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The Influence of Interest Rates on Resource Prices

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Errata for CFDP No. 407

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Page 9. The equation for A_4 should read

$$A_4 = (a_1 a_2 - 2a_1) / \theta$$

Page 18. The expressions for B_1, B_2 and B_3 should read

$$B_1 = 1 - A_1 - A_2 , \; B_2 = A_1 + A_2 , \; B_3 = A_2 .$$

Page 18. Six lines from the end, $A_1 + A_2 \neq 0$

should be replaced by $A_1 + A_2 \neq 1$.

THE INFLUENCE OF INTEREST RATES ON RESOURCE PRICES*

by

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I. Introduction

In recent years there have been many analyses of the rate of resource depletion, both with a view to defining an optimal depletion rate (as in [2], [3]) and also with a view to analyzing the depletion rate that one might expect to result from market forces (as in [1], [12], [13]). It is easily established (see [3], [12]) that a necessary condition for a finite stock of an exhaustible resource to be allocated efficiently over time is that its price, net of extraction costs, should rise at a rate equal to the rate of return on other assets. And, not surprisingly, competitive markets will under certain circumstances realize this condition. In particular, if owners of the resource regard it as a capital asset constituting an element of their portfolio, then they will hold it just as long as the return that it gives them (the rate of increase of the net price) is no less than the returns available elsewhere. Equilibrium in the asset market will then imply the realization of the necessary condition mentioned earlier.

This simple but convincing theorizing clearly implies that if resource markets are functioning efficiently, there will be a strong

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association between the rates of change of resource prices and the rates of return on other assets. In particular, as certain commodities (for example, copper, lead and zinc) are exhaustible resources, the theory would predict that in an efficient allocation the rates of change of their prices would be related to rates of return on other assets. My aim in this paper is simply to conduct an empirical test of this prediction, and then to use the results of this to make some very tentative deductions about the intertemporal efficiency of resource markets.

II. The Model

A naive approach to this issue might be just to regress the rates of change of appropriate commodity prices on variables representing the returns available elsewhere. I have preferred instead to construct a model which incorporates an element of arbitrage between the resource market under consideration and a capital market, because the model constructed can give one prior information about the lag structures and stochastic specifications likely to be encountered, and these turn out to be important considerations. It is also true that the model tested, and in some considerable measure validated, has some interesting and precise implications about the validity of the theoretical predictions referred to earlier. These will be considered in some detail subsequently.

The model considered has a fairly obvious structure. It is supposed that the resource price always adjusts so that supply and demand are equated. If p is the current price and p' a weighted average of past prices, and likewise y is current income and y' a weighted average of past incomes, supply is just taken to depend on p' and y' : $S(p', y')$. The rationale for including p' is that supply responds to price changes with a lag: y' is included in case the level of economic activity affects investment in the extension of extractive and refining capacity directly, rather than via the price of the output.

The demand function is more complex and contains two distinct elements: one is a log-linear function of price and income, and this is multiplied by a term which depends on the ratio of the expected rate of capital gain from the resource to the expected rate of capital gain attainable on other assets.

$$D = p^{\alpha} y^{\beta'} \left[\frac{p''/p}{0''/0}\right]^{a_1}$$

where α and β' are of course price and income elasticities, p'' is the resource price expected to rule at some future date, 0 is the price of some other asset, and $0''$ is again the price this is expected to exchange for at the same future date. The motivation underlying this functional form is clear: demand consists of a "normal" or "user" element depending in the obvious way on price and income, and this is scaled up or down according to whether or not the resource is expected to be a good investment in the near future. Thus if its price is expected to rise at a rate in excess of those of other assets, demand is increased, and vice versa. The multiplicative term is introducing an element of arbitrage between resource and capital markets into the model, and my aim is to assess the importance of this effect. Obviously, realization of the efficiency conditions mentioned in the introduction would require very effective arbitrage.

An alternative interpretation of the demand function may be worth mentioning. This is that traders and speculators are distinct agents in the market, with trader demand depending on $p^{\alpha}y^{\beta'}$ and speculator demand conditioned by $(p''/p)/(0''/0)$, but with a multiplicative rather than additive interaction. This has the implication that, given a set of expectations about rates of return, speculators are more willing to enter a market the greater is the level of regular or user demand in that market.

It is clear that, whatever interpretation one takes, there is an element of "ad hocery" about this demand function. For example, it cannot be derived from a model of stochastic dynamic optimization on the part of market participants, though ideally this is the foundation on which one would like to base the model. But unfortunately my attempts to derive a demand function from such a basis suggest that nothing estimable will emerge without very strong assumptions. (Perhaps it is worth noting that the demand function resulting from a dynamic stochastic optimization exercise does reduce to something close to the one used if the variance and all higher moments of the probability distribution of future prices are set to zero: obviously this corresponds to assuming a form of certainty --equivalent behavior.) As the present formulation has no completely rigorous justification, it is clearly open to the objection that there are alternative and apparently equally plausible formulations. One could for example argue that though it is plausible that demand should be scaled up or down according to anticipated return differentials, there is no reason why one should work with the ratio rather than the difference, or some other function. A response to such criticism is that the exact formulation of the multiplicative term in anticipated return differentials seems not to be crucial: alternatives such as working with differences rather than ratios lead eventually to rather similar equations to be estimated and imply similar restrictions on the coefficients. Hence what seems to be important is the general principle that demand depends on anticipated returns (and that these anticipations depend on past experience) rather than the particular detail in which the hypothesis is embodied.

Taking the demand and supply functions together, market clearing implies that

$$(1) \qquad S(p', y') = p^{\alpha} y^{\beta'} \left[\frac{p''/p}{0''/0}\right]^{a_1} e_1(t)$$

where $e_1(t)$ is a lognormally distributed serially independent error process. An obvious response to such an equation is to enquire why the term in anticipated returns appears only on the right-hand side: why should suppliers not also modify their behavior according expected price changes? The answer is clearly that one can imagine a term identical to that in square brackets appearing on the L.H.S., raised perhaps to a power b_1. But it is then abundantly clear that a_1 and b_1 could not both be estimated: we therefore imagine the multiplicative terms of this type concentrated on the R.H.S. with a_1 the net exponent.

Differentiating (1) logarithmically w.r.t. time and using the following notation:

$$\dot{p}/p = r_c \ , \quad \dot{p}''/p'' = r_c'' \ , \quad \dot{0}/0 = r \ , \quad \dot{0}''/0'' = r'' \ , \quad \dot{y}/y = g$$

we have

$$\dot{S}/S = a_1(r_c'' - r'') - a_1(r_c - r) + \alpha r_c + \beta' g + \frac{d}{dt}\log e_1 \ .$$

In order to make further progress, it is necessary to specify how the anticipated values r_c'' and r'' are formed. Consider first the term

p''/p . Introducing the time argument explicitly, this might be written $p''(t+h)/p(t)$, where $t+h$ is the time to which expectations formed at t refer. Using this notation, the term $\dot{p}''/p''$ can be written

$$\dot{p}''/p'' = \lim_{\Delta t \to 0} \left\{ \frac{p''(t+h+\Delta t) - p''(t+h)}{\Delta t \cdot p''(t+h)} \right\} .$$

As it is not unreasonable to assume that speculators' expectations in commodity markets are of a very short-term type, we shall also let h tend to zero, and define

$$\dot{p}''/p'' = \lim_{\Delta t \to 0} \lim_{h \to 0} \left\{ \frac{p''(t+\Delta t+h) - p''(t+h)}{\Delta t \cdot p''(t+h)} \right\} .$$

Clearly $\lim_{h \to 0} p''(t+h) = p''(t)$, and it is obvious to assume that $p''(t) = p(t)$.

Hence

$$\dot{p}''/p'' = \lim_{\Delta t \to 0} \left\{ \frac{p''(t+\Delta t) - p(t)}{\Delta t \cdot p(t)} \right\} .$$

Now, a reasonable first-order approximation to $p''(t+\Delta t)$ is clearly

$$p''(t+\Delta t) = p(t) + p(t) r_c^e(t) \Delta t$$

where $r_c^e(t)$ is the expected rate of price change at time t , so that

$$\frac{p''(t+\Delta t) - p(t)}{\Delta t \cdot p(t)} = r_c^e(t) .$$

Although it is reasonable to assume that the current price level $p(t)$ can be observed accurately, one would clearly not wish to make this assumption about its current rate of change $r_c(t)$: an approximation to this has to be built up from past observations, and it is assumed that an agent's best approximation to $r_c(t)$ is given by the distributed lag form $a_2 r_c(t)/(D+a_2)$, where D is the differential operator. Hence in (2) we can make the substitution

$$r_c'' = a_2 r_c/(D+a_2) + \epsilon_2(t) \tag{3}$$

where $\epsilon_2(t)$ is a white noise error process, and by similar arguments one can justify the assumption that

$$r'' = a_3 r/(D+a_3) + \epsilon_3(t) \; . \tag{3'}$$

In order to make (2) operational, it is necessary to specify the form of the supply function. This is assumed to take the very simple form $S(p', y') = p'^{a_4} y'^{\beta''}$, with p' and y' defined by the lag processes

$$p'(t) = \frac{3\lambda p(t)}{(D+\lambda)^3} , \quad y'(t) = \frac{\mu y(t)}{D+\mu} \; . \tag{4}$$

Substituting from (3), (3') and (4) into (2) yields the following second-order differential equation, which contains only observable variables:

$$\text{(5)} \quad \begin{cases} \ddot{r}_c(a_1 + a_4 - \alpha) + \dot{r}_c(a_1 a_3 + a_2 a_4 + a_3 a_4 - \alpha a_2 - \alpha a_3) \\ + r_c(a_2 a_3 a_4 - \alpha a_2 a_3) = \ddot{r} a_1 + \dot{r} a_1 a_2 + \ddot{g}\beta + \dot{g}(a_2 + a_3)\beta \\ + g a_2 a_3 \beta + \epsilon \; . \end{cases}$$

In this equation, $\beta = \beta' - \beta''$ and is a net income elasticity. It could thus be zero even though the income variable exerted a significant influence on both sides of the equation. The error process ϵ will exhibit third-order serial correlation, and if one believes the stochastic specifications

(1), (3) and (3') this will be of the moving-average type. But in fact if we operate with a short time-period, as will be the case, the assumption that the errors in (1), (3) and (3') are uncorrelated is unreasonable. They are likely to exhibit substantial positive serial correlation, of the sort that will lead to a mixed autoregressive-moving average error process in (5).

III. Estimation

Rather than estimate the differential equation (5), I have chosen to estimate a difference equation approximation to it. There is a growing literature on the estimation of stochastic differential equations and of discrete forms of these, and the merits of different approaches have been discussed *inter alia* by Sargan [10] and Phillips [8]. The transformation applied to (5) is one discussed by these authors, and seems to have desirable properties: it is

$$\dot{x}(t) = x(t+1) - x(t)$$

$$\ddot{x}(t) = x(t+2) - 2x(t+1) + x(t)$$

$$x(t) = \frac{1}{2}(x(t+1) + x(t)) \; .$$

Applying this transformation, (5) becomes

$$(6) \quad \begin{cases} r_c(t) = A_1 r_c(t-1) + A_2 r_c(t-2) + A_3 r(t) + A_4 r(t-1) + A_5 r(t-2) \\ \qquad + A_6 g(t) + A_7 g(t-1) + A_8 g(t-2) + \epsilon(t) \end{cases}$$

where the arguments denote values of variables in particular time-periods. The coefficients in (6) are related to the original parameters by the formulae

$$A_1 = 2 - a_3 - a_2a_4/\theta + \alpha a_2/\theta - a_2a_3a_4/2\theta + \alpha a_2a_3/2\theta$$

$$A_2 = -1 + a_3 + a_2a_4/\theta - \alpha a_2/\theta - a_2a_3a_4/\theta + \alpha a_2a_3/\theta$$

$$A_3 = a_1/\theta$$

$$A_4 = (a_1a_4 - 2a_1)/\theta$$

$$A_5 = (a_1 - a_1a_2)/\theta$$

$$A_6 = \beta/\theta$$

$$A_7 = -2\beta/\theta + \beta a_2/\theta + \beta a_3/\theta + \beta a_2a_3/2\theta$$

$$A_8 = \beta/\theta - \beta a_2/\theta - \beta a_3/\theta + \beta a_2a_3/2\theta$$

where $\theta = a_1 + a_4 - \alpha$.

Obviously estimating (6) is not entirely straightforward: the equation contains lagged endogenous variables, groups of variables which will be collinear, autocorrelated errors, and has coefficients which are complex non-linear functions of the parameters of the original model. It is also true that some parameters are under and some over-identified. I have in fact used two different approaches to estimating (6). The first estimates the coefficients A_1 to A_8 without any attempt to impose on them the restrictions implicit in the formulae relating them to the parameters. The estimation method, a member of the class of generalized instrumental variable estimators (GIVE), was developed by Hendry [4] on the basis of work by Sargan [9], and produces asymptotically efficient, normally distributed and consistent estimates of the coefficients of an equation with lagged endogenous variables and an autoregressive error process. Of course, the error process in (6) is probably not purely autoregressive, but is a mixed autoregressive-moving average process, but Monte Carlo studies by Hendry and Trivedi [7] suggest that the biases produced in approximating a moving average process by an autoregressive one of similar order are not large. Indeed, subsequent analytical results due to Hendry [5] confirm that in some simple cases the biases in the coefficients are unimportant, and that if the true error process is mixed autoregressive-moving average, then a pure autoregressive process is a very good approximation.

Fortunately one of the important constraints implicit in the coefficient-parameter relationships has a very simple form and is easily tested against the unconstrained estimates: it is that

$$A_3 + A_4 + A_5 = 0 \ .$$

Obviously this can be tested by seeing whether the sum of the interest rate coefficients is significantly different from zero. This constraint is in fact satisfied to a very high degree of accuracy--a very interesting finding whose implications are considered in some detail below.

The second approach to estimating the model is to estimate the parameters of the original system directly, which means estimating (6) subject to non-linear constraints on the coefficients. In fact, as already mentioned, some of the parameters are underidentified and instead of being able to estimate all six parameters α , β , a_1 , a_2 , a_3 , a_4 , it is only possible to estimate:

$$\theta_1 = a_1/\theta , \quad \theta_2 = a_2 , \quad \theta_3 = a_3 , \quad \theta_4 = \beta/\theta .$$

Hence the two lag coefficients of the expectation formation equations (3) and (3') can be estimated directly, but the remaining parameters can only be identified in combinations. An autoregressive maximum likelihood estimation procedure was used: its theoretical basis is to be found in an article by Hendry [6], who also developed the program.

IV. Results

The model described in equation (6) has been estimated against data from zinc, lead and copper markets on the London Metal Exchange over the period December 1965 to December 1973: this includes both a relatively tranquil period for commodity markets, and also the commodity price boom of the early 1970s. A variety of prices are available (spot, settlement, forward), but the results seem insensitive to the particular choice made. The three-month forward prices give the best fits, and as this is also the market with the greatest trading volume I concentrate below on results for prices of this maturity. (Of course, if one buys a three-month contract on January 1st, then the relevant selling price on February 1st is the two-month price: but a two-month contract can be resold at the prevailing three-month price if the seller will bear storage costs, which are typically very low. Hence there is a strong link between two and three month prices, etc., and a sequence of prices of constant futivity does give an accurate picture of the possibilities open to a trader in the forward market.) The remaining data used are easily described: $r(t)$, the return on an alternative asset, was taken to be the return on 91-day U.K. Treasury Bills, and Y, the income variable whose growth rate is given by g, was taken to be the O.E.C.D. Index of Industrial Production. All data were on a monthly basis, giving at least 100 observations in all runs, and all variables except the index of industrial production can be assumed to be measured accurately. The monthly changes in this latter are likely to be of the same order of magnitude as its measurement error, although the fact that they are positively serially correlated may mean that the first-differenced series is measured to a higher degree of accuracy than the original.

A final point to mention about the data is that all equations were run with both money values and real values of variables, with the deflator being the U.K. retail price index. Again, there was little to choose between the sets of results, but the equations estimated in real terms usually gave slightly better fits and are reported here.

It is probably best to begin by discussing the results of the constrained estimates of the parameter combinations θ_1 to θ_4 : these are given in Tables 1, 2 and 3. The figure in brackets following each estimate is the t-statistic: this needs to be at least 1.96 for the coefficient to be significantly in excess of zero at the 95% confidence level. As $\alpha > 0$, one would clearly expect $0 < \theta_1 < 1$, and this expectation is borne out in every case, with estimates significantly in excess of zero but nevertheless much below one. a_2 and a_3 must certainly be non-negative, and in all cases they are significantly so: the estimates imply reasonably rapid responses of expectations to observations, with more rapid responses in the commodity than bond markets. In all cases, β/θ is not significantly different from zero--a finding which, if taken at its face value, implies that the prices of the metals concerned have not been significantly affected by levels of industrial production. Recall that $\beta = \beta' - \beta''$ where β' is the exponent of y in the demand function and β'' is the exponent of y' in the supply function, so that a possible interpretation of the insignificance of β is that these two coefficients are approximately equal. This would then imply that prices are independent of y because this affects both demand and supply equally. Of course, there are alternative interpretations. One, alluded to before, is that the series g is dominated by errors. Certainly y contains errors, but in monthly data these are likely to be sufficiently serially correlated for differencing to improve the accuracy of the resulting time series substantially. Another possibility is that the existence of inventories implies substantial time-lags between changes in production and changes in demand. However, this latter hypothesis is slightly discredited by the fact that even when a number of lags are applied to Y before generating g, it is still not possible to record significant values of β/θ. Hence one is left with the conclusion that either g is dominated by errors, or the influence of industrial production on demand is less than widely supposed. In passing, one might note that regressions of P on Y will almost certainly record spuriously high values of R^2 because both variables are so strongly trended: by specifying the relationship between differences, the present model is applying more stringent tests than normal. A final point to mention about the constrained estimates is that all of them, as expected, show significant second-order autocorrelation. In general, it seems fair to say that the constrained estimates are all eminently reasonable, and give one no strong grounds for wishing to query the model.

The unconstrained estimates of the coefficients of equation (6) are more complex, and are presented in Tables 4, 5 and 6. In each case three different forms of the equation have been estimated, with the coefficients reported in the three columns of each table and the relevant t-statistics in brackets. In the first column are the results of estimating (6) on the assumption that the errors are N.I.D., and in the third column are the results of estimating (6) with a second-order autoregressive error. The second column contains the results of estimating the unrestricted second-order autoregressive transform of the first column, and hence features a larger number of lagged variables than the others. Analysis of the likelihood values corresponding to these three forms enable one to test the validity of the dynamic specification of (6) (for more details, see Hendry [4]). If L_1, L_2 and L_3 are the likelihood values corresponding to the three forms, it is clear that $L_2 \geqq L_3 \geqq L_1$, with $L_2 = L_3$ if (6) has the correct dynamic specification and $L_3 = L_1$ if the errors in (6) are in fact N.I.D.: we can therefore test the appropriateness of the alternative specifications by seeing whether the various likelihood ratios are significantly in excess of unity, and also by testing whether the residual correlograms are random. Each table reports chi-squared tests of the hypotheses that $L_2/L_3 > 1$ and $L_3/L_1 > 1$, and, in the bottom line, of the hypothesis that the residual correlogram is significantly non-random. In addition, R^2 and the standard error of the estimate are supplied where appropriate.

Turning now to the numerical results and looking first at the case of zinc, we note most immediately that the first and second equation forms produce values of R^2 which are very respectable for time series in first-difference form. The third equation is nearest to having a random pattern of residuals, but L_2/L_3 is significantly in excess of unity whereas L_2/L_1 is not. This suggests that the best possible fit might be provided by an equation of the same general form as (6) but with two more lagged values for each predetermined variable: such an equation could easily arise from the same model if one specified higher-order lags in the expectation-formation equations. But in spite of this possible scope for improvement, the results corresponding to equation (6) for zinc are sufficiently good to merit closer examination. It will be recalled that one of the constraints implicit in the model is that the sum of the coefficients of current and lagged interest rates should be zero, and in the third column of Table 4 these coefficients are -0.680, +0.540, +0.147, giving a sum of +0.007, clearly not significantly different from zero. Indeed, the sums of the interest rate coefficients in the first and second columns are also +0.007, again not significantly different from zero. The summing to zero of the interest rate coefficients is, as we shall see, a theoretically crucial qualitative characteristic of the model, and it is a characteristic that is clearly strongly supported by data from the zinc market. It should of course be emphasized that the coefficients are not summing to zero just because each differs insignificantly from zero: many of these coefficients are in excess of 0.5 and highly significant. We conclude therefore that interest rates do have a significant effect on zinc prices, and that this effect is as specified by the model.

Looking at the remaining coefficients in any of the columns of Table 4, we see that the once-lagged endogenous variable is highly significant (and the fact that it has a positive sign implies that we are not merely picking out the obvious common-variable correlation between $p_t/p_{t-1} - 1$ and $p_{t-1}/p_{t-2} - 1$), and that the growth of industrial production is always insignificant. This is obviously in keeping with the insignificance of θ_4 in the restricted estimates.

The results for lead and copper are very similar to those for zinc, except that in both cases the third column is clearly the best fit. In all cases, individual interest rate variables record significant t-statistics but the coefficients sum to zero--thus for the three forms of the lead equation these sums are -0.001, -0.001 and 0.000. For copper, the corresponding sums are -0.001, -0.003 and -0.01.

V. Conclusions

The model described by equation (6) has now been tested exhaustively against data from three different resource markets, and many of its features have received striking confirmation: the only one of its predictions that is not borne out is that concerning the role of g, and this may well be explicable in terms of the quality of the data.

These findings place us in a position to comment on the issues discussed in Section I, where it was noted that if exhaustible resources are allocated efficiently over time, their net price should rise at a rate equal to the return elsewhere. Certainly, we have found a strong relationship between the rate of change of resource prices and returns elsewhere--and this is a relationship that we would probably not even have sought had the relevant theory not existed. But the adding-up property of the interest rate coefficients, so strikingly confirmed in the unrestricted estimates, makes it clear that this relationship is not of the form that would be necessary for efficient intertemporal resource allocation. The point is that an equation with this adding-up property can be written:

$$r_c(t) = b_1 r(t) + b_2 r(t-1) - (b_1 + b_2) r(t-2)$$

or

$$r_c(t) = b_1(r(t) - r(t-1)) + (b_1 + b_2)(r(t-1) - r(t-2))$$

or

$$r_c(t) = b_1 \Delta r(t) + b_2' \Delta r(t-1) \ .$$

The rate of change of the resource price is therefore seen to depend not on the level but on the rate of change of the interest rate: the actual relationship differs from the necessary conditions by one time-derivative. As this property was predicted by the model of equation (1), it should be possible to use that model to provide some explanation. The result seems to depend on two features of the model:

(i) that demand and supply conditions depend both on the level and on the rate of change of the resource price, whereas they depend only on the rate of change of the other price.

(ii) there are lags in the formation of expectations about rates of return.

The importance of the first of these features can be checked by looking at the expressions for A_1 to A_8 following equation (6): if the demand and supply conditions for the resource depended only on the rate of change of its price, this would imply that α (the exponent of p in the demand function) and a_4 (the exponent of p' in the supply function) were both zero, in which case $A_1 = 2 - a_3$ and $A_2 = -1 + a_3$. Hence $A_1 + A_2 = 1$, and the sum of the coefficients on $r_c(t)$, $r_c(t-1)$ and $r_c(t-2)$ would be zero. One would then have a symmetrical relationship between changes in r_c and changes in r, and integration would yield the relationship that would characterize an efficient allocation. That the actual relationship estimated reflects the asymmetry of treatment in (1) can be seen by noting that the terms $r_c(t)$, $A_1 r_c(t-1)$ and $A_2 r_c(t-2)$ can be grouped together on the L.H.S. as

$$B_1 r_c(t) + B_2 \Delta r_c(t) + B_3 \Delta r_c(t-1)$$

where

$$B_1 = 1 - \frac{A_1}{2} - \frac{A_2}{2}, \quad B_2 = \frac{A_1 + A_2}{2}, \quad B_3 = \frac{A_2 - A_1}{2},$$

$$\Delta r_c(t) = r_c(t) - r_c(t-1), \quad \text{etc.}$$

Equation (6) then takes the form

$$(7) \qquad B_1 r_c(t) + B_2 \Delta r_c(t) + B_3 \Delta r_c(t-1)$$

$$= b_1 \Delta r(t) + b_2 \Delta r(t-1) + A_6 g(t) + A_7 g(t-1) + A_8 g(t-2) + e(t)$$

and gives a relationship between the level and rate of change of the return to the resource, and the rate of change of the alternative return--exactly what is implied in derivative form in equation (5), and reflecting in differenced form the ways in which these variables enter the original specification. It is worth noting that the fact that $A_1 + A_2 \neq 0$ clearly implies that the presence of both levels and rates of change of the resource price in demand and supply functions is justified.

The importance of lags in expectation formation in establishing the adding-up property of interest rate coefficients is best seen by referring to (2): suppose that traders had perfect myopic foresight about rates of return. Then $r''_c = r_c$ and $r'' = r$, and all terms in the return on other assets would vanish from the equation.

There is one minor complication concerning the data that should be faced before finally relating our results to the basic theory: this is that the latter requires the price of a resource <u>net of extraction costs but before refining</u> to rise at rate r, whereas our data refer to the price of the refined resource. Clearly one can write

$$p_R = p_0 + EC + RC$$

where p_R is the price of the refined metal, and p_0 is the imputed net price of the ore--i.e. the value of a unit of unextracted and unrefined one. It is this price to which the theory of Section I refers. EC and RC refer to extraction and refining costs. Letting $EC + RC = C$, rearranging and differentiating,

$$\frac{\dot{p}_0}{p_0} = \left\{\frac{\dot{p}_R}{p_R} - \frac{\dot{C}}{p_R}\right\}\frac{1}{1 - C/p_R} .$$

This shows that the rate of change of the net price of the ore is easily related to the rate of change of refined prices: one simply subtracts cost changes as a fraction of refined price and multiplies by a factor which approaches unity for high-grade ores where extraction and refining costs are a small fraction of refined price, but which could be very large for low-grade ores. It should be clear from this that the relationship between $\dot{p}_0/p_0$ and r will not be qualitatively different from that between $\dot{p}_R/p_R$ and r --indeed if $\dot{C}$ is small, the two will be very similar. So we can reasonably suppose the relationships we find between r_c and

r to hold also for $\dot{p}_0/p_0$ and r .

Given this similarity, we can legitimately use the results established so far as a test of the basic theory of Section I, and ask whether there is evidence that markets allocate the finite resources studied efficiently over time. The answer seems to be no: the relationship that the data suggests between resource prices and interest rates is that given in difference equation form in equation (7), or in differential equation form in (5). The relevant parts of this latter could be written as

$$c_1\ddot{r}_c + c_2\dot{r}_c + c_3 r_c = c_4\ddot{r} + c_5\dot{r}$$

which integrates to

$$c_1\dot{r}_c + c_2 r_c + c_3 \log p = c_4\dot{r} + c_5 r \ .$$

There is therefore a relationship between r_c and r implicit in these results, but one that is considerably more complex than the simple one that efficiency would require. And this departure from the relatively simple dictates of efficiency occurs because demands for the resources depend not only on the expected returns associated with their ownership, but also on their prices. This level-dependent element in the demand functions is of course a derived demand, reflecting demand conditions for the products produced from the resource and the availability of substitutes in the production process. For efficient allocation the resource should be valued purely as an asset--except at the initial moment of the resource--allocation problem, when the correct price level has to be chosen.

As the resources studied seem not to be allocated efficiently over time, it naturally follows that they must be depleted "too fast" or "too slowly." It would obviously be very interesting to know which--but that seems to require considerably more research.

TABLE 1. Zinc Forward Dec 65/Dec 73

$$\theta_1 = \frac{a_1}{a_1 + a_4 - \alpha} = 0.044 \quad (6.25)$$

$$\theta_2 = a_2 = 1.068 \quad (8.37)$$

$$\theta_3 = a_3 = 0.377 \quad (3.19)$$

$$\theta_4 = \frac{\beta}{a_1 + a_4 - \alpha} = -0.109 \quad (1.25)$$

Autoregressive Parameters: -0.000, -0.0327

S = 0.001

TABLE 2. Lead Forward Dec 65/Dec 73

$$\theta_1 = \frac{a_1}{a_1 + a_4 - \alpha} = 0.018 \quad (2.50)$$

$$\theta_2 = a_2 = 0.888 \quad (4.79)$$

$$\theta_3 = a_3 = 0.425 \quad (2.41)$$

$$\theta_4 = \frac{\beta}{a_1 + a_4 - \alpha} = 0.021 \quad (0.77)$$

Autoregressive Parameters: -0.000, -0.245

S = 0.001

TABLE 3. Copper Forward Dec 65/Dec 73

$$\theta_1 = \frac{a_1}{a_1 + a_4 - \alpha} = 0.028 \;\; (2.05)$$

$$\theta_2 = a_2 = 0.716 \;\; (5.04)$$

$$\theta_3 = a_3 = 0.549 \;\; (4.59)$$

$$\theta_4 = \frac{\beta}{a_1 + a_4 - \alpha} = 0.040 \;\; (0.67)$$

Autoregressive Parameters: -0.0002, -0.229

$S = 0.004$

KEY TO TABLES 4, 5 AND 6

The variables to which coefficients refer are listed in the left hand margin. r_c is the return to holding the commodity, r is the return on an alternative asset, and g is the rate of growth of y . The number after a variable represents the length of lag: thus r is the current return on the other asset, r1 is the same lagged one and r2 indicated a two-period lag. In all tables, δ is the standard error of the estimate.

TABLE 4. Zinc Forward Dec 65/Dec 73. Real Variables (Deflator: U.K.R.P.I.)

r_c1	0.691 (6.09)	0.647 (5.87)	0.768 (8.25)
r_c2	0.037 (0.22)	-0.099 (0.58)	0.0007 (0.18)
g	-0.138 (1.11)	-0.240 (1.82)	-0.131 (1.05)
g1	0.131 (1.21)	0.071 (0.59)	0.153 (1.32)
g2	-0.076 (0.62)	-0.140 (0.98)	-0.071 (0.566)
r	0.568 (6.82)	0.542 (5.81)	0.540 (6.48)
r1	-0.677 (5.49)	-0.639 (5.48)	-0.680 (5.60)
r2	0.116 (1.12)	-0.067 (0.463)	0.147 (1.46)
δ			-0.122 (0.94)
r_c3		0.541 (3.32)	
r_c4	$R^2 = 0.66$	-0.001 (0.66)	L_2/L_3:
g3	S = 0.037	-0.024 (0.205)	$\chi^2(5) = 17.24$
g4		-0.042 (0.335)	L_3/L_1:
r3		0.056 (0.342)	$\chi^2 = 1.62$
r4		0.115 (0.985)	S = 0.036
		$R^2 = 0.73$	
		S = 0.035	
	$\chi^2(12) = 14.92$	$\chi^2(11) = 16.21$	$\chi^2(12) = 13.49$

TABLE 5. <u>Copper Forward</u> Dec 65/Dec 73. Real Variables (Deflator: U.K.R.P.I.)

r_c1	0.382 (3.69)	0.445 (4.30)	0.528 (5.81)
r_c2	-0.255 (2.48)	-0.437 (4.07)	0.003 (0.98)
g	0.357 (1.33)	0.300 (1.05)	0.33 (1.29)
g1	0.111 (0.47)	0.039 (0.15)	-0.003 (0.01)
g2	0.154 (0.58)	0.425 (1.37)	0.24 (0.93)
r	0.460 (2.60)	0.386 (2.27)	0.41 (2.59)
r1	-0.237 (0.94)	-0.192 (0.80)	-0.21 (0.92)
r2	-0.224 (1.21)	0.098 (0.34)	-0.21 (1.26)
δ			-0.50 (5.00)
r_c3	$R^2 = 0.24$	0.365 (3.57)	
r_c4	S = 0.079	0.005 (1.44)	L_2/L_3:
g3		0.002 (0.006)	$\chi^2(5) = 5.08$
g4		0.341 (1.25)	L_3/L_4:
r3		-0.011 (0.032)	$\chi^2 = 12.84$
r4		-0.284 (1.12)	
		$R^2 = 0.37$	
		S = 0.075	
	$\chi^2(12) = 20.27$	$\chi^2(11) = 5.02$	$\chi^2(12) = 8.17$

TABLE 6. Lead Forward Dec 65/Dec 73. Real Variables (Deflator: U.K.R.P.I.)

$r_c 1$	0.516 (4.62)	0.609 (5.44)	0.576 (5.85)
$r_c 2$	-0.22 (1.98)	-0.339 (2.67)	-0.0005(0.33)
g	-0.121 (1.22)	-0.202 (1.91)	-0.116 (1.19)
g1	0.049 (0.58)	0.092 (0.93)	0.086 (0.90)
g2	-0.056 (0.59)	-0.129 (1.13)	-0.054 (0.56)
r	0.241 (3.79)	0.201 (3.19)	0.210 (3.41)
r1	-0.369 (4.01)	-0.356 (3.98)	-0.326 (3.74)
r2	0.127 (1.84)	0.116 (1.06)	0.116 (1.77)
δ			-0.307 (2.60)
$r_c 3$		0.224 (1.97)	
$r_c 4$	$R^2 = 0.34$	0.001 (0.59)	L_2/L_3:
g3	S = 0.028	0.122 (1.29)	$\chi^2(5) = 8.75$
g4		-0.075 (0.76)	L_3/L_1:
r3		0.172 (1.31)	$\chi^2 = 4.21$
r4		-0.134 (1.47)	
		$R^2 = 0.42$	
		S = 0.027	
	$\chi^2(12) = 10.49$	$\chi^2(11) = 8.9$	$\chi^2(12) = 6.39$

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