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### On Disequilibrium Economic Dynamics. Part II. Wicksellian Disequilibrium Dynamics, Say's Law and the End of the Natural Rate Theory of Unemployment

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**COWLES FOUNDATION DISCUSSION PAPER NO. 386**

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**ON DISEQUILIBRIUM ECONOMIC DYNAMICS**

**PART II**

**WICKSELLIAN DISEQUILIBRIUM DYNAMICS, SAY'S LAW**

**AND THE END OF THE NATURAL RATE THEORY OF UNEMPLOYMENT**

**by**

**Katsuhito Iwai**

**January 10, 1975**

ON DISEQUILIBRIUM ECONOMIC DYNAMICS

PART II

WICKSELLIAN DISEQUILIBRIUM DYNAMICS, SAY'S LAW  
AND THE END OF THE NATURAL RATE THEORY OF UNEMPLOYMENT\*

by

Katsuhito Iwai

1. On Short-Run Theory of Expectation-Formation

In Part I of this series of papers, we studied a micro-dynamic model of the firm whose entrepreneur must make sequential decisions on product price, money wage, labor employment and product sale in an economy in which firms are competing with each other both monopolistically and monopsonistically. We are now in a position of presenting our short-run macro dynamic theory which is firmly founded upon the micro-dynamic theory of the firm.

Let us summarize the  $i^{\text{th}}$  firm's optimal product price, optimal money wage, optimal sales and optimal labor employment policies derived in Part I as follows:

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\*This is Part II of a series of papers on disequilibrium economic dynamics. Part I of this series, "Microfoundations of Wicksellian Disequilibrium Dynamics," Cowles Foundation Discussion Paper No. 385, should be read before this paper. Part III, "Microfoundations of Keynesian Disequilibrium Dynamics," and Part IV, "Keynesian Disequilibrium Dynamics and the Theory of Long-Run Phillips Curve," are forthcoming as Cowles Foundation Discussion Papers. Research for this paper was in part supported by grants from the National Science Foundation and the Ford Foundation.

$$\begin{aligned}
(1) \quad p_t^*(i) &= \frac{i\hat{p}_t}{t} + \{ \frac{i\hat{y}_t^d}{t} - y_t^s(i) - i g^* \} / i\eta, \\
(2) \quad w_t^*(i) &= \{ i\eta(i\varphi - \delta\tau) + (i\eta - 1)k_t(i) + i\eta \frac{i\hat{p}_{t+\tau}}{t} + i\sigma^i \epsilon_t^i \frac{i\hat{w}_t}{t} \\
&\quad + ( \frac{i\hat{y}_{t+\tau}^d}{t} - i g^* ) - i\sigma ( \frac{i\hat{N}_t^s}{t} - i h^* ) \} / (i\eta + i\sigma^i \epsilon), \\
(3) \quad y_t(i) &= \min[ y_t^s(i), y_t^d(i) ] , \\
(4) \quad n_t^*(i) &= \min[ n_t^s(i), n_t^d(i) ] , \quad \text{where} \\
(5) \quad n_t^d(i) &\equiv \{ i\eta(i\varphi - \delta\tau) + (i\eta - 1)k_t(i) + i\eta \frac{i\hat{p}_{t+\tau}}{t} - w_t^*(i) \\
&\quad + ( \frac{i\hat{y}_{t+\tau}^d}{t} - i g^* ) \} / i\sigma \\
&= \{ i\eta^i \epsilon (i\varphi - \tau\delta) + i\epsilon (i\eta - 1)k_t(i) + i\eta^i \epsilon ( \frac{i\hat{p}_{t+\tau}}{t} - \frac{i\hat{w}_t}{t} ) \\
&\quad + i\epsilon ( \frac{i\hat{y}_{t+\tau}^d}{t} - i g^* ) + i\eta ( \frac{i\hat{N}_t^s}{t} - i h^* ) \} / (i\eta + i\epsilon^i \sigma) .
\end{aligned}$$

(Note that all the variables are measured in logarithms.) The superscript  $i$  on the left shoulder of a variable or a parameter stands for the  $i^{\text{th}}$  entrepreneur's subjective evaluation of the variable or the parameter in question. For instance,  $\frac{i\hat{p}_t}{t}$  denotes the  $i^{\text{th}}$  entrepreneur's subjective expectation of the aggregate product price  $P_t$  conditional on his available information at the beginning of period  $t$ ,  $i\eta$  denotes the  $i^{\text{th}}$  entrepreneur's subjective demand-elasticity in his subjective product demand function, and  $i g^*$  denotes the normal rate of excess product demand for the  $i^{\text{th}}$  entrepreneur whose constant value is determined solely by  $i\eta$  and his subjective probability distribution  $i\psi(z)$ .

Alternatively, we can characterize the  $i^{\text{th}}$  entrepreneur's optimal product price and money wage policy as follows:

$$(6) \quad {}_t\hat{g}_t(i) \equiv E_t^i \{y_t^d(i) - y_t^s(i)\} = i_g^* ,$$

$$(7) \quad {}_t\hat{j}_t(i) \equiv E_t^i \{\max\{y_t^d(i) - y_t^s(i), 0\}\} = i_j^* ,$$

$$(8) \quad {}_t\hat{h}_t(i) \equiv E_t^i \{n_t^s(i) - n_t^d(i)\} = i_h^* ,$$

$$(9) \quad {}_t\hat{u}_t(i) \equiv E_t^i \{\max\{n_t^s(i) - n_t^d(i), 0\}\} = i_u^* .$$

That is to say, he must choose  $p_t^*(i)$  and  $w_t^*(i)$  so that his subjective expectations of the rate of excess product demand, the rate of unfilled-orders, the rate of excess labor supply and the rate of involuntary unemployment, given by  ${}_t\hat{g}_t(i)$ ,  ${}_t\hat{j}_t(i)$ ,  ${}_t\hat{h}_t(i)$  and  ${}_t\hat{u}_t(i)$ , might coincide with their constant normal rates,  $i_g^*$ ,  $i_j^*$ ,  $i_h^*$  and  $i_u^*$ , respectively. The values of these normal rates are determined solely by given subjective market elasticities,  $i_\eta$  and  $i_\epsilon$ , by a given technological elasticity,  $\gamma$ , and by given subjective probability distributions,  $i_\Psi(z)$  and  $i_\Phi(z)$ .

In the Wicksellian economy in which the entrepreneur can adjust product price,  $p_t(i)$ , money wage,  $w_t(i)$ , and labor employment,  $n_t(i)$  costlessly at the beginning of every period, nothing will prevent him from setting them equal to their optimal values  $p_t^*(i)$ ,  $w_t^*(i)$  and  $n_t^*(i)$  given above. Therefore, in what follows we do not have to distinguish between the actual and optimal product price, money wage and labor employment. But in the more general economy called the Keynesian economy in which it is costly for the entrepreneur to adjust money wage even at the beginning of the period, we must distinguish between the actual and optimal money wage. We shall postpone the exploration of the Keynesian economy later in Part III and Part IV of this series.

However, our product price, money wage and labor demand functions given above remain incomplete as a dynamic description of the  $i^{\text{th}}$  entrepreneur's price, wage and employment adjustment behaviors until we have specified how he revises his subjective expectations,  $i_t^{\hat{P}}$ ,  $i_{t+\tau}^{\hat{P}}$ ,  $i_t^{\hat{W}}$ ,  $i_t^{\hat{Y}^d}$ ,  $i_{t+\tau}^{\hat{Y}^d}$  and  $i_t^{\hat{N}^s}$ , of the relevant aggregate variables over time as he accumulates new information from his actions and observations in the markets. In other words, they must be complemented by some model of the entrepreneur's expectation-formation. In general, we can conceive as many alternative expectation-formation models as we want, from the simplest rule-of-thumb to the most elaborate ones. However, what we have in mind as a workable short-run theory of expectation-formation is the following simple one.

We suppose that the entrepreneur has his own 'econometric model,' however crude or distorted it might be, which is designed to explain the dynamic movement of the relevant aggregate variables such as  $P_t$ ,  $W_t$ ,  $Y_t^d$  and  $N_t^s$ . In the short-run, he regards his econometric model as a correct specification of the actual working of the economy and estimates its structural econometric equations utilizing all the necessary information available to him. His subjective conditional expectations of the relevant aggregate variables such as  $i_{t+\theta}^{\hat{P}}$ ,  $i_{t+\theta}^{\hat{W}}$ ,  $i_{t+\theta}^{\hat{Y}^d}$  and  $i_{t+\theta}^{\hat{N}^s}$ ,  $\theta \geq 0$ , are nothing but these variables' unbiased predictors, which can be computed from his estimated econometric model. It should be also noted that from his stochastic specification of the structural econometric equations he can also characterize the stochastic properties of his prediction errors of the relevant variables. His subjective probability distributions  $i_{\Psi}(z)$ ,  $i_{\Phi}(z)$  and  $i_{\Psi(\tau)}(z)$  of the deviations of the random variables  $\alpha_t(i)$ ,  $\beta_t(i)$  and  $i_{t+\tau}^{\hat{\alpha}}(i)$  from their subjective expectations  $i_t^{\hat{\alpha}}(i)$ ,

$\hat{\beta}_t^i(i)$  and  $\hat{\alpha}_{t+\tau}^i(i)$ , given by (I-17), (I-32) and (I-46), are nothing but their summary representations. As a matter of fact, we expect our entrepreneur to behave as a trained econometrician.

As time goes on, he acquires new data from his observations in his market activities and other information-gathering activities. Then, he reestimates his econometric equations using the same specification as before, and recomputes the unbiased predictors of the relevant variables. Therefore, once the entrepreneur's perception of the economy, or his 'economic theory,' is given to us in the form of a well-specified econometric model, his short-run expectation-formation behavior can be deduced by a simple econometric exercise. In practice, however, we usually do not know his econometric model, so that our short-run expectation-formation theory must rely on our ad hoc specification of his econometric model. In the present paper, we shall not pursue this short-run theory of expectation-formation any further, and the more detailed investigation of this interesting topic will be left for another opportunity.<sup>1</sup>

## 2. Rational Expectations and the Natural Rate Theory of Unemployment

In the long-run, the entrepreneur's economic theory itself is subject to change. If the values of the subjective demand-elasticity  $\hat{\eta}^i$  and subjective supply-elasticity  $\hat{\epsilon}^i$  in his subjective product demand and labor supply schedules are different from their true and uniform values,  $\eta$  and  $\epsilon$ , then in the light of his accumulated experiences in his own

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<sup>1</sup>In fact, the last half of our previous paper, Iwai [8], developed a short-run theory of expectation-formation, employing the simplest possible econometric model of the entrepreneur. In that paper, we were able to derive a short-run Phillips curve relation.

markets and in other markets he may be persistently and systematically disappointed and be led to revise their values sooner or later. By the same token, if his econometric model of the economy is a distorted picture of the actual macroeconomic relations and their stochastic properties, his predictors of the relevant aggregate variables computed from it may persistently and systematically disappoint him as he accumulates actual macro data and analyzes their statistical properties. Sooner or later he may come to realize its misspecification, and then endeavor to construct the more consistent econometric equations with the more consistent stochastic specifications. In this manner, our entrepreneur will learn from his experiences and revise his economic theory as time goes on.

If the entrepreneur's economic theory is consistent with the actual workings of the economy and his expectations based upon it are on the average not liable to disappointment, we say that he has rational expectations. In our model this requires the following three conditions be satisfied. Firstly, his subjective product demand and labor supply schedules correctly specify their true schedules, that is, the values of the subjective demand-elasticity and supply-elasticity are equal to the true and uniform elasticity values:

$$(10) \quad {}^i\eta = \eta, \quad {}^i\epsilon = \epsilon.$$

Secondly, his subjective conditional expectations of the relevant aggregate variables coincide with their true (or objective) conditional expectations; that is, we have



$$(11) \quad \begin{aligned} {}_t^i \hat{P}_{t+\theta} &= \mathcal{E}_t(P_{t+\theta}), & {}_t^i \hat{W}_{t+\theta} &= \mathcal{E}_t(W_{t+\theta}), \\ {}_t^i \hat{Y}_{t+\theta}^d &= \mathcal{E}_t(Y_{t+\theta}^d), & {}_t^i \hat{N}_{t+\theta}^s &= \mathcal{E}_t(N_{t+\theta}^s), \end{aligned}$$

for all  $t$  and  $\theta \geq 0$ ; where  $\mathcal{E}_t(z_{t+\theta})$  denotes the true (or objective) expected value of a random variable  $z_{t+\theta}$  conditional upon the economy's history up to the beginning of period  $t$ . Thirdly, his subjective probability distributions of the deviations of the actual values of the relevant random variables from their subjective expected values correctly represent the true (or objective) probability distributions of their deviations from the true (or objective) expected values; that is, we have:

$$(12) \quad \begin{aligned} \Pr^i \{ \alpha_t(1) - {}_t^i \hat{\alpha}_t(1) \leq z \mid {}_t^i \hat{\alpha}_t(1) \} &\equiv {}^i \Psi(z) \\ &= \Pr \{ \alpha_t(1) - \mathcal{E}_t \{ \alpha_t(1) \} \leq z \} \equiv \Psi(z), \\ \Pr^i \{ \beta_t(1) - {}_t^i \hat{\beta}_t(1) \leq z \mid {}_t^i \hat{\beta}_t(1) \} &\equiv {}^i \Phi(z), \\ &= \Pr \{ \beta_t(1) - \mathcal{E}_t \{ \beta_t(1) \} \leq z \} \equiv \Phi(z), \\ \Pr^i \{ {}_{t+\tau}^i \hat{\alpha}_{t+\tau}(1) - {}_t^i \hat{\alpha}_{t+\tau}(1) \leq z \mid {}_t^i \hat{\alpha}_{t+\tau}(1) \} &\equiv {}^i \Psi^{(\tau)}(z) \\ &= \Pr \{ {}_{t+\tau}^i \hat{\alpha}_{t+\tau}(1) - \mathcal{E}_t \{ {}_{t+\tau}^i \hat{\alpha}_{t+\tau}(1) \} \leq z \} \equiv \Psi^{(\tau)}(z), \end{aligned}$$

where we have implicitly assumed that the objective probability distributions  $\Psi(z)$ ,  $\Phi(z)$  and  $\Psi^{(\tau)}(z)$ , are invariant over time. The conditions given by (11) and (12) together imply that the entrepreneur knows the true probability distributions of the random variables  $\alpha_t(1)$ ,  $\beta_t(1)$  and  ${}_{t+\tau}^i \hat{\alpha}_{t+\tau}(1)$  at the beginning of period  $t$ . An entrepreneur who

happens to have rational expectations in the sense of (10), (11) and (12) has no reason to revise his functional and stochastic specification of the macro econometric model nor his specification of the subjective product demand and labor supply schedules. Everything is on average foreseen by him from the beginning and nothing will on average surprise him in the present as well as in the future. All of his new market observations will only confirm the correctness of his economic theory and he will learn nothing from his experiences. This is clearly the generalization of the classical notion of the perfect foresight in a world of uncertainty.

If every entrepreneur in the economy happens to have rational expectations, we simply say that the economy has economy-wide rational expectations. The concept of rational expectations was first coined by J. Muth as a testable hypothesis in his study of price movements in a single competitive market, though his original concept requires much less stringent conditions on economic agents' perception of their market environment.<sup>2</sup> Since his study, the hypothesis of rational expectations and its variations have been extensively used in various areas of economics. In particular, in the recent controversy on the existence of permanent trade-off between unemployment and inflation in macroeconomics, the proponents of the so-called natural rate theory of unemployment forcefully build their case against the existence of permanent trade-off, which

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<sup>2</sup> See Muth [19]. His definition of rational expectations is that the market-wide average of individual firms' subjective expectations of some economic variable is equal to the objective expectation of that variable. Therefore, he does not postulate that all the individual firms' subjective expectations are equal to the objective expectation, nor does his definition require that the subjective probability distribution coincides with the objective probability distribution. Unfortunately, his concept of rational expectations is too weak for our model to produce any useful results, because of the existence of some non-linear relationships among variables in our model. See also Samuelson [25].

is often characterized by the so-called (long-run) Phillips curve, upon the hypothesis of rational expectations.<sup>3</sup> According to this theory, "there is some level of unemployment which has the property that it is consistent with equilibrium in the structure of real wage rates."<sup>4</sup> The real wage rates are in equilibrium if the actual and anticipated changes in prices (and other relevant variables) are on average equal; in other words, the real wage rates are in equilibrium if they are consistent with rational expectations. The rate of unemployment which is consistent with this equilibrium structure of real wage rates is called the natural rate of unemployment, and its level is considered to be invariant under changes in the time-pattern of the rate of inflation. The actual unemployment rate can deviate temporarily from the natural rate if the actual real wage rates deviate from the equilibrium level in response to a sudden and unexpected change in the rate of inflation. However, sooner or later, the public will learn about the inflation and be able to form the expectation about it correctly; as a result, the real wage rates will return to their initial equilibrium level and the natural rate of unemployment will be restored in the labor market. Milton Friedman, for example, maintains that "there is always a temporary trade-off between inflation and unemployment; there is no permanent trade-off," and that "the temporary trade-off comes not from inflation per se, but from unanticipated inflation, which generally means, from a rising rate of inflation."<sup>5</sup> More recently,

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<sup>3</sup> The original proponents of the natural rate theory are Friedman [5] and Phelps [20, 21]. Recently their idea has been pursued further by Lucas [14, 15, 16], Lucas-Rapping [17], Lucas-Prescott [18], Sargent [26], and Sargent-Wallace [27].

<sup>4</sup> Friedman [5], p. 8.

<sup>5</sup> Friedman [5], p. 11.

Robert Lucas even asserts that "rational expectations are equivalent to the existence of a natural [rate of unemployment]." <sup>6</sup>

The logic of the natural rate theory based upon the powerful hypothesis of rational expectations is very robust, even to the point that they can be regarded as logically equivalent as is asserted by Lucas. Indeed, we can reaffirm their argument fairly easily and in a sense trivially within a framework of our microdynamic theory of a single firm, which is more general than the microeconomic models used by most of the natural rate theorists.

In order to see this, let us first evaluate the deviation of the actual rate of excess product demand from its subjective expectation in the  $i^{\text{th}}$  firm's market as follows:

$$\begin{aligned}
 (13) \quad g_t(i) - \hat{g}_t(i) &= \{-\eta(p_t(i) - P_t) + Y_t^d + a_t(i) - y_t^s(i)\} \\
 &\quad - \{-\hat{\eta}(p_t(i) - \hat{P}_t) + \hat{Y}_t^d + \hat{a}_t(i) - y_t^s(i)\} \\
 &= \hat{\eta}(P_t - \hat{P}_t) + (Y_t^d - \hat{Y}_t^d) + (\eta - \hat{\eta})P_t + a_t(i) .
 \end{aligned}$$

Thus the entrepreneur's prediction error of  $g_t(i)$  can be attributed partly to his prediction errors of  $P_t$  and  $Y_t^d$ , partly to his misspecification of the subjective product demand schedule, measured by  $(\eta - \hat{\eta})$ , and partly to the random disturbance  $a_t(i)$  specific to his firm. In the same way, we can attribute the deviation of  $h_t(i)$  from his subjective expectation  $\hat{h}_t(i)$  partly to his prediction errors of  $W_t$  and  $N_t^s$ , partly to his misspecification of the subjective labor supply schedule, measured by  $(\epsilon - \hat{\epsilon})$ , and partly to the firm-specific random

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<sup>6</sup> Lucas [15], p. 54.

disturbance  $b_t(i)$  ; that is, we have:

$$(14) \quad h_t(i) - \frac{i}{t} \hat{h}_t(i) = -\epsilon^i (W_t - \frac{i}{t} \hat{W}_t) + (N_t^s - \frac{i}{t} \hat{N}_t^s) - (\epsilon - \epsilon^i) W_t + b_t(i) .$$

Evaluating the objective expectations of equations (13) and (14) conditional upon the economy's history up to the beginning of period, say,  $t-\theta$  ( $\theta \geq 0$ ) , we have the following two equations:

$$(15) \quad \begin{aligned} & \mathbb{E}_{t-\theta} \{ g_t(i) - \frac{i}{t} \hat{g}_t(i) \} \\ &= \epsilon^i \cdot \mathbb{E}_{t-\theta} (P_t - \frac{i}{t} \hat{P}_t) + \mathbb{E}_{t-\theta} (Y_t^d - \frac{i}{t} \hat{Y}_t^d) + (\eta - \epsilon^i \eta) \mathbb{E}_{t-\theta} (P_t) \\ &= \epsilon^i \cdot \mathbb{E}_{t-\theta} \{ \mathbb{E}_t (P_t) - \frac{i}{t} \hat{P}_t \} + \mathbb{E}_{t-\theta} \{ \mathbb{E}_t (Y_t^d) - \frac{i}{t} \hat{Y}_t^d \} + (\eta - \epsilon^i \eta) \mathbb{E}_{t-\theta} (P_t) , \end{aligned}$$

$$(16) \quad \begin{aligned} & \mathbb{E}_{t-\theta} \{ h_t(i) - \frac{i}{t} \hat{h}_t(i) \} \\ &= -\epsilon^i \cdot \mathbb{E}_{t-\theta} \{ \mathbb{E}_t (W_t) - \frac{i}{t} \hat{W}_t \} + \mathbb{E}_{t-\theta} \{ \mathbb{E}_t (N_t^s) - \frac{i}{t} \hat{N}_t^s \} - (\epsilon - \epsilon^i) \mathbb{E}_{t-\theta} (W_t) . \end{aligned}$$

Now suppose that the  $i^{\text{th}}$  entrepreneur happens to have rational expectations. Then, it is clear from the first condition of rational expectations (10) that the third terms in the right-hand-sides of both equations (15) and (16) must vanish; and it is also clear from the second conditions (11) that the first and second terms must also vanish. Therefore we have:

$$(17) \quad \mathbb{E}_{t-\theta} \{ g_t(i) - \frac{i}{t} \hat{g}_t(i) \} = 0 ,$$

$$(18) \quad \mathbb{E}_{t-\theta} \{ h_t(i) - \frac{i}{t} \hat{h}_t(i) \} = 0 .$$

Since we are assuming in the present paper that each entrepreneur can adjust both product price and money wage costlessly at the beginning of

every period, by (6) and (8) his optimal price and wage policy is to set  $\hat{g}_t(i)$  and  $\hat{h}_t(i)$  equal to their constant normal rates  $g^*$  and  $h^*$ . Moreover under the first and third conditions (10) and (12) of rational expectations it is easy to see from (I-22) and (I-50) that these individual normal rates become equal to the rates  $g^*$  and  $h^*$  uniform for all firms and given by:

$$(19) \quad (\eta-1) \int_{-\infty}^{-g^*} \exp(z+g^*) d\Psi(z) = \{1 - \Psi(-g^*)\} ,$$

$$(20) \quad \{1 - \Phi(-h^*)\} + (1+\epsilon) \int_{-\infty}^{-h^*} \exp(z+h^*) d\Phi(z) = \epsilon \int_{-\infty}^{-h^*} \exp\left\{\frac{\eta-\sigma}{\eta}(z+h^*)\right\} d\Phi(z) .$$

We shall call these uniform rates the natural rates of excess product demand and excess labor supply respectively. Therefore, we have in fact obtained the following important results for all  $t$  and  $\theta \geq 0$  :

$$(21) \quad \mathbb{E}_{t-\theta}\{g_t(i)\} = g^* ,$$

$$(22) \quad \mathbb{E}_{t-\theta}\{h_t(i)\} = h^* .$$

In words, if in the Wicksellian economy an individual entrepreneur has rational expectations, then the actual rates of excess product demand and excess labor supply in his market, given by  $g_t(i)$  and  $h_t(i)$ , are on average equal to the constant natural rates, given by  $g^*$  and  $h^*$ , whose values are determined solely by given objective elasticities,  $\eta$ ,  $\epsilon$  and  $\gamma$ , and given objective probability distributions,  $\Psi(z)$  and  $\Phi(z)$ .

Under rational expectations, we can also show that the objective expectations of the discrepancies between expected and actual rates of unfilled-orders and involuntary unemployment in the  $i^{\text{th}}$  firm's markets

are both equal to zero. That is, in view of (12), (17) and (18), we have for all  $t$  and  $\theta \geq 0$  :

$$\begin{aligned}
 (23) \quad & \mathbb{E}_{t-\theta} \{j_t(i) - \hat{j}_t(i)\} = \mathbb{E}_{t-\theta} \{j_t(i) - E_t^i(\max[g_t(i), 0])\} \\
 & = \mathbb{E}_{t-\theta} \{j_t(i) - E_t^i(\max[\hat{g}_t(i) + \{\alpha_t(i) - \hat{\alpha}_t(i)\}, 0])\} \\
 & = \mathbb{E}_{t-\theta} \{j_t(i) - \mathbb{E}_t(\max[\mathbb{E}_t(g_t(i)) + \{\alpha_t(i) - \mathbb{E}_t(g_t(i))\}, 0])\} \\
 & = \mathbb{E}_{t-\theta} \{\mathbb{E}_t(j_t(i)) - \mathbb{E}_t(\max[g_t(i), 0])\} \\
 & = 0 ,
 \end{aligned}$$

$$(24) \quad \mathbb{E}_{t-\theta} \{u_t(i) - \hat{u}_t(i)\} = 0 .$$

Again by (7) and (9) our entrepreneur equalizes  $\hat{j}_t(i)$  and  $\hat{u}_t(i)$  with the constant normal rates,  $j^*$  and  $u^*$ , respectively. Under the conditions (10) and (12) for rational expectations these individual normal rates defined by (I-28) and (I-57) again become uniform for all firms and given by

$$(25) \quad j^* = \int_{-g^*}^{\infty} z d\Psi(z) + g^* \{1 - \Psi(-g^*)\}$$

$$(26) \quad u^* = \int_{-h^*}^{\infty} z d\Phi(z) + h^* \{1 - \Phi(-h^*)\} .$$

We shall call  $j^*$  and  $u^*$  the natural rates of unfilled-orders and involuntary unemployment respectively. Thus, combining (23) and (24) with (25) and (26), we obtain the following results for all  $t$  and  $\theta \geq 0$  :

$$(27) \quad E_{t-\theta}\{j_t(1)\} = j^* ,$$

$$(28) \quad E_{t-\theta}\{u_t(1)\} = u^* .$$

We can conclude from this that in the Wicksellian economy the hypothesis of rational expectations imply that the actual rates of unfilled-orders and involuntary unemployment are on the average equal to the given natural rates  $j^*$  and  $u^*$  whose values are determined only by given objective elasticities,  $\eta$ ,  $\epsilon$  and  $\gamma$ , and by given objective probability distributions,  $\Psi(z)$  and  $\Phi(z)$ .

The equation (28) supports the claim of natural rate theory that under the hypothesis of rational expectations the rate of unemployment is on the average equal to the natural rate, independently of the time-patterns of the rate of inflation and other monetary variables in the economy. Does this mean that the natural rate theory is now elaborated as an unrefutable scientific truth? Our answer to this question turns out to be negative, and the rest of this paper will be devoted to the explanation of this negative answer.

### 3. Say's Law and the Consistency of Rational Expectations

Nothing is the graver sin to professional economists than to commit the fallacy of composition: a fallacy in which what is true of a part is, on that account alone, alleged to be also true of the whole.<sup>7</sup> Our starting point for the critique of the natural rate theory of unemployment in the Wicksellian economy is an observation that the deduction of the natural rate result (28) from the hypothesis of rational expectations

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<sup>7</sup> Samuelson [24], p. 13.



relies solely upon microeconomic relationships derived from our model of the single firm developed in Part I of this series of papers, but is totally independent of the economy's aggregative relationships.

In the following, it is convenient to assume at the outset that the first and third conditions (10) and (12) of rational expectations are already satisfied in our economy. Then, firms in it can differ from each other only in their different endowments of capital stock and in their different subjective conditional expectations of the relevant aggregate variables. The hypothesis of economy-wide rational expectations is, therefore, reduced to the simple condition given by (11) that these subjective conditional expectations coincide with their objective conditional expectations for all firms.

Notice that under the conditions (10) and (12), all the individual entrepreneur's normal rates,  $i_g^*$ ,  $i_h^*$ ,  $i_j^*$  and  $i_u^*$ , become equal to their uniform natural rates,  $g^*$ ,  $h^*$ ,  $j^*$  and  $u^*$ , respectively, and that all the coefficients and constant terms of the individual optimal product price, optimal money wage and optimal labor demand equations, given by (1), (2) and (5), become also uniform for all firms. Thus, we can proceed most of our analysis of the economy's macrorelationships in terms of aggregate variables.

Let us denote by  $N_t$ ,  $N_t^d$ ,  $Y_t$ ,  $Y_t^s$ ,  $G_t$ ,  $J_t$ ,  $H_t$ ,  $U_t$  and  $K_t$  the economy-wide geometrical averages of individual firms' labor employments, labor demands, product sales, product supplies, rates of excess product demand, rates of unfilled-orders, rates of excess labor supply, rates of involuntary unemployment and indices of capital stock in period  $t$ , respectively; that is, we define

$$(29) \quad N_t \equiv \sum_i n_t(i)/m, \quad N_t^d \equiv \sum_i n_t^d(i)/m, \quad Y_t \equiv \sum_i y_t(i)/m,$$

$$Y_t^s \equiv \sum_i y_t^s(i)/m, \quad G_t \equiv \sum_i g_t(i)/m, \quad J_t \equiv \sum_i j_t(i)/m,$$

$$H_t \equiv \sum_i h_t(i)/m, \quad U_t \equiv \sum_i u_t(i)/m, \quad K_t \equiv \sum_i k_t(i)/m.$$

Let us also denote by  ${}_{t-\theta}\hat{p}_t$ ,  ${}_{t-\theta}\hat{w}_t$ ,  ${}_{t-\theta}\hat{y}_t^d$  and  ${}_{t-\theta}\hat{N}_t^s$ , the economy-wide averages of individual entrepreneurs' subjective expectations of  $P_t$ ,  $W_t$ ,  $Y_t^d$  and  $N_t^s$  conditional upon their individual information available at the beginning of period  $t-\theta$  ( $\theta \geq 0$ ); that is, we define

$$(30) \quad {}_{t-\theta}\hat{p}_t \equiv \sum_i {}_{t-\theta}^i \hat{p}_t / m, \quad {}_{t-\theta}\hat{w}_t \equiv \sum_i {}_{t-\theta}^i \hat{w}_t / m,$$

$${}_{t-\theta}\hat{y}_t^d \equiv \sum_i {}_{t-\theta}^i \hat{y}_t^d / m, \quad {}_{t-\theta}\hat{N}_t^s \equiv \sum_i {}_{t-\theta}^i \hat{N}_t^s / m.$$

We have already defined by  $P_t$  and  $W_t$  the economy-wide averages of product prices  $p_t(i)$  and money wages  $w_t(i)$  in (I-2) and (I-5), and we have shown in (I-3) and (I-6) that the economy-wide averages of demands in individual product markets  $y_t^d(i)$  and supplies in individual labor markets  $n_t^s(i)$  are equal to the real aggregate demand  $Y_t^d$  and the aggregate labor supply  $N_t^s$  respectively. It is easy to see then that some of these aggregate variables are related with each other in the following obvious manners:

$$(31) \quad G_t = Y_t^d - Y_t^s, \quad J_t = Y_t^d - Y_t, \quad H_t = N_t^s - N_t^d, \quad U_t = N_t^s - N_t,$$

$$(32) \quad Y_{t+\tau}^s = K_t + \gamma N_t = K_t + \gamma(N_t^s - U_t).$$

Now, let us aggregate the individual firms' optimal product price equations, given by (1), over all firms. Then, noting that under the conditions (10) and (12) all  ${}^i\eta$ 's equal  $\eta$  and all  ${}^i g^*$ 's equal  $g^*$ , we obtain the aggregate price equation:

$$(33) \quad P_t = {}_t\hat{P}_t + \frac{1}{\eta}({}_t\hat{Y}_t^d - Y_t^s - g^*) .$$

Since we have assumed throughout the present series of papers that entrepreneurs can adjust their product prices costlessly at the beginning of every period, the above equation determines the actual dynamic movement of the aggregate product price for all periods. Evaluating its objective conditional expectations and rearranging terms, we have for all  $t$  and  $\theta \geq 0$  :

$$(34) \quad E_{t-\theta}(P_t - {}_t\hat{P}_t) + \frac{1}{\eta}E_{t-\theta}({}_tY_t^d - {}_t\hat{Y}_t^d) = \frac{1}{\eta}E_{t-\theta}({}_tY_t^d - Y_t^s - g^*) .$$

If the economy happens to have economy-wide rational expectations, it is clear that the two terms in the left-hand-side of the above equation both vanish for all  $t$  and  $\theta \geq 0$ . In consequence, its right-hand-side must vanish for all  $t$  and  $\theta \geq 0$ . We have thus obtained the following equation as a necessary condition for the consistency of the hypothesis of economy-wide rational expectations with the economy's aggregative pricing behavior:

$$(35) \quad E_{t-\theta}({}_tY_t^d - Y_t^s) = g^* \quad \text{for all } t \text{ and } \theta \geq 0 .$$

This crucial equation, maintaining that the objective conditional expectation of the rate of gap between the real aggregate demand and the aggregate

supply is always equal to the natural rate of excess product demand, can be called Say's equation and the principle that it is universally satisfied can be regarded as the modern version of the classical Say's law that "supply creates its own demand." Therefore, we can conclude that the economy, (whether Wicksellian or Keynesian,) is capable of possessing economy-wide rational expectations only if Say's equation holds.

In a monetary economy in which money or any other durable asset functions as "a link between the present and the future,"<sup>8</sup> Say's law cannot be the universal truth. Of course, in an extremely well-organized barter-exchange economy like the one described by the celebrated Arrow-Debreu model of Walrasian general equilibrium, in which all the current as well as future commodities have complete markets, there need not exist any durable asset which function as a 'store of value.' Say's law is valid in such an economy. However, so long as some futures markets are missing and at least one durable asset (usually, a means of payment = money) functions as a store of value (or as a "temporary abode of general purchasing power"), the spending units in the economy--households primarily as buyers of currently supplied consumption goods, firms mostly as buyers of currently supplied investment goods and governmental and other economic institutions as buyers of either of them or both--can decide to spend greater or smaller amounts of money, or other stores of value, on current products (or on future delivery of some commodities if there are well-organized futures markets for them) than their current incomes; in other words, they can dishoard or hoard by decumulating or accumulating the durable asset which functions as a store of value. The act of hoarding is a completely different act from placing orders to some well-organized futures markets for the future delivery of specific products. While the

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<sup>8</sup> Keynes [10], p. 293.

latter constitutes a part of the current, real aggregate demand, the former does not. The current dishoarding only means the disappearance of a part of the current, real aggregate demand, and it will reappear in the products markets in the form of real aggregate demand only in some unspecified future periods. Money or any other store of value is a link between the present and future, but it is only a "liquid" link. In any case, if the spending units as a whole can make their hoarding-dishoarding decisions at their discretion, the current, real aggregate demand can exceed or fall short of the current, aggregate income and, hence, the current, aggregate supply; in other words, Say's equation can be disturbed. This seems to be pretty much a common-sense story since the days of Keynes. But only quite recently the importance of this common-sense story has begun to be seriously reappraised by theoretical economists.<sup>9</sup>

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<sup>9</sup>We owe to Clower [1] for this recent reappraisal of the role of Say's law in Keynesian or disequilibrium economics. See also Leijonhufvud [12, 13]. Clower put the essence of the problem in the following phrase: "either Walras' law is incompatible with Keynesian economics, or Keynes had nothing fundamentally new to add to orthodox economic theory," (p. 110). Unfortunately, he somehow named what we call Say's law 'Walras' law' and what we generally call Walras' law 'Say's principle,' and has led to some confusions. His "Say's principle" and our Walras' law are nothing but the economy-wide summation of individual household's budget equations, which is satisfied in almost all economies. Our definition of Say's law is in complete accordance with Keynes' usage of that word. In his subsequent papers [2, 3], however, Clower shifted the analytical focus, and emphasized the importance of the 'means-of-payment' role of money in the theory of monetary disequilibrium. We believe that this has only confounded the issue. For the fundamental cause of disequilibrium in a monetary economy lies in the disturbance of Say's law, which is possible only because money functions as a 'store of value'--"a subtle device for linking the present and the future" (Keynes [10], p. 294). That money is a means-of-payment seems to be rather a side-issue at least for the purpose of disequilibrium dynamics. Keynes emphasized this point very clearly in the following passages ([10], p. 294):

We cannot get rid of money even by abolishing gold and silver and legal tender instruments. So long as there exists any durable asset, it is capable of possessing monetary attributes [i.e., a link between the present and the future] and, therefore, of giving rise to the characteristic problems of a monetary economy.

We know that in the Wicksellian economy the hypothesis of rational expectations implies the natural rate proposition (28). In view of this and (32), we can rephrase Say's equation (35) as follows:

$$\begin{aligned}
 (36) \quad & \mathbb{E}_{t-\theta}[Y_t^d - \{K_{t-\tau} + \gamma(N_{t-\tau}^s - U_{t-\tau})\}] \\
 &= \mathbb{E}_{t-\theta}[Y_t^d - \{K_{t-\tau} + \gamma(N_{t-\tau}^s - \mathbb{E}_{t-\tau}(U_{t-\tau}))\}] , \text{ for } \theta \geq \tau , \\
 &= \mathbb{E}_{t-\theta}[Y_t^d - \{K_{t-\tau} + \gamma(N_{t-\tau}^s - u^*)\}] \\
 &= \mathbb{E}_{t-\theta}(Y_t^d - Y_t^{s*}) = g^* \text{ for all } t \text{ and } \theta \geq \tau ;
 \end{aligned}$$

where  $Y_t^{s*}$  is defined by

$$(37) \quad Y_t^{s*} \equiv K_{t-\tau} + \gamma(N_{t-\tau}^s - u^*) ,$$

and called the natural aggregate supply in period  $t$ . It is the economy-wide geometrical average of outputs in period  $t$  which could be supplied in the product markets if the aggregate rate of unemployment had been equal to the natural rate  $u^*$  when the entrepreneurs committed themselves to starting the production process which would produce them. The crucial observation for us is that both  $Y_t^d$  and  $Y_t^{s*}$  in this new form of Say's equation (36) are givens to our entrepreneurs in period  $t$ . The real aggregate demand  $Y_t^d$  is determined by households and other agents in the economy through their aggregative spending decisions on current products, based upon current as well as past price and wage structure; upon their expectations of future prices and wages; upon current as well as past business conditions, upon their expectations of future business conditions; upon term-structure of interest rates; upon their liquidity

preferences; upon portfolios of various assets in the economy; and upon all sorts of other variables which may appear in certain macroeconomic models. The natural aggregate supply  $Y_t^{s*}$  defined by (37) is, on the other hand, determined by a given natural rate of unemployment  $u^*$ , by a given aggregate capital  $K_{t-\tau}$  endowed in the economy in period  $t-\tau$  and by an aggregate labor supply  $N_{t-\tau}^s$  in period  $t-\tau$ . The aggregate labor supply  $N_{t-\tau}^s$  is in turn determined by households through their work-leisure choices based primarily upon the economy's current as well as past real wage structure. Of course, both  $Y_t^d$  and  $N_t^s$  are influenced by aggregate product prices and aggregate money wages decided by our entrepreneurs as a whole in the present and in the past. But, the crucial fact is that there is no a priori reason why the entrepreneurs' unconcerted price and wage decisions, each acting for his own advantage, always influence both the real aggregate demand and the aggregate supply in such a way that their expected rate of gap is instantaneously brought into equality with the constant natural rate  $g^*$  at each moment of time. All we need in our theory is that in a monetary economy Say's equation can be disturbed at least in the short-run.

If  $Y_t^d - Y_t^s > g^*$  we say that the economy has a price-inflation gap or a positive aggregate demand gap in period  $t$ , and if  $Y_t^d - Y_t^s < g^*$  we say that it has a price-deflation gap or a negative aggregate demand gap in period  $t$ . Unless the economy's real aggregate demand  $Y_t^d$  keeps track of a time path that on average eliminates these gaps, not all the entrepreneurs can have rational expectations simultaneously. Of course, some entrepreneurs may be lucky enough to have rational expectations. But so long as there exists a price-inflation or price-deflation gap in the economy, which reveals the inconsistency between the spending sector's

aggregative spending decisions and the production sector's aggregate supply of products, at least some entrepreneurs' expectations are bound to be falsified by their own aggregative pricing behavior!

Note in passing that Say's equation is not a sufficient condition for the hypothesis of rational expectations. We can easily imagine a state of the economy in which Say's equation holds, yet not all entrepreneurs entertain rational expectations. The disequilibrium treated by the natural rate theory of unemployment seems to correspond to this rather special situation.

We can sum up our proposition as follows: the hypothesis of economy-wide rational expectations and its derivative in the Wicksellian economy --the natural rate theory of unemployment--are contingent upon the validity of the modern Say's law that the objective conditional expectations of aggregate demand gap  $\xi_{t-\theta}(Y_t^d - Y_t^s - g^*)$  are always equal to zero. In other words, we maintain that to postulate the hypothesis of economy-wide rational expectations at the outset is to assume this modern Say's law. It is Keynes who asserted that Say's law should be regarded as the classical employment theory's "axiom of parallels."<sup>10</sup> By the same token, we can regard our modern Say's law (31) as the "axiom of parallels" for the natural rate theory of unemployment--the heir of the classical theory of unemployment.

In order to be complete, let us also aggregate the individual firms' optimal money wage equations given by (2) and their labor demand functions given by (5) over all firms. Then, noting that under conditions (10) and (12) all the coefficients and constant terms are uniform for all firms,

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<sup>10</sup> Keynes [10], p. 21.



we obtain the aggregate money wage equation and the aggregate labor demand function as follows:

$$(38) \quad W_t = \frac{\eta}{\eta + \sigma \epsilon} \left\{ \varphi - \delta \tau + \frac{\eta - 1}{\eta} K_t + {}_t \hat{P}_{t+\tau} + \frac{\sigma \epsilon}{\eta} {}_t \hat{W}_t \right. \\ \left. + ({}_t \hat{Y}_{t+\tau}^d - g^*) - \sigma ({}_t \hat{N}_t^s - h^*) \right\},$$

$$(39) \quad N_t^d = \frac{\eta \epsilon}{\eta + \sigma \epsilon} \left\{ \varphi - \delta \tau + \frac{\eta - 1}{\eta} K_t + {}_t \hat{P}_{t+\tau} - {}_t \hat{W}_t \right. \\ \left. + \frac{1}{\eta} ({}_t \hat{Y}_{t+\tau}^d - g^*) + \frac{1}{\epsilon} ({}_t \hat{N}_t^s - h^*) \right\}.$$

Substituting (39) into (38), rearranging terms and evaluating the objective expectations, we have for all  $t$  and  $\theta \geq 0$ :

$$(40) \quad E_{t-\theta} (W_t - {}_t \hat{W}_t) - \frac{1}{\epsilon} E_{t-\theta} (N_t^s - {}_t \hat{N}_t^s) = \frac{1}{\epsilon} E_{t-\theta} (N_t^d - N_t^s + h^*).$$

Therefore, in the same way as before, we can obtain the following equation as a necessary condition for the consistency of the hypothesis of economy-wide rational expectations with the economy's aggregative money wage determination:

$$(41) \quad E_{t-\theta} (N_t^s - N_t^d) = h^* \quad \text{for all } t \text{ and } \theta \geq 0.$$

This equation, which maintains that the objective conditional expectation of the rate of gap between the aggregate labor supply and the aggregate labor demand is always equal to the natural rate of excess labor supply  $h^*$ , may be regarded as Say's equation in labor markets. Accordingly, if  $N_t^s - N_t^d > h^*$  we say that the economy has a wage-deflation gap in period  $t$ , and if  $N_t^s - N_t^d < h^*$  we say that it has a wage-inflation gap in period  $t$ .

However, different from Say's equation in the product markets, given by (35),  $N_t^d$  in equation (41) is not a 'given' to the firm's sector of the economy. On the contrary, as is seen from equation (39), it is determined by the entrepreneurs as a whole on the basis of their endowed capital stocks and of their subjective expectations of  $P_{t+\tau}$ ,  $W_t$ ,  $Y_{t+\tau}^d$  and  $N_t^s$ . It is necessary, therefore, to evaluate the left-hand side of (41) more carefully. If we substitute (33) and (38) into (39), then in view of the aggregative relations (31) and (32) we have

$$\begin{aligned}
 (42) \quad & E_{t-\theta} (N_t^s - N_t^d - h^*) \\
 &= \frac{\eta E}{\sigma E_{t-\theta}} \{ (\hat{P}_{t+\tau} - {}_{t+\tau}\hat{P}_{t+\tau}) + \frac{1}{\eta} ({}_{t+\tau}\hat{Y}_{t+\tau}^d - {}_{t+\tau}\hat{Y}_{t+\tau}^d) \} \\
 &+ \frac{\eta E}{\sigma E_{t-\theta}} \{ P_{t+\tau} + Y_{t+\tau} - W_t - N_t \} - \frac{\eta E}{\sigma E_{t-\theta}} \{ G_{t+\tau} - J_{t+\tau} - \frac{\sigma}{\eta} (H_t - U_t) - \varphi + \delta\tau \} .
 \end{aligned}$$

Now, if the economy had economy-wide rational expectations, then the first term in the right-hand-side of the above equation would vanish and the third term would coincide with the normal (or natural) rate of profit:  $\rho^* = g^* - j^* - \sigma(h^* - u^*)/\eta - \varphi - \delta\tau$ , defined by (I-58). Since the second term in the right-hand-side is nothing but the objective conditional expectation of the economy-wide rate of profit, it would be equal to the third term under the economy-wide rational expectations. Hence the left-hand-side of (42) would vanish; and as a result, we can say that Say's equation in labor markets, given by (41), would hold if the economy had economy-wide rational expectations. However, we already know that the economy-wide rational expectations are possible only if Say's equation in product markets, given by (35), is satisfied. Therefore, it can be concluded that Say's equation in labor markets is conditional upon Say's equation

in product markets, and that if the latter is disturbed the former is necessarily disturbed and if the latter is satisfied the former is satisfied under economy-wide rational expectations. This one-way causal relationship from product markets to labor markets is due to the intertemporal nature of firms' production activities. This causal relationship is reversed in the households' sector, and by putting both firms and households together we can close a full circle.

#### 4. Long-Run Equilibrium

That we do not believe in the existence of a monetary economy in which Say's law is a universal truth does not and should not imply that we do not believe in the existence of a state of the economy in which Say's equation happens to hold. In such a state of the economy, if any, all the entrepreneurs are capable of possessing rational expectations simultaneously. Then, we define long-run equilibrium as a state of the economy in which both Say's equation, given by (35), and economy-wide rational expectations, characterized by conditions (10), (11) and (12), hold. It is important to note that the definition of long-run equilibrium requires the fulfillment of Say's equation in the first place; for without which the hypothesis of economy-wide rational expectations would be inconsistent.

One can regard our notion of long-run equilibrium as equivalent to that of "equilibrium over time" proposed by J. R. Hicks in his dynamic Walrasian general equilibrium theory. He defined it as a state of the economy in which "the prices realized on the second Monday are the same as those which were previously expected to rule at that date."<sup>11</sup> Hicks

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<sup>11</sup>Hicks [7], p. 132.

did not treat the problems of uncertainty in a satisfactory manner, but his notion of equilibrium over time is refined and extended to the economy under uncertainty by several mathematical economists in their extensions of the Arrow-Debreu model of the Walrasian general equilibrium theory.<sup>12</sup> However, since the natural rate theory of unemployment might be interpreted as its 'special' case, we can apply almost the same criticism as those given to the natural rate theory to these models of dynamic Walrasian general equilibrium theory.

In the Wicksellian economy the natural rate theory is unrivaled in long-run equilibrium. The average rate of involuntary unemployment as well as average rates of excess product demand, excess labor supply and unfilled-orders are all determined by given market and technological characteristics and by given stochastic properties of the disturbance parts of the relevant aggregate variables; they are independent of time patterns of changes in the monetary variables in the economy such as the rate of inflation, monetary aggregates and fiscal policy variables, provided that Say's equation (35) are not disturbed by changes in these nominal variables. In this sense, we can say that the Phillips curve in the Wicksellian economy is on average vertical in a state of long-run equilibrium;<sup>13</sup>

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<sup>12</sup> See Radner [22, 23] and Hahn [6].

<sup>13</sup> Note that by this we do not necessarily mean that the 'quantity theory of money' or its modern version, 'monetarism,' holds in a long-run equilibrium state. Although we have not specified any macroeconomic model that can determine the real aggregate demand, it is true that in the Wicksellian economy a simultaneous increase of all the exogenous monetary variables by the same proportion will not affect the values of real variables in long-run equilibrium. This proposition is a trivial consequence of the absence of money illusion. However, the 'quantity theory' usually claims more than that; it asserts that real variables in long-run equilibrium are independent of the nominal quantity of money. We do not believe that this stronger proposition holds in the modern economy

although there may exist some short-run correlation between unanticipated changes in the magnitudes of some monetary variables and actual rate of unemployment due to the entrepreneurs' inability of distinguishing real from monetary disturbances in the short-run. It should be emphasized, however, that this long-run verticality of the Phillips curve is only a characteristic of the long-run equilibrium state, which would break down whenever Say's equation were disturbed by changes in monetary forces in the economy.

A question arises immediately: is there always a long-run equilibrium in our Wicksellian economy? In order to give a satisfactory answer to this existence problem, so to speak, we must, of course, develop a fully specified macroeconomic or monetary general (dis)equilibrium model of our own, which we shall not do in the present paper. Fortunately, some of the works of the natural rate theorists and a few recent results in the general equilibrium theory can be interpreted as proofs of the existence of a long-run equilibrium in a certain class of economies; though all of them are assuming perfect competition.<sup>14</sup> In any case, we want to bypass the existence problem here and simply assume that for any given set of monetary and fiscal policy variables and other exogeneous variables we can always find a corresponding long-run equilibrium in the economy we are dealing with.

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where there exist various kinds of governmental liabilities other than 'money.' See Tobin [28] for the more complete discussion of this point. In any case, in the Keynesian economy in which money wage adjustments are costly (dynamic money illusion!?), even the first weaker proposition would break down. Real variables in long-run equilibrium in the Keynesian economy are affected by the time pattern of changes in monetary variables.

<sup>14</sup> See, for example, Lucas [14] and Radner [22, 23].

Then, another and the much more important question arises: how will the economy behave if it is thrown out of long-run equilibrium? More specifically, will Say's equation and rational expectations be restored automatically and the economy be brought back to a state of long-run equilibrium? Since the natural rate theory of unemployment may be interpreted as a statement of some properties of the long-run equilibrium state of our Wicksellian economy, such a 'stability problem' can have an important bearing to its empirical relevancy. However, in order to analyze fully the economy's behavior out of long-run equilibrium, we must face two problems. Firstly, as in the problem of existence of a long-run equilibrium, we must fully specify the economy's macroeconomic relations. Secondly and more seriously, we must specify our entrepreneurs' expectation-formation processes on a rather ad hoc basis. For, the powerful hypothesis of rational expectations can never be imposed upon when we are concerned with the economy's out-of-equilibrium states. What we need is a short-run model of expectation formation like the one we discussed briefly at the beginning of this paper. However, in this series of papers, we shall neither give a precise specification of the macroeconomic relations nor construct any explicit expectation-formation model of entrepreneurs. In what follows, we shall only give a rather heuristic description of the economy's out-of-equilibrium behaviors, leaving their more detailed and precise analysis to another opportunity.

##### 5. Cumulative Inflation Process

Let us rewrite the aggregate price equation (33), the aggregate money wage equation (38) and the aggregate labor demand function (39) as follows:

$$(43) \quad P_t - {}_t\hat{P}_t = \frac{1}{\eta}({}_t\hat{Y}_t^d - Y_t^s - g^*),$$

$$(44) \quad W_t - {}_t\hat{W}_t = \frac{1}{\epsilon}(N_t^d - {}_t\hat{N}_t^s + h^*),$$

$$(45) \quad N_t^d - {}_t\hat{N}_t^s + h^* = \frac{\eta\epsilon}{\eta + \sigma\epsilon} \left\{ \varphi - \delta\tau + {}_t\hat{P}_{t+\tau} - {}_t\hat{W}_t + \frac{\eta-1}{\eta}K_t \right. \\ \left. + \frac{1}{\eta}({}_t\hat{Y}_{t+\tau}^d - g^*) - \frac{\sigma}{\eta}({}_t\hat{N}_t^s - h^*) \right\}.$$

(Note that we are still assuming the conditions (10) and (12).) Equation (43) states that when the entrepreneurs on average expect a price-inflation gap at the beginning of period  $t$ , i.e.,  ${}_t\hat{Y}_t^d - Y_t^s > g^*$ ,<sup>15</sup> the aggregate product price  $P_t$  determined by them as a whole in that period will necessarily exceed the economy-wide average of their own expectations of it, given by  ${}_t\hat{P}_t$ ; and that when they on average expect a price-deflation gap, i.e.,  ${}_t\hat{Y}_t^d - Y_t^s < g^*$ ,  $P_t$  will necessarily fall short of  ${}_t\hat{P}_t$ .  $P_t$  can coincide with  ${}_t\hat{P}_t$  if and only if the entrepreneurs on average expect that there is neither price-inflation gap nor price-deflation gap in the economy, i.e.,  ${}_t\hat{Y}_t^d - Y_t^s = g^*$ . Similarly, equation (44) states that, when the entrepreneurs on average expect a wage-inflation gap at the beginning of period  $t$ , i.e.,  $N_t^d - {}_t\hat{N}_t^s > -h^*$ , the aggregate money wage  $W_t$  determined by them as a whole will necessarily exceed the economy-wide average of their own expectations of it, given by  ${}_t\hat{W}_t$  and that when they on average expect a wage-deflation gap, i.e.,  $N_t^d - {}_t\hat{N}_t^s < -h^*$ ,  $W_t$  will necessarily fall short of  ${}_t\hat{W}_t$ . Again,  $W_t$  can coincide with  ${}_t\hat{W}_t$  if and only if the entrepreneurs on average expect that there is neither

<sup>15</sup>This statement is a little bit unprecise. To be rigorous, we should put it as follows: when a gap between economy-wide average of individual entrepreneur's expectations of  $Y_t^d$ , given by  ${}_t\hat{Y}_t^d$ , and the aggregate supply  $Y_t^s$  exceeds the natural rate of excess product demand  $g^*$ , ... .

wage-inflation gap nor wage-deflation gap in the economy, i.e.,  
 $N_t^d - \hat{N}_t^s = -h^*$ . One can hardly fail to notice a certain similarity between the above aggregate price and money wage equations and the well-known law of supply and demand that the price of a commodity rises when there is an excess demand and falls where there is an excess supply in its market.

Suppose that the economy has been in a happy state of long-run equilibrium for a long, long time. Suddenly, the real aggregate demand  $Y_t^d$  increased relative to its past trend level in period  $t = 0$ , and then resumed its past average growth rate from the next period on. This unanticipated increase in  $Y_t^d$  may be due to an upward shift of firms' marginal efficiency schedules of capital; or to a decrease in consumers' time-preferences; or to a decline in wealth-owners' liquidity preferences; or to an expansion of governmental expenditures; or to an injection of money into financial markets through the central bank's open market operation; and so on. Whatever the cause, this will certainly create a price-inflation gap in the economy, for the aggregate supply  $Y_t^s$  will more or less follow the trend in the previous long-run equilibrium at least for some time; that is, we have for some  $t \geq 0$  :

$$(46) \quad Y_t^d - Y_t^s > g^* .$$

The emergence of a price-inflation gap will be at first perceived by most entrepreneurs as an unexpected influx of demand for their products. However, so long as they regard this unexpected increase in  $y_t^d(i)$  as a transient phenomenon due to a temporary increase in the firm-specific random disturbance  $a_t(i)$ , nothing will happen to the economy except



for their customers' frustrations; both  $P_t$  and  $W_t$  will still keep the track of the trend paths which they have been following in the previous long-run equilibrium. But this is only a lull before a storm. Sooner or later our entrepreneurs, facing a continuous demand influx, will realize that this is not a temporary good luck but a persistent one caused by an increase in the real aggregate demand relative to the previous trend level, and begin to revise their subjective expectations of  $Y_t^d$  upwards relative to the trend level. As a consequence, a positive gap between  ${}_t\hat{Y}_t^d - Y_t^s$  and  $g^*$  will be newly created, and the underlying price-inflation gap (46) has come to be reflected in the entrepreneurs' subjective expectations. The economy will start to lose the track of the trend path followed in the previous long-run equilibrium from then on.

It follows from the new form of the aggregate price equation (43) that this creation of a positive gap between  ${}_t\hat{Y}_t^d - Y_t^s$  and  $g^*$  at the beginning of period, say,  $t$  will lead the entrepreneurs as a whole to choose  $P_t$  in that period higher than the economy-wide average of their own subjective expectations of  $P_t$  itself, which is given by  ${}_t\hat{P}_t$  and evaluated at the beginning of period  $t$ . Hence, at the end of period  $t$  they will find themselves to have underestimated  $P_t$  determined by themselves! This unexpected rise in  $P_t$  will presumably induce the entrepreneurs as a whole to revise their subjective expectations of the aggregate price in the next period, whose average is given by  ${}_{t+1}\hat{P}_{t+1}$ , relative to the past trend level. But this will be of no help. The rise in  ${}_{t+1}\hat{P}_{t+1}$  above the past trend level will be offset simultaneously by an equal further rise in  $P_{t+1}$ . (Recall that the aggregate price equation (43) has the 'money-neutrality property' in the sense that it is

homogeneous of degree one with respect to  $\hat{P}_t$  .) This process will keep going on, for so long as the gap between  $\hat{Y}_t^d$  and  $Y_t^s$  remains greater than  $g^*$  as a reflection of the underlying price-inflation gap (46),  $P_t$  will necessarily exceed  $\hat{P}_t$  by the amount proportional to the excess rate of this gap. A cumulative price inflation in excess of long-run equilibrium price inflation or a price-price spiral has been thus triggered off. This cumulative price inflation process will aggregate prices and economy-wide average expectations of them chasing one another upwards will never stop so long as there exists a price-inflation gap in the economy. No matter how our entrepreneurs revise their expectations of aggregate product price, their own aggregative pricing behavior will inevitably falsify their own expectations, and they will be left with unextinguishable frustrations as before.

When the entrepreneurs revise their expectations of the current aggregate real demand,  $\hat{Y}_t^d$ , in response to the continuous inflow of demand for their products caused by the creation of a price-inflation gap (46), presumably they will also revise their expectations of the aggregate real demands in the future, given by  $\hat{Y}_{t+\theta}^d$  ( $\theta \geq 1$ ), in particular  $\hat{Y}_{t+\tau}^d$ , upwards relative to their previous trend levels. According to (45), this rise in  $\hat{Y}_{t+\tau}^d$  above the trend level will increase the aggregate labor demand  $N_t^d$  above the aggregate labor supply  $N_t^s$  minus the natural rate of excess labor supply  $h^*$ . In other words, the basic price-inflation gap will be transmitted into labor markets and create a wage-inflation gap:

$$(47) \quad N_t^d - N_t^s + h^* > 0 .$$

Since we can expect that  ${}_t\hat{N}_t^s$  will remain a good estimate of the aggregate labor supply  $N_t^s$ , which is still following the past trend path, for some time, the left-hand-side of the new form of the aggregate money wage equation (44), given by  $N_t^d - {}_t\hat{N}_t^s + h^*$ , will become positive. Then, in the exactly same way as the cumulative price inflation process, a cumulative wage inflation in excess of long-run equilibrium wage inflation or a wage-wage spiral will be triggered off. The entrepreneurs as a whole raise the money wage rate  $W_t$  above the economy-wide average expectation of it, given by  ${}_t\hat{W}_t$ , by the rate proportional to the gap:  $N_t^d - {}_t\hat{N}_t^s + g^*$ ; which will at the beginning of the next period lead them to revise  ${}_{t+1}\hat{W}_{t+1}$  upwards relative to the past trend level. But this rise in  ${}_{t+1}\hat{W}_{t+1}$  will be offset simultaneously by an equal further rise of  $W_{t+1}$ ; and this process will keep going on. The cumulative wage-inflation process thus triggered off will not stop so long as there exists a wage-inflation gap in the economy.

In this cumulative price-cum-wage inflation process, both  $P_t$  and  $W_t$  will grow at rates higher than the previous long-run equilibrium rates. But note that they may grow at accelerating or decelerating or steady rates depending upon the underlying expectation-formation mechanism and other adjustment forces. An interesting question which immediately arises is--which will grow faster; or put it in another way, how 'real wage rates,'  $W_t - P_t$ , will behave during this cumulative inflation process? To examine this, suppose that in response to the continuous inflow of demand for their products our entrepreneurs as a whole raise  ${}_t\hat{Y}_t^d$  and  ${}_{t+1}\hat{Y}_{t+1}^d$  equally relative to their past trend levels. Now it is easily seen from (43) that a positive gap between  $P_t$  and  ${}_t\hat{P}_t$  which is equal to  $1/\eta$  times the increment of  ${}_t\hat{Y}_t^d$  will be created in product

markets, and from (44) and (45) or directly from (39) that a positive gap between  $W_t$  and  ${}_t\hat{W}_t$  which is equal to  $1/(\eta + \sigma\epsilon)$  times the increment of  ${}_t\hat{Y}_{t+\tau}^d$  will be created in labor markets. Therefore, if the expectation-formation process of  ${}_t\hat{P}_t$  and  ${}_t\hat{W}_t$  are similar, we can expect that  $P_t$  tends to grow faster than  $W_t$ , and that real wage rates will start to decline. However, this decline in real wage rates will eventually reverse itself. There are two forces which will bring about this reversal. Firstly, the existence of a positive wage-inflation gap (47), which is by (31) equivalent to the decline of the economy-wide average rate of excess labor supply  $H_t$  below its natural rate  $h^*$ , will 'presumably' lower the economy-wide average rate of unemployment  $U_t$  below the natural rate  $u^*$ . We say 'presumably' because out of long-run equilibrium  $U_t$  is not necessarily correlated positively with  $H_t$ . Since  $u_t(i) = \max[h_t(i), 0]$  is a non-decreasing but non-linear function, its economy-wide average  $U_t$  depends not only on  $H_t$  but also on how  $h_t(i)$ 's are dispersed cross-sectionally, and this so-called aggregation effect may upset the non-negative correlation between  $u_t(i)$  and  $h_t(i)$ . In any case, however, if  $U_t$  actually declines below  $u^*$ , then it is clear from (32) that the aggregate supply  $Y_{t+\tau}^s$   $\tau$  periods later will decrease relative to the trend level. Then, other things being equal, this increase in  $Y_{t+\tau}^s$  will directly narrow the price-inflation gap and accordingly slow down the speed of the cumulative price-inflation. Unfortunately, other things are not equal. We must take into account the effect of the decline of  $U_t$  on the real aggregate demand  $Y_t^d$  in the current as well as future product markets. Needless to say, the decline of  $U_t$  will raise wage incomes of households, part of which will be sooner or later released in the future products markets as an increase in  $Y_t^d$ . In addition to this,  $Y_t^d$  will also be raised

by a possible increase of profit incomes during the process of cumulative inflation. Moreover, since the existence of a price-inflation gap is equivalent to the rise of the economy-wide average rate of excess product demand  $G_t$  above the natural rate  $g^*$ , the economy-wide average rate of unfilled-orders  $J_t$  will 'presumably' increase above its natural rate  $j^*$ , and part of this frustrated demand left in the spending units' pockets as unspent cash balance will be certainly added to the real aggregate demand in the future. Thus, the so-called multiplier process will be started. However, as is well-known, if the spending sector's marginal propensity to spend out of its aggregate income, properly defined, is less than one, this multiplier process will eventually stop and the net result will be still the curtailment of the price-inflation gap in the economy. In this case, the cumulative rise of prices at higher rates than the long-run equilibrium growth rates will gradually slow down. Secondly, since the aggregate labor demand  $N_t^d$  is a decreasing function of  ${}_t\hat{W}_t - {}_t\hat{P}_{t+\tau}$  and the aggregate labor supply  $N_t^s$  is probably an increasing function of real wage rates, the initial decline in real wages will gradually widen out the wage-inflation gap in labor markets. Then, the aggregate money wage  $W_t$  will start to catch up with  $P_t$ . In consequence, the above two forces will work together to stop the initial drop in real wage rates and at some point begin to raise real wage rates. But this rise of real wages will sooner or later start to widen the price-inflation gap and to narrow the wage-inflation gap, and again reverse its own trend. In this way, real wage rates will keep oscillating possibly with dampened intensities during the cumulative inflation process.

## 6. Stability of Long-Run Equilibrium

The salient feature of our cumulative inflation process is that the cumulative rises of prices and wages in excess of their long-run equilibrium growth rates will never cease so long as the cause which gave rise to it continues to operate; in other words, so long as Say's equation remains disturbed and the price-inflation gap persists in the economy. Therefore, our question whether the economy will eventually return to a long-run equilibrium or not hinges crucially upon whether or not the cumulative inflation process itself has a force to restore Say's equation.

There is some possibility that Say's equation will be restored temporarily by an increase in the aggregate supply  $Y_t^s$  through a decline of the aggregate rate of involuntary unemployment  $U_{t-\tau}$   $\tau$  periods before, if it is not offset by the increase in the real aggregate demand  $Y_t^d$  induced by the accompanied rises in wage and profit incomes as well as unspent incomes, that is, if the underlying income multiplier process is stable. Keynes would have called this state as an 'overemployment equilibrium,' as a counterpart of underemployment equilibrium in depressions. However, in the Wicksellian economy in which entrepreneurs can adjust prices and money wages freely at the beginning of every period, such restoration of Say's equation through the adjustment of employment, output and income will be only transient, though it may prolong for a considerably long period. Adjustment of prices and money wages will eventually take over this initial quantity adjustment and recreate the price-inflation gap as was already discussed in our examination of the movement of real wage rates during the process of cumulative inflation. The Wicksellian economy is fairly a classical and flexible economy, in which price and wage adjustments still play the major role in the economy's adjustment

process out of long-run equilibrium.

As a consequence, our "stability" problem has been reduced to a simple question: will the rise of prices and money wages at higher rates than their long-run equilibrium rates depress the initial upsurge of the real aggregate demand? This is nothing but the money wage problem of Keynes, though his concern was rather whether 'in depressions' fall of prices and money wages would work as an equilibrator of the economy.<sup>16</sup> Since this paper is not the complete treatise of monetary theory, we do not want to be dragged into the muddle of this complicated problem. The only thing we can say is the following. Rising prices and wages will reduce the real purchasing power of monetary stocks and work to discourage the spending sector's expenditures on current products either by raising interest rates in money markets and thus hindering firms' investment expenditures or by directly depressing households' consumption expenditures through real balance effects. This is the stabilizing factor. However, if money supply responds "with positive conformity to the needs of trade," then this stabilizing force may be nullified. On the other hand, if prices and wages keep rising, the spending units in the economy will come to expect that they will continue to rise in the future. This price expectation effect will tend to stimulate their current expenditures at least temporarily by provoking intertemporal substitution of consumptions. Moreover, if price and wage inflation proceeds at accelerating speeds, it may lead to expectations of further and aggravating inflation, and ultimately the public may begin to lose their trust on current monetary institutions and rush to exchange monetary assets for real goods. These

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<sup>16</sup> See Keynes [10], Chapter 19.

expectation effects are clearly destabilizing factors. Therefore, the stability of long-run equilibrium depends upon the relative strength of these stabilizing and destabilizing forces during the process of cumulative inflation.

### 7. The Case of Stagflation

So far we have only discussed the process of cumulative inflation when an upsurge of real aggregate demand relative to the past trend path creates a price-inflation gap in product markets. For, our model of the Wicksellian economy seems more suited to the analysis of inflationary situation than that of depression in which downward rigidity of money wages would play an essential role. In any case, the situation where there is a price-deflation gap in product markets can be analyzed in the symmetrical way in the Wicksellian economy; in most cases, we only have to reverse the direction of changes in variables from those in our analysis of the price-inflation gap situation.<sup>17</sup>

Quite often, the initiating cause of disturbance comes not from product markets but from factor markets. The aggregate labor supply  $N_t^s$  may suddenly rise or fall relative to its past trend level, or the aggregate capital stock  $K_t$  may suddenly increase or decrease relative

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<sup>17</sup>There is one important caveat. In the case of cumulative inflation, there exists an upper limit on the level of aggregate product supply  $Y_t^s$ , which is equal to the level:  $K_{t-\tau} + \gamma N_{t-\tau}^s$ . Hence, the possibility of a temporary restoration of Say's equation via increase in  $Y_t^s$  seems rather weak in this situation. However, in the case of cumulative deflation, there is no such lower limit on  $Y_t^s$ , and, therefore, the possibility of the temporary restoration of Say's equation via decrease in  $Y_t^s$  becomes very strong. This situation is nothing but what Keynes called 'underemployment equilibrium.'



to its secular trend. As an example, let us briefly consider the case where  $K_t$  suddenly decreased in period  $t = 0$  below its secular trend level and resumed its past average growth rate from the next period on. Its impact will first show up in labor markets. Faced with this decline of  $K_t$  our entrepreneurs will try to curtail their demands for labor, so that the aggregate labor demand  $N_t^d$  will decrease and a wage-deflation gap will emerge in labor markets. Sooner or later money wages will start to fall cumulatively relative to their long-run equilibrium growth rates. On the other hand, the decline in  $K_t$  will also affect product markets directly. It will certainly depress the aggregate supply  $Y_{t+\tau}^s$   $\tau$  periods later and create a price-inflation gap in product markets from that period on. Sooner or later product prices will rise cumulatively at higher rates than their long-run equilibrium growth rates.

Therefore, in this example, we will have the wage-deflation gap and the price-inflation gap simultaneously; in other words, the decline of the aggregate capital stock will bring forth the stagflation situation. Whether this example captures the essential features of our current experiences is yet to be resolved.<sup>18</sup>

In the Wicksellian economy in which prices and money wages are flexible, this stagflation situation will give rise to a rapid decline of real wage rates. Then, responding to this, our entrepreneurs will rapidly increase their demands for labor, and at the same time workers will probably reduce their labor supplies to firms. The wage-deflation gap will be sooner or later closed and, instead, a wage-inflation gap

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<sup>18</sup> Since we have amalgamated all the productive factors other than variable labor-services into an index  $K_t$ , a decrease in  $K_t$  might have been brought about by 'oil crisis,' 'food shortage,' and all sorts of disturbances we are now experiencing.

will be created in labor markets. At this point, the situation will become similar to our first example in which an initial upsurge of the real aggregate demand creates a price-inflation gap in product markets and a wage-inflation gap in labor markets and triggers off a cumulative price-cum-wage inflation process. We shall not repeat the discussions given there any further.

#### 8. Comparison with WickSELL, Keynes and Friedman

It should be clear by now that our description of the economy's behaviors out of long-run equilibrium is closely akin to WickSELL's well-known analysis of cumulative process in his "Interest and Prices."<sup>19</sup> His idea is summarized succinctly by him in the following passage:

At any moment and in every economic situation there is a certain level of the average rate of interest which is such that the general level of prices has no tendency to move either upwards or downwards. This we call the normal rate of interest. Its magnitude is determined by the current level of the natural capital rate, and rises and falls with it. If, for any reason whatever, the average rate of interest is set and maintained below this normal level, no matter how small the gap, prices will rise and will go on rising; or if they were already in process of falling, they will fall more slowly and eventually begin to rise.

If, on the other hand, the rate of interest is maintained no matter how little above the current level of the natural rate, prices will fall continuously and without limit.<sup>20</sup>

Therefore, his stability analysis is reduced to the consideration: "whether it is possible for credit institutions to maintain their rates of interest

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<sup>19</sup> WickSELL [20]; in particular, pp. 81-121. Or, see also WickSELL [30], pp. 190-208.

<sup>20</sup> WickSELL [29], p. 120.

at any desired level, or whether they are obliged sooner or later, as a result of the operation on the money market of the forces of supply and demand, to come into line with the natural rate."<sup>21</sup> To this stability question he answers in the following manner:

The only possible explanation [for the eventual restoration of the equality between the market rate of interest on money and the normal rate of interest] lies in the influence which is exerted on prices by the difference between the two rates of interest. When the money rate of interest is relatively too low all prices rise. The demand for money loans is consequently increased, and as a result of a greater need for cash holdings, the supply is diminished. The consequence is that the rate of interest is soon restored to its normal level, so that it again coincides with the natural rate.

At the same time it is clear that in an elastic monetary system, where there is only a small reaction against an alteration in prices, a fairly constant difference between the two rates of interest could be maintained for a long time, and the effect on prices might be considerable.<sup>22</sup>

Keynes was Wicksellian when he was writing "A Treatise on Money," and even after his publication of "General Theory," his analysis of inflation remained Wicksellian.<sup>23</sup> His theory is stated in the "Treatise" as follows:

...the long-period or equilibrium norm of the Purchasing Power of Money is given by the money-rate of efficiency earnings of the Factors of Production; whilst the actual Purchasing power oscillates below or above this equilibrium level according as the cost of current investment is running ahead of, or falling behind, savings.<sup>24</sup>

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<sup>21</sup>Wicksell [29], pp. 107 and 108.

<sup>22</sup>Wicksell [29], pp. 109 and 110.

<sup>23</sup>His analysis of inflation after "General Theory" can be found in Keynes [11].

<sup>24</sup>Keynes [9], p. 137.

His 'stability' analysis in the "Treatise" is very detailed and complicated, but the essential idea seems similar to Wicksell's.

Now it is an easy matter to translate both Wicksell's equilibrium condition that the market rate of interest be equal to the normal rate of interest and Keynes' equilibrium condition that the cost of current investment be equal to savings into our Say's equation that gaps between the aggregate real demand and the aggregate supply are on average equal to the natural rate of excess product demand; although Say's equation is only a necessary condition for our long-run equilibrium. In this sense, we can call our disequilibrium economic dynamics Wicksellian (and pre-General Theory Keynesian) disequilibrium dynamics.

It should be also apparent to the reader that our picture of the economy's out-of-equilibrium behaviors is at variance with that of the natural rate theorists. The most authoritative statement of this school's thoughts can be found in Milton Friedman's presidential address for the American Economic Association.<sup>25</sup> There, he maintains, after having described the process in which an increase in the rate of monetary growth leads to rises of income, output and employment:

But it describes only the initial effects. Because selling prices of products typically respond to an unanticipated rise in nominal demand faster than prices of factors of production, real wages received have gone down--though real wages anticipated by employees went up, since employees implicitly evaluated the wages offered at the earlier price level. Indeed, the simultaneous fall ex post in real wages to employers and rise ex ante in real wages to employees is what enabled employment to increase. But the decline ex post in real wages will soon come to affect anticipations. Employees will start to

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<sup>25</sup>Friedman [5].

reckon on rising prices of the things they buy and to demand higher nominal wages for the future. "Market" unemployment is below the "natural" level. There is an excess demand for labor so real wages will tend to rise towards their initial level.<sup>26</sup>

In this analysis, the disequilibrium manifests itself only in the discrepancy between ex post real wage rates determined by employers and ex ante real wage rates anticipated by employees, and the 'stabilizing' force lies solely in the employees' error-learning process about inflation. Consequently, the process of adjustment towards a new long-run equilibrium is completely confined to the adjustment between ex ante and ex post real wage rates in labor markets, and is totally independent of the effect of cumulatively rising prices on the economy's real aggregate demand and on other aggregate variables. It seems that Friedman implicitly assumes Say's law in his analysis of the economy's out-of-equilibrium behavior. In contrast with his theory, the fundamental observation in our 'stability' analysis given in the previous section, which conforms to that of Wicksell and Keynes, is that the adjustment towards a new long-run equilibrium involves not only the adjustment of real wage rates in labor markets but also the whole monetary variables in the economy. Indeed, the adjustment of real wage rates brought by the employees' error-learning about inflation in labor markets plays rather a minor role, and the stability of the long-run equilibrium hinges crucially on whether or not cumulative rise or fall of prices and wages in excess of their long-run equilibrium growth rates, provoked by the disturbance of Say's equation, will cool off or stimulate the real aggregate demand and eventually restore Say's equation itself. To assume Say's equation from the start is like throwing the

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<sup>26</sup> Friedman [5], p. 10.

baby out with the bathwater.

#### 9. Concluding Remarks to Part I and Part II

We have defined long-run equilibrium as a state of the economy in which Say's equation holds and every entrepreneur has rational expectations. In this long-run equilibrium, average rate of unemployment is pegged at the natural rate of unemployment, whose level is totally independent of the time pattern of the rate of inflation.

A state of long-run equilibrium can be upset in two ways. First, entrepreneurs' expectations may go astray even if Say's equation remains unperturbed. If this is the source of disequilibrium, the adjustment process towards long-run equilibrium is simply the entrepreneurs' error-learning process motivated by the discrepancies between actual and anticipated magnitudes of relevant variables. Sooner or later this error-learning process will bring the economy back to the long-run equilibrium, albeit it may overshoot the target and lead to some oscillations. In this situation we can rely fairly confidently on the economy's automatic tendency towards long-run equilibrium. The disequilibrium situations examined by the natural rate theory of unemployment and by the dynamic Walrasian general equilibrium theory have been confined to this rather well-behaved case. In consequence, the policy prescription given by them is typically that the monetary authority causes money supply to grow at some steady rate which is expected to bring forth the desired rate of inflation; it should not 'lean against the wind.'<sup>27</sup> Since the source

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<sup>27</sup> See, Friedman [5].

of disequilibrium is found only in the economic agents' inability of distinguishing real from monetary disturbances, the more predictable monetary policy is the less probable will the economy be thrown out of equilibrium.<sup>28</sup>

However, the more fundamental cause of disequilibrium lies in the disturbance of Say's equation. If a price-inflation gap is initially created in the economy, the average rate of excess product demand exceeds its natural rate and the average rate of unfilled-orders will presumably increase above its natural rate. Prices will start rising cumulatively at higher rates than their long-run equilibrium rates. This price-inflation gap will be soon transmitted to labor markets, and a wage-inflation gap will emerge in them. The average rate of excess labor supply will become lower than its natural rate, and the average rate of involuntary unemployment will also go down below its natural rate. Money wages will start rising cumulatively at higher rates than long-run equilibrium rates. If, on the other hand, a price-deflation gap is initially created in the economy, prices and money wages will start growing at slower rates than long-run equilibrium rates and eventually begin to decline cumulatively. We can also conceive a stagflation situation, where a price-inflation gap and a wage-inflation gap are simultaneously created at the initial stage, and various other situations. The main motive force behind these cumulative processes is again the entrepreneurs' error-learning process in response to the discrepancies between actual and anticipated magnitudes of relevant economic variables, but the crucial difference from the first kind of disequilibrium is that the entrepreneurs' expect-

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<sup>28</sup>This argument is advanced by Lucas [14] and Sargent-Wallace [27].

tations can never be fulfilled simultaneously so long as the fundamental cause of disequilibrium--the disturbance of Say's equation--continues to operate. Therefore, whether the economy will automatically tend towards long-run equilibrium or not depends upon whether the cumulative inflation or deflation itself can restore Say's equation or not. The long-run equilibrium may not be stable, and even if it is stable the process of adjustment towards it, which usually provokes the multiplier process, will be very painful. Therefore, since Say's equation is in constant danger of being upset by unforeseen changes of economic and non-economic environment, the policy prescription for such a volatile economy would be that the authority should try to maintain Say's equation by effectively employing fiscal and monetary policies in order to counteract major shocks which endanger it. This 'lean-against-the-wind' policy is nothing but the standard Keynesian policy prescription.

It should be noted that our preceding analysis of the economy's out-of-equilibrium behaviors has also invalidated the quantity theory of money and its peculiar modern version, 'monetarism,' which claim that the quantity of money (appropriately defined, of course) is the chief regulating factor of the rate of inflation in the long-run and also to a lesser extent in the short-run.<sup>29</sup> It is true that the long-run equilibrium rate of inflation is determined as the rate of inflation which keeps the balance between the real aggregate demand and the aggregate supply for given time patterns of monetary and fiscal policy variables as well as other exogeneous variables such as the growth rate of labor force and the rate of capital accumulation. In the long-run equilibrium, therefore, the quantity of

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<sup>29</sup>See, for example, Friedman [4] for a clear statement of this proposition.



money is certainly one of the important determinants of the rate of inflation. However, once Say's equation is disturbed and the economy is thrown out of long-run equilibrium, then Wicksellian cumulative process starts and the rate of inflation deviates from the long-run equilibrium rate. Thus, the aggregate demand gap (i.e., price-inflation or price-deflation gap) emerges as the main determinant of the motion of the rate of inflation out of long-run equilibrium, and the quantity of money can no longer be regarded as its chief regulating factor. The latter can only affect the rate of inflation either through its indirect influence on the former or through its direct influence on the entrepreneurs' expectations about the rate of inflation. Just as the classical theory of unemployment and the classical quantity theory of money stood and fell together, the natural rate theory of unemployment and the monetarism--their modern revivals--stand and fall together.

If the economy's automatic tendency towards long-run equilibrium were very strong, or if the counter-cyclical policy of the fiscal and monetary authority were very successful, the economy would fluctuate around long-run equilibrium without deviating too far from it. If this were to be the case, then in the Wicksellian economy the motions of the actual economy would become very close to those of the economy described by the natural rate theory of unemployment; in particular, the average rate of unemployment would be approximately equal to the natural rate of unemployment, whose level can be affected by neither monetary policies nor fiscal policies adopted by the government. The task of Part III and Part IV of this series is to destroy this last neo-classical element in our disequilibrium economic dynamics. There would come out a complete Keynesian picture of the economy.

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