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Social Choice on Pollution Management: The Genossenschaften

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1. Introduction: The Genossenschaften

The provision of a public good or a public service raises several important questions. How much of the good or service should be produced? Who should bear the costs of its provision? And, more fundamentally, what mechanisms should be used to decide these issues? These questions become particularly important and acute when the good or service is used to "repair" a public resource in a situation where both the damage done to the resource and the benefits reaped from it vary across different segments of the public.

A specific case of such a service is water quality management and water pollution control. Consider, for example, a region composed of firms and households located on a river. Different firms will discharge varying amounts of pollutants into the river, and these amounts will, in turn, differ from the various levels of waste discharge of the households. At the same time, the cost of bringing the water to a quality standard

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appropriate for industrial use is far lower than the cost of making it suitable for domestic or recreational use by households. Moreover, the benefits of pollution control accruing to firms and households located upstream are significantly smaller than those accruing to their downstream neighbors.

One rather unique arrangement for coping with problems of water resource management is the *Genossenschaften* of the Ruhr industrial area of West Germany. Each of these river associations is a basin-wide agency responsible for water quality management in its region. While the several *Genossenschaften* (the *Ruhrverband*, the *Emschergenossenschaft*, the *Lippeverband*, the *Wupperverband*, the *Niersverband*, and the *Erftverband*) differ from each other with respect to institutional details, they have a common basic decision-making structure that resembles a hybrid of a cooperative and a corporate shareholders meeting. Abstracting considerably from the wealth of detailed structural features and the multiplicity of functions of the river associations, the "typical" *Genossenschaft* has the authority to set water quality standards, to raise revenue from its members by exacting (with the force of public law) contributions which resemble effluent charges, and to use the revenue so raised for maintaining and improving water quality, primarily via treatment facilities.

The members of the typical *Genossenschaft* are primarily the industrial enterprises, the coal mines, the cities and communities, and the waterworks associations in the river association's area. There is a plenary representative body--a General Assembly of Members--and a day-to-day decisionmaking body--a Board of Directors--which is elected by the General Assembly and which, in turn, has a technical staff to help it manage the association's daily affairs. Ultimate authority within the Genossenschaft thus rests with the General Assembly composed of representatives of the firms, the cities and communities, and the waterworks. A member's voting strength in the Assembly is approximately proportional to his financial contribution to the association, and decisions are reached on the basis of an absolute majority. The "contribution" required of a member is, in effect, a tax on waste discharge, and hence depends primarily upon the quantity and the quality of the member's effluent. Closing the circle, then, the largest polluters have the most votes in determining the water quality standard to be enforced in the association's river.

The specific criteria for determining who will be compelled to be a member of the association, and the details of the procedure for allocating votes in the assembly differ from one association to another. The precise relationship between the assembly and the board of directors--specifically, how members of the latter will be elected by the former--also varies across associations, as does the exact way in which members' contributions are computed. Nevertheless, the common characteristics of these agencies overshadow their differences, and this core of common features forms the basis for this description of a "typical" Genossenschaft and the focus of our paper.[1]

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[1] For a thorough description of the Genossenschaften and the intricate details of their politics and daily decisionmaking, the reader is referred to the forthcoming monograph by William A. Irwin, Charges on Effluents in the United States and Europe. Mr. Irwin's work was supported by a grant from the Council on Law-Related Studies, Cambridge, Massachusetts. A briefer discussion of the Genossenschaften, with particular emphasis on the different types of effluent charge systems used by the associations, appears in Allen V. Kneese and Blair T. Bower, Managing Water Quality: Economics, Technology, Institutions (published for Resources for the Future by) Johns Hopkins Press, Baltimore, 1968, Chapters 12 and 13.

The purpose of the present paper is to investigate the consequences of the typical Genossenschaften representation scheme by means of some simple mathematical voting models. Consider, for a moment, one's intuition about a system in which the standard established by a law is the result of a vote in a legislature where members receive ballots in proportion to their violation of the law. Would an equilibrium exist in such a system? If a standard did exist from which there were no tendency to move, would it be binding on people's actions? One can imagine a scenario in which, given an arbitrary standard, the heaviest violators gain control of the legislature and lower the standard drastically, perhaps even abolish it. But now, with the new legal standard, their degree of violation is no longer so great relative to those who were more law abiding under the original, tougher law, so the latter group now has more power in the assembly and raises the standard of conduct, and we start through the process once again.[1] The scenario appears to bode ill for the existence of an equilibrium.

Nevertheless, on the whole, the Genossenschaften seem to have functioned well, though problems have arisen in passing the budget in the assembly of one of the Genossenschaften--the Ruhrverband--because of the voting power of upstream interests in that body.[2] This paper tries to present some insights into the associations' favorable experience by deriving conditions for the existence of an equilibrium under such a representation scheme.

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[1] Of course, this description imputes a pseudodynamics to the process of finding an equilibrium, and this is irrelevant to the issue of existence of such an equilibrium. But the description does help to focus on problems one might anticipate in resolving the question of existence.

[2] Irwin, op.cit., pp. 48-52.

Our results also bear on the problems encountered in the Ruhrverband, as they help to explain how the representation system generates such difficulties. The results also help to explain some aspects of the representation-allocation methods adopted by several of the Genossenschaften--for example, why one of the river associations gives downstream interests a bloc of 75 votes out of a total of 300 votes before distributing the remainder on the basis of members' contributions,[1] and why another limits the mining companies to a total of 40 percent of all votes in the Assembly.[2] But this is getting ahead of the story.

In Section 2, we present an abstract model of a Genossenschaft, ignoring many of the complexities of the real river associations. The following section presents sufficient conditions for the existence of a global equilibrium for the model under specific assumptions about the technologies of firms and pollution control and the job opportunities of households. Section 4 relaxes these assumptions to allow for other kinds of technologies and to take into account some general-equilibrium aspects of the model, and it provides sufficient conditions for the existence of a local equilibrium. The implications these theorems have for the organization of the Genossenschaften are explored in Section 5. This section also compares the equilibria the Genossenschaften representation scheme yields with those produced by alternative voting procedures, and it considers the effect of technological change in pollution control on the Genossenschaften equilibria.

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[1] Irwin, op.cit., p. 54.

[2] Section 10 Paragraph 8 of the Act establishing the Lippeverband, 19 January 1926, as cited in a document provided by W.A. Irwin, which describes the representation system of the Lippeverband.

2. A Stylized Genossenschaft

The genossenschaft, or water board, is responsible for maintaining the quality of water in a given region or management area. The board has authority to impose and collect effluent charges on the m firms and n households located in the region, and to use the revenue so raised to construct and operate pollution treatment facilities. Waste emitted by firms and households is treated in the facility, but if aggregate waste emissions exceed the capacity of the facility, the excess is discharged directly into the water, causing a reduction in water quality. The board must determine what quality standard is to prevail in the area, or equivalently, how large a treatment facility must be constructed so that untreated "excess" discharges will be reduced to an acceptable level. Effluent charges which will yield revenue sufficient to construct a facility of the agreed-upon size are determined and imposed upon households and firms alike. The board ultimately responsible for these decisions consists of representatives of households--who are interested in water quality for domestic and recreational uses, but who also must bear a share of the cost of maintaining the quality standard--and of firms, who are primarily concerned with the cost implications of water quality decisions. This board operates by weighted majority vote, in which each member's voting strength is proportional to its financial contribution.

An important assumption underlying our analysis is that water quality can be measured and characterized by a single variable. Thus, though the concentration and physical composition of different firms' waste emissions may differ considerably, each firm's discharge can be characterized in terms of a "standard pollution units" equivalent, and this equivalent is used to determine the effluent charges imposed on the firm.[1] The water quality prevailing in the area can be similarly characterized by a single measure or standard, and this standard varies monotonically with the total untreated discharge, measured in standard pollution units per unit of time.[2] The period

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[1]The various Genossenschaften do, in fact, seem to use something approximating this approach for computing effluent charges, though the standard of measurement varies from one Genossenschaft to another. See Kneese and Bower, op.cit., pp. 244-251.

[2]Thus, in particular, we ignore variations in the rate of discharge or in water flow within the time period, as well as upstream-downstream complications.

One interpretation of this is that the water resource is a lake (or perhaps a tidal estuary), with no predominant downstream current, but with sufficient circulation to ensure relatively even dispersal of pollutants throughout the lake. Water quality in this case depends on the total stock of pollutants remaining in the lake, which increases with new discharges and decreases over time according to the natural recovery capacity of the lake (or flushing rate of the estuary). If the recovery rate is ρ (so that in the absence of new discharges, the stock of pollutants decreases during each period to $1-\rho$ of the level at the beginning of the period) then in the steady-state situation where new emissions are at a constant rate of S units per period, the stock approaches $\frac{1-\rho}{\rho} S$ as the number of periods becomes large.

Alternatively, we can think of the resource as a river, with the major polluters located upstream in relation to the places where water is used for domestic and recreational purposes. In this case, water quality as measured by the concentration of pollutants in the water used for consumption purposes will vary directly with the rate of upstream discharge.

More generally, when firms and consumption locations are interspersed along the river, we can think of water quality as the average concentration of pollutants in the river, averaged over measurements taken at a number of representative locations along the river. This quality standard need not vary monotonically with the aggregate volume of untreated discharges (per period), since upstream and downstream discharges will affect the averaged standard differently. Hence, the interpretation in this case is a bit different. At any effluent tax rate τ the board (or its technical staff) can determine what use of the revenue produced at that rate will achieve the highest possible water quality, or lowest possible (average) concentration of pollutants. We could define $S(\tau)$ as this best-achievable standard, and recast the analysis that follows in terms of the $S(\tau)$ functions themselves, without attempting to interpret these functions in terms of aggregate discharges. For expositional convenience, however, we shall retain the "aggregate discharge" interpretation in the body of the paper.

will be taken as fixed, say at one year, throughout. The quantity S is defined as the total untreated discharge during this period, and for simplicity, we shall refer to S as the "quality standard" of the water (though it is more accurately an inverse standard).

To give more structure to this picture, we now turn to a more detailed description of the principal agents involved.

Firms

Each firm is assumed to be a price taker in both the input and output markets. For simplicity, assume the i^{th} firm uses a single factor, labor, which it hires at a fixed wage w per unit (the existence of other factors of production is immaterial for our purposes), and it sells its output q_i at a fixed price p_i per unit. The prices received by the firms may differ because different firms are producing different outputs. In producing its output, the i^{th} firm faces its own generalized production relationship F^i between its output level q_i, the labor it hires L_i, and the waste or garbage G_i it emits to the water as a byproduct:[1]

$$(1)\quad \begin{cases} q_i = F^i(L_i, G_i) \text{ with } F^i_1 = \dfrac{\partial F^i}{\partial L_i} > 0 , \quad F^i_2 = \dfrac{\partial F^i}{\partial G_i} > 0 , \quad F^i(0, G_i) = 0 \\ \qquad\qquad\qquad\qquad \text{for all } G_i . \\ F^i(L_i, G_i) \text{ strictly concave, } i = 1, \ldots, m . \end{cases}$$

Thus each firm's production process shows decreasing returns to scale. In addition, as indicated by (1), a ceteris paribus increase in the labor employed increases output and a positive output requires a positive labor input. Finally, holding the number of workers fixed, and positive, the firm can pro-

[1]The firm's production relationship and all other primary functions introduced will be assumed to be twice continuously differentiable, unless indicated otherwise.

duce more if it pollutes more (that is, increases G_i) or conversely its output falls if it is obliged to emit less waste. One interpretation of this is that the firm hires labor to produce its product, with waste materials being generated as a byproduct. The firm can either emit this garbage into the water or it can pretreat its effluent. But pretreatment requires labor input. Hence, a reduction in G_i , holding L_i fixed, means that some workers previously employed to produce the firm's commodity must be allocated to clean-up operations instead. As a result, q_i must fall.

Each firm is assumed to be a profit-maximizer. Hence, if the effluent charge set by the genossenschaft is τ per unit of waste discharged into the water, the i^{th} firm's problem is to choose L_i and G_i to

(2) $$\underset{L_i,\ G_i}{\text{Maximize}} \quad \pi_i = p_i F^i(L_i, G_i) - wL_i - \tau G_i \ .$$

More specific assumptions about the firm's production relationship will be introduced when they are needed. For the present, two features of the firm's problem (2) should be emphasized. First, the firm does not care about the water quality standard as such. The firm's only motivation for reducing its own waste emissions is that it is charged a price or tax τ for each unit of waste it releases. The firm is only concerned with the overall standard of water quality insofar as the latter influences the emission tax it must pay.[1] Second, while the exact substitution possibilities between labor and

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[1]This assumption is not terribly unrealistic. In most cases, the resources required to bring water up to the quality needed for industrial purposes are minimal. Kneese and Bower, op.cit., pp. 36-38.

pollution have not been delineated yet, note that each firm can control the amount of waste it discharges. It can change G_i directly by altering its output level q_i while holding its labor input L_i fixed or by changing its labor input with a constant production level or by varying both L_i and q_i.

We shall assume that at every feasible tax rate τ (and the set of feasible τ will be specified presently), each firm has a positive waste emission. From this and the other assumptions embodied in (1) and (2), two conclusions can be drawn about firm behavior. First, each firm prefers lower to higher effluent charges, since under our assumptions its profit declines as τ increases. Second, each firm's demand function for waste disposal (supply function of waste discharge) is downward-sloping; that is, for all i

$$\frac{\partial G_i}{\partial \tau} < 0 .$$

Thus, as the emission tax increases, each profit-maximizing firm will pollute less.

<u>Households</u>

Households, in contrast to firms, do care about the water quality standard itself--not just about the cost of the effluent charges they must pay to maintain the standard. Moreover, also in contrast to firms, they cannot regulate their waste discharges because households do not have available to them the substitution possibilities firms command. The utility level of the j^{th} household, U^j, is assumed to be a function of its consumption level C_j and a measure of the prevailing water quality, S, defined as the amount of untreated waste discharged during the year when the treatment plants are operating. Thus the j^{th} household has an ordinal utility function of the form:

$$(3)\begin{cases} U^j = U^j(C_j, S) \quad \text{with} \quad U^j_1 = \dfrac{\partial U^j}{\partial C_j} > 0 \ ; \ U^j_2 = \dfrac{\partial U^j}{\partial S} < 0 \ ; \ S \geqq 0 \ , \ C_j \geqq 0 \ ; \\ U^j \quad \text{quasi-concave,} \quad j = 1, 2, \ldots, n \ . \end{cases}$$

Each household's welfare rises with ceteris paribus increases in its consumption level and with ceteris paribus increases in water quality, which correspond to decreases in S. The utility function $U^j(C_j, S)$ is assumed to be quasi-concave, which implies a diminishing marginal rate of substitution between consumption and (positively measured) water quality, and this is true for all j.

If members of a particular household j are employed by firms in the region, the household's labor income, and hence its gross income, will depend on the production functions $F^i(\cdot)$, the output prices p_i, the wage rate w, and the effluent charge τ (as well as the prices of any other inputs). The production functions are fixed, however, and we also assume that all input and output prices, including the wage rate, are determined in a much larger setting than the region and that activity in the region is too small to have any perceptible effect on them. Hence, the household's gross income $Y_j(\tau)$ depends only on the effluent charge τ.

The household's gross income $Y_j(\tau)$ can be allocated to the several consumption goods available only after the household has paid its effluent charge. It is assumed that household j discharges an amount of garbage Γ_j into the water, and the household cannot alter this amount. Hence, the j^{th} household's consumption level--its gross income minus the levy for its waste outflow--is:

$$C_j(\tau) = Y_j(\tau) - \tau\Gamma_j \ . \qquad (4)$$

Each household would like to reach the highest utility level possible. The j^{th} household's objective is to maximize (3), subject to the constraint (4). But its opportunities for doing so are limited in the current framework. Given the assumptions about how Y_j and Γ_j are determined, the only way the household can increase its welfare is through whatever influence it has on τ and S. This brings us to the remaining agent on the scene, the water board--the Genossenschaft of the present model.

The Water Board

The effluent charges paid by households and firms determine the total amount of pollution control the board can undertake. The treatment operations undertaken by the board are funded entirely by the effluent charges described. Hence, with a tax rate τ, the board's revenue R and thus the maximum amount that can be spent on treatment is

$$R(\tau) = \tau \sum_{i=1}^{m} G_i(\tau) + \tau \sum_{j=1}^{n} \Gamma_j \ . \qquad (5)$$

The functional notation $G_i(\tau)$ is used to indicate that the optimal solution to the i^{th} firm's problem in (2) depends on τ.[1] At the same time, the total waste discharged into the water when the effluent tax is τ, denoted $J(\tau)$, is

(6) $$J(\tau) = \sum_{i=1}^{m} G_i(\tau) + \sum_{j=1}^{n} \Gamma_j \ .$$

The genossenschaft uses the revenue $R(\tau)$ to reduce the pollution level of the water. In doing so, the board faces a treatment technology function ϕ that relates the amount of waste removed, K, to the amount of money expended, E:

(7) $$K = \phi(E) \ , \quad \phi'(E) > 0 \ , \quad \phi(0) = 0 \ .$$

The function in (7) is simply the inverse of a cost function and the assumption about its first derivative means that an extra dollar spent on pollution control has a beneficial effect--it cleans up more garbage. The fact that $\phi(0)$ equals zero means there is no clean-up unless positive expenditures are made. For the time being, no further assumptions are made concerning the clean-up technology. The theorem in Section 3 will employ a further assumption about the nature of returns to scale in this technology, but Section 4 will relax this requirement.

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[1]Since p_i and w are assumed to be fixed throughout the paper, we suppress the fact that the optimal G_i and L_i values for (2) also depend on these parameters. That is, we should properly write $G_i(\tau, p_i, w)$ but with p_i and w fixed throughout, $G_i(\tau)$ suffices.

The quality standard of the water, S, is defined as the excess of aggregate waste emissions over treatment capacity. Since the board must operate subject to the constraint $E \leqq R$, that is, it cannot spend more than the revenue it raises, the best standard (the lowest S) it can achieve depends on the effluent tax τ. Denoting the best standard achievable with tax rate τ by $S(\tau)$, we have:

$$(8) \quad S(\tau) = \begin{cases} J(\tau) - \phi(R(\tau)) = J(\tau) - \phi(\tau J(\tau)) & \text{if } J(\tau) > \phi(\tau J(\tau)), \\ 0 & \text{otherwise.} \end{cases}$$

The basic decision variable for the board is the effluent charge τ. By assumption, it is nonnegative and bounded from above. If there is a rate $\tilde{\tau}$ at which total clean-up occurs, that is, $\phi(\tilde{\tau}J(\tilde{\tau})) = J(\tilde{\tau})$, the lowest such rate is a suitable bound, since there is no reason for the board to ever consider higher rates.[1] Even if total clean-up is not achieveable, there is some maximum feasible tax rate, such as the rate at which some significant number of firms are driven out of business, or some significant number of households are reduced to a below-subsistence consumption level. The set of feasible effluent tax rates thus constitutes a closed interval: $[0, \tau_m]$. Clearly not every standard of water quality will be achievable. A given quality standard $\bar{S}$ is feasible if there is some feasible effluent tax rate $\bar{\tau}$ that will yield revenue sufficient to clean up enough of the

[1] Note that if there are constant returns to scale in pollution control so that $\phi''(E) = 0$, then such a $\tilde{\tau}$ exists. Specifically, if $\phi(E) = \alpha E$, then $\tilde{\tau} = \frac{1}{\alpha}$, because $\phi\left(\frac{1}{\alpha} J\left(\frac{1}{\alpha}\right)\right) = \alpha\left[\frac{1}{\alpha}J\left(\frac{1}{\alpha}\right)\right] = J\left(\frac{1}{\alpha}\right)$.

waste discharged when $\bar{\tau}$ prevails to meet the standard $\bar{S}$; that is, if there exists a $\bar{\tau} \epsilon [0, \tau_m]$ such that $S(\bar{\tau}) = \bar{S}$.

The board is responsible for setting the effluent charge τ and hence the implied quality standard $S(\tau)$. It is composed of representatives of the m firms and n households in the region. Each unit, whether firm or household, receives a proportion of the total vote equal to its share of all contributions to the association. More precisely, with an effluent tax of τ per unit of waste, the j^{th} household receives $\frac{\Gamma_j}{\sum_{i=1}^{m} G_i(\tau) + \sum_{j=1}^{n} \Gamma_j}$ of the votes, while the i^{th} firm receives the fraction $\frac{G_i(\tau)}{\sum_{i=1}^{m} G_i(\tau) + \sum_{j=1}^{n} \Gamma_j}$ of the total number of votes. Note carefully that a change in the effluent tax τ affects a firm's voting strength in two ways. It changes the amount of waste the particular firm discharges, as shown by the $G_i(\tau)$ function, and it changes the total waste emitted into the water, $\sum_{i=1}^{m} G_i(\tau) + \sum_{j=1}^{n} \Gamma_j$. Since each household's waste emission is fixed, a change in τ affects a household's voting strength only via the second route, namely, by altering the total waste discharged.

The effluent charge τ established by the board is determined by a weighted majority vote in which the number of votes allocated to each firm and each household is determined in the manner described above. For a given distribution of voting strengths, a particular tax rate τ^o [and its implied quality standard $S(\tau^o)$] is a voting equilibrium if there is no other rate τ' which is preferred to τ^o by a weighted majority (weighted according to the given distribution of votes) of representatives. An equilibrium is thus invulnerable to proposals to change, while conversely, a tax rate which does not constitute a voting equilibrium is vulnerable, in the sense that some proposed change from that rate will command weighted majority support on the board. Which rates are voting equilibria in this sense thus depends, in part, upon the current distribution of votes. In particular, if the current rate is τ' , and τ' itself is not an equilibrium with respect to the distribution of votes implied by τ' , then the current rate, and hence the distribution of votes itself, will be unstable. We are interested in whether there exists some tax rate which is not unstable in this sense. Such a value will be referred to as a "superequilibrium," since it is both a voting equilibrium with respect to the current distribution of votes, and the vote distribution it implies is identical to the current distribution.

The next section establishes conditions for the existence of such a superequilibrium and explores some of its properties. Certain conditions on which this argument rests, such as the assumption that the technology of pollution treatment displays non-increasing returns to scale, are somewhat restrictive. When these assumptions are relaxed, the existence of a "superequilibrium" is no longer ensured. We shall show in Section 4, however, that under quite general conditions there still exists a "local" type of superequilibrium, that is, a superequilibrium which is stable against proposals for "small" changes in the current rate. To avoid ambiguity, we shall employ the following definitions throughout: a tax rate τ^* is a global superequilibrium if there is no other rate τ' preferred to τ^* by a weighted majority vote when voting strengths are determined by τ^* . In contrast, a tax rate τ^+

will be said to be a <u>local superequilibrium</u> if there exists some nondegenerate neighborhood $(\tau^+ - \delta, \tau^+ + \delta)$ of τ^+ which contains no rate τ'' preferred by a weighted majority vote to τ^+ when the distribution of votes is determined by τ^+.

3. The Existence of a Global Superequilibrium

The production functions of the firms in the region were specified quite generally in (1): decreasing returns to scale, a positive labor requirement, and positive marginal products for labor and waste discharges. From this general specification and the firms' price-taking behavior, it follows that each firm's demand function for waste disposal (supply function of waste discharge) is downward sloping:

$$(9') \qquad G_i'(\tau) < 0 \quad \text{for each} \quad i = 1, \ldots, m .$$

One further assumption will be made for the purpose of Section 3. It will be assumed that the firm's supply curve of waste discharge is convex as well as downward sloping so that for all i,

$$(10') \qquad G_i''(\tau) \geqq 0 .$$

While this goes beyond the restrictions customarily imposed on factor demand curves, some hint of plausibility (or lack of implausibility) may come from the fact that if $F^i(L_i, G_i)$ took the modified Cobb-Douglas form $G_i^a L_i^b$ with $a + b < 1$, $G_i(\tau)$ would be convex. Since $J(\tau) = \sum_{i=1}^{m} G_i(\tau) + \sum_{j=1}^{n} \Gamma_j$ and Γ_j is fixed for all j, (9') and (10') imply (9) and (10), respectively:

(9) $$J'(\tau) < 0$$

(10) $$J''(\tau) \gtreqqless 0 \ .$$

For the purposes of this section, we make a rather strong assumption about the anti-pollution technology. Namely, the function $\phi(E)$ is assumed to have either constant or decreasing returns to scale:

(11) $$\phi''(E) \leqq 0 \ .$$

Though this assumption is restrictive, two points should be made about it. First, the public-good nature of pollution control is logically separable from the increasing-returns-to-scale aspect of the problem, and in the present section we wish to focus on the former. Second, while impressive economies of scale are sometimes achievable, this is not always the case.[1] Hence, in at least some instances, the nonincreasing-returns-to-scale assumption may be appropriate.

Finally, we assume that the households' gross incomes are fixed, and are large enough so that every household has positive consumption at every feasible tax rate. No matter how the firms respond to changes in the effluent tax, the gross income of the j^{th} household is assumed to be constant at $\bar{Y}_j > \tau_m \Gamma_j$. For example, if an increase in τ leads firms to lay-off or fire workers in the region, these workers can find jobs elsewhere so that each household has the same gross income as before the increase in τ . With this assumption, the j^{th} household's consumption is:

(4') $$C_j = \bar{Y}_j - \tau \Gamma_j \ .$$

[1] See, for example, Irwin, op.cit., p. 46.

Taking all our assumptions together, it follows that over the relevant range $S(\tau)$ is a nonnegative strictly decreasing, strictly convex function. That is, for all $\tau \in [0, \tau_m]$,

(12) $$S(\tau) \geqq 0 , \quad S'(\tau) < 0 , \quad S''(\tau) > 0 .$$

The first statement, $S(\tau) \geqq 0$, follows from the definition in (8). Note next that $S(\tau) \geqq 0$ is equivalent to $J(\tau) \geqq \phi(\tau J(\tau))$. But nonincreasing returns to scale in pollution control, $\phi''(E) \leqq 0$, implies $\phi(R) \geqq R\phi'(R)$ for all $R \geqq 0$, or $\phi(\tau J(\tau)) \geqq \tau J(\tau)\phi'(\tau J(\tau))$. Combining these two inequalities yields $J(\tau) \geqq \tau J(\tau)\phi'(\tau J(\tau))$ or[1]

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[1]The inequality in (13) shows that unless the social anti-pollution technology is marked by constant returns to scale, the <u>Genossenschaften</u> method is a socially inefficient approach to pollution control. The difficulty essentially rests with the fact that the <u>Genossenschaften</u> use an average-cost pricing mechanism to allocate the cost of waste removal. The firms carry their own treatment activities to the point where $\tau = p_i \frac{\partial F^i}{\partial G_i}$, taking τ as the private cost of treatment. The social cost of clean-up activities is, however, $\frac{1}{\phi'(\tau J(\tau))}$. The statement in (13) shows that if decreasing returns to scale characterize the treatment technology, then $\frac{1}{\phi'(\tau J(\tau))} > \tau$ so that the social cost of pollution control exceeds the private cost of treatment and inefficiency results. Similarly, if the social pollution treatment technology is marked by increasing returns to scale, the direction of the inequality is reversed: the social cost of pollution control is less than the private cost of such activity and inefficiency results again. Only if there are constant returns to scale in social pollution control will the private cost equal the social cost: $\tau = \frac{1}{\phi'(\tau J(\tau))}$.

$$(13) \qquad 1 \geqq \tau\emptyset'(\tau J(\tau)) .$$

Differentiating $S(\tau)$, we have

$$(14) \quad S'(\tau) = J'(\tau) - \emptyset'(\circ)[J(\tau) + \tau J'(\tau)] = \overset{(+)}{[1 - \tau\emptyset'(\circ)]}\overset{(-)}{J'(\tau)} - \overset{(+)}{\emptyset'(\circ)}\overset{(+)}{J(\tau)} ,$$

where $(\circ) = (\tau J(\tau))$. But then, using (7), (9), and (13), we have $S'(\tau) < 0$ as asserted in the second part of (12). It follows that $S(\tau)$ does not have an interior relative minimum for $\tau \geqq 0$. The best achievable water quality simply increases, that is, $S(\tau)$ decreases, with every increase in τ. Differentiating $S'(\tau)$ in (14), one obtains

$$(15) \quad S''(\tau) = \overset{(+)}{[1 - \tau\emptyset'(\circ)]}\overset{(+)}{J''(\tau)} - 2\overset{(+)}{\emptyset'(\circ)}\overset{(-)}{J'(\tau)} - \overset{(-)}{\emptyset''(\circ)}\overset{(+)}{[J(\tau) + \tau J'(\tau)]^2} .$$

Using (7), (9), (10), (11), and (13), it follows that $S''(\tau) > 0$ and $S(\tau)$ is a strictly convex function as stated in the last part of (12).

The strict monotonicity and strict convexity of the $S(\tau)$ function have strong implications for the voting preferences and behavior of each of the n households. Consider the j^{th} household. From the assumption in (4') that $C_j(\tau) = \bar{Y}_j - \tau\Gamma_j$, with $\bar{Y}_j$ and Γ_j fixed, and $S'(\tau) < 0$, it follows that there is a one-to-one relationship between (S, C_j) pairs and τ values. The strict convexity of $S(\tau)$ and the linear relationship between C_j and τ given by (4') also imply that the set of (S, C_j) pairs available to the j^{th} consumer is strictly convex. This opportunity set, denoted A_j, is defined as $A_j = \{(S, C_j) | \exists\, \tau \in [0, \tau_m]$ for which $S \geqq S(\tau)$ and $C_j \leqq \bar{Y}_j - \tau\Gamma_j\}$.

To see that A_j is strictly convex, suppose $(S^1, C_j^1) \in A_j$ and $(S^2, C_j^2) \in A_j$, and consider $(\lambda S^1 + (1-\lambda)S^2, \lambda C_j^1 + (1-\lambda)C_j^2)$ for $0 \leqq \lambda \leqq 1$. Since $(S^1, C_j^1) \in A_j$, there exists a $\tau_1 \in [0, \tau_m]$ such that $S^1 \geqq S(\tau_1)$ and $C_j^1 \leqq C_j(\tau_1)$, and similarly for $(S^2, C_j^2) \in A_j$, there exists a $\tau_2 \in [0, \tau_m]$ such that $S^2 \geqq S(\tau_2)$ and $C_j^2 \leqq C_j(\tau_2)$. Let $\tau(\lambda) = \lambda\tau_1 + (1-\lambda)\tau_2$. Clearly, $\tau(\lambda) \in [0, \tau_m]$. Since $S(\tau)$ is strictly convex, $S(\tau(\lambda)) < \lambda S(\tau_1) + (1-\lambda)S(\tau_2) \leqq \lambda S^1 + (1-\lambda)S^2$. In addition, from the linearity of $C_j(\tau)$, $C_j(\tau(\lambda)) = \lambda C_j(\tau_1) + (1-\lambda)C_j(\tau_2) \geqq \lambda C_j^1 + (1-\lambda)C_j^2$. Taking these two statements together with (4'), we have

$$(16) \qquad \lambda S^1 + (1-\lambda)S^2 > S(\tau(\lambda)) \quad \text{and} \quad \lambda C_j^1 + (1-\lambda)C_j^2 \leqq \bar{Y}_j - \tau(\lambda)\Gamma_j .$$

Hence, $(\lambda S^1 + (1-\lambda)S^2, \lambda C_j^1 + (1-\lambda)C_j^2) \in A_j$ so that A_j is convex. Moreover, from the strict inequality in the first part of (16), it follows that if (S^1, C_j^1) and (S^2, C_j^2) are on the boundary of A_j, the point $(\lambda S^1 + (1-\lambda)S^2, \lambda C_j^1 + (1-\lambda)C_j^2)$ is in the interior of A_j. Thus, A_j is a strictly convex set.

The set A_j is derived in the first quadrant of Figure 1 for the case where there exists a tax rate at which total clean-up occurs, that is, $S(\tau_m) = 0$. In the second quadrant of the figure, the household's consumption level C_j is graphed on the vertical axis as a function of the effluent tax τ plotted on the horizontal axis. The fourth quadrant displays the relationship between the standard S on the horizontal axis and the tax τ on the vertical axis. The third quadrant simply contains a 45° line that is used to help derive the boundary of the opportunity set A_j, which is the shaded region displayed in the first quadrant.

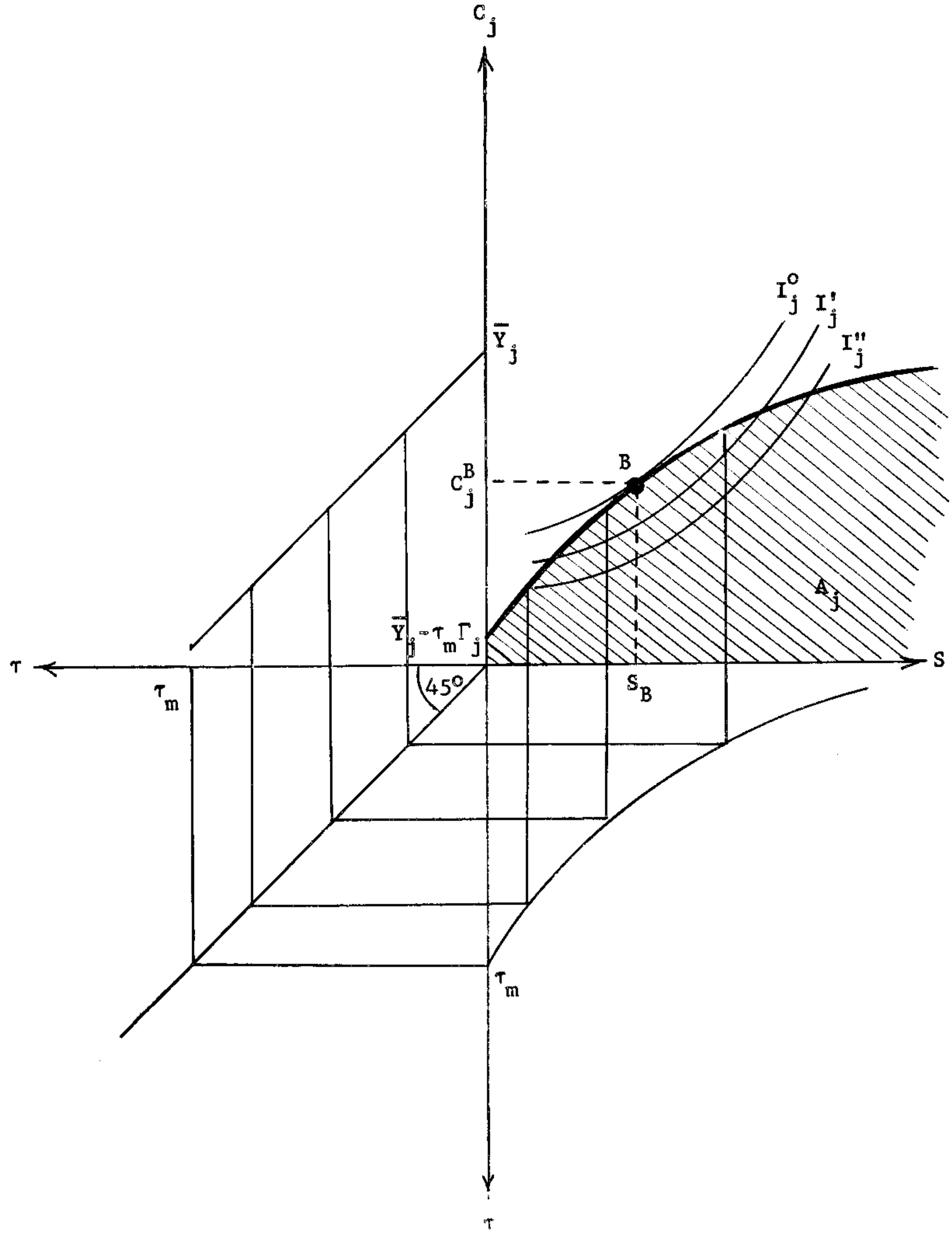

FIGURE 1

The Optimization Decision of Household j

While the j^{th} household's S, C_j opportunity set is given by A_j, the household's preferences for consumption and water quality are represented by the quasi-concave utility function in (3). Several of the indifference curves of this function are shown in Figure 1 as I_j^o, I_j', I_j''. If the household could choose its S, C_j position individualistically, it would go to its utility-maximizing point B, with standard S^B and consumption C_j^B, which is on the highest indifference curve the household can attain given the opportunity set A_j. In terms of the decision variable τ, household j's most-preferred tax rate τ_j^* is simply the one corresponding to the pair S^B, C_j^B. The household can rank all possible effluent tax rates by locating, for each tax rate, the corresponding point on the boundary of the opportunity set A_j, and associating with the given tax rate the level of the consumption-water quality indifference curve passing through that point. Since S and C_j vary monotonically with the tax rate, it follows that as the tax rate moves away, in either direction, from the j^{th} household's optimum τ_j^*, the household's ordinal utility level decreases.

Hence, each household's preferences with respect to τ can be represented by an ordinal utility index $U^j(C_j(\tau), S(\tau)) \equiv u_j(\tau)$ which has a single maximum or peak at $\tau = \tau_j^*$, is strictly increasing for $\tau < \tau_j^*$, and strictly decreasing for $\tau > \tau_j^*$. This is illustrated in Figure 2a.

(The specific assumptions made in this section, namely, (4'), (10'), and (11) were used to ensure this last conclusion about households' rankings. Any other set of assumptions that yielded this result would serve equally well in providing sufficient conditions for a global superequilibrium. The ones presented here are, however, the most transparent ones to use.)

Firms' rankings are even easier to describe. Under the assumptions introduced in the last section, each firm's profit is strictly decreasing with respect to τ . The tax rate $\tau = 0$ is highest in each firm's preference ordering on alternative social states, and higher rates are less preferred. This is illustrated in Figure 2b.

From the results on the firms' and households' preferences over social states, it follows that each voter's preference ranking with respect to effluent tax rates has a single peak. In short, for any given tax rate $\bar{\tau}$, the "single-peakedness" condition introduced by Duncan Black is met.

Hence, from Black's results on voting with single-peaked preferences, we can conclude that for any distribution of votes on the board there exists a voting equilibrium, and moreover that this equilibrium tax rate is a median with respect to the weighted distribution of most-preferred tax rates.[1]

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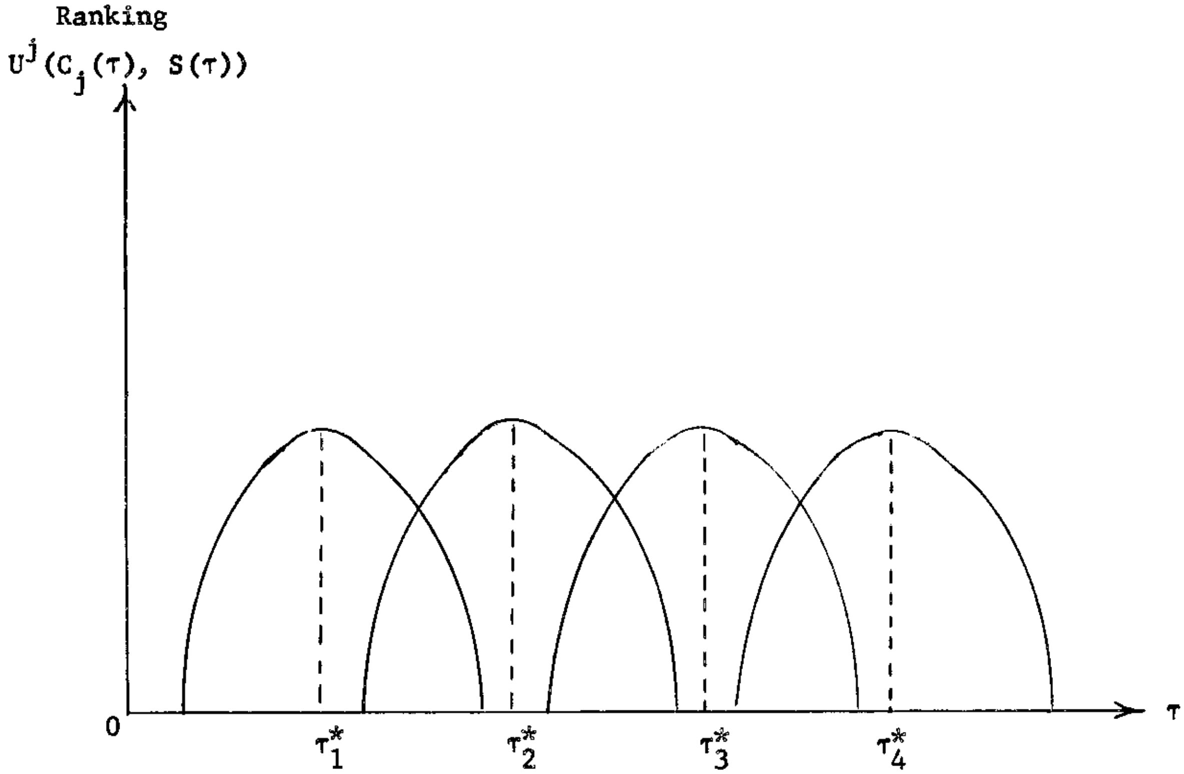
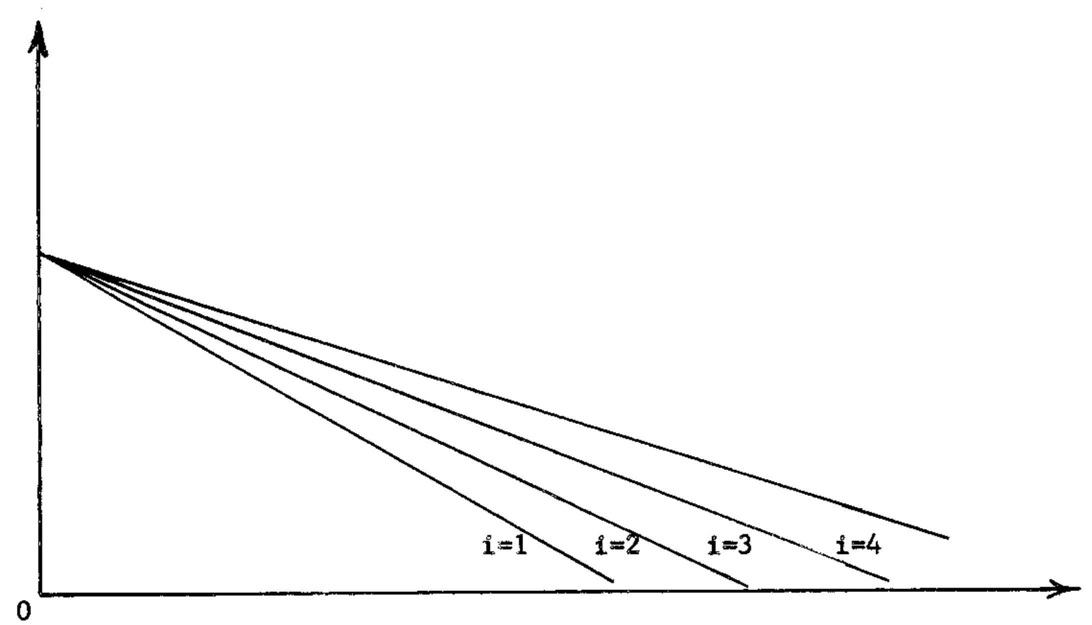
[1] Duncan Black, The Theory of Committees and Elections, Cambridge University Press, Cambridge, U.K., 1958, Chapters 2-4. Also see Kenneth J. Arrow, Social Choice and Individual Values, Second Edition, Yale University Press, 1963, pp. 75-80. Our voting problem differs in minor respects from the type considered by Black. Our genossenschaft operates by weighted vote whereas Black's committee used a one-man--one-vote rule. This distinction is, however, clearly inessential. For example, a representative with v votes can be regarded, for our purposes, as v representatives with the same preferences and one vote each. Black's definition of majority rule is also slightly different from ours. In Black's sense, an alternative τ' defeats another, τ'' , by majority vote if and only if the number of voters who prefer τ' to τ'' is strictly greater than the number who prefer τ'' to τ' . In contrast, in our sense τ' defeats τ'' if and only if the number who prefer τ' to τ'' is strictly greater than half the total number of voters. (The two definitions are equivalent when no voter is indifferent between τ' and τ'' .) A voting equilibrium--that is, an alternative which is not defeated by any other alternative--under Black's definition is clearly also an equilibrium under our definition; moreover it is easily shown that when all voters have single-peaked preferences (but not in general) the converse is also true.

Rather than appeal to Black's results, we could instead employ Theorem 2 of Gerald H. Kramer and Alvin K. Klevorick, "Existence of a 'Local' Cooperative Equilibrium in a Class of Voting Games," Cowles Foundation Discussion Paper No. 343, August 1972, to establish the existence of a voting equilibrium. It is straightforward to verify directly the proposition in (17) below.



Ranking

2a. Household Rankings



2b. Firm Rankings

FIGURE 2

The Rankings of Alternative Motions by Households and Firms

To be more precise, we shall say a tax rate t is a <u>global majority equilibrium</u> (g.m.e.) with respect to a given distribution $v(\tau)$ of votes if and only if there exists no other feasible tax rate τ' such that the set of voters (whether firms or households) who prefer τ' to t commands an absolute majority of the votes on the board. Under the assumptions of this section there is at least one g.m.e. for any distribution of voting strengths on the board. Moreover, every such equilibrium will satisfy the following necessary and sufficient condition:

(17) t is a global majority equilibrium with respect to τ if and only if

(a) $\sum_{\{k \mid \tau_k^* \leqq t\}} v_k(\tau) \geqq .5$ <u>and</u> (b) $\sum_{\{k \mid \tau_k^* \geqq t\}} v_k(\tau) \geqq .5$,

where $v_k(\tau)$ equals the fraction of the total number of votes that the k^{th} member of the board (household or firm) has when the tax rate is τ , and τ_k^* is the tax rate most preferred by the k^{th} member of the board.

Consider now what happens as τ changes; for concreteness, suppose τ increases from $\bar{\tau}$ to $\bar{\bar{\tau}}$. First, the tax-rate preferences of each member of the board remain unchanged. The households' and firms' rankings of alternative effluent tax rates do not depend on the prevailing rate. As a result, the single-peakedness condition is maintained. Hence, there also exists a global majority equilibrium for the vote distribution implied by the new higher tax rate.

What does happen as the tax rate increases is that the relative voting strength of the households increases and, correspondingly, the relative voting strength of the firms decreases. To see this, denote by $V(\tau)$ the fraction of votes all the households together have when the tax rate is τ.

Then, $V(\tau) = \frac{\sum_{j=1}^{n} \Gamma_j}{J(\tau)}$ and $V'(\tau) = \frac{-J'(\tau) \sum_{j=1}^{n} \Gamma_j}{[J(\tau)]^2}$. But, from (9), $J'(\tau) < 0$, and thus $V'(\tau) > 0$. Indeed, each household's relative voting strength rises in the same proportion as the aggregate household relative voting strength. Specifically, since $v_j(\tau)$ is the relative voting strength of household j at tax rate τ, $\frac{v_j'(\tau)}{v_j(\tau)} = \frac{V'(\tau)}{V(\tau)} = -\frac{J'(\tau)}{J(\tau)}$. What happens to a particular firm's relative voting strength depends on how the tax-rate change affects its waste discharges vis-à-vis the effect the change has on total waste discharges into the water. Since each firm's most-preferred tax rate is $\tau = 0$, while the most-preferred tax rate of at least some (in the most probable case, of all) households is positive, the increase from $\bar{\tau}$ to $\bar{\bar{\tau}}$ shifts voting strength from those units preferring lower tax rates to those units preferring higher tax rates. Intuitively, the net result of the tax-rate increase from $\bar{\tau}$ to $\bar{\bar{\tau}}$ is to increase the weighted median most-preferred tax rate(s) and hence to lead to a higher g.m.e. tax rate.

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[1] $v_i(\tau) = \frac{G_i(\tau)}{J(\tau)}$ for the i^{th} firm, and $v_i'(\tau) = \frac{J(\tau)G_i'(\tau) - J'(\tau)G_i(\tau)}{[J(\tau)]^2}$. Since $G_i'(\tau)$ and $J'(\tau)$ are both negative, whether the i^{th} firm's relative voting strength rises or falls depends on how $\frac{G_i'(\tau)}{G_i(\tau)}$ compares with $\frac{J'(\tau)}{J(\tau)}$.

The heuristic argument can be illustrated using Figure 3 where a <u>genossenschaft</u> consisting of four firms and five households is pictured. Assume that each of the five households discharges the same amount of waste, $\Gamma_j = \Gamma$ for $j = 1, \ldots, 5$, and has the same income, $\bar{Y}_j = \bar{Y}$ for $j = 1, \ldots, 5$, but that the tastes of the households differ. Hence, each household receives exactly 1/5 of the voting strength of the households, that is, $\frac{v_j(\tau)}{V(\tau)} = \frac{1}{5}$ for each j, but each has a different most-preferred tax rate, as indicated in the figure. Furthermore, in this example, each τ_j^* is assumed to be strictly positive.

Suppose that $\bar{\tau}$ is sufficiently low so that the firms have 60% of the votes when $\tau = \bar{\tau}$, that is $V(\bar{\tau}) = .40$. Then it follows from condition (17) that the only g.m.e. is $t = 0$. That is, with an effluent charge of $\bar{\tau}$, the resulting waste discharges of Γ for each household and $G_i(\bar{\tau})$ for the i^{th} firm lead to a vote distribution that produces zero as the weighted majority voting equilibrium tax rate. As τ is increased, $V(\tau)$ increases, and suppose that for $\tau = \bar{\bar{\tau}}$, $V(\bar{\bar{\tau}}) = .60$. Now each household has 12 percent of the total vote, while the firms in the aggregate have 40 percent of the votes. As can be easily verified, the new g.m.e. is τ_1^*. Finally, if we consider an even higher tax rate, $V(\tau)$ rises again so that, for example, with $\bar{\bar{\bar{\tau}}}$, one finds $V(\bar{\bar{\bar{\tau}}}) = .80$. Each household now has 16 percent of the votes, while the firms share 20 percent of the votes. The new g.m.e. is τ_2^*. It should also be noted that while zero is the lowest possible global majority equilibrium, the highest possible g.m.e. is τ_3^*, the tax rate that would result from a weighted majority vote among the households alone.

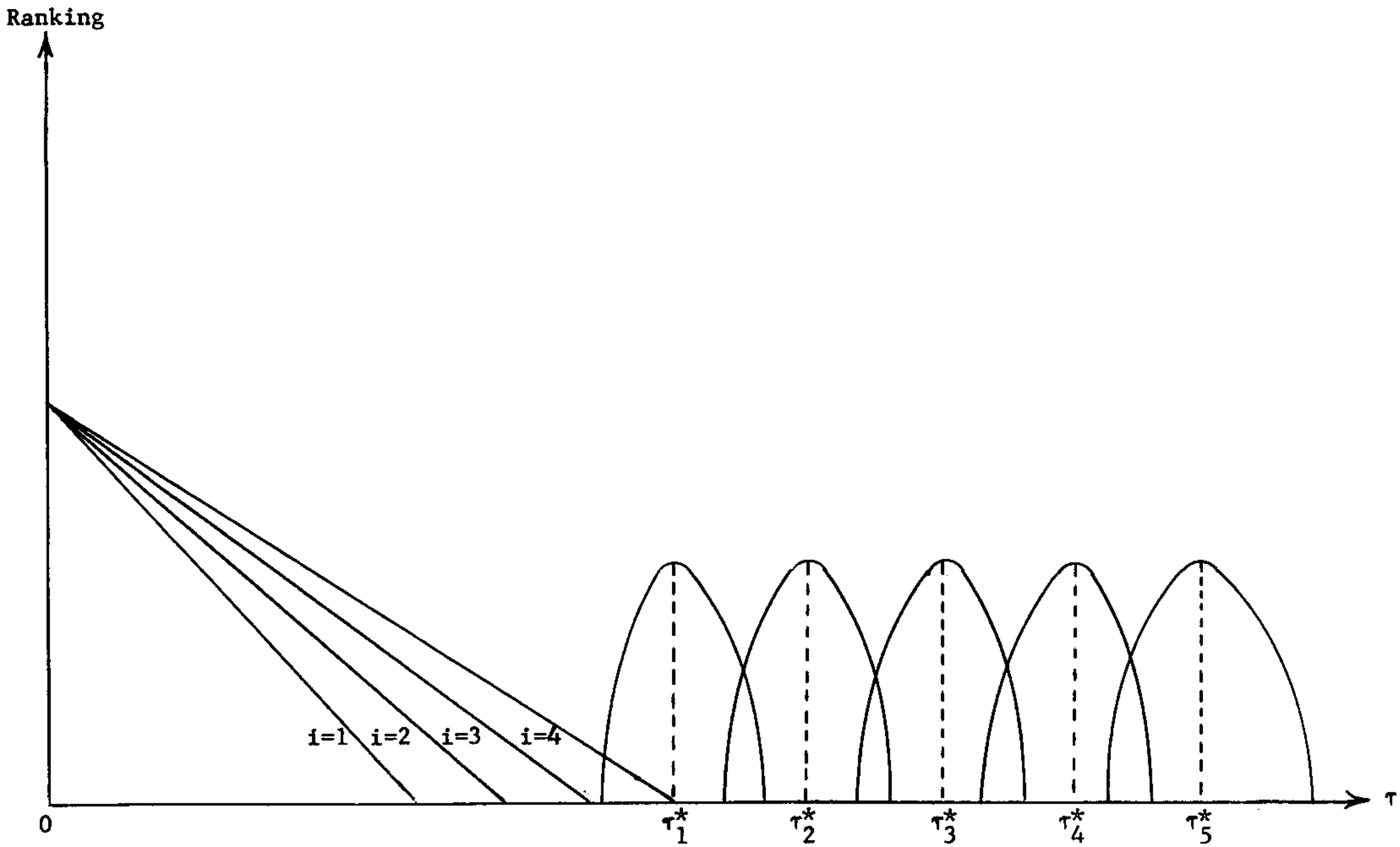

FIGURE 3

A *Genossenschaft* with Four Firms and Five Households

The manner in which the g.m.e. varies with respect to τ is actually a bit more complicated than this discussion indicates. For example, there may be more than one g.m.e. for a particular value of τ . If, for instance, $V(\tau') = .625$, then $\sum_{\{k|\tau_k^* \leqq t\}} v_k(\tau') \geqq .5$ and $\sum_{\{k|\tau_k^* \geqq t\}} v_k(\tau') \geqq .5$ for all t in the closed interval $[\tau_1^*, \tau_2^*]$. Hence, all t in this interval satisfy condition (17) and there is an infinity of global majority equilibria for $\tau = \tau'$. In the presence of this nonuniqueness, how can we discuss "monotonicity" of the g.m.e. with respect to τ ? Furthermore, the heuristic discussion of the example provides no information about whether the changes in the set of global majority equilibria occur "smoothly" as τ varies. In particular, denoting the set of g.m.e. at τ by $\psi(\tau)$, is the correspondence $\psi(\tau)$ upper semicontinuous? The following two results are addressed to these questions.

The domain of the mapping $\psi(\tau)$ is clearly the set of feasible tax rates. That is, the domain of $\psi(\tau)$ is the set T defined as

$$T = [0, \tau_m] . \tag{18}$$

Assuming that only feasible tax rates will be brought before the board for consideration, the range of $\psi(\tau)$ is some subset of T . Consider first whether the image of τ under the mapping $\psi : T \to T$ changes smoothly as τ changes. Lemma 1 answers this question in the affirmative.

<u>Lemma 1</u>. The correspondence $\psi : T \to T$ is upper semicontinuous on T .

<u>Proof</u>. Suppose (τ^ν) is a sequence of tax rates in T such that $(\tau^\nu) \to \tau^o$, and suppose t^ν is a g.m.e. with respect to τ^ν for each ν and $(t^\nu) \to t^o$. The mapping $\psi : T \to T$ is upper semicontinuous on T if and only if t^o is a global majority equilibrium with respect to τ^o; that is, if and only if $t^o \in \psi(\tau^o)$. Form the following three subsequences of (t^ν):

$$(t^{\nu'}) = \{(t^\nu) | t^\nu = t^o\}$$

$$(t^{\nu''}) = \{(t^\nu) | t^\nu < t^o\}$$

$$(t^{\nu'''}) = \{(t^\nu) | t^\nu > t^o\} .$$

Since $(t^\nu) \to t^o$, at least one of these three subsequences must be an infinite sequence converging to t^o. If $(t^{\nu'})$ is an infinite sequence with t^o as a limit, then it is trivial that $\sum_{\{k | \tau_k^* \leqq t^o\}} v_k(\tau^o) \geqq .5$ and $\sum_{\{k | \tau_k^* \geqq t^o\}} v_k(\tau^o) \geqq .5$ so that t^o is a g.m.e. with respect to τ^o.

Suppose $(t^{\nu''})$ is an infinite sequence that converges to t^o. Since each $t^{\nu''}$ is a g.m.e. for the corresponding $\tau^{\nu''}$, it follows from (17) that (α) $\sum_{\{k | \tau_k^* \leqq t^{\nu''}\}} v_k(\tau^{\nu''}) \geqq .5$ and (β) $\sum_{\{k | \tau_k^* \geqq t^{\nu''}\}} v_k(\tau^{\nu''}) \geqq .5$. (For notational simplicity, let us temporarily omit the double primes on ν.) From (α) and the fact that $t^o > t^\nu$, we have $\sum_{\{k | \tau_k^* \leqq t^o\}} v_k(\tau^\nu) \geqq .5$ for all ν. Taking limits and recalling that the $v_k(\tau)$ functions are continuous (indeed, continuously differentiable), we have $.5 \leqq \lim_{\nu \to \infty} \sum_{\{k | \tau_k^* \leqq t^o\}} v_k(\tau^\nu) = \sum_{\{k | \tau_k^* \leqq t^o\}} v_k(\tau^o)$. Hence, <u>condition (a)</u> in (17) obtains for t^o when $\tau = \tau^o$.

Now let τ_U^* be the largest $\tau_k^* < t^o$. Since there is a finite number of voters (m firms and n households), such a τ_U^* must exist. But since $(t^{\nu''})$ is an infinite sequence converging to t^o with $t^{\nu''} < t^o$ for all $t^{\nu''} \epsilon (t^{\nu''})$, there must exist a subsequence (t^η) of $(t^{\nu''})$ such that $\tau_U^* < t^\eta < t^o$ for all $t^\eta \epsilon (t^\eta)$, $(t^\eta) \to t^o$ and $(\tau^\eta) \to \tau^o$. From (β) and the construction of the η subsequence, for each η, $.5 \leqq \sum_{\{k|\tau_k^* \geqq t^\eta\}} v_k(\tau^\eta) = \sum_{\{k|\tau_k^* \geqq t^o\}} v_k(\tau^\eta)$. But, then, taking limits and using the continuity of the $v_k(\tau)$ functions, it follows that $.5 \leqq \lim_{\eta\to\infty} \sum_{\{k|\tau_k^* \geqq t^o\}} v_k(\tau^\eta) = \sum_{\{k|\tau_k^* \geqq t^o\}} v_k(\tau^o)$. Thus, condition (b) in (17) obtains for t^o when $\tau = \tau^o$.

Hence, we have shown that if $(t^{\nu''})$ is an infinite sequence converging to t^o, it follows that conditions (a) and (b) in (17) obtain for t^o when $\tau = \tau^o$. The tax rate t^o is then a global majority equilibrium with respect to τ^o. The remaining possibility is that $(t^{\nu'''})$ is the only convergent subsequence of (t^ν). This case is argued in a manner analogous to that used for the case where $(t^{\nu''})$ is a convergent subsequence of (t^ν).

Since at least one of the three subsequences $(t^{\nu'})$, $(t^{\nu''})$, $(t^{\nu'''})$ must be an infinite sequence converging to t^o, it follows that $(\tau^\nu) \to \tau^o$, $t^\nu \epsilon \psi(\tau^\nu)$ for each τ^ν, and $(t^\nu) \to t^o$ imply $t^o \epsilon \psi(\tau^o)$. The proof is complete. Q.E.D.

The next result makes precise the sense in which the correspondence $\psi : T \to T$ is monotonic. Two new definitions will be helpful in stating the result. Denote by $t^L(\tau)$ the minimum value of $t \epsilon \psi(\tau)$ and by $t^U(\tau)$ the maximum value of $t \epsilon \psi(\tau)$. Since the mapping $\psi : T \to T$

is upper semicontinuous and T is bounded by 0 and τ_m, the set of global majority equilibria with respect to any given tax rate τ, $\psi(\tau) \subseteq T$, is a closed, bounded nonempty set, and hence contains minimum and maximum members. The following lemma implies that $t^L(\tau)$ and $t^U(\tau)$ are both nondecreasing functions of τ.

<u>Lemma 2</u>. If $\tau_1 < \tau_2$, then $t^U(\tau_1) \leqq t^L(\tau_2)$.

<u>Proof</u>. Suppose the contrary, namely, that $\tau_1 < \tau_2$ but $t^U(\tau_1) > t^L(\tau_2)$. Denote by A the index set $A = \{k | \tau_k^* \geqq t^U(\tau_1)\}$. Since $t^U(\tau_1) > t^L(\tau_2) \geqq 0$, every member k of A has a strictly positive most-preferred tax rate τ_k^*. Given our assumptions on firms' and households' preferences, this implies A consists only of households. For any household j, however, $v_j(\tau) = \frac{\Gamma_j}{J(\tau)}$, and $v_j'(\tau) = \frac{-\Gamma_j}{[J(\tau)]^2} J'(\tau)$, which from (9) is strictly positive. Hence every household's vote share $v_j(\tau)$ is strictly increasing in τ, so $\sum_{k \in A} v_k(\tau_2) > \sum_{k \in A} v_k(\tau_1)$.

Since $t^U(\tau_1)$ is a global majority equilibrium with respect to τ_1 it must satisfy (17b) for τ_1; that is, $\sum_{k \in A} v_k(\tau_1) \geqq .5$, whence $\sum_{k \in A} v_k(\tau_2) > .5$. This implies, however, that $\sum_{k \in \tilde{A}} v_k(\tau_2) < .5$ where $\tilde{A} = \{k | \tau_k^* < t^U(\tau_1)\}$ (since $\sum_A v_k(\tau) + \sum_{\tilde{A}} v_k(\tau) = 1$ for all τ). Moreover, since $t^U(\tau_1) > t^L(\tau_2)$ by hypothesis, it must be true that

$$\sum_{\{k | \tau_k^* \leqq t^L(\tau_2)\}} v_k(\tau_2) \leqq \sum_{\{k | \tau_k^* < t^U(\tau_1)\}} v_k(\tau_2) \equiv \sum_{k \in \tilde{A}} v_k(\tau_2) .$$

This fact together with the previous inequality implies that

$\sum_{\{k | \tau_k^* \leqq t^L(\tau_2)\}} v_k(\tau_2) < .5$, and hence that $t^L(\tau_2)$ does not satisfy (17a) for $\tau = \tau_2$. Since $t^L(\tau_2)$ is a g.m.e. with respect to τ_2 , however, (17a) must hold. This is a contradiction. Hence, $t^U(\tau_1) \leqq t^L(\tau_2)$.

Q.E.D.

Lemma 2 and the continuity of the $v_k(\tau)$ functions can be used to show that the mapping $\psi : T \to T$ is a specific type of upper semicontinuous correspondence. We omit the proof, but it can be shown that for only a finite number of tax rates τ does the image set $\psi(\tau)$ contain more than one element. Hence, Figure 4a or 4b could be a graph of the $\psi(\tau)$ mapping, but Figure 5 could not.

Making use of Lemmas 1 and 2, we can now answer the questions about the existence and nature of global superequilibria in the Genossenschaften model. The principal results are summarized in Theorem 1.

Theorem 1.

(i) Under the assumptions of Section 2 and the further assumptions of (4'), (10'), and (11) in Section 3, there exists a global superequilibrium [G.S.], τ^* .

(ii) A necessary condition for there to exist a positive global superequilibrium, that is, one with $\tau^* > 0$, is that there exist some positive rate $\tau' > 0$ which is preferred to the zero rate $\tau = 0$ by a weighted majority of households with the weight of the j^{th} household being $\Gamma_j / \sum_{j=1}^{n} \Gamma_j$.

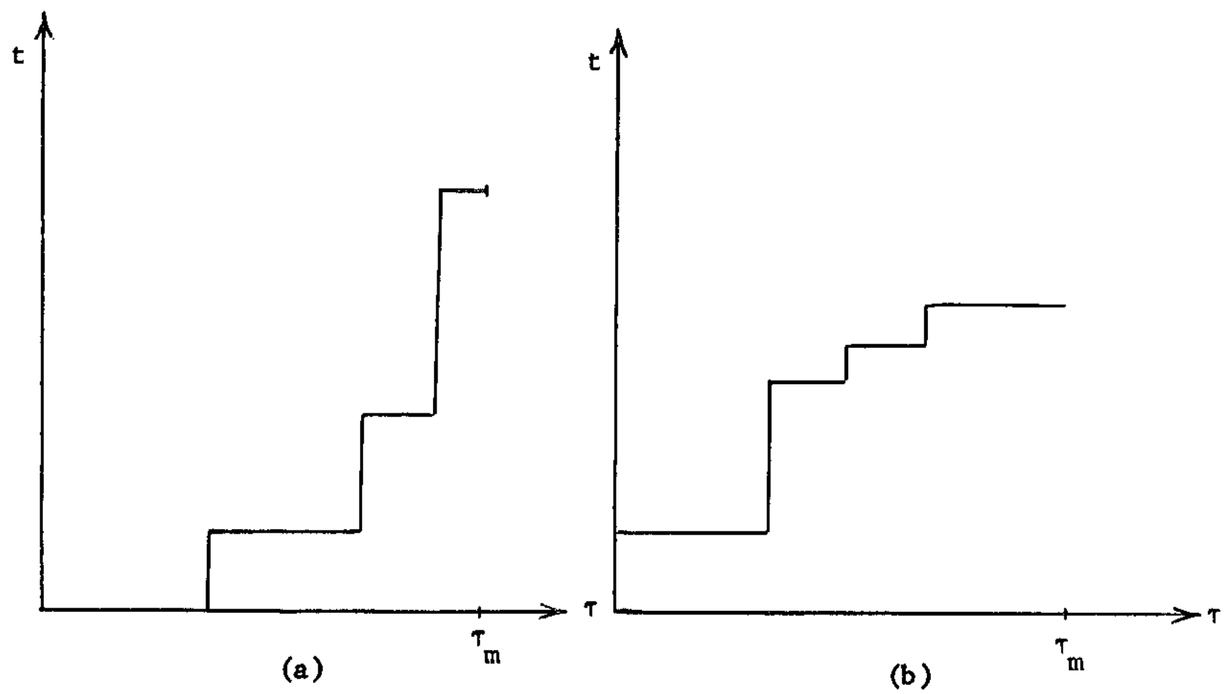

FIGURE 4

Possible Graphs of $\psi(\tau)$

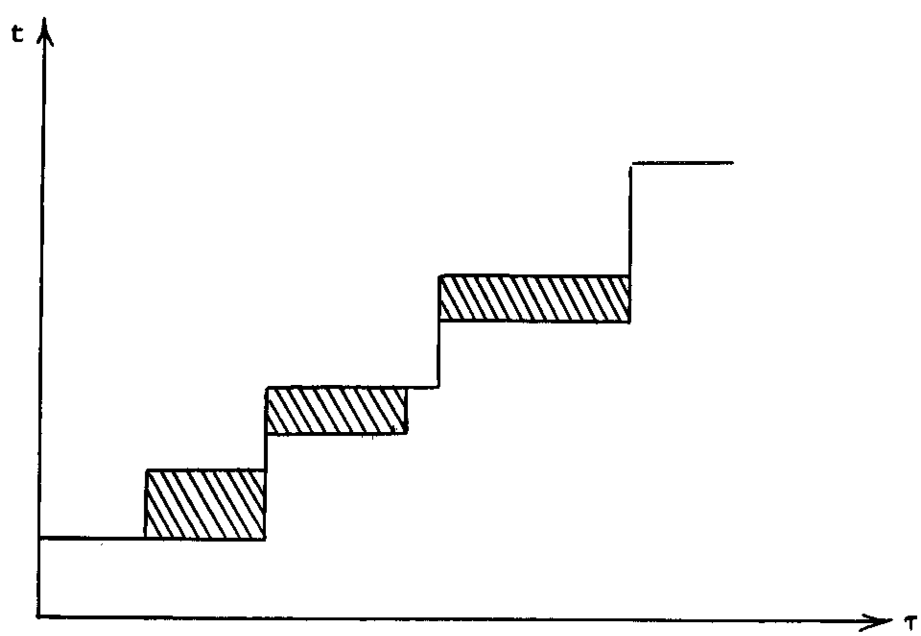

FIGURE 5

A Graph of an Upper Semicontinuous Correspondence Other Than $\psi(\tau)$

(iii) A sufficient condition for <u>all</u> global superequilibria to be positive is that each household's most-preferred tax rate be positive and that the households collectively control the board at $\tau = 0$, that is, $\sum_{j=1}^{n} \Gamma_j > \sum_{i=1}^{m} G_i(0)$.

(iv) A sufficient condition for there to exist a positive global superequilibrium $\tau^* > 0$ is that there exist some $\hat{\tau} > 0$ such that with votes distributed according to the waste discharges emitted when $\tau = \hat{\tau}$, there exists some $\hat{t} \in \psi(\hat{\tau})$ such that $\hat{t} \geqq \hat{\tau}$.

Proof.

(i) The assumptions of Section 2 together with (4'), (10'), and (11) ensure that for any $\tau \in T$ there exists a nonempty set of global majority equilibria given by the correspondence $\psi(\tau)$. Lemma 1 established that ψ is an upper semicontinuous correspondence mapping T into T, and from (18) T is clearly a nonempty, compact convex subset of the real line. Finally, it is clear from the conditions in (17) that $\psi(\tau)$ is convex for all $\tau \in T$. Hence, all the conditions of the Kakutani fixed-point theorem are met, and it follows that there exists a fixed point, that is, a τ^* such that $\tau^* \in \psi(\tau^*)$. The tax rate τ^* is a global majority equilibrium when voting strengths are determined by τ^*, and thus it is a global superequilibrium.

(ii) Suppose there were no positive tax rate preferred to the zero rate by a weighted majority of the households alone, with the j^{th} household's weight equal to $\frac{\Gamma_j}{\sum_{j=1}^{n} \Gamma_j}$. Denote by $\mathcal{H}$ the set of households.

Then $\tau = 0$ would be a g.m.e. with respect to the above vote distribution, and since $v_k(\tau) = \frac{\Gamma_k}{J(\tau)}$ for each $k \in \mathcal{H}$, it follows from (17) that for every τ, $\frac{J(\tau)}{\sum_{j=1}^{n} \Gamma_j} \sum_{\{k\in\mathcal{H} | \tau_k^* \leqq 0\}} v_k(\tau) \geqq .5$ and

$\frac{J(\tau)}{\sum_{j=1}^{n} \Gamma_j} \sum_{\{k\in\mathcal{H} | \tau_k^* \geqq 0\}} v_k(\tau) \geqq .5$. The first of these inequalities implies that $\sum_{\{k\in\mathcal{H} | \tau_k^* > 0\}} v_k(\tau) \leqq .5 \frac{\sum_{j=1}^{n} \Gamma_j}{J(\tau)}$. But denoting the set of firms by $\mathcal{F}$, we know that $\tau_k^* = 0$ for all $k \in \mathcal{F}$, and hence $\sum_{\{k\in\mathcal{F} | \tau_k^* > 0\}} v_k(\tau) = 0$ for all τ . Then for any τ and any $t > 0$, $\sum_{\{k | \tau_k^* \geqq t\}} v_k(\tau) \leqq .5 \frac{\sum_{j=1}^{n} \Gamma_j}{J(\tau)} < .5$. Hence, from (17b) no positive tax rate can be a g.m.e. for any τ . The set $\phi(\tau) = \{0\}$ for all $\tau \in T$, and the only global superequilibrium is $\tau^* = 0$.

(iii) If each household's most-preferred tax rate is positive, then for any τ , $\sum_{\{k | \tau_k^* \leqq 0\}} v_k(\tau) = \sum_{i=1}^{m} \frac{G_i(\tau)}{J(\tau)}$. Under the assumption that $\sum_{j=1}^{n} \Gamma_j > \sum_{i=1}^{m} G_i(0)$, it follows that $\sum_{\{k | \tau_k^* \leqq 0\}} v_k(0) = \frac{\sum_{i=1}^{m} G_i(0)}{J(0)} < .5$. Moreover, since $\frac{\sum_{i=1}^{m} G_i(\tau)}{J(\tau)}$ is a strictly decreasing function of τ , this implies $\sum_{\{k | \tau_k^* \leqq 0\}} v_k(\tau) < .5$ for all $\tau \geqq 0$. Hence, $t = 0$ does not satisfy condition (17a) for a g.m.e. with respect to <u>any</u> τ , and thus it cannot be a global superequilibrium. It follows that every global superequilibrium tax rate is positive.

(iv) If $\hat{\tau} \in \psi(\hat{\tau})$, then $\hat{\tau}$ is a G.S. and since $\hat{\tau} > 0$ by assumption, there exists a positive global superequilibrium. Suppose that $\hat{\tau} \notin \psi(\hat{\tau})$ but that $\hat{t} > \hat{\tau}$ for some $\hat{t} \in \psi(\hat{\tau})$. Then consider the nonempty, compact, convex set $\hat{T} = [\hat{\tau}, \tau_m]$. The assumptions in (i) imply that for any $\tau \in \hat{T}$ there exists a nonempty set of global majority equilibria given by the correspondence ψ and $\psi(\tau)$ is convex for all $\tau \in \hat{T}$. In addition, Lemma 1 shows the correspondence ψ is upper semicontinuous on $\hat{T} \subset T$. Furthermore, from Lemma 2, $t^L(\tau_2) \geqq t^U(\tau_1)$ for $\tau_2 > \tau_1$, so that $t \geqq t^U(\hat{\tau}) \geqq \hat{t}$ for all $t \in \bigcup_{\tau \in \hat{T}} \{\psi(\tau)\}$. Thus, ψ maps $\hat{T}$ into $\hat{T}$. All the conditions of the Kakutani fixed-point theorem are met, and hence there exists a $\tau^* \in \hat{T}$ such that $\tau^* \in \psi(\tau^*)$. That is, there exists a global superequilibrium tax rate $\tau^* \in \hat{T}$. Since all $\tau \in \hat{T}$ are positive, the proof is complete. Q.E.D.

Several alternative possibilities for global superequilibria are illustrated in Figure 6. Assume for convenience that, in fact, $S(\tau_m) = 0$ in each case. Figure 6a represents a case in which the only G.S. tax rate is $\tau = 0$. In Figures 6b and 6c, $\tau = 0$ is still a G.S. tax rate, but there are also positive global superequilibrium tax rates. In 6b, there are two other global superequilibrium charges τ_{b1} and τ_{b2} , while in the case of 6c, there is only one positive G.S. tax rate: τ_{c1} . Figures 6b and 6c illustrate that condition (iii) in Theorem 1 is sufficient but not necessary for the existence of a positive τ^* : even if the decision units desiring the lowest possible rate, $\tau = 0$, control the board at low tax rates, a positive global superequilibrium may still exist. The remaining

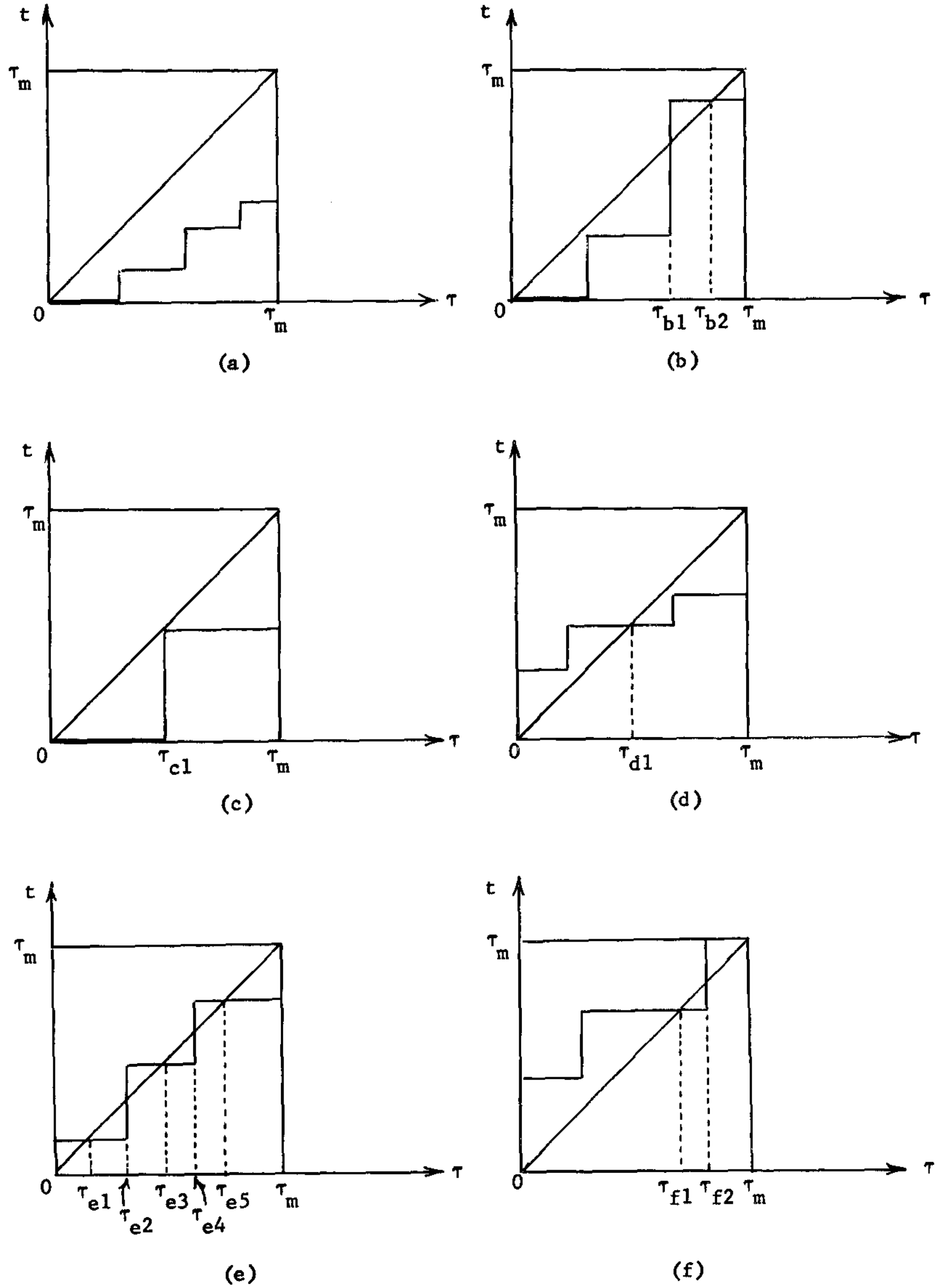

FIGURE 6

Alternative Global Superequilibrium Situations three diagrams, 6d-6f, depict cases in which the households with positive tax preferences control the board when $\tau = 0$. In each of these cases, there is no $\tau \epsilon T$ for which zero is a global majority equilibrium, and hence zero is not a G.S. tax rate. Figure 6d represents a case in which there is a unique positive global superequilibrium tax rate τ_{d1} , while 6e and 6f are instances of multiple positive global superequilibria. In particular, in the case depicted in Figure 6f, the maximum clean-up tax rate τ_m is a G.S. tax rate and $S^* = 0$ or total clean-up of all waste is the resulting standard.

4. The Existence of Equilibrium under More General Conditions

The results obtained in Section 3 were based on the assumption of non-increasing returns to scale in the technology of pollution treatment and other somewhat restrictive premises. It was assumed that the effluent charges imposed on the firms, which are the principal employers in the region, have no effect on the households' incomes, and that each firm's strictly positive waste emission decreases at a decreasing rate as the effluent charge increases. In this section we shall relax these assumptions--specifically, (4'), (10'), and (11)--and investigate the equilibrium properties of the Genossenschaften voting system under the more general assumptions presented in Section 2. Specifically, the curve describing a firm's supply of waste discharge as a function of the tax rate is unrestricted except that it must be downward sloping and positive for all feasible τ (which is implied by the assumptions of Section 2). The only restrictions on the water board's treatment technology are those given in (7): positive marginal productivity of an extra dollar and zero clean-up for zero expenditure. The treatment technology may, for example, have ranges of increasing returns to scale. Finally, the gross income of each household now may depend on the firms' hiring decisions, which in turn depend on the effluent charge τ. (Household income $Y_j(\tau)$ is assumed to be a twice continuously differentiable function of τ, for all j.) This introduces some general-equilibrium aspects into the model. In deciding on its tax-rate preferences, each household must not only take account of how the tax rate affects water quality and the household's net income via the effluent charge it must pay but also how the tax rate affects the household's <u>gross</u> income since any tax-rate change may now alter the labor income of the household.

Relaxing the Section 3 assumptions in this way has several implications for the earlier analysis. The $S(\tau)$ function defined in (8) need no longer be convex or even strictly decreasing. Because of this, and the dependence of households' gross incomes on τ, the households' opportunity sets, A_j for $j = 1, \ldots, n$, are not in general (strictly) convex. This, in turn, implies that each household's preferences over tax rates are not necessarily single-peaked. Figure 7 illustrates the j^{th} household's situation for the relatively simple case in which $S(\tau)$ is still a strictly decreasing function, $S(\tau_m) = 0$, and household incomes are still fixed, but $S(\tau)$ is no longer a convex function. Allowing $S(\tau)$ to be nonmonotonic and the j^{th} household's gross income to vary with τ would complicate matters even further. It is clear from Figure 7 that the nonconvex $S(\tau)$ function generates an opportunity set A_j for household j (the striped area in the figure) which is nonconvex, and hence the household's tax-rate

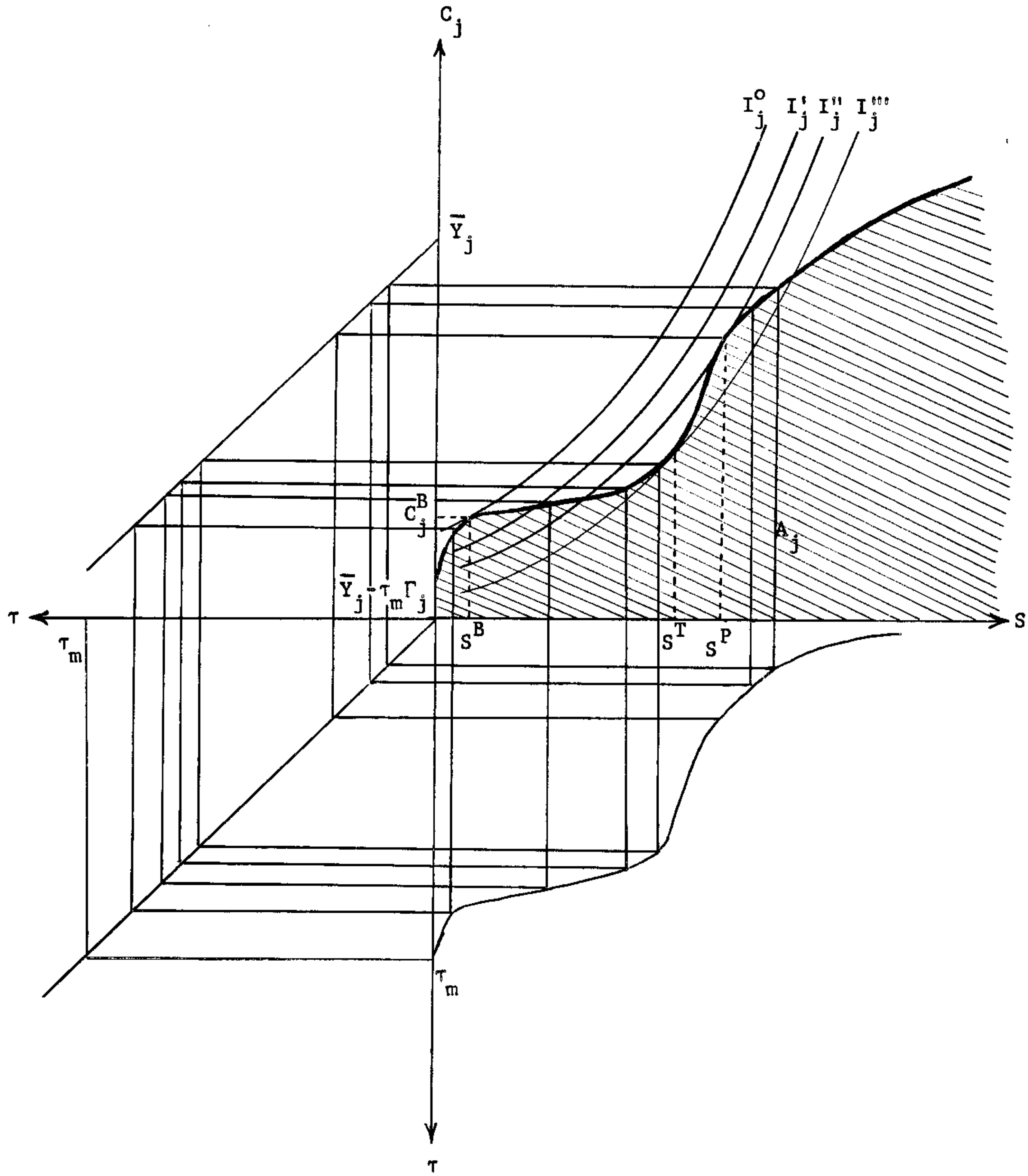

FIGURE 7

The Optimization Decision of Household j preferences need no longer be single-peaked. For the particular set of indifference curves shown in Figure 7, the tax-rate preferences of household j are clearly not single-peaked. Instead, they take the form shown in Figure 8, where τ_j^* is the tax rate corresponding to the household's most-preferred feasible point (S^B, C_j^B) in Figure 7. The tax rate τ_j^P in Figure 8 corresponds to the standard S^P in Figure 7, and similarly, τ_j^T in Figure 8 corresponds to standard S^T in Figure 7.

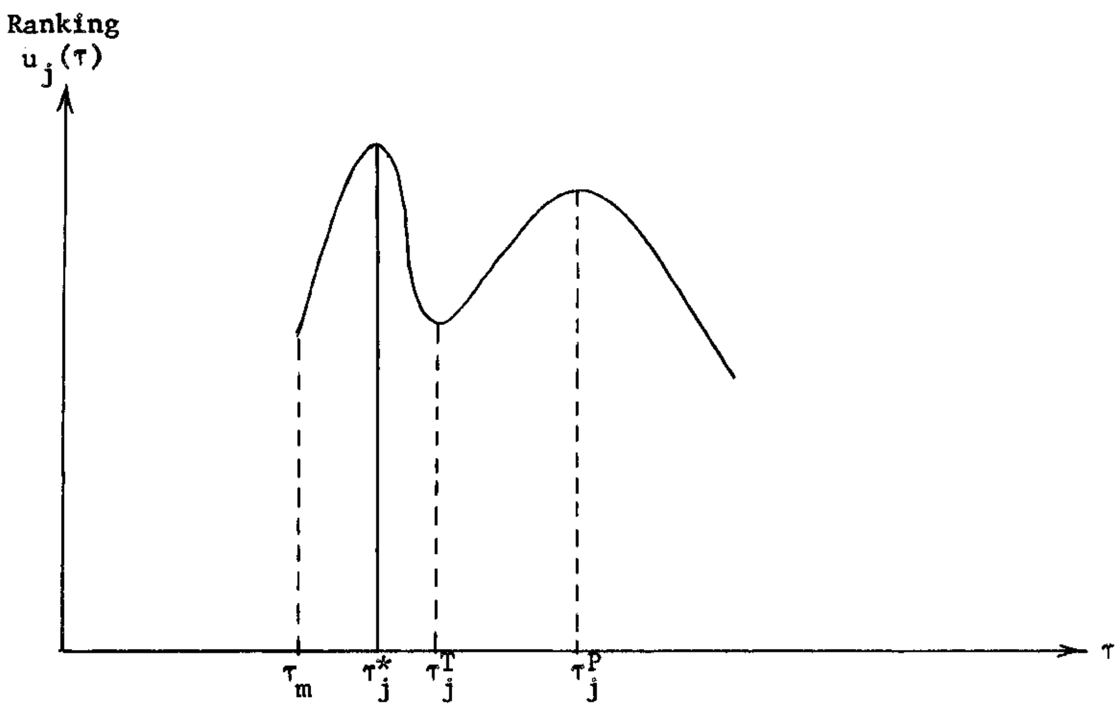

FIGURE 8

Tax-Rate Preferences of Household j of Figure 7

While the tax-rate preferences of the firms remain exactly as shown in Figure 2b, and hence single-peaked, the fact that the households' preferences need no longer have that property means that we can no longer appeal to the Black single-peakedness result to establish the existence of a voting equilibrium. Indeed, it is easily shown by example that a global majority equilibrium need no longer exist with non-single-peaked preferences: $\psi(\tau)$ may be empty for some (perhaps, for all) τ . Hence, the approach of Section 3 cannot be used to demonstrate the existence of a global superequilibrium. With one additional, rather weak assumption, however, we can demonstrate the existence of a local superequilibrium.

A local equilibrium, it will be recalled from Section 2, is an effluent tax rate τ^o for which there exists no "nearby" rate τ' preferred by a weighted majority to τ . More precisely, a feasible rate τ^o is a local majority equilibrium (l.m.e.) with respect to the vote distribution $v(\tau)$ if and only if there exists a neighborhood $(\tau^o - \delta, \tau^o + \delta)$ of τ^o with $\delta > 0$ which contains no feasible rate τ' preferred by a weighted majority, weights being given by the distribution $v(\tau)$, to τ^o . Let us denote by $\xi(\tau)$ the set of local majority equilibria with respect to the vote distribution $v(\tau)$. Evidently ξ is a correspondence from the set of feasible rates T into T .

Under one additional assumption, a local majority equilibrium exists with respect to any feasible vote distribution; that is, the set $\xi(\tau)$ is nonempty for all $\tau \in T$. The additional assumption required for this result is a rather weak restriction on voter preferences with respect to alternative effluent tax rates.

Under the assumptions of Section 2, each profit-maximizing firm will prefer a lower tax rate to a higher one, and every firm prefers $\tau = 0$ to any higher rate. The situation for households is more complex. The j^{th}

household's preferences over alternative tax rates are represented by

$$u_j(\tau) = U^j(C_j(\tau),\ S(\tau)) = U^j(Y_j(\tau) - \tau\Gamma_j,\ S(\tau)) .$$

Since the functions $U^j(\cdot)$, $Y_j(\tau)$, and $S(\tau)$ are each continuous (twice continuously differentiable, in fact), $u_j(\tau)$ is continuous. But without further assumptions we can conclude nothing about the shape of $u_j(\tau)$, and, in particular, nothing about the number of "peaks" it has on the set T .

Our additional premise is, essentially, that each $u_j(\tau)$ has only finitely many "peaks" on T , a natural extension of the "single-peakedness" condition invoked in the previous section. To formulate this premise more precisely, we must first define a "peak." A continuous function defined on an interval will be said to have a <u>proper relative maximum</u> at a point x in the interval if and only if it has a relative maximum at x <u>and</u> does not have a relative minimum at x . Our "finite-peakedness" assumption is then:

(19) For all households $j = 1, 2, \ldots, n$, the utility function $u_j(\tau)$ has only a finite number of proper relative maxima on T .

Since each firm's tax-rate preferences are single-peaked, each firm's utility function $u_i(\tau)$ also satisfies this finite-peakedness condition. Results we have proven elsewhere[1] can now be directly applied to the current voting problem. The following theorem applies:

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[1] G.H. Kramer and A.K. Klevorick, <u>op.cit</u>. In that paper, the domain of definition of the utility functions is the closed interval $[0,1]$. The results clearly apply for any homeomorph of $[0,1]$, and, in particular, for any other closed interval like T .

<u>Theorem 2</u>. If the utility function $u_k(\tau)$ of each decisionmaker is continuous on T and has only a finite number of proper relative maxima on T, then for all $\tau \in T$,

(i) there exists a local majority equilibrium with respect to τ;

(ii) the tax rate $x^u(\tau) \equiv \sup\{x \in \xi(\tau)\}$ is itself a local majority equilibrium with respect to τ; that is, $x^u(\tau) \in \xi(\tau)$;

(iii) if $x^u(\tau) < \tau_m$, it is a proper relative maximum of the utility function $u^k(\tau)$ for some $k \in \mathcal{Y} \cup \mathcal{H}$;

(iv) if $z \in T$ and $z > x^u(\tau)$, then when voting strengths are determined by τ, z will be defeated by tax rates slightly smaller than z; that is, for some $\delta > 0$, $\sum_{\{k|u_k(z')>u_k(z)\}} v_k(\tau) > .5$ for all $z' \in (z-\delta, z)$.

Theorem 2 establishes, <u>inter alia</u>, that for every $\tau \in T$, there exists a local majority equilibrium: $\xi(\tau)$ is not empty. It remains to be shown that there exists a local superequilibrium $\tau^+ \in \xi(\tau^+)$; that is, a tax rate τ^+ which is a local majority equilibrium with respect to the vote distribution $v(\tau^+)$. Although such a rate τ^+ is clearly a fixed point of the correspondence ξ, we cannot follow the last section's approach and use the Kakutani fixed-point theorem to establish the existence of such a point because the set $\xi(\tau)$ is not convex for all $\tau \in T$ and indeed the correspondence $\xi : T \to T$ is not in general upper semicontinuous.

To prove the existence of a local superequilibrium we will need two preliminary results, analogues of Lemmas 1 and 2 of Section 3, and they, in turn, require some further definitions. Let the preference ordering over tax rates of the k^{th} decisionmaking unit (whether firm or household) be represented by the utility function $u_k(\tau)$. Then, for any $\tau \in T$, define two sets of decisionmaking units, the <u>increase group</u> $I(\tau)$, and the <u>decrease group</u> $D(\tau)$, as follows:

(20)
- (a) $k \in I(\tau)$ if and only if there exists some $\delta > 0$ such that $u_k(\tau') > u_k(\tau)$ for all feasible τ' in the interval $(\tau, \tau+\delta)$.
- (b) $k \in D(\tau)$ if and only if there exists some $\delta' > 0$ such that $u_k(\tau') > u_k(\tau)$ for all feasible τ' in the interval $(\tau-\delta', \tau)$.

Thus $I(\tau)$ consists of those voters who "prefer small increases" from τ , that is, voters for whom τ is inferior to any slightly larger value τ' . The set $D(\tau)$ can be interpreted analogously. For certain values of τ , some households may belong to both of the sets $I(\tau)$, $D(\tau)$ and some households may belong to neither. Since firms always prefer lower to higher tax rates, however, it follows that

$$
(21)\quad \begin{cases} \mathfrak{F} \subseteq D(\tau) & \text{for all } \tau \in [0, \tau_m] \text{ and} \\ \mathfrak{F} \not\subseteq I(\tau) & \text{for any } \tau \in [0, \tau_m) . \end{cases}
$$

A local majority equilibrium can be readily defined in terms of the $I(\tau)$ and $D(\tau)$ sets. Specifically, a feasible effluent tax rate τ is a l.m.e. with respect to the distribution of votes $v(\tau')$ if and only if

$$(22)\begin{cases} \text{(a)} \quad \sum_{k \in D(\tau)} v_k(\tau') \leqq .5 \quad \text{if} \quad \tau > 0 \\ \text{and (b)} \quad \sum_{k \in I(\tau)} v_k(\tau') \leqq .5 \quad \text{if} \quad \tau < \tau_m \,. \end{cases}$$

We shall now use these definitions and facts to establish the following two lemmas.

Lemma 3. Let (τ^ν) be a sequence of feasible tax rates such that $(\tau^\nu) \to \tau^o$, and (x^ν) the sequence defined by $x^\nu \equiv x^u(\tau^\nu)$ for all ν. Then if $(x^\nu) \to r^o$, the limiting point r^o is a local majority equilibrium with respect to the vote distribution $v(\tau^o)$; that is, $r^o \in \xi(\tau^o)$.

Proof. From result (iii) of Theorem 2, for any τ, $x^u(\tau)$ must be a proper relative maximum of $u^k(\tau)$ for some $k \in \mathcal{J} \cup \mathcal{H}$ or $x^u(\tau) = \tau_m$. But from assumption (19) and the single peakedness of the $u_k(\tau)$ functions for $k \in \mathcal{J}$, it follows that the number of proper relative maxima for all $u^k(\tau)$ for all $k \in \mathcal{J} \cup \mathcal{H}$ is finite. Hence, (x^ν) must contain a subsequence (x^η) all of whose terms are equal to some common value, and this value must be r^o. In addition, it follows from result (ii) of Theorem 2 that $x^\eta = r^o$ is a local majority equilibrium with respect to τ^η. Thus, the definition in (22) implies that $.5 \geqq \sum_{D(x^\eta)} v_k(\tau^\eta) = \sum_{D(r^o)} v_k(\tau^\eta)$ and $.5 \geqq \sum_{I(x^\eta)} v_k(\tau^\eta) = \sum_{I(r^o)} v_k(\tau^\eta)$ must hold for all η. Since the $v_k(\tau)$ functions are continuous in τ, taking limits we have:

$$.5 \geqq \lim_{\eta\to\infty} \sum_{D(r^o)} v_k(\tau^\eta) = \sum_{D(r^o)} v_k(\tau^o) \quad \text{and} \quad .5 \geqq \lim_{\eta\to\infty} \sum_{I(r^o)} v_k(\tau^\eta) = \sum_{I(r^o)} v_k(\tau^o) \,.$$

Hence, conditions (a) and (b) in (22) are met by r^o when $\tau = \tau^o$, and r^o is a local majority equilibrium with respect to τ^o. Q.E.D.

Thus, although the correspondence ξ is not in general upper semicontinuous, it nevertheless does have a certain continuity property as expressed in the above lemma. Lemma 4 establishes a monotonicity property of the mapping ξ.

Lemma 4. The supremum local majority equilibrium tax rate $x^u(\tau)$ is monotonically increasing in τ.

Proof. Let $\tau' > \tau$ and for notational simplicity denote $x^u(\tau)$ by r. If $r = 0$ then clearly $x^u(\tau') \geqq r$. Suppose, then, that $r > 0$. Denoting $\sum_{k \in B}$ by $\sum_{B}$ for any set B and recalling that the set of firms is denoted by $\mathcal{F}$ and the set of households by $\mathcal{H}$, we have

$$\sum_{D(r)} v_k(\tau) = \sum_{\mathcal{F} \cap D(r)} v_k(\tau) + \sum_{\mathcal{H} \cap D(r)} v_k(\tau)$$

$$= \sum_{\mathcal{F}} v_k(\tau) + \sum_{\mathcal{H} \cap D(r)} v_k(\tau) \;, \quad \text{using (21).}$$

But the j^{th} household's share of the total voting strength of all households is fixed at $\dfrac{\Gamma_j}{\sum_{j=1}^{n} \Gamma_j}$, independent of the tax rate determining each unit's voting strength in the whole body, and this is true for all $j \in \mathcal{H}$. It follows that for any group of firms and households $B \subseteq \mathcal{F} \cup \mathcal{H}$,

$\dfrac{\sum_{B\cap\mathcal{H}} v_k(\tau)}{\sum_{\mathcal{H}} v_k(\tau)} = \gamma(B)$ for all $\tau \in T$, with the fraction γ depending only on the set B. Thus, $\sum_{D(r)} v_k(\tau) = \sum_{\mathcal{J}} v_k(\tau) + \gamma_0 \sum_{\mathcal{H}} v_k(\tau)$ with $\gamma_0 \equiv \gamma(D(r))$.

$= \gamma_0 + (1-\gamma_0)\sum_{\mathcal{J}} v_k(\tau)$ since $\sum_{\mathcal{J}} v_k(\tau) + \sum_{\mathcal{H}} v_k(\tau) \equiv 1$.

$> \gamma_0 + (1-\gamma_0)\sum_{\mathcal{J}} v_k(\tau')$ because, as noted earlier, $\sum_{\mathcal{H}} v_k(\tau)$ increases and $\sum_{\mathcal{J}} v_k(\tau)$ decreases as τ increases. But $\gamma_0 + (1-\gamma_0)\sum_{\mathcal{J}} v_k(\tau') = \sum_{\mathcal{J}} v_k(\tau') + \gamma_0 \sum_{\mathcal{H}} v_k(\tau') = \sum_{D(r)} v_k(\tau')$ from (21), $\sum_{\mathcal{J}} v_k(\tau) + \sum_{\mathcal{H}} v_k(\tau) \equiv 1$, and the fact that $\gamma_0 = \gamma(D(r))$ depends only on $D(r)$ and not on τ. Therefore, $\sum_{D(r)} v_k(\tau) > \sum_{D(r)} v_k(\tau')$.

Suppose $r > x^u(\tau')$. Then result (iv) of Theorem 2 implies $\sum_{D(r)} v_k(\tau') > .5$, and hence $\sum_{k\in D(r)} v_k(\tau) > \sum_{k\in D(r)} v_k(\tau') > .5$. In this case r fails to meet condition (a) of (22), and is not a local majority equilibrium with respect to τ. But this contradicts result (ii) of Theorem 2. Hence we must have $x^u(\tau') \geqq r$, and thus $\tau' > \tau$ implies $x^u(\tau') \geqq x^u(\tau)$: $x^u(\tau)$ is monotonically increasing in τ. Q.E.D.

With these definitions and preliminary results in hand, we can now turn to the question of the existence and nature of local superequilibria in the Genossenschaften model. The results to be established comprise the following theorem:

Theorem 3.

(i) Under the assumptions of Section 2 and the further assumption (19), there exists a local superequilibrium, τ^+ .

(ii) A sufficient condition for the existence of a positive local superequilibrium $\tau^+ > 0$ is that $x^u(\tau) > \tau$ for some feasible $\tau \in T$.

(iii) A sufficient condition for the existence of a positive local superequilibrium is that for each household $j \in \mathcal{H}$ there be a $\delta_j > 0$ such that any τ' in the interval $(0, \delta_j)$ is strictly preferred to $\tau = 0$ by household j and the households control the board at $\tau = 0$, that is $\sum_{j=1}^{n} \Gamma_j > \sum_{i=1}^{m} G_i(0)$.

Proof. It will be convenient to prove (ii) first.

(ii) We must show that if $x^u(\tau) > \tau$ for some $\tau \in T$, there exists a strictly positive local superequilibrium. Let $\tau^+ = \sup\{\tau \in T | x^u(\tau) > \tau\}$. Clearly, there exists a monotonically increasing sequence (τ^ν) such that $(\tau^\nu) \to \tau^+$ and $x^u(\tau^\nu) > \tau^\nu$ for all ν . Once again, denote $x^u(\tau^\nu)$ by x^ν . From Lemma 4, $x^u(\tau)$ is monotonically increasing in τ so that (x^ν) is a bounded monotonic sequence. Hence, it converges to some limit x^+ , and from Lemma 3, x^+ is a local majority equilibrium with respect to τ^+ : $x^+ \in \xi(\tau^+)$. Moreover $x^\nu > \tau^\nu$ for all ν in the sequence implies that $x^+ \geqq \tau^+$.

In fact, $x^+ = \tau^+$. Suppose the contrary, namely, $x^+ > \tau^+$. Define $\rho = x^+ - \tau^+ > 0$ and let $\tilde{\tau} = \tau^+ + \frac{\rho}{2}$. With $\tilde{\tau} > \tau^+$, Lemma 4 implies $x^u(\tilde{\tau}) \geqq x^u(\tau^+)$. Since $x^+ \in \xi(\tau^+)$ implies $x^u(\tau^+) \geqq x^+$, we have

$x^u(\tilde{\tau}) \geqq x^+$. Therefore, $x^u(\tilde{\tau}) - \tilde{\tau} \geqq x^+ - \tilde{\tau} = x^+ - (\tau^+ + \frac{\rho}{2}) = \frac{\rho}{2} > 0$. Then we have $\tilde{\tau} > \tau^+$ and $x^u(\tilde{\tau}) > \tilde{\tau}$, contradicting the definition of τ^+ . Hence, $x^+ = \tau^+$, and we have established the existence of a positive tax rate, namely τ^+ , such that τ^+ is a local majority equilibrium when voting strengths are determined by τ^+ . Thus, τ^+ is a positive local superequilibrium.

(i) A local superequilibrium (not necessarily positive) must always exist. Given the above proof of (ii), the only case remaining has $x^u(\tau) \leqq \tau$ for all $\tau \in T$. But since $0 \in T$, this means $x^u(0) = 0$ and 0 is then a local superequilibrium tax rate.

(iii) From (20) and the hypothesis of (iii), it is clear that

$I(0) = \mathcal{H}$. Thus, $\sum_{I(0)} v_k(0) = \frac{\sum_{j=1}^{n} \Gamma_j}{J(0)} > .5$ because of the assumption that $\sum_{j=1}^{n} \Gamma_j > \sum_{i=1}^{m} G_i(0)$. Hence, $\tau = 0$ does not satisfy condition (b) of (22) for the distribution of votes $v_k(0)$. Since it follows from Theorem 2 that there exists a local majority equilibrium for any $\tau \in T$, $x^u(0) > 0$, and the condition of (ii) of Theorem 3 is satisfied for $\tau = 0$. Q.E.D.[1]

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[1] It might be noted that while Theorem 1 for the existence of a global superequilibrium was provèd by means of a familiar result, the Kakutani fixed-point theorem, it could instead have been established using the approach of Theorem 3.

5. Implications of the Results

The existence theorems of Sections 3 and 4 shed light on the operation of the Genossenschaften. First, the theorems demonstrate that only very mild conditions are needed to ensure the existence of a local superequilibrium. Of course, to ensure the existence of a global superequilibrium, one needs further assumptions--ones that ensure single-peakedness of consumer household and firm preferences over tax rates--and these are more restrictive. The theorems not only demonstrate the existence of superequilibria, but they also show how a positive superequilibrium tax rate can arise. Despite the fact that votes are distributed in proportion to how much each member "breaks the law"--the biggest polluters receiving the most votes--there exist equilibrium positions for the system in which an "effective law" exists, that is, in which the tax rate is positive. Moreover, as Figures 6b and 6c illustrate, the characteristics of the firms, the households, and the treatment technology available to the genossenschaft can admit a positive tax rate as a superequilibrium even if a zero tax rate--and hence no waste clean-up--is also a superequilibrium.

Second, the theorems demonstrate that positive tax rates can emerge as superequilibria, and provide sufficient conditions for this type of superequilibrium to exist. Indeed, result (iii) of Theorem 1 provides a condition under which all global superequilibrium tax rates are positive. And, with a little further work--but no further assumptions--part (iii) of Theorem 3 could be strengthened to show that under the condition given there, <u>all</u> local superequilibrium tax rates are positive.[1]

In viewing how successful the Ruhr area <u>Genossenschaften</u> have been in providing water suitable for the needs of households as well as mines and factories, it is interesting to note how institutional features of several of the river associations seem aimed at achieving the vote-distribution condition in Theorems 1(iii) and 3(iii). Specifically, these sufficient conditions require that the households control the board when votes are based on the zero-tax effluents (that is, when the firms' power is greatest): $\sum_{i=1}^{m} G_i(0) < \sum_{j=1}^{n} \Gamma_j$. Compare this with the <u>Lippeverband</u>'s requirement that mining companies must not have more than 40% of all votes in the General Assembly.[2] And, while the model presented here has abstracted from the question of upstream versus downstream interests, it is reasonable to identify upstream groups as low-tax preferrers and downstream groups as high-tax preferrers. Hence, the <u>Niersverband</u>'s by-laws giving downstream interests a bloc of 75 votes before the remaining 225 votes in the association are distributed according to the size of members' contributions[3] also resemble

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[1]The additional work involves proving results analogous to (ii) and (iv) of Theorem 2 for $x^L(\tau) \equiv \inf\{x \in \xi(\tau)\}$. An argument analogous to that used in proving Lemma 4 then shows that $x^L(\tau)$ is monotonically increasing in τ. Since the proof of (iii) of Theorem 3 shows that zero is not a local majority equilibrium tax rate with respect to $\tau = 0$, we have $x^L(0) > 0$ and $x^L(\tau) > 0$ for all $\tau \in T$. Hence, all local superequilibria will have positive tax rates.

[2]Section 10 Paragraph 8 of the Act establishing the <u>Lippeverband</u>, 19 January 1926, as cited in a document provided by W.A. Irwin which describes the representation system of the <u>Lippeverband</u>.

[3]Irwin, <u>op.cit</u>., p. 54.

an effort to attain the real-world analogue of our zero-tax vote-distribution condition: the households have more votes than the firms when $\tau = 0$.

Third, it is interesting to compare the water-quality standard the Genossenschaften voting procedure yields with the standard that emerges under other voting mechanisms. We restrict ourselves to some comparisons of the Genossenschaften approach with simple majority voting. To simplify matters, suppose that the single-peakedness condition is met (so that a global superequilibrium exists for the Genossenschaften method), and also suppose that the number of decisionmakers is odd: $m+n$ is odd. Under these conditions, it follows from Black's work that simple majority voting will yield the median most-preferred tax rate as the unique voting equilibrium. Obviously, if there are more firms than communities[1] $(m > n)$, simple majority rule will lead to a zero tax rate. In this case, the water-quality standard in the Genossenschaften global superequilibrium must be at least as high as that under simple majority rule. If, moreover, one of the sufficient conditions in Theorem 1 is met, then the Genossenschaften method will definitely lead to higher water quality as its G.S. will have $\tau^* > 0$.

On the other hand, if communities outnumber firms $(n > m)$, the conclusion is less clear. First, it is obvious that for simple majority voting to yield a positive tax rate, the number of communities with zero as their most-preferred tax rate (τ_j^*) must be strictly less than $\frac{n-m}{2}$. If

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[1]While the basic decisionmaking units in our model are firms and households, the basic agents in the Genossenschaften are, as noted in Section 1, firms and communities. Hence we frame this comparison--where the relative numbers of different types of agents is critical--in terms of the real decisionmaking units: firms and communities.

more than $\frac{n-m}{2}$ communities prefer zero to any other tax rate, the discussion of the $m > n$ case applies again. Consider the case where the number of communities with $\tau_j^* = 0$ is strictly less than $\frac{n-m}{2}$ and, for simplicity, suppose that each community's most-preferred tax rate is positive. If communities are indexed in order of increasing most-preferred tax rate, the simple majority voting equilibrium will be the τ_j^* of the community with index $j = \frac{n-m+1}{2}$. This tax rate may be lower or higher than the global superequilibrium tax rate for the <u>genossenschaft</u>.

For example, if the firms control the board when voting strengths are determined by $\tau = 0$, and if there does not exist a G.S. with a positive tax rate, as in Figure 6a, then clearly simple majority voting leads to a higher tax rate and higher water quality than does the <u>genossenschaft</u> rule. On the other hand, suppose the following circumstances exist: (a) $n = m+1$; (b) the community with index 1 pollutes very little relative to <u>any</u> other community so that $\frac{\Gamma_i}{\sum_{j=1}^{n} \Gamma_j}$ is much smaller than $\frac{1}{n}$; (c) the most-preferred tax rate of community 1, τ_1^*, is very low relative to the desires of the other communities; and (d) the communities control the board when votes are determined by the zero-tax effluent levels. In this case, the minimum value for a global majority equilibrium at $\tau = 0$ is greater than or equal to τ_1^*. But then, by the results of Theorem 1, the global superequilibrium tax rate will be greater than or equal to τ_1^*, and if $\frac{\Gamma_1}{\sum_{j=1}^{n} \Gamma_j}$ is small enough, the global superequilibrium will definitely have

$\tau^* > \tau_1^*$. The genossenschaft's water quality standard $S(\tau^*)$ will be superior to that produced by simple majority voting $S(\tau_1^*)$. More generally, whether water quality is higher under simple majority voting or under genossenschaft voting depends on all the characteristics of the region involved: the $G(\tau) = \sum_{i=1}^{m} G_i(\tau)$ function (specifically, the magnitude of $G'(\tau)$ for $\tau \in T$), the utility functions of the communities, the waste discharges of the communities (specifically, the magnitude of $\sum_{j=1}^{n} \Gamma_j$ relative to $G(\tau)$ at various values of $\tau \in T$ and the importance of the different communities in the community voting bloc, as measured by $\frac{\Gamma_j}{\sum_{j=1}^{n} \Gamma_j}$ for each j), and the treatment technology of the genossenschaft (as it affects $S(\tau)$ and hence the most-preferred tax rates of the various communities.

Finally, our analysis of the Genossenschaften produces some interesting implications for the effect of improvements in the association's treatment technology. We restrict our attention to the model in Section 3 for which a global superequilibrium exists. Consider a change in the pollution technology that increases the amount of waste removed for each positive level of expenditure but is still marked by decreasing or constant returns to scale. The genossenschaft now possesses a technology $K = \hat{\phi}(E)$ with $\hat{\phi}(E) > \phi(E)$ for all $E > 0$ but $\hat{\phi}''(E) \leqq 0$. As a result, at each positive tax rate τ for which $S(\tau) > 0$, there is now less untreated waste than there was under the old technology: $\hat{S}(\tau) < S(\tau)$ for all $\tau > 0$ such that $S(\tau) > 0$. The best water quality standard available for any given positive tax rate has improved.

This change has no effect on the distribution of votes for any given tax rate τ . When tax rate τ prevails, the i^{th} firm still has the fraction $\frac{G_i(\tau)}{J(\tau)}$ of the total vote and the j^{th} household still has the fraction $\frac{\Gamma_j}{J(\tau)}$ of the total vote, and Γ_j , $G_i(\tau)$, and hence $J(\tau)$ are the same as before. Furthermore, the improvement in technology does not alter firms' tax-rate preferences. They remain single-peaked with the peak at $\tau = 0$. Lastly, the technological change maintains the convexity of the opportunity set A_j facing the j^{th} household, and this is true for all $j = 1, \ldots, n$. Hence, each household's tax-rate preferences remain single-peaked.

What does change is the tax rate at which each household's preference ordering reaches its peak. One illustration of the change effected by technological improvement is presented in Figure 9. The j^{th} household's pre-improvement and post-improvement opportunity sets are derived and its optimal positions in the two situations are shown. The symbols without carets and the solid curves indicate the pre-technological improvement situation; the symbols with carets and the dashed curves show the situation after the change. In the case shown in Figure 9, the household's optimum after the technological change shows a higher water quality standard than before the improvement ($\hat{S}^B < S^B$) and a higher level of consumption than in the preimprovement situation $(\hat{C}_j^B > C_j^B)$. Since $C_j = \overline{Y}_j - \tau\Gamma_j$ and $\overline{Y}_j$ and Γ_j are fixed, the higher level of consumption in the new optimum means that the household's most-preferred tax rate has decreased: $\hat{\tau}_j^* < \tau_j^*$ as the figure shows.

But Figure 9 presents only one possible effect of technological change. The improvement may lead to an increase in a household's most-preferred tax

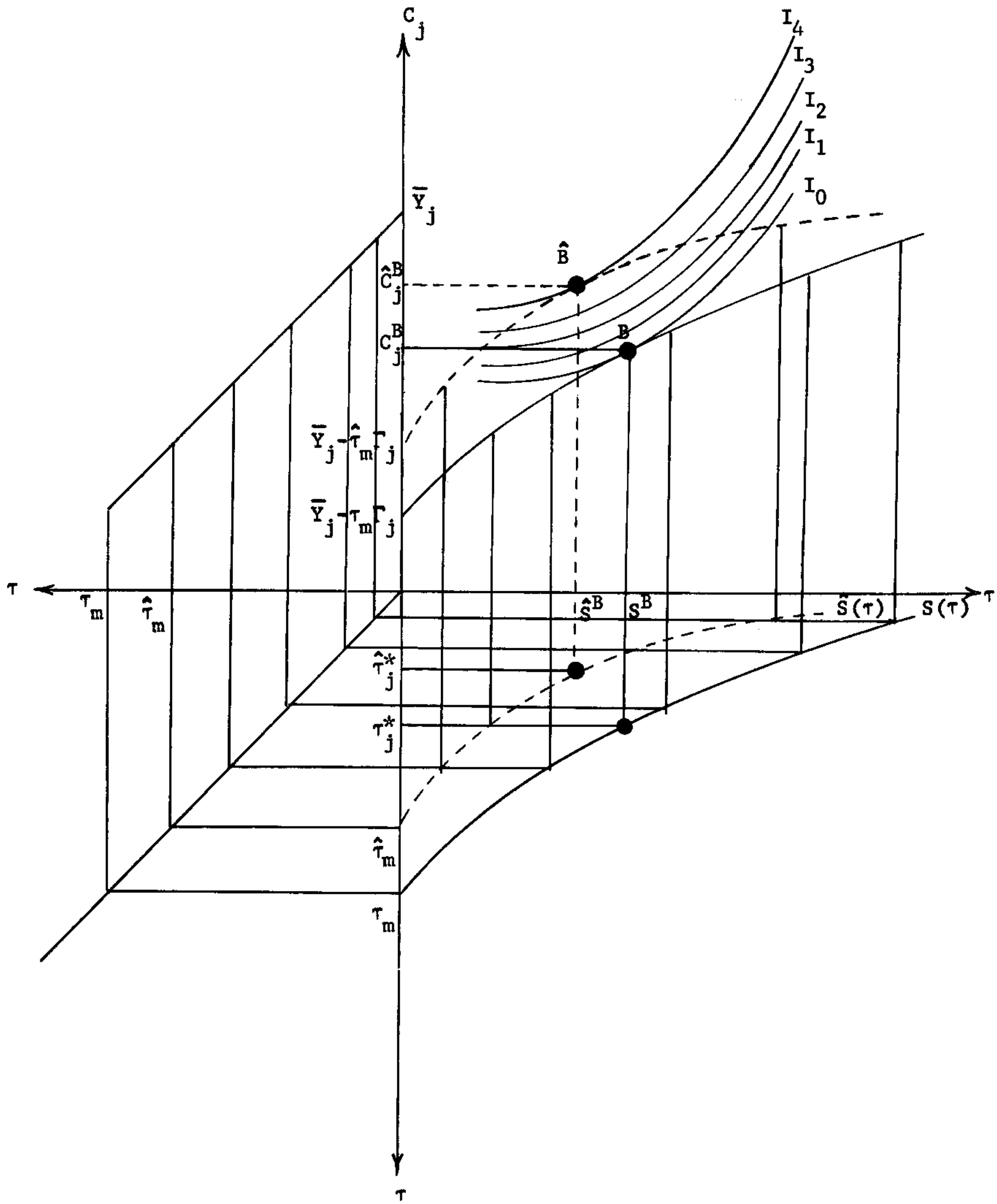

FIGURE 9

The Position of the j^{th} Household Before and After
The Technological Change

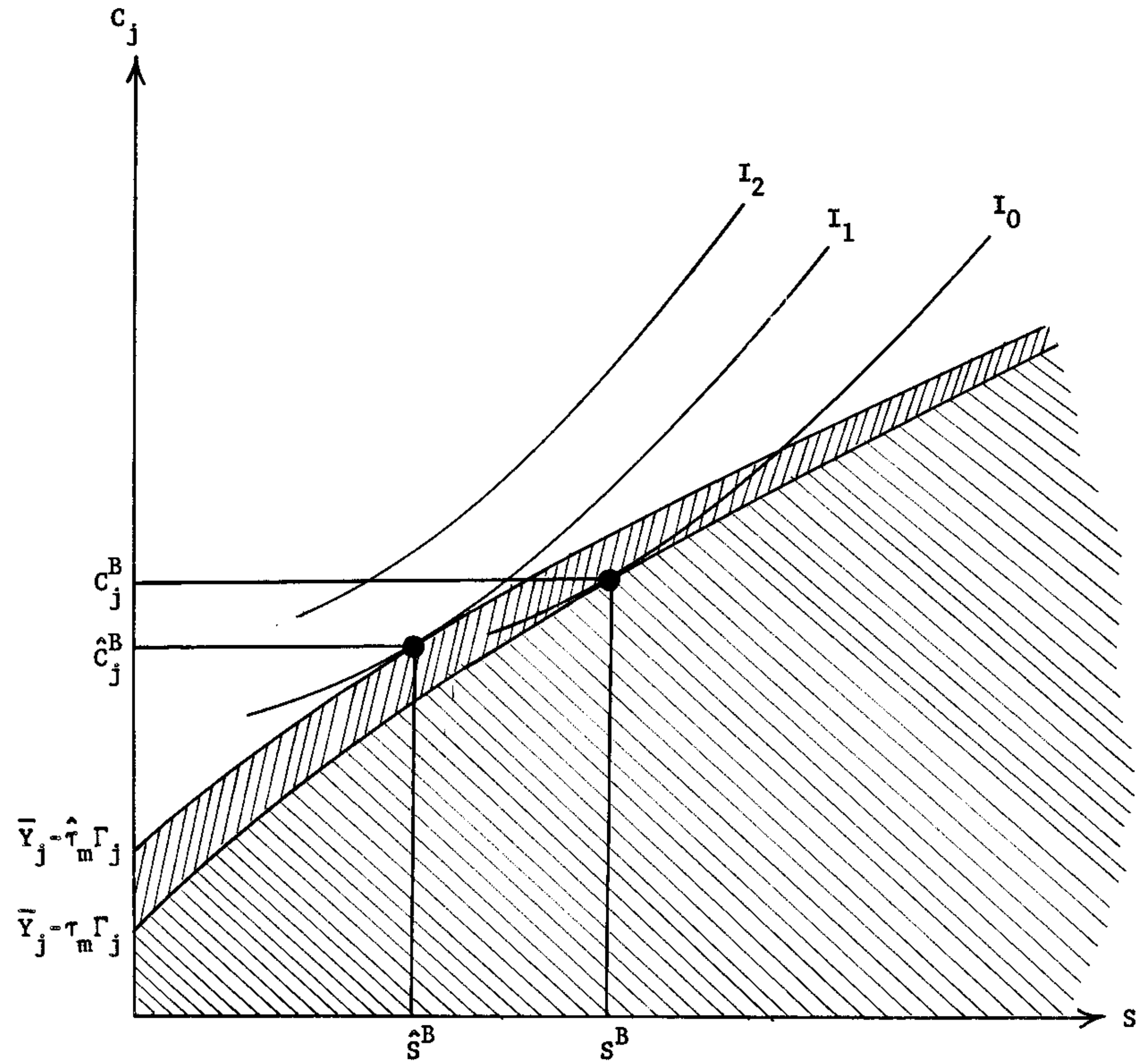

FIGURE 10

Another Example of the Effect of Technological Change rate. This possibility is illustrated in Figure 10, where only the first quadrant of our usual four-quadrant diagram is shown. The technological improvement is reflected in the expansion of the household's opportunity set from the striped area to the striped area plus the shaded area. For this household, the change shown leads to an improvement in its optimal water-quality standard $(\hat{S}^B < S^B)$ but a decrease in its optimal level of consumption $(\hat{C}_j^B < C_j^B)$. Since the household's gross income and its waste dischange are fixed, and since $C_j = \overline{Y}_j - \tau\Gamma_j$, this corresponds to an increase in the household's most-preferred tax rate.

The effect technological improvement has on the global superequilibrium tax rate(s) depends on how each household's most-preferred tax rate changes as a result of the improvement and on the weight each household has within the household voting bloc. The G.S. tax rate(s) may rise or fall. Consider the two extreme cases. First, suppose that each household is like the household in Figure 10 so that the technological progress increases each household's most-preferred tax rate. The net result is that for every $\tau \in T$ with $0 < t^L(\tau) < \tau_m$, $t^L(\tau)$ increases and for every $\tau \in T$ with $0 < t^U(\tau) < \tau_m$, $t^U(\tau)$ increases.[1] Hence, except for the part (if any) of the graph lying along the horizontal axis and the part (if any) of the graph coincident with the horizontal line drawn at τ_m, the entire graph of the matting $\psi : T \to T$ is shifted upward, as shown in Figures 11a and 11b. In each case, the original situation is shown by the solid lines while the situation after technological progress occurs is depicted by the broken lines.

[1] Recall that $t^L(\tau)$ is the minimum g.m.e. with respect to τ and $t^U(\tau)$ is the maximum g.m.e. with respect to τ.

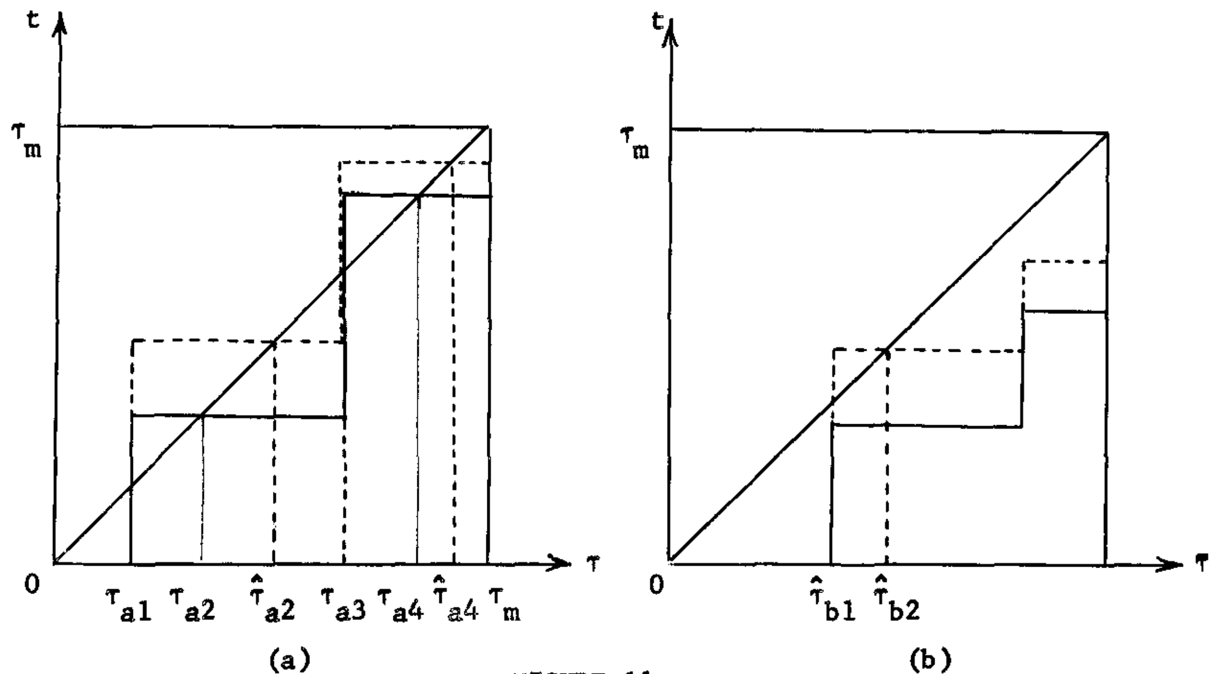

FIGURE 11

The Effect of Technological Change When All Households Are as in Figure 10

In Figure 11a, the original global superequilibrium tax rates were 0, τ_{a1}, τ_{a2}, τ_{a3}, and τ_{a4}. After the technological improvement, 0, τ_{a1}, and τ_{a3} are still G.S. tax rates but τ_{a2} and τ_{a4} are not. The latter two are replaced by $\hat{\tau}_{a2} > \tau_{a2}$ and $\hat{\tau}_{a4} > \tau_{a4}$. Technical change might reduce the number of global superequilibria. For example, if the graph of $\psi(\tau)$ had shifted even further upward while at the same time the difference between $t^U(\tau_{a3})$ and $t^L(\tau_{a3})$ had been reduced sufficiently, τ_{a3} would not be a G.S. effluent charge under the new technology. Figure 11b shows just the opposite situation. Technological progress creates two positive global superequilibrium tax rates $\hat{\tau}_{b1}$ and $\hat{\tau}_{b2}$ whereas $\tau = 0$ was the only G.S. before the improvement. Whether the number of global superequilibria increases, decreases, or remains the same when technology changes, so long as each household's situation is similar to that in Figure 10, one general result obtains. Namely, the improvement in the treatment technology <u>increases</u> or leaves unchanged the minimum and maximum global superequilibrium tax rates. The minimum is left unchanged only if it was originally zero, and the maximum is left unchanged if and only if it was originally equal to the maximum value in T .

Turning to the other extreme case, assume that all households resemble the one in Figure 9. Technological change of the type shown there reduces the most-preferred tax rate of every household. The net result is that for every feasible τ such that $0 < t^L(\tau) < \tau_m$, $t^L(\tau)$ decreases, and for every $\tau \in T$ such that $0 < t^U(\tau) < \tau_m$, $t^U(\tau)$ decreases. If $\tau = 0$ is a global majority equilibrium with respect to any τ under the original technology, it is a g.m.e. with respect to that τ under the new, improved technology. On the other hand, the new global majority equilibria for tax rates in the interval (if there is one) for which τ_m is the original g.m.e. depend on exactly how the households' most-preferred tax rates change and on each household's voting strength in the household bloc.

Figures 12a and 12b illustrate two possible situations resulting from technological change when all households are like the one in Figure 9. Once again, the original conditions generate the solid-line graphs while the post-improvement conditions generate the broken-line graphs. In Figure 12a, the original global superequilibrium tax rates are 0 , $\tau_{\alpha 1}$, $\tau_{\alpha 2}$, $\tau_{\alpha 3}$, and $\tau_{\alpha 4}$. The technical change leaves 0 , $\tau_{\alpha 1}$, and $\tau_{\alpha 3}$ as G.S. rates but it replaces $\tau_{\alpha 2}$ and $\tau_{\alpha 4}$ by $\hat{\tau}_{\alpha 2} < \tau_{\alpha 2}$ and $\hat{\tau}_{\alpha 2} < \tau_{\alpha 2}$, respectively. This illustrates the general result that obtains when all households are

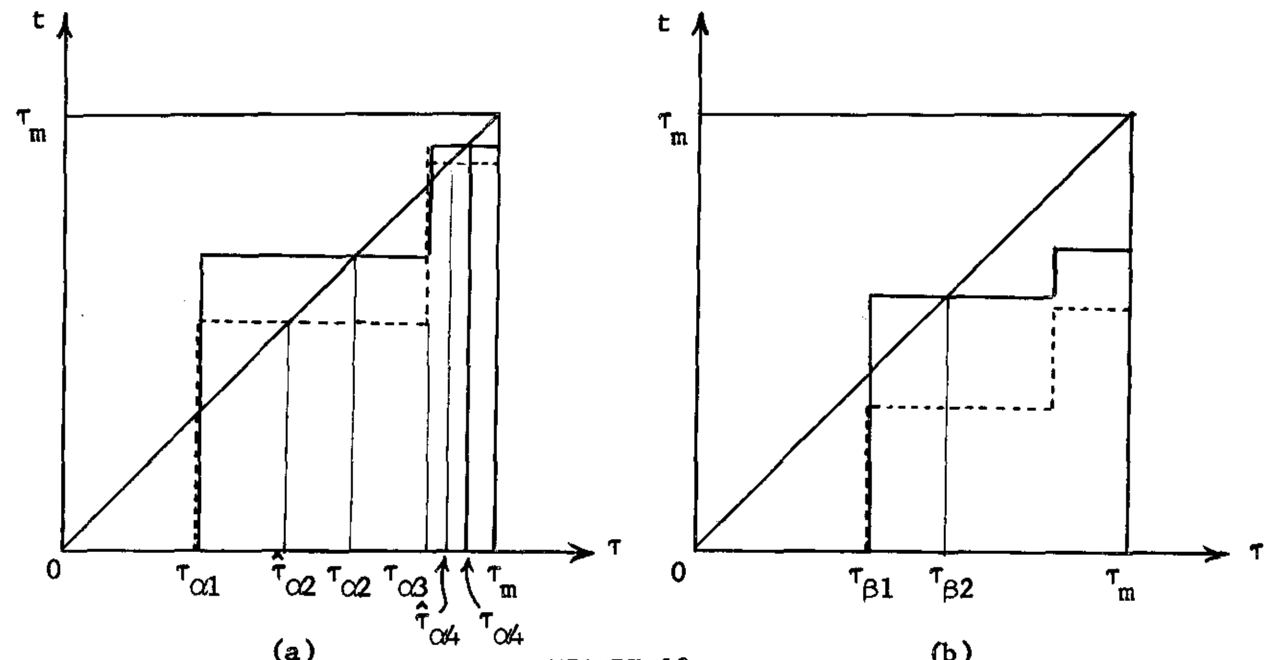

FIGURE 12

The Effect of Technological Change When All Households
Are as in Figure 9

positioned as in Figure 9. The improvement in the treatment technology <u>decreases</u> or leaves unchanged the minimum and maximum global superequilibrium tax rates, with no change taking place in the minimum if and only if it was zero originally, and no change in the maximum only if it was originally the maximum value in T , τ_m .

As was the case when Figure 10 described each household's position, the technological progress may increase the number of global superequilibria. This would occur, for example, if the original $\phi(\tau)$ graph in Figure 12a had another "step" lying totally above the 45^o line. The shifting downward of the graph might, then, generate a new G.S. on this "step."

What creates a problem, however, for those concerned with water quality is the possibility that when the households are like those in Figure 9, technological progress in treating waste can reduce the number of global superequilibrium tax rates and lead to a lowering of water quality. The potential difficulties are illustrated by Figure 12b. If the vote were taken before the technological improvement in treating waste, there would exist two positive global superequilibrium tax rates: $\tau_{\beta 1}$ and $\tau_{\beta 2}$. But after technological progress has altered each household's opportunity set and decreased each one's most-preferred tax rate, the only G.S. tax rate is zero. There is no treatment of waste discharges, and firm waste emissions are at their maximum levels. Hence, under the circumstances shown in Figure 12b, the *Genossenschaften* voting mechanism translates an increased ability to clean up waste for any given positive expenditure level into the lowest possible water quality standard: $S(0)$.