can apply all the propositions concerning the individual firm's long-run average behaviors in its own labor market, established in Part III of this series and summarized in Section 3 of the present paper, to the analysis of the relationships among the above aggregate variables, simply by substituting the cross-sectional averages for the long-run averages.

First of all, Proposition 2 in Section 3 maintains that the (subjective) long-run average rate of change in the actual money wage of an individual firm is equal to the (subjective) expected rate of change in the "optimal" money wage. Within the macroeconomic context this proposition can be translated as follows: in the long-run macro-equilibrium the (objective) rate of change in the aggregate money wage, $\Delta\underset{\sim}{W}_t \equiv \underset{\sim}{W}_t - \underset{\sim}{W}_{t-1}$, is approximately equal to the uniform (objective) expected rate of change in the "optimal" money wage, μ. By the law of large numbers, the latter is in turn approximately equal to the (objective) rate of change in the aggregate "optimal" money wage, $\Delta\underset{\sim}{W}^*_t \equiv \underset{\sim}{W}^*_t - \underset{\sim}{W}^*_{t-1}$. Thus, we have

$$\Delta\underset{\sim}{W}_t \cong \mu \cong \Delta\underset{\sim}{W}^*_t \,. \tag{40}$$

Second, our Proposition 3 states that the long-run average deviation of the individual firm's actual money wages from the "optimal" money wages tends to be zero. Again within the macroeconomic context, this proposition means that in the long-run macro-equilibrium the aggregate money wage is approximately equal to the aggregate "optimal" money wage:

$$\underset{\sim}{W}_t \cong \underset{\sim}{W}^*_t \quad . \tag{41}$$

This equation, along with equation (40), says that the level of the aggregate money wage that would prevail in the economy where no money wage adjustment costs exist has now rehabilitated its position as the actual aggregate money wage in the state of long-run macro-equilibrium of the economy with money wage rigidity.

Third, by the conditions of self-fulfilling expectations, in particular (35) and (36), we can easily show that in the state of long-run equilibrium the individual entrepreneur's subjective expected rate of excess labor supply given by (29) coincides with its objective expected value conditional upon the same information available to him. Therefore, we have

$$\mathcal{E}(h_{xt} | {}^{x}\theta_t) = {}^{x}_{t}\hat{h}_{xt} \quad . \tag{42}$$

Since by (34) the deviation of ${}^{x}_{t}\hat{h}_{xt}$ from the natural rate of excess labor supply h^* is proportional to $\lambda_0^* - x_t$, we have

$$\mathcal{E}(h_{xt} | {}^{x}\theta_t) - h^* \propto \frac{\eta+\gamma\epsilon+\eta\epsilon(1-\gamma)}{\eta(1-\gamma)+\gamma} \cdot (\lambda_0^* - x_t) \quad . \tag{43}$$

Aggregating the above relation in terms of the steady-state distribution $\underset{\sim}{\Pi}^0(x)$ and applying Proposition 3 to it again, we obtain:

$$\int_{\lambda_2^*}^{\lambda_1^*} \mathcal{E}(h_{xt} | {}^{x}\theta_t) \cdot d\underset{\sim}{\Pi}^0(x) = h^* \quad . \tag{44}$$

Therefore, if we take the objective expectation of the left-hand-side of (43), we finally obtain the following important result:

$$(45) \qquad E(\underset{\sim}{H}_t) \cong E\{\int_{\lambda_2^*}^{\lambda_1^*} h_{xt} \cdot d\underset{\sim}{\Pi}^0(x)\} = \mathcal{E}\{\int_{\lambda_2^*}^{\lambda_1^*} \mathcal{E}(h_{xt} | {}^x\theta_x) \cdot d\underset{\sim}{\Pi}^0(x)\}$$

$$= h^* ,$$

where we have used the well-known fact that the expectation of a conditional expectation is the expectation. Thus, we have indeed shown that the aggregate rate of excess labor supply is on the average equal to the natural rate of excess labor supply which is a constant determined solely by the real characteristics of the economy and independent of its monetary characteristics.

Does this mean that the "natural rate theory of unemployment," or more generally the "neutrality of money" in the neoclassical equilibrium theory, has reincarnated in the economy with money wage rigidity as a characteristic nature of the state of long-run macro-equilibrium? The answer to this question still remains negative, even if we put aside our criticism of the short-run aspects of the natural rate theory of unemployment (as well as those of the neoclassical equilibrium theory) presented in Part II of the series. That the aggregate rate of excess labor supply equals the natural rate of excess labor supply means only that the aggregate rate of involuntary unemployment in excess of its natural rate equals the aggregate rate of unfilled vacancies in excess of its natural rate. If money wage adjustment is costly, it by no means implies that the aggregate rate of involuntary unemployment coincides with the natural rate of involuntary unemployment.

8. The Downward-Sloping Long-Run Phillips Curve

Then, what determines the aggregate rate of involuntary unemployment in the state of stochastic macro-equilibrium? In order to give a satisfactory answer to this question, let us recall equation (30) which expresses the individual firm's subjective expected rate of involuntary unemployment as an increasing and convex function of its subjective expected rate of excess labor supply. We can easily establish an analogous relationship between the objective expected rate of involuntary unemployment and the objective expected rate of excess labor supply, both conditional upon the information set available to the individual entrepreneur. Thus, we have

$$(46) \qquad \mathcal{E}(u_{xt}|^{x}\theta_t) = \int_{-\mathcal{E}(h_{xt}|^{x}\theta_t)}^{\infty} z \cdot d\Phi(z) + \mathcal{E}(h_{xt}|^{x}\theta_t) \cdot \{1 - \Phi[-\mathcal{E}(h_{xt}|^{x}\theta_t)]\} .$$

A Taylor expansion of the right-hand-side around $\mathcal{E}(h_{xt}|^{x}\theta_t) = h^*$ leads to

$$(47) \qquad \mathcal{E}(u_{xt}|^{x}\theta_t) \cong \int_{-h^*}^{\infty} z \cdot d\Phi(z) + h^*\{1 - \Phi(-h^*)\} + \{1 - \Phi(-h^*)\} \cdot \{\mathcal{E}(h_{xt}|^{x}\theta_t) - h^*\} + \frac{1}{2}\Phi'(-h^*) \cdot \{\mathcal{E}(h_{xt}|^{x}\theta_t) - h^*\}^2 ,$$

which by (32) and (43) we can rewrite as:

$$(47)' \qquad \mathcal{E}(u_{xt}|^{x}\theta_t) \cong u^* + \frac{\eta + \varepsilon\{\eta(1-\gamma)+\gamma\}}{\eta(1-\gamma)+\gamma} \cdot \{1 - \Phi(-h^*)\}(\lambda_0^* - x_t) + \frac{1}{2}\left[\frac{\eta + \varepsilon\{\eta(1-\gamma)+\gamma\}}{\eta(1-\gamma)+\gamma}\right]^2 \cdot \Phi'(-h^*)(\lambda_0^* - x_t)^2 .$$

Aggregating the above relation in terms of the steady-state distribution

$\underset{\sim}{\Pi}^0(x)$ and applying Proposition 3 once again, we have

$$(48) \qquad \int_{\lambda_2^*}^{\lambda_1^*} \mathcal{E}(u_{xt}|^x\theta_t)d\underset{\sim}{\Pi}^0(x) \cong u^* + \frac{1}{2}\left[\frac{\eta+\varepsilon[\eta(1-\gamma)+\gamma]}{\eta(1-\gamma)+\gamma}\right]^2 \cdot \Phi'(-h^*) \cdot \mathrm{Var}^0(\underset{\sim}{x}) \ ;$$

where $\mathrm{Var}^0(\underset{\sim}{x})$ is the objective steady-state variance of the variable index x_t. If we take the objective expectation of the left-hand-side and noting that the expectation of a conditional expectation is the expectation, we finally obtain the following fundamental aggregative relation in the state of long-run macro-equilibrium:

$$(49) \qquad \mathcal{E}(\underset{\sim}{U}_t) \cong \mathcal{E}\{\int_{\lambda_2^*}^{\lambda_1^*} u_{xt} \cdot d\underset{\sim}{\Pi}^0(x)\} = \mathcal{E}\{\int_{\lambda_2^*}^{\lambda_1^*} \mathcal{E}(u_{xt}|^x\theta_t) \cdot d\underset{\sim}{\Pi}^0(x)\}$$

$$\cong u^* + \frac{1}{2}\left[\frac{\eta+\varepsilon[\eta(1-\gamma)+\gamma]}{\eta(1-\gamma)+\gamma}\right]^2 \cdot \Phi'(-h^*) \cdot \mathrm{Var}^0(\underset{\sim}{x}) \ .$$

This relation claims that the expected aggregate rate of involuntary unemployment in a state of long-run macro-equilibrium $\mathcal{E}(\underset{\sim}{U}_t)$ exceeds the natural rate of involuntary unemployment u^* by the amount approximately proportional to the steady-state variance of the variable index x_t. Since the variance of x_t, which is also equal to the variance of $x_t - \lambda_0^*$, summarizes the extent of dispersion of ex ante measures of labor market disequilibrium among firms, we can state our result as follows: The more dispersed ex ante labor market disequilibria across firms the larger the gap between the aggregate rate of involuntary unemployment and its natural rate in the state of long-run macro-equilibrium. In the economy in which money wage adjustment is a costly economic activity, perpetual inter-firm flux of labor supply, product demand, capital accumu-

lation and technological change will never let the dispersion of ex ante labor market disequilibria shrink to zero. As a result, even though the aggregate rate of excess labor supply were approximately brought into equality with its natural rate in long-run macro-equilibrium, the aggregate rate of involuntary unemployment would never be reduced to its natural rate level.

We are then left with an obvious question: what are the determinants of the steady-state variance $\text{Var}^0(\underset{\sim}{x})$? It is clear from our discussions in Part III of this series, briefly outlined in Section 3, that $\text{Var}^0(\underset{\sim}{x})$ in long-run macro-equilibrium is determined by all sorts of basic structural parameters in our economy: the (objective) elasticities of labor supply and product demand, the output elasticity of labor input in production, the (objective) probability distributions of the entrepreneur's expectation errors, the (objective) probability distribution of the rate of change in the "optimal" money wage and the cost structure of money wage adjustment activity. The most important determinant is, of course, the (objective) average rate of change in the "optimal" money wage μ , which was demonstrated in (40) approximately equal to the rate of change in the aggregate money wage $\Delta \underset{\sim}{W}_t$. Thus, we can immediately conclude that the gap between the aggregate rate of involuntary unemployment and its natural rate in the state of long-run macro-equilibrium is correlated with the rate of change in the aggregate money wage.

Money in our economy is therefore no longer "neutral" even in the state of long-run macro-equilibrium! It is clear that this long-run non-neutrality of money is solely due to the existence of lump-sum adjustment costs of "money" wage. If this were regarded as a manifestation of a kind of "money illusion" involved in wage adjustment activities in labor markets,

then it would be this illusion which is raison d'être for the "monetary economy" in which economic transactions are carried out primarily through the intermediary of certain monetary assets.

In reality, this "money illusion" is asymmetric between upward adjustment and downward adjustment. Workers, conscious about their "relative" wages both within a firm and across firms, are likely to oppose any cut in the level of money wage. Money wage is more rigid downwards than upwards. In our Keynesian model of money wage adjustment this downward rigidity of money wage may be specified by the condition that the lump-sum adjustment cost of a money wage cut c_2 is higher than that of a money wage increase c_1. Now, this assumption of downward rigidity of money wage would bear a crucial implication for the long-run relationship between the aggregate rate of involuntary unemployment and the rate of change in the aggregate money wage.

In our Proposition 4 of Section 3, we gave an explicit expression of the steady-state variance $\mathrm{Var}(\underset{\sim}{x})$ for the special Keynesian model in which (i) money wage is absolutely rigid downwards and (ii) the "optimal" money wage is fluctuating according to the Bernouilli trial multiplicative random walk model. If we substitute equation (15) into our aggregate relation (49) and substitute objective for subjective parameters, we can deduce the following approximate relationship between the aggregate rate of involuntary unemployment and the rate of change in the aggregate money wage in the state of long-run macro-equilibrium of this special Keynesian economy:

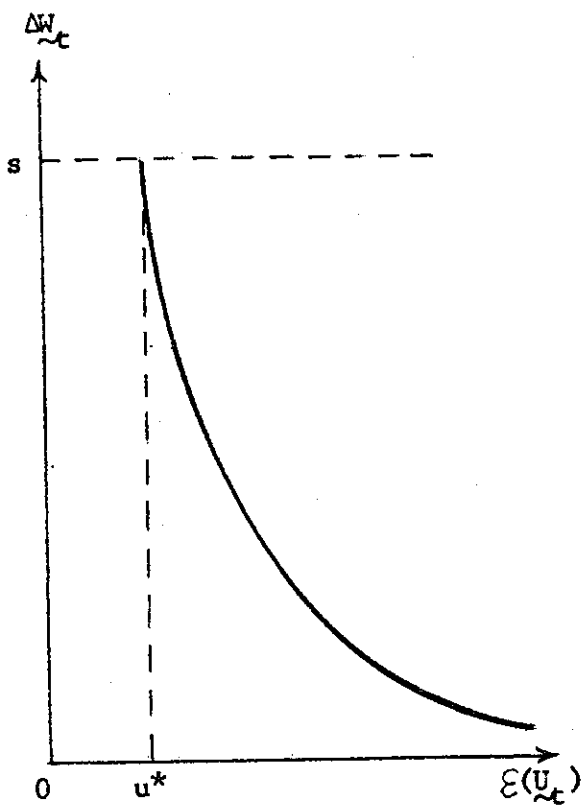
$$(50) \qquad \mathcal{E}(\underset{\sim}{U}_t) - u^* \propto 48^{-1/3}\left\{\frac{2\mu c_1}{-\rho''(0)}\right\}^{2/3} - \frac{1}{3}s^2 + \frac{1}{4}\left(\frac{s^2}{\mu}\right)^2$$

$$\text{when} \quad s \geq \mu > \frac{-s^3\rho''(0)}{12c_1}$$

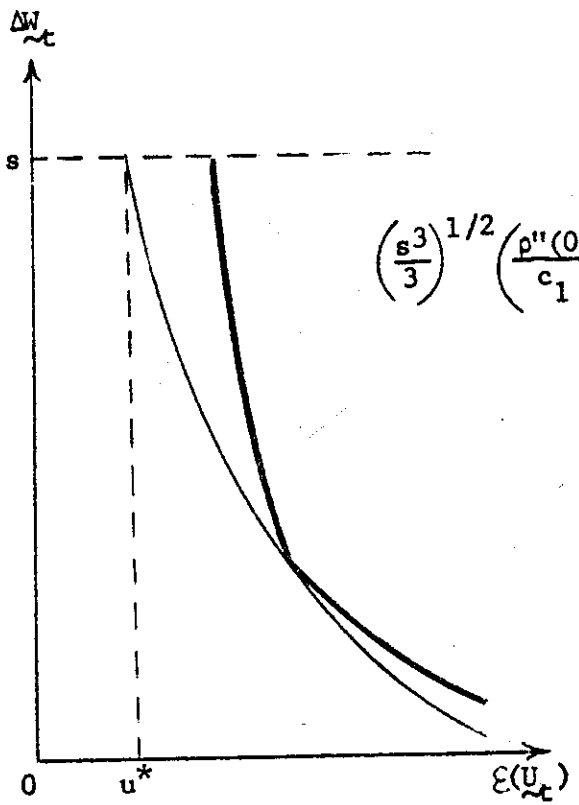
$$\propto \frac{s^2}{4}\left(\frac{s^2}{\mu^2} - 1\right) \qquad \text{when} \quad \frac{-s^3\rho''(0)}{12c_1} \geq \mu \geq 0 \ ,$$

where $\mu \cong \underset{\sim}{\Delta W}_t$. The diagram below depicts three possible cases of this relationship. They demonstrate that <u>if money wage is (absolutely) rigid downwards the aggregate rate of involuntary unemployment in the state of long-run macro-equilibrium is "negatively" correlated with the rate of change in the aggregate money wage at least for the relevant range of the possible values of the latter</u>. They are nothing but the "long-run Phillips curve" which minatains that there exists a trade-off between involuntary unemployment and wage inflation--"a cruel dilemma"--even in the long-run macro-equilibrium.

A state of long-run macro-equilibrium is maintained by offsetting motions of firms constantly thrown into disequilibrium by perpetual inter-firm random disturbances. The firm whose expected rate of excess labor supply exceeds the natural rate has a "drive" to eliminate its own <u>ex ante</u> disequilibrium by lowering its money wage (relative to the other firms); and the firm whose expected rate of excess labor supply falls short of the natural rate has an opposite "drive." However, the existence of money wage adjustment costs prevents the firm from satisfying its drive and attaining its equilibrium position. In particular, under the assumption of absolute downward rigidity of money wage, the firm whose expected rate of excess labor supply is above the natural rate cannot adjust its money wage immediately and will be forced to stay in a frustrating state until some

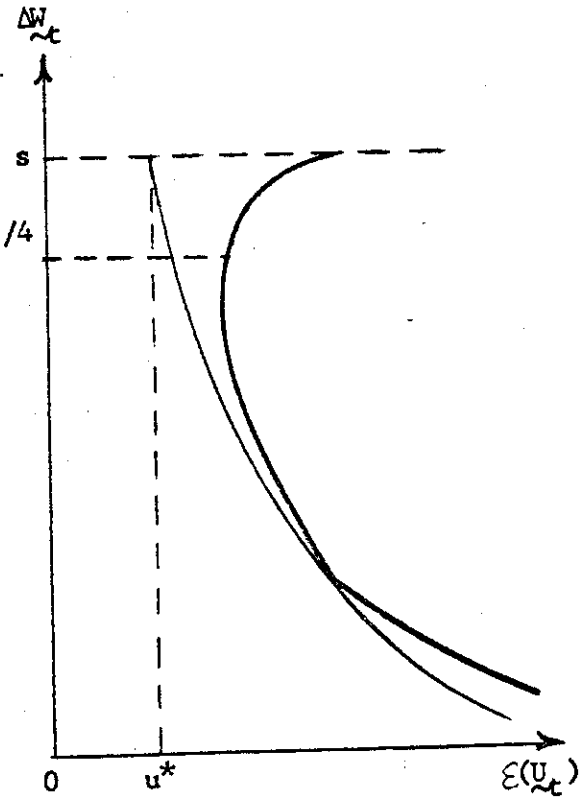


(i) $s \geq \left(\frac{-12c_1}{\rho''(0)}\right)^{1/2}$



(ii) $\left(\frac{-12c_1}{\rho''(0)}\right)^{1/2} > s \geq \left(\frac{-9c_1}{\rho''(0)}\right)^{1/2}$

$\left(\frac{s^3}{3}\right)^{1/2}\left(\frac{\rho''(0)}{c_1}\right)^{1/4}$



(iii) $\left(\frac{-9c_1}{\rho''(0)}\right)^{1/2} > s > 0$

DIAGRAM: Three Possible Shapes of Long-Run Phillips Curve

favorable events pull down its expected rate of excess labor supply below the natural rate. Clearly, the larger the number of such frustrated firms the higher the aggregate rate of involuntary unemployment. Then, if we compare two long-run macro-equilibrium states, one with a high and the other with a low growth rate of aggregate money wage, the firm which is trapped in a frustrating situation has the higher probability of satisfying its drive in the near future in the former than in the latter. For in the long-run macro-equilibrium with a high money wage inflation rate the frustrated firm can extricate itself from disequilibrium simply by waiting for the other firms to raise their money wages relative to its own. (Of course, in order to make this comparison meaningful, we must also take into account the possible compensating change of the firm's money wage adjustment rule itself; but the relation (50) assures us that this compensating adjustment is not large enough to offset the initial effect.)[31] In other words, we can say that the money wage inflation has a lubrication effect on the adjustment of individual money wage adjustments. It is this lubrication effect that generates the downward slope of the long-run Phillips curve in our special economy.[32]

It is a safe bet to conjecture that we can also deduce the downward

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[31] More concretely, by the compensating adjustment we mean the firm's attempt of widening the gap between the floor and ceiling thresholds as a probable response to an increased inflation rate. As was pointed out by Charles Wilson, it is of some interest to decompose the effect of an increase in inflation rate on the long-run macro-equilibrium unemployment rate into positive and negative effects and compare their relative magnitudes in the more systematic way.

[32] Rees [17] has made a similar point.

sloping long-run Phillips curve (at least within certain relevant range of the rates of inflation) under the more general setting that only assumes relative downward rigidity of money wage.

Before leaving this section we must make an obvious but nonetheless important remark. It is that our long-run Phillips curve is constructed by tracing out all the equilibrium pairs of the aggregate rate of involuntary unemployment and the rate of change in the aggregate money wage. Each point along the curve corresponds to one imagined long-run macro-equilibrium with a given rate of wage inflation, and the curve itself tells us nothing about the transient path from one long-run macro-equilibrium to another, nor does it tell us whether such a transition is stable or even possible. It is beyond the scope of the present paper to investigate the behavior of the economy when it is thrown out of a given long-run macro-equilibrium. We can only point out here that, if the cause of dis-equilibrium lies in the inflation gap--the excess of aggregate demand over aggregate supply, the economy will undergo a cumulative inflation process similar to the one we examined in Part II of this series. If, on the other hand, the cause turns out to be the deflation gap--the deficiency of aggregate demand relative to aggregate supply, the downward rigidity of money wage will hinder the deployment of cumulative deflation process. Instead, in this situation, involuntary unemployment will spread over the whole economy and may stubbornly persist until the central authority succeeds in raising the level of aggregate demand to the level of aggregate supply or some automatic stabilizing forces consolidates their power to restore the balance between aggregate demand and supply.[33] Since the aim of this

[33] This situation may be identified as the so-called "unemployment equilibrium" à la Keynes, though it may not be an equilibrium in the genuine sense of the word.

paper is to show that the dynamic economy with downward money wage rigidity will never lose its Keynesian features even in the long-run, it would be no surprise that it should exhibit much stronger Keynesian features in the short-run. We shall, however, leave the detailed analysis of the economy out of long-run macro-equilibrium to other occasions.

9. A Brief Note on Wage Indexation

In recent years, workers become conscious not only about the movement of their money wages but also about the movement of their real wages. Labor unions sometimes go on strike to secure the purchasing power of their money wages against rising prices of their consumption baskets. Recent proposals for the indexation of money wages can be interpreted as an attempt to institutionalize this general upsurge of the consciousness of the movement of real wages. Hence,it seems of some interest to give a brief consideration to the implication of wage indexation.

Under the complete indexation scheme, the money wage adjustment rule (1) should be replaced by

$$(51)\qquad \begin{cases} w_t = w_{t-1} + (PI_t - PI_{t-1}) & \text{if } \lambda_2 < w_t^* + \lambda_0 - (w_{t-1} + PI_t - PI_{t-1}) < \lambda_1 , \\ w_t = w_t^* + \lambda_0 & \text{otherwise} \end{cases}$$

where $\lambda_0 \gtreqless 0$, $\lambda_1 > 0$, $\lambda_2 < 0$ and PI_t is the log of the price index in period t agreed by both employees and employers as the appropriate denominator of real wage rate. It is then obvious that this rule is formally equivalent to the original rule (1) once we define $v_t \equiv w_t - PI_t$ and $v_t^* \equiv w_t^* - PI_t$; where v_t is the log of the real wage in period t and v_t^* is the log of the "optimal" real wage in period t . Therefore, all

the propositions deduced in this article can be applied to this case only with appropriate reinterpretation of variables. In particular, our long-run Phillips curve (50) in the previous section must be now interpreted as the negative correlation between the aggregate rate of involuntary unemployment and the rate of change in the aggregate real wage in long-run macro-equilibrium. In general, we can expect that the long-run equilibrium growth rate of aggregate real wage is governed by the growth rate of aggregate "labor productivity."[34] It then follows that the aggregate rate of involuntary unemployment in the completely indexed economy is chiefly determined by the labor productivity growth rate; independently of the movement of nominal variables except for their indirect influences on the labor productivity. The complete indexation would therefore eliminate the "long-run non-neutrality of money" simply by removing a form of "money illusion" implicit in the rigidity of "money" wages. However, we must emphasize that whether the complete elimination of long-run non-neutrality of money is desirable or not is an entirely different matter. For, under the complete indexation scheme, the long-run macro-equilibrium rate of involuntary unemployment becomes at the mercy of changes in labor productivity that are beyond the effective control of either monetary or fiscal authority. Moreover, from the short-run point of view the economy with complete indexation appears to be more vulnerable to disturbances which force the realignment of real wage structure than the economy without; although the former appears to be more robust to disturbances caused by changes in aggregate demand than the latter. In any case, the economy without any wage indexation and the economy with the complete wage indexation

[34] We must, of course, assume that income distribution remains more or less stable in the long-run.

are two poles of a spectrum of various economies. Actual economies seem to be located somewhere in the middle of this spectrum. Furthermore, a transition from the non-indexation scheme to the indexation scheme and vice versa is often carried out on the basis of economic claculations which must take account of transactions costs associated with the transition itself. It is therefore reasonable to expect that the extent of indexation be incomplete as long as the rate of price inflation is moderate. We can then conclude that in the "relevant" range of the inflation rate the long-run aggregate rate of involuntary unemployment in excess of its constant natural rate is governed both by the rate of money wage inflation and by the rate of labor productivity growth, and their relative importance is in turn determined by the extent of wage indexation in the economy.

10. A Conclusion

Equilibrium is all but an indispensable notion in economic analysis. According to the neoclassical economics, the static economy in which wants, resources and technologies are unchanging is said to be in a state of static equilibrium "if every person is acting in such a way as to reach his most preferred position, subject to the opportunities open to him."[35] In the case of dynamic economy in which wants, resources and technologies are changing over time, expectations about the future play the central role in the notion of equilibrium. Such a dynamic economy is said to be in equilibrium at a point of time or simply in temporary equilibrium if every individual is reaching the most preferred position, subject to the constraints by which they are bound, and with respect to the expectations

[35] Hicks [5], p. 58. See also Hicks [6].

that they have at the point of time; and it is said to be in equilibrium over time if (i) it is in equilibrium in every point of time and (ii) the expectations on which it is based, in each single period, is consistent with one another and with what actually happens.[36] Common to all these neoclassical equilibrium notions is the idea that in order for the economy as a whole to be in equilibrium all individuals must be in their own equilibrium positions simultaneously. In consequence, the whole edifice of the neoclassical equilibrium would collapse whenever any one of the individuals in it were displaced from her own 'equilibrium' point.

In the present paper, however, we have demonstrated that even if the central authority succeeds in keeping balance between aggregate demand and aggregate supply and even if all the entrepreneurs in the economy happen to have expectations which are consistent with what actually happens, the dynamic economy under the condition of money wage rigidity has no tendency to approach any neoclassical equilibrium. It can only achieve, at best, a state of stochastic macro-equilibrium--an equilibrium of the economy as a whole which is maintained by offsetting motions of a large number of firms perpetually thrown out of their equilibrium positions by incessant stochastic disturbances. The stochastic macro-equilibrium is, in other words, a macroscopic equilibrium of microscopic disequilibria. It differs fundamentally from the neoclassical notion of equilibrium which can be simply characterized as a collection of mutually compatible microscopic equilibria.

Involuntary unemployment is, of course, a disequilibrium phenomenon in labor markets. Consequently, it should not exist in any neoclassical

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[36] Hicks [5, 6].

equilibrium. However, if the dynamic economy with money wage rigidity will never approach any neoclassical equilibrium position, as we have seen, the phenomenon of involuntary unemployment will never disappear no matter how long the economy is run. The explanation of persistent involuntary unemployment as a long-run phenomenon becomes possible only when we have freed ourselves from the rigid equilibrium notion of neoclassical economics. In fact, its careless import has been the source of much confusion in the conventional macroeconomics.

One of the fundamental attributes of neoclassical equilibrium is its money-neutrality property. The so-called natural rate theory of unemployment, which claims that there is no permanent trade-off between inflation and unemployment, is a sophisticated restatement of this neoclassical property. However, if the neoclassical equilibrium is not even an asymptotic state of the dynamic economy, then the natural rate theory of unemployment loses its ground. In fact, we have shown in this paper that the average rate of involuntary unemployment in excess of its natural rate in long-run-cum-macro-equilibrium is higher, the more dispersed labor market disequilibria across firms; and we have also proved that the labor market disequilibria in turn become more dispersed, the lower the average growth rate of money wage, under the condition of downward rigidity of money wage. If we combine these two propositions, we can derive a negative relation between the average rate of involuntary unemployment and the steady-state rate of money wage inflation even in a state of long-run macro-equilibrium. This is nothing but the downward-sloping long-run Phillips curve whose theoretical foundation macroeconomics has been looking for.

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