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The Estimation of a Dynamic Equation Following A Preliminary Test for Autocorrelation

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Section 1

This paper considers the procedure of estimating a dynamic linear equation including testing for the possibility that the disturbance terms in the relationship have positive first-order serial correlation. That is, we investigate procedures which consist of first testing for serial correlation; then either estimating the relationship using Ordinary Least Squares if the hypothesis of no serial correlation is accepted or estimating the relationship using an estimator which takes serial correlation into account if the hypothesis is rejected.

The model considered is the simplest possible which allows for variation in the factors known to be at least asymptotically important. The equation to be estimated is

(1) $$y_t = \alpha y_{t-1} + \beta x_t + \gamma + u_t \ , \quad t = 1, \ldots, T$$

with an error process

(2) $$u_t = \rho_u u_{t-1} + \eta_t \ .$$

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In obvious vector notation $y = \alpha y_{-1} + \beta x + u$. x is strictly exogenous $0 \leq \alpha < 1$, $0 \leq \rho_u < 1$, and $E(\eta|x) = 0$. The random vector η is assumed to be distributed as $N(0, \sigma_\eta^2 I_T)$.

If ρ_u is zero, it is known that OLS is asymptotically unbiased, consistent, and efficient although the presence of the autoregression in y causes a small sample bias in the estimates (see Malinvaud, [10], Peck, [11], Hurwicz, [12]). When ρ_u is different from zero, however, OLS is known to be inconsistent and is likely to give poor estimates in finite samples. Thus the usual procedure is to guard against this possibility by testing for first-order serial correlation before accepting the OLS estimates.[1] A variety of at least consistent estimators are available when ρ_u is different from zero.

The researcher's estimation strategy thus has three components. First, which test should be used; second, at what nominal significance level should the test be performed, and third, what estimator should be employed if serial correlation is found? These are the questions investigated in this paper.[2] In studying these questions, I shall be concerned only with their effect on the final estimates. I am not concerned here with the prediction problem, for which the value of ρ_u is of interest, nor with distortions in inference about the model which could arise because significance tests on coefficients in the model would not have their nominal significance.

There is no reason to think that testing at the one or five percent levels, which is customary in classical testing theory, is optimal

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[1] I have assumed here that first-order serial correlation is the only possible difficulty with the disturbances. This is not meant to suggest that, say, higher-order serial correlation need not be tested for in practice.

for the problem addressed in this paper. In fact, one might suspect that never testing at all and simply assuming the existence of serial correlation is a better strategy. The presence of autocorrelation is only of interest here because it affects the properties of the estimators. If serial correlation is truly absent in the population but a Type I error is made in testing, this is unimportant if the alternative estimator does well, and serious if correcting for nonexistent correlation gives much worse estimates than OLS. Similarly, if correlation is present, but a Type II error is made, this matters only if this error, leading to the use of OLS, gives poorer estimates than does an alternative estimator on the same sample. Ordinary Least Squares estimates for this model are very bad when ρ_u is large, but some alternative procedures do well even when the true ρ_u is zero. Thus, on one hand a testing procedure at an (algebraically) high significance level seems to reflect better the consequences of the two possible errors. On the other hand, even a five percent test may have high power against large values of ρ_u where the consequences of ignoring serial correlation are most serious. It is also possible that which alternative estimator is best depends on the test and significance level chosen. We are concerned with the choice of a procedure which has three components, a test, a nominal significance level for that test, and an alternative estimator to ordinary least squares to be used when the null hypothesis of no correlation is rejected: denoted as (t, sl, e) . Specifying a procedure for each parameter point

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[2]The test for serial correlation is a pretest procedure but is in effect performed on the unobservable variable u_{t-1} . The outcome of the test, however, affects both the final specification and the choice of estimator.

gives a complete strategy for this simple problem. Given an evaluation criterion, the ranking of procedures will vary with unobservable dimensions of the parameter space, but we explore these rankings and look for good strategies based only on the observable dimensions of the problem; i.e., we attempt to find an operational decision rule for someone faced with the problem of estimating equation (1).

Four tests and five alternative estimators are investigated using Monte Carlo methods. Section 2 describes the tests and estimators used; Section 3 specifies the design of the experiments, and Section 4 discusses the results.

Section 2

First, the four tests are discussed: they are the Durbin-Watson d statistic, the Durbin test 1, Durbin test 2, and the likelihood ratio test. All tests are used as one-sided tests for positive serial correlation. The OLS residuals are denoted by $\hat{u}$.

1) The Durbin-Watson test (DW) based on the statistic

$$d = \frac{\sum_{2}^{T}(\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{2}^{T}\hat{u}_t^2}$$

is known to be biased toward accepting the null hypothesis when y_{-1} is a regressor, but it is routinely calculated by regression programs and might still be useful, especially if a larger than usual nominal significance level were used. The distribution of d depends on the regressors; thus to specify a precise testing strategy using this test, the exact significance level must be calculated for each sample. We have used the original procedure suggested by Durbin and Watson to approximate this (see Durbin and Watson [1]) of fitting a Beta distribution to $\frac{1}{4}d$ with moments depending on the sample. This distribution is then numerically integrated to obtain the significance level. In this procedure, of course, y_{-1} is inappropriately treated as an exogenous regressor. In so far as this approximation affects the behavior of this test, it should be regarded as part of the definition of this test.

2) Durbin test 1 (D1) is performed by calculating

$$h = \frac{\sum_{2}^{T} \hat{u}_t \hat{u}_{t-1}}{\sum_{2}^{T} \hat{u}_t^2} \sqrt{\frac{T}{1 - TV(\hat{\alpha})}}$$

where $V(\hat{\alpha})$ is the estimated variance of the OLS estimate of α and the positive square root is taken. Durbin [2] shows that h is asymptotically standard normal if $\rho_u = 0$, which is used to calculate the appropriate critical region for the desired significance level of h . If the denominator $1 - TV(\hat{\alpha})$ is negative, the test cannot be computed. In these cases our rule is to default to using Durbin's second test in order to always obtain a test outcome. For most parameter points, the alternative computation is infrequently needed.

3) Durbin test 2 (D2) is computed by regressing $\hat{u}$ on $\hat{u}_{-1}$, x , y_{-1} and the constant term and applying an asymptotically valid t-test to the regression coefficient of $\hat{u}_{-1}$.

4) The likelihood ratio test (LRT) requires the computation of the maximum likelihood estimator allowing $\rho_u \geq 0$. Then -2 times the log likelihood ratio for this model against the model constraining ρ_u to be zero is asymptotically $\chi^2(1)$. Computing this test is essentially as much work as is computing the maximum likelihood estimates and thus affords no computational savings, but it has a strong asymptotic justification.

Another procedure which could be used as a test would be to compute an estimate of ρ_u from the OLS residuals and to compare this $\hat{\rho}_u$ to some prechosen number. It is, therefore, not a conventional test, but if it is the magnitude of the sample correlation of the disturbances which affects the properties of OLS, this test would have some intuitive appeal even though the serial correlation coefficient computed from the residuals is biased toward zero. We shall not examine the properties of this test here, however.

Six estimators, including OLS, are examined. All except for OLS are consistent in the presence of serial correlation, but most are not asymptotically efficient. They differ considerably in the amount of computation required, but none is too expensive to be practical in most situations. The estimators are OLS, Maximum Likelihood, Durbin's estimator, Wallis' instrumental variables and generalized least squares, instrumental variables alone, and Hatanaka's residual-adjusted generalized least squares. Each is discussed in turn. Generalized least squares using the true ρ_u, might be the best estimator in this problem, but it would be nonsensical to study it in the context of testing for serial correlation since it uses the known value of ρ_u.

1) Ordinary Least Squares (OLS) has been discussed above and needs no further comment.

2) The Maximum Likelihood Estimator (MLE) is computed by an iterative process. It is the Cochrane-Orcutt procedure on the assumption that the initial value of y, y_1 in the sample is fixed. The estimator is consistent and asymptotically efficient as long as convergence to the global maximum is achieved. Iterations were stopped when the residual sum of squares changed by less than .01. In a very small number of cases convergence was not achieved in the specified maximum number of iterations, but the last iteration was taken as the estimate anyway.

3) Durbin's estimator as extended by Malinvaud (see Durbin [3], and Malinvaud [10], p. 565) consists in first applying OLS to the equation

$$y_t = d_1 y_{t-1} + d_2 y_{t-2} + d_3 x_t + d_4 x_{t-1} + d_5 + v_t \;;$$

estimating ρ_u as $\hat{d}_4/\hat{d}_3$, and then performing generalized least squares with this estimate. In all cases when GLS is used in this paper, the exact factorization of the covariance matrix of the disturbances is used; no observations are lost.

4) Instrumental Variables (IV) is performed using the lagged values of the strictly exogenous regressors as instruments. This is suggested because y_{t-1} is correlated with u_t when $\rho_u > 0$, and x_{t-1} and y_{t-1} are systematically related. This estimator thus corrects this source of inconsistency in OLS, but does not use sample information about ρ_u to improve the efficiency of the estimates.

5) Wallis proposed a two-step estimator (see Wallis [13]) consisting of first estimating equation (1) by instrumental variables, then forming an estimate of ρ_u from the IV residuals, and finally performing GLS with this estimate of ρ_u. This estimator is denoted IVGLS. Here, as elsewhere when ρ_u is estimated from the residuals, a small correction for bias is used. The estimator is

$$\hat{\rho}_u = \frac{T}{T-1} \frac{\sum_{2}^{T} \hat{u}_t \hat{u}_{t-1}}{\sum_{2}^{T} \hat{u}_{t-1}^2} + \frac{K}{T}$$

where K is the number of exogenous regressors and $\{\hat{u}_t\}$ are the residuals. This adjustment to the usual formula is numerically insignificant for the parameters used in this study.

6) The last estimator considered is the residual-adjusted generalized least squares (RAGLS) method due to Hatanaka (see [5]). This two step method consists of applying instrumental variables to equation (1) in the first step and calculating an estimate ρ_u^* of ρ_u from the residuals $\hat{u}$. The second step is to compute a modified generalized least squares estimate. The equation estimated is

$$y_t - \rho_u^* y_{t-1} = \alpha(y_{t-1} - \rho_u^* y_{t-2}) + \beta(x_t - \rho_u^* x_{t-1}) + \gamma(1 - \rho_u^*) + \delta\hat{u}_{t-1} + v_t , \quad t = 3, \ldots, T .$$

This method loses one observation in the second step because of the presence of $\hat{u}_{t-1}$ as a regressor. No first observation correction for exact GLS, thus, applies. The estimator is shown by Hatanaka to be consistent and asymptotically efficient without iteration.

Of these estimators only the maximum likelihood method and RAGLS are asymptotically efficient. All the estimators have finite-sample bias, however, and asymptotic results may be an insufficient guide in typical samples, especially if ρ_u is large. Only asymptotic distributions for the tests are available. The next section discusses the design of our small-sample experiments to explore the properties of these estimators and preliminary test procedures.

Section 3

This section describes the design of the Monte Carlo experiments. The available asymptotic distribution theory (see, e.g. Malinvaud [10]) suggests that four factors are important in determining the behavior of estimators for the equation

$$(2) \qquad y_t = \alpha y_{t-1} + \beta x_t + \gamma + u_t$$

$$u_t = \rho_t u_{t-1} + \eta_t \ .$$

They are α, ρ_u, the pattern of correlation of x and the signal-noise ratio. The signal-noise ratio is defined as $\frac{V(x\beta)}{V(u)}$ which is $\frac{\beta^2 V(x)}{\sigma_\eta^2/(1-\rho_u^2)}$. Note that a signal-noise ratio of zero corresponds to an equation containing no strictly exogenous regressors. To reduce the variation to be explored further, the exogenous variable is assumed itself to be first-order autoregressive, $x_t = \rho_x x_{t-1} + \varepsilon_t$, $v(\varepsilon) = \sigma_\varepsilon^2$. This allows a substantial range of behavior for x, but rules out more complicated kinds of behavior which may commonly occur. With x autoregressive, the signal-noise ratio can be written as

$$\beta^2 \frac{\sigma_\varepsilon^2(1 - \rho_u^2)}{\sigma_\eta^2(1 - \rho_x^2)} \ .$$

In varying these four parameters, the coefficients β and γ are set to 1. The sample size for all experiments is T = 50, which means 49 available observations for the regression (48 for the Hatanaka second step). Table 1 shows the values of α, ρ_x, ρ_u and S/N used in the experiment.

A full factoral design was employed.

TABLE 1

$\alpha =$ 0, .3, .6, .9

$\rho_x =$ 0, .5, .9

$S/N =$ .25, 2

$\rho_u =$ 0, .2, .4, .7, .9

Note that only positive values of the parameters have been used. Five values for ρ_u were used as testing for $\rho_u > 0$ precedes estimation and thus the behavior of the procedure should be sensitive to ρ_u. The case $\alpha = 0$ corresponds to a misspecified equation, and $\rho_x = 0$ corresponds to random x.

It was desired to minimize the error in comparisons among estimators. Because of the nature of the problem, standard variance reduction techniques (see, e.g. Hendry and Harrison [6]) are difficult to employ. However, the underlying random errors were kept the same for replications at different parameter points and x was fixed in repeated realizations of the y process. Therefore the different tests and estimators will tend to be positively correlated and their differences will have smaller variances than if the experiments were uncorrelated. Two hundred replications were used at each parameter point. All random numbers were generated using the McGill "Super-Duper" random number generator for normal random numbers. The generator was subjected to several tests for randomness and passed them all except that a very small amount of fourth-order serial correlation was found.

To start the x autoregressive process the first twenty observations from realizations begun at the process mean were discarded. The next twenty observations were used to generate values of y which were also discarded. The first y_t actually kept as part of the data y, was kept fixed in all realizations at a particular parameter point.

Since the finite-sample moments of the estimators of equation (2) do not exist even though the limiting distributions are well behaved the results of the Monte Carlo study are summarized using nonparametric statistics.[3] A number of statistics were computed in this study, but only results based on the median of the absolute errors (MAE) of the estimators were reported. This statistic can be considered to be a nonparametric analog of the mean squared error of an estimator, being an increasing function of bias and dispersion of an estimator.

Section 4

In this section the results of the experiments are repeated. As in most Monte Carlo studies it is impossible to summarize concisely all of the results. The plan of the discussion is as follows. We discuss first the comparative behavior of each test for autocorrelation at various significance levels and each estimator used unconditionally without testing; then we consider the tests, significance levels, and estimators as components of procedures, i.e. we examine the behavior of the components of the procedure vector conditional on the other components of the vector. In attempting to rank procedures, we consider whether it is possible to

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[3] The sample moments may be of interest even though the population quantities do not exist. Calculations using them in this study did not lead to any different conclusions.

choose a good test, significance level, or estimator independently of the other components. Finally, we consider some particular procedures and compare their performance at each parameter point with the best possible procedure. In order to condense the discussion of estimators, results are only reported for the parameters α and β ; the constant term (which was, of course, estimated) is ignored.

We look first at the serial correlation tests. Table 2 shows the percentage of samples in which the Durbin-Watson test, the Durbin tests one and two, and the likelihood ratio test caused the null hypothesis of no serial correlation to be rejected at significance levels of .01, .05, .20 and .50 at the higher signal-noise ratio. Each group of five columns corresponds to a value of ρ_x . within each group ρ_u varies from 0 to .9. Each page shows a different value of α .

We consider first the behavior of the tests when α is zero. The true significance level of the Durbin-Watson test is usually below its nominal value for all levels except 50%, but the significance level rises as ρ_x increases. At $\rho_x = .9$ the test is quite close to its true level at all four nominal levels, while for random x , it is accurate only at the 50% level. In fact for $\rho_x = .9$, the significance level of the Durbin-Watson test is close to correct at all levels. As ρ_u increases, the power increases with the increase being faster for more correlated x series. A 5% test, however, never gives power above .5 except for $\rho_x = .9$ and $\rho_u = .7$ or .9 . Thus, much higher nominal significance levels than are commonly used are necessary for adequate power of this test.

The pair of Durbin tests have substantially higher power than does

TABLE 2

Percentage of Rejections of $H_0 : \rho_u = 0$

Test $\alpha = 0$	Significance Level	$\rho_x = 0$					$\rho_x = .5$					$\rho_x = .9$				
		$\rho_u = 0$	.2	.4	.7	.9	0	.2	.4	.7	.9	0	.2	.4	.7	.9
Durbin-Watson	.01	0	0	0	0	.005	0	0	0	.015	.010	0	.025	.135	.190	.070
	.05	0	0	.03	.075	.080	.005	.025	.075	.125	.095	.020	.18	.440	.555	.215
	.20	.065	.175	.355	.390	.355	.125	.240	.445	.455	.385	.200	.525	.790	.910	.540
	.50	.545	.770	.865	.835	.730	.555	.810	.900	.895	.730	.550	.825	.985	.985	.860
Durbin Test 1	.01	.05	.095	.185	.100	.025	.055	.135	.260	.145	.030	.015	.120	.365	.510	.150
	.05	.095	.245	.415	.300	.135	.125	.285	.430	.325	.140	.065	.280	.640	.750	.275
	.20	.285	.520	.635	.535	.385	.305	.500	.700	.610	.375	.225	.585	.835	.900	.525
	.50	.560	.760	.850	.800	.650	.515	.765	.870	.835	.665	.500	.800	.965	.970	.805
Durbin Test 2	.01	.010	.025	.055	.030	.015	0	.050	.095	.075	.010	.010	.045	.230	.375	.085
	.05	.040	.100	.185	.155	.080	.05	.125	.260	.200	.110	.025	.225	.540	.630	.185
	.20	.180	.325	.490	.425	.315	.175	.375	.545	.480	.290	.152	.465	.775	.875	.440
	.50	.450	.690	.805	.740	.605	.445	.685	.785	.765	.590	.415	.735	.950	.965	.735
Likelihood Ratio	.01	.005	.015	.050	.100	.045	.005	.010	.060	.140	.085	.010	.02	.25	.635	.450
	.05	.050	.055	.160	.300	.175	.025	.085	.195	.410	.195	.040	.135	.415	.810	.675
	.20	.205	.240	.420	.650	.425	.210	.250	.500	.690	.500	.215	.340	.685	.910	.800
	.50	.535	.560	.725	.805	.630	.525	.600	.730	.855	.650	.570	.650	.840	.970	.895

TABLE 2 (continued)

Test $\alpha = .3$	Significance Level	$\rho_x = 0$					$\rho_x = .5$					$\rho_x = .9$				
	$\rho_u =$	0	.2	.4	.7	.9	0	.2	.4	.7	.9	0	.2	.4	.7	.9
Durbin-Watson	.01	0	0	.015	.205	.410	0	.015	.055	.260	.415	0	.065	.350	.645	.495
	.05	0	.05	.165	.435	.625	.015	.100	.290	.535	.645	.025	.265	.625	.900	.765
	.20	.095	.330	.560	.820	.885	.160	.425	.700	.870	.885	.215	.605	.870	.990	.945
	.50	.610	.820	.925	.970	.975	.605	.850	.945	.980	.975	.545	.865	.985	.990	.990
Durbin Test 1	.01	.050	.125	.260	.390	.460	.045	.140	.310	.405	.455	.010	.130	.440	.750	.555
	.05	.130	.300	.460	.595	.665	.105	.310	.530	.645	.655	.055	.290	.665	.895	.775
	.20	.315	.580	.755	.840	.890	.280	.545	.770	.880	.870	.210	.565	.870	.980	.940
	.50	.595	.805	.915	.965	.950	.535	.825	.910	.965	.955	.470	.820	.975	.990	.985
Durbin Test 2	.01	.015	.045	.120	.295	.450	0	.075	.170	.310	.375	0	.065	.345	.665	.470
	.05	.045	.135	.320	.520	.600	.055	.195	.375	.530	.605	.020	.235	.605	.845	.705
	.20	.170	.415	.620	.805	.830	.195	.425	.680	.820	.840	.145	.515	.835	.960	.905
	.50	.480	.735	.850	.945	.950	.455	.720	.875	.960	.950	.415	.765	.970	.990	.975
Likelihood Ratio	.01	.005	.015	.060	.260	.355	0	.020	.095	.345	.390	.010	.020	.280	.760	.710
	.05	.040	.085	.210	.490	.540	.055	.095	.290	.585	.560	.050	.175	.530	.875	.810
	.20	.180	.275	.525	.730	.735	.215	.330	.555	.780	.835	.215	.365	.720	.955	.900
	.50	.480	.610	.780	.865	.800	.535	.615	.785	.890	.785	.550	.635	.880	.980	.930

TABLE 2 (continued)

Test $\alpha = .6$	Significance Level	$\rho_x = 0$					$\rho_x = .5$					$\rho_x = .9$				
		$\rho_u = 0$	.2	.4	.7	.9	0	.2	.4	.7	.9	0	.2	.4	.7	.9
Durbin-Watson	.01	0	.030	.225	.710	.905	0	.055	.320	.775	.905	0	.120	.505	.925	.950
	.05	.020	.165	.490	.890	.990	.04	.251	.555	.905	.990	.05	.345	.745	.975	.975
	.20	.180	.505	.825	.980	.995	.24	.570	.865	.985	.995	.195	.655	.915	.990	.990
	.50	.645	.895	.970	.995	1.00	.635	.895	.975	1.00	1.00	.525	.885	.985	1.00	1.00
Durbin Test 1	.01	.040	.130	.405	.760	.915	.015	.135	.430	.790	.895	0	.100	.480	.895	.930
	.05	.100	.335	.640	.920	.985	.080	.330	.655	.915	.965	.035	.320	.720	.965	.975
	.20	.320	.640	.855	.960	.995	.255	.610	.870	.975	.995	.155	.560	.890	.990	.990
	.50	.610	.850	.965	.995	1.00	.550	.860	.965	1.00	1.00	.425	.835	.965	1.00	1.00
Durbin Test 2	.01	.005	.090	.315	.720	.905	0	.095	.345	.740	.875	0	.075	.410	.885	.915
	.05	.050	.255	.550	.880	.970	.055	.265	.565	.880	.940	.015	.240	.645	.955	.965
	.20	.195	.535	.800	.950	.995	.195	.520	.840	.960	.990	.130	.490	.860	.985	.990
	.50	.520	.795	.960	.995	1.00	.451	.800	.950	1.00	1.00	.375	.805	.965	1.00	1.00
Likelihood Ratio	.01	.005	.035	.22	.670	.875	.010	.060	.260	.685	.860	.020	.040	.355	.870	.910
	.05	.05	.135	.445	.840	.915	.040	.155	.480	.865	.920	.060	.170	.575	.950	.960
	.20	.19	.390	.700	.930	.950	.225	.390	.705	.930	.945	.245	.405	.780	.975	.975
	.50	.495	.680	.870	.965	.965	.495	.645	.865	.970	.950	.555	.650	.900	.990	.975

TABLE 2 (continued)

Test α = .9	Significance Level	$\rho_x = 0$					$\rho_x = .5$					$\rho_x = .9$				
		$\rho_u = 0$	.2	.4	.7	.9	0	.2	.4	.7	.9	0	.2	.4	.7	.9
Durbin-Watson	.01	.010	.155	.585	.990	1.00	.005	.160	.590	.980	1.00	0	.135	.545	.965	.995
	.05	.070	.410	.840	.995	1.00	.080	.405	.825	.995	1.00	.050	.350	.780	.990	1.00
	.20	.255	.735	.935	1.00	1.00	.235	.725	.925	1.00	1.00	.180	.645	.915	1.00	1.00
	.50	.645	.930	.995	1.00	1.00	.580	.900	.990	1.00	1.00	.480	.865	.980	1.00	1.00
Durbin Test 1	.01	.015	.205	.625	.985	1.00	0	.145	.590	.960	.995	0	.080	.470	.945	.995
	.05	.115	.465	.845	.990	1.00	.065	.380	.800	.990	1.00	.025	.255	.705	.975	.995
	.20	.285	.715	.935	1.00	1.00	.205	.655	.915	1.00	1.00	.135	.560	.875	.995	1.00
	.50	.600	.915	.995	1.00	1.00	.490	.875	.985	1.00	1.00	.355	.800	.980	1.00	1.00
Durbin Test 2	.01	.010	.145	.560	.975	1.00	0	.125	.550	.955	.995	0	.070	.430	.945	.995
	.05	.060	.365	.755	.995	1.00	.03	.295	.735	.980	1.00	.005	.225	.660	.965	.995
	.20	.200	.645	.925	1.00	1.00	.155	.585	.895	1.00	1.00	.100	.490	.845	.995	1.00
	.50	.525	.890	.990	1.00	1.00	.440	.850	.980	1.00	1.00	.325	.765	.965	1.00	1.00
Likelihood Ratio	.01	.010	.140	.505	.960	1.00	.015	.105	.475	.940	1.00	.025	.050	.375	.915	.995
	.05	.065	.275	.695	.990	1.00	.050	.210	.660	.990	1.00	.060	.135	.545	.965	.995
	.20	.215	.545	.880	1.00	1.00	.235	.470	.855	.995	1.00	.250	.365	.775	.990	.995
	.50	.495	.760	.945	1.00	1.00	.470	.715	.915	1.00	1.00	.515	.650	.890	.995	.995

the Durbin-Watson test except at the 50% significance level. However, the true significance level of the Durbin tests is much closer to the nominal levels than is the Durbin-Watson test except at the 50% level. The true significance level of both tests is lowest for large ρ_x. This has the expected consequence that test 1 is generally more powerful than the other test when compared at the same nominal level, especially for small values of ρ_u.

The likelihood ratio test is quite close to its nominal significance level. Its power is generally below the power of the Durbin tests and, for some values, the Durbin-Watson test when ρ_u is small, but it does well at large values of ρ_u. The largest differences among all these tests stems from discrepancies between nominal significance and true significance. If, however, for $\rho_x = 0$ the Durbin-Watson test at the 20% nominal level with actual significance near 5% is compared with the Durbin test 1 at the 1% nominal and 5% true levels and the other two tests at the 5% nominal and true levels, the Durbin-Watson test is the most powerful.

The relationship between nominal and true significance, varies with the (unobservable) value of α as well as the (observable) value of ρ_x. The significance level of the Durbin-Watson test rises as α rises. At the 20% nominal level, it rises from .065 to .255 as α rises from 0 to .9, when $\rho_x = 0$, but there is hardly any change for $\rho_x = .9$, where the nominal level is essentially correct. In contrast, the two Durbin tests show little variation in true significance level as α varies, test 1 remaining above its nominal level and test 2 below when ρ_x is small. For large values of α, however, and large values of ρ_x, Durbin's test 1 falls below its nominal level. The likelihood ratio test remains close to its nominal significance for all values of α, although it tends to be slightly too high for large ρ_x and slightly too small for low ρ_x. Durbin's tests tend to decline in significance level as ρ_x increases.

When α is large, all four tests have good power. With $\alpha = .6$, all four tests show the power approaching one as ρ_u rises, and the power is over 50% for all tests and nominal significance levels above .01 when ρ_u is at least .4. At $\rho_u = .4$ either the Durbin-Watson or Durbin test 1 has highest power for significance .20 while the likelihood ratio test is always worst. The ranking of the Durbin-Watson and Durbin test 1 varies somewhat with significance level, but the likelihood ratio test is always lowest in power. This test is, in fact, the lowest in power for virtually all cases when α is greater than or equal to .3. For large values of α even the worst test has reasonably good power, however. In summary, the Durbin-Watson test seems to perform better than expected especially at higher significance levels and with high values of ρ_x. The Durbin tests are stronger for low ρ_x and the likelihood ratio test seems to perform below expectations.

We consider next the behavior of the various estimators when they are used without an autocorrelation test. Available asymptotic theory (see, e.g. [9], [10]) suggests that OLS will behave worst, i.e., have largest inconsistency for small signal-noise ratios, large values of ρ_u, and large values of ρ_x. Also, the larger is α, the larger the inconsistency. For positive serial correlation OLS is expected to overestimate α and underestimate β when $\rho_x \geq 0$.

Since the statistics produced in this study are voluminous, and since we are primarily interested in comparisons of these procedures we report only rankings of the estimators by median absolute error at each parameter point. Table 3 shows the rank ordering of the six estimators for the estimates of β and α at each parameter point. Six one-letter abbreviations, standing for the six estimators, are shown in order of increasing MAE at each point. The abbreviations for the estimators are as follows:

Abbreviations	Estimator
O	Ordinary Least Squares
I	Instrumental Variables
D	Durbin Estimator
W	Wallis Instrumental Variables and GLS
H	Hatanaka Residual Adjusted GLS
M	Maximum Likelihood

In examining these rankings it should be recalled that as ρ_u increases, the gap between the MAE of inconsistent OLS and the MAE of the consistent estimators increases substantially. The medians of all of the estimators of α except for IV are below the true α when ρ_u is zero and all increase with the value of ρ_u. OLS shows the most dramatic increase in median, overstating α by as much as .81 when α is zero! Some idea of the cost of wrongly rejecting H_0 is found in the observation that the MLE, which generally has the smallest interquartile range (IQR) when ρ_u is greater than zero has an IQR ranging from 97% to 123% of the OLS IQR when ρ_u is zero. A typical value is 118%. The MLE bias when ρ_u is zero is also slightly larger than the bias of OLS. Thus the strategy of not testing would generally increase the dispersion of the estimates only moderately if the MLE were used, even when a test would seem most

TABLE 3

Ranking of Estimators by Median ABS Error

ρ_x	α	S/N	ρ_u = 0 β	0 α	.2 β	.2 α	.4 β	.4 α	.7 β	.7 α	.9 β	.9 α
0	0	2	OMDWHI	IOMHDW	MDWIHO	HIMDOW	MWIDHO	MDWOHI	MWHDIO	MWHDOI	MHWDIO	MHWDOI
		.25	OMHWDI	HDOWMI	MHOWDI	HMIODW	MWHDIO	WMHDOI	MWHDIO	MWHDOI	MDHWIO	MWHDOI
	.3	2	ODWMHI	ODIMHW	MDWIHO	IOHMDW	MWIDHO	HOMDWI	MWDHIO	MWDHOI	MWHDIO	MWHDOI
		.25	OMWDHI	DHOMWI	OMHWDI	HWDMOI	MHWDIO	MHDWOI	MWHDIO	MWDHOI	MDWHOI	MWHDOI
	.6	2	ODMWHI	DMWOHI	OWDMHI	OHDIMW	MWHDIO	MHWODI	MWHDIO	MDWOHI	MWDHIO	MWHDOI
		.25	ODMWHI	DWOHMI	OMWDHI	HMOWDI	MWHDOI	MHWODI	MHWDOI	MWHDOI	MWHDOI	MWHDOI
	.9	2	DOMWHI	MHDOWI	ODMHWI	MHWODI	ODMWHI	HMWDOI	MODWHI	MDWHOI	WMDHOI	WMDHIO
		.25	ODMHWI	HMOWDI	ODMWHI	MDOWHI	ODMWHI	MWDHOI	MODWHI	WMHDIO	WMODHI	HMWDIO
.5	0	2	OMWDIH	MHIDOW	MDWOHI	OMHWID	MDHWIO	HMWDIO	MWDHIO	MHWDIO	MHWDIO	MWHDOI
		.25	OMHWDI	OHMWDI	MHWODI	OMHIWD	MWHDIO	HWODMI	MWHDIO	MWDHOI	MDHWIO	MHWODI
	.3	2	OMIDWH	IOMHDW	OMDWIH	OWDHMI	MWHDIO	MHWDOI	MWHDIO	MWHDIO	MWHDIO	MWDHOI
		.25	OMWDIH	MOWHID	OMHWDI	OMWDHI	MHWDIO	HMWODI	MWHDIO	MWHDOI	MDWHIO	MHDWOI
	.6	2	OMWDHI	OMHWID	OMIDHW	OIMWDH	MDIWHO	WMDHOI	MHWDIO	MWDHOI	MWHDIO	MDHWOI
		.25	OWMDHI	OMHDWI	OWDMHI	OMWHDI	MWHDOI	WMDOHI	MHWDIO	DWMHOI	MWHDOI	MWHDOI
	.9	2	MDHOWI	OIMHWD	OMDHWI	IOMHWD	OMWDHI	MWDHOI	MWDOHI	WMHDOI	MWDHOI	WMDHOI
		.25	ODMHWI	OHMDWI	ODMWHI	MOWHDI	ODMWHI	MHWDOI	OMDWHI	WHMDOI	WMDOHI	WMHDIO
.9	0	2	OMIWHD	MOIHDW	OMIHDW	MDWOHI	MHWDOI	MHWDIO	MDHWIO	MDHWIO	MHDWIO	MDHWIO
		.25	OMHDWI	OMHWDI	OMHWDI	WMHDOI	MWHDIO	MWDHIO	MWDHIO	WHMDIO	MDHWIO	MDWHIO
	.3	2	OMWHID	HMOIWD	OMIWDH	IOMWHD	IWHDMO	MWDHIO	MHDWIO	MHWDIO	MHWDIO	DMHWIO
		.25	MOHDWI	OMHWDI	OMDWHI	MDWOHI	MDHWIO	MWDHIO	MHDWIO	WMDHIO	MWDHIO	WDHMIO
	.6	2	WHDMOI	MOWHDI	MHWODI	MWOHDI	MIOWDH	MWDHIO	DMHWIO	MHWDIO	MHWDIO	HWDMIO
		.25	WDMHOI	OMWDHI	OWMHDI	MOWHDI	MOWDIH	MWDOHI	HMDWIO	MWHDIO	MWDHIO	WDHMOI
	.9	2	HDOMWI	OHMWDI	OMWDHI	MWHDOI	WMHDOI	WMHDIO	HMDWOI	MDHWIO	WMDOHI	MHDWIO
		.25	MOWHDI	OWMHDI	OHWMDI	OMWHDI	OWDMHI	WHMDIO	OWDMHI	MWHDIO	MWDHOI	MHWDOI

clearly useful, at least for the parameter values examined in this study.

The most striking phenomenon shown in Table 3 is the uniform mediocrity of IV. The estimator is ranked last or nearly last not only for large values of ρ_u but also for most small and zero values of ρ_u, where one might expect it to be about as good as an efficient estimator. This is true for both α and β. When ρ_u is zero OLS is usually best, but ML is most often the second choice, especially for the smaller values of α. The Durbin estimator does better at low values of ρ_x than at higher ones, as one might expect. RAGLS, although asymptotically as efficient as ML is usually inferior to it, particularly for β. IVGLS is rarely best but is usually superior to RAGLS and IV alone.

As the value of ρ_u increases, OLS becomes a relatively less attractive estimator although it remains the best estimator overall for β when ρ_u is .2. An interesting phenomenon which occurs throughout this table and in the other results in this paper is that the rankings of the estimators and procedures differ substantially for β and α. As will be seen below, it is even true that the best test for autocorrelation, considered as part of a procedure, depends on which coefficient is being considered. Thus whether or not one should take account of serial correlation in estimation can depend on whether accuracy in α or β is more important!

As ρ_u increases, ML becomes more clearly the best estimator. When ρ_u is .9, ML is best in thirty-six cases, IVGLS is best nine times, and RAGLS is best twice. RAGLS and IVGLS are frequently second best. The cases in which IVGLS is best are mostly for large α. Clearly, when serial correlation is very high, it is important to use an efficient estimator.

At the intermediate values of ρ_u , ML is most frequently the best estimator; the OLS, IV, and Durbin estimators do least well (except for OLS at $\rho = .2$); RAGLS and IVGLS are inbetween. While ML is usually best for both coefficients, when $\rho_u = .2$ OLS does better for β but not for α . IVGLS seems to be relatively better for α than for β .

While ML seems clearly superior when ρ_u is positive, if it is excluded from the list of possibilities, IVGLS is most often best when $\rho_u > .2$ for both coefficients; RAGLS seems to be the next best estimator. Correspondingly, when ρ_u is zero, among the estimators which are consistent in the presence of autocorrelation (but ignoring IV), ML is the best, while IVGLS is least often best. It seems, therefore, that ML is less sensitive to an autocorrelation misspecification than is IVGLS, and one would expect the quality of the autocorrelation test to be more important for IVGLS than for ML.

Having discussed testing and estimation in isolation from each other, we now examine the behavior of procedures, i.e. triples of the form (t, sl, e) . For the four tests and five alternative estimators considered, I have tried seven significance levels in these procedures. The significance levels, which extend (algebraically) much higher than those usually considered, are .01, .05, .10, .20, .35, .50, and .75. Hence, there are one hundred and forty possible procedures. Table 4 shows the best procedures at each parameter point. The entries are of the form tse for the three components of the procedure. The estimator abbreviations are the same as the ones used in Table 3. The test abbreviations are as follows:

Abbreviation	Test
W	Durbin-Watson
D	Durbin Test 1
E	Durbin Test 2
L	Likelihood Ratio Test

For example, the entry L35H stands for a likelihood ratio test at the 35 percent significance level followed by the RAGLS estimator if H_0 is rejected.

In a number of cases there are several ties among procedures. Usually this is either because the best test-significance level combination almost always accepted H_0, and hence the alternative estimator was rarely chosen or because all the tests usually rejected H_0 and were, therefore, equivalent. When all possibilities for an element of a procedure were tied, an asterisk is used. The shorthand DW05M stands for D05M and W05M. Similarly DW05WM stands for D05W, D05M, W05W, and W05M. "Factorizations" are also used for the significance levels.

The first approach to the analysis of the behavior of these procedures is to see which tests, significance levels and estimators occur frequently as components of the best procedures at various parameter points: a sort of marginal analysis of the joint behavior of the procedures studied. The best combinations of two elements of procedures are considered, and the best single procedure or cluster of procedures is found.

Tables 5a, 5b, and 5c show further data on the performance of the tests, significance levels and estimators, respectively, as components of the best procedures. Table 5a shows the ranking by MAE of the tests, 5b shows rankings for the significance levels, and Table 5c shows rank-

TABLE 4

Minimum MAE Test-Significance Level-Estimator Procedures

Signal-Noise Ratio = 2

ρ_u		$\alpha = 0$					$\alpha = .3$				
		0	.2	.4	.7	.9	0	.2	.4	.7	.9
$\rho_x = 0$	$\hat{\beta}$	L35H L20H	L35H W20H	D10M W20M	E20M	L10M	L05HWM	W75I	L20M	L20-10M	L50M L20M
	$\hat{\alpha}$	L05I	L20I	DEW75M D50M	DEW50–75M	W75M	L01I	L50M	DEW75M	DEW75-35M	DEW75,50M
$\rho_x = .5$	$\hat{\beta}$	L20M	L35I EW10I D20I	L20H	L35M D20M	L75-05M	L50H L35H	DEW35I	D01M	DEW75-20M L75-35M	DE75M W75-50M
	$\hat{\alpha}$	L05IDW	L75M	DW5075M E75M	DEW35,50 75M L5075M	L35-75M DEW75M	L01I	DEW7550M	E50M	DEW75-35M L75-50M	DW75-20M E75-35M
$\rho_x = .9$	$\hat{\beta}$	L01DHW	DEW75M	L50M W20M	E05M	L10-01M	L05HW L01HWD	E50I DW75I	W20M	E05M	L75-05D E75-35D DW75-20D
	$\hat{\alpha}$	L01I	EL01IM	W35-75M D50-75M E75M	DLW75-10M E75-20M	L75-35M DE75M W7550M	L01I	DE01M	L75-35I DEW75-20I	L75-10M DE75-35M W75-20M	L75-10M E75-35M DW75-20M

TABLE 4 (continued)

ρ_u		$\alpha = .6$					$\alpha = .9$				
		0	.2	.4	.7	.9	0	.2	.4	.7	.9
$\rho_x = 0$	$\hat{\beta}$	L75-50D	E05M	L20M E10M	L75,10M E75-35,05M D75-35M W75-35,05M	DEW75-35M	L10I	L50M E20M	L20M DEW10M	DL01M	L75-05W DEW*W
	$\hat{\alpha}$	D75D	D05M	DE7550M W75-35M	DEW75-35M	DE75-10M W75-05M	L75-35D	L01I	L01M	DELW75-05M	L75-05W DEW*W
$\rho_x = .5$	$\hat{\beta}$	W75I	L10I E10-05I D05I	L35,10D D10-05D W05D	L75-20M E75-10M D*M W*M	DE10-05H W05H	W75I	DEW75I	L75M E75,50M DW75-35M	L20,10W DE10,05W W05W	DEL01W
	$\hat{\alpha}$	L10-05I	L10I	L75,50M DE75-35M W75-20M	L75,50M E75-35M DW75-20M	L75-35M E75-20M DW75-10M	L05D	L10D L10M	L10,01M E05M	DEL75-05M W*M	DEL75-05M W*M
$\rho_x = .9$	$\hat{\beta}$	L01HWD	DE75M	L75M DEW75-35M	DEL75-10M W05M	EL75-20H DW75-10H	DE10W	D75M	L75W E75,50W DW75-35W	L35M E20M W10-05M	L75-10M DEW75-05M
	$\hat{\alpha}$	L20DW	D20M	L75M E75,50M DW75-35M	DEL05-01D W01D	EL75-20M DW75-10M	W75,50H	L01M	E75,50W DW75-35W	L75-35H DE75-20H W75-05H	L*W E75-10W DW75-05W

TABLE 4 (continued)

Signal-Noise Ratio = .25

ρ_x \ ρ_u		$\alpha = 0$					$\alpha = .3$				
		0	.2	.4	.7	.9	0	.2	.4	.7	.9
$\rho_x = 0$	$\hat{\beta}$	L75H	WL75H	DE75M	L50M	W75M	W75D	DLW75H	L35M	L50-35M	DEW75M
	$\hat{\alpha}$	W10-01* E01M	D35M	W50M	W75M	W75M	W05,01*	W20M	E75M W75-50M	DEW75-50M	DE75M W75-50M
$\rho_x = .5$	$\hat{\beta}$	L20DW	E01HWM	D10H	L75M	W75M	L10M	L05HWM E01HWM D05H	L35W	L75-35W E35W	E75M W50M
	$\hat{\alpha}$	EW01*	E50M	DW75M	W75M	W75M	LEW01*	W20H	E50M W75M	E75-50M DW75-35M	DEW75M
$\rho_x = .9$	$\hat{\beta}$	L05,01HWDM	L50W	E75M DW75,50M	E75W DW75,50W	W75M	L05HDW	L75M	D35M	L75W DE75-35W W75-20W	DEW75-50W
	$\hat{\alpha}$	EW01* L01I L05WM	L01HDM	DEW75,50M	DW75M	W75M	L20M	E20M	D35M	W75M	DEW75-35M

TABLE 4 (continued)

ρ_u		α = .6					α = .9				
		0	.2	.4	.7	.9	0	.2	.4	.7	.9
ρ_x = 0	$\hat{\beta}$	DW75D	L75,35M	L10H E05H W20H	E75,35M DW75M	DEW75-35M	E75-50M D75-35M W50-35M	D10M	L20M E10M	LE75-20W DW75-10W	**H
	$\hat{\alpha}$	EW01*	W01W	W20M	E75-50M D75-35M W75-20M	L75-35M DEW75-05M	L01HWDM	W01DM	L01D	D01M	**W
ρ_x = .5	$\hat{\beta}$	L10H	D01D	L50-20D D35-20D D20-10D	E20,10D DW10D	E20,10M DW10M	L10M D05D	L05D	L75M DEW75-35M	DEL75-05W W75-10W	**W
	$\hat{\alpha}$	EW01*	W01HIM	L35M	E75,50M DW75-35M	DE75-10M W75-05M	L20M L05,01HWDM DE01*	W01M	L01M	*75-05M	**W
ρ_x = .9	$\hat{\beta}$	L05,01H L05,01W	L75M	DEW50,35M	L75-35M DE75-20M W75-10M	E75-20W DW75-10W	L05I	DE75,50M W50,35M	L75,10W E75,50W DW75-35W	L75M DE75-35M W75-20M	L*M DE05-01M
	$\hat{\alpha}$	L75W	W01DWM	L75,01M E75,50M DW75-35M	L10,05H E05H	L75M E75,50M DW75-35M	DE75D	D05HDM	L01H	L75,10,01W E75-35W D75-35,05W W75-20W	DE75-10M W75-05M

TABLE 5a

Ranking of Autocorrelation Tests by MAE
as Components of Optimal Procedures

α	ρ_u	S/N	β				
			$\rho_u = 0$	.2	.4	.7	.9
0	0	2	L/EDW/L/ED	LW/L/EDW/E	DW/EDW/LE/	E/LEDW/L/E	L/W/L/W/L/
		.25	L/ED/W/LED	LW/L/ED/L/	ED/LW/L/ED	L/W/L/W/L/	W/E/L/D/W/
	.5	2	L/EDW/L/W/	LEDW/LEDW/	L/E/L/EDW/	LD/LEDW/E/	L/W/ED/L/W
		.25	L/EWLD/L/E	E/D/EL/ED/	D/ED/ED/ED	L/W/LED/L/	W/L/W/EW/L
	.9	2	L/EDWL	EDW/L/DW/L	EW/EDW/LD/	E/LEDW/LED	L/EDW/EDW/
		.25	L/EWL/EDWL	L/EDW/EDW/	EDW/LEDW/D	EDW/EDW/ED	W/L/ED/W/L
.3	0	2	L/EDWL/L/D	W/L/EDW/DW	L/W/LDW/LE	L/EW/EDW/D	L/EDW/E/DW
		.25	W/L/W/L/EW	LDW/E/L/D/	L/W/L/EDW/	L/ED/WL/EW	EDW/ED/W/E
	.5	2	L/W/L/DW/E	EDW/ED/ED/	D/LD/LEDW/	LEDW/L/DW/	EDW/ED/W/L
		.25	L/ED/L/EWL	LED/DLE/WL	L/D/LED/LD	LE/L/E/LD/	EW/DW/EW/D
	.9	2	L/W/L/D/WL	EDW/E/DW/D	W/LED/LEDW	E/LEDW/LED	LEDW/L/EDW
		.25	L/EDWL/LDW	L/E/L/ELWD	D/LD/LEDW/	LEDW/LEDW/	EDW/EDW/ED
.6	0	2	L/EW/L/D/L	E/D/LEDW/D	LE/W/D/LE/	LEDW/LEDW/	EDW/EDW/E/
		.25	DW/E/D/EDW	LW/LE/D/L/	LEW/LEDW/D	EDW/EDW/ED	EDW/LEDW/L
	.5	2	W/L/EDWL/E	LED/W/L/DW	LDW/LW/ED/	LEDW/LED/L	EDW/EDW/ED
		.25	L/EDWL	D/E/D/LE/L	LED/E/LEDW	EDW/LEDW/L	EDW/EDW/D/
	.9	2	L/EDWL/EDL	ED/LW/EDW/	LEDW/LW/LE	LEDW/LED/W	LEDW/LEDW/
		.25	L/EDWL/LED	L/EDW/LEDW	EDW/LEDW/L	LEDW/LDW/L	EDW/EDW/ED
.9	0	2	L/EWD/L/E/	LE/LED/W/L	LEDW/L/EDW	LD/LEDW/EW	LEDW/L/EDW
		.25	EDW/E/LEDW	D/LEDW/LED	LE/D/W/LED	LEDW/LEDW/	LEDW/LED
	.5	2	W/D/LEDW/L	EDW/LED/EW	LEDW/LEDW/	LEDW/LEDW/	LED/LEDW/L
		.25	DL/EDWL/LE	L/W/EDW/LE	LEDW/LEDW/	LEDW/LED/L	LEDW/LED
	.9	2	ED/WEDL/EW	D/LEW/EW/L	LEDW/E/DW/	LEW/LEDW/L	LEDW/LEDW/
		.25	L/EDL/EDW/	EDW/EDWL/E	LEDW/LEDW/	LEDW/LEDW/	LEDW/EDW/L

TABLE 5a (continued)

α	ρ_x	S/N	α				
			ρ_u = 0	.2	.4	.7	.9
0	0	2	L/WL/EDWL/	L/ED/L/WL/	EDW/E/W/D/	EDW/L/DW/L	W/LED/L/W/
		.25	WE/LE/ED/L	D/ED/W/EW/	W/EDW/D/E/	W/D/E/W/D/	W/D/E/W/ED
	.5	2	L/EDWL/L/E	L/DW/EDW/D	EDW/EW/D/L	LEDW/LDW/L	LEDW/L/W/L
		.25	EW/LW/LWD/	E/LD/E/W/E	DW/E/W/D/L	W/D/E/W/L/	W/D/E/W/D/
	.9	2	L/EDW	LE/D/LD/EW	EDW/LEDW/E	LEDW/LEDW/	LEDW/LEDW/
		.25	EWL/L/DWL/	L/E/WL/DW/	EDW/DW/L/E	DW/LEW/LD/	W/ED/L/W/L
.3	0	2	L/EDWL/E/D	L/E/W/LE/D	EDW/EDW/ED	EDW/L/DW/E	EDW/DW/EDW
		.25	W/LEW/EW/E	W/E/W/L/ED	EW/D/E/D/L	EDW/DW/E/L	EDW/ED/EDW
	.5	2	L/EDWL/L/D	EDW/DW/D/L	E/DW/LD/ED	LEDW/LEDW/	EDW/E/LW/L
		.25	LEW/LW/L/D	W/LE/D/L/W	EW/EDW/L/W	EDW/E/D/LW	EDW/EDW/E/
	.9	2	L/EDWL/D/L	ED/W/L/ELD	LEDW/LEDW/	LEDW/EDW/L	LEDW/LEDW/
		.25	L/DL/EDWL	E/L/EW/EW/	D/LD/LEDW/	W/LEDW/E/L	EDW/D/LEW/
.6	0	2	D/LEDW/E/W	D/LE/W/L/D	EDW/LED/W/	EDW/L/W/LE	EDW/LED/LE
		.25	EW/LW/W/DL	W/L/EL/ELW	W/DW/LE/LE	EDW/LED/ED	LEDW/LEDW/
	.5	2	L/WLD/W/LE	L/W/L/EDLW	LEDW/DW/ED	LEDW/LEW/L	LEDW/LEDW/
		.25	EW/D/LDW/E	W/L/ED/ELW	L/W/ED/LED	EDW/LEW/LE	EDW/LD/LEW
	.9	2	L/E/LW/DW/	D/EDW/L/ED	LEDW/E/LED	LEDW/LEDW/	LEDW/LEDW/
		.25	L/W/ED/L/W	W/LEW/E/LE	LEDW/E/L/E	LE/LEDW/ED	LEDW/LEDW/
.9	0	2	L/EDWL/L/E	L/E/WE/L/W	L/WE/LEDW/	LEDW/LDW/E	LEDW/L/EDW
		.25	L/ELDW/DWL	W/E/DE/W/L	L/E/W/D/L/	D/LEDW/E/L	LEDW/LED
	.5	2	L/DL/W/L/E	L/EDW/EDWL	LE/D/W/L/D	LEDW/ED/L/	LEDW/LED/L
		.25	LEDW/WL/L/	W/LE/WLE/L	L/E/W/D/LE	LEDW/EDW/L	LEDW/LED
	.9	2	W/EDWL/EDW	L/ED/WLED/	EDW/LEDW/E	LEDW/LEDW/	LEDW/EDW/L
		.25	ED/L/EDW	D/LW/EWDL/	L/E/LDW/L/	LEDW/LEDW/	EDW/LEDW/L

TABLE 5b

Ranking of Significance Levels by MAE as Components of Optimal Procedures

α	ρ_x	S/N	β				
			ρ_u = 0	.2	.4	.7	.9
0	0	2	35 20/05/10 05/	35 20/50/35 20/	10 20/75 50/35 20	20/50 35 20 75/75	10/50/05/50/
		.25	75/50/35 20/75	75/50/75/35/	75/10 20/75/50	50/75 35/50 35/	75/50/75/35/
	.5	2	20/05/35/75 50	35 10 20/05 20 10/	20/10/20/75 50	35 20/75 50 35 20/	75 50 35 20 10 05/75
		.25	20/35/20 10/35	01/10/01 05/20	10/35 20/10/50	75/50 35/75/10	75/01/50/75/
	.9	2	01/10/50 35 20 10	75/50/75/50/	50 20/35 50/50 20	05/10 05/75 50 35	10 05 01/75 50 35 20
		.25	05 01/05/10/01	50/75 35 50 20/75	75 50/75 50/35/	75 50/75 50/75 50	75/50/75 50 20/
.3	0	2	05/20/01 05 10/	75/50/05/35 20	20/10/35 20 10 05	20 10/05/75 50 35	50 20/75 35/20/
		.25	75/50 35/50/35	75/50/75/01 20	35/50/75 50 35/	50 35/75/50 75/	75/50/35/20/
	.5	2	50 35/50/75/20	35/05/50/75/	01/50 20/75 50 35	75 50 35 20/10/05	75 50/35/75/50
		.25	10/20 05/20/10	05 01/05 01/05 01	35/75 20/35/20	75 35/75 35/50 05	75 50/75 50/50 35
	.9	2	05 01/75/01/05	50 75/50 35/75/	20/75 50 35 20/75	05/20 10 05/75 50	75 50 35 20 10 05/05
		.25	05/01 05/10/01	75/50/35/20 35	35/75 35/75 50 35	75 50 35 20/75 50 35	75 50/75 50/35/
.6	0	2	75 50/35/50/05	05/10/05 10/05	20 10/05/20 10/	75 10 50 35 05/50 35	75 50 35/20 10 05/
		.25	75/50 35/50/75	75/35/50 35/50 35	10 05 20/10 05/05	75 35/50 10 35 20/	75 50 05 35/75 50 35
	.5	2	75/10/35 20 10/	10 05/75/01/75	35 10 05/35 05/01	75 50 35 20 10 01 05/	10 05/10 05/10 05
		.25	10/05/35 20/10	01/50/01/05/	50 35 20 10/35/75	20 10/75 50 35 20 05	20 10 05/75 50 35 20
	.9	2	01/05/75 01 50 35	75/20/50/35 50	75 50 35/50 20/35	75 50 35 20 10 05/01	75 50 35 20 10/05 01
		.25	05 01/05 01 05 01	75/35/50 35/50	50 35/75/50 20/	75 50 35 20 10/20 10	75 50 35 20 10/75 50
.9	0	2	10/50/05 35 10 75	50 20/35 05 10/20	20 10/75 50 35 20/	01/75 50 35 20 10 05	75 50 35 20 10 05 01/
		.25	75 50 35/05 10/20	10/20 10/35 10 05	20 10/75 50 35 20/	75 50 35 20 10/05 10	75 50 35 20 10 05 01/
	.5	2	75/01 10 05/05 01	75/01/10 01/75	75 50 35/50 35 20/	20 10 05/75 50 35 05	01/75 50 35 20 10 05
		.25	05 10/05 01 10/20	05/01/20/01 10	75 50 35/20 50 35 10	75 50 35 10 05 20/05	75 50 35 20 10 05 01/
	.9	2	10/75 10 20 05/20	75/50 35/50/35	75 50 35/75 50 35/	35 20 10 05/75 50 20	75 50 35 20 10 05/01
		.25	05/10/05 10/50	75 50 35/75/10/	75 10 50 35/20 10 05	75 50 35 20/50 35 20	75 50 35 20 10 05 01/

TABLE 5b (continued)

			α				
α	ρ_u	S/N	ρ_u = 0	.2	.4	.7	.9
0	0	2	05/75 05/75 50 05	20/35/50/35 20	75 50/50/35/50	75 50/75/50 35/	75/50 35 75/20/
		.25	10 05 01/05 01/05	35/75/50/75/	50/75/50/35/	75/50/75/50/	75/50/75/50/
	.5	2	05/10/05/01 05	75/50/75 50/35	75 50/35/75/50	75 50 35/20/10/	75 50 35/20/10/
		.25	01/10 05 01/10 05	50/75 35 20/75	75/50/75/35/	75/50/75/50/	75/50/75/50/
	.9	2	01/10 01 05 35 20 50	01/05/01/05 10	75 50 35/75 50 35/	75 50 35 20 10/05 10	75 50 35/20 50 35/
		.25	01 05/05/10 05/	01/50 10/01/05	75 50/35/75/35	75/50/35/20/	75/50/35/20/
.3	0	2	01/05 01/75/05	50/35/75/20/	75/50/75 50 35/	75 50 35/75 50/35	75 50/35/20/75
		.25	05 01/10 01/10/01	20/10/50/10 20	75 50/75/50/35	75 50/35/75/50	75 50/35 20/20/
	.5	2	01/05 01/10/35	75 50/50/35/05	50/75 35 50/75 35	75 50 35/20/10/	75 50 35 20/75 50 10
		.25	01/05/01/10/	20/10 05/20/35	50 75/50 35/75/	75 50 35/20/75 20	75/50/35/20/
	.9	2	01/05 01 50 35 20 10	01/05/20/01 10	75 50 35 20/75 50 35	75 50 35 20 10/10 20	75 50 35 20 10/05 20
		.25	20/10 35 75 50 20/	20/35/01/20/	35/75 35/75 50 35	75/50 35 75 20/10	75 50 35/20/75 20
.6	0	2	75/0, 05/75/05	05/20 10/05/10	75 50 35/75 35/20	75 50 35/75/20/	75 50 35 20 10 05/75
		.25	01/05 01/05/01	01/05 01/05 01/	20/50 35/35 20/	75 50 35 20/75 50 35	75 50 35 20 10 05/20
	.5	2	10 05/01 35 05/20	10/01/05/10/	75 50 35 20/10/75	75 50 35 20/35 20 10	75 50 35 20 10/20 10
		.25	01/05 01/10 05 20	01/05/01/05/	35/10/35/75 50	75 50 35/75 35 20/	75 50 35 20 10 05/75
	.9	2	20/75/10 05 50/	20/50 35 20/75 50	75 50 35/20 10 05/	05 01/75 50 35 20 10	75 50 35 20 10/05 01
		.25	75/50/35/75/	01/10 01/01/05	75 01 50 35/20/10	10 05/75 50 35 20 10	75 50 35/50 35 20 10
.9	0	2	50 35 75/01/05 01	01/05/01/10/	01/75 50 35 20 10 05	75 50 35 20 10 05/01	75 50 35 20 10 05 01/
		.25	01/10 05/50/05	01/05/01/05/	01/05/01/05/	01/75 50 35 20 10 05	75 50 35 20 10 05 01/
	.5	2	05/50 75/50 35/	10/05 01/01 05/	10 01 05/10 05/10	75 50 35 20 10 05 01/	75 50 35 20 10 05 01/
		.25	05 01 20/05 01 10/	01/05/10/05/	01/05 10/50 35/	75 50 35 20 10 05/01	75 50 35 20 10 05 01/
	.9	2	75 50/35 20 05 50/	01/05 10 01/20 10	75 50 35/20 10 05/	75 50 35 20 10 05/75	75 50 35 20 10 05 01/
		.25	75/20 10 05 35 50 75	05/10 05/10 05/	01/05 01/75 50 35	75 10 01 50 35 05 20/	75 50 35 20 10 05/75

TABLE 5c

Ranking of Estimators by MAE as Components of Optimal Procedures

α	ρ_x	S/N	β $\rho_u = 0$	.2	.4	.7	.9
0	0	2	H/M/H/I/H/	H/M/H/M/I/	M/D/M/D/W/	M/W/M/WM/W	M/H/M/H/M/
		.25	H/I/D/I/M/	H/M/H/M/HM	M/W/M/HM/W	M/H/W/H/W/	M/H/W/M/H/
	.5	2	M/H/M/I/W/	I/M/IM/I/H	H/W/H/M/H/	M/H/W	M/H/M/W/M/
		.25	WD/H/W/M/H	HWM/H/IDM/	H/W/H/D/W/	M/W/H/M/W/	M/H/WDM/W/
	.9	2	HWD/W/MHIW	M/W/M/W/D/	M/H/D/W/D	M/D/W/M/H/	M/D/H
		.25	HWDM/D/HIW	W/M/W/M/W/	M/W/M/W/M/	W/M/H/W/H/	M/D/M/D/M/
.3	0	2	HWM/W/D/H/	I/H/I/HIM/	M/H/M/H/M/	M/D/W/D/M	M/W/M/W/H/
		.25	D/W/H/I/H/	H/M/H/M/D/	M/H/W/M/H/	M/W/M/W/M/	M/W/M/W/M/
	.5	2	H/W/I/H/I/	I/HI/HIWDM	M/W/H/M/H/	M/W/H/W/D	M/W/M/W/M/
		.25	M/HM/H/W/H	HWM/HWDM/H	W/H/M/H/M/	W/M/D/W/M/	M/H/M/H/D/
	.9	2	HWD/H/I/DM	I/M/I/M/I/	M/W/M/W/HW	M/H	D/M/DMW/M/
		.25	HWD/HWDM/H	M/D/W/D/M/	M/W/M/W/M/	W/M/W/M/W/	W/D/W/H/D/
.6	0	2	D/W/D/W/M/	M/WDM/WDM/	M/H/M/H/M/	M/D/	M/W/
		.25	D/W/D/W/I/	M/H/M/HM/H	H/W/H/M/HW	M/W/M/W/M/	M/W/
	.5	2	I/H/IDM/W/	I/HIWDM/I/	D/M/D/W/MH	M/W/	H/D/M/D/
		.25	H/WDM/HIWD	D/HWM/HWDM	D/W/M/W/D/	D/W/D/W/M/	M/W/M/H/MW
	.9	2	HWD/M/HWD/	M/IW/H/W/I	M/W/M/W/M/	M/H/W	H/W/D
		.25	HW/M/D/I/H	M/I/HWDM/H	M/W/M/D/W/	M/W/M/W/H	W/D/W/D/W/
.9	0	2	I/W/D/IM/W	M/H/M/H/M/	M/H/W/H/M/	M/D	W/M
		.25	M/HWDM/W/H	M/W/M/HWM/	M/W/M/W/M/	W/M	H/M
	.5	2	I/HIWDM/HI	I/MD/I/D/M	M/W/M/W/H/	W/M/	W/M/
		.25	DM/HIWDM/H	D/M/DM/W/M	M/H/M/D/M/	W/H/W/H/W/	W/M
	.9	2	W/HIW/H/W/	M/W/H/DW/D	W/M/W/M/W/	M/D/	M/H
		.25	I/HWDM/W/H	M/H/IWM/W/	W/H/M/H/	M/W/H/	M/H/W

TABLE 5c (continued)

α	ρ_x	S/N	ρ_u 0	.2	.4	.7	.9
			α				
0	0	2	I/MD/HIWM/	I/M/ID/M/I	M/D/M/I/WI	M/W/M/W/M/	M/H/M/H/M/
		.25	HIWDM/HIWD	M/W/H/WDM/	M/W/H/W/H/	M/W/H/W/M/	M/D/H/M/D/
	.5	2	IWD/HM/HIW	M/H/M/H/M/	M/D/M/D/H/	M/W/M/W/D	M/H/MH/M/H
		.25	HIWDM/HIWD	M/H/M/H/M/	M/W/M/W/D/	M/W/M/W/H/	M/D/H/D/M/
	.9	2	I/HIWDM	IM/HIWD/HW	M/H/M/W/M/	M/D/M/H	M/H/M/H/D
		.25	HIWDM/I/D/	HDM/HIW/DM	M/W/H/W/H/	M/W/M/W/D/	M/D/H/D/H/
.3	0	2	I/HIWDM/D/	M/D/M/W/D/	M/W/I/W/D/	M/W/M/W/M/	M/W/M/W/M/
		.25	HIWDM/HIWD	M/HM/W/M/D	M/H/M/H/M/	M/W/M/W/M/	M/D/M/W/D/
	.5	2	I/HIWDM/I/	M/D/HWM/D/	M/W/M/W/M/	M/W/M/W/H	M/W/M/H
		.25	HIWDMWHIWD	H/M/H/M/H/	M/H/W/H/W/	M/W/M/W/M/	M/D/W/D/H/
	.9	2	I/HIWDM/H/	M/IM/HWM/D	I/W/I/W/HD	M/H/	M/H/W
		.25	M/W/HDM/IW	M/IWM/HIWD	M/H/M/D/H/	M/H/D	M/W/D/W/D/
.6	0	2	D/HIWDM/D/	M/D/WM/DM/	M/HM/D/W/H	M/W/M/W/H/	M/W/D
		.25	HIWDM/HIWD	W/IM/HD/H/	M/H/W/M/H/	M/H/	M/W/
	.5	2	I/HIWD/M/I	I/M/W/H/IW	M/D/I/M/D/	M/H/	M/W/
		.25	HIWDM/I/HI	HIM/W/D/HI	M/W/M/W/H/	M/H/W	M/W/H
	.9	2	WD/H/WD/HW	M/IW/M/WHD	M/I/M/I/M/	D/I/H/I/W/	M/H
		.25	W/D/M/H/WD	WDM/HIWM/I	M/I/M/I/M/	H/M/	M/W/
.9	0	2	D/HID/W/HI	I/HWDM/D/H	M/D/M/D/M/	M/D	W/M
		.25	HWDM/HIWDM	DM/D/I/DM/	D/M/D/M/D/	M/D/	W/M
	.5	2	D/H/D/MI/H	DM/HDM/HWD	M/W/M/WD/W	M/W	M/W
		.25	HIWDM/HIWD	M/D/HWDM/H	M/D/W/DM/D	M/D	W/M
	.9	2	H/WD/HWD/H	M/HDM/HIWD	W/H/WM/HWM	H/M/H/D	W/M
		.25	D/WDM/HIWD	HDM/DHWM/H	H/M/W/HD/M	W/M/D/	M/W

ings for the estimators. The abbreviations used are the same as in Table 4. A slash (/) is used to show differences in rankings. For example, L/EDW/ indicates that the LRT was the test component of all of the best procedures at that point and tests D1, D2, and DW were each used among the procedures ranked second. These tables are based on the fifty best procedures at each point.

Over all parameter points with ρ_u equalling zero, the likelihood ratio test and significance levels of ten percent or less are most often the best choices as components of best procedures in the estimation of β. On the other hand, the Durbin-Watson test and the 01 significance level are the best choices in estimating α with the LRT the second choice. The choice of alternative estimator is less clear in this case as it is often not chosen when ρ_u is zero, but RAGLS for β and the Durbin estimator for α are overall, most frequently part of the best procedure. ML is rarely more than second, however. Only IV is substantially worse than the others for β and it is about as good as the other for estimating α at the higher signal-noise ratio.

At intermediate values of ρ_u, the two Durbin tests are most often part of the best procedures for estimating β, but the Durbin-Watson test is the best choice for α, and Durbin test 1 is second best. The 75% significance level is best for both coefficients followed by 50%. The ML estimator is overwhelmingly the best in estimating both coefficients, but the IVGLS estimator is a clear second choice for β though no clear second choice can be made for α. At the highest value of ρ_u, the Durbin-Watson test is best overall for both coefficients and ML is the best estimator, followed by IVGLS. The 75% significance level, as might be expected, is preferred most often at this point.

Over the entire parameter space, the LRT, the 75% significance level, and the ML are most often best for β, and the Durbin-Watson, the 75% significance level and the ML are best for α. Of course, such simple head-counting conclusions are affected by the selection of parameter points.

Examining the procedures by the other dimensions of the parameter space, the signal/noise ratio has little effect on the relative performance of procedures except that at the lower ratio, IVGLS is superior to ML in estimating β (but not α). The ML estimator is the first choice and IVGLS the second for all values of ρ_x. Either the Durbin-Watson or the LRT is best for each value of ρ_x but when $\rho_x = .5$ there is no clear indication of the best significance level. The only significant effect on these conclusions as α varies is that for the combination of low α and high signal-noise ratio the IVGLS estimator does very poorly for both coefficients, relative to the other estimators.

Having considered the components of procedures separately, we shall now consider the relative behavior of procedures as a whole. When ρ_u is zero, the best procedures for β use the LRT at the 01 or 05 level and IVGLS or RAGLS as the alternative estimator. The best procedure for α is W01 with any estimator. This is essentially OLS. At intermediate and large values of ρ_u ML is usually the best estimator component for both coefficients, and 75% the best significance level component. All of the tests are about equal for β but the Durbin-Watson test is ranked first for α and the LRT does least well. At low ρ_u values and ρ_x equal to zero, (L,75,H) and (W,75,D or H) are good for β, but (W,01,*) are best for α. In contrast (L,01 or 05, W or H) and (L,01,I or M) are best for β and α respectively when ρ_x is .9.

At higher values of ρ_u, ML is the best estimator component at all values of ρ_x. For ρ_x of zero (D,E, or W,75,M) and (L,20,M) are good for β and (W,75,M) is best for α. When ρ_x is .5, (L,E, or W,75,M) and (W,50,M) are ranked highest for β and (W,75,M) remains best for α. At $\rho_x = .9$, (W,75,M) is best for both β and α but is tied with (D,E, or W,50,M) for β. The IVGLS estimator also does well for β, but the LRT is not among the best procedure components at any of the ρ_x values. The Durbin-Watson test performs better at the lower signal-noise ratio than at the higher, where the LRT does well in estimating α, especially at low ρ_u values.

Assuming that the best estimator for each procedure is used, we briefly consider which combinations of test and significance level fare best. When $\rho_u \leq .2$ the combinations most frequently best for β are 1. (L,05) ; 2. (L,01) ; 3. (W,75) ; 4. (D,75) ; and 5. (L,75) . The corresponding ranking for α is 1. (W,01) ; 2. (E,01) ; 3. (L,01) ; 4. (L,05) and 5. (W,05) . For the cases when $\rho_u \geq .4$, the rankings are 1. (W,75) ; 2. (E,75) ; 3. (D,75) ; 4. (W,50) ; and 5. (D,50) for β and 1. (W,75) ; 2. (D,75) ; 3. (E,75) ; 4. (W,50) ; and 5. (D,50) . These rankings are sensitive to ρ_x, however, as (L,75) and (W,75) are best for β when ρ_x is zero even for low values of ρ_u.

It is clear from the discussion above that no single procedure will be best at all parameter points, and that the best procedure will generally depend on unknown parameters. The optimal procedure will, therefore, rarely be chosen even if attention is restricted to procedures which are generally good. As the last part of this analysis we compare some particular operational procedures and groups of procedures to the best procedure at each point in order to see if there are some operational rules which are not too much worse than the best (but nonoperational) procedures at each point. The estimators are generally dominated by ML; comparisons of procedures to the optimal procedure for each point confirmed this again. Therefore, only comparisons among procedures including the ML estimator are shown here. Table 6 shows the performance of four of the best groups of procedures at each parameter point: (W or L , low significance level, M) and (W or L , high significance level, M). The table shows for each procedure group the ranking of the best member of that group at each parameter point for each coefficient. The entry 1, for example, indicates that the best member of the procedure group was no worse than the best procedure at that point. An asterisk indicates that the best group member was not among the top best fifty procedures. In computing these rankings, no adjustment has been made for ties; thus, for example, a procedure ranked second could be inferior to several superior procedures which were tied with each other.

Table 6 shows clearly where the choice of significance level is most important. The conventional significance levels do relatively well when ρ_u is zero and when ρ_u is .9, but fare less well at the intermediate ρ_u values. At the high ρ_u values even the tests heavily weighted to accept H_0 have good power; at low ρ_u values high power is undiserable, but in between the power is relatively low when high power is better. It is apparent from Table 6 , however, that the Durbin-Watson test procedures are generally inferior to the LRT at significance levels of 01-10, but at the 20-35 levels the comparison is much closer. It is once again apparent that the choice of test and significance level depends upon the parameters and upon which coefficient is of most interest, but

TABLE 6

Ranking of Procedures W01M, W05M, W10M

ρ_u		S/N Ratio = 2										S/N Ratio = .25									
		0		.2		.4		.7		.9		0		.2		.4		.7		.9	
α	ρ_x	β	α	β	α	β	α	β	α	β	α	β	α	β	α	β	α	β	α	β	α
0	0	*	5	8	*	5	*	5	*	*	*	*	1	19	35	*	*	*	*	*	*
0	.5	*	4	5	18	11	*	*	*	*	*	6	1	6	*	*	*	*	*	8	*
0	.9	*	2	10	4	*	8	2	1	*	6	4	1	*	4	*	*	*	*	*	*
.3	0	5	2	6	21	2	*	2	9	5	7	*	1	*	2	*	29	15	17	8	8
.3	.5	*	2	7	6	*	*	3	3	*	3	*	1	3	8	*	*	*	12	*	8
.3	.9	*	2	14	2	*	*	2	2	*	2	4	4	*	3	*	*	*	5	*	4
.6	0	*	2	3	3	2	4	1	4	2	1	14	1	8	2	6	8	2	3	2	1
.6	.5	5	5	7	7	2	2	1	2	3	1	*	1	7	1	*	2	9	3	1	1
.6	.9	4	*	14	8	6	3	1	*	*	1	5	17	7	1	5	9	1	*	*	2
.9	0	10	4	7	4	1	3	2	1	*	*	3	2	2	1	3	7	*	2	*	*
.9	.5	3	*	*	2	4	3	4	1	*	1	*	2	4	1	6	16	*	1	*	*
.9	.9	*	*	*	3	12	*	1	2	1	*	*	*	9	2	*	7	2	*	1	1

TABLE 6 (continued)

Ranking of Procedures W20M, W35M

ρ_u		S/N Ratio = 2										S/N Ratio = .25									
		0		.2		.4		.7		.9		0		.2		.4		.7		.9	
α	ρ_u	β	α	β	α	β	α	β	α	β	α	β	α	β	α	β	α	β	α	β	α
0	0	*	3	3	15	1	12	2	3	10	13	12	5	16	5	14	9	32	36	21	24
0	.5	*	6	3	7	8	2	2	1	*	12	*	*	*	5	*	8	*	14	*	19
0	.9	*	2	15	*	1	1	3	1	3	2	*	*	9	8	9	2	6	4	36	9
.3	0	*	*	7	10	4	10	6	1	4	2	*	6	*	1	3	8	5	2	4	3
.3	.5	*	*	*	3	3	2	1	1	3	1	*	*	*	3	19	4	11	1	3	3
.3	.9	*	2	11	4	1	*	3	1	5	1	*	4	10	8	3	3	2	2	17	1
.6	0	7	*	*	*	8	1	1	1	1	1	12	*	1	13	6	1	2	1	1	1
.6	.5	*	*	*	*	6	1	1	1	5	1	*	*	*	*	10	4	*	1	2	1
.6	.9	4	*	3	2	1	1	1	*	*	1	*	*	3	*	1	1	1	6	*	1
.9	0	10	*	3	*	3	16	2	1	*	*	1	*	2	*	4	*	*	2	*	*
.9	.5	*	*	14	19	1	10	5	1	*	1	*	*	3	*	1	*	*	1	*	*
.9	.9	*	*	3	6	3	4	2	2	1	3	*	*	1	8	*	7	1	*	2	1

TABLE 6 (continued)

Ranking of Procedures L01M, L05M, L10M

ρ_u		S/N Ratio = 2										S/N Ratio = .25									
		0		.2		.4		.7		.9		0		.2		.4		.7		.9	
α	ρ_u	β	α	β	α	β	α	β	α	β	α	β	α	β	α	β	α	β	α	β	α
0	0	2	2	6	14	6	*	5	9	1	5	*	2	8	34	7	*	18	*	*	*
0	.5	10	2	5	22	*	*	6	5	1	3	4	2	4	*	13	*	5	*	*	*
0	.9	3	*	14	1	*	*	2	1	1	3	1	1	*	1	*	*	*	10	9	19
.3	0	1	2	6	19	10	*	1	8	7	10	*	2	9	7	*	*	15	15	21	19
.3	.5	*	2	7	6	6	*	4	6	9	6	1	1	1	10	21	*	9	18	22	22
.3	.9	5	*	11	4	*	*	2	1	2	1	2	4	12	8	*	*	*	6	*	9
.6	0	7	5	3	4	15	*	1	5	5	2	14	2	8	5	6	11	11	6	5	2
.6	.5	3	3	3	3	9	*	2	3	6	2	2	3	4	4	*	16	*	5	8	2
.6	.9	2	7	*	8	6	5	1	*	*	2	2	7	7	2	*	1	3	5	*	2
.9	0	*	4	7	2	2	1	1	1	3	3	3	1	3	8	7	5	5	2	2	2
.9	.5	3	4	*	1	*	1	4	1	3	1	1	1	8	3	*	1	*	1	2	2
.9	.9	*	*	*	1	*	*	2	*	1	3	3	2	4	2	5	4	*	3	1	2

TABLE 6 (continued)

Ranking of Procedures L20M, L35M

	ρ_u	S/N Ratio = 2										S/N Ratio = .25									
		0		.2		.4		.7		.9		0		.2		.4		.7		.9	
α	ρ_u	β	α	β	α	β	α	β	α	β	α	β	α	β	α	β	α	β	α	β	α
0	0	*	*	*	8	3	*	2	3	6	2	*	*	5	28	4	21	2	14	11	15
0	.5	1	5	5	16	4	12	1	2	1	1	4	5	*	18	*	15	2	10	17	18
0	.9	3	*	11	10	5	5	3	1	2	1	9	4	12	*	16	7	19	6	5	7
.3	0	9	*	*	2	1	*	1	3	1	6	9	*	9	12	1	11	1	8	9	10
.3	.5	10	*	*	6	6	17	1	2	7	4	2	*	*	7	3	9	2	7	12	12
.3	.9	5	*	*	3	10	*	2	1	5	1	*	1	8	2	*	*	2	2	*	5
.6	0	7	*	4	2	1	4	2	4	4	2	*	*	10	18	6	3	5	2	2	1
.6	.5	*	9	*	*	2	8	1	2	6	1	4	4	*	17	8	1	*	3	6	2
.6	.9	4	7	*	5	3	3	1	*	*	1	6	5	2	7	5	3	1	6	*	2
.9	0	14	*	2	16	1	16	2	1	3	3	8	6	2	31	1	27	5	2	2	2
.9	.5	*	4	18	6	2	8	4	1	4	1	4	1	9	22	6	*	*	1	2	2
.9	.9	*	*	*	3	*	*	1	2	1	3	*	2	*	5	*	*	2	*	1	2

if one had to choose one procedure to use at all parameter points examined in this study, good choices would be W20M,W35M or L20M,L35M , that is, to test with the Durbin-Watson or likelihood ratio tests using a significance level much more likely to reject H_0 when it is true and to use the maximum likelihood estimator when H_0 is rejected.

In conclusion, we have considered the process of testing for serial correlation and estimating the coefficients of the model as a three component procedure involving a choice of test, significance level and estimator. The maximum likelihood estimator was found to be generally superior as a component of good procedures, and the Durbin-Watson and likelihood ratio tests used at unconventional significance levels to be superior. This is in contrast to the finding that the LRT is inferior to other tests in terms of power when considered alone. For the Durbin-Watson test the choice of algebraically high significance levels is particularly important.

As with any Monte Carlo study, one must be careful in generalizing the conclusions outside of the parameter values studied. Not all parameters of importance for this problem have been varied, and many plausible values for the parameters which were varied were not examined. Further, the compressed presentation of results as rankings makes it difficult to decide just how big losses really are in adopting a particular strategy. Even random variation may account for some of the observed behavior. But, this study can at least be viewed as a demonstration that for plausible but simple problems the strategies which are usually adopted are not optimal compared to computationally feasible alternatives.

One final puzzle which is left with the reader is why the RAGLS estimator, which is asymptotically equivalent to the maximum likelihood estimator and is asymptotically efficient, fared so poorly as a procedure component, being in fact inferior to the less efficient but similar IVGLS estimator?

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