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### Further Evidence on the Estimation of Dynamic Economic Relations from a Time Series of Cross-Sections

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COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

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FURTHER EVIDENCE ON THE ESTIMATION OF DYNAMIC ECONOMIC RELATIONS  
FROM A TIME SERIES OF CROSS-SECTIONS

Marc Nerlove

November 6, 1968

FURTHER EVIDENCE ON THE ESTIMATION OF DYNAMIC ECONOMIC RELATIONS  
FROM A TIME SERIES OF CROSS-SECTIONS\*

by

Marc Nerlove  
Yale University

"... the Dodo suddenly called out,  
'the race is over!' and they all  
crowded round it, panting, and  
asking, 'But who has won?'"

Alice's Adventures in Wonderland

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I am indebted to Mrs. E. Bockelman for programming assistance and to William Raduchel for his considerable help in the design and programming of the gradient procedure used to obtain the maximum-likelihood estimates. I have also benefited greatly from the critical comments of G.S. Maddala contained in his unpublished paper, "Pooling Cross-Section and Time-Series Data -- Comments on the Balestra-Nerlove Approach," although Maddala would, I believe, continue to disagree with many of the ideas here set forth.

## 1. Introduction

Data on a number of individual units (firms, households, geographical areas, etc.) over several periods of time are becoming increasingly available. Very often we would like to use such data to estimate a behavior relationship containing an autoregressive component, due, possibly, to a distributed lag or other dynamic factor affecting economic behavior. In an earlier study [3], Balestra and the present author studied the demand for natural gas using data on 36 states of the United States, over a six year period. We encountered a number of rather serious methodological problems in attempting to estimate a distributed lag model which appear to be of more general interest in view of the growing availability of data for individual units over several time periods. This paper reports the second of a series of experimental studies designed to explore the general methodological issues involved in studies of this type.

The first series of experiments, reported in [16], dealt with estimation of the following simple model:

$$(1) \quad y_{it} = \alpha y_{it-1} + u_{it}, \quad i = 1, \dots, N, \quad t = 1, \dots, T,$$

where the  $u_{it}$  are unobserved random variables such that

$$(2) \left\{ \begin{array}{l} u_{it} = \mu_i + v_{it} \\ E\mu_i = Ev_{it} = 0, \text{ all } i \text{ and } t, \\ E\mu_i v_{i't} = 0, \text{ all } i, i', \text{ and } t, \\ E\mu_i \mu_{i'} = \begin{cases} \sigma_\mu^2, & i = i' \\ 0, & i \neq i' \end{cases} \\ Ev_{it} v_{i't'} = \begin{cases} \sigma_v^2, & i = i', t = t' \\ 0, & \text{otherwise.} \end{cases} \end{array} \right.$$

In typical economic application the number of individuals,  $N$ , is much larger than the number of time periods,  $T$ , and in the first set of experiments and in those reported here  $N$  has been chosen two and one-half times as large as  $T$ :

$$N = 25$$

$$T = 10.$$

The random variables  $u_{it}$ , which disturb the autoregressive relationship (1), are supposed to represent the net effects of variables it has not been possible to include explicitly in the analysis. When one considers a pure cross-section and fits, for example, a simple linear relationship to the data, it is standard practice to assume that the large number of factors which affect the individuals in the sample and the value of the dependent variable observed for each of them, but which have not been explicitly included

as independent variables, may be appropriately summarized by a random disturbance.<sup>1/</sup> The assumed stochastic mechanism generating the disturbances becomes, then, a basis for econometric analysis. To be sure, it is important to consider carefully just what the disturbances may or may not represent, but their stochastic nature is a necessary assumption for econometric analysis.<sup>2/</sup>

When time-series data are considered a similar argument is made for the inclusion of stochastic disturbances in the relation or relations to be estimated. So much is well-known and widely understood, but when numerous individual units are observed over time the problem of specifying the stochastic nature of the disturbances becomes conceptually more difficult. It is clear in the abstract that some of the left-out variables will represent factors peculiar to both the individual units and the time periods for which observations are obtained, while other variables reflect individual differences which tend to affect the observations for a given individual in more-or-less the same fashion over more than one, and perhaps all periods of time. Still other variables may reflect factors peculiar to specific time periods but affecting individual units more-or-less equally.

A three-component model naturally suggests itself:

$$u_{it} = \mu_i + \lambda_t + v_{it} ,$$

$\mu_i$  represents the more-or-less time-invariant, individual effects,

$\lambda_t$  represents the period-specific and more-or-less individual-in-

variant effects and  $v_{it}$  represents the remaining effects which are

assumed to vary over both individuals and time periods. The fundamental question which must be answered prior to econometric analysis is whether or not to treat  $\mu_i$  and  $\lambda_t$  as parameters or as random variables. If we treat them as parameters, we may or may not wish to estimate their values explicitly; if we treat them as random variables we may, of course, only estimate certain moments of their joint distribution. This question is not a trivial one, for it makes a surprising amount of difference in the estimates of the other parameters in dynamic relations depending upon which approach is adopted.

The model, (1) and (2), investigated in the first series of experiments assumed period, individual-invariant effects,  $\lambda_t$ , to be absent, and that  $\mu_i$  and  $v_{it}$  were uncorrelated random variables with zero means. The assumed absence of period effects is largely a question of judgment in any particular application; however, what happens to the estimates when such effects are erroneously assumed to be absent will be the subject of further investigation. That  $\mu_i$  and  $v_{it}$  are assumed uncorrelated is essentially definitional and involves no loss of generality. The assumed absence of serial correlation among the  $v_{it}$  is more fundamental. When the random variables  $u_{it}$  are arranged in vector form, first by individuals, then according to period,

$$u = (u_{11}, \dots, u_{1T}, u_{21}, \dots, u_{2T}, \dots, u_{N1}, \dots, u_{NT})'$$

it can be seen that the variance-covariance matrix of these variables takes the following form:

$$(3) \quad E u u' = \sigma^2 \begin{bmatrix} A & 0 & \dots & 0 \\ 0 & A & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & A \end{bmatrix}$$

where

$$\sigma^2 = \sigma_{\mu}^2 + \sigma_v^2,$$

$$A = \begin{bmatrix} 1 & \rho & \dots & \rho \\ \rho & 1 & \dots & \rho \\ \vdots & \vdots & & \vdots \\ \rho & \rho & \dots & 1 \end{bmatrix},$$

and

$$\rho = \sigma_{\mu}^2 / \sigma^2$$

The parameter  $\rho$  is the so-called "intra-class correlation coefficient" of the classical random-effects model in the analysis of variance [5, pp. 222-26]. (The model in this form was used by Kuh [13] in his work on the relation between time-series and cross-section estimates, except that he did not assume that the individual effects,  $\mu_i$ , and the time-varying effects,  $v_{it}$ , were uncorrelated.) It can be seen that the assumptions made concerning the  $u_{it}$  amount to the assumption of a very specific form of serial correlation; indeed, one might argue that the model is simply an approximation to a more



realistic one which allows rather pronounced serial dependence among the disturbances for the same individual, but only a negligible amount of dependence among disturbances for different individuals. To achieve a good approximation, it may be necessary to allow, for example, some negative serial correlation among the disturbances  $v_{it}$  for the same individual to counter-balance the exceptionally rigid formulation involving the random effect  $\mu_i$ , assumed to persist for all time periods over which a given individual is observed. The effects of mis-specification in this direction will also be investigated at a later date.

The question of whether or not the individual effects may be treated as constant parameters for the purpose of statistical analysis is related, not only to the nature of the approximation discussed above, but also to the underlying mechanism assumed and the purpose of the analysis. Thus, for example, consider a number of individuals; for each of these we suppose some stochastic mechanism chooses certain individual effects,  $\mu_i$  and another mechanism chooses a number of further disturbances,  $v_{it}$ , for each. These two effects are then summed to give us  $NT$  disturbances,  $T$  for each of  $N$  individuals. If we think of extending the sample by  $\theta$  periods for the same individual units by choosing  $\theta$  more  $v_{it}$ 's, but retaining the same  $\mu_i$  with which we began, it is apparent that something useful can be said about the new observations by explicitly estimating the values  $\mu_i$ ,  $i = 1, \dots, N$ , i.e. treating these as parameters

in the problem. This is analogous to the problem of predicting from a regression equation in which the disturbances are known to be serially correlated. For such prediction it is helpful not only to have good estimates of the coefficients (i.e., estimates which take account of the serial correlation of the disturbances), but estimates of past values of the disturbances themselves.<sup>3/</sup> Estimation, however, is not quite the same as prediction and it is far from clear that treating the  $\mu_1$ 's as parameters will lead to good estimates of the true parameters of the system. Indeed, it was found in the series of experiments reported in [16] that estimates of  $\alpha$  biased downwards were obtained when each  $\mu_1$  was estimated explicitly. If the standard econometric approach is taken to the disturbances  $u_{it}$ , no part, not even the  $\mu_1$  should be regarded as non-stochastic or fixed in repeated samples. Treating part as if it were fixed, but unobserved, is one possible approach to estimation, but it is only one and it is far from clear a priori that it is better than some other approach.

Several recent investigations have examined the question of estimation in models similar to (1) and (2). Wallace and Hussain [19] and Hussain [11] consider estimation of non-dynamic relationships from cross-sections over time; in the latter, various approaches applicable to simultaneous equations are considered; the former, and evidently more recent paper, compares analytically a method equivalent to treating  $\mu_1$  (and  $\lambda_t$ ) as parameters, and one based explicitly on notion that these are random variables. In the second case, estimates of the moments of  $\mu_1$  and  $v_{it}$  (and  $\lambda_t$ ) are obtained from the calculated residuals of an ordinary least squares, and the estimated

variance proportions are then used to transform the data before running a second regression. Wallace and Hussain show that, provided the moment matrix of the independent variables (all assumed non-stochastic) tends to a positive definite matrix as both  $N$  and  $T$  tend to infinity, both methods lead to asymptotically equivalent estimates with asymptotically equal variance-covariance matrices. Under suitable additional assumptions, both methods yield asymptotically normal estimates. Wallace and Hussain do not consider situations in which a lagged value of the dependent variable is one of the explanatory variables. Unfortunately, examples of this sort are exceptionally common in the analysis of economic data, and a great deal of complexity is introduced into the problem when a dynamic structure is postulated.

Amemiya [1] has considered the problem of estimating a model such as (1) and (2), i.e., one including a lagged value of the dependent variable. His model also includes truly exogenous variables as explanatory as well. Amemiya obtains the asymptotic distribution for fixed  $N$  and  $T \rightarrow \infty$  for the following three estimates of the regression coefficients: (1) Maximum-likelihood; (2) least-squares, assuming the  $\mu_1$  are fixed constants; and (3) generalized least-squares based on a consistently estimated value of  $\rho$  (what I have called the two-round estimates) or even one which is arbitrarily guessed. Not unexpectedly Amemiya shows that it is impossible to obtain separate consistent estimates of  $\rho$  and  $\sigma^2$  when  $N$  is fixed. That this is so should be intuitively obvious since the distributional properties of the random variables can hardly be determined with greater precision

as the sample size is increased as long as the sample size is not permitted to increase in a direction which could yield more information in this connection, i.e., with respect to the number of individuals. When both  $N$  and  $T$  are allowed to tend to infinity, Amemiya shows that all three estimates are consistent and asymptotically efficient provided that the moment matrix of the independent variables converges to a positive definite matrix, regardless of the way in which  $N$  and  $T$  increase.<sup>4/</sup>

Asymptotic results are useful largely to the extent that they can serve as a guide to what we may find in the samples of small or moderate size with which we typically deal in econometrics.<sup>5/</sup> It is apparent that the results of Wallace and Hussain and Amemiya may not serve us well in this respect since, asymptotically, they do not distinguish between various estimation procedures. Indeed Amemiya's results suggest ordinary least-squares, with no allowance for unobserved individual differences are asymptotically as good as full maximum-likelihood, two-round procedures using an estimated value of  $\rho$ , and methods based on treating the  $\mu_i$  as constant parameters to be estimated. Since it was found in [16] that ordinary least-squares, either ignoring the  $\mu_i$ 's or treating them as constants to be estimated, gave exceedingly poor estimates of  $\alpha$  for relevant sample sizes, it is apparent that existing large-sample asymptotic results are not helpful in problems of this sort. Consequently, I believe that Monte Carlo experiments of the sort described here and in [16]

may serve a useful purpose in increasing our understanding of the problems of estimation of dynamic economic relations from a time series of cross-sections.

In the experiments reported in [16] no exogenous variable was included. This not only made it impossible to ascertain the properties of instrumental variable estimates and related two-round procedures, but may very well have significantly affected the properties of those estimates whose distributions were investigated.<sup>6/</sup> Indeed, as we shall see below the relative performance of full-maximum likelihood and two-round procedures is affected by the inclusion of an exogenous variable.

Five methods of estimation were compared in the initial set of experiments:

- (a) Generalized least-squares estimates employing the true value of  $\rho$  (used to generate the observations).
- (b) Ordinary least-squares estimates, which are the same as (a) for  $\rho = 0$ .
- (c) Least-squares estimates of  $\alpha$  and an over-all constant term obtained from regressions containing individual constant terms, i.e., analyses treating the  $\mu_i$  as parameters to be estimated.
- (d) Two-round estimates based upon a value of  $\rho$  estimated by means of the individual constants from (c) by computing a

variance of these constants and forming the ratio of this estimate to the sum of this estimate and the estimated residual variance of the regression, i.e., the same as (a)

$$\text{but using } \rho = \frac{\hat{\sigma}_\mu^2}{\hat{\sigma}_\mu^2 + \hat{\sigma}_\nu^2} .$$

- (e) Maximum-likelihood estimates based on a search procedure for  $\rho$  in the interval  $[0, 1)$ .

In [16] it was noted that, for fixed N, the estimates obtained by (a) and those obtained by (c) tend to a common value. For  $N, T \rightarrow \infty$  such that  $N/T \rightarrow 0$ , estimates (a), (d) and (e) tend to a common value in probability, albeit the estimates may have different asymptotic distributions. Thus, asymptotic theory casts relatively little light on the comparative small sample properties of the estimates. In the experiments reported in [16], the following conclusions regarding these small sample properties emerged:

1. Despite the small number of observations over time, the small sample bias known to exist in the estimation of autoregressive schemes is surprisingly small for generalized least-squares using the true value of  $\rho$ . While this method is not one which may be used in practice, the result does indicate the importance of information concerning the value of  $\rho$  and the great relevance of that information in estimation.
2. Ordinary least-squares estimates of  $\alpha$  are seriously biased upwards when  $\rho$  is different from zero. The corresponding

- estimates of  $\sigma^2$  based on the calculated residuals from an ordinary least-squares regression are strongly biased towards zero.
3. Least squares with individual constant terms yields estimates of  $\alpha$  biased towards zero. Furthermore, the implied estimate of  $\rho$ , based on the "variance" of the constant terms and the estimated residual variance, is biased upwards despite the fact that the implied estimate of  $\sigma^2 = \sigma_{\mu}^2 + \sigma_{\nu}^2$  is also biased upwards.
  4. The distribution of the two-round estimates of  $\alpha$  and the distribution of the maximum likelihood estimates are very similar. There is some bias downwards for large true  $\alpha$  and some upwards for small true  $\alpha$ . The estimates of  $\sigma^2$  are highly erratic for the maximum-likelihood method and some what less so for the two-round method. The very implausible behavior of the maximum-likelihood method found in the study of the demand for natural gas [5] was not, however, reproduced in the experiments.

The experiments reported below differ from the earlier series in three respects: First, an exogenous variable is introduced in the relationship to be estimated. Second, the presence of an exogenous variable allows us to explore the behavior of instrumental-variable estimates and of two-round estimates based on an instrumental variable first stage. Third, a gradient procedure for obtaining the maximum-

likelihood estimates is used. The findings confirm the bias of the ordinary least-squares estimates and those based on least-squares with individual constant terms. Furthermore, contrary to the conjecture expressed in [16] that the curious behavior of the maximum-likelihood estimates found in [3] could probably not be reproduced without the introduction of specification error, we find here that such behavior occurs with non-negligible frequency when exogenous variables with the sorts of time paths assumed here are introduced.



## 2. Design of the Experiments

Generation of the observations. -- Except for inclusion of an exogenous variable, the model used to generate the observations was that previously employed. Arranging the observations first by individual, then according to period, and defining

$$y = (y_{11}, \dots, y_{1T}, \dots, y_{N1}, \dots, y_{NT})' ,$$

$$y_{-1} = (y_{10}, \dots, y_{1T-1}, \dots, y_{N0}, \dots, y_{NT-1})' ,$$

$$x = (x_{11}, \dots, x_{1T}, \dots, x_{N1}, \dots, x_{NT})' ,$$

$$u = (u_{11}, \dots, u_{1T}, \dots, u_{N1}, \dots, u_{NT})' ,$$

the model becomes

$$(1) \quad y = \alpha y_{-1} + \beta x + u .$$

$$(2) \quad E u u' = \sigma^2 \begin{bmatrix} A & 0 & \dots & 0 \\ 0 & A & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & A \end{bmatrix} = \sigma^2 \Omega ,$$

where

$$A = \begin{bmatrix} 1 & \rho & \dots & \rho \\ \rho & 1 & \dots & \rho \\ \vdots & \vdots & & \vdots \\ \rho & \rho & \dots & 1 \end{bmatrix} .$$

The first step in the generation of the observations for various sets of parameter values  $\alpha$  ,  $\beta$  ,  $\rho$  , and  $\sigma^2$  was the

generation of a set of exogenous variables, held fixed through the entire set of experiments. While a set of real exogenous variables, say state employment figures over the last 10 or twenty years, could have been used, there are several advantages in using an artificially generated series. The exact characteristics of the exogenous variables may be controlled if they are artificially generated. This means that the series may have many of the properties of real exogenous variables, serial correlation, trend, and the like, without some of the undesirable properties found in particular series. To be sure we have to live with these undesirable properties in real estimation problems, but, in real estimation problems, we generally have more than one exogenous variable. The extreme smoothness and trend-like character of, say, state employment figures is not characteristic of some linear combination of, for example, these, prices, and weather variables. Since the nature of the exogenous variables present may quite profoundly affect that character of the estimates obtained by various methods, it is important, not only to know this nature exactly, but to be able to vary it systematically. Such variation is not attempted here, but will form the basis of subsequent experiments.

The exogenous variables were generated by choosing a random variable  $\omega_{it}$ , uniformly distributed on the interval  $[-1/2, 1/2]$  and forming

$$(3) \quad x_{it} = 0.1t + 0.5x_{it-1} + \omega_{it} .$$

TABLE 2.1: Values of the Exogenous Variables

| T= | 1  | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11   | 12   | 13   | 14   | 15   | 16   | 17   | 18   | 19   | 20   |      |
|----|----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| I= | 1  | 1.36 | 0.77 | 1.11 | 0.66 | 1.22 | 0.95 | 1.01 | 1.28 | 1.75 | 2.14 | 1.99 | 2.34 | 2.25 | 2.33 | 2.94 | 2.57 | 2.49 | 2.74 | 3.53 | 3.67 |
|    | 2  | 3.58 | 1.58 | 1.05 | 0.87 | 0.67 | 0.84 | 1.58 | 1.52 | 2.01 | 2.18 | 1.70 | 2.13 | 2.34 | 2.66 | 2.86 | 2.73 | 3.25 | 3.55 | 3.43 | 3.54 |
|    | 3  | 8.50 | 4.35 | 2.55 | 1.19 | 1.20 | 1.53 | 1.09 | 0.93 | 1.23 | 1.54 | 1.96 | 2.00 | 2.74 | 2.48 | 2.62 | 3.17 | 3.16 | 3.64 | 4.11 | 4.51 |
|    | 4  | 5.59 | 2.57 | 1.47 | 0.78 | 1.14 | 0.68 | 0.89 | 1.30 | 2.03 | 1.88 | 2.20 | 2.75 | 2.83 | 2.77 | 2.99 | 3.52 | 3.56 | 3.67 | 3.67 | 3.63 |
|    | 5  | 0.33 | 0.43 | 0.67 | 0.65 | 0.43 | 0.99 | 0.72 | 1.47 | 1.81 | 2.23 | 2.51 | 2.39 | 2.06 | 2.34 | 2.35 | 2.43 | 2.45 | 3.24 | 3.11 | 3.93 |
|    | 6  | 6.24 | 3.65 | 1.93 | 1.08 | 1.15 | 0.79 | 0.77 | 1.29 | 1.43 | 1.54 | 1.97 | 2.35 | 2.06 | 2.83 | 2.81 | 2.58 | 2.93 | 3.29 | 3.13 | 3.28 |
|    | 7  | 9.92 | 5.53 | 3.09 | 2.26 | 1.72 | 1.91 | 1.75 | 1.45 | 1.28 | 2.06 | 2.35 | 1.96 | 2.43 | 2.59 | 2.86 | 2.77 | 3.31 | 3.46 | 4.12 | 4.02 |
|    | 8  | 9.96 | 5.27 | 3.29 | 1.75 | 1.77 | 1.36 | 0.90 | 1.31 | 1.23 | 1.47 | 1.71 | 1.89 | 1.98 | 2.53 | 2.69 | 3.12 | 3.50 | 3.33 | 3.74 | 4.02 |
|    | 9  | 7.55 | 3.56 | 1.98 | 1.41 | 0.97 | 1.39 | 1.32 | 1.35 | 1.62 | 2.01 | 2.36 | 1.90 | 2.15 | 2.71 | 3.16 | 3.64 | 3.12 | 3.50 | 3.58 | 3.32 |
|    | 10 | 2.00 | 1.05 | 1.10 | 0.77 | 0.73 | 0.77 | 0.89 | 0.98 | 1.45 | 1.54 | 1.68 | 2.47 | 2.29 | 2.87 | 2.96 | 3.50 | 3.76 | 3.59 | 3.65 | 3.63 |
|    | 11 | 1.49 | 1.19 | 0.54 | 1.10 | 1.00 | 1.58 | 1.38 | 1.96 | 2.03 | 1.58 | 1.65 | 2.04 | 1.91 | 2.10 | 2.35 | 2.31 | 2.67 | 3.11 | 3.41 | 3.53 |
|    | 12 | 4.99 | 2.48 | 1.28 | 0.85 | 1.30 | 1.70 | 1.78 | 1.55 | 1.99 | 2.43 | 2.64 | 2.90 | 2.43 | 2.69 | 3.17 | 3.66 | 3.97 | 3.61 | 4.11 | 3.83 |
|    | 13 | 7.83 | 4.29 | 2.83 | 2.25 | 1.20 | 1.05 | 0.78 | 0.83 | 0.97 | 1.20 | 1.84 | 2.07 | 2.25 | 2.76 | 2.46 | 2.78 | 3.19 | 3.40 | 3.97 | 3.82 |
|    | 14 | 9.61 | 4.61 | 2.44 | 1.37 | 1.49 | 0.97 | 1.09 | 0.94 | 1.43 | 1.83 | 1.52 | 2.39 | 2.01 | 2.14 | 2.46 | 2.40 | 2.46 | 3.07 | 3.18 | 3.11 |
|    | 15 | 9.37 | 4.43 | 2.11 | 1.67 | 1.43 | 1.35 | 1.15 | 1.41 | 1.34 | 1.86 | 1.55 | 2.19 | 2.00 | 2.61 | 2.66 | 3.26 | 3.55 | 3.32 | 3.55 | 3.60 |
|    | 16 | 9.18 | 4.79 | 2.88 | 1.35 | 0.83 | 0.68 | 1.13 | 1.50 | 1.72 | 1.66 | 1.59 | 2.35 | 2.73 | 3.18 | 3.13 | 3.53 | 3.68 | 4.03 | 3.52 | 3.53 |
|    | 17 | 3.58 | 2.40 | 1.49 | 1.38 | 0.94 | 1.20 | 1.67 | 1.41 | 1.28 | 1.87 | 2.21 | 2.68 | 2.68 | 2.69 | 2.65 | 3.00 | 3.00 | 3.40 | 3.14 | 3.24 |
|    | 18 | 8.49 | 4.57 | 2.32 | 1.60 | 1.55 | 1.11 | 0.93 | 1.04 | 1.82 | 1.70 | 2.21 | 2.64 | 2.45 | 2.92 | 3.27 | 2.77 | 3.17 | 3.67 | 3.69 | 3.86 |
|    | 19 | 9.63 | 4.85 | 2.74 | 1.32 | 1.52 | 1.63 | 1.58 | 1.80 | 2.14 | 1.95 | 2.57 | 2.12 | 2.63 | 2.34 | 3.14 | 3.44 | 3.03 | 3.64 | 3.90 | 4.41 |
|    | 20 | 6.52 | 3.53 | 1.78 | 1.74 | 1.18 | 0.81 | 1.34 | 1.60 | 1.65 | 1.51 | 1.67 | 2.26 | 2.33 | 2.82 | 2.95 | 2.87 | 2.87 | 3.31 | 2.57 | 3.80 |
|    | 21 | 4.48 | 1.96 | 1.25 | 1.37 | 1.59 | 1.85 | 2.10 | 2.31 | 1.59 | 2.29 | 1.93 | 2.23 | 2.83 | 3.21 | 2.95 | 3.41 | 3.15 | 3.77 | 3.55 | 4.19 |
|    | 22 | 3.38 | 2.29 | 1.17 | 0.83 | 0.76 | 1.11 | 1.02 | 1.03 | 1.84 | 2.22 | 2.28 | 1.90 | 1.85 | 2.57 | 2.46 | 2.86 | 3.06 | 3.70 | 4.21 | 3.92 |
|    | 23 | 8.00 | 4.18 | 2.21 | 1.39 | 0.73 | 1.19 | 1.64 | 1.33 | 1.94 | 1.77 | 2.08 | 2.41 | 2.94 | 2.45 | 2.80 | 2.57 | 2.68 | 3.18 | 3.16 | 3.62 |
|    | 24 | 1.19 | 0.49 | 0.53 | 0.62 | 0.35 | 0.73 | 1.14 | 1.15 | 1.23 | 1.26 | 1.98 | 2.47 | 2.73 | 2.73 | 2.56 | 2.68 | 3.31 | 3.81 | 3.58 | 3.60 |
|    | 25 | 7.21 | 4.04 | 2.00 | 0.95 | 0.55 | 1.32 | 1.22 | 1.75 | 2.18 | 1.61 | 2.13 | 1.85 | 2.63 | 2.57 | 2.64 | 3.25 | 3.65 | 3.89 | 4.18 | 3.67 |

where the value  $x_{10}$  was chosen as  $5 + 10\omega_{10}$ . This was done for  $i = 1, \dots, 25$  and  $t = 1, \dots, 20$ . In the experiments reported below  $N = 25$ , but  $T = 10$ . The additional 10 exogenous variables were used in a manner described below. Values of the exogenous variables used in all experiments and for all parameter values are given in Table 2.1.

The next step in each repetition was the generation of random variables  $u_{it}$  with variance-covariance matrix given in (2). Since the maximum-likelihood estimates, the properties of which are explored below, are based on the assumption of normality, and since specification error was not being investigated in this series of experiments the  $u_{it}$ 's were assumed to follow a multivariate normal distribution. Given values for  $\sigma^2$  and  $\rho$  there are two ways in which random variables  $u_{it}$  may be computed starting with independent random variables uniformly distributed on the interval  $[0, 1]$ . From the uniformly distributed random variables it is easy to obtain independent normal variables with zero mean and unit variance.<sup>1</sup> At this point there are two possibilities: Suppose we wish to generate 500 values  $u_{it}$ ,  $i = 1, \dots, 25$ ,  $t = 1, \dots, 20$ . We may first choose 25  $\mu_i$ 's such that

$$\mu_i \sim N(0, \rho\sigma^2),$$

then obtain 500  $v_{it}$ 's such that

$$v_{it} \sim N(0, (1-\rho)\sigma^2),$$

and finally form

$$u_{it} = \mu_i + v_{it} , \quad i = 1, \dots, 25 , \quad t = 1, \dots, 20 .$$

An alternative to the foregoing is to transform the sequence of independent normal, zero mean, unit variance random variables, call these  $w_j$ , directly into a vector  $u$  having the desired variance-covariance matrix,  $\sigma^2 \Omega$  in (2).

In general, the best way in which to obtain a vector  $u$  coming from a population  $N(\theta, \Omega)$  on the basis of a series of random variables  $w_1 \sim n(0, 1)$  is to make use of the unique decomposition of any real, positive semi-definite matrix into the product of a lower triangular matrix and its transpose:

$$(3) \quad \sigma^2 \Omega = \sigma^2 T T' .$$

Then  $u = Tw + \theta \sim N(\theta, \sigma^2 \Omega)$  where each element of  $w$  is  $n(0, 1)$ . The elements of  $T$  may be found by a simple system of recursions. Such a general solution to the problem, however, does not permit one to take advantage of the very simple structure of  $\Omega$ , in particular of the fact that it depends on only two parameters. Thus, it is preferable in this case to make use of the square root of  $\Omega$  defined as the matrix with characteristic roots equal to the square roots of the characteristic roots of  $\Omega$  (all real and positive because of the positive definiteness of  $\Omega$ ).

To obtain  $\Omega_{21}^{-1}$ , one makes use of the orthogonal transformation

$$(4) \quad C = \begin{bmatrix} e'/\sqrt{T} \\ C_1 \end{bmatrix}$$

where  $e$  is a  $T \times 1$  vector consisting entirely of ones and  $C_1$  is a  $T-1 \times T$  matrix such that

$$(5) \quad \begin{cases} C_1 e = 0 \\ C_1 C_1' = I_{T-1} \\ C_1' C_1 = I_T - ee'/T \end{cases}$$

As shown in [3]:

$$(6) \quad \sigma^2 CAC' = \begin{bmatrix} \xi & 0 & \dots & 0 \\ 0 & \eta & & \\ \vdots & \vdots & & \\ 0 & 0 & \dots & \eta \end{bmatrix}$$

where

$$(7) \quad \begin{cases} \xi = \sigma^2 \{ (1 - \rho) + T\rho \} \\ \eta = \sigma^2 (1 - \rho) . \end{cases}$$

Thus we have

$$(8) \quad \Omega_{21}^{-1} = \begin{bmatrix} B & 0 & \dots & 0 \\ 0 & B & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & B \end{bmatrix}$$

where each of the  $N$  blocks  $B$  is a  $T \times T$  matrix of the form

$$(9) \quad B = C' \text{diag}(\sqrt{\xi}, \sqrt{\eta}, \dots, \sqrt{\eta})C \\ = \sqrt{\eta}I_T + \frac{(\sqrt{\xi} - \sqrt{\eta})}{T} ee',$$

Thus, an element of the  $NT \times 1$  vector  $u$  is obtained as

$$(10) \quad u_{it} = \frac{\sqrt{\xi} - \sqrt{\eta}}{T} \sum_{k=1}^T w_k + \sqrt{\eta}w_t, \quad t = 1, \dots, T,$$

where  $w_1, \dots, w_T$  is a sequence of independent normal variables with zero mean and unit variance. Note that  $N$  such sets of  $T$ 's are required to generate the full  $NT \times 1$  vector  $u$ .

It is important to note that the  $u_{it}$ 's generated in either of the two ways have exactly the same distribution. Given a particular starting point for the random number generator used to obtain the initial uniformly distributed pseudorandom numbers with which both methods begin, the actual  $u_{it}$ 's generated will differ in value (since the original pseudorandom numbers enter in each somewhat differently), however, the distributions will both be  $N(0, \Omega)$ . Hence, there is no way one could distinguish between two sets of such numbers, one generated one way, the other in the other fashion. The only ground for choice is thus computational convenience. Obtaining the  $u_{it}$ 's by the first method requires the computation of  $N + NT$  values of two different random normal variables and  $NT$  additions, whereas

the computation of the  $u_{it}$ 's by the second method requires only  $NT$  values of a random zero-mean, unit-variance, normal variable and  $N + NT$  additions. Thus, the second method trades  $N$  additions for the generation of  $N$  additional random values. Given the complexity of the pseudorandom number routines it is obviously a considerable computational saving to use the second method.

Given the values of the exogenous variables, fixed for the entire series of experiments, and given the disturbances, computed for each repetition of a single experiment involving certain parameter values, the next step is to compute the values of the series  $y$  and  $y_{-1}$  for each individual and for the number of time periods,  $T$ , involved in the experiments. Were the relationship to be estimated a pure autoregression, as in the first series of experiments reported in [16], it would be easy to choose the starting value  $y_{i0}$  in such a way that its variance would be exactly the same as the variance of any other  $y_{it}$  for the same individual. However, the presence of an exogenous variable complicates matters. The initial value  $y_{i1}$  was taken to be

$$(11) \quad y_{i1} = \beta x_{i1} + \frac{u_{i1}}{\sqrt{1 - \alpha^2}}$$

which would yield an appropriate value for  $\beta = 0$ , i.e., no exogenous variable present. Then, however, 19 more values were generated and the first 10 discarded in forming the vector  $y$ , the first 9 in



forming the vector  $y_{-1}$ , and correspondingly the first 10 values of  $x_{it}$  were also discarded in forming  $x$ .

The procedure described was repeated 50 times for each set of parameter values,  $\alpha$ ,  $\beta$ ,  $\rho$ , and  $\sigma^2$  chosen, giving, for each repetition, three vectors  $y$ ,  $y_{-1}$ , and  $x$ , each  $250 \times 1$ .

Parameter values. -- As indicated above, in all experiments reported here  $T = 10$  and  $N = 25$ . The parameter  $\alpha$  was taken to have values 0.10, 0.30, 0.50, 0.70, and 0.90. For each such value, three values of  $\beta$  were examined:  $\beta = 0.0$ , 0.5, and 1.0. For each of the 15 combinations of  $\alpha$  and  $\beta$ , the following values of  $\rho$  were investigated:  $\rho = 0.00, 0.15, 0.30, 0.45, 0.60, 0.75, 0.90,$  and 0.95. Throughout  $\sigma^2$  was set equal to 1. Although a constant term was estimated, the true constant was assumed to be zero. Thus, altogether 120 parameter combinations were examined in the experiments.

Methods of estimation. -- For each of the 120 parameter combinations chosen, six methods of estimation were considered:

- (a) Generalized least-squares estimates employing the true value of  $\rho$  used in the generation of the observations. An estimate of the residual variance,  $\hat{\sigma}^2$ , was obtained by a slightly modified form of the standard formula for an unbiased estimator of  $\sigma^2$ .<sup>8/</sup> The modification was simply not to make the usual adjustment for loss of degrees of freedom. This led to an estimate slightly (but very slightly since  $NT = 250$ )

biased downwards, but comparable with the maximum-likelihood estimate of  $\sigma^2$ . Generalized least-squares is, of course, not a practical method of estimation since the true value of  $\rho$  is unknown; however, it provides a useful standard of comparison. We refer below to this method as GLS.

(b) Ordinary least-squares estimates of the relation (1) of this section, i.e., not including individual constant terms. Again  $\sigma^2$  was estimated without adjustment for lost degrees of freedom. We refer to this method below as OLS.

(c) Least-squares estimates of the parameters in

$$(12) \quad y = \alpha y_{-1} + \beta x + \sum_{i=1}^N \mu_i t_i + v,$$

where the  $\mu_i$  are  $N$  constants,  $t_i$  is an  $NT \times 1$  vector consisting entirely of zeros except for the  $i^{\text{th}}$  individual and then of ones, and  $v = (v_{11}, \dots, v_{1T}, \dots, v_{N1}, \dots, v_{NT})'$ .

An estimate of  $\sigma^2$  was obtained from the usual estimate of the residual variance  $\sigma_v^2$  in (12), unadjusted, however, for loss of degrees of freedom, and the estimates of the parameters

$$(13) \quad \mu_i, \text{ by } \hat{\sigma}^2 = \frac{\sum_i \left\{ \hat{\mu}_i - \frac{\sum \hat{\mu}_i}{N} \right\}^2}{N} + \hat{\sigma}_v^2.$$

An estimate of  $\rho$  was obtained as

$$(14) \quad r_c = \frac{\frac{1}{N} \left\{ \frac{\sum \hat{\mu}_i}{N} - \frac{1}{N} \right\}^2}{\hat{\sigma}^2}.$$

We refer to this method below as LSC.

- (d) Instrumental-variable estimates using  $x_{it-1}$  as an instrument for  $y_{it-1}$ . The intraclass correlation coefficient was estimated from the calculated residuals,  $\hat{u}_{it}$  by

$$(15) \quad r_i = \frac{\frac{1}{T} \sum_{i=1}^N \left( \sum_{t=1}^T \hat{u}_{it} \right)^2 - \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \hat{u}_{it}^2}{NT\hat{\sigma}^2}$$

where

$$(16) \quad \hat{\sigma}^2 = \frac{\sum_{i=1}^N \sum_{t=1}^T \hat{u}_{it}^2}{NT} \cdot 9/$$

Standard errors of the estimates were computed from an estimate of the asymptotic variance-covariance matrix suggested by Sargan [17, pp. 396-97]. We refer to this method below as IV.

- (e) "Two-round" estimates equivalent to generalized least squares using not the true value of  $\rho$  but the estimate  $r_c$  obtained in (c), a least-squares regression with individual constant terms. Standard errors for these estimates were not computed.<sup>10/</sup> We refer to this method below as 2RC.

(f) "Two-round" estimates similar to (e) but using  $r_i$ , an estimate of  $\rho$  based on the calculated residuals from the instrumental variable estimate of equation (1) as given in (15) above. Standard errors for these estimates were not computed.<sup>10/</sup> We refer to this method below as IV.

(g) Maximum-likelihood estimates obtained by maximizing the logarithmic likelihood function:

$$(17) \quad L(\alpha, \beta, \gamma, \rho, \sigma^2) = -\frac{NT}{2} \log 2\pi - \frac{NT}{2} \log \sigma^2 - \frac{N}{2} \log[1 - \rho + T\rho] \\ - \frac{N(T-1)}{2} \log(1 - \rho) - \frac{1}{2\sigma^2} \sum_{i=1}^N \left\{ \frac{1}{T[1-\rho+T\rho]} \left( \sum_{t=1}^T u_{it} \right)^2 \right. \\ \left. - \frac{1}{T(1-\rho)} \left( \sum_{t=1}^T u_{it} \right)^2 + \frac{1}{(1-\rho)} \sum_{t=1}^T u_{it}^2 \right\},$$

where  $T = 10$ ,  $N = 25$ , and

$$u_{it} = y_{it} - \alpha y_{it-1} - \beta x_{it} - \gamma,$$

with respect to all the unknown parameters. Note that a non-zero constant term,  $\gamma$ , is allowed for. Asymptotic standard errors were obtained from the standard result for the asymptotic variance-covariance matrix of maximum-likelihood estimates:

$$\begin{bmatrix} \frac{\partial^2 L}{\partial \alpha^2} & \frac{\partial^2 L}{\partial \alpha \partial \beta} & \frac{\partial^2 L}{\partial \alpha \partial \gamma} & \frac{\partial^2 L}{\partial \alpha \partial \rho} & \frac{\partial^2 L}{\partial \alpha \partial \sigma^2} \\ \frac{\partial^2 L}{\partial \alpha \partial \beta} & \frac{\partial^2 L}{\partial \beta^2} & \frac{\partial^2 L}{\partial \beta \partial \gamma} & \frac{\partial^2 L}{\partial \beta \partial \rho} & \frac{\partial^2 L}{\partial \beta \partial \sigma^2} \\ \frac{\partial^2 L}{\partial \alpha \partial \gamma} & \frac{\partial^2 L}{\partial \beta \partial \gamma} & \frac{\partial^2 L}{\partial \gamma^2} & \frac{\partial^2 L}{\partial \gamma \partial \rho} & \frac{\partial^2 L}{\partial \gamma \partial \sigma^2} \\ \frac{\partial^2 L}{\partial \alpha \partial \rho} & \frac{\partial^2 L}{\partial \beta \partial \rho} & \frac{\partial^2 L}{\partial \gamma \partial \rho} & \frac{\partial^2 L}{\partial \rho^2} & \frac{\partial^2 L}{\partial \rho \partial \sigma^2} \\ \frac{\partial^2 L}{\partial \alpha \partial \sigma^2} & \frac{\partial^2 L}{\partial \beta \partial \sigma^2} & \frac{\partial^2 L}{\partial \gamma \partial \sigma^2} & \frac{\partial^2 L}{\partial \rho \partial \sigma^2} & \frac{\partial^2 L}{\partial (\sigma^2)^2} \end{bmatrix}^{-1}$$

where the partial derivatives of the logarithmic likelihood function are evaluated at the maximum-likelihood estimates  $\hat{\alpha}$ ,  $\hat{\beta}$ ,  $\hat{\gamma}$ ,  $\hat{\rho}$ , and  $\hat{\sigma}^2$ . The method used to obtain the maximum of the likelihood function was that suggested by Fletcher and Powell [6] and programmed in ALGOL by Wells [20] with the modification suggested by Box [4] for incorporating the constraint that  $\rho$  lie in the interval  $[0, 1)$ . To ensure that  $\hat{\sigma}^2$  would be positive the simple device of maximizing  $L$  with respect to  $\sigma$  instead of  $\sigma^2$  was adopted. The method of bounding  $\hat{\rho}$  was to maximize  $L$  with respect to

$$\theta = [\arcsin \sqrt{\rho}]$$

or

$$\rho = \sin^2 \theta .$$

Clearly  $\theta$  may range over the entire real line whereas  $\rho = \sin^2 \theta$  must lie between 0 and 1. The problem is that  $L$  will have multiple maxima (albeit widely separated) with respect to  $\theta$  since  $\sin \theta$  is a periodic function. However, the method of Fletcher and Powell is a local procedure and the estimates were taken to be the first point at which a local maximum of  $L$  was obtained. As a starting point for the maximum-likelihood estimation procedure the estimates of  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\sigma^2$  from 2RC, method (e) above, and the estimate of  $\rho$ ,  $r_c$ , from method (c) above, were used. We refer to this method below as ML.

When a boundary point maximum of the likelihood function occurs the usual formula for the asymptotic variance-covariance matrix given above is no longer valid. Indeed the matrix of second-derivatives need not be negative definite at a boundary maximum. It is possible in such cases that the inverse of the information matrix has negative diagonal elements. Hence, in all cases (and there are a great many for  $\beta = 0$ ) in which a boundary maximum (always  $\hat{\rho} = 0$ ) occurred, the computation of asymptotic standard errors was omitted. To check many of the boundary maxima a search procedure similar to that used in [16] was employed. The solution by the method of Fletcher and Powell was validated in every case checked by search.

### 3. Comparative Properties of Different Estimation Procedures

For each of the methods of estimation described in the preceding section and each choice of parameters, 50 sets of observations and estimates were generated in the manner described. To characterize the distributions of coefficient estimates, the mean, mean-square error, minimum, first quartile, median, third quartile, and maximum were computed. When estimated standard errors were computed, their distributions were characterized in the same way except that the variance of the distribution was computed rather than the mean-square error. The reason for computing the variance rather than mean-square error in these cases was simply that the "true" standard error was generally unknown.<sup>11/</sup>

Appendix Tables A.1-12, B.1-12, and C.1-12 summarize these results. The sets of tables A, B, and C refer to the value of  $\beta$  chosen, respectively 0.0, 0.5, and 1.0. The case  $\beta = 0.0$  is an especially interesting one inasmuch as the specification error involved, namely including a variable when it should be excluded, is one seldom analyzed. The first seven tables of each set present the means and mean-square errors of the estimates of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\sigma^2$  and, where appropriate,  $\rho$ , for various combinations of the true values of the parameters  $\alpha$  and  $\rho$ . Each table covers a different estimation procedure:

1. Generalized least-squares estimates (GLS);

2. Ordinary least-squares estimates (OLS);
3. Least-squares estimates with individual constant terms (LSC);
4. Two-round estimates using  $\hat{\rho}$  from LSC (2RC);
5. Instrumental-variable estimates using  $x_{-1}$  as the instrument for  $y_{-1}$  (IV);
6. Two-round estimates using  $\hat{\rho}$  from IV (2RI);
7. Maximum-likelihood estimates (ML).

The remaining five tables of each set give the means and variances of the computed standard errors of the estimates of  $\alpha$ ,  $\beta$ , and  $\gamma$  for the corresponding combinations of the true values of  $\alpha$  and  $\rho$ . Standard errors were computed only for five of the seven types of estimates considered: GLS, OLS, LSC, IV, and ML. Maxima, minima, and quartiles of the distributions are not reported in this paper although they were computed.

To facilitate comparison of estimation procedures, the results are discussed under three headings: bias, relative mean-square error, and characteristics of the maximum-likelihood estimates. The computed standard errors are discussed only tangentially and then with reference to how well they serve, or do not serve, to indicate the actual variability of the estimates.

Bias. -- Although small-sample bias is known to exist in all cases in which a lagged value of the dependent variables is one of the explanatory variables,<sup>12/</sup> it is apparent that such bias is very slight for the GLS estimates reported in appendix Tables A.1,



B.1, and C.1. The means of the GLS estimates of  $\alpha$  are particularly close to the true values, irrespective of the value of  $\beta$ . The residual variance is also well estimated by GLS.  $\beta$  and  $\gamma$  are somewhat less well estimated, the bias in  $\gamma$  being slightly larger, the larger the true value of  $\beta$ . Perhaps the intuitive rationale of the virtually negligible small-sample bias, is that when  $\rho$  is known a cross-section over time of the sort considered here, in fact provides a large number of observations.

The picture is quite different when ordinary least-squares estimates are considered. Except when  $\rho = 0$ , in which case the GLS and OLS estimates coincide, the OLS estimate of  $\alpha$  is severely biased upwards. Indeed, for large values of  $\rho$ , the OLS estimate of  $\alpha$  is a better estimate of  $\rho$  than it is of  $\alpha$ . The coefficient of the exogenous variable,  $\beta$ , is strongly biased downwards for large values of  $\rho$  except when the true value of  $\beta$  is zero; in this case, the bias is comparable to that of the GLS estimate and essentially negligible. The OLS estimate of  $\sigma^2$  is strongly biased downwards for large values of  $\rho$  and all values of  $\beta$ . Turning to Tables A.9, B.9, and C.9 it is apparent that the estimated standard errors are greatly affected by the under estimation of the residual variance. The OLS estimate of the constant term is slightly but distinctly biased downwards for  $\beta = 0$ , and greatly biased downwards for large values of  $\beta$  and  $\rho$ .

As was the case in the experiments reported in [16], the introduction of individual constant terms in an attempt to remove the

time-invariant individual effects produces estimates of  $\alpha$  which are markedly too low. This is true regardless of the value of  $\beta$  or  $\rho$ , but the LSC estimates of  $\alpha$  are noticeably less biased downwards the higher the value of  $\beta$  and  $\rho$ ; indeed, for  $\beta = 1.0$  and  $\rho = .9$ , the bias in the LSC estimate of  $\alpha$  is negligible when  $\alpha = .9$  but increases as the true value of  $\alpha$  falls. For small values of  $\alpha$ ,  $\rho$ , and  $\beta$ , the bias is 50 percent or more of the true parameter value. Rather surprisingly, the LSC estimate of  $\beta$  is biased upwards irrespective of the true value of  $\beta$ . The bias is greater the greater  $\alpha$ , and less the greater  $\rho$ . When  $\beta$  is not zero and  $\alpha$  is large the LSC estimate of  $\gamma$  is biased upwards, especially as when the true value of  $\rho$  is small. The estimated value of  $\rho$  is biased upwards, the more severely the lower the true value of  $\rho$ . On the other hand, the LSC estimate of  $\sigma^2$ , which depends on the "variance" of the constant terms as well as the estimate of the remaining residual variance, is biased upwards but generally less so the lower the true value of  $\rho$ .

In [16, p. 46] the following argument was given for expecting the LSC estimate of  $\alpha$  (and  $\beta$ ) to be biased downwards:

"We are not, in fact, interested in the individual  $\mu_1$ 's but rather only in  $\rho$  or  $\sigma_\mu^2$ , i.e., the extent to which variation in the  $\mu_1$ 's causes  $y_{it}$  to vary. By treating the  $\mu_1$ 's as constants and estimating them, we must inevitably overestimate  $\sigma_\mu^2$  on account of errors of estimation of each individual  $\mu_1$ . For  $T$  very large relative to  $N$  this effect should be very slight, but typically  $N$  will be substantially larger than  $T$  in applications of economic relevance. If such

overestimation of  $\sigma_{\mu}^2$  does not result in a compensating underestimation of  $\sigma_v^2$ , there is simply too little variation in  $y_{it}$  left over to be explained by variation in  $y_{it-1}$  and  $x_{it}$ . Thus  $\alpha$  and  $\beta$  should tend to be underestimated in value."

It was left, however, as an open question as to why  $\sigma_v^2$  was not underestimated. Far from clearing up the puzzle, the present series of Monte Carlo experiments have only deepened the mystery, for, as we have seen, the LSC estimate of  $\beta$  is not biased downwards but upwards! While it is likely that the extent and nature of the bias has a good deal to do with the serial correlation in the  $x_{it}$  series and the degree to which the levels differ from individual to individual, it is evident that so simple an explanation as offered in [16] cannot be adequate.

Despite the upward bias in the LSC estimate of  $\rho$  (larger the smaller the true value of  $\rho$ ), two-round estimates of  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\sigma^2$  based on generalized least-squares estimates assuming the LSC  $\hat{\rho}$  are not seriously biased in comparison with either the OLS or LSC estimates. Indeed, as we shall see, these two-round estimates appear to be superior over a wide range of true parameter values to all the other estimates considered. The 2RC estimate of  $\alpha$  is biased downwards but this bias is greatly reduced as the true value of  $\hat{\rho}$  rises (presumably because the LSC estimate  $\hat{\rho}$  is less biased. The estimate of  $\beta$  is biased upwards, but considerably less than the corresponding LSC estimate. Perhaps the principle source of difficulty is the distinct upward bias in the 2RC estimate of  $\sigma^2$ . Exceptionally

erratic behavior was noted in [16] for a similar estimate. The presence of an exogenous variable appears greatly to mitigate this difficulty (when  $\beta$ , of course is not equal to zero), but it does not remove it entirely.

The instrumental variable estimates illustrate clearly the implications of asymptotic efficiency and consistency for the behavior of estimates in small samples. It is often stated that large sample properties are of no relevance to those who must of necessity be content with small samples; yet such properties do serve as some guide to what we may expect in the finite case even if the sample is not really large. When the true value of  $\beta$  is zero, the IV estimates are inconsistent. In this case, we might expect highly erratic behavior as indeed occurs. Both the means and mean-square errors of all estimates vary greatly and are often extremely large. When  $\beta = 1.0$ , on the other hand the bias in the IV estimates of all the parameters is slight. The variances, however, of the distributions of the IV estimates are large, giving rise to fairly high mean-square errors despite the lack of bias. When  $\beta = 0.5$ , the IV estimates are considerably more erratic than for  $\beta = 1.0$ . To some extent the increased variability affects the means of the distributions. (This is especially true for  $\hat{\sigma}^2$ .) Since the IV estimates are less efficient (asymptotically) the smaller  $\beta$ , but not inconsistent as long as  $\beta$  does not actually equal zero (or  $x_{it}$  and  $x_{it-1}$  are not perfectly correlated), we might expect little difference in the results for distributions of the estimates based on very large numbers

of repetitions of each experiment. It is clear, however, that when only 50 repetitions of each experiment are performed, halving  $\beta$  greatly affects the results. It is also clear that this represents an important general implication both for Monte Carlo methods of investigation and for the reliability of results in actual estimation problems (where, in effect, only one repetition is performed). The large-sample property of inefficiency implies erratic small sample behavior.

When  $\beta = 0.0$ , the two-round estimates which use the IV estimate of  $\rho$  to transform the data are greatly affected by the extreme variability of the estimates of  $\rho$ . However, there surprisingly seem certain systematic tendencies. The bias of  $\hat{\alpha}$ , for example, is negative for small values of  $\rho$ , but becomes large and positive as  $\rho$  increases. The bias of  $\hat{\sigma}^2$  is erratic but does definitely appear to shift from positive to negative as  $\rho$  increases. When  $\beta = 0.5$  or  $1.0$ , the 2RI estimates are much better behaved. Only for  $\rho = 0$ , for example, do the biases of  $\alpha$  appear non-negligible. The estimates of  $\sigma^2$  show the greatest bias and behave somewhat erratically for  $\beta = 0.5$ .

Perhaps the most unexpected result of the present investigation was the exceedingly poor performance of the maximum-likelihood method under certain circumstances. This finding is so important that it is discussed separately below. Here, however, we merely note that the biases in cases where a large number of boundary solutions

(always  $\hat{\rho} = 0$ ) did not occur are slight. A great many such solutions occurred for  $\beta = 0.0$ , and for  $\alpha = 0.7$  even when  $\beta = 0.5$  or  $1.0$ . In these cases the means of the distribution of the ML estimates were greatly affected.

On the whole the 2RC estimates appear to have less bias over a wider range than do the other estimates examined.

Relative mean-square errors. -- Tables 3.1-3.9 give the ratios of the mean-square errors of the OLS, LSC, 2RC, 2RI and ML estimates of  $\alpha$ ,  $\beta$ , and  $\gamma$  to the corresponding mean-square error of the GLS estimate. In some sense, the GLS estimate represents an ideal, and, because the different estimation techniques may be affected by the vicissitudes of the particular experiments, examining their behavior relative to the GLS estimates may be helpful in avoiding the difficulties due to too small a number of repetitions.

When the true  $\rho$  is zero, ordinary least-squares and generalized least squares coincide and the MSE is 1.00. Curiously the OLS estimates of  $\beta$  and  $\gamma$  have smaller mean square errors than the GLS estimates in certain cases. When  $\beta = 0.0$  these are frequent, especially for small  $\alpha$  and large  $\rho$ ; this phenomenon occurs only for  $\hat{\gamma}$  and not for  $\hat{\beta}$  when  $\beta = 1.0$  or  $0.5$ . Needless to say, the GLS estimates are best only among unbiased linear estimates, and the phenomenon here observed is perfectly possible when comparison among biased estimates is made.

The instrumental variable estimates are terrible, as expected,

TABLE 3.1: Ratios of MSEs for Various Estimators of  $\alpha$  to Those of the GLS Estimator. Various Values of  $\alpha$  and  $\rho$ .  
 $\beta = 0.0$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$ ,  $T = 10$ ,  $N = 25$

| $\beta =$      | 0.0 | $\rho = 0.0$       | $\rho = 0.15$      | $\rho = 0.30$      | $\rho = 0.45$      | $\rho = 0.60$      | $\rho = 0.75$      | $\rho = 0.90$      | $\rho = 0.95$      |
|----------------|-----|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| $\alpha = 0.1$ | OLS | 1.00               | 5.77               | 24.59              | 39.96              | 80.97              | 100.46             | 129.78             | 230.31             |
|                | LSC | 4.79               | 5.26               | 4.38               | 3.71               | 4.31               | 4.34               | 3.00               | 4.59               |
|                | IV  | $6.05 \times 10^3$ | $1.35 \times 10^4$ | $7.12 \times 10^3$ | $1.42 \times 10^3$ | $2.57 \times 10^4$ | $1.87 \times 10^5$ | $2.19 \times 10^4$ | $2.48 \times 10^6$ |
|                | 2RC | 2.31               | 2.02               | 1.49               | 1.38               | 1.33               | 1.66               | 1.14               | 1.06               |
|                | 2RI | 2.20               | 2.41               | 3.92               | 4.78               | 6.49               | 13.84              | 27.88              | 50.12              |
|                | ML  | 1.13               | 1.41               | 1.51               | 1.71               | 1.36               | 1.68               | 1.55               | 1.25               |
| $\alpha = 0.3$ | OLS | 1.00               | 5.22               | 25.84              | 33.73              | 103.83             | 112.00             | 129.36             | 131.28             |
|                | LSC | 4.86               | 7.32               | 7.27               | 4.90               | 11.00              | 7.06               | 8.58               | 7.17               |
|                | IV  | $1.10 \times 10^6$ | $1.17 \times 10^5$ | $1.18 \times 10^4$ | $6.80 \times 10^3$ | $6.31 \times 10^5$ | $5.95 \times 10^3$ | *                  | $3.63 \times 10^2$ |
|                | 2RC | 2.33               | 2.71               | 2.24               | 1.62               | 2.83               | 1.68               | 2.30               | 2.00               |
|                | 2RI | 2.12               | 3.68               | 3.70               | 5.88               | 11.00              | 14.81              | 39.58              | 48.37              |
|                | ML  | 1.08               | 1.58               | 1.94               | 2.58               | 2.33               | 2.29               | 2.12               | 2.40               |
| $\alpha = 0.5$ | OLS | 1.00               | 8.03               | 24.31              | 35.17              | 44.94              | 63.87              | 58.12              | 66.67              |
|                | LSC | 16.63              | 9.84               | 13.17              | 9.63               | 9.31               | 10.94              | 9.48               | 8.17               |
|                | IV  | $7.14 \times 10^3$ | $3.35 \times 10^4$ | $3.83 \times 10^4$ | $1.85 \times 10^4$ | $1.61 \times 10^4$ | $8.45 \times 10^3$ | $4.55 \times 10^2$ | $9.61 \times 10^5$ |
|                | 2RC | 6.79               | 3.16               | 4.00               | 2.54               | 2.43               | 2.84               | 2.80               | 1.97               |
|                | 2RI | 6.74               | 4.00               | 8.52               | 7.74               | 6.57               | 10.61              | 13.38              | 26.64              |
|                | ML  | 1.58               | 2.65               | 6.14               | 14.80              | 16.63              | 24.42              | 27.85              | 34.33              |
| $\alpha = 0.7$ | OLS | 1.00               | 10.50              | 12.91              | 25.86              | 23.96              | 36.04              | 24.86              | 38.74              |
|                | LSC | 19.87              | 27.30              | 13.35              | 23.14              | 15.34              | 20.23              | 13.17              | 24.26              |
|                | IV  | $4.07 \times 10^4$ | $9.29 \times 10^3$ | $3.82 \times 10^3$ | $3.61 \times 10^4$ | $8.10 \times 10^3$ | $1.79 \times 10^4$ | $6.90 \times 10^3$ | $1.87 \times 10^2$ |
|                | 2RC | 8.56               | 9.95               | 3.82               | 6.95               | 4.10               | 5.82               | 3.57               | 7.74               |
|                | 2RI | 10.83              | 11.90              | 5.65               | 9.73               | 8.28               | 8.36               | 9.00               | 19.00              |
|                | ML  | 1.78               | 9.75               | 12.15              | 24.23              | 23.14              | 34.32              | 24.11              | 36.00              |
| $\alpha = 0.9$ | OLS | 1.00               | 3.63               | 4.61               | 5.71               | 4.82               | 7.10               | 7.84               | 15.10              |
|                | LSC | 56.67              | 29.89              | 23.96              | 24.71              | 13.28              | 14.45              | 3.84               | 3.20               |
|                | IV  | $1.74 \times 10^4$ | $1.24 \times 10^5$ | $1.31 \times 10^3$ | $4.77 \times 10^3$ | $1.08 \times 10^3$ | $1.01 \times 10^4$ | $5.82 \times 10^2$ | $8.31 \times 10^1$ |
|                | 2RC | 25.08              | 10.68              | 8.39               | 8.05               | 3.71               | 3.90               | 0.63               | 0.70               |
|                | 2RI | 29.17              | 12.63              | 9.09               | 6.19               | 4.04               | 4.25               | 3.47               | 5.70               |
|                | ML  | 1.50               | 3.47               | 4.61               | 5.71               | 4.82               | 7.00               | 7.74               | 14.50              |

\* Format field exceeded.









TABLE 3.5; Ratios of MSEs for Various estimators of  $\beta$  to those of the GLS Estimator. Various Values of  $\alpha$  and  $\rho$ .  $\beta=0.5$ ,  $\gamma=0.0$ ,  $\sigma^2 = 1.0$ ,  $T=10$   $N=25$ .

| $\beta$      | $\rho=0.0$   | $\rho=0.15$ | $\rho=0.30$ | $\rho=0.45$ | $\rho=0.60$ | $\rho=0.75$ | $\rho=0.90$ | $\rho=0.95$ |        |
|--------------|--------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|--------|
| $\alpha=0.1$ | 0.5          |             |             |             |             |             |             |             |        |
|              | OLS          | 1.00        | 1.21        | 3.24        | 6.84        | 17.54       | 27.30       | 91.29       | 131.77 |
|              | LSC          | 1.54        | 1.56        | 2.07        | 1.41        | 1.86        | 1.68        | 2.24        | 1.62   |
|              | IV           | 9.60        | 3.30        | 5.88        | 2.35        | 4.50        | 5.42        | 10.24       | 11.69  |
|              | 2RC          | 1.22        | 1.12        | 1.26        | 1.04        | 1.09        | 1.10        | 1.12        | 0.92   |
|              | 2RI          | 1.15        | 1.06        | 1.51        | 0.98        | 1.45        | 1.68        | 1.24        | 1.15   |
|              | ML           | 1.03        | 0.94        | 1.04        | 1.01        | 1.02        | 1.12        | 1.18        | 1.08   |
| $\alpha=0.3$ | OLS          | 1.00        | 1.60        | 5.46        | 10.79       | 14.27       | 31.44       | 73.18       | 122.28 |
|              | LSC          | 2.26        | 2.53        | 1.56        | 1.73        | 2.55        | 2.56        | 2.77        | 3.14   |
|              | IV           | 6.74        | 13.15       | 6.07        | 5.27        | 9.75        | 7.18        | 7.64        | 14.07  |
|              | 2RC          | 1.53        | 1.46        | 0.93        | 1.12        | 1.28        | 1.15        | 0.95        | 1.28   |
|              | 2RI          | 1.37        | 1.37        | 1.90        | 1.92        | 1.91        | 1.38        | 1.09        | 1.21   |
|              | ML           | 1.02        | 1.12        | 1.05        | 1.30        | 1.60        | 1.10        | 0.91        | 1.21   |
|              | $\alpha=0.5$ | OLS         | 1.00        | 2.74        | 7.05        | 10.10       | 22.16       | 31.88       | 75.81  |
| LSC          |              | 3.58        | 2.75        | 3.46        | 3.33        | 4.28        | 4.41        | 3.28        | 2.06   |
| IV           |              | 8.98        | 5.29        | 8.65        | 4.08        | 4.76        | 5.51        | 6.57        | 9.65   |
| 2RC          |              | 2.04        | 1.43        | 1.43        | 1.41        | 1.40        | 1.51        | 1.14        | 0.94   |
| 2RI          |              | 1.88        | 1.49        | 2.09        | 1.99        | 1.96        | 1.76        | 1.90        | 1.70   |
| ML           |              | 1.14        | 1.47        | 2.69        | 2.69        | 5.24        | 6.93        | 4.48        | 1.59   |
| $\alpha=0.7$ |              | OLS         | 1.00        | 3.08        | 5.08        | 8.68        | 14.26       | 19.08       | 34.41  |
|              | LSC          | 5.90        | 5.57        | 4.57        | 4.93        | 7.38        | 4.95        | 3.20        | 2.16   |
|              | IV           | 6.37        | 3.35        | 4.98        | 4.94        | 6.69        | 7.48        | 7.85        | 73.48  |
|              | 2RC          | 2.99        | 2.21        | 1.66        | 1.86        | 2.36        | 1.27        | 0.97        | 1.04   |
|              | 2RI          | 2.21        | 1.56        | 1.60        | 2.56        | 3.01        | 1.98        | 2.62        | 3.04   |
|              | ML           | 1.34        | 2.82        | 4.64        | 7.69        | 13.26       | 16.36       | 20.64       | 8.80   |
|              | $\alpha=0.9$ | OLS         | 1.00        | 2.30        | 3.35        | 5.15        | 6.09        | 8.43        | 14.48  |
| LSC          |              | 13.90       | 8.92        | 5.58        | 7.44        | 6.51        | 3.10        | 1.41        | 2.50   |
| IV           |              | 13.27       | 10.30       | 12.37       | 24.70       | 16.38       | 11.02       | 268.26      | 428.19 |
| 2RC          |              | 5.95        | 2.75        | 1.52        | 1.91        | 1.49        | 0.58        | 0.63        | 1.06   |
| 2RI          |              | 3.57        | 1.50        | 1.74        | 2.68        | 2.09        | 1.84        | 1.85        | 3.62   |
| ML           |              | 1.10        | 2.19        | 3.02        | 4.90        | 5.89        | 7.73        | 11.70       | 14.94  |





TABLE 3.8;

Ratios of MSE s for Various Estimators of  $\beta$  to those of the GLS Estimator. Various Values of  $\alpha$  and  $\rho$ .  $\beta=1.0$   $\gamma=0.0$ ,  $\sigma^2=1.0$ ,  $T=10$ ,  $N=25$ .

| $\beta = 1.0$ | $\rho=0.0$ | $\rho=0.15$ | $\rho=0.30$ | $\rho=0.45$ | $\rho=0.60$ | $\rho=0.75$ | $\rho=0.90$ | $\rho=0.95$ |        |
|---------------|------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|--------|
| $\alpha=0.1$  | OLS        | 1.00        | 2.32        | 6.22        | 15.98       | 34.37       | 57.03       | 198.00      | 292.20 |
|               | LSC        | 1.71        | 1.52        | 2.40        | 2.17        | 1.96        | 2.12        | 1.50        | 1.50   |
|               | IV         | 2.68        | 3.46        | 3.21        | 3.19        | 4.78        | 2.97        | 4.96        | 53.35  |
|               | 2RC        | 1.23        | 1.01        | 1.24        | 1.14        | 1.14        | 1.30        | .96         | 1.05   |
|               | 2RI        | 1.17        | 1.08        | 1.11        | 1.06        | 1.44        | 1.38        | 1.11        | 1.00   |
|               | ML         | 1.03        | 1.01        | .99         | 1.09        | 1.22        | 1.35        | 1.11        | 1.05   |
|               | OLS        | 1.00        | 4.26        | 12.02       | 22.93       | 33.77       | 76.66       | 222.16      | 282.48 |
|               | LSC        | 2.60        | 2.58        | 2.14        | 2.22        | 2.84        | 2.76        | 3.24        | 1.90   |
|               | IV         | 4.02        | 3.30        | 2.49        | 3.56        | 2.61        | 3.67        | 5.92        | 7.81   |
| $\alpha=0.3$  | 2RC        | 1.57        | 1.16        | .89         | .96         | 1.30        | 1.05        | 1.32        | 1.05   |
|               | 2RI        | 1.50        | 1.11        | 1.02        | 1.22        | 1.35        | 1.21        | 1.32        | 1.00   |
|               | ML         | 1.13        | 1.22        | 1.27        | 1.15        | 1.36        | 1.10        | 1.28        | 1.00   |
|               | OLS        | 1.00        | 4.52        | 13.00       | 31.13       | 45.04       | 87.48       | 185.50      | 305.42 |
|               | LSC        | 3.92        | 3.22        | 4.66        | 4.99        | 3.02        | 4.57        | 2.33        | 2.16   |
|               | IV         | 3.36        | 3.10        | 4.36        | 3.85        | 4.06        | 4.06        | 5.10        | 9.74   |
|               | 2RC        | 1.98        | 1.27        | 1.64        | 1.52        | .88         | 1.33        | .90         | 1.05   |
|               | 2RI        | 1.75        | 1.29        | 1.76        | 1.64        | 2.06        | 1.35        | 1.17        | 1.21   |
|               | ML         | 1.15        | 1.43        | 1.96        | 1.90        | 2.59        | 1.42        | 1.10        | 1.05   |
| $\alpha=0.5$  | OLS        | 1.00        | 6.38        | 12.18       | 16.24       | 28.30       | 39.11       | 132.40      | 178.00 |
|               | LSC        | 9.56        | 6.31        | 9.22        | 3.68        | 3.89        | 2.61        | 2.46        | 1.50   |
|               | IV         | 5.30        | 3.33        | 5.67        | 2.33        | 3.43        | 1.92        | 7.06        | 8.38   |
|               | 2RC        | 4.30        | 1.72        | 3.16        | 1.01        | 1.10        | .92         | 1.26        | .96    |
|               | 2RI        | 3.06        | 1.36        | 2.86        | 1.50        | 2.07        | 1.50        | 1.91        | 1.31   |
|               | ML         | 1.35        | 4.10        | 7.70        | 10.19       | 16.63       | 9.74        | 2.14        | 1.15   |
|               | OLS        | 1.00        | 3.03        | 4.42        | 6.44        | 8.48        | 15.21       | 35.88       | 127.06 |
|               | LSC        | 6.05        | 8.71        | 2.93        | 3.05        | 1.99        | 1.25        | 1.03        | 1.24   |
|               | ML         | 4.23        | 7.19        | 2.96        | 2.30        | 4.03        | 2.55        | 7.10        | 27.59  |
| $\alpha=0.9$  | 2RC        | 1.99        | 2.33        | .60         | .76         | .70         | .67         | .80         | 1.00   |
|               | 2RI        | 1.54        | 1.53        | 1.21        | 1.20        | 1.80        | 1.69        | 1.58        | 1.82   |
|               | ML         | 1.04        | 1.80        | 2.96        | 3.77        | 16.00       | 6.19        | 2.05        | 1.59   |



when  $\beta = 0.0$  . However, the mean-square errors fall markedly as  $\beta$  increases and are respectable, if not comparatively low, for  $\beta = 1.0$  .

Estimates of  $\alpha$  or  $\beta$  based on least-squares regressions including individual constant terms are generally superior to OLS or IV estimates, except that in certain cases when  $\rho$  is small their mean-square errors are greater than the OLS estimates. The LSC estimates of  $\gamma$  are frequently worse than the OLS estimates.

An unexpected result of the present investigation was the comparative behavior of the maximum-likelihood method and the two-round estimates using  $\hat{\rho}$  from the LSC regressions. When  $\hat{\rho}$  from an IV first round is used we would expect the corresponding two-round estimates to be quite erratic when  $\beta = 0.0$  simply because the underlying estimates of  $\rho$  are so poor. However, even when  $\beta = 0.5$  or  $1.0$  , the 2RC estimates have lower mean-square errors except when  $\rho$  is very small. Furthermore the 2RC estimates, especially of  $\alpha$  and  $\beta$  , very frequently have smaller relative mean-square errors than the ML estimates. One might have anticipated this in situations in which a large number of boundary solutions occur for values of  $\rho$  different from zero, but it also occurs even in situations in which the ML method might be expected to perform well.

On the basis of the criterion of minimum mean-square error, the 2RC estimates also compare favorably with all other estimates, including the ML, over a wide range of parameter values.

Characteristics of the maximum-likelihood estimates. -- Table 3.10





shows, for each set of true parameter values, what percentage of the 50 repetitions converged, and, for convergent cases, in what percentage boundary maxima were obtained. Since  $\rho$  and  $\sigma^2$  were the only bounded parameters, only  $\hat{\rho}$  and  $\hat{\sigma}^2$  might be found at a boundary. In fact, all boundary points found were points for which  $\hat{\rho} = 0$ .

In only one case, out of the many thousands estimated, did convergence of the maximization procedure used fail to occur: One of the fifty repetitions for the parameter values  $\alpha = 0.9$ ,  $\beta = 1.0$ ,  $\rho = 0.75$ . Further investigation of this case suggested the presence of two local maxima very close together, but, in view of the generally smooth behavior of the likelihood functions, this case requires still further study.

More disconcerting was the large number of boundary solutions obtained. Naturally, these are to be expected when the true value of  $\rho$  is zero or close to zero, but such solutions are far more widespread. They occur in great numbers for  $\alpha = 0.7$  when  $\beta = 0.0$  and  $\beta = 1.0$ , and when  $\beta = 0.5$  such solutions occur for nearly all values of  $\rho$  and  $\alpha = 0.5, 0.7, \text{ or } 0.9$ . The results do not appear to be due to the method of Fletcher and Powell, modified to permit bounding of  $\hat{\rho}$ , for, in a large number of cases in which a boundary solution occurred, the maximum-likelihood estimates were reestimated using the slower search procedure described in [16, pp. 54-55] with exactly the same results. Comparison of cases in which the method converged to interior solutions with those in which it converged to a boundary solution, revealed that, despite a large true

$\rho$  , the likelihood function was extremely flat in the  $\rho$  direction. Small variations in the sample of random disturbances, thus, frequently produced a boundary solution. As noted by Konijn [12], there are circumstances (namely, when the individual means over time of the dependent variables pertaining to any individual are equal for any pair of individuals) when a boundary solution must inevitably occur. However, as pointed out in [16, p. 48, footnote 6], the crucial assumption cannot be valid when one of the explanatory variables is a lagged value of the dependent variable. What the present results appear to indicate is that when there is also an exogenous variable with certain serial properties, these characteristics in association with certain definite characteristics of the autoregressive part of the relationship being estimated can interact to produce a situation close to the one Konijn described. When one is calculating, in contrast to a situation in which one is theorizing, being close to a "bad" case is quite as difficult as actually having the problem. It is clear that further analytic work is required on this point in order to establish the validity of the conjecture based on the analysis of the output of the ML search procedure in selected cases of boundary and non-boundary solutions.

#### 4. Conclusions

While the results of the present investigation are not entirely unambiguous, the Dodo might well answer the question, "But who has won?", by saying, "The 2RC method, of course!" Both in terms of relative bias and mean-square error, over a wide range of parameter values, the two-round procedure, using a value of  $\hat{\rho}$  estimated from first-round regressions including individual constant terms, compares favorably with all the other estimation techniques investigated, including maximum likelihood, which method has considerable intuitive appeal for most econometricians. All studies of this sort, however, have many dangling loose ends; further investigation of which is required. Among those noted in preceding pages are the following:

1. The sensitivity of the various methods investigated to the presence of specification error, especially to the presence of serial correlation in the "remainder" effects  $v_{it}$  and/or the presence of a individual-invariant, "period effect,"  $\lambda_t$ .
2. The sensitivity of the various methods, particularly the maximum-likelihood method, to the choice of exogenous variables with different characteristics.
3. The question of whether the Monte Carlo results on bias and mean-square error can be well approximated by the so-called small- $\sigma$  asymptotics of J. Kadane, i.e., approximations to

finite sample moments obtained by letting the variance of the disturbance term tend to zero.

4. The reason or reasons why least-squares with individual constant terms underestimates  $\alpha$  and overestimates  $\beta$  and  $\rho$ .
5. Finally, the question of the shape of the likelihood function particularly its flatness or peakedness in certain directions and the possibility of more than one local maximum and the implications of this shape for the convergence of gradient maximization procedures.

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FOOTNOTES

1/ This elementary and old point (see Haavelmo [8, pp. 50-51]) is stressed here because the nature of the disturbances in the analysis of cross-sections over time has proved to be a source of considerable confusion.

2/ It is also standard to assume that the relationship to be estimated is homogeneous from observation to observation, even though disturbed, that is to say, the parameters excepting the constant term do not vary from individual to individual in, for example, a cross-section. It is possible, of course, to adopt a more general specification in which the coefficients in the relationship to be estimated are also random variables and we estimate only certain of their moments (see Hildreth, C., and J.P. Hauck, "Linear Models with Random Coefficients," unpublished mimeo. (March 27, 1967) and [18], for example), but such models have not proved highly productive. The simplification that the disturbing elements affect only the level, and not the form of the functional relationship, is the basis of nearly all econometric work.

3/ Of course, the problem can always be cast in the form of a prediction conditional upon past values of the dependent variable and the independent variables alone, but the calculated residuals provide equivalent information if the regression coefficients have been correctly calculated.

4/ I find this result counter-intuitive on at least two counts: First, it is puzzling that the choice of an initial estimate of  $\rho$  in a two-round procedure can be completely arbitrary, and, indeed may be zero, so that ordinary least-squares are used in the second-round, without in any way impairing the consistency or asymptotic efficiency of the estimates. Second, it is difficult to believe that if  $N$  were to increase at a much greater rate than  $T$ , estimation of each  $\mu_i$  could possibly be as efficient a procedure as one based on some characteristics of the distribution of the  $\mu_i$ . Since, however, the proof of Amemiya's assertions in the case when both  $N$  and  $T$  tend to infinity consists merely of the assumption of convergence of the moment matrix of the independent variables and the following sentence: "Then, extending our results of Section 4, it is easy to show that, regardless of the way  $N$  and  $T$  go to infinity ...", [1, p. 14], it is difficult for me to find an error in his derivation.

5/ Large-sample asymptotic results are, in fact, only one form of asymptotic theory. Essentially, what we try to do is to approximate the finite sample moments of our estimators, or some function thereof, by the first few terms in an expansion in powers of the reciprocal



of sample size. It is possible to make similar calculations using expansions in powers of a scalar factor in the variance-covariance matrix of disturbances. This amounts to an asymptotic result for the variance of the disturbances tending to zero rather than sample size to infinity. Often these results provide a better approximation to the finite sample distributions obtained in Monte Carlo experiments than do the large-sample asymptotics. An explanation of the method and application to certain problems in the estimation of simultaneous equations is contained in a forthcoming Cowles Foundation Discussion Paper by J.B. Kadane. At a later date an attempt will be made to derive asymptotic results of this sort for the present model and compare them with the experimental sampling distributions reported here.

5/ For example, Malinvaud [15, pp. 463-64] shows that the asymptotic bias of the ordinary least-squares estimate of the coefficient of a lagged dependent variable in a regression containing exogenous variables is smaller than when no exogenous variables are present.

7/ Only a true random process may, of course, produce real random numbers. To approximate such a process digitally in machines with a 36-bit word recursion relations of the form

$$R_{n+1} = CR_n \pmod{2^{35}}$$

are used to produce a sequence of numbers  $R_0, R_1, R_2, \dots$  which simulate the behavior of a true random sequence, i.e., so-called "pseudorandom numbers." (For a discussion of various methods for generating pseudorandom numbers and the problems arising, see [9, pp. 25-42].) Not only do such sequences repeat after a certain point, but they tend to exhibit certain forms of nonrandom behavior. Hence when generating long sequences of such numbers it is desirable to use several generators, i.e., relations of the form given above with differently chosen  $C$  and  $R_0$ . In the calculations reported here, five such generators were used in such a way that no generator was ever started at a previous starting point and in such a way that the particular generator used to produce a number in the ultimate sequence was itself determined through the use of one of the generators. It is believed that the sequence of numbers thus obtained are subject to very few of the difficulties and problems by which most sequences of pseudorandom numbers are beset. The numbers resulting were transformed into pseudorandom variables uniformly distributed on the interval  $[0, 1]$ . Pairs of such numbers were in turn transformed to independent normal variates with mean zero and variance one by means of the transformation.

$$\begin{cases} w_1 = (-2 \log v_1)^{\frac{1}{2}} \cos(2\pi v_2) \\ w_2 = (-2 \log v_1)^{\frac{1}{2}} \sin(2\pi v_2) \end{cases},$$

where  $(v_1, v_2)$  is a pair of uniformly and independently distributed random variables [9, p. 39].

8/ See, for example, [ , p. 234].

9/ Although (15) is a natural formula for estimating  $\rho$  from the calculated residuals, it differs from the maximum-likelihood estimate of  $\rho$  from observed  $\hat{u}_{it}$ , which is a common estimate in components of variance analysis. The maximum-likelihood estimate would be

$$\frac{\sum_{i=1}^N \left\{ \left[ \sum_{t=1}^T \hat{u}_{it} \right]^2 - \sum_{t=1}^T \hat{u}_{it}^2 \right\}}{NT(T-1)\hat{\sigma}^2}$$

where  $\hat{\sigma}^2$  is given by (16).

Except when  $\rho$  is very small or very large the two estimates tend to be quite close. The maximum-likelihood estimate, however, does have one major defect for the use proposed below: namely, it may be slightly negative, whereas  $r_i$  in (15) cannot be negative. Below it is proposed to use  $r_i$  in place of  $\rho$  in a "second-round" generalized least-squares estimation procedure. The possibility of a negative value has obvious disadvantages.

10/ Madansky [14, p. 52] gives an extremely general formula for the computation of an asymptotic variance-covariance matrix of estimates in a generalizes instrumental variable estimation context. He later interprets two- and three-stage least-squares as instrumental variable estimates and uses his result to evaluate the efficiency of three-stage least-squares. If the validity of Madansky's result is accepted for three-stage least squares, then it should also be applicable to the two-round estimates provided the estimates  $r_c$  and  $r_i$  are consistent estimates of  $\rho$ . Although they both are, estimated asymptotic standard errors have not been computed for either of the two-round estimates. Madansky's results as well as those of Sargan are for the case in which no lagged value of the dependent variable is present as one of the explanatory variables. On the basis of earlier results of Amemiya and Fuller [2] which showed such two-round estimates to be inefficient in comparison to maximum-likelihood estimates, the Madansky standard errors were never programmed. Later results of Amemiya [1], however, suggest their asymptotic validity.

11/ Except, of course, in the case of the generalized least-squares estimates.

12/ See Hurwicz [10].

APPENDIX

Means and Mean-Square Errors of Various Estimates of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\sigma^2$  and  $\rho$  and Selected Estimates of Standard Errors for Various Values of the Parameters  $\alpha$ ,  $\beta$ , and  $\rho$ .











TABLE A.3: LSC. Least Squares with Individual Constant Terms.  
 Estimates of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\sigma^2$  and  $\rho$ .  
 Means and Mean-Square Errors. Various Values of  $\alpha$  and  $\rho$ .  
 $\beta = 0.0$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$ .

Sample size:  
 T=10 N=25

|                | $\beta =$        | 0.0    | $\rho = 0.0$ | $\rho = 0.15$ | $\rho = 0.30$ | $\rho = 0.45$ | $\rho = 0.60$ | $\rho = 0.75$ | $\rho = 0.90$ | $\rho = 0.95$ |
|----------------|------------------|--------|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\alpha = 0.1$ | $\hat{\alpha}$   | Mean   | -.0239       | -.0274        | -.0145        | -.0083        | -.0146        | -.0256        | -.0016        | -.0100        |
|                |                  | MSE    | 0.0187       | 0.0205        | 0.0171        | 0.0167        | 0.0168        | 0.0217        | 0.0153        | 0.0147        |
|                | $\hat{\beta}$    | Mean   | -.0001       | -.0118        | -.0131        | 0.0240        | -.0092        | 0.0207        | 0.0020        | -.0028        |
|                |                  | MSE    | 0.0086       | 0.0104        | 0.0088        | 0.0067        | 0.0060        | 0.0034        | 0.0013        | 0.0005        |
|                | $\hat{\gamma}$   | Mean   | -.0143       | 0.0460        | 0.0306        | -.0942        | -.0143        | -.0741        | -.0443        | -.0409        |
|                | MSE              | 0.0877 | 0.0895       | 0.1111        | 0.0717        | 0.0958        | 0.0805        | 0.0456        | 0.0700        |               |
|                | $\hat{\sigma}^2$ | Mean   | 0.9937       | 1.0414        | 1.0654        | 1.0709        | 1.1062        | 1.3212        | 1.1108        | 1.1922        |
|                |                  | MSE    | 0.0050       | 0.0124        | 0.0250        | 0.0416        | 0.0643        | 0.2399        | 0.1292        | 0.1458        |
|                | $\hat{\rho}$     | Mean   | 0.1263       | 0.2748        | 0.4232        | 0.5271        | 0.6750        | 0.8186        | 0.9142        | 0.9595        |
|                |                  | MSE    | 0.0170       | 0.0194        | 0.0202        | 0.0121        | 0.0101        | 0.0083        | 0.0008        | 0.0002        |
| $\alpha = 0.3$ | $\hat{\alpha}$   | Mean   | 0.1576       | 0.1406        | 0.1500        | 0.1662        | 0.1479        | 0.1638        | 0.1439        | 0.1566        |
|                |                  | MSE    | 0.0238       | 0.0300        | 0.0269        | 0.0235        | 0.0264        | 0.0219        | 0.0283        | 0.0251        |
|                | $\hat{\beta}$    | Mean   | 0.0305       | 0.0172        | 0.0086        | -.0044        | 0.0079        | 0.0068        | 0.0068        | 0.0031        |
|                |                  | MSE    | 0.0150       | 0.0097        | 0.0049        | 0.0049        | 0.0065        | 0.0027        | 0.0011        | 0.0007        |
|                | $\hat{\gamma}$   | Mean   | -.0821       | -.0603        | -.0260        | 0.0435        | -.0536        | -.0244        | 0.0142        | 0.0375        |
|                | MSE              | 0.1314 | 0.1073       | 0.0730        | 0.0950        | 0.1217        | 0.0650        | 0.0778        | 0.0428        |               |
|                | $\hat{\sigma}^2$ | Mean   | 1.0275       | 1.0489        | 1.1521        | 1.1659        | 1.2813        | 1.3160        | 1.3018        | 1.3673        |
|                |                  | MSE    | 0.0086       | 0.0160        | 0.0506        | 0.0764        | 0.1830        | 0.2663        | 0.2115        | 0.2978        |
|                | $\hat{\rho}$     | Mean   | 0.1241       | 0.2917        | 0.4640        | 0.5653        | 0.7109        | 0.8234        | 0.9296        | 0.9652        |
|                |                  | MSE    | 0.0165       | 0.0244        | 0.0347        | 0.0215        | 0.0166        | 0.0091        | 0.0013        | 0.0004        |
| $\alpha = 0.5$ | $\hat{\alpha}$   | Mean   | 0.3303       | 0.3189        | 0.3153        | 0.3271        | 0.3324        | 0.3270        | 0.3205        | 0.3425        |
|                |                  | MSE    | 0.0316       | 0.0364        | 0.0382        | 0.0337        | 0.0326        | 0.0339        | 0.0379        | 0.0294        |
|                | $\hat{\beta}$    | Mean   | 0.0080       | 0.0020        | -.0101        | -.0105        | 0.0211        | -.0082        | 0.0133        | 0.0018        |
|                |                  | MSE    | 0.0123       | 0.0083        | 0.0098        | 0.0080        | 0.0056        | 0.0042        | 0.0016        | 0.0008        |
|                | $\hat{\gamma}$   | Mean   | -.0242       | -.0384        | 0.0482        | 0.0182        | -.0942        | -.0325        | -.0540        | -.0482        |
|                | MSE              | 0.1052 | 0.0992       | 0.0924        | 0.1246        | 0.0745        | 0.1092        | 0.0939        | 0.0567        |               |
|                | $\hat{\sigma}^2$ | Mean   | 1.0068       | 1.1428        | 1.2618        | 1.3559        | 1.3949        | 1.5215        | 1.9413        | 1.5602        |
|                |                  | MSE    | 0.0083       | 0.0408        | 0.1111        | 0.1957        | 0.2943        | 0.4531        | 1.3890        | 0.6674        |
|                | $\hat{\rho}$     | Mean   | 0.1619       | 0.3604        | 0.5072        | 0.6426        | 0.7316        | 0.8497        | 0.9483        | 0.9685        |
|                |                  | MSE    | 0.0286       | 0.0493        | 0.0505        | 0.0425        | 0.0218        | 0.0119        | 0.0029        | 0.0005        |

TABLE A.3 (Continued)

| $\beta =$      |                  | 0.0  | $\rho = 0.0$ | $\rho = 0.15$ | $\rho = 0.30$ | $\rho = 0.45$ | $\rho = 0.60$ | $\rho = 0.75$ | $\rho = 0.90$ | $\rho = 0.95$ |
|----------------|------------------|------|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\alpha = 0.7$ | $\hat{\alpha}$   | Mean | 0.4933       | 0.4767        | 0.4937        | 0.4831        | 0.4980        | 0.4993        | 0.4933        | 0.4759        |
|                |                  | MSE  | 0.0457       | 0.0546        | 0.0454        | 0.0509        | 0.0445        | 0.0445        | 0.0461        | 0.0558        |
|                | $\hat{\beta}$    | Mean | -.0076       | -.0107        | -.0097        | -.0108        | 0.0066        | -.0053        | -.0014        | 0.0013        |
|                |                  | MSE  | 0.0123       | 0.0109        | 0.0096        | 0.0080        | 0.0038        | 0.0034        | 0.0019        | 0.0008        |
|                | $\hat{\gamma}$   | Mean | 0.0186       | 0.0553        | -.0178        | -.0162        | 0.0242        | -.0028        | -.0101        | -.0580        |
|                |                  | MSE  | 0.1239       | 0.1354        | 0.1353        | 0.1280        | 0.0701        | 0.1382        | 0.1586        | 0.1104        |
|                | $\hat{\sigma}^2$ | Mean | 1.0730       | 1.3740        | 1.6260        | 1.8520        | 2.0122        | 2.0728        | 2.5033        | 2.7421        |
|                |                  | MSE  | 0.0138       | 0.1785        | 0.5400        | 1.0427        | 1.4714        | 1.6397        | 2.8747        | 4.4503        |
|                | $\hat{\rho}$     | Mean | 0.1965       | 0.4729        | 0.6119        | 0.7197        | 0.8111        | 0.8848        | 0.9623        | 0.9817        |
|                |                  | MSE  | 0.0412       | 0.1124        | 0.1055        | 0.0801        | 0.0495        | 0.0199        | 0.0041        | 0.0011        |
| $\alpha = 0.9$ | $\hat{\alpha}$   | Mean | 0.6474       | 0.6685        | 0.6724        | 0.6772        | 0.7138        | 0.7366        | 0.8201        | 0.8502        |
|                |                  | MSE  | 0.0680       | 0.0568        | 0.0551        | 0.0519        | 0.0372        | 0.0289        | 0.0073        | 0.0032        |
|                | $\hat{\beta}$    | Mean | -.0082       | -.0011        | -.0044        | -.0148        | 0.0032        | -.0058        | -.0027        | 0.0017        |
|                |                  | MSE  | 0.0159       | 0.0134        | 0.0114        | 0.0078        | 0.0040        | 0.0041        | 0.0011        | 0.0004        |
|                | $\hat{\gamma}$   | Mean | 0.0428       | 0.0424        | 0.0263        | 0.1085        | -.0007        | 0.0562        | -.0207        | 0.0166        |
|                |                  | MSE  | 0.1383       | 0.1514        | 0.1614        | 0.1905        | 0.1728        | 0.1576        | 0.0669        | 0.0547        |
|                | $\hat{\sigma}^2$ | Mean | 1.2920       | 2.2604        | 3.2662        | 3.8995        | 4.1332        | 3.9901        | 2.6043        | 1.8891        |
|                |                  | MSE  | 0.1196       | 1.9296        | 6.0954        | 10.113        | 11.173        | 10.920        | 3.1733        | 1.1089        |
|                | $\hat{\rho}$     | Mean | 0.3766       | 0.6752        | 0.8078        | 0.8690        | 0.9137        | 0.9407        | 0.9632        | 0.9745        |
|                |                  | MSE  | 0.1508       | 0.2826        | 0.2622        | 0.1784        | 0.0991        | 0.0369        | 0.0042        | 0.0007        |





TABLE A.5: IV. Instrumental-Variable Estimates of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\sigma^2$  and  $\rho$ . Means and Mean-Square Errors. Various Values of  $\alpha$  and  $\rho$ .  $\beta = 0.0$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$ .

Sample size:  
T=10 N=25

|                | $\beta =$        | 0.0         | $\rho = 0.0$      | $\rho = 0.15$     | $\rho = 0.30$     | $\rho = 0.45$    | $\rho = 0.60$     | $\rho = 0.75$    | $\rho = 0.90$     | $\rho = 0.95$     |
|----------------|------------------|-------------|-------------------|-------------------|-------------------|------------------|-------------------|------------------|-------------------|-------------------|
| $\alpha = 0.1$ | $\hat{\alpha}$   | Mean<br>MSE | 0.8800<br>23.582  | 0.1655<br>52.501  | -1.2629<br>27.770 | 0.1642<br>6.3932 | 0.6264<br>100.30  | 3.8689<br>935.42 | -.4638<br>111.59  | 8.9559<br>7936.6  |
|                | $\hat{\beta}$    | Mean<br>MSE | 0.0724<br>0.1742  | 0.0507<br>0.1997  | -.0548<br>0.0546  | 0.0408<br>0.0571 | -.0488<br>0.8579  | 0.0391<br>0.2469 | -.0445<br>0.0460  | 0.8369<br>20.440  |
|                | $\hat{\gamma}$   | Mean<br>MSE | -.2089<br>1.7832  | -.1321<br>1.7705  | 0.1922<br>1.5594  | -.1559<br>0.4259 | -.1017<br>10.709  | 0.8737<br>39.652 | 0.3252<br>1.7732  | -.0012<br>150.63  |
|                | $\hat{\sigma}^2$ | Mean<br>MSE | 25.023<br>4657.2  | 47.944<br>*       | 27.048<br>*       | 7.8398<br>251.00 | 82.865<br>*       | 690.46<br>*      | 114.60<br>*       | 9397.9<br>*       |
|                | $\hat{\rho}$     | Mean<br>MSE | 0.1240<br>0.0220  | 0.2553<br>0.0359  | 0.3477<br>0.0472  | 0.4926<br>0.0519 | 0.5737<br>0.0601  | 0.6262<br>0.0853 | 0.6543<br>0.1524  | 0.7395<br>0.1406  |
| $\alpha = 0.3$ | $\hat{\alpha}$   | Mean<br>MSE | 9.9142<br>5406.7  | -4.6209<br>480.52 | -.2671<br>43.804  | 0.8693<br>32.636 | -3.7855<br>1513.8 | 1.3991<br>18.449 | -24.016<br>*      | 0.9230<br>1.2713  |
|                | $\hat{\beta}$    | Mean<br>MSE | 0.5771<br>24.498  | 0.2004<br>4.3233  | 0.1049<br>0.3871  | 0.1598<br>0.9231 | -.3429<br>1.9332  | 0.1012<br>0.4702 | -3.7910<br>706.03 | 0.0182<br>0.0121  |
|                | $\hat{\gamma}$   | Mean<br>MSE | -1.1171<br>108.66 | -.2607<br>32.386  | -.2528<br>3.4776  | -.3430<br>8.9002 | 1.8783<br>104.98  | -.3577<br>5.7928 | 11.939<br>7092.1  | -.0381<br>0.3029  |
|                | $\hat{\sigma}^2$ | Mean<br>MSE | 5569.1<br>*       | 533.00<br>*       | 66.690<br>*       | 44.932<br>*      | 3040.6<br>*       | 32.282<br>*      | *<br>*            | 1.8102<br>18.509  |
|                | $\hat{\rho}$     | Mean<br>MSE | 0.1283<br>0.0246  | 0.2394<br>0.0388  | 0.4934<br>0.0777  | 0.4355<br>0.0642 | 0.6320<br>0.0552  | 0.6288<br>0.0753 | 0.6204<br>0.1912  | 0.6415<br>0.1919  |
| $\alpha = 0.5$ | $\hat{\alpha}$   | Mean<br>MSE | 0.1516<br>13.575  | -.4242<br>124.02  | 0.8772<br>111.20  | 0.3992<br>64.658 | 0.4944<br>56.302  | 0.2019<br>26.204 | 1.0933<br>1.8190  | 7.5752<br>3459.7  |
|                | $\hat{\beta}$    | Mean<br>MSE | 0.0282<br>0.2965  | 0.2754<br>1.6786  | 0.0512<br>2.2119  | 0.0625<br>0.5780 | -.0161<br>0.2649  | -.0029<br>0.5039 | 0.0068<br>0.1130  | -.7137<br>14.267  |
|                | $\hat{\gamma}$   | Mean<br>MSE | -.1004<br>2.4541  | -.8012<br>14.156  | -.2840<br>16.748  | 0.1703<br>4.1878 | 0.4370<br>9.8415  | 0.2286<br>12.440 | -.1080<br>1.2681  | -4.3673<br>792.79 |
|                | $\hat{\sigma}^2$ | Mean<br>MSE | 19.025<br>5499.9  | 208.43<br>*       | 199.56<br>*       | 148.95<br>*      | 106.72<br>*       | 65.286<br>*      | 6.0410<br>265.79  | *<br>*            |
|                | $\hat{\rho}$     | Mean<br>MSE | 0.1985<br>0.0565  | 0.3739<br>0.0935  | 0.4538<br>0.0970  | 0.5119<br>0.0942 | 0.6595<br>0.0685  | 0.6928<br>0.0772 | 0.7407<br>0.0839  | 0.6467<br>0.2177  |

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TABLE A.5 (Continued)

| $\beta =$      | 0.0              | $\rho = 0.0$ | $\rho = 0.15$    | $\rho = 0.30$     | $\rho = 0.45$    | $\rho = 0.60$     | $\rho = 0.75$    | $\rho = 0.90$    | $\rho = 0.95$    |                  |
|----------------|------------------|--------------|------------------|-------------------|------------------|-------------------|------------------|------------------|------------------|------------------|
| $\alpha = 0.7$ | $\hat{\alpha}$   | Mean<br>MSE  | -.9152<br>93.643 | 0.9499<br>18.582  | 0.3132<br>12.985 | 0.0023<br>79.494  | 1.4831<br>23.496 | 0.4316<br>39.344 | 0.4853<br>24.160 | 1.0496<br>0.4304 |
|                | $\hat{\beta}$    | Mean<br>MSE  | 0.1980<br>4.8416 | 0.0011<br>0.4605  | 0.2127<br>1.2255 | -.1653<br>1.4816  | 0.4039<br>8.0884 | -.1403<br>1.0613 | 0.1262<br>0.7150 | 0.0373<br>0.0391 |
|                | $\hat{\gamma}$   | Mean<br>MSE  | -.0451<br>11.018 | 0.0408<br>6.0805  | -.6860<br>15.611 | -.0146<br>24.652  | -.7295<br>25.945 | 1.6781<br>129.18 | -.2757<br>7.5572 | -.1292<br>0.7107 |
|                | $\hat{\sigma}^2$ | Mean<br>MSE  | 190.67<br>*      | 66.825<br>*       | 54.129<br>*      | 411.15<br>*       | 164.33<br>*      | 245.13<br>*      | 222.88<br>*      | 2.6274<br>71.648 |
|                | $\hat{\rho}$     | Mean<br>MSE  | 0.3039<br>0.1186 | 0.5229<br>0.2078  | 0.5681<br>0.1469 | 0.6362<br>0.1103  | 0.6744<br>0.1022 | 0.7381<br>0.0636 | 0.7118<br>0.1313 | 0.6648<br>0.1798 |
| $\alpha = 0.9$ | $\hat{\alpha}$   | Mean<br>MSE  | 0.4518<br>20.914 | -1.9992<br>235.43 | 1.1263<br>3.0190 | 1.4361<br>10.013  | 0.9925<br>3.0368 | 1.4808<br>20.241 | 0.9311<br>1.1061 | 0.9967<br>0.0831 |
|                | $\hat{\beta}$    | Mean<br>MSE  | -.2256<br>1.1961 | 0.0731<br>4.2174  | 0.0667<br>0.3237 | 0.1525<br>1.2712  | 0.1150<br>0.3791 | 0.1952<br>1.7318 | 0.0594<br>0.1937 | -.0167<br>0.0116 |
|                | $\hat{\gamma}$   | Mean<br>MSE  | 0.7940<br>9.2106 | 0.7869<br>205.16  | -.1635<br>5.3338 | -1.0861<br>54.356 | -.5771<br>5.1724 | -.9599<br>50.591 | -.3852<br>3.8067 | 0.0787<br>0.2135 |
|                | $\hat{\sigma}^2$ | Mean<br>MSE  | 97.977<br>*      | 3663.6<br>*       | 70.057<br>*      | 333.73<br>*       | 135.54<br>*      | 837.68<br>*      | 89.028<br>*      | 4.9427<br>280.16 |
|                | $\hat{\rho}$     | Mean<br>MSE  | 0.4753<br>0.2763 | 0.6264<br>0.3176  | 0.6736<br>0.2201 | 0.6238<br>0.1268  | 0.7316<br>0.1040 | 0.6788<br>0.1013 | 0.6972<br>0.1327 | 0.7822<br>0.0877 |

\* Format field exceeded.







TABLE A.7: ML. Maximum-Likelihood Estimates of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\sigma^2$  and  $\rho$ . Means and Mean-Square Errors. Various Values of  $\alpha$  and  $\rho$ .  $\beta = 0.0$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$ .

Sample size:  
T=10 N=25

|                | $\beta =$        | 0.0  | $\rho = 0.0$ | $\rho = 0.15$ | $\rho = 0.30$ | $\rho = 0.45$ | $\rho = 0.60$ | $\rho = 0.75$ | $\rho = 0.90$ | $\rho = 0.95$ |
|----------------|------------------|------|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\alpha = 0.1$ | $\hat{\alpha}$   | Mean | 0.0764       | 0.0921        | 0.1107        | 0.1216        | 0.1097        | 0.1015        | 0.1296        | 0.1180        |
|                |                  | MSE  | 0.0044       | 0.0055        | 0.0059        | 0.0077        | 0.0053        | 0.0084        | 0.0079        | 0.0040        |
|                | $\hat{\beta}$    | Mean | 0.0007       | -.0108        | -.0151        | 0.0237        | -.0064        | 0.0204        | 0.0019        | -.0028        |
|                |                  | MSE  | 0.0074       | 0.0092        | 0.0074        | 0.0055        | 0.0050        | 0.0029        | 0.0011        | 0.0004        |
|                | $\hat{\gamma}$   | Mean | -.0150       | 0.0410        | 0.0376        | -.0894        | -.0184        | -.0713        | -.0385        | -.0345        |
|                |                  | MSE  | 0.0727       | 0.0788        | 0.0917        | 0.0579        | 0.0770        | 0.0642        | 0.0360        | 0.0531        |
|                | $\hat{\sigma}^2$ | Mean | 0.9797       | 0.9905        | 0.9709        | 0.9426        | 0.9398        | 1.0742        | 0.8627        | 0.9227        |
|                |                  | MSE  | 0.0048       | 0.0086        | 0.0157        | 0.0283        | 0.0401        | 0.1020        | 0.0929        | 0.0761        |
|                | $\hat{\rho}$     | Mean | 0.0140       | 0.1417        | 0.2874        | 0.3947        | 0.5692        | 0.7471        | 0.8744        | 0.9405        |
|                |                  | MSE  | 0.0006       | 0.0042        | 0.0066        | 0.0124        | 0.0082        | 0.0073        | 0.0021        | 0.0004        |
| $\alpha = 0.3$ | $\hat{\alpha}$   | Mean | 0.2710       | 0.2878        | 0.3095        | 0.3328        | 0.3064        | 0.3305        | 0.3083        | 0.3240        |
|                |                  | MSE  | 0.0053       | 0.0065        | 0.0072        | 0.0124        | 0.0056        | 0.0071        | 0.0070        | 0.0084        |
|                | $\hat{\beta}$    | Mean | 0.0248       | 0.0113        | 0.0031        | 0.0016        | 0.0041        | 0.0060        | 0.0035        | 0.0027        |
|                |                  | MSE  | 0.0115       | 0.0078        | 0.0041        | 0.0044        | 0.0050        | 0.0023        | 0.0008        | 0.0005        |
|                | $\hat{\gamma}$   | Mean | -.0665       | -.0402        | -.0099        | 0.0186        | -.0350        | -0.0221       | 0.0171        | 0.0313        |
|                |                  | MSE  | 0.1001       | 0.0854        | 0.0586        | 0.0722        | 0.0871        | 0.0467        | 0.0522        | 0.0265        |
|                | $\hat{\sigma}^2$ | Mean | 1.0095       | 0.1347        | 0.9883        | 0.9476        | 0.9853        | 0.9370        | 0.8893        | 0.9038        |
|                |                  | MSE  | 0.0076       | 0.0047        | 0.0155        | 0.0323        | 0.0637        | 0.0978        | 0.0897        | 0.1070        |
|                | $\hat{\rho}$     | Mean | 0.0084       | 0.9733        | 0.2907        | 0.3889        | 0.5707        | 0.7139        | 0.8791        | 0.9374        |
|                |                  | MSE  | 0.0003       | 0.0108        | 0.0113        | 0.0203        | 0.0107        | 0.0114        | 0.0020        | 0.0008        |
| $\alpha = 0.5$ | $\hat{\alpha}$   | Mean | 0.4750       | 0.5286        | 0.5521        | 0.6341        | 0.6456        | 0.6623        | 0.7084        | 0.7539        |
|                |                  | MSE  | 0.0030       | 0.0098        | 0.0178        | 0.0518        | 0.0582        | 0.0757        | 0.1114        | 0.1236        |
|                | $\hat{\beta}$    | Mean | 0.0128       | 0.0013        | -.0089        | -.0161        | 0.0187        | -.0021        | 0.0123        | 0.0024        |
|                |                  | MSE  | 0.0085       | 0.0062        | 0.0076        | 0.0058        | 0.0036        | 0.0026        | 0.0014        | 0.0005        |
|                | $\hat{\gamma}$   | Mean | -.0375       | -.0253        | 0.0370        | 0.0388        | -.0706        | -.0286        | -.0300        | -.0100        |
|                |                  | MSE  | 0.0724       | 0.0674        | 0.0633        | 0.0691        | 0.0382        | 0.0577        | 0.0378        | 0.0192        |
|                | $\hat{\sigma}^2$ | Mean | 0.9697       | 0.9694        | 0.9519        | 0.8540        | 0.7783        | 0.7280        | 0.7368        | 0.4682        |
|                |                  | MSE  | 0.0082       | 0.0080        | 0.0222        | 0.0627        | 0.1232        | 0.2238        | 0.5264        | 0.5261        |
|                | $\hat{\rho}$     | Mean | 0.0190       | 0.1247        | 0.2265        | 0.2724        | 0.3559        | 0.4630        | 0.4939        | 0.4680        |
|                |                  | MSE  | 0.0013       | 0.0066        | 0.0254        | 0.0826        | 0.1321        | 0.2061        | 0.3674        | 0.4427        |

TABLE A.7 (Continued)

| $\beta =$      | 0.0              | $\rho = 0.0$ | $\rho = 0.15$ | $\rho = 0.30$ | $\rho = 0.45$ | $\rho = 0.60$ | $\rho = 0.75$ | $\rho = 0.90$ | $\rho = 0.95$ |        |
|----------------|------------------|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|--------|
| $\alpha = 0.7$ | $\hat{\alpha}$   | Mean         | 0.6630        | 0.7981        | 0.8868        | 0.9176        | 0.9515        | 0.9564        | 0.9850        | 0.9690 |
|                |                  | MSE          | 0.0041        | 0.0195        | 0.0413        | 0.0533        | 0.0671        | 0.0755        | 0.0844        | 0.0828 |
|                | $\hat{\beta}$    | Mean         | -.0186        | -.0094        | 0.0088        | -.0022        | 0.0043        | -.0006        | 0.0002        | 0.0063 |
|                |                  | MSE          | 0.0081        | 0.0075        | 0.0056        | 0.0041        | 0.0033        | 0.0031        | 0.0009        | 0.0004 |
|                | $\hat{\gamma}$   | Mean         | 0.0532        | 0.0391        | -.0420        | -.0024        | -.0132        | -.0005        | -.0037        | -.0346 |
|                |                  | MSE          | 0.0782        | 0.0733        | 0.0536        | 0.0398        | 0.0282        | 0.0484        | 0.0105        | 0.0132 |
|                | $\hat{\sigma}^2$ | Mean         | 1.0022        | 0.9483        | 0.8137        | 0.6776        | 0.4872        | 0.3603        | 0.1501        | 0.1842 |
|                |                  | MSE          | 0.0059        | 0.0145        | 0.0430        | 0.1328        | 0.3037        | 0.4888        | 0.7564        | 0.8971 |
|                | $\hat{\rho}$     | Mean         | 0.0186        | 0.0539        | 0.0272        | 0.0364        | 0.0246        | 0.0502        | 0.0422        | 0.0983 |
|                |                  | MSE          | 0.0010        | 0.0220        | 0.0816        | 0.1901        | 0.3468        | 0.5290        | 0.7652        | 0.8023 |
| $\alpha = 0.6$ | $\hat{\alpha}$   | Mean         | 0.8823        | 0.9796        | 1.0024        | 1.0093        | 1.0118        | 1.0177        | 1.0189        | 1.0178 |
|                |                  | MSE          | 0.0018        | 0.0066        | 0.0106        | 0.0120        | 0.0135        | 0.0140        | 0.0147        | 0.0145 |
|                | $\hat{\beta}$    | Mean         | -.0146        | -.0073        | -.0037        | -.0181        | -.0019        | -.0133        | 0.0019        | 0.0008 |
|                |                  | MSE          | 0.0119        | 0.0081        | 0.0058        | 0.0038        | 0.0033        | 0.0030        | 0.0012        | 0.0009 |
|                | $\hat{\gamma}$   | Mean         | 0.0396        | 0.0286        | 0.0112        | 0.0532        | -.0069        | 0.0341        | -.0034        | -.0019 |
|                |                  | MSE          | 0.1015        | 0.0681        | 0.0521        | 0.0331        | 0.0283        | 0.0254        | 0.0100        | 0.0072 |
|                | $\hat{\sigma}^2$ | Mean         | 0.9732        | 0.8908        | 0.7446        | 0.5910        | 0.4937        | 0.2765        | 0.1522        | 0.1208 |
|                |                  | MSE          | 0.0071        | 0.0166        | 0.0698        | 0.1706        | 0.4977        | 0.5262        | 0.8014        | 0.8492 |
|                | $\hat{\rho}$     | Mean         | 0.0176        | 0.0052        | 0.0005        | 0.0011        | 0.0241        | 0.0113        | 0.0235        | 0.0765 |
|                |                  | MSE          | 0.0011        | 0.0212        | 0.0897        | 0.2016        | 0.3488        | 0.5516        | 0.7863        | 0.8188 |

TABLE A-8: GLS Standard Errors of the Generalized Least-Squares Estimates of  $\alpha$ ,  $\beta$ , and  $\gamma$   
 Sample Size: Means and Variances. Various Values of  $\rho$ .  $\beta \leq 0.0$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$   
 $T = 10$ ,  $N = 25$

|                | $\beta =$              | 0.0      | $\rho = 0.0$          | $\rho = 0.15$         | $\rho = 0.30$         | $\rho = 0.45$         | $\rho = 0.60$         | $\rho = 0.75$         | $\rho = 0.90$         | $\rho = 0.95$         |
|----------------|------------------------|----------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $\alpha = 0.1$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0629                | 0.0626                | 0.0629                | 0.0630                | 0.0625                | 0.0629                | 0.0628                | 0.0627                |
|                |                        | Variance | $1.42 \times 10^{-6}$ | $1.51 \times 10^{-6}$ | $1.35 \times 10^{-6}$ | $1.66 \times 10^{-6}$ | $1.94 \times 10^{-6}$ | $1.69 \times 10^{-6}$ | $2.04 \times 10^{-6}$ | $1.36 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean     | 0.0962                | 0.0921                | 0.0835                | 0.0752                | 0.0633                | 0.0510                | 0.0319                | 0.0228                |
|                |                        | Variance | $1.04 \times 10^{-5}$ | $1.61 \times 10^{-5}$ | $1.72 \times 10^{-5}$ | $1.03 \times 10^{-5}$ | $8.42 \times 10^{-6}$ | $4.22 \times 10^{-6}$ | $2.99 \times 10^{-6}$ | $7.75 \times 10^{-7}$ |
|                | $\hat{\gamma}$         | Mean     | 0.2869                | 0.2848                | 0.2711                | 0.2607                | 0.2426                | 0.2305                | 0.2101                | 0.2072                |
|                |                        | Variance | $9.22 \times 10^{-5}$ | $1.58 \times 10^{-4}$ | $1.82 \times 10^{-4}$ | $1.22 \times 10^{-4}$ | $1.26 \times 10^{-4}$ | $8.48 \times 10^{-5}$ | $1.27 \times 10^{-4}$ | $6.41 \times 10^{-5}$ |
| $\alpha = 0.3$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0606                | 0.0606                | 0.0602                | 0.0597                | 0.0598                | 0.0598                | 0.0605                | 0.0601                |
|                |                        | Variance | $2.94 \times 10^{-6}$ | $2.76 \times 10^{-6}$ | $3.01 \times 10^{-6}$ | $3.40 \times 10^{-6}$ | $2.20 \times 10^{-6}$ | $2.04 \times 10^{-6}$ | $2.91 \times 10^{-6}$ | $2.69 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean     | 0.0978                | 0.0915                | 0.0839                | 0.0752                | 0.0644                | 0.0498                | 0.0317                | 0.0226                |
|                |                        | Variance | $1.87 \times 10^{-5}$ | $2.10 \times 10^{-5}$ | $9.48 \times 10^{-6}$ | $1.18 \times 10^{-5}$ | $1.00 \times 10^{-5}$ | $5.85 \times 10^{-6}$ | $1.73 \times 10^{-6}$ | $1.03 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean     | 0.2915                | 0.2830                | 0.2723                | 0.2613                | 0.2467                | 0.2252                | 0.2094                | 0.2045                |
|                |                        | Variance | $1.65 \times 10^{-4}$ | $2.04 \times 10^{-4}$ | $1.00 \times 10^{-4}$ | $1.41 \times 10^{-4}$ | $1.43 \times 10^{-4}$ | $1.27 \times 10^{-4}$ | $7.67 \times 10^{-5}$ | $8.66 \times 10^{-5}$ |
| $\alpha = 0.5$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0549                | 0.0543                | 0.0543                | 0.0542                | 0.0545                | 0.0538                | 0.0536                | 0.0540                |
|                |                        | Variance | $4.10 \times 10^{-6}$ | $5.60 \times 10^{-6}$ | $5.31 \times 10^{-6}$ | $3.94 \times 10^{-6}$ | $4.56 \times 10^{-6}$ | $5.21 \times 10^{-6}$ | $5.60 \times 10^{-6}$ | $5.91 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean     | 0.0959                | 0.0915                | 0.0845                | 0.0744                | 0.0643                | 0.0501                | 0.0323                | 0.0226                |
|                |                        | Variance | $1.74 \times 10^{-5}$ | $9.53 \times 10^{-6}$ | $1.35 \times 10^{-5}$ | $1.36 \times 10^{-5}$ | $7.65 \times 10^{-6}$ | $5.26 \times 10^{-6}$ | $1.69 \times 10^{-6}$ | $1.51 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean     | 0.2857                | 0.2830                | 0.2741                | 0.2583                | 0.2463                | 0.2273                | 0.2137                | 0.2044                |
|                |                        | Variance | $1.54 \times 10^{-4}$ | $9.23 \times 10^{-5}$ | $1.48 \times 10^{-4}$ | $1.61 \times 10^{-4}$ | $1.11 \times 10^{-4}$ | $1.17 \times 10^{-4}$ | $6.97 \times 10^{-5}$ | $1.20 \times 10^{-4}$ |

TABLE A-8:GLS (continued)

T = 10, N = 25

|                | $\beta =$              | 0.0      | $\rho = 0.0$          | $\rho = 0.15$         | $\rho = 0.30$         | $\rho = 0.45$         | $\rho = 0.60$         | $\rho = 0.75$         | $\rho = 0.90$         | $\rho = 0.95$         |
|----------------|------------------------|----------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $\alpha = 0.7$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0460                | 0.0433                | 0.0431                | 0.0438                | 0.0431                | 0.0435                | 0.0429                | 0.0437                |
|                |                        | Variance | $5.81 \times 10^{-6}$ | $6.15 \times 10^{-6}$ | $1.02 \times 10^{-5}$ | $8.62 \times 10^{-6}$ | $1.14 \times 10^{-5}$ | $6.40 \times 10^{-6}$ | $1.05 \times 10^{-5}$ | $7.58 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean     | 0.0974                | 0.0918                | 0.0849                | 0.0757                | 0.0638                | 0.0506                | 0.0321                | 0.0229                |
|                |                        | Variance | $1.37 \times 10^{-5}$ | $1.03 \times 10^{-5}$ | $2.14 \times 10^{-5}$ | $8.98 \times 10^{-6}$ | $6.42 \times 10^{-6}$ | $4.62 \times 10^{-6}$ | $2.63 \times 10^{-6}$ | $8.29 \times 10^{-7}$ |
|                | $\hat{\gamma}$         | Mean     | 0.2905                | 0.2840                | 0.2760                | 0.2628                | 0.2448                | 0.2296                | 0.2132                | 0.2081                |
|                |                        | Variance | $1.21 \times 10^{-4}$ | $1.00 \times 10^{-4}$ | $2.40 \times 10^{-4}$ | $1.12 \times 10^{-4}$ | $9.51 \times 10^{-5}$ | $1.01 \times 10^{-4}$ | $1.46 \times 10^{-4}$ | $7.54 \times 10^{-5}$ |
| $\alpha = 0.9$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0285                | 0.0244                | 0.0233                | 0.0237                | 0.0221                | 0.0221                | 0.0190                | 0.0170                |
|                |                        | Variance | $1.00 \times 10^{-5}$ | $7.88 \times 10^{-6}$ | $6.70 \times 10^{-6}$ | $8.97 \times 10^{-6}$ | $7.38 \times 10^{-6}$ | $9.17 \times 10^{-6}$ | $3.85 \times 10^{-6}$ | $5.42 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean     | 0.0959                | 0.0914                | 0.0836                | 0.0746                | 0.0631                | 0.0502                | 0.0318                | 0.0227                |
|                |                        | Variance | $1.65 \times 10^{-5}$ | $1.33 \times 10^{-5}$ | $1.54 \times 10^{-5}$ | $1.19 \times 10^{-5}$ | $7.32 \times 10^{-6}$ | $5.28 \times 10^{-6}$ | $1.90 \times 10^{-6}$ | $1.22 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean     | 0.2859                | 0.2829                | 0.2715                | 0.2593                | 0.2422                | 0.2270                | 0.2101                | 0.2047                |
|                |                        | Variance | $1.43 \times 10^{-4}$ | $1.35 \times 10^{-4}$ | $1.64 \times 10^{-4}$ | $1.42 \times 10^{-4}$ | $1.14 \times 10^{-4}$ | $1.04 \times 10^{-4}$ | $8.52 \times 10^{-5}$ | $9.58 \times 10^{-5}$ |

TABLE A-9: OLS Standard Errors of the Ordinary Least-Squares Estimators of  $\alpha$ ,  $\beta$ , and  $\lambda$ . Means and Variances. Various Values of  $\alpha$  and  $\rho$ :  $\beta=0.0$ ,  $\lambda=0.0$ ,  $\sigma^2=1.0$

Sample Size:  
T = 10, N = 25

|                | $\beta =$              | 0.0      | $\rho=0.0$            | $\rho=0.15$           | $\rho=0.30$           | $\rho=0.45$           | $\rho=0.60$           | $\rho=0.75$           | $\rho=0.90$           | $\rho=0.95$           |
|----------------|------------------------|----------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $\alpha = 0.1$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0629                | 0.0610                | 0.0578                | 0.0539                | 0.0474                | 0.0370                | 0.0255                | 0.0180                |
|                |                        | Variance | $1.42 \times 10^{-6}$ | $2.79 \times 10^{-6}$ | $5.25 \times 10^{-6}$ | $8.43 \times 10^{-6}$ | $1.37 \times 10^{-5}$ | $1.89 \times 10^{-5}$ | $1.35 \times 10^{-5}$ | $6.45 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean     | 0.0962                | 0.0955                | 0.0908                | 0.0847                | 0.0749                | 0.0624                | 0.0400                | 0.0292                |
|                |                        | Variance | $1.04 \times 10^{-5}$ | $1.78 \times 10^{-5}$ | $2.15 \times 10^{-5}$ | $1.55 \times 10^{-5}$ | $1.48 \times 10^{-5}$ | $9.84 \times 10^{-6}$ | $5.25 \times 10^{-6}$ | $2.18 \times 10^{-6}$ |
|                | $\hat{\lambda}$        | Mean     | 0.2869                | 0.2847                | 0.2707                | 0.2525                | 0.2234                | 0.1861                | 0.1194                | 0.0870                |
|                |                        | Variance | $9.22 \times 10^{-5}$ | $1.61 \times 10^{-4}$ | $1.92 \times 10^{-4}$ | $1.36 \times 10^{-4}$ | $1.32 \times 10^{-4}$ | $8.81 \times 10^{-5}$ | $4.70 \times 10^{-5}$ | $1.91 \times 10^{-5}$ |
| $\alpha = 0.3$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0606                | 0.0569                | 0.0503                | 0.0449                | 0.0377                | 0.0288                | 0.0188                | 0.0129                |
|                |                        | Variance | $2.94 \times 10^{-6}$ | $5.07 \times 10^{-6}$ | $1.14 \times 10^{-5}$ | $1.77 \times 10^{-5}$ | $1.15 \times 10^{-5}$ | $1.45 \times 10^{-5}$ | $5.51 \times 10^{-6}$ | $2.97 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean     | 0.0978                | 0.0944                | 0.0898                | 0.0827                | 0.0730                | 0.0573                | 0.0374                | 0.0266                |
|                |                        | Variance | $1.87 \times 10^{-5}$ | $2.19 \times 10^{-5}$ | $1.09 \times 10^{-5}$ | $1.74 \times 10^{-5}$ | $1.52 \times 10^{-5}$ | $8.99 \times 10^{-6}$ | $2.69 \times 10^{-6}$ | $1.52 \times 10^{-6}$ |
|                | $\hat{\lambda}$        | Mean     | 0.2915                | 0.2816                | 0.2679                | 0.2469                | 0.2179                | 0.1710                | 0.1115                | 0.0795                |
|                |                        | Variance | $1.65 \times 10^{-4}$ | $1.97 \times 10^{-4}$ | $9.67 \times 10^{-5}$ | $1.52 \times 10^{-4}$ | $1.35 \times 10^{-4}$ | $8.21 \times 10^{-5}$ | $2.46 \times 10^{-5}$ | $1.36 \times 10^{-5}$ |
| $\alpha = 0.5$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0549                | 0.0471                | 0.0405                | 0.0334                | 0.0280                | 0.0209                | 0.0118                | 0.0089                |
|                |                        | Variance | $4.10 \times 10^{-6}$ | $1.23 \times 10^{-5}$ | $1.27 \times 10^{-5}$ | $1.18 \times 10^{-5}$ | $1.04 \times 10^{-5}$ | $7.26 \times 10^{-6}$ | $3.38 \times 10^{-6}$ | $1.94 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean     | 0.0959                | 0.0938                | 0.0886                | 0.0792                | 0.0694                | 0.0550                | 0.0353                | 0.0248                |
|                |                        | Variance | $1.74 \times 10^{-5}$ | $1.02 \times 10^{-5}$ | $1.49 \times 10^{-5}$ | $1.58 \times 10^{-5}$ | $8.47 \times 10^{-6}$ | $7.41 \times 10^{-6}$ | $2.52 \times 10^{-6}$ | $1.91 \times 10^{-6}$ |
|                | $\hat{\lambda}$        | Mean     | 0.2857                | 0.2796                | 0.2642                | 0.2364                | 0.2068                | 0.1641                | 0.1054                | 0.0739                |
|                |                        | Variance | $1.54 \times 10^{-4}$ | $8.93 \times 10^{-5}$ | $1.35 \times 10^{-4}$ | $1.42 \times 10^{-4}$ | $7.55 \times 10^{-5}$ | $6.70 \times 10^{-5}$ | $2.26 \times 10^{-5}$ | $1.71 \times 10^{-5}$ |

TABLE A-8 : OLS (continued)

Sample Size:

T = 10, N = 25

|                | $\beta =$              | 0.0      | $\rho = 0.0$          | $\rho = 0.15$         | $\rho = 0.30$         | $\rho = 0.45$         | $\rho = 0.60$         | $\rho = 0.75$         | $\rho = 0.90$         | $\rho = 0.95$         |
|----------------|------------------------|----------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $\alpha = 0.7$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0460                | 0.0336                | 0.0269                | 0.0220                | 0.0167                | 0.0128                | 0.0073                | 0.0051                |
|                |                        | Variance | $5.81 \times 10^{-6}$ | $7.24 \times 10^{-6}$ | $1.27 \times 10^{-5}$ | $8.59 \times 10^{-6}$ | $6.70 \times 10^{-6}$ | $2.51 \times 10^{-6}$ | $1.61 \times 10^{-6}$ | $4.61 \times 10^{-7}$ |
|                | $\hat{\beta}$          | Mean     | 0.0974                | 0.0931                | 0.0870                | 0.0778                | 0.0657                | 0.0526                | 0.0334                | 0.0238                |
|                |                        | Variance | $1.37 \times 10^{-5}$ | $1.12 \times 10^{-5}$ | $2.07 \times 10^{-5}$ | $8.39 \times 10^{-6}$ | $7.83 \times 10^{-6}$ | $5.70 \times 10^{-6}$ | $3.07 \times 10^{-6}$ | $8.44 \times 10^{-7}$ |
|                | $\hat{\lambda}$        | Mean     | 0.2905                | 0.2775                | 0.2596                | 0.2320                | 0.1960                | 0.1568                | 0.0995                | 0.0711                |
|                |                        | Variance | $1.21 \times 10^{-4}$ | $1.02 \times 10^{-4}$ | $1.90 \times 10^{-4}$ | $7.73 \times 10^{-5}$ | $6.88 \times 10^{-5}$ | $5.15 \times 10^{-5}$ | $2.88 \times 10^{-5}$ | $7.82 \times 10^{-6}$ |
| $\alpha = 0.9$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0285                | 0.0165                | 0.0117                | 0.0093                | 0.0066                | 0.0050                | 0.0028                | 0.0020                |
|                |                        | Variance | $1.00 \times 10^{-5}$ | $4.87 \times 10^{-6}$ | $2.58 \times 10^{-6}$ | $2.22 \times 10^{-6}$ | $9.34 \times 10^{-7}$ | $7.38 \times 10^{-7}$ | $1.31 \times 10^{-7}$ | $7.27 \times 10^{-8}$ |
|                | $\hat{\beta}$          | Mean     | 0.0959                | 0.0918                | 0.0839                | 0.0749                | 0.0632                | 0.0506                | 0.0324                | 0.0235                |
|                |                        | Variance | $1.65 \times 10^{-5}$ | $1.27 \times 10^{-5}$ | $1.49 \times 10^{-5}$ | $1.23 \times 10^{-5}$ | $6.80 \times 10^{-6}$ | $5.20 \times 10^{-6}$ | $2.02 \times 10^{-6}$ | $1.31 \times 10^{-6}$ |
|                | $\hat{\lambda}$        | Mean     | 0.2859                | 0.2735                | 0.2500                | 0.2233                | 0.1886                | 0.1507                | 0.0966                | 0.0700                |
|                |                        | Variance | $1.43 \times 10^{-4}$ | $1.17 \times 10^{-4}$ | $1.34 \times 10^{-4}$ | $1.08 \times 10^{-4}$ | $6.17 \times 10^{-5}$ | $4.54 \times 10^{-5}$ | $1.82 \times 10^{-5}$ | $1.16 \times 10^{-5}$ |

TABLE A-10: LSC Standard Errors of the Estimates of  $\alpha$  and  $\beta$  Obtained by Least-Squares with Individual Constant Terms. Means and Variances. Various Values of Sample size:  $T = 10, N = 25$   $\alpha$  and  $\rho$ .  $\beta = 0.0, \gamma = 0.0, \sigma^2 = 1.0$

|                | $\beta =$              | 0.0                   | $\rho = 0.0$          | $\rho = 0.15$         | $\rho = 0.30$         | $\rho = 0.45$         | $\rho = 0.60$         | $\rho = 0.75$         | $\rho = 0.90$         | $\rho = 0.95$         |
|----------------|------------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $\alpha = 0.1$ | S.E. of $\hat{\alpha}$ | Mean                  | 0.0631                | 0.0628                | 0.0633                | 0.0634                | 0.0627                | 0.0633                | 0.0633                | 0.0631                |
|                |                        | Variance              | $2.04 \times 10^{-6}$ | $1.56 \times 10^{-6}$ | $1.87 \times 10^{-6}$ | $2.35 \times 10^{-6}$ | $2.72 \times 10^{-6}$ | $2.26 \times 10^{-6}$ | $2.46 \times 10^{-6}$ | $1.58 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean                  | 0.0951                | 0.0885                | 0.0795                | 0.0717                | 0.0600                | 0.0478                | 0.0302                | 0.0216                |
|                | Variance               | $9.43 \times 10^{-6}$ | $1.76 \times 10^{-5}$ | $1.72 \times 10^{-5}$ | $1.04 \times 10^{-5}$ | $8.22 \times 10^{-6}$ | $4.65 \times 10^{-6}$ | $2.91 \times 10^{-6}$ | $6.95 \times 10^{-7}$ |                       |
|                | $\hat{\gamma}$         | Mean                  | —                     | —                     | —                     | —                     | —                     | —                     | —                     | —                     |
|                |                        | Variance              | —                     | —                     | —                     | —                     | —                     | —                     | —                     | —                     |
| $\alpha = 0.3$ | S.E. of $\hat{\alpha}$ | Mean                  | 0.0623                | 0.0624                | 0.0625                | 0.0619                | 0.0619                | 0.0622                | 0.0626                | 0.0623                |
|                |                        | Variance              | $2.16 \times 10^{-6}$ | $2.36 \times 10^{-6}$ | $2.42 \times 10^{-6}$ | $2.37 \times 10^{-6}$ | $2.44 \times 10^{-6}$ | $2.43 \times 10^{-6}$ | $3.03 \times 10^{-6}$ | $2.86 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean                  | 0.0969                | 0.0877                | 0.0794                | 0.0714                | 0.0607                | 0.0468                | 0.0299                | 0.0213                |
|                | Variance               | $2.07 \times 10^{-5}$ | $2.30 \times 10^{-5}$ | $1.19 \times 10^{-5}$ | $1.30 \times 10^{-5}$ | $8.56 \times 10^{-6}$ | $4.52 \times 10^{-6}$ | $1.56 \times 10^{-6}$ | $1.03 \times 10^{-6}$ |                       |
|                | $\hat{\gamma}$         | Mean                  | —                     | —                     | —                     | —                     | —                     | —                     | —                     | —                     |
|                |                        | Variance              | —                     | —                     | —                     | —                     | —                     | —                     | —                     | —                     |
| $\alpha = 0.5$ | S.E. of $\hat{\alpha}$ | Mean                  | 0.0596                | 0.0598                | 0.0596                | 0.0599                | 0.0598                | 0.0589                | 0.0598                | 0.0594                |
|                |                        | Variance              | $3.91 \times 10^{-6}$ | $3.66 \times 10^{-6}$ | $3.74 \times 10^{-6}$ | $2.92 \times 10^{-6}$ | $4.82 \times 10^{-6}$ | $5.13 \times 10^{-6}$ | $4.21 \times 10^{-6}$ | $6.39 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean                  | 0.0939                | 0.0869                | 0.0797                | 0.0700                | 0.0607                | 0.0470                | 0.0301                | 0.0212                |
|                | Variance               | $1.91 \times 10^{-5}$ | $8.36 \times 10^{-6}$ | $1.46 \times 10^{-5}$ | $1.36 \times 10^{-5}$ | $7.69 \times 10^{-6}$ | $5.02 \times 10^{-6}$ | $1.83 \times 10^{-6}$ | $1.47 \times 10^{-6}$ |                       |
|                | $\hat{\gamma}$         | Mean                  | —                     | —                     | —                     | —                     | —                     | —                     | —                     | —                     |
|                |                        | Variance              | —                     | —                     | —                     | —                     | —                     | —                     | —                     | —                     |

Table A-10:LSC (continued)

Sample size:

T = 10, N = 25

|                | $\beta =$              | 0.0      | $\rho = 0.0$          | $\rho = 0.15$         | $\rho = 0.30$         | $\rho = 0.45$         | $\rho = 0.60$         | $\rho = 0.75$         | $\rho = 0.90$         | $\rho = 0.95$         |
|----------------|------------------------|----------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $\alpha = 0.7$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0548                | 0.0549                | 0.0546                | 0.0553                | 0.0550                | 0.0541                | 0.0545                | 0.0554                |
|                |                        | Variance | $7.52 \times 10^{-6}$ | $5.83 \times 10^{-6}$ | $8.44 \times 10^{-6}$ | $6.91 \times 10^{-6}$ | $7.60 \times 10^{-6}$ | $4.90 \times 10^{-6}$ | $4.31 \times 10^{-6}$ | $1.16 \times 10^{-5}$ |
|                | $\hat{\beta}$          | Mean     | 0.0950                | 0.0862                | 0.0794                | 0.0708                | 0.0596                | 0.0473                | 0.0299                | 0.0212                |
|                |                        | Variance | $1.40 \times 10^{-5}$ | $1.44 \times 10^{-5}$ | $2.22 \times 10^{-5}$ | $1.05 \times 10^{-5}$ | $7.21 \times 10^{-6}$ | $5.03 \times 10^{-6}$ | $2.61 \times 10^{-6}$ | $7.60 \times 10^{-7}$ |
|                | $\hat{\gamma}$         | Mean     | —                     | —                     | —                     | —                     | —                     | —                     | —                     | —                     |
|                |                        | Variance | —                     | —                     | —                     | —                     | —                     | —                     | —                     | —                     |
| $\alpha = 0.9$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0483                | 0.0465                | 0.0456                | 0.0454                | 0.0424                | 0.0382                | 0.0282                | 0.0217                |
|                |                        | Variance | $1.01 \times 10^{-5}$ | $1.02 \times 10^{-5}$ | $9.61 \times 10^{-6}$ | $1.19 \times 10^{-5}$ | $1.31 \times 10^{-5}$ | $1.47 \times 10^{-5}$ | $7.16 \times 10^{-6}$ | $1.02 \times 10^{-5}$ |
|                | $\hat{\beta}$          | Mean     | 0.0914                | 0.0853                | 0.0776                | 0.0693                | 0.0590                | 0.0471                | 0.0301                | 0.0216                |
|                |                        | Variance | $1.92 \times 10^{-5}$ | $1.42 \times 10^{-5}$ | $1.44 \times 10^{-5}$ | $1.23 \times 10^{-5}$ | $8.12 \times 10^{-6}$ | $5.76 \times 10^{-6}$ | $1.80 \times 10^{-6}$ | $1.40 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean     | —                     | —                     | —                     | —                     | —                     | —                     | —                     | —                     |
|                |                        | Variance | —                     | —                     | —                     | —                     | —                     | —                     | —                     | —                     |



Table A-11 : IV Standard Errors of the Instrumental-Variable Estimates of  $\alpha$ ,  $\beta$ , and  $\gamma$ . Means and Variances. Various Values of  $\alpha$  and  $\rho$ .  $\beta = 0.0$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$

Sample size:

T = 10, N = 25

|                | $\beta =$              | 0.0      | $\rho = 0.0$          | $\rho = 0.15$      | $\rho = 0.30$      | $\rho = 0.45$      | $\rho = 0.60$         | $\rho = 0.75$         | $\rho = 0.90$         | $\rho = 0.95$         |
|----------------|------------------------|----------|-----------------------|--------------------|--------------------|--------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $\alpha = 0.1$ | S.E. of $\hat{\alpha}$ | Mean     | 174.60                | 283.46             | 2736.3             | 133.33             | 1322.6                | *                     | 1770.0                | *                     |
|                |                        | Variance | $1.74 \times 10^5$    | $1.21 \times 10^6$ | $3.39 \times 10^8$ | $1.33 \times 10^5$ | $3.38 \times 10^7$    | $8.04 \times 10^9$    | $1.37 \times 10^8$    | $3.66 \times 10^{14}$ |
|                | $\hat{\beta}$          | Mean     | 195.39                | 317.23             | 3062.3             | 149.21             | 1480.1                | *                     | 1980.8                | *                     |
|                |                        | Variance | $2.18 \times 10^5$    | $1.52 \times 10^6$ | $4.24 \times 10^8$ | $1.66 \times 10^5$ | $4.23 \times 10^7$    | $1.01 \times 10^{10}$ | $1.72 \times 10^8$    | $4.59 \times 10^{14}$ |
|                | $\hat{\gamma}$         | Mean     | 23.066                | 37.417             | 360.73             | 17.622             | 174.35                | 1783.2                | 233.32                | *                     |
|                |                        | Variance | $3.03 \times 10^3$    | $2.11 \times 10^4$ | $5.88 \times 10^6$ | $2.31 \times 10^3$ | $5.87 \times 10^5$    | $1.40 \times 10^8$    | $2.39 \times 10^6$    | $6.36 \times 10^{12}$ |
| $\alpha = 0.3$ | S.E. of $\hat{\alpha}$ | Mean     | *                     | 2696.9             | 361.59             | 288.63             | *                     | 235.22                | *                     | 39.469                |
|                |                        | Variance | $2.93 \times 10^{11}$ | $1.54 \times 10^8$ | $1.16 \times 10^6$ | $9.92 \times 10^5$ | $1.06 \times 10^{10}$ | $5.12 \times 10^5$    | $1.55 \times 10^{13}$ | $8.77 \times 10^3$    |
|                | $\hat{\beta}$          | Mean     | *                     | 3018.1             | 404.66             | 323.02             | *                     | 263.23                | *                     | 44.173                |
|                |                        | Variance | $3.68 \times 10^{11}$ | $1.93 \times 10^8$ | $1.45 \times 10^6$ | $1.24 \times 10^6$ | $1.32 \times 10^{10}$ | $6.42 \times 10^5$    | $1.94 \times 10^{13}$ | $1.10 \times 10^4$    |
|                | $\hat{\gamma}$         | Mean     | *                     | 355.54             | 47.720             | 38.099             | 2059.8                | 31.022                | *                     | 5.2152                |
|                |                        | Variance | $5.10 \times 10^9$    | $2.68 \times 10^6$ | $2.01 \times 10^4$ | $1.72 \times 10^4$ | $1.84 \times 10^8$    | $8.89 \times 10^3$    | $2.69 \times 10^{11}$ | $1.52 \times 10^2$    |
| $\alpha = 0.5$ | S.E. of $\hat{\alpha}$ | Mean     | 201.98                | 1067.8             | 3756.4             | 864.29             | 2945.5                | 407.85                | 93.797                | *                     |
|                |                        | Variance | $4.85 \times 10^5$    | $1.37 \times 10^7$ | $5.36 \times 10^8$ | $2.22 \times 10^7$ | $3.69 \times 10^8$    | $2.73 \times 10^6$    | $1.30 \times 10^5$    | $6.68 \times 10^{11}$ |
|                | $\hat{\beta}$          | Mean     | 226.06                | 1195.0             | 4204.0             | 967.22             | 3296.4                | 456.47                | 104.96                | *                     |
|                |                        | Variance | $6.08 \times 10^5$    | $1.72 \times 10^7$ | $6.71 \times 10^8$ | $2.78 \times 10^7$ | $4.62 \times 10^8$    | $3.42 \times 10^6$    | $1.62 \times 10^5$    | $8.37 \times 10^{11}$ |
|                | $\hat{\gamma}$         | Mean     | 26.670                | 140.82             | 495.19             | 113.96             | 388.30                | 53.804                | 12.387                | *                     |
|                |                        | Variance | $8.43 \times 10^3$    | $2.38 \times 10^5$ | $9.31 \times 10^6$ | $3.86 \times 10^5$ | $6.41 \times 10^6$    | $4.74 \times 10^4$    | $2.25 \times 10^3$    | $1.16 \times 10^{10}$ |

Table A-11 (Continued)

:IV

T = 10, N = 25

|                | $\beta =$              | 0.0      | $\rho = 0.0$       | $\rho = 0.15$      | $\rho = 0.30$      | $\rho = 0.45$      | $\rho = 0.60$      | $\rho = 0.75$      | $\rho = 0.90$      | $\rho = 0.95$      |
|----------------|------------------------|----------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| $\alpha = 0.7$ | S.E. of $\hat{\alpha}$ | Mean     | 1320.9             | 301.47             | 218.13             | 1352.0             | 1254.9             | 774.50             | 1161.0             | 47.327             |
|                |                        | Variance | $7.20 \times 10^7$ | $8.04 \times 10^5$ | $6.68 \times 10^5$ | $6.91 \times 10^7$ | $6.95 \times 10^7$ | $1.99 \times 10^7$ | $1.66 \times 10^7$ | $3.48 \times 10^4$ |
|                | $\hat{\beta}$          | Mean     | 1478.3             | 337.36             | 244.10             | 1513.0             | 1404.5             | 866.67             | 1299.3             | 52.959             |
|                |                        | Variance | $9.02 \times 10^7$ | $1.01 \times 10^6$ | $8.36 \times 10^5$ | $8.66 \times 10^7$ | $8.71 \times 10^7$ | $2.49 \times 10^7$ | $2.08 \times 10^7$ | $4.35 \times 10^4$ |
|                | $\hat{\gamma}$         | Mean     | 174.16             | 39.824             | 28.800             | 178.33             | 165.49             | 101.95             | 153.05             | 6.2621             |
|                |                        | Variance | $1.25 \times 10^6$ | $1.40 \times 10^4$ | $1.16 \times 10^4$ | $1.20 \times 10^6$ | $1.21 \times 10^6$ | $3.45 \times 10^5$ | $2.89 \times 10^5$ | $6.03 \times 10^2$ |
| $\alpha = 0.9$ | S.E. of $\hat{\alpha}$ | Mean     | 251.38             | 9904.5             | 171.25             | 437.61             | 157.12             | 1187.2             | 247.32             | 13.966             |
|                |                        | Variance | $1.29 \times 10^6$ | $3.02 \times 10^9$ | $2.52 \times 10^5$ | $8.24 \times 10^5$ | $3.31 \times 10^5$ | $6.54 \times 10^7$ | $1.88 \times 10^6$ | $1.18 \times 10^3$ |
|                | $\hat{\beta}$          | Mean     | 281.35             | *                  | 191.63             | 489.61             | 175.89             | 1328.7             | 276.81             | 15.645             |
|                |                        | Variance | $1.61 \times 10^6$ | $3.78 \times 10^9$ | $3.15 \times 10^5$ | $1.03 \times 10^7$ | $4.14 \times 10^5$ | $8.19 \times 10^7$ | $2.35 \times 10^6$ | $1.48 \times 10^3$ |
|                | $\hat{\gamma}$         | Mean     | 33.217             | 1305.7             | 22.704             | 57.600             | 20.963             | 156.74             | 32.641             | 1.9305             |
|                |                        | Variance | $2.24 \times 10^4$ | $5.24 \times 10^7$ | $4.37 \times 10^3$ | $1.42 \times 10^5$ | $5.78 \times 10^3$ | $1.14 \times 10^6$ | $3.25 \times 10^4$ | $2.08 \times 10^1$ |

Table A-12 : ML Standard Errors of the Maximum-Likelihood Estimates of  $\alpha$ ,  $\beta$ , and  $\gamma$ . Means and Variances. Various Values of  $\alpha$  and  $\rho$ .  $\beta = 0.0$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$

Sample Size:

T = 10, N = 25

|                | $\beta =$              | 0.0      | $\rho = 0.0$          | $\rho = 0.15$         | $\rho = 0.30$         | $\rho = 0.45$         | $\rho = 0.60$         | $\rho = 0.75$         | $\rho = 0.90$         | $\rho = 0.95$         |
|----------------|------------------------|----------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $\alpha = 0.1$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0713                | 0.0722                | 0.0736                | 0.0746                | 0.0732                | 0.0741                | 0.0747                | 0.0741                |
|                |                        | Variance | $9.20 \times 10^{-6}$ | $5.33 \times 10^{-6}$ | $8.00 \times 10^{-6}$ | $1.29 \times 10^{-5}$ | $1.08 \times 10^{-5}$ | $1.08 \times 10^{-5}$ | $9.89 \times 10^{-6}$ | $6.00 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean     | 0.0968                | 0.0922                | 0.0837                | 0.0757                | 0.0636                | 0.0508                | 0.0321                | 0.0230                |
|                |                        | Variance | $9.13 \times 10^{-6}$ | $1.77 \times 10^{-5}$ | $1.84 \times 10^{-5}$ | $1.14 \times 10^{-5}$ | $9.18 \times 10^{-6}$ | $5.28 \times 10^{-6}$ | $3.34 \times 10^{-6}$ | $7.77 \times 10^{-7}$ |
|                | $\hat{\gamma}$         | Mean     | 0.2908                | 0.2849                | 0.2712                | 0.2571                | 0.2403                | 0.2356                | 0.1979                | 0.1981                |
|                |                        | Variance | $8.56 \times 10^{-5}$ | $1.60 \times 10^{-4}$ | $1.94 \times 10^{-4}$ | $1.98 \times 10^{-4}$ | $2.86 \times 10^{-4}$ | $6.54 \times 10^{-4}$ | $7.17 \times 10^{-4}$ | $6.89 \times 10^{-4}$ |
| $\alpha = 0.3$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0720                | 0.0756                | 0.0786                | 0.0791                | 0.0787                | 0.0806                | 0.0805                | 0.0813                |
|                |                        | Variance | $8.47 \times 10^{-6}$ | $1.14 \times 10^{-5}$ | $2.54 \times 10^{-5}$ | $3.69 \times 10^{-5}$ | $7.34 \times 10^{-5}$ | $4.42 \times 10^{-5}$ | $4.47 \times 10^{-5}$ | $9.60 \times 10^{-5}$ |
|                | $\hat{\beta}$          | Mean     | 0.0974                | 0.0915                | 0.0841                | 0.0758                | 0.0647                | 0.0500                | 0.0320                | 0.0228                |
|                |                        | Variance | $3.17 \times 10^{-5}$ | $2.25 \times 10^{-5}$ | $1.30 \times 10^{-5}$ | $1.48 \times 10^{-5}$ | $9.99 \times 10^{-6}$ | $5.25 \times 10^{-6}$ | $1.93 \times 10^{-6}$ | $1.32 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean     | 0.2921                | 0.2826                | 0.2732                | 0.2590                | 0.2462                | 0.2227                | 0.2015                | 0.1958                |
|                |                        | Variance | $2.79 \times 10^{-4}$ | $2.12 \times 10^{-4}$ | $1.23 \times 10^{-4}$ | $2.13 \times 10^{-4}$ | $4.18 \times 10^{-4}$ | $7.80 \times 10^{-4}$ | $7.30 \times 10^{-4}$ | $9.48 \times 10^{-4}$ |
| $\alpha = 0.6$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0735                | 0.0918                | 0.0969                | 0.0963                | 0.1049                | 0.0936                | 0.0944                | 0.1098                |
|                |                        | Variance | $1.37 \times 10^{-5}$ | $4.77 \times 10^{-4}$ | $5.18 \times 10^{-4}$ | $7.68 \times 10^{-4}$ | $1.77 \times 10^{-3}$ | $4.92 \times 10^{-4}$ | $1.62 \times 10^{-3}$ | $5.09 \times 10^{-3}$ |
|                | $\hat{\beta}$          | Mean     | 0.0967                | 0.0924                | 0.0854                | 0.0757                | 0.0651                | 0.0515                | 0.0325                | 0.0234                |
|                |                        | Variance | $3.16 \times 10^{-5}$ | $1.58 \times 10^{-5}$ | $1.77 \times 10^{-5}$ | $2.59 \times 10^{-5}$ | $7.77 \times 10^{-6}$ | $7.21 \times 10^{-6}$ | $3.32 \times 10^{-6}$ | $3.09 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean     | 0.2911                | 0.2854                | 0.2753                | 0.2615                | 0.2422                | 0.2294                | 0.2287                | 0.1889                |
|                |                        | Variance | $2.72 \times 10^{-4}$ | $1.62 \times 10^{-4}$ | $1.84 \times 10^{-4}$ | $2.06 \times 10^{-4}$ | $3.17 \times 10^{-4}$ | $6.09 \times 10^{-4}$ | $2.66 \times 10^{-3}$ | $2.21 \times 10^{-3}$ |

Table A-12 : ML

T = 10, N = 25

|                | $\beta =$              | 0.0      | $\rho = 0.0$          | $\rho = 0.15$         | $\rho = 0.30$         | $\rho = 0.45$ | $\rho = 0.60$ | $\rho = 0.75$ | $\rho = 0.90$ | $\rho = 0.95$         |
|----------------|------------------------|----------|-----------------------|-----------------------|-----------------------|---------------|---------------|---------------|---------------|-----------------------|
| $\alpha = 0.7$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0688                | 0.0938                | 0.1043                | Not Listed    | Not Listed    | Not Listed    | Not Listed    | 0.0931                |
|                |                        | Variance | $3.65 \times 10^{-5}$ | $2.67 \times 10^{-4}$ | $5.44 \times 10^{-4}$ | Not Listed    | Not Listed    | Not Listed    | Not Listed    | $2.81 \times 10^{-3}$ |
|                | $\hat{\beta}$          | Mean     | 0.0978                | 0.0917                | 0.0836                | Not Listed    | Not Listed    | Not Listed    | Not Listed    | 0.0236                |
|                |                        | Variance | $1.38 \times 10^{-5}$ | $2.02 \times 10^{-5}$ | $2.43 \times 10^{-5}$ | Not Listed    | Not Listed    | Not Listed    | Not Listed    | $9.66 \times 10^{-7}$ |
|                | $\hat{\gamma}$         | Mean     | 0.2948                | 0.2921                | 0.2710                | Not Listed    | Not Listed    | Not Listed    | Not Listed    | 0.2109                |
|                |                        | Variance | $1.43 \times 10^{-4}$ | $1.26 \times 10^{-4}$ | $6.56 \times 10^{-4}$ | Not Listed    | Not Listed    | Not Listed    | Not Listed    | $9.44 \times 10^{-3}$ |
| $\alpha = 0.9$ | S.E. of $\hat{\alpha}$ | Mean     | 0.0520                | 0.0228                | Not Listed            | Not Listed    | Not Listed    | Not Listed    | Not Listed    | 0.0161                |
|                |                        | Variance | $1.26 \times 10^{-4}$ | $1.92 \times 10^{-5}$ | Not Listed            | Not Listed    | Not Listed    | Not Listed    | Not Listed    | $2.26 \times 10^{-4}$ |
|                | $\hat{\beta}$          | Mean     | 0.0975                | 0.0899                | Not Listed            | Not Listed    | Not Listed    | Not Listed    | Not Listed    | 0.0234                |
|                |                        | Variance | $1.37 \times 10^{-5}$ | $1.25 \times 10^{-5}$ | Not Listed            | Not Listed    | Not Listed    | Not Listed    | Not Listed    | $2.96 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean     | 0.2945                | 0.2704                | Not Listed            | Not Listed    | Not Listed    | Not Listed    | Not Listed    | 0.1351                |
|                |                        | Variance | $1.22 \times 10^{-4}$ | $9.56 \times 10^{-5}$ | Not Listed            | Not Listed    | Not Listed    | Not Listed    | Not Listed    | $5.45 \times 10^{-3}$ |

Not Listed cases are nearly all boundary solutions











TABLE B.3: LSC. Least Squares with Individual Constant Terms.  
 Estimates of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\sigma^2$  and  $\rho$ .  
 Means and Mean-Square Errors. Various Values of  $\alpha$  and  $\rho$ .  
 $\beta = 0.5$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$

Sample size:  
 T=10 N=25

|                  | $\beta =$      | $\rho = 0.0$ | $\rho = 0.15$ | $\rho = 0.30$ | $\rho = 0.45$ | $\rho = 0.60$ | $\rho = 0.75$ | $\rho = 0.90$ | $\rho = 0.95$ |        |
|------------------|----------------|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|--------|
| $\alpha = 0.1$   | $\hat{\alpha}$ | Mean         | -.0221        | -.0200        | -.0002        | -.0038        | -.0139        | -.0070        | 0.0139        | 0.0352 |
|                  |                | MSE          | 0.0182        | 0.0174        | 0.0148        | 0.0152        | 0.0163        | 0.0146        | 0.0109        | 0.0076 |
|                  | $\hat{\beta}$  | Mean         | 0.5544        | 0.5691        | 0.5806        | 0.5467        | 0.5585        | 0.5478        | 0.5461        | 0.5311 |
|                  |                | MSE          | 0.0134        | 0.0173        | 0.0141        | 0.0113        | 0.0082        | 0.0067        | 0.0038        | 0.0021 |
|                  | $\hat{\gamma}$ | Mean         | 0.0304        | -.0129        | -.0935        | 0.0307        | 0.0257        | 0.0138        | -.0009        | -.0206 |
| MSE              |                | 0.0870       | 0.1283        | 0.0820        | 0.0941        | 0.0742        | 0.0629        | 0.0557        | 0.0424        |        |
| $\hat{\sigma}^2$ | Mean           | 0.9962       | 1.0324        | 1.0463        | 1.1556        | 1.1358        | 1.1604        | 1.0901        | 1.1346        |        |
|                  | MSE            | 0.0080       | 0.0164        | 0.0309        | 0.0688        | 0.0787        | 0.0983        | 0.1169        | 0.1183        |        |
| $\hat{\rho}$     | Mean           | 0.1217       | 0.2782        | 0.4098        | 0.5604        | 0.6748        | 0.7994        | 0.9132        | 0.9580        |        |
|                  | MSE            | 0.0158       | 0.0191        | 0.0171        | 0.0202        | 0.0106        | 0.0054        | 0.0009        | 0.0002        |        |
| $\alpha = 0.3$   | $\hat{\alpha}$ | Mean         | 0.1664        | 0.1575        | 0.1831        | 0.1651        | 0.1644        | 0.1721        | 0.1999        | 0.2153 |
|                  |                | MSE          | 0.0224        | 0.0247        | 0.0170        | 0.0225        | 0.0214        | 0.0192        | 0.0117        | 0.0094 |
|                  | $\hat{\beta}$  | Mean         | 0.5864        | 0.6041        | 0.5700        | 0.5591        | 0.6016        | 0.5788        | 0.5666        | 0.5540 |
|                  |                | MSE          | 0.0203        | 0.0218        | 0.0114        | 0.0116        | 0.0171        | 0.0100        | 0.0061        | 0.0044 |
|                  | $\hat{\gamma}$ | Mean         | -.0151        | -.0250        | 0.0269        | 0.0599        | -.0484        | -.0077        | -.0210        | -.0655 |
| MSE              |                | 0.1016       | 0.0996        | 0.0800        | 0.1026        | 0.0651        | 0.0754        | 0.0557        | 0.0358        |        |
| $\hat{\sigma}^2$ | Mean           | 1.0064       | 1.0677        | 1.0929        | 1.1502        | 1.1858        | 1.2321        | 1.2317        | 1.2059        |        |
|                  | MSE            | 0.0084       | 0.0176        | 0.0422        | 0.0656        | 0.0823        | 0.1583        | 0.1588        | 0.1619        |        |
| $\hat{\rho}$     | Mean           | 0.1245       | 0.2987        | 0.4216        | 0.5774        | 0.6908        | 0.8120        | 0.9219        | 0.9593        |        |
|                  | MSE            | 0.0169       | 0.0276        | 0.0231        | 0.0224        | 0.0134        | 0.0063        | 0.0009        | 0.0003        |        |
| $\alpha = 0.5$   | $\hat{\alpha}$ | Mean         | 0.3537        | 0.3337        | 0.3472        | 0.3487        | 0.3649        | 0.3713        | 0.4090        | 0.4448 |
|                  |                | MSE          | 0.0246        | 0.0316        | 0.0267        | 0.0267        | 0.0210        | 0.0190        | 0.0109        | 0.0048 |
|                  | $\hat{\beta}$  | Mean         | 0.6351        | 0.6214        | 0.6305        | 0.6275        | 0.6260        | 0.6161        | 0.5674        | 0.5456 |
|                  |                | MSE          | 0.0304        | 0.0292        | 0.0256        | 0.0260        | 0.0214        | 0.0181        | 0.0069        | 0.0035 |
|                  | $\hat{\gamma}$ | Mean         | -.0178        | 0.0690        | -.0381        | -.0181        | -.0162        | -.0459        | -.0197        | -.0005 |
| MSE              |                | 0.1040       | 0.1271        | 0.0986        | 0.0912        | 0.0563        | 0.0903        | 0.0558        | 0.0391        |        |
| $\hat{\sigma}^2$ | Mean           | 1.0224       | 1.1248        | 1.2186        | 1.2720        | 1.3084        | 1.4479        | 1.2881        | 1.1813        |        |
|                  | MSE            | 0.0081       | 0.0375        | 0.0852        | 0.1270        | 0.1768        | 0.4409        | 0.2119        | 0.1062        |        |
| $\hat{\rho}$     | Mean           | 0.1447       | 0.3279        | 0.4841        | 0.6058        | 0.7212        | 0.8329        | 0.9248        | 0.9603        |        |
|                  | MSE            | 0.0225       | 0.0391        | 0.0404        | 0.0294        | 0.0185        | 0.0098        | 0.0012        | 0.0002        |        |

TABLE B.3 (Continued)

| $\beta =$      | 0.               | $\rho = 0.0$ | $\rho = 0.15$ | $\rho = 0.30$ | $\rho = 0.45$ | $\rho = 0.60$ | $\rho = 0.75$ | $\rho = 0.90$ | $\rho = 0.95$ |        |
|----------------|------------------|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|--------|
| $\alpha = 0.7$ | $\hat{\alpha}$   | Mean         | 0.5204        | 0.5284        | 0.5163        | 0.5245        | 0.5449        | 0.5682        | 0.6197        | 0.6561 |
|                |                  | MSE          | 0.0359        | 0.0322        | 0.0360        | 0.0338        | 0.0267        | 0.0192        | 0.0077        | 0.0029 |
|                | $\hat{\beta}$    | Mean         | 0.7131        | 0.7257        | 0.7247        | 0.6974        | 0.7101        | 0.6620        | 0.5966        | 0.5565 |
|                |                  | MSE          | 0.0620        | 0.0663        | 0.0644        | 0.0547        | 0.0531        | 0.0312        | 0.0125        | 0.0054 |
|                | $\hat{\gamma}$   | Mean         | 0.0685        | 0.0668        | 0.0624        | 0.1137        | 0.0640        | 0.0227        | 0.0211        | 0.0182 |
|                |                  | MSE          | 0.1290        | 0.1549        | 0.1121        | 0.1535        | 0.1018        | 0.0723        | 0.0836        | 0.0506 |
|                | $\hat{\sigma}^2$ | Mean         | 1.0562        | 1.2764        | 1.4372        | 1.6207        | 1.6718        | 1.8127        | 1.5083        | 1.1784 |
|                |                  | MSE          | 0.0131        | 0.1103        | 0.2773        | 0.5129        | 0.6457        | 0.9500        | 0.5157        | 0.1832 |
|                | $\hat{\rho}$     | Mean         | 0.1928        | 0.4182        | 0.5745        | 0.6926        | 0.7828        | 0.8741        | 0.9343        | 0.9580 |
|                |                  | MSE          | 0.0410        | 0.0800        | 0.0835        | 0.0643        | 0.0364        | 0.0167        | 0.0020        | 0.0002 |
| $\alpha = 0.9$ | $\hat{\alpha}$   | Mean         | 0.7264        | 0.7441        | 0.7672        | 0.7732        | 0.7913        | 0.8323        | 0.8658        | 0.8793 |
|                |                  | MSE          | 0.0317        | 0.0263        | 0.0198        | 0.0171        | 0.0130        | 0.0053        | 0.0015        | 0.0007 |
|                | $\hat{\beta}$    | Mean         | 0.8538        | 0.8224        | 0.7689        | 0.7593        | 0.7256        | 0.6392        | 0.5653        | 0.5486 |
|                |                  | MSE          | 0.1446        | 0.1195        | 0.0870        | 0.0781        | 0.0579        | 0.0239        | 0.0065        | 0.0040 |
|                | $\hat{\gamma}$   | Mean         | 0.5805        | 0.5259        | 0.3846        | 0.4207        | 0.3399        | 0.2616        | 0.1167        | 0.0216 |
|                |                  | MSE          | 0.4649        | 0.4378        | 0.3183        | 0.2729        | 0.2183        | 0.1265        | 0.0544        | 0.0426 |
|                | $\hat{\sigma}^2$ | Mean         | 1.2326        | 1.7225        | 2.0619        | 2.3776        | 2.4919        | 1.9084        | 1.5171        | 1.2458 |
|                |                  | MSE          | 0.0703        | 0.6718        | 1.6541        | 2.4836        | 2.7240        | 1.3286        | 0.4324        | 0.1968 |
|                | $\hat{\rho}$     | Mean         | 0.3195        | 0.5611        | 0.6820        | 0.7838        | 0.8472        | 0.8723        | 0.9350        | 0.9603 |
|                |                  | MSE          | 0.1077        | 0.1795        | 0.1553        | 0.1155        | 0.0638        | 0.0172        | 0.0016        | 0.0003 |





TABLE B.5: IV. Instrumental-Variable Estimates of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\sigma^2$  and  $\rho$ . Means and Mean-Square Errors. Various Values of  $\alpha$  and  $\rho$ .  $\beta = 0.5$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$ .

Sample size:  
T=10 N=25

|                |                | $\beta =$ | $\rho = 0.0$ | $\rho = 0.15$ | $\rho = 0.30$ | $\rho = 0.45$ | $\rho = 0.60$ | $\rho = 0.75$ | $\rho = 0.90$ | $\rho = 0.95$ |
|----------------|----------------|-----------|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\alpha = 0.1$ | $\hat{\alpha}$ | Mean      | 0.1143       | 0.0724        | 0.0784        | 0.0248        | 0.1261        | 0.0727        | 0.1032        | 0.0970        |
|                |                | MSE       | 0.3351       | 0.1521        | 0.1026        | 0.0675        | 0.0694        | 0.0802        | 0.0358        | 0.0279        |
|                | $\hat{\beta}$  | Mean      | 0.4816       | 0.5317        | 0.5273        | 0.5261        | 0.4850        | 0.5065        | 0.5081        | 0.4901        |
|                |                | MSE       | 0.0835       | 0.0366        | 0.0400        | 0.0188        | 0.0198        | 0.0217        | 0.0174        | 0.0152        |
|                | $\hat{\gamma}$ | Mean      | 0.0415       | -.0303        | -.0574        | 0.0500        | 0.0304        | 0.0156        | -.0187        | 0.0083        |
|                | MSE            | 0.0791    | 0.1372       | 0.0886        | 0.1129        | 0.0936        | 0.1338        | 0.1572        | 0.0943        |               |
|                | $\sigma^2$     | Mean      | 1.3441       | 1.1453        | 1.0937        | 1.1803        | 1.0431        | 1.0903        | 0.9644        | 1.0316        |
|                |                | MSE       | 1.6358       | 0.1155        | 0.1409        | 0.1832        | 0.2625        | 0.3819        | 0.3196        | 0.2432        |
|                | $\hat{\rho}$   | Mean      | 0.0945       | 0.2340        | 0.3458        | 0.5140        | 0.5605        | 0.7192        | 0.8737        | 0.9387        |
|                |                | MSE       | 0.0134       | 0.0208        | 0.0249        | 0.0241        | 0.0307        | 0.0233        | 0.0034        | 0.0006        |
| $\alpha = 0.3$ | $\hat{\alpha}$ | Mean      | 0.2960       | 0.4405        | 0.4245        | 0.2580        | 0.2830        | 0.2517        | 0.3006        | 0.2146        |
|                |                | MSE       | 0.1624       | 0.3255        | 0.0836        | 0.0846        | 0.1067        | 0.0490        | 0.0235        | 0.0348        |
|                | $\hat{\beta}$  | Mean      | 0.5049       | 0.4244        | 0.4248        | 0.4801        | 0.4950        | 0.5371        | 0.4962        | 0.5437        |
|                |                | MSE       | 0.0607       | 0.1131        | 0.0443        | 0.0353        | 0.0653        | 0.0280        | 0.0168        | 0.0197        |
|                | $\hat{\gamma}$ | Mean      | -.0137       | -.0323        | -.0080        | 0.1375        | 0.0403        | -.0317        | 0.0048        | -.0362        |
|                | MSE            | 0.1217    | 0.0917       | 0.0823        | 0.2292        | 0.1461        | 0.1474        | 0.0852        | 0.1112        |               |
|                | $\sigma^2$     | Mean      | 1.1700       | 1.3351        | 0.9803        | 1.1204        | 1.1248        | 1.1206        | 0.9846        | 1.2885        |
|                |                | MSE       | 0.1986       | 3.3637        | 0.0484        | 0.1809        | 0.2678        | 0.5301        | 0.2088        | 0.7712        |
|                | $\hat{\rho}$   | Mean      | 0.1004       | 0.1916        | 0.2542        | 0.4796        | 0.5848        | 0.7407        | 0.8801        | 0.9454        |
|                |                | MSE       | 0.0133       | 0.0172        | 0.0294        | 0.0380        | 0.0332        | 0.0105        | 0.0024        | 0.0007        |
| $\alpha = 0.5$ | $\hat{\alpha}$ | Mean      | 0.5214       | 0.5251        | 0.4861        | 0.4987        | 0.4927        | 0.4887        | 0.4554        | 0.4955        |
|                |                | MSE       | 0.0849       | 0.0682        | 0.0674        | 0.0437        | 0.0241        | 0.0241        | 0.0333        | 0.0140        |
|                | $\hat{\beta}$  | Mean      | 0.4976       | 0.4441        | 0.5272        | 0.5066        | 0.5017        | 0.5284        | 0.5117        | 0.5171        |
|                |                | MSE       | 0.0763       | 0.0561        | 0.0640        | 0.0318        | 0.0238        | 0.0226        | 0.0138        | 0.0164        |
|                | $\hat{\gamma}$ | Mean      | -.0402       | 0.0975        | -.0872        | -.0461        | 0.0163        | -.0751        | 0.0295        | -.0494        |
|                | MSE            | 0.0844    | 0.0993       | 0.1479        | 0.0914        | 0.1175        | 0.2038        | 0.1379        | 0.0854        |               |
|                | $\sigma^2$     | Mean      | 1.1071       | 1.0734        | 1.1342        | 1.0880        | 1.0250        | 1.1095        | 1.1931        | 1.0047        |
|                |                | MSE       | 0.0616       | 0.0401        | 0.1718        | 0.3174        | 0.1647        | 0.4257        | 0.7645        | 0.2333        |
|                | $\hat{\rho}$   | Mean      | 0.1020       | 0.2097        | 0.3468        | 0.4386        | 0.5815        | 0.7200        | 0.8835        | 0.9384        |
|                |                | MSE       | 0.0144       | 0.0193        | 0.0389        | 0.0366        | 0.0288        | 0.0198        | 0.0034        | 0.0007        |

TABLE B.5 (Continued)

| $\beta =$      | 0.5              | $\rho = 0.0$ | $\rho = 0.15$    | $\rho = 0.30$    | $\rho = 0.45$    | $\rho = 0.60$    | $\rho = 0.75$    | $\rho = 0.90$    | $\rho = 0.95$    |                  |
|----------------|------------------|--------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| $\alpha = 0.7$ | $\hat{\alpha}$   | Mean<br>MSE  | 0.6976<br>0.0472 | 0.7010<br>0.0171 | 0.6669<br>0.0387 | 0.6970<br>0.0311 | 0.6697<br>0.0269 | 0.6523<br>0.0314 | 0.6617<br>0.0345 | 0.6118<br>0.2675 |
|                | $\hat{\beta}$    | Mean<br>MSE  | 0.4962<br>0.0669 | 0.5146<br>0.0399 | 0.5380<br>0.0703 | 0.4865<br>0.0549 | 0.5500<br>0.0482 | 0.5396<br>0.0471 | 0.5042<br>0.0306 | 0.5959<br>0.1837 |
|                | $\hat{\gamma}$   | Mean<br>MSE  | 0.0048<br>0.1089 | -.0048<br>0.1127 | 0.0058<br>0.1457 | 0.0449<br>0.1172 | 0.0227<br>0.0944 | 0.0640<br>0.1485 | 0.1269<br>0.2049 | 0.0816<br>0.8540 |
|                | $\hat{\sigma}^2$ | Mean<br>MSE  | 1.0817<br>0.0255 | 1.0424<br>0.0204 | 1.1570<br>0.2371 | 1.1670<br>0.5335 | 1.2250<br>0.7043 | 1.5029<br>3.2750 | 1.5522<br>4.4640 | 4.3449<br>486.84 |
|                | $\hat{\rho}$     | Mean<br>MSE  | 0.1258<br>0.0220 | 0.2338<br>0.0246 | 0.3772<br>0.0460 | 0.4317<br>0.0571 | 0.5864<br>0.0482 | 0.7530<br>0.0253 | 0.8608<br>0.0150 | 0.9199<br>0.0036 |
|                | $\hat{\alpha}$   | Mean<br>MSE  | 0.8811<br>0.0250 | 0.9256<br>0.0200 | 0.8518<br>0.0292 | 0.6493<br>2.2613 | 0.8580<br>0.0274 | 0.8808<br>0.0286 | 0.8129<br>0.3176 | 0.9346<br>0.2421 |
|                | $\hat{\beta}$    | Mean<br>MSE  | 0.5594<br>0.1380 | 0.4629<br>0.1381 | 0.5836<br>0.1930 | 1.0030<br>9.7093 | 0.5814<br>0.1458 | 0.5534<br>0.0849 | 0.6060<br>1.2340 | 0.4485<br>0.6851 |
|                | $\hat{\gamma}$   | Mean<br>MSE  | -.0113<br>0.2904 | -.1209<br>0.2254 | 0.1660<br>0.3129 | 0.7608<br>19.253 | 0.1336<br>0.4218 | 0.0275<br>0.6017 | 0.4984<br>4.9415 | -.2546<br>6.5966 |
|                | $\hat{\sigma}^2$ | Mean<br>MSE  | 1.1683<br>0.0761 | 1.2409<br>0.3117 | 1.9032<br>8.8161 | 125.68<br>*      | 2.6347<br>28.739 | 2.5533<br>23.620 | 29.455<br>*      | 22.123<br>*      |
|                | $\hat{\rho}$     | Mean<br>MSE  | 0.1879<br>0.0471 | 0.2748<br>0.0488 | 0.4516<br>0.0868 | 0.5236<br>0.0876 | 0.6012<br>0.0745 | 0.6944<br>0.0621 | 0.8596<br>0.0177 | 0.9105<br>0.0078 |
| $\alpha = 0.0$ | $\hat{\alpha}$   | Mean<br>MSE  | 0.8811<br>0.0250 | 0.9256<br>0.0200 | 0.8518<br>0.0292 | 0.6493<br>2.2613 | 0.8580<br>0.0274 | 0.8808<br>0.0286 | 0.8129<br>0.3176 | 0.9346<br>0.2421 |
|                | $\hat{\beta}$    | Mean<br>MSE  | 0.5594<br>0.1380 | 0.4629<br>0.1381 | 0.5836<br>0.1930 | 1.0030<br>9.7093 | 0.5814<br>0.1458 | 0.5534<br>0.0849 | 0.6060<br>1.2340 | 0.4485<br>0.6851 |
|                | $\hat{\gamma}$   | Mean<br>MSE  | -.0113<br>0.2904 | -.1209<br>0.2254 | 0.1660<br>0.3129 | 0.7608<br>19.253 | 0.1336<br>0.4218 | 0.0275<br>0.6017 | 0.4984<br>4.9415 | -.2546<br>6.5966 |
|                | $\hat{\sigma}^2$ | Mean<br>MSE  | 1.1683<br>0.0761 | 1.2409<br>0.3117 | 1.9032<br>8.8161 | 125.68<br>*      | 2.6347<br>28.739 | 2.5533<br>23.620 | 29.455<br>*      | 22.123<br>*      |
|                | $\hat{\rho}$     | Mean<br>MSE  | 0.1879<br>0.0471 | 0.2748<br>0.0488 | 0.4516<br>0.0868 | 0.5236<br>0.0876 | 0.6012<br>0.0745 | 0.6944<br>0.0621 | 0.8596<br>0.0177 | 0.9105<br>0.0078 |

\* Format field exceeded.







TABLE B.7: ML. Maximum-Likelihood Estimates of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\sigma^2$  and  $\rho$ .  
Means and Mean-Square Errors. Various Values of  $\alpha$  and  $\rho$ .  
 $\beta = 0.5$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$

Sample size:  
T=10 N=25

|                | $\beta =$        | $\rho = 0.0$ | $\rho = 0.15$    | $\rho = 0.30$    | $\rho = 0.45$    | $\rho = 0.60$    | $\rho = 0.75$    | $\rho = 0.90$    | $\rho = 0.95$    |                  |
|----------------|------------------|--------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| $\alpha = 0.1$ | $\hat{\alpha}$   | Mean<br>MSE  | 0.0722<br>0.0042 | 0.0977<br>0.0041 | 0.1193<br>0.0068 | 0.1148<br>0.0061 | 0.1021<br>0.0047 | 0.1050<br>0.0047 | 0.1104<br>0.0048 | 0.1131<br>0.0043 |
|                | $\hat{\beta}$    | Mean<br>MSE  | 0.5102<br>0.0090 | 0.5122<br>0.0105 | 0.5182<br>0.0071 | 0.4895<br>0.0081 | 0.5011<br>0.0045 | 0.4940<br>0.0045 | 0.4981<br>0.0020 | 0.4929<br>0.0014 |
|                | $\hat{\gamma}$   | Mean<br>MSE  | 0.0173<br>0.0720 | -.0252<br>0.1054 | -.0894<br>0.0707 | 0.0178<br>0.0746 | 0.0165<br>0.0586 | 0.0031<br>0.0517 | -.0073<br>0.0470 | -.0234<br>0.0358 |
|                | $\hat{\sigma}^2$ | Mean<br>MSE  | 0.9843<br>0.0079 | 0.9805<br>0.0128 | 0.9586<br>0.0226 | 0.0171<br>0.0293 | 0.9748<br>0.0415 | 0.9678<br>0.0508 | 0.9065<br>0.0886 | 0.9672<br>0.0799 |
|                | $\hat{\rho}$     | Mean<br>MSE  | 0.0125<br>0.0006 | 0.1442<br>0.0029 | 0.2753<br>0.0069 | 0.4402<br>0.0110 | 0.5748<br>0.0080 | 0.7288<br>0.0060 | 0.8820<br>0.0017 | 0.9445<br>0.0002 |
|                | $\hat{\alpha}$   | Mean<br>MSE  | 0.2747<br>0.0053 | 0.3023<br>0.0065 | 0.3365<br>0.0065 | 0.3124<br>0.0060 | 0.3149<br>0.0094 | 0.3090<br>0.0045 | 0.3108<br>0.0026 | 0.2991<br>0.0027 |
|                | $\hat{\beta}$    | Mean<br>MSE  | 0.5208<br>0.0092 | 0.5123<br>0.0096 | 0.4747<br>0.0077 | 0.4701<br>0.0087 | 0.5022<br>0.0107 | 0.4946<br>0.0043 | 0.4967<br>0.0020 | 0.5018<br>0.0017 |
|                | $\hat{\gamma}$   | Mean<br>MSE  | -.0270<br>0.0745 | -.0309<br>0.0767 | 0.0141<br>0.0681 | 0.0452<br>0.0771 | -.0408<br>0.0550 | -.0167<br>0.0553 | -.0234<br>0.0421 | -.0630<br>0.0291 |
|                | $\hat{\sigma}^2$ | Mean<br>MSE  | 0.9882<br>0.0078 | 0.9862<br>0.0088 | 0.9534<br>0.0203 | 0.9478<br>0.0281 | 0.9319<br>0.0340 | 0.9319<br>0.0659 | 0.9388<br>0.0663 | 0.9749<br>0.0863 |
|                | $\hat{\rho}$     | Mean<br>MSE  | 0.0111<br>0.0006 | 0.1395<br>0.0057 | 0.2489<br>0.0134 | 0.4184<br>0.0112 | 0.5485<br>0.0172 | 0.7176<br>0.0064 | 0.8840<br>0.0013 | 0.9429<br>0.0005 |
| $\alpha = 0.3$ | $\hat{\alpha}$   | Mean<br>MSE  | 0.4837<br>0.0029 | 0.5153<br>0.0081 | 0.5647<br>0.0189 | 0.5763<br>0.0266 | 0.6050<br>0.0345 | 0.5953<br>0.0378 | 0.5535<br>0.0128 | 0.5325<br>0.0036 |
|                | $\hat{\beta}$    | Mean<br>MSE  | 0.5263<br>0.0097 | 0.4649<br>0.0156 | 0.4435<br>0.0199 | 0.4382<br>0.0210 | 0.4200<br>0.0262 | 0.4257<br>0.0284 | 0.4476<br>0.0094 | 0.4716<br>0.0027 |
|                | $\hat{\gamma}$   | Mean<br>MSE  | -.0315<br>0.0711 | 0.0648<br>0.0886 | -.0277<br>0.0631 | -.0306<br>0.0591 | -.0241<br>0.0316 | -.0511<br>0.0534 | -.0223<br>0.0360 | -.0046<br>0.0289 |
|                | $\hat{\sigma}^2$ | Mean<br>MSE  | 0.9924<br>0.0073 | 0.9901<br>0.0126 | 0.9478<br>0.0260 | 0.8894<br>0.0424 | 0.8032<br>0.0869 | 0.8443<br>0.1673 | 0.7994<br>0.1332 | 0.8554<br>0.0678 |
|                | $\hat{\rho}$     | Mean<br>MSE  | 0.0124<br>0.0007 | 0.1216<br>0.0096 | 0.2143<br>0.0258 | 0.3193<br>0.0469 | 0.4229<br>0.0801 | 0.5828<br>0.1090 | 0.8375<br>0.0223 | 0.9366<br>0.0006 |

TABLE B.7 (Continued)

| $\beta =$      | 0.5              | $\rho = 0.0$ | $\rho = 0.15$    | $\rho = 0.30$    | $\rho = 0.45$    | $\rho = 0.60$    | $\rho = 0.75$    | $\rho = 0.90$    | $\rho = 0.95$    |                  |
|----------------|------------------|--------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| $\alpha = 0.7$ | $\hat{\alpha}$   | Mean<br>MSE  | 0.6691<br>0.0042 | 0.8121<br>0.0171 | 0.8649<br>0.0334 | 0.8893<br>0.0465 | 0.9358<br>0.0630 | 0.9358<br>0.0670 | 0.8762<br>0.0487 | 0.7740<br>0.0141 |
|                | $\hat{\beta}$    | Mean<br>MSE  | 0.5360<br>0.0141 | 0.3708<br>0.0336 | 0.2882<br>0.0654 | 0.2516<br>0.0854 | 0.2165<br>0.0955 | 0.2101<br>0.1031 | 0.2771<br>0.0805 | 0.4135<br>0.0220 |
|                | $\hat{\gamma}$   | Mean<br>MSE  | 0.0060<br>0.0825 | -.0289<br>0.0897 | -.0145<br>0.0659 | -.0247<br>0.0611 | -.0739<br>0.0466 | -.0794<br>0.0357 | -.0318<br>0.0290 | -.0166<br>0.0270 |
|                | $\hat{\sigma}^2$ | Mean<br>MSE  | 0.9903<br>0.0069 | 0.9331<br>0.0114 | 0.7997<br>0.0474 | 0.7192<br>0.1220 | 0.5246<br>0.2929 | 0.4090<br>0.4379 | 0.4262<br>0.4939 | 0.6231<br>0.2685 |
|                | $\hat{\rho}$     | Mean<br>MSE  | 0.0232<br>0.0017 | 0.0309<br>0.0186 | 0.0396<br>0.0793 | 0.0807<br>0.1726 | 0.0447<br>0.3339 | 0.1149<br>0.4748 | 0.3812<br>0.4499 | 0.8021<br>0.1057 |
| $\alpha = 0.9$ | $\hat{\alpha}$   | Mean<br>MSE  | 0.8907<br>0.0007 | 0.9651<br>0.0045 | 0.9867<br>0.0082 | 1.0023<br>0.0105 | 1.0024<br>0.0118 | 1.0071<br>0.0123 | 1.0083<br>0.0120 | 0.9611<br>0.0060 |
|                | $\hat{\beta}$    | Mean<br>MSE  | 0.5362<br>0.0114 | 0.3574<br>0.0293 | 0.3015<br>0.0472 | 0.2817<br>0.0515 | 0.2897<br>0.0524 | 0.2698<br>0.0595 | 0.2723<br>0.0538 | 0.3832<br>0.0239 |
|                | $\hat{\gamma}$   | Mean<br>MSE  | -.0339<br>0.0803 | -.1959<br>0.0994 | -.2292<br>0.1754 | -.3269<br>0.1400 | -.3576<br>0.1733 | -.3451<br>0.1435 | -.3561<br>0.1427 | -.2272<br>0.1019 |
|                | $\hat{\sigma}^2$ | Mean<br>MSE  | 0.9947<br>0.0053 | 0.9041<br>0.0158 | 0.7772<br>0.0633 | 0.6197<br>0.1476 | 0.5327<br>0.4479 | 0.3557<br>0.5661 | 0.1732<br>0.6924 | 0.4210<br>0.4663 |
|                | $\hat{\rho}$     | Mean<br>MSE  | 0.0124<br>0.0006 | 0.0093<br>0.0200 | 0.0276<br>0.0795 | 0.0152<br>0.1895 | 0.0426<br>0.3274 | 0.0757<br>0.4713 | 0.1962<br>0.5174 | 0.6094<br>0.2187 |

TABLE B.8: GLS. Standard Errors of the Generalized Least-Squares Estimates of  $\alpha$ ,  $\beta$ , and  $\gamma$ . Means and Variances. Various Values of  $\alpha$  and  $\rho$ .  $\beta = 0.5$ ,  $\gamma = 0.00$ ,  $\sigma^2 = 1.0$

Sample size:  
T=10, N=25

|                | $\beta =$              | 0.5              | $\rho = 0.00$                   | $\rho = 0.15$                   | $\rho = 0.30$                   | $\rho = 0.45$                   | $\rho = 0.60$                   | $\rho = 0.75$                   | $\rho = 0.90$                   | $\rho = 0.95$                   |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.1$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0619<br>$1.90 \times 10^{-6}$ | 0.0618<br>$2.57 \times 10^{-6}$ | 0.0611<br>$2.06 \times 10^{-6}$ | 0.0610<br>$1.23 \times 10^{-6}$ | 0.0609<br>$3.26 \times 10^{-6}$ | 0.0595<br>$3.27 \times 10^{-6}$ | 0.0554<br>$3.56 \times 10^{-6}$ | 0.0501<br>$4.24 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1011<br>$2.24 \times 10^{-5}$ | 0.0965<br>$1.90 \times 10^{-5}$ | 0.0887<br>$1.82 \times 10^{-5}$ | 0.0810<br>$7.48 \times 10^{-6}$ | 0.0710<br>$1.20 \times 10^{-5}$ | 0.0581<br>$8.32 \times 10^{-6}$ | 0.0421<br>$3.38 \times 10^{-6}$ | 0.0334<br>$1.92 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2875<br>$1.70 \times 10^{-4}$ | 0.2831<br>$1.82 \times 10^{-4}$ | 0.2704<br>$2.00 \times 10^{-4}$ | 0.2621<br>$9.21 \times 10^{-5}$ | 0.2458<br>$1.40 \times 10^{-4}$ | 0.2286<br>$1.24 \times 10^{-4}$ | 0.2096<br>$1.10 \times 10^{-4}$ | 0.2062<br>$1.02 \times 10^{-4}$ |
| $\alpha = 0.3$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0588<br>$3.91 \times 10^{-6}$ | 0.0588<br>$2.80 \times 10^{-6}$ | 0.0585<br>$3.53 \times 10^{-6}$ | 0.0578<br>$3.24 \times 10^{-6}$ | 0.0577<br>$3.37 \times 10^{-6}$ | 0.0561<br>$2.76 \times 10^{-6}$ | 0.0512<br>$2.38 \times 10^{-6}$ | 0.0457<br>$3.27 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1030<br>$2.30 \times 10^{-5}$ | 0.0993<br>$1.65 \times 10^{-5}$ | 0.0917<br>$1.74 \times 10^{-5}$ | 0.0818<br>$1.62 \times 10^{-5}$ | 0.0740<br>$1.24 \times 10^{-5}$ | 0.0610<br>$7.56 \times 10^{-6}$ | 0.0457<br>$2.94 \times 10^{-6}$ | 0.0365<br>$3.36 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2881<br>$1.66 \times 10^{-4}$ | 0.2842<br>$1.24 \times 10^{-4}$ | 0.2731<br>$1.68 \times 10^{-4}$ | 0.2575<br>$1.28 \times 10^{-4}$ | 0.2453<br>$1.09 \times 10^{-4}$ | 0.2281<br>$1.05 \times 10^{-4}$ | 0.2134<br>$9.11 \times 10^{-5}$ | 0.2069<br>$8.52 \times 10^{-5}$ |
| $\alpha = 0.5$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0529<br>$5.60 \times 10^{-6}$ | 0.0533<br>$5.54 \times 10^{-6}$ | 0.0522<br>$5.58 \times 10^{-6}$ | 0.0518<br>$4.70 \times 10^{-6}$ | 0.0504<br>$4.42 \times 10^{-6}$ | 0.0487<br>$3.58 \times 10^{-6}$ | 0.0439<br>$4.10 \times 10^{-6}$ | 0.0372<br>$2.50 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1068<br>$1.79 \times 10^{-5}$ | 0.1021<br>$2.01 \times 10^{-5}$ | 0.0955<br>$2.02 \times 10^{-5}$ | 0.0874<br>$1.82 \times 10^{-5}$ | 0.0773<br>$1.28 \times 10^{-5}$ | 0.0655<br>$8.44 \times 10^{-6}$ | 0.0486<br>$5.18 \times 10^{-6}$ | 0.0387<br>$2.07 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2889<br>$1.62 \times 10^{-4}$ | 0.2859<br>$1.17 \times 10^{-4}$ | 0.2748<br>$1.32 \times 10^{-4}$ | 0.2615<br>$1.29 \times 10^{-4}$ | 0.2447<br>$7.11 \times 10^{-5}$ | 0.2299<br>$8.85 \times 10^{-5}$ | 0.2129<br>$9.99 \times 10^{-5}$ | 0.2052<br>$8.28 \times 10^{-5}$ |

TABLE B.8 (Continued)

|                | $\beta =$              | 0.5              | $\rho = 0.00$                   | $\rho = 0.15$                   | $\rho = 0.30$                   | $\rho = 0.45$                   | $\rho = 0.60$                   | $\rho = 0.75$                   | $\rho = 0.90$                   | $\rho = 0.95$                   |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.7$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0434<br>$7.05 \times 10^{-6}$ | 0.0414<br>$6.62 \times 10^{-6}$ | 0.0415<br>$6.95 \times 10^{-6}$ | 0.0411<br>$6.36 \times 10^{-6}$ | 0.0395<br>$5.28 \times 10^{-6}$ | 0.0370<br>$5.62 \times 10^{-6}$ | 0.0319<br>$2.31 \times 10^{-6}$ | 0.0270<br>$1.82 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1094<br>$3.04 \times 10^{-5}$ | 0.1064<br>$3.24 \times 10^{-5}$ | 0.0982<br>$2.22 \times 10^{-5}$ | 0.0903<br>$2.55 \times 10^{-5}$ | 0.0811<br>$2.00 \times 10^{-5}$ | 0.0675<br>$1.74 \times 10^{-5}$ | 0.0510<br>$4.68 \times 10^{-6}$ | 0.0401<br>$3.32 \times 10^{-6}$ |
|                | $\hat{Y}$              | Mean<br>Variance | 0.2892<br>$1.54 \times 10^{-4}$ | 0.2852<br>$1.19 \times 10^{-4}$ | 0.2714<br>$1.07 \times 10^{-4}$ | 0.2618<br>$1.01 \times 10^{-4}$ | 0.2450<br>$1.27 \times 10^{-4}$ | 0.2272<br>$1.15 \times 10^{-4}$ | 0.2121<br>$6.29 \times 10^{-5}$ | 0.2066<br>$6.05 \times 10^{-5}$ |
| $\alpha = 0.9$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0227<br>$4.44 \times 10^{-6}$ | 0.0225<br>$7.98 \times 10^{-6}$ | 0.0217<br>$6.37 \times 10^{-6}$ | 0.0213<br>$5.80 \times 10^{-6}$ | 0.0202<br>$4.28 \times 10^{-6}$ | 0.0189<br>$3.07 \times 10^{-6}$ | 0.0154<br>$1.75 \times 10^{-6}$ | 0.0124<br>$9.54 \times 10^{-7}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1086<br>$1.65 \times 10^{-5}$ | 0.1033<br>$3.71 \times 10^{-5}$ | 0.0951<br>$3.38 \times 10^{-5}$ | 0.0866<br>$2.55 \times 10^{-5}$ | 0.0758<br>$2.43 \times 10^{-5}$ | 0.0632<br>$1.71 \times 10^{-5}$ | 0.0449<br>$8.06 \times 10^{-6}$ | 0.0338<br>$4.37 \times 10^{-6}$ |
|                | $\hat{Y}$              | Mean<br>Variance | 0.2975<br>$1.57 \times 10^{-4}$ | 0.2930<br>$2.13 \times 10^{-4}$ | 0.2793<br>$2.29 \times 10^{-4}$ | 0.2681<br>$1.85 \times 10^{-4}$ | 0.2546<br>$1.57 \times 10^{-4}$ | 0.2357<br>$1.67 \times 10^{-4}$ | 0.2208<br>$9.18 \times 10^{-5}$ | 0.2100<br>$1.17 \times 10^{-4}$ |

TABLE B.9: OLS. Standard Errors of the Ordinary Least-Squares Estimates of  $\alpha$ ,  $\beta$ , and  $\gamma$  Means and Variances. Various Values of  $\rho$ .  $\beta = 0.5$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$

Sample size:  
T=10, N=25

|                | $\beta = 0.5$          | $\rho = 0.00$    | $\rho = 0.15$                   | $\rho = 0.30$                   | $\rho = 0.45$                   | $\rho = 0.60$                   | $\rho = 0.75$                   | $\rho = 0.90$                   | $\rho = 0.95$                   |                                 |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.1$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0619<br>$1.90 \times 10^{-6}$ | 0.0603<br>$3.54 \times 10^{-6}$ | 0.0567<br>$4.88 \times 10^{-6}$ | 0.0520<br>$1.29 \times 10^{-5}$ | 0.0472<br>$1.35 \times 10^{-5}$ | 0.0384<br>$1.66 \times 10^{-5}$ | 0.0264<br>$1.32 \times 10^{-5}$ | 0.0193<br>$7.35 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1011<br>$2.24 \times 10^{-5}$ | 0.0996<br>$2.39 \times 10^{-5}$ | 0.0946<br>$2.40 \times 10^{-5}$ | 0.0896<br>$1.22 \times 10^{-5}$ | 0.0801<br>$1.56 \times 10^{-5}$ | 0.0658<br>$1.19 \times 10^{-5}$ | 0.0445<br>$8.41 \times 10^{-6}$ | 0.0336<br>$5.55 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2875<br>$1.70 \times 10^{-4}$ | 0.2832<br>$2.13 \times 10^{-4}$ | 0.2698<br>$2.57 \times 10^{-4}$ | 0.2564<br>$1.17 \times 10^{-4}$ | 0.2280<br>$1.45 \times 10^{-4}$ | 0.1879<br>$8.56 \times 10^{-5}$ | 0.1263<br>$5.81 \times 10^{-5}$ | 0.0963<br>$4.09 \times 10^{-5}$ |
| $\alpha = 0.3$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0588<br>$3.91 \times 10^{-6}$ | 0.0550<br>$5.84 \times 10^{-6}$ | 0.0501<br>$1.24 \times 10^{-5}$ | 0.0438<br>$9.05 \times 10^{-6}$ | 0.0382<br>$1.49 \times 10^{-5}$ | 0.0299<br>$1.34 \times 10^{-5}$ | 0.0194<br>$7.65 \times 10^{-6}$ | 0.0145<br>$4.35 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1030<br>$2.30 \times 10^{-5}$ | 0.1012<br>$1.71 \times 10^{-5}$ | 0.0952<br>$1.87 \times 10^{-5}$ | 0.0862<br>$1.89 \times 10^{-5}$ | 0.0771<br>$1.13 \times 10^{-5}$ | 0.0623<br>$8.85 \times 10^{-6}$ | 0.0421<br>$3.92 \times 10^{-6}$ | 0.0316<br>$3.04 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2881<br>$1.66 \times 10^{-4}$ | 0.2827<br>$1.32 \times 10^{-4}$ | 0.2677<br>$1.67 \times 10^{-4}$ | 0.2453<br>$1.28 \times 10^{-4}$ | 0.2183<br>$7.66 \times 10^{-5}$ | 0.1772<br>$6.59 \times 10^{-5}$ | 0.1199<br>$2.26 \times 10^{-5}$ | 0.0904<br>$2.12 \times 10^{-5}$ |
| $\alpha = 0.5$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0529<br>$5.60 \times 10^{-6}$ | 0.0472<br>$1.03 \times 10^{-5}$ | 0.0399<br>$1.28 \times 10^{-5}$ | 0.0340<br>$1.31 \times 10^{-5}$ | 0.0273<br>$8.75 \times 10^{-6}$ | 0.0207<br>$1.06 \times 10^{-5}$ | 0.0132<br>$3.32 \times 10^{-6}$ | 0.0095<br>$1.15 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1068<br>$1.79 \times 10^{-5}$ | 0.1020<br>$1.76 \times 10^{-5}$ | 0.0953<br>$2.06 \times 10^{-5}$ | 0.0863<br>$1.82 \times 10^{-5}$ | 0.0736<br>$8.29 \times 10^{-6}$ | 0.0596<br>$8.00 \times 10^{-6}$ | 0.0391<br>$2.71 \times 10^{-6}$ | 0.0293<br>$1.45 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2889<br>$1.62 \times 10^{-4}$ | 0.2826<br>$1.27 \times 10^{-4}$ | 0.2657<br>$1.44 \times 10^{-4}$ | 0.2413<br>$1.28 \times 10^{-4}$ | 0.2080<br>$4.37 \times 10^{-5}$ | 0.1690<br>$5.68 \times 10^{-5}$ | 0.1119<br>$2.56 \times 10^{-5}$ | 0.0836<br>$1.10 \times 10^{-5}$ |

TABLE B.9 (Continued)

|                | $\beta$                | $\rho = 0.5$     | $\rho = 0.00$                   | $\rho = 0.15$                   | $\rho = 0.30$                   | $\rho = 0.45$                   | $\rho = 0.60$                   | $\rho = 0.75$                   | $\rho = 0.90$                   | $\rho = 0.95$                   |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.7$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0434<br>$7.05 \times 10^{-6}$ | 0.0329<br>$8.71 \times 10^{-6}$ | 0.0269<br>$1.04 \times 10^{-5}$ | 0.0216<br>$5.06 \times 10^{-6}$ | 0.0170<br>$4.80 \times 10^{-6}$ | 0.0120<br>$3.16 \times 10^{-6}$ | 0.0075<br>$1.35 \times 10^{-6}$ | 0.0057<br>$6.09 \times 10^{-7}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1094<br>$3.04 \times 10^{-5}$ | 0.1029<br>$2.90 \times 10^{-5}$ | 0.0920<br>$1.67 \times 10^{-5}$ | 0.0824<br>$1.63 \times 10^{-5}$ | 0.0704<br>$1.16 \times 10^{-5}$ | 0.0551<br>$8.78 \times 10^{-6}$ | 0.0364<br>$2.26 \times 10^{-6}$ | 0.0276<br>$1.89 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2892<br>$1.54 \times 10^{-4}$ | 0.2786<br>$1.27 \times 10^{-4}$ | 0.2561<br>$9.51 \times 10^{-5}$ | 0.2332<br>$8.26 \times 10^{-5}$ | 0.1997<br>$8.99 \times 10^{-5}$ | 0.1588<br>$6.34 \times 10^{-5}$ | 0.1054<br>$1.52 \times 10^{-5}$ | 0.0792<br>$1.39 \times 10^{-5}$ |
| $\alpha = 0.9$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0227<br>$4.44 \times 10^{-6}$ | 0.0156<br>$5.70 \times 10^{-6}$ | 0.0116<br>$2.98 \times 10^{-6}$ | 0.0091<br>$2.16 \times 10^{-6}$ | 0.0068<br>$9.74 \times 10^{-7}$ | 0.0052<br>$5.87 \times 10^{-7}$ | 0.0032<br>$1.98 \times 10^{-7}$ | 0.0026<br>$1.18 \times 10^{-7}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1086<br>$1.65 \times 10^{-5}$ | 0.0985<br>$2.85 \times 10^{-5}$ | 0.0887<br>$2.16 \times 10^{-5}$ | 0.0790<br>$1.42 \times 10^{-5}$ | 0.0677<br>$1.13 \times 10^{-5}$ | 0.0544<br>$6.39 \times 10^{-6}$ | 0.0382<br>$2.92 \times 10^{-6}$ | 0.0301<br>$2.01 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2975<br>$1.57 \times 10^{-4}$ | 0.2799<br>$1.83 \times 10^{-4}$ | 0.2562<br>$1.52 \times 10^{-4}$ | 0.2295<br>$1.05 \times 10^{-4}$ | 0.1989<br>$8.49 \times 10^{-5}$ | 0.1595<br>$6.07 \times 10^{-5}$ | 0.1127<br>$1.91 \times 10^{-5}$ | 0.0887<br>$1.67 \times 10^{-5}$ |

TABLE B.10: .LSC. Standard Errors of the Estimates of  $\alpha$  and  $\beta$ . Obtained by Least-Squares with Individual Constant Terms. Means and Variances. Various Values of  $\alpha$  and  $\rho$ .  $\beta = 0.5$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$

Sample size:  
T=10, N=25

|                | $\beta =$              | 0.5              | $\rho = 0.00$                   | $\rho = 0.15$                   | $\rho = 0.30$                   | $\rho = 0.45$                   | $\rho = 0.60$                   | $\rho = 0.75$                   | $\rho = 0.90$                   | $\rho = 0.95$                   |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.1$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0620<br>$2.57 \times 10^{-6}$ | 0.0621<br>$3.14 \times 10^{-6}$ | 0.0613<br>$2.37 \times 10^{-6}$ | 0.0613<br>$1.36 \times 10^{-6}$ | 0.0610<br>$4.03 \times 10^{-6}$ | 0.0594<br>$4.36 \times 10^{-6}$ | 0.0550<br>$4.60 \times 10^{-6}$ | 0.0494<br>$4.88 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1001<br>$2.42 \times 10^{-5}$ | 0.0930<br>$1.62 \times 10^{-5}$ | 0.0852<br>$1.59 \times 10^{-5}$ | 0.0773<br>$8.36 \times 10^{-6}$ | 0.0679<br>$1.16 \times 10^{-5}$ | 0.0557<br>$9.72 \times 10^{-6}$ | 0.0407<br>$3.42 \times 10^{-6}$ | 0.0324<br>$2.02 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | —                               | —                               | —                               | —                               | —                               | —                               | —                               | —                               |
| $\alpha = 0.3$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0606<br>$3.53 \times 10^{-6}$ | 0.0607<br>$2.69 \times 10^{-6}$ | 0.0604<br>$3.79 \times 10^{-6}$ | 0.0596<br>$3.20 \times 10^{-6}$ | 0.0593<br>$4.13 \times 10^{-6}$ | 0.0575<br>$3.39 \times 10^{-6}$ | 0.0518<br>$3.06 \times 10^{-6}$ | 0.0456<br>$3.64 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1023<br>$2.39 \times 10^{-5}$ | 0.0961<br>$2.00 \times 10^{-5}$ | 0.0887<br>$2.07 \times 10^{-5}$ | 0.0787<br>$1.55 \times 10^{-5}$ | 0.0717<br>$1.56 \times 10^{-5}$ | 0.0592<br>$8.23 \times 10^{-6}$ | 0.0447<br>$3.46 \times 10^{-6}$ | 0.0356<br>$3.46 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | —                               | —                               | —                               | —                               | —                               | —                               | —                               | —                               |
| $\alpha = 0.5$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0570<br>$6.61 \times 10^{-6}$ | 0.0574<br>$3.87 \times 10^{-6}$ | 0.0565<br>$4.86 \times 10^{-6}$ | 0.0560<br>$4.01 \times 10^{-6}$ | 0.0544<br>$5.27 \times 10^{-6}$ | 0.0522<br>$3.58 \times 10^{-6}$ | 0.0458<br>$5.66 \times 10^{-6}$ | 0.0378<br>$2.82 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1068<br>$2.06 \times 10^{-5}$ | 0.1002<br>$2.64 \times 10^{-5}$ | 0.0933<br>$2.42 \times 10^{-5}$ | 0.0857<br>$1.95 \times 10^{-5}$ | 0.0764<br>$1.59 \times 10^{-5}$ | 0.0652<br>$9.82 \times 10^{-6}$ | 0.0487<br>$6.47 \times 10^{-6}$ | 0.0385<br>$2.44 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | —                               | —                               | —                               | —                               | —                               | —                               | —                               | —                               |

TABLE B.10 (Continued)

|                | $\beta =$              | 0.5              | $\rho = 0.00$                   | $\rho = 0.15$                   | $\rho = 0.30$                   | $\rho = 0.45$                   | $\rho = 0.60$                   | $\rho = 0.75$                   | $\rho = 0.90$                   | $\rho = 0.95$                   |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.7$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0513<br>$6.30 \times 10^{-6}$ | 0.0509<br>$7.64 \times 10^{-6}$ | 0.0507<br>$7.90 \times 10^{-6}$ | 0.0497<br>$6.32 \times 10^{-6}$ | 0.0471<br>$5.12 \times 10^{-6}$ | 0.0439<br>$7.49 \times 10^{-6}$ | 0.0353<br>$3.72 \times 10^{-6}$ | 0.0284<br>$2.22 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1118<br>$4.29 \times 10^{-5}$ | 0.1087<br>$4.45 \times 10^{-5}$ | 0.1010<br>$3.87 \times 10^{-5}$ | 0.0929<br>$3.74 \times 10^{-5}$ | 0.0844<br>$2.58 \times 10^{-5}$ | 0.0713<br>$2.37 \times 10^{-5}$ | 0.0533<br>$6.96 \times 10^{-6}$ | 0.0408<br>$3.92 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | —                               | —                               | —                               | —                               | —                               | —                               | —                               | —                               |
| $\alpha = 0.9$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0397<br>$6.44 \times 10^{-6}$ | 0.0394<br>$6.27 \times 10^{-6}$ | 0.0369<br>$9.13 \times 10^{-6}$ | 0.0347<br>$6.89 \times 10^{-6}$ | 0.0316<br>$6.42 \times 10^{-6}$ | 0.0262<br>$6.65 \times 10^{-6}$ | 0.0184<br>$3.42 \times 10^{-6}$ | 0.0134<br>$1.42 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1217<br>$2.70 \times 10^{-5}$ | 0.1189<br>$5.72 \times 10^{-5}$ | 0.1098<br>$8.05 \times 10^{-5}$ | 0.0996<br>$4.15 \times 10^{-5}$ | 0.0874<br>$4.42 \times 10^{-5}$ | 0.0717<br>$3.66 \times 10^{-5}$ | 0.0484<br>$1.52 \times 10^{-5}$ | 0.0347<br>$5.84 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | —                               | —                               | —                               | —                               | —                               | —                               | —                               | —                               |



TABLE B.11: IV. Standard Errors of the Instrumental-Variable Estimates of  $\alpha$ ,  $\beta$ , and  $\gamma$ . Means and Variances. Various Values of  $\alpha$  and  $\rho$ .  $\beta = 0.5$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$

Sample size:  
T=10, N=25

|                | $\beta =$              | 0.5              | $\rho = 0.00$                   | $\rho = 0.15$                   | $\rho = 0.30$                   | $\rho = 0.45$                   | $\rho = 0.60$                   | $\rho = 0.75$                   | $\rho = 0.90$                   | $\rho = 0.95$                   |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.1$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 1.9403<br>$8.50 \times 10^0$    | 1.6517<br>$6.00 \times 10^{-1}$ | 1.3786<br>$2.67 \times 10^{-1}$ | 1.4465<br>$3.19 \times 10^{-1}$ | 1.4803<br>$6.41 \times 10^{-1}$ | 1.5544<br>$4.80 \times 10^{-1}$ | 1.3442<br>$4.51 \times 10^{-1}$ | 1.4110<br>$3.50 \times 10^{-1}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 2.1918<br>$1.07 \times 10^1$    | 1.8682<br>$7.58 \times 10^{-1}$ | 1.5616<br>$3.38 \times 10^{-1}$ | 1.6392<br>$4.04 \times 10^{-1}$ | 1.6750<br>$8.08 \times 10^{-1}$ | 1.7592<br>$6.10 \times 10^{-1}$ | 1.5214<br>$5.72 \times 10^{-1}$ | 1.5979<br>$4.43 \times 10^{-1}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.4279<br>$1.42 \times 10^{-1}$ | 0.3821<br>$8.28 \times 10^{-3}$ | 0.3535<br>$4.51 \times 10^{-3}$ | 0.3692<br>$6.01 \times 10^{-3}$ | 0.3545<br>$1.12 \times 10^{-2}$ | 0.3634<br>$1.15 \times 10^{-2}$ | 0.3294<br>$1.14 \times 10^{-2}$ | 0.3445<br>$8.55 \times 10^{-3}$ |
| $\alpha = 0.3$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.9440<br>$4.37 \times 10^{-1}$ | 1.4396<br>$1.72 \times 10^1$    | 0.9094<br>$1.39 \times 10^{-1}$ | 0.8916<br>$1.03 \times 10^{-1}$ | 0.9752<br>$3.44 \times 10^{-1}$ | 1.0265<br>$1.23 \times 10^0$    | 0.8104<br>$1.18 \times 10^{-1}$ | 1.1147<br>$3.02 \times 10^{-1}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 1.0729<br>$5.53 \times 10^{-1}$ | 1.6262<br>$2.17 \times 10^1$    | 1.0327<br>$1.74 \times 10^{-1}$ | 1.0165<br>$1.32 \times 10^{-1}$ | 1.1070<br>$4.33 \times 10^{-1}$ | 1.1640<br>$1.55 \times 10^0$    | 0.9221<br>$1.49 \times 10^{-1}$ | 1.2639<br>$3.86 \times 10^{-1}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.3379<br>$7.19 \times 10^{-3}$ | 0.3965<br>$2.95 \times 10^{-1}$ | 0.3107<br>$1.91 \times 10^{-3}$ | 0.3281<br>$4.02 \times 10^{-3}$ | 0.3292<br>$7.25 \times 10^{-3}$ | 0.3329<br>$2.08 \times 10^{-2}$ | 0.3015<br>$5.25 \times 10^{-3}$ | 0.3499<br>$1.21 \times 10^{-2}$ |
| $\alpha = 0.5$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.3159<br>$1.72 \times 10^{-2}$ | 0.3498<br>$2.70 \times 10^{-2}$ | 0.3830<br>$3.12 \times 10^{-2}$ | 0.3780<br>$3.79 \times 10^{-2}$ | 0.3537<br>$1.71 \times 10^{-2}$ | 0.4174<br>$8.00 \times 10^{-2}$ | 0.5051<br>$3.17 \times 10^{-1}$ | 0.3720<br>$3.28 \times 10^{-2}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.3434<br>$2.34 \times 10^{-2}$ | 0.3896<br>$3.69 \times 10^{-2}$ | 0.4194<br>$3.95 \times 10^{-2}$ | 0.4094<br>$5.32 \times 10^{-2}$ | 0.3878<br>$2.43 \times 10^{-2}$ | 0.4561<br>$1.03 \times 10^{-1}$ | 0.5622<br>$4.16 \times 10^{-1}$ | 0.4048<br>$4.18 \times 10^{-2}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.3055<br>$1.20 \times 10^{-3}$ | 0.3057<br>$9.05 \times 10^{-4}$ | 0.3061<br>$2.43 \times 10^{-3}$ | 0.2965<br>$4.76 \times 10^{-3}$ | 0.2919<br>$3.27 \times 10^{-3}$ | 0.2966<br>$7.86 \times 10^{-3}$ | 0.3129<br>$1.60 \times 10^{-2}$ | 0.2843<br>$4.20 \times 10^{-3}$ |

TABLE B.11 (Continued)

|                | $\beta =$              | 0.5              | $\rho = 0.00$                   | $\rho = 0.15$                   | $\rho = 0.30$                   | $\rho = 0.45$                   | $\rho = 0.60$                   | $\rho = 0.75$                   | $\rho = 0.90$                   | $\rho = 0.95$                |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|------------------------------|
| $\alpha = 0.7$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.7525<br>$4.11 \times 10^{-2}$ | 0.7957<br>$1.08 \times 10^{-1}$ | 0.9081<br>$8.36 \times 10^{-1}$ | 0.8605<br>$1.39 \times 10^{-1}$ | 0.8637<br>$2.48 \times 10^{-1}$ | 0.9691<br>$1.41 \times 10^0$    | 1.0627<br>$1.25 \times 10^0$    | 3.6662<br>$3.35 \times 10^2$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.8174<br>$5.42 \times 10^{-2}$ | 0.8596<br>$1.38 \times 10^{-1}$ | 0.9854<br>$1.06 \times 10^0$    | 0.9357<br>$1.79 \times 10^{-1}$ | 0.9370<br>$3.12 \times 10^{-1}$ | 1.0573<br>$1.76 \times 10^0$    | 1.1620<br>$1.59 \times 10^0$    | 4.0752<br>$4.20 \times 10^2$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.3225<br>$8.97 \times 10^{-4}$ | 0.3149<br>$1.06 \times 10^{-3}$ | 0.3395<br>$1.41 \times 10^{-2}$ | 0.3304<br>$7.47 \times 10^{-3}$ | 0.3327<br>$1.21 \times 10^{-2}$ | 0.3635<br>$4.40 \times 10^{-2}$ | 0.3668<br>$5.60 \times 10^{-2}$ | 0.6989<br>$6.61 \times 10^0$ |
| $\alpha = 0.9$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 2.9853<br>$6.13 \times 10^{-1}$ | 3.2955<br>$2.01 \times 10^0$    | 4.0124<br>$3.85 \times 10^1$    | 290.97<br>$3.99 \times 10^6$    | 5.2520<br>$7.95 \times 10^1$    | 5.4060<br>$6.84 \times 10^1$    | 61.165<br>$1.11 \times 10^5$    | 36.282<br>$4.49 \times 10^4$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 3.3241<br>$7.70 \times 10^{-1}$ | 3.6709<br>$2.52 \times 10^0$    | 4.4701<br>$4.81 \times 10^1$    | 325.64<br>$5.00 \times 10^6$    | 5.8594<br>$9.94 \times 10^1$    | 6.0315<br>$8.56 \times 10^1$    | 68.435<br>$1.38 \times 10^5$    | 40.593<br>$5.63 \times 10^4$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.5259<br>$9.45 \times 10^{-3}$ | 0.5656<br>$3.37 \times 10^{-2}$ | 0.6810<br>$6.89 \times 10^{-1}$ | 38.460<br>$6.92 \times 10^4$    | 0.8497<br>$1.46 \times 10^0$    | 0.8422<br>$1.16 \times 10^0$    | 8.2296<br>$1.94 \times 10^3$    | 4.8848<br>$7.70 \times 10^2$ |

TABLE B.12: ML. Standard Errors of the Maximum-Likelihood Estimates of  $\alpha$ ,  $\beta$ , and  $\gamma$ . Means and Variances. Various Values of  $\alpha$  and  $\rho$ .  $\beta = 0.5$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$

Sample size:  
T=10, N=25

|                | $\beta =$              | 0.5              | $\rho = 0.00$                   | $\rho = 0.15$                   | $\rho = 0.30$                   | $\rho = 0.45$                   | $\rho = 0.60$                   | $\rho = 0.75$                   | $\rho = 0.90$                   | $\rho = 0.95$                   |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.1$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0686<br>$8.29 \times 10^{-6}$ | 0.0715<br>$1.08 \times 10^{-5}$ | 0.0711<br>$8.26 \times 10^{-6}$ | 0.0710<br>$4.58 \times 10^{-6}$ | 0.0705<br>$1.06 \times 10^{-5}$ | 0.0686<br>$1.23 \times 10^{-5}$ | 0.0628<br>$1.09 \times 10^{-5}$ | 0.0555<br>$8.53 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1014<br>$2.51 \times 10^{-5}$ | 0.0983<br>$1.97 \times 10^{-5}$ | 0.0910<br>$2.03 \times 10^{-5}$ | 0.0829<br>$1.06 \times 10^{-5}$ | 0.0733<br>$1.52 \times 10^{-5}$ | 0.0605<br>$1.35 \times 10^{-5}$ | 0.0447<br>$4.93 \times 10^{-6}$ | 0.0354<br>$3.00 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2874<br>$2.82 \times 10^{-4}$ | 0.2837<br>$2.03 \times 10^{-4}$ | 0.2697<br>$2.64 \times 10^{-4}$ | 0.2629<br>$1.86 \times 10^{-4}$ | 0.2442<br>$3.22 \times 10^{-4}$ | 0.2267<br>$3.99 \times 10^{-4}$ | 0.2021<br>$7.30 \times 10^{-4}$ | 0.2019<br>$7.00 \times 10^{-4}$ |
| $\alpha = 0.3$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0710<br>$1.73 \times 10^{-5}$ | 0.0737<br>$1.40 \times 10^{-5}$ | 0.0757<br>$3.67 \times 10^{-5}$ | 0.0740<br>$1.03 \times 10^{-5}$ | 0.0731<br>$2.23 \times 10^{-5}$ | 0.0707<br>$1.68 \times 10^{-5}$ | 0.0617<br>$1.04 \times 10^{-5}$ | 0.0524<br>$6.54 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1055<br>$2.96 \times 10^{-5}$ | 0.1035<br>$2.91 \times 10^{-5}$ | 0.0970<br>$3.05 \times 10^{-5}$ | 0.0867<br>$2.10 \times 10^{-5}$ | 0.0797<br>$2.28 \times 10^{-5}$ | 0.0667<br>$1.74 \times 10^{-5}$ | 0.0506<br>$6.90 \times 10^{-6}$ | 0.0398<br>$5.06 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2922<br>$1.24 \times 10^{-4}$ | 0.2847<br>$1.36 \times 10^{-4}$ | 0.2714<br>$1.86 \times 10^{-4}$ | 0.2565<br>$2.11 \times 10^{-4}$ | 0.2417<br>$1.72 \times 10^{-4}$ | 0.2238<br>$4.82 \times 10^{-4}$ | 0.2064<br>$5.35 \times 10^{-4}$ | 0.2023<br>$8.47 \times 10^{-4}$ |
| $\alpha = 0.5$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0675<br>$1.74 \times 10^{-5}$ | 0.0782<br>$9.53 \times 10^{-5}$ | 0.0852<br>$4.50 \times 10^{-4}$ | 0.0838<br>$1.38 \times 10^{-4}$ | 0.0898<br>$7.52 \times 10^{-4}$ | 0.0735<br>$1.01 \times 10^{-4}$ | 0.0629<br>$9.21 \times 10^{-5}$ | 0.0462<br>$9.96 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1135<br>$2.76 \times 10^{-5}$ | 0.1125<br>$6.19 \times 10^{-5}$ | 0.1132<br>$3.72 \times 10^{-4}$ | 0.1043<br>$8.28 \times 10^{-5}$ | 0.1025<br>$4.99 \times 10^{-4}$ | 0.0805<br>$7.11 \times 10^{-5}$ | 0.0615<br>$5.56 \times 10^{-5}$ | 0.0451<br>$7.43 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2924<br>$1.66 \times 10^{-4}$ | 0.2866<br>$1.56 \times 10^{-4}$ | 0.2754<br>$1.81 \times 10^{-4}$ | 0.2587<br>$2.40 \times 10^{-4}$ | 0.2386<br>$2.10 \times 10^{-4}$ | 0.2251<br>$8.70 \times 10^{-4}$ | 0.1949<br>$8.06 \times 10^{-4}$ | 0.1912<br>$5.15 \times 10^{-4}$ |

TABLE B.12 (Continued)

|                | $\hat{\beta}$          | $\rho = 0.5$     | $\rho = 0.00$                   | $\rho = 0.15$                   | $\rho = 0.30$                   | $\rho = 0.45$                   | $\rho = 0.60$                   | $\rho = 0.75$                   | $\rho = 0.90$                   | $\rho = 0.95$                   |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.7$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0646<br>$8.05 \times 10^{-5}$ | 0.1009<br>$3.64 \times 10^{-3}$ | 0.1074<br>$1.49 \times 10^{-3}$ | 0.0993<br>$7.02 \times 10^{-4}$ | 0.0600<br>$5.44 \times 10^{-4}$ | 0.0812<br>$4.11 \times 10^{-4}$ | 0.0644<br>$2.46 \times 10^{-3}$ | 0.0429<br>$1.75 \times 10^{-4}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1228<br>$5.71 \times 10^{-5}$ | 0.1685<br>$5.02 \times 10^{-3}$ | 0.1620<br>$2.46 \times 10^{-3}$ | 0.1396<br>$7.74 \times 10^{-4}$ | 0.1064<br>$6.38 \times 10^{-4}$ | 0.1107<br>$6.63 \times 10^{-4}$ | 0.0881<br>$3.25 \times 10^{-3}$ | 0.0578<br>$2.35 \times 10^{-4}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2921<br>$1.56 \times 10^{-4}$ | 0.3010<br>$7.45 \times 10^{-4}$ | 0.2707<br>$2.02 \times 10^{-4}$ | 0.2824<br>$3.65 \times 10^{-4}$ | 0.2459<br>$2.53 \times 10^{-3}$ | 0.2292<br>$1.34 \times 10^{-3}$ | 0.1932<br>$1.67 \times 10^{-3}$ | 0.1741<br>$1.18 \times 10^{-3}$ |
| $\alpha = 0.9$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0345<br>$6.23 \times 10^{-5}$ | 0.0247<br>$4.84 \times 10^{-5}$ | 0.0171<br>$5.82 \times 10^{-5}$ | 0.0122<br>$6.32 \times 10^{-6}$ | 0.0095<br>$2.82 \times 10^{-5}$ | 0.0078<br>$1.60 \times 10^{-5}$ | 0.0082<br>$1.81 \times 10^{-4}$ | 0.0132<br>$6.62 \times 10^{-5}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1223<br>$1.61 \times 10^{-4}$ | 0.1067<br>$1.42 \times 10^{-4}$ | 0.0942<br>$1.70 \times 10^{-4}$ | 0.0808<br>$1.24 \times 10^{-5}$ | 0.0693<br>$2.86 \times 10^{-5}$ | 0.0551<br>$1.69 \times 10^{-5}$ | 0.0420<br>$5.24 \times 10^{-4}$ | 0.0385<br>$1.34 \times 10^{-4}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.3107<br>$1.23 \times 10^{-4}$ | 0.2883<br>$5.75 \times 10^{-4}$ | 0.2626<br>$7.93 \times 10^{-4}$ | 0.2314<br>$8.78 \times 10^{-5}$ | 0.2086<br>$2.03 \times 10^{-3}$ | 0.1645<br>$1.10 \times 10^{-3}$ | 0.1208<br>$1.54 \times 10^{-3}$ | 0.1401<br>$3.07 \times 10^{-3}$ |











TABLE C.3: LSC. Least Squares with Individual Constant Terms.  
 Estimates of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\sigma^2$  and  $\rho$ .  
 Means and Mean-Square Errors. Various Values of  $\alpha$  and  $\rho$ .  
 $\beta = 1.0$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$ .

Sample size:  
 T=10 N=25

|     |                  | $\beta =$ | $\rho = 0.0$ | $\rho = 0.15$ | $\rho = 0.30$ | $\rho = 0.45$ | $\rho = 0.60$ | $\rho = 0.75$ | $\rho = 0.90$ | $\rho = 0.95$ |
|-----|------------------|-----------|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 0.1 | $\hat{\alpha}$   | Mean      | 0.0005       | 0.0065        | -0.0113       | 0.0024        | 0.0185        | 0.0298        | 0.0486        | 0.0671        |
|     |                  | MSE       | 0.0131       | 0.0119        | 0.0158        | 0.0128        | 0.0097        | 0.0079        | 0.0043        | 0.0022        |
|     | $\hat{\beta}$    | Mean      | 1.1078       | 1.0872        | 1.1246        | 1.1026        | 1.0790        | 1.0798        | 1.0419        | 1.0314        |
|     |                  | MSE       | 0.0315       | 0.0212        | 0.0240        | 0.0191        | 0.0143        | 0.0146        | 0.0042        | 0.0030        |
|     | $\hat{\gamma}$   | Mean      | -0.0010      | -0.0033       | -0.0603       | -0.0159       | 0.0082        | 0.0103        | 0.0502        | 0.0073        |
|     |                  | MSE       | 0.1312       | 0.1117        | 0.0873        | 0.0854        | 0.0878        | 0.0529        | 0.0408        | 0.0538        |
|     | $\hat{\sigma}^2$ | Mean      | 1.0173       | 1.0293        | 1.0424        | 1.0250        | 1.0603        | 1.1348        | 1.0488        | 1.0369        |
|     |                  | MSE       | 0.0075       | 0.0128        | 0.0244        | 0.0388        | 0.0377        | 0.1153        | 0.0976        | 0.0877        |
|     | $\hat{\rho}$     | Mean      | 0.1230       | 0.2539        | 0.4100        | 0.5158        | 0.6556        | 0.7941        | 0.9084        | 0.9548        |
|     |                  | MSE       | 0.0165       | 0.0146        | 0.0180        | 0.0110        | 0.0080        | 0.0047        | 0.0008        | 0.0002        |
| 0.3 | $\hat{\alpha}$   | Mean      | 0.1741       | 0.1840        | 0.2006        | 0.1937        | 0.1964        | 0.2149        | 0.2468        | 0.2646        |
|     |                  | MSE       | 0.0196       | 0.0173        | 0.0146        | 0.0141        | 0.0142        | 0.0096        | 0.0042        | 0.0021        |
|     | $\hat{\beta}$    | Mean      | 1.1585       | 1.1349        | 1.1233        | 1.1173        | 1.1269        | 1.1019        | 1.0734        | 1.0470        |
|     |                  | MSE       | 0.0434       | 0.0281        | 0.0248        | 0.0226        | 0.0261        | 0.0160        | 0.0081        | 0.0040        |
|     | $\hat{\gamma}$   | Mean      | 0.0077       | 0.0381        | 0.0122        | 0.0635        | 0.0573        | -0.0280       | -0.0038       | 0.0270        |
|     |                  | MSE       | 0.1068       | 0.0738        | 0.0899        | 0.0767        | 0.0807        | 0.0644        | 0.9123        | 0.0808        |
|     | $\hat{\sigma}^2$ | Mean      | 1.0135       | 1.0333        | 1.1050        | 1.1557        | 1.1252        | 1.1493        | 1.0881        | 1.0557        |
|     |                  | MSE       | 0.0099       | 0.0135        | 0.0385        | 0.0657        | 0.0767        | 0.0734        | 0.1140        | 0.1179        |
|     | $\hat{\rho}$     | Mean      | 0.1318       | 0.2761        | 0.4329        | 0.5641        | 0.6732        | 0.7996        | 0.9123        | 0.9535        |
|     |                  | MSE       | 0.0184       | 0.0201        | 0.0232        | 0.0184        | 0.0112        | 0.0048        | 0.0006        | 0.0002        |
| 0.5 | $\hat{\alpha}$   | Mean      | 0.3716       | 0.3816        | 0.3936        | 0.3888        | 0.4071        | 0.4203        | 0.4589        | 0.4711        |
|     |                  | MSE       | 0.0188       | 0.0164        | 0.0131        | 0.0138        | 0.0104        | 0.0079        | 0.0024        | 0.0013        |
|     | $\hat{\beta}$    | Mean      | 1.1987       | 1.1986        | 1.1995        | 1.1825        | 1.1432        | 1.1392        | 1.0675        | 1.0499        |
|     |                  | MSE       | 0.0573       | 0.0570        | 0.0559        | 0.0444        | 0.0275        | 0.0247        | 0.0070        | 0.0041        |
|     | $\hat{\gamma}$   | Mean      | 0.0838       | 0.0318        | -0.0244       | -0.0187       | 0.1073        | -0.0185       | -0.0220       | 0.0033        |
|     |                  | MSE       | 0.1331       | 0.1318        | 0.1008        | 0.0740        | 0.0703        | 0.0776        | 0.0509        | 0.0410        |
|     | $\hat{\sigma}^2$ | Mean      | 0.9994       | 1.0807        | 1.1400        | 1.1691        | 1.2817        | 1.2688        | 1.0971        | 1.1025        |
|     |                  | MSE       | 0.0084       | 0.0247        | 0.0528        | 0.0697        | 0.1479        | 0.1682        | 0.0931        | 0.1046        |
|     | $\hat{\rho}$     | Mean      | 0.1380       | 0.3193        | 0.4514        | 0.5809        | 0.7139        | 0.8199        | 0.9151        | 0.9564        |
|     |                  | MSE       | 0.0206       | 0.0344        | 0.0310        | 0.0223        | 0.0166        | 0.0076        | 0.0008        | 0.0002        |

TABLE C.3 (Continued)

| $\beta =$      |                  | 1.0  | $\rho = 0.0$ | $\rho = 0.15$ | $\rho = 0.30$ | $\rho = 0.45$ | $\rho = 0.60$ | $\rho = 0.75$ | $\rho = 0.90$ | $\rho = 0.95$ |
|----------------|------------------|------|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\alpha = 0.7$ | $\hat{\alpha}$   | Mean | 0.5609       | 0.5778        | 0.5720        | 0.6058        | 0.6234        | 0.6459        | 0.6721        | 0.6849        |
|                |                  | MSE  | 0.0211       | 0.0166        | 0.0191        | 0.0101        | 0.0073        | 0.0037        | 0.0012        | 0.0005        |
|                | $\hat{\beta}$    | Mean | 1.3334       | 1.2850        | 1.3376        | 1.2291        | 1.1887        | 1.1383        | 1.0700        | 1.0403        |
|                |                  | MSE  | 0.1319       | 0.0953        | 0.1337        | 0.0662        | 0.0479        | 0.0279        | 0.0086        | 0.0039        |
|                | $\hat{\gamma}$   | Mean | 0.1075       | 0.1162        | 0.0494        | 0.0528        | 0.0366        | 0.0210        | 0.0724        | 0.0044        |
|                |                  | MSE  | 0.1490       | 0.1001        | 0.1317        | 0.0653        | 0.0905        | 0.0652        | 0.0671        | 0.0354        |
|                | $\hat{\sigma}^2$ | Mean | 1.0304       | 1.1564        | 1.3131        | 1.2984        | 1.3371        | 1.3338        | 1.1759        | 0.9830        |
|                |                  | MSE  | 0.0116       | 0.0626        | 0.1599        | 0.1775        | 0.2722        | 0.2753        | 0.1949        | 0.0572        |
|                | $\hat{\rho}$     | Mean | 0.1684       | 0.3607        | 0.5262        | 0.6172        | 0.7099        | 0.8179        | 0.9150        | 0.9522        |
|                |                  | MSE  | 0.0305       | 0.0524        | 0.0588        | 0.0365        | 0.0203        | 0.0083        | 0.0011        | 0.0001        |
| $\alpha = 0.9$ | $\hat{\alpha}$   | Mean | 0.8190       | 0.8245        | 0.8353        | 0.8422        | 0.8604        | 0.8735        | 0.8874        | 0.8925        |
|                |                  | MSE  | 0.0074       | 0.0071        | 0.0048        | 0.0038        | 0.0020        | 0.0010        | 0.0003        | 0.0001        |
|                | $\hat{\beta}$    | Mean | 1.3134       | 1.3240        | 1.2334        | 1.2262        | 1.1622        | 1.0984        | 1.0484        | 1.0267        |
|                |                  | MSE  | 0.1192       | 0.1280        | 0.0706        | 0.0674        | 0.0378        | 0.0168        | 0.0061        | 0.0021        |
|                | $\hat{\gamma}$   | Mean | 0.6154       | 0.4857        | 0.5643        | 0.3915        | 0.2742        | 0.2300        | 0.0705        | 0.0540        |
|                |                  | MSE  | 0.5675       | 0.4084        | 0.4320        | 0.2561        | 0.1894        | 0.1135        | 0.0540        | 0.0556        |
|                | $\hat{\sigma}^2$ | Mean | 1.0641       | 1.3148        | 1.3971        | 1.4846        | 1.4321        | 1.3713        | 1.2103        | 1.1088        |
|                |                  | MSE  | 0.0166       | 0.1659        | 0.2467        | 0.3723        | 0.3649        | 0.3093        | 0.1916        | 0.1271        |
|                | $\hat{\rho}$     | Mean | 0.2008       | 0.4285        | 0.5539        | 0.6577        | 0.7331        | 0.8222        | 0.9185        | 0.9546        |
|                |                  | MSE  | 0.0445       | 0.0905        | 0.0726        | 0.0505        | 0.0230        | 0.0084        | 0.0014        | 0.0004        |

TABLE C.4: 2RC. Two-Round Estimates of  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\sigma^2$   
 Using  $\hat{\rho}$  Estimated by LSC. Means and Mean-Square Errors.  
 Various Values of  $\alpha$  and  $\rho$ .  
 $\beta = 1.0$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$

Sample size:  
 T=10 N=25

|                  |                | $\beta =$ | $\rho = 0.0$ | $\rho = 0.15$ | $\rho = 0.30$ | $\rho = 0.45$ | $\rho = 0.60$ | $\rho = 0.75$ | $\rho = 0.90$ | $\rho = 0.95$ |
|------------------|----------------|-----------|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\alpha = 0.1$   | $\hat{\alpha}$ | Mean      | 0.0446       | 0.0677        | 0.0546        | 0.0707        | 0.0859        | 0.0913        | 0.0942        | 0.0957        |
|                  |                | MSE       | 0.0063       | 0.0044        | 0.0058        | 0.0045        | 0.0039        | 0.0035        | 0.0019        | 0.0012        |
|                  | $\hat{\beta}$  | Mean      | 1.0644       | 1.0273        | 1.0616        | 1.0338        | 1.0133        | 1.0187        | 0.9969        | 1.0035        |
|                  |                | MSE       | 0.0227       | 0.0140        | 0.0124        | 0.0100        | 0.0083        | 0.0090        | 0.0027        | 0.0021        |
|                  | $\hat{\gamma}$ | Mean      | -.0074       | -.0101        | -.0719        | -.0193        | -.0017        | 0.0020        | 0.0439        | 0.0025        |
| MSE              |                | 0.1180    | 0.1008       | 0.0796        | 0.0796        | 0.0781        | 0.0465        | 0.0367        | 0.0503        |               |
| $\hat{\sigma}^2$ | Mean           | 1.0729    | 1.1032       | 1.1275        | 1.1123        | 1.1542        | 1.2386        | 1.1477        | 1.1369        |               |
|                  | MSE            | 0.0134    | 0.0249       | 0.0436        | 0.0585        | 0.0646        | 0.1736        | 0.1361        | 0.1226        |               |
|                  | $\hat{\rho}$   | Mean      | —            | —             | —             | —             | —             | —             | —             |               |
|                  |                | MSE       | —            | —             | —             | —             | —             | —             | —             |               |
| $\alpha = 0.3$   | $\hat{\alpha}$ | Mean      | 0.2241       | 0.2533        | 0.2768        | 0.2692        | 0.2686        | 0.2793        | 0.2894        | 0.2924        |
|                  |                | MSE       | 0.0096       | 0.0062        | 0.0056        | 0.0041        | 0.0052        | 0.0029        | 0.0016        | 0.0010        |
|                  | $\hat{\beta}$  | Mean      | 1.0938       | 1.0483        | 1.02162       | 1.0230        | 1.0370        | 1.0216        | 1.0200        | 1.0123        |
|                  |                | MSE       | 0.0263       | 0.0127        | 0.0103        | 0.0098        | 0.0120        | 0.0061        | 0.0033        | 0.0022        |
|                  | $\hat{\gamma}$ | Mean      | 0.0083       | 0.0298        | 0.0089        | 0.0544        | 0.0451        | -.0325        | -.0089        | 0.0223        |
| MSE              |                | 0.0909    | 0.0674       | 0.0769        | 0.0668        | 0.0672        | 0.0560        | 0.0410        | 0.0744        |               |
| $\hat{\sigma}^2$ | Mean           | 1.0703    | 1.1075       | 1.1932        | 1.2531        | 1.2230        | 1.2522        | 1.1898        | 1.1566        |               |
|                  | MSE            | 0.0160    | 0.0264       | 0.0703        | 0.1136        | 0.1228        | 0.1248        | 0.1636        | 0.1625        |               |
|                  | $\hat{\rho}$   | Mean      | —            | —             | —             | —             | —             | —             | —             |               |
|                  |                | MSE       | —            | —             | —             | —             | —             | —             | —             |               |
| $\alpha = 0.5$   | $\hat{\alpha}$ | Mean      | 0.4236       | 0.4540        | 0.4702        | 0.4647        | 0.4803        | 0.4801        | 0.4967        | 0.4946        |
|                  |                | MSE       | 0.0080       | 0.0045        | 0.0028        | 0.0028        | 0.0028        | 0.0021        | 0.0008        | 0.0006        |
|                  | $\hat{\beta}$  | Mean      | 1.1114       | 1.0751        | 1.0699        | 1.0531        | 1.0201        | 1.0380        | 1.0036        | 1.01040       |
|                  |                | MSE       | 0.0289       | 0.0225        | 0.0197        | 0.0135        | 0.0080        | 0.0072        | 0.0027        | 0.0020        |
|                  | $\hat{\gamma}$ | Mean      | 0.0745       | 0.0257        | -.0363        | -.0188        | 0.0892        | -.0235        | -.0245        | -.0004        |
| MSE              |                | 0.1128    | 0.1100       | 0.0774        | 0.0588        | 0.0552        | 0.0660        | 0.0445        | 0.0377        |               |
| $\hat{\sigma}^2$ | Mean           | 1.0552    | 1.1590       | 1.2289        | 1.2648        | 1.3898        | 1.3803        | 1.1982        | 1.2073        |               |
|                  | MSE            | 0.0128    | 0.0471       | 0.0922        | 0.1193        | 0.2333        | 0.2590        | 0.1394        | 0.1559        |               |
|                  | $\hat{\rho}$   | Mean      | —            | —             | —             | —             | —             | —             | —             |               |
|                  |                | MSE       | —            | —             | —             | —             | —             | —             | —             |               |



TABLE C.5: IV. Instrumental-Variable Estimates of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\sigma^2$  and  $\rho$ . Means and Mean-Square Errors. Various Values of  $\alpha$  and  $\rho$ .  $\beta = 1.0$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$ .

Sample size:  
T=10 N=25

|                  | $\beta =$      | 1.0    | $\rho = 0.0$ | $\rho = 0.15$ | $\rho = 0.30$ | $\rho = 0.45$ | $\rho = 0.60$ | $\rho = 0.75$ | $\rho = 0.90$ | $\rho = 0.95$ |
|------------------|----------------|--------|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\alpha = 0.1$   | $\hat{\alpha}$ | Mean   | 0.1132       | 0.1356        | 0.1227        | 0.1152        | 0.1151        | 0.1035        | 0.0928        | 0.0932        |
|                  |                | MSE    | 0.0348       | 0.0324        | 0.0312        | 0.0187        | 0.0269        | 0.0141        | 0.0126        | 0.0064        |
|                  | $\hat{\beta}$  | Mean   | 0.9953       | 0.9629        | 1.0020        | 0.9815        | 0.9809        | 1.0018        | 0.9740        | 1.0200        |
|                  |                | MSE    | 0.0493       | 0.0481        | 0.0321        | 0.0281        | 0.0349        | 0.0205        | 0.0139        | 0.1067        |
|                  | $\hat{\gamma}$ | Mean   | -.0120       | -.0231        | -.1011        | -.0004        | 0.0035        | 0.0133        | 0.1111        | -.0427        |
| MSE              |                | 0.1074 | 0.1069       | 0.0999        | 0.1242        | 0.1102        | 0.0926        | 0.0966        | 0.1067        |               |
| $\hat{\sigma}^2$ | Mean           | 1.0475 | 1.0074       | 0.9748        | 0.9367        | 0.9677        | 1.0208        | 0.9765        | 0.9853        |               |
|                  | MSE            | 0.0137 | 0.0106       | 0.0256        | 0.0463        | 0.0599        | 0.1390        | 0.1433        | 0.1046        |               |
| $\hat{\rho}$     | Mean           | 0.0982 | 0.1952       | 0.3293        | 0.4381        | 0.5835        | 0.7503        | 0.8870        | 0.9434        |               |
|                  | MSE            | 0.0125 | 0.0076       | 0.0124        | 0.0122        | 0.0182        | 0.0054        | 0.0016        | 0.0003        |               |
| $\alpha = 0.3$   | $\hat{\alpha}$ | Mean   | 0.3115       | 0.2963        | 0.3308        | 0.3254        | 0.2789        | 0.2940        | 0.2678        | 0.2979        |
|                  |                | MSE    | 0.0335       | 0.0174        | 0.0163        | 0.0159        | 0.0132        | 0.0099        | 0.0111        | 0.0043        |
|                  | $\hat{\beta}$  | Mean   | 0.9826       | 0.9928        | 0.9489        | 0.9474        | 1.0244        | 0.9939        | 1.0178        | 1.0104        |
|                  |                | MSE    | 0.0671       | 0.0360        | 0.0289        | 0.0363        | 0.0240        | 0.0213        | 0.0148        | 0.0164        |
|                  | $\hat{\gamma}$ | Mean   | 0.0030       | 0.0327        | 0.0338        | 0.0574        | 0.0413        | -.0102        | 0.0760        | 0.0014        |
| MSE              |                | 0.0720 | 0.0849       | 0.0918        | 0.1081        | 0.1141        | 0.0928        | 0.1657        | 0.1212        |               |
| $\hat{\sigma}^2$ | Mean           | 1.0298 | 0.9920       | 0.9805        | 0.9893        | 0.9914        | 0.9764        | 1.0671        | 0.9687        |               |
|                  | MSE            | 0.0116 | 0.0138       | 0.0190        | 0.0450        | 0.0707        | 0.0596        | 0.2399        | 0.1446        |               |
| $\hat{\rho}$     | Mean           | 0.0955 | 0.2135       | 0.3337        | 0.4569        | 0.6053        | 0.7465        | 0.8930        | 0.9390        |               |
|                  | MSE            | 0.0115 | 0.0098       | 0.0091        | 0.0134        | 0.0113        | 0.0048        | 0.0014        | 0.0004        |               |
| $\alpha = 0.5$   | $\hat{\alpha}$ | Mean   | 0.4935       | 0.4857        | 0.5202        | 0.4911        | 0.5188        | 0.4930        | 0.4837        | 0.4854        |
|                  |                | MSE    | 0.0120       | 0.0169        | 0.0115        | 0.0088        | 0.0109        | 0.0062        | 0.0054        | 0.0057        |
|                  | $\hat{\beta}$  | Mean   | 1.0001       | 1.0092        | 0.9927        | 1.0006        | 0.9684        | 1.0193        | 1.0068        | 1.0290        |
|                  |                | MSE    | 0.0490       | 0.0549        | 0.0523        | 0.0343        | 0.0370        | 0.0219        | 0.0153        | 0.0185        |
|                  | $\hat{\gamma}$ | Mean   | 0.0456       | 0.0608        | -.0646        | 0.0066        | 0.0412        | -.0369        | 0.0341        | -.0099        |
| MSE              |                | 0.0913 | 0.1315       | 0.0787        | 0.0647        | 0.0637        | 0.1055        | 0.1430        | 0.1110        |               |
| $\hat{\sigma}^2$ | Mean           | 0.9928 | 1.0111       | 0.9921        | 0.9829        | 0.9951        | 1.0379        | 1.0332        | 1.0477        |               |
|                  | MSE            | 0.0070 | 0.0205       | 0.0370        | 0.0297        | 0.0663        | 0.1182        | 0.2018        | 0.1498        |               |
| $\hat{\rho}$     | Mean           | 0.0958 | 0.2407       | 0.3312        | 0.4764        | 0.5955        | 0.7583        | 0.8939        | 0.9431        |               |
|                  | MSE            | 0.0111 | 0.0186       | 0.0182        | 0.0141        | 0.0165        | 0.0068        | 0.0014        | 0.0004        |               |

TABLE C.5 (Continued)

| $\beta =$      |                  | 1.0  | $\rho = 0.0$ | $\rho = 0.15$ | $\rho = 0.30$ | $\rho = 0.45$ | $\rho = 0.60$ | $\rho = 0.75$ | $\rho = 0.90$ | $\rho = 0.95$ |
|----------------|------------------|------|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\alpha = 0.7$ | $\hat{\alpha}$   | Mean | 0.6892       | 0.7067        | 0.6822        | 0.7108        | 0.7040        | 0.7098        | 0.6943        | 0.6976        |
|                |                  | MSE  | 0.0098       | 0.0079        | 0.0113        | 0.0054        | 0.0065        | 0.0026        | 0.0028        | 0.0026        |
|                | $\hat{\beta}$    | Mean | 1.0185       | 0.9756        | 1.0685        | 0.9693        | 1.0102        | 0.9819        | 0.9980        | 1.0039        |
|                |                  | MSE  | 0.0731       | 0.0503        | 0.0822        | 0.0419        | 0.0422        | 0.0205        | 0.0247        | 0.0218        |
|                | $\hat{\gamma}$   | Mean | 0.0260       | 0.0170        | -.0238        | 0.0002        | -.0617        | -.0241        | 0.1056        | 0.0087        |
|                |                  | MSE  | 0.1065       | 0.0797        | 0.1146        | 0.0718        | 0.1528        | 0.1544        | 0.1578        | 0.1137        |
|                | $\hat{\sigma}^2$ | Mean | 1.0090       | 0.9977        | 1.0647        | 0.9711        | 1.0377        | 0.9931        | 1.0677        | 0.9178        |
|                |                  | MSE  | 0.0117       | 0.0295        | 0.0755        | 0.0696        | 0.2408        | 0.1481        | 0.3048        | 0.1187        |
|                | $\hat{\rho}$     | Mean | 0.1039       | 0.2208        | 0.3698        | 0.4518        | 0.5741        | 0.7329        | 0.8861        | 0.9352        |
|                |                  | MSE  | 0.0139       | 0.0171        | 0.0292        | 0.0227        | 0.0248        | 0.0099        | 0.0030        | 0.0006        |
| $\alpha = 0.9$ | $\hat{\alpha}$   | Mean | 0.9026       | 0.8975        | 0.9151        | 0.8923        | 0.9037        | 0.9012        | 0.8888        | 0.8887        |
|                |                  | MSE  | 0.0032       | 0.0048        | 0.0032        | 0.0024        | 0.0041        | 0.0021        | 0.0024        | 0.0026        |
|                | $\hat{\beta}$    | Mean | 0.9697       | 1.0318        | 0.9278        | 1.0289        | 0.9887        | 0.9729        | 1.0366        | 1.0267        |
|                |                  | MSE  | 0.0834       | 0.1057        | 0.0714        | 0.0509        | 0.0765        | 0.0342        | 0.0419        | 0.0469        |
|                | $\hat{\gamma}$   | Mean | 0.0387       | -.0504        | -.0614        | 0.0236        | -.0281        | 0.0797        | 0.0763        | 0.1034        |
|                |                  | MSE  | 0.1999       | 0.1976        | 0.1917        | 0.1960        | 0.3407        | 0.2633        | 0.2891        | 0.1806        |
|                | $\hat{\sigma}^2$ | Mean | 1.0080       | 1.0912        | 0.9940        | 1.0784        | 1.1201        | 1.1226        | 1.3890        | 1.4591        |
|                |                  | MSE  | 0.0089       | 0.0665        | 0.0626        | 0.1952        | 0.6071        | 0.5985        | 1.8741        | 4.1116        |
|                | $\hat{\rho}$     | Mean | 0.1170       | 0.2742        | 0.3237        | 0.4774        | 0.5584        | 0.7123        | 0.8826        | 0.9367        |
|                |                  | MSE  | 0.0158       | 0.0349        | 0.0255        | 0.0278        | 0.0399        | 0.0212        | 0.0064        | 0.0015        |







TABLE C.7: ML. Maximum-Likelihood Estimates of  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\sigma^2$  and  $\rho$ . Means and Mean-Square Errors. Various Values of  $\alpha$  and  $\rho$ .  $\beta = 1.0$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$ .

Sample size:  
T=10 N=25

|                |                  | $\beta = 1.0$ | $\rho = 0.0$     | $\rho = 0.15$    | $\rho = 0.30$    | $\rho = 0.45$    | $\rho = 0.60$    | $\rho = 0.75$    | $\rho = 0.90$    | $\rho = 0.95$    |
|----------------|------------------|---------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| $\alpha = 0.1$ | $\hat{\alpha}$   | Mean<br>MSE   | 0.0871<br>0.0038 | 0.1145<br>0.0044 | 0.0905<br>0.0046 | 0.1036<br>0.0043 | 0.1141<br>0.0046 | 0.1130<br>0.0041 | 0.1066<br>0.0021 | 0.1017<br>0.0012 |
|                | $\hat{\beta}$    | Mean<br>MSE   | 1.0220<br>0.0189 | 0.9816<br>0.0140 | 1.0271<br>0.0099 | 1.0006<br>0.0096 | 0.9858<br>0.0089 | 0.9971<br>0.0093 | 0.9845<br>0.0031 | 0.9977<br>0.0021 |
|                | $\hat{\gamma}$   | Mean<br>MSE   | -.0122<br>0.1078 | -.0157<br>0.0945 | -.0778<br>0.0769 | -.0208<br>0.0773 | -.0055<br>0.0745 | -.0010<br>0.0443 | 0.0423<br>0.0357 | 0.0015<br>0.0496 |
|                | $\hat{\sigma}^2$ | Mean<br>MSE   | 1.0052<br>0.0072 | 0.9860<br>0.0099 | 0.9683<br>0.0177 | 0.9276<br>0.0321 | 0.9340<br>0.0280 | 0.9900<br>0.0767 | 0.9355<br>0.0799 | 0.9644<br>0.0766 |
|                | $\hat{\rho}$     | Mean<br>MSE   | 0.0175<br>0.0011 | 0.1243<br>0.0045 | 0.2872<br>0.0075 | 0.4005<br>0.0116 | 0.5617<br>0.0087 | 0.7342<br>0.0051 | 0.8849<br>0.0014 | 0.9456<br>0.0003 |
| $\alpha = 0.3$ | $\hat{\alpha}$   | Mean<br>MSE   | 0.2780<br>0.0047 | 0.3142<br>0.0056 | 0.3327<br>0.0084 | 0.3136<br>0.0041 | 0.3066<br>0.0052 | 0.3074<br>0.0029 | 0.3022<br>0.0017 | 0.2988<br>0.0010 |
|                | $\hat{\beta}$    | Mean<br>MSE   | 1.0239<br>0.0189 | 0.9723<br>0.0133 | 0.9545<br>0.0147 | 0.9673<br>0.0117 | 0.9896<br>0.0125 | 0.9867<br>0.0064 | 1.0040<br>0.0032 | 1.0042<br>0.0021 |
|                | $\hat{\gamma}$   | Mean<br>MSE   | 0.0086<br>0.0797 | 0.0231<br>0.0641 | 0.0079<br>0.0699 | 0.0494<br>0.0620 | 0.0397<br>0.0606 | -.0346<br>0.0527 | -.0099<br>0.0399 | 0.0214<br>0.0731 |
|                | $\hat{\sigma}^2$ | Mean<br>MSE   | 0.9955<br>0.0092 | 0.9669<br>0.0106 | 0.9739<br>0.0182 | 0.9847<br>0.0253 | 0.9365<br>0.0440 | 0.9472<br>0.0371 | 0.9502<br>0.0888 | 0.9633<br>0.0985 |
|                | $\hat{\rho}$     | Mean<br>MSE   | 0.0126<br>0.0006 | 0.1239<br>0.0051 | 0.2710<br>0.0086 | 0.4236<br>0.0084 | 0.5570<br>0.0120 | 0.7259<br>0.0048 | 0.8870<br>0.0010 | 0.9429<br>0.0003 |
| $\alpha = 0.5$ | $\hat{\alpha}$   | Mean<br>MSE   | 0.4802<br>0.0025 | 0.5310<br>0.0050 | 0.5452<br>0.0058 | 0.5274<br>0.0036 | 0.5404<br>0.0070 | 0.5134<br>0.0025 | 0.5099<br>0.0010 | 0.5005<br>0.0006 |
|                | $\hat{\beta}$    | Mean<br>MSE   | 1.0166<br>0.0168 | 0.9435<br>0.0253 | 0.9430<br>0.0235 | 0.9468<br>0.0169 | 0.9188<br>0.0236 | 0.9816<br>0.0077 | 0.9814<br>0.0033 | 1.0005<br>0.0020 |
|                | $\hat{\gamma}$   | Mean<br>MSE   | 0.0633<br>0.0968 | 0.0205<br>0.0924 | -.0471<br>0.0606 | -.0201<br>0.0479 | 0.0740<br>0.0467 | -.0256<br>0.0607 | -.0254<br>0.0424 | -.0014<br>0.0371 |
|                | $\hat{\sigma}^2$ | Mean<br>MSE   | 0.9746<br>0.0082 | 0.9643<br>0.0120 | 0.9438<br>0.0204 | 0.9158<br>0.0285 | 0.9351<br>0.0411 | 0.9677<br>0.0607 | 0.9179<br>0.0657 | 0.9890<br>0.0763 |
|                | $\hat{\rho}$     | Mean<br>MSE   | 0.0118<br>0.0006 | 0.1268<br>0.0064 | 0.2406<br>0.0148 | 0.3875<br>0.0142 | 0.5433<br>0.0188 | 0.7306<br>0.0067 | 0.8857<br>0.0013 | 0.9456<br>0.0003 |

TABLE C.7 (Continued)

| $\beta =$      | 1.0              | $\rho = 0.0$ | $\rho = 0.15$ | $\rho = 0.30$ | $\rho = 0.45$ | $\rho = 0.60$ | $\rho = 0.75$ | $\rho = 0.90$ | $\rho = 0.95$ |        |
|----------------|------------------|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|--------|
| $\alpha = 0.7$ | $\hat{\alpha}$   | Mean         | 0.6720        | 0.7731        | 0.7808        | 0.8371        | 0.8493        | 0.7856        | 0.7184        | 0.7085 |
|                |                  | MSE          | 0.0024        | 0.0090        | 0.0181        | 0.0285        | 0.0343        | 0.0162        | 0.0010        | 0.0004 |
|                | $\hat{\beta}$    | Mean         | 1.0584        | 0.8112        | 0.8172        | 0.6601        | 0.6386        | 0.7949        | 0.9558        | 0.9824 |
|                |                  | MSE          | 0.0186        | 0.0619        | 0.1116        | 0.1835        | 0.2046        | 0.1042        | 0.0075        | 0.0030 |
|                | $\hat{\gamma}$   | Mean         | 0.0458        | -.0248        | -.0759        | -.0829        | -.1021        | -.0597        | 0.0366        | -.0100 |
|                |                  | MSE          | 0.1086        | 0.0632        | 0.0852        | 0.0401        | 0.0575        | 0.0497        | 0.0498        | 0.0311 |
|                | $\hat{\sigma}^2$ | Mean         | 0.9865        | 0.9327        | 0.9140        | 0.7551        | 0.6664        | 0.7512        | 0.9050        | 0.8519 |
|                |                  | MSE          | 0.0090        | 0.0163        | 0.0344        | 0.0953        | 0.1753        | 0.1588        | 0.1211        | 0.0683 |
|                | $\hat{\rho}$     | Mean         | 0.0192        | 0.0581        | 0.1588        | 0.1593        | 0.2198        | 0.5379        | 0.8722        | 0.9379 |
|                |                  | MSE          | 0.0012        | 0.0155        | 0.0515        | 0.1275        | 0.2138        | 0.1283        | 0.0033        | 0.0003 |
| $\alpha = 0.9$ | $\hat{\alpha}$   | Mean         | 0.8931        | 0.9319        | 0.9532        | 0.9635        | 0.9646        | 0.9621        | 0.9173        | 0.9064 |
|                |                  | MSE          | 0.0004        | 0.0015        | 0.0031        | 0.0043        | 0.0049        | 0.0044        | 0.0006        | 0.0001 |
|                | $\hat{\beta}$    | Mean         | 1.0096        | 0.8823        | 0.7513        | 0.7301        | 0.7373        | 0.7321        | 0.9262        | 0.9695 |
|                |                  | MSE          | 0.0206        | 0.0265        | 0.0714        | 0.0834        | 0.3040        | 0.0830        | 0.0121        | 0.0027 |
|                | $\hat{\gamma}$   | Mean         | 0.1028        | -.2502        | -.2651        | -.4282        | -.4392        | -.3834        | -.1224        | -.0342 |
|                |                  | MSE          | 0.1352        | 0.1530        | 0.1609        | 0.2683        | 0.1497        | 0.2019        | 0.0720        | 0.0530 |
|                | $\hat{\sigma}^2$ | Mean         | 0.9683        | 0.9371        | 0.8094        | 0.6936        | 0.5893        | 0.5010        | 0.8121        | 0.9088 |
|                |                  | MSE          | 0.0082        | 0.0139        | 0.0424        | 0.0993        | 0.1845        | 0.2777        | 0.1471        | 0.0943 |
|                | $\hat{\rho}$     | Mean         | 0.0101        | 0.0797        | 0.1003        | 0.1437        | 0.2349        | 0.3954        | 0.8474        | 0.9365 |
|                |                  | MSE          | 0.0004        | 0.0102        | 0.0473        | 0.0992        | 0.1497        | 0.1558        | 0.0114        | 0.0010 |

TABLE C.8 GLS Standard Errors of the Generalized Least-Squares Estimates of  $\alpha$ ,  $\beta$ , and  $\gamma$ . Means and Variances. Various Values of  $\alpha$  and  $\rho$ .  $\beta=1.0$ ,  $\gamma=0.0$ ,  $\sigma^2=1.0$

Sample size:  
T=10, n=25

|                | $\beta=$               | 1.0              | $\rho=0.0$                      | $\rho=0.15$                     | $\rho=0.30$                     | $\rho=0.45$                     | $\rho=0.60$                     | $\rho=0.75$                     | $\rho=0.90$                     | $\rho=0.95$                     |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.1$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0597<br>$2.64 \times 10^{-6}$ | 0.0591<br>$2.56 \times 10^{-6}$ | 0.0579<br>$2.66 \times 10^{-6}$ | 0.0571<br>$3.31 \times 10^{-6}$ | 0.0552<br>$3.66 \times 10^{-6}$ | 0.0517<br>$3.72 \times 10^{-6}$ | 0.0438<br>$4.75 \times 10^{-6}$ | 0.0345<br>$2.40 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1143<br>$2.89 \times 10^{-5}$ | 0.1087<br>$3.0 \times 10^{-5}$  | 0.1012<br>$1.98 \times 10^{-5}$ | 0.0935<br>$1.56 \times 10^{-5}$ | 0.0824<br>$1.46 \times 10^{-5}$ | 0.0719<br>$1.27 \times 10^{-5}$ | 0.0536<br>$7.34 \times 10^{-6}$ | 0.0404<br>$3.42 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2904<br>$1.40 \times 10^{-4}$ | 0.2860<br>$1.73 \times 10^{-4}$ | 0.2705<br>$1.49 \times 10^{-4}$ | 0.2575<br>$1.20 \times 10^{-4}$ | 0.2445<br>$9.17 \times 10^{-5}$ | 0.2283<br>$1.21 \times 10^{-4}$ | 0.2112<br>$9.51 \times 10^{-5}$ | 0.2029<br>$8.97 \times 10^{-5}$ |
| $\alpha = 0.3$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0561<br>$3.56 \times 10^{-6}$ | 0.0556<br>$3.60 \times 10^{-6}$ | 0.0541<br>$5.89 \times 10^{-6}$ | 0.0533<br>$2.47 \times 10^{-6}$ | 0.0512<br>$3.04 \times 10^{-6}$ | 0.0474<br>$3.96 \times 10^{-6}$ | 0.0378<br>$2.29 \times 10^{-6}$ | 0.0301<br>$1.79 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1190<br>$2.70 \times 10^{-5}$ | 0.1149<br>$2.19 \times 10^{-5}$ | 0.1083<br>$3.48 \times 10^{-4}$ | 0.0999<br>$1.82 \times 10^{-5}$ | 0.0902<br>$1.40 \times 10^{-5}$ | 0.0777<br>$1.15 \times 10^{-5}$ | 0.0570<br>$5.27 \times 10^{-6}$ | 0.0439<br>$4.20 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2891<br>$1.98 \times 10^{-4}$ | 0.2828<br>$1.73 \times 10^{-4}$ | 0.2737<br>$1.53 \times 10^{-4}$ | 0.2617<br>$1.27 \times 10^{-4}$ | 0.2442<br>$9.89 \times 10^{-5}$ | 0.2288<br>$9.69 \times 10^{-5}$ | 0.2118<br>$7.04 \times 10^{-5}$ | 0.2066<br>$8.56 \times 10^{-5}$ |
| $\alpha = 0.5$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0492<br>$3.69 \times 10^{-6}$ | 0.0477<br>$4.45 \times 10^{-6}$ | 0.0469<br>$3.72 \times 10^{-6}$ | 0.0458<br>$2.95 \times 10^{-6}$ | 0.0435<br>$4.66 \times 10^{-6}$ | 0.0389<br>$3.54 \times 10^{-6}$ | 0.0301<br>$2.10 \times 10^{-6}$ | 0.0235<br>$1.09 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1261<br>$3.80 \times 10^{-5}$ | 0.1213<br>$5.00 \times 10^{-5}$ | 0.1158<br>$3.84 \times 10^{-5}$ | 0.1076<br>$2.50 \times 10^{-5}$ | 0.0977<br>$2.33 \times 10^{-5}$ | 0.0828<br>$1.73 \times 10^{-5}$ | 0.0598<br>$7.51 \times 10^{-6}$ | 0.0456<br>$4.07 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2862<br>$1.59 \times 10^{-4}$ | 0.2824<br>$1.96 \times 10^{-4}$ | 0.2735<br>$1.83 \times 10^{-4}$ | 0.2584<br>$8.89 \times 10^{-5}$ | 0.2463<br>$1.17 \times 10^{-4}$ | 0.2269<br>$1.09 \times 10^{-4}$ | 0.2096<br>$8.36 \times 10^{-5}$ | 0.2058<br>$8.14 \times 10^{-5}$ |

TABLE C.8 GLS

Sample size:  
T=10, N=25

| $\beta =$      | 1.0                    | $\rho=0.0$  | $\rho=0.15$                     | $\rho=0.30$                     | $\rho=0.45$                     | $\rho=0.60$                     | $\rho=0.75$                     | $\rho=0.90$                     | $\rho=0.95$                     |
|----------------|------------------------|---|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.7$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance<br>0.0388<br>$6.91 \times 10^{-6}$ | 0.0370<br>$6.43 \times 10^{-6}$ | 0.0358<br>$5.04 \times 10^{-6}$ | 0.0341<br>$5.21 \times 10^{-6}$ | 0.0324<br>$3.07 \times 10^{-6}$ | 0.0285<br>$2.59 \times 10^{-6}$ | 0.0212<br>$8.70 \times 10^{-7}$ | 0.0159<br>$4.98 \times 10^{-7}$ |
|                | $\hat{\beta}$          | Mean<br>Variance<br>0.1359<br>$4.91 \times 10^{-5}$ | 0.1283<br>$5.80 \times 10^{-5}$ | 0.1226<br>$5.00 \times 10^{-5}$ | 0.1122<br>$3.55 \times 10^{-5}$ | 0.1021<br>$1.80 \times 10^{-5}$ | 0.0867<br>$2.17 \times 10^{-5}$ | 0.0614<br>$7.00 \times 10^{-6}$ | 0.0452<br>$3.49 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance<br>0.2888<br>$1.99 \times 10^{-4}$ | 0.2830<br>$1.66 \times 10^{-4}$ | 0.2741<br>$1.46 \times 10^{-4}$ | 0.2582<br>$1.12 \times 10^{-4}$ | 0.2477<br>$1.28 \times 10^{-4}$ | 0.2301<br>$1.37 \times 10^{-4}$ | 0.2130<br>$8.63 \times 10^{-5}$ | 0.2052<br>$6.63 \times 10^{-5}$ |
| $\alpha = 0.9$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance<br>0.0165<br>$3.16 \times 10^{-6}$ | 0.0175<br>$1.98 \times 10^{-6}$ | 0.0174<br>$1.76 \times 10^{-6}$ | 0.0172<br>$2.00 \times 10^{-6}$ | 0.0157<br>$1.13 \times 10^{-6}$ | 0.0140<br>$6.71 \times 10^{-7}$ | 0.0101<br>$4.45 \times 10^{-6}$ | 0.0077<br>$2.46 \times 10^{-7}$ |
|                | $\hat{\beta}$          | Mean<br>Variance<br>0.1186<br>$5.82 \times 10^{-5}$ | 0.1169<br>$3.43 \times 10^{-5}$ | 0.1101<br>$3.97 \times 10^{-5}$ | 0.1023<br>$3.99 \times 10^{-5}$ | 0.0906<br>$2.68 \times 10^{-5}$ | 0.0765<br>$1.19 \times 10^{-5}$ | 0.0521<br>$5.73 \times 10^{-6}$ | 0.0386<br>$3.75 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance<br>0.3039<br>$2.40 \times 10^{-4}$ | 0.3044<br>$1.94 \times 10^{-4}$ | 0.2944<br>$2.31 \times 10^{-4}$ | 0.2834<br>$1.78 \times 10^{-4}$ | 0.2676<br>$1.74 \times 10^{-4}$ | 0.2491<br>$1.17 \times 10^{-4}$ | 0.2203<br>$7.25 \times 10^{-5}$ | 0.2128<br>$8.48 \times 10^{-5}$ |

TABLE C.9

OLS Standard Errors of the ordinary Least-Squares  
Estimates of  $\alpha$ ,  $\beta$ , and  $\gamma$ . Means and Variances.  
Various Values of  $\alpha$  and  $\rho$ .  $\beta=1.0$ ,  $\gamma=0.0$ ,  
 $\sigma^2 = 1.0$ .

Sample sizes:  
T=10, N=25

|                | $\beta =$              | 1.0              | $\rho=0.0$                      | $\rho=0.15$                     | $\rho=0.30$                     | $\rho=0.45$                     | $\rho=0.60$                     | $\rho=0.75$                     | $\rho=0.90$                     | $\rho=0.95$                     |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.1$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0597<br>$2.64 \times 10^{-6}$ | 0.0580<br>$3.15 \times 10^{-6}$ | 0.0553<br>$3.90 \times 10^{-6}$ | 0.0519<br>$6.59 \times 10^{-6}$ | 0.0460<br>$1.25 \times 10^{-5}$ | 0.0378<br>$9.05 \times 10^{-6}$ | 0.0281<br>$1.07 \times 10^{-5}$ | 0.0230<br>$7.76 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1143<br>$2.89 \times 10^{-5}$ | 0.1109<br>$3.18 \times 10^{-5}$ | 0.1069<br>$2.15 \times 10^{-5}$ | 0.0994<br>$1.89 \times 10^{-5}$ | 0.0892<br>$1.45 \times 10^{-5}$ | 0.0755<br>$1.87 \times 10^{-5}$ | 0.0547<br>$7.26 \times 10^{-6}$ | 0.0457<br>$4.64 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2904<br>$1.40 \times 10^{-4}$ | 0.2852<br>$1.80 \times 10^{-4}$ | 0.2719<br>$1.85 \times 10^{-4}$ | 0.2532<br>$1.53 \times 10^{-4}$ | 0.2302<br>$7.55 \times 10^{-5}$ | 0.1956<br>$1.33 \times 10^{-4}$ | 0.1415<br>$3.58 \times 10^{-5}$ | 0.1176<br>$2.61 \times 10^{-5}$ |
| $\alpha = 0.3$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0561<br>$3.56 \times 10^{-6}$ | 0.0526<br>$4.96 \times 10^{-6}$ | 0.0471<br>$1.17 \times 10^{-5}$ | 0.0429<br>$9.57 \times 10^{-6}$ | 0.0374<br>$1.23 \times 10^{-5}$ | 0.0298<br>$9.18 \times 10^{-6}$ | 0.0214<br>$4.65 \times 10^{-6}$ | 0.0174<br>$5.36 \times 10^{-5}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1190<br>$2.70 \times 10^{-5}$ | 0.1152<br>$2.51 \times 10^{-5}$ | 0.1080<br>$3.70 \times 10^{-5}$ | 0.0995<br>$2.22 \times 10^{-5}$ | 0.0880<br>$1.70 \times 10^{-5}$ | 0.0727<br>$1.69 \times 10^{-5}$ | 0.0528<br>$5.87 \times 10^{-6}$ | 0.0436<br>$4.71 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2891<br>$1.98 \times 10^{-4}$ | 0.2812<br>$1.73 \times 10^{-4}$ | 0.2702<br>$1.89 \times 10^{-4}$ | 0.2525<br>$1.45 \times 10^{-4}$ | 0.2223<br>$1.08 \times 10^{-4}$ | 0.1865<br>$6.66 \times 10^{-5}$ | 0.1364<br>$3.89 \times 10^{-5}$ | 0.1122<br>$1.51 \times 10^{-5}$ |
| $\alpha = 0.5$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0492<br>$3.69 \times 10^{-6}$ | 0.0428<br>$6.52 \times 10^{-6}$ | 0.0377<br>$9.16 \times 10^{-6}$ | 0.0328<br>$7.46 \times 10^{-6}$ | 0.0261<br>$6.11 \times 10^{-6}$ | 0.0206<br>$6.18 \times 10^{-6}$ | 0.0146<br>$3.35 \times 10^{-6}$ | 0.0118<br>$2.49 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1261<br>$3.80 \times 10^{-5}$ | 0.1180<br>$5.16 \times 10^{-5}$ | 0.1099<br>$3.01 \times 10^{-5}$ | 0.0984<br>$2.45 \times 10^{-5}$ | 0.0851<br>$1.82 \times 10^{-5}$ | 0.0693<br>$1.59 \times 10^{-5}$ | 0.0494<br>$7.21 \times 10^{-6}$ | 0.0412<br>$4.27 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2862<br>$1.60 \times 10^{-4}$ | 0.2797<br>$1.94 \times 10^{-4}$ | 0.2656<br>$1.64 \times 10^{-4}$ | 0.2427<br>$8.69 \times 10^{-5}$ | 0.2150<br>$7.33 \times 10^{-5}$ | 0.1776<br>$6.13 \times 10^{-5}$ | 0.1280<br>$2.78 \times 10^{-5}$ | 0.1070<br>$1.81 \times 10^{-4}$ |

TABLE C.9 OLS

Sample size:  
T=10, N =25

|                |                        | $\beta =$        | 1.0                             | $\rho = 0.0$                    | $\rho = 0.15$                   | $\rho = 0.30$                   | $\rho = 0.45$                   | $\rho = 0.60$                   | $\rho = 0.75$                   | $\rho = 0.90$                   | $\rho = 0.95$ |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------|
| $\alpha = 0.7$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0388<br>$6.91 \times 10^{-6}$ | 0.0309<br>$8.90 \times 10^{-6}$ | 0.0252<br>$5.16 \times 10^{-6}$ | 0.0206<br>$8.71 \times 10^{-6}$ | 0.0167<br>$6.45 \times 10^{-6}$ | 0.0123<br>$2.36 \times 10^{-6}$ | 0.0084<br>$1.16 \times 10^{-6}$ | 0.0070<br>$6.17 \times 10^{-7}$ |               |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1359<br>$491. \times 10^{-5}$ | 0.1198<br>$6.23 \times 10^{-5}$ | 0.1083<br>$3.75 \times 10^{-5}$ | 0.0939<br>$4.51 \times 10^{-5}$ | 0.0813<br>$1.26 \times 10^{-5}$ | 0.0648<br>$1.35 \times 10^{-5}$ | 0.0457<br>$5.70 \times 10^{-6}$ | 0.0377<br>$3.33 \times 10^{-6}$ |               |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2888<br>$1.99 \times 10^{-4}$ | 0.2775<br>$1.61 \times 10^{-4}$ | 0.2624<br>$1.31 \times 10^{-4}$ | 0.2344<br>$8.91 \times 10^{-5}$ | 0.2074<br>$6.86 \times 10^{-5}$ | 0.1698<br>$7.42 \times 10^{-5}$ | 0.1220<br>$2.74 \times 10^{-5}$ | 0.1001<br>$1.37 \times 10^{-5}$ |               |
| $\alpha = 0.9$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0165<br>$3.16 \times 10^{-6}$ | 0.0127<br>$1.65 \times 10^{-6}$ | 0.0102<br>$1.32 \times 10^{-6}$ | 0.0086<br>$1.72 \times 10^{-6}$ | 0.0068<br>$1.17 \times 10^{-6}$ | 0.0053<br>$6.08 \times 10^{-7}$ | 0.0038<br>$4.64 \times 10^{-7}$ | 0.0033<br>$3.17 \times 10^{-7}$ |               |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1186<br>$5.83 \times 10^{-5}$ | 0.1086<br>$2.57 \times 10^{-5}$ | 0.0980<br>$2.69 \times 10^{-5}$ | 0.0882<br>$2.96 \times 10^{-5}$ | 0.0773<br>$2.23 \times 10^{-5}$ | 0.0643<br>$8.88 \times 10^{-6}$ | 0.0496<br>$8.09 \times 10^{-6}$ | 0.0437<br>$6.78 \times 10^{-6}$ |               |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.3039<br>$2.40 \times 10^{-4}$ | 0.2883<br>$1.76 \times 10^{-4}$ | 0.2653<br>$1.81 \times 10^{-4}$ | 0.2433<br>$1.23 \times 10^{-4}$ | 0.2163<br>$1.31 \times 10^{-4}$ | 0.1824<br>$7.25 \times 10^{-5}$ | 0.1411<br>$6.87 \times 10^{-5}$ | 0.1252<br>$5.36 \times 10^{-5}$ |               |

TABLE C.10

LSC Standard Errors of the Estimates of  $\alpha$  and  $\beta$  Obtained by least-squares with Individual Constant Terms. Means and variances. Various values of  $\alpha$  and  $\rho$ .  $\beta=1.0$   $\gamma = 0.0$

Sample sizes:  
T=10, N=25

$$\sigma^2 = 1.0$$

|                | $\beta=$               | 1.0              | $\rho=0.0$                      | $\rho=0.15$                     | $\rho=0.30$                     | $\rho=0.45$                     | $\rho=0.60$                     | $\rho=0.75$                     | $\rho=0.90$                     | $\rho=0.95$                     |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.1$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0596<br>$3.36 \times 10^{-6}$ | 0.0591<br>$3.20 \times 10^{-6}$ | 0.0576<br>$3.21 \times 10^{-6}$ | 0.0568<br>$4.04 \times 10^{-6}$ | 0.0548<br>$4.71 \times 10^{-6}$ | 0.0510<br>$4.52 \times 10^{-6}$ | 0.0428<br>$5.52 \times 10^{-6}$ | 0.0333<br>$2.57 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1132<br>$3.19 \times 10^{-5}$ | 0.1060<br>$3.59 \times 10^{-5}$ | 0.0977<br>$2.18 \times 10^{-5}$ | 0.0907<br>$1.75 \times 10^{-5}$ | 0.0808<br>$1.61 \times 10^{-5}$ | 0.0695<br>$1.23 \times 10^{-5}$ | 0.0519<br>$8.17 \times 10^{-6}$ | 0.0388<br>$3.44 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | —<br>—                          | —<br>—                          | —<br>—                          | —<br>—                          | —<br>—                          | —<br>—                          | —<br>—                          | —<br>—                          |
| $\alpha = 0.3$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0573<br>$3.85 \times 10^{-6}$ | 0.0569<br>$3.86 \times 10^{-6}$ | 0.0555<br>$5.90 \times 10^{-6}$ | 0.0542<br>$2.97 \times 10^{-6}$ | 0.0518<br>$3.80 \times 10^{-6}$ | 0.0475<br>$4.29 \times 10^{-6}$ | 0.0371<br>$2.61 \times 10^{-6}$ | 0.0292<br>$2.00 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1189<br>$2.82 \times 10^{-5}$ | 0.1132<br>$2.23 \times 10^{-5}$ | 0.1064<br>$3.85 \times 10^{-5}$ | 0.0978<br>$1.85 \times 10^{-4}$ | 0.0884<br>$1.66 \times 10^{-5}$ | 0.0761<br>$1.27 \times 10^{-5}$ | 0.0554<br>$5.94 \times 10^{-6}$ | 0.0423<br>$4.65 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | —<br>—                          | —<br>—                          | —<br>—                          | —<br>—                          | —<br>—                          | —<br>—                          | —<br>—                          | —<br>—                          |
| $\alpha = 0.5$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0520<br>$3.92 \times 10^{-6}$ | 0.0510<br>$5.37 \times 10^{-6}$ | 0.0497<br>$3.59 \times 10^{-6}$ | 0.0481<br>$3.25 \times 10^{-6}$ | 0.0455<br>$6.23 \times 10^{-6}$ | 0.0398<br>$4.32 \times 10^{-6}$ | 0.0298<br>$2.40 \times 10^{-6}$ | 0.0228<br>$1.24 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1284<br>$4.26 \times 10^{-5}$ | 0.1223<br>$5.64 \times 10^{-5}$ | 0.1162<br>$4.69 \times 10^{-5}$ | 0.1079<br>$2.73 \times 10^{-5}$ | 0.0980<br>$2.83 \times 10^{-5}$ | 0.0823<br>$2.09 \times 10^{-5}$ | 0.0585<br>$8.55 \times 10^{-6}$ | 0.0441<br>$4.53 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | —<br>—                          | —<br>—                          | —<br>—                          | —<br>—                          | —<br>—                          | —<br>—                          | —<br>—                          | —<br>—                          |

TABLE C.10 LSC

Simple sizes:  
T=10, N = 25

| $\beta =$      |                        | 1.0              | $\rho=0.0$                      | $\rho=0.15$                     | $\rho=0.30$                     | $\rho=0.45$                     | $\rho=0.60$                     | $\rho=0.75$                     | $\rho=0.90$                     | $\rho=0.95$                     |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.7$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0439<br>$7.01 \times 10^{-6}$ | 0.0428<br>$6.38 \times 10^{-6}$ | 0.0408<br>$7.54 \times 10^{-6}$ | 0.0384<br>$4.63 \times 10^{-6}$ | 0.0358<br>$3.46 \times 10^{-6}$ | 0.0306<br>$3.87 \times 10^{-6}$ | 0.0215<br>$1.08 \times 10^{-6}$ | 0.0157<br>$5.64 \times 10^{-7}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1432<br>$6.69 \times 10^{-5}$ | 0.1356<br>$6.31 \times 10^{-5}$ | 0.1287<br>$7.01 \times 10^{-5}$ | 0.1179<br>$3.61 \times 10^{-5}$ | 0.1065<br>$2.57 \times 10^{-5}$ | 0.0893<br>$2.81 \times 10^{-5}$ | 0.0610<br>$8.34 \times 10^{-6}$ | 0.0442<br>$3.90 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | —                               | —                               | —                               | —                               | —                               | —                               | —                               | —                               |
| $\alpha = 0.9$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0284<br>$3.92 \times 10^{-6}$ | 0.0272<br>$3.15 \times 10^{-6}$ | 0.0253<br>$2.51 \times 10^{-6}$ | 0.0235<br>$1.15 \times 10^{-6}$ | 0.0202<br>$1.83 \times 10^{-6}$ | 0.0164<br>$9.45 \times 10^{-7}$ | 0.0106<br>$6.26 \times 10^{-7}$ | 0.0077<br>$3.56 \times 10^{-7}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1483<br>$5.95 \times 10^{-5}$ | 0.1409<br>$8.35 \times 10^{-5}$ | 0.1301<br>$6.84 \times 10^{-5}$ | 0.1181<br>$4.04 \times 10^{-5}$ | 0.1019<br>$3.15 \times 10^{-5}$ | 0.0827<br>$1.98 \times 10^{-5}$ | 0.0528<br>$8.46 \times 10^{-6}$ | 0.0380<br>$4.99 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | —                               | —                               | —                               | —                               | —                               | —                               | —                               | —                               |



TABLE C.11  
 Sample Size:  
 T=10 , N=25

IV Standard Errors of the Instrumental-Variable Estimates of  $\alpha$ ,  $\beta$ , and  $\gamma$ . Means and Variances. Various Values of  $\alpha$  and  $\rho$ .  $\beta=1.0$ ,  $\gamma=0.0$ ,  $\sigma^2=1.0$

|                | $\beta=$               | 1.0           | $\rho=0.0$                      | $\rho=0.15$                     | $\rho=0.30$                     | $\rho=0.45$                     | $\rho=0.60$                     | $\rho=0.75$                     | $\rho=0.90$                     | $\rho=0.95$                     |
|----------------|------------------------|---------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.1$ | S.E. of $\hat{\alpha}$ | Mean Variance | 0.2807<br>$2.57 \times 10^{-3}$ | 0.2665<br>$2.86 \times 10^{-3}$ | 0.2576<br>$2.96 \times 10^{-3}$ | 0.2586<br>$3.48 \times 10^{-3}$ | 0.2635<br>$3.83 \times 10^{-3}$ | 0.2784<br>$6.09 \times 10^{-3}$ | 0.2773<br>$6.66 \times 10^{-3}$ | 0.2730<br>$6.23 \times 10^{-3}$ |
|                | $\hat{\beta}$          | Mean Variance | 0.2757<br>$3.77 \times 10^{-3}$ | 0.2646<br>$4.23 \times 10^{-3}$ | 0.2488<br>$3.99 \times 10^{-3}$ | 0.2546<br>$4.91 \times 10^{-3}$ | 0.2626<br>$5.00 \times 10^{-3}$ | 0.2763<br>$7.36 \times 10^{-3}$ | 0.2816<br>$8.80 \times 10^{-3}$ | 0.2697<br>$8.33 \times 10^{-3}$ |
|                | $\hat{\gamma}$         | Mean Variance | 0.2974<br>$2.15 \times 10^{-4}$ | 0.2924<br>$2.81 \times 10^{-4}$ | 0.2850<br>$6.05 \times 10^{-4}$ | 0.2800<br>$9.39 \times 10^{-4}$ | 0.2848<br>$1.33 \times 10^{-3}$ | 0.2903<br>$2.44 \times 10^{-3}$ | 0.2849<br>$2.92 \times 10^{-3}$ | 0.2850<br>$2.20 \times 10^{-3}$ |
| $\alpha = 0.3$ | S.E. of $\hat{\alpha}$ | Mean Variance | 0.5456<br>$1.40 \times 10^{-2}$ | 0.5455<br>$6.00 \times 10^{-3}$ | 0.5309<br>$1.11 \times 10^{-2}$ | 0.5302<br>$1.30 \times 10^{-2}$ | 0.5561<br>$1.94 \times 10^{-2}$ | 0.4944<br>$1.41 \times 10^{-2}$ | 0.5590<br>$4.63 \times 10^{-2}$ | 0.5296<br>$2.77 \times 10^{-2}$ |
|                | $\hat{\beta}$          | Mean Variance | 0.5789<br>$1.84 \times 10^{-2}$ | 0.5782<br>$7.84 \times 10^{-3}$ | 0.5616<br>$1.48 \times 10^{-2}$ | 0.5623<br>$1.65 \times 10^{-2}$ | 0.5910<br>$2.54 \times 10^{-2}$ | 0.5199<br>$1.86 \times 10^{-2}$ | 0.5949<br>$5.95 \times 10^{-2}$ | 0.5602<br>$3.53 \times 10^{-2}$ |
|                | $\hat{\gamma}$         | Mean Variance | 0.3025<br>$3.14 \times 10^{-4}$ | 0.2957<br>$3.62 \times 10^{-4}$ | 0.2937<br>$5.28 \times 10^{-4}$ | 0.2950<br>$9.08 \times 10^{-4}$ | 0.2950<br>$1.77 \times 10^{-3}$ | 0.2905<br>$1.38 \times 10^{-3}$ | 0.3033<br>$4.99 \times 10^{-3}$ | 0.2887<br>$2.57 \times 10^{-3}$ |
| $\alpha = 0.5$ | S.E. of $\hat{\alpha}$ | Mean Variance | 0.9007<br>$1.18 \times 10^{-2}$ | 0.8979<br>$1.62 \times 10^{-2}$ | 0.8998<br>$2.57 \times 10^{-2}$ | 0.8762<br>$3.01 \times 10^{-2}$ | 0.9355<br>$5.29 \times 10^{-2}$ | 0.8823<br>$5.16 \times 10^{-2}$ | 0.9072<br>$1.29 \times 10^{-1}$ | 0.9249<br>$7.23 \times 10^{-2}$ |
|                | $\hat{\beta}$          | Mean Variance | 0.9799<br>$1.51 \times 10^{-2}$ | 0.9761<br>$2.05 \times 10^{-2}$ | 0.9777<br>$3.27 \times 10^{-2}$ | 0.9508<br>$3.83 \times 10^{-2}$ | 1.0181<br>$6.62 \times 10^{-2}$ | 0.9576<br>$6.42 \times 10^{-2}$ | 0.9861<br>$1.62 \times 10^{-1}$ | 1.0054<br>$9.01 \times 10^{-2}$ |
|                | $\hat{\gamma}$         | Mean Variance | 0.3117<br>$1.94 \times 10^{-4}$ | 0.3132<br>$5.80 \times 10^{-3}$ | 0.3096<br>$9.31 \times 10^{-4}$ | 0.3070<br>$9.43 \times 10^{-4}$ | 0.3113<br>$1.94 \times 10^{-3}$ | 0.3120<br>$2.79 \times 10^{-3}$ | 0.3131<br>$5.60 \times 10^{-2}$ | 0.3155<br>$3.80 \times 10^{-3}$ |

TABLE C.11 IV

Sample size:  
T=10, N=25

|                | $\beta =$              | 1.0              | $\rho = 0.0$                    | $\rho = 0.15$                   | $\rho = 0.30$                   | $\rho = 0.45$                   | $\rho = 0.60$                   | $\rho = 0.75$                   | $\rho = 0.90$                   | $\rho = 0.95$                   |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.7$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 1.4311<br>$3.35 \times 10^{-2}$ | 1.4260<br>$4.10 \times 10^{-2}$ | 1.4386<br>$8.27 \times 10^{-2}$ | 1.3741<br>$7.04 \times 10^{-2}$ | 1.4775<br>$2.42 \times 10^{-1}$ | 1.3942<br>$1.23 \times 10^{-1}$ | 1.5426<br>$2.98 \times 10^{-1}$ | 1.3908<br>$2.07 \times 10^{-1}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 1.5762<br>$4.23 \times 10^{-2}$ | 1.5711<br>$5.15 \times 10^{-2}$ | 1.5839<br>$1.03 \times 10^{-1}$ | 1.5126<br>$8.80 \times 10^{-2}$ | 1.6274<br>$3.00 \times 10^{-1}$ | 1.5351<br>$1.54 \times 10^{-1}$ | 1.7023<br>$3.70 \times 10^{-1}$ | 1.5327<br>$2.56 \times 10^{-1}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.3481<br>$5.82 \times 10^{-4}$ | 0.3466<br>$9.46 \times 10^{-4}$ | 0.3541<br>$2.37 \times 10^{-3}$ | 0.3376<br>$2.16 \times 10^{-3}$ | 0.3486<br>$8.16 \times 10^{-3}$ | 0.3398<br>$4.89 \times 10^{-3}$ | 0.3598<br>$9.83 \times 10^{-3}$ | 0.3314<br>$5.44 \times 10^{-3}$ |
| $\alpha = 0.9$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 3.3492<br>$1.96 \times 10^{-1}$ | 3.3006<br>$2.40 \times 10^{-1}$ | 3.2801<br>$4.63 \times 10^{-1}$ | 3.2734<br>$9.36 \times 10^{-1}$ | 3.6321<br>$5.48 \times 10^0$    | 3.4602<br>$3.59 \times 10^0$    | 4.1497<br>$1.15 \times 10^1$    | 4.2805<br>$1.74 \times 10^1$    |
|                | $\hat{\beta}$          | Mean<br>Variance | 3.7319<br>$2.46 \times 10^{-1}$ | 3.6763<br>$3.00 \times 10^{-1}$ | 3.6539<br>$5.79 \times 10^{-1}$ | 3.6461<br>$1.17 \times 10^0$    | 4.0478<br>$6.85 \times 10^0$    | 3.8560<br>$4.49 \times 10^0$    | 4.6262<br>$1.44 \times 10^1$    | 4.7728<br>$2.17 \times 10^1$    |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.5476<br>$2.99 \times 10^{-3}$ | 0.5472<br>$4.68 \times 10^{-3}$ | 0.5362<br>$8.49 \times 10^{-3}$ | 0.5420<br>$1.89 \times 10^{-2}$ | 0.5840<br>$1.04 \times 10^{-1}$ | 0.5650<br>$7.18 \times 10^{-2}$ | 0.6578<br>$2.23 \times 10^{-1}$ | 0.6745<br>$3.28 \times 10^{-1}$ |

TABLE C.12: ML. Standard Errors of the Maximum Likelihood Estimates of  $\alpha$ ,  $\beta$ , and  $\gamma$ . Means and Variances. Various Values of  $\alpha$  and  $\rho$ .  $\beta = 1.0$ ,  $\gamma = 0.0$ ,  $\sigma^2 = 1.0$ .

Sample Size:  
T=10, N=25

|                | $\beta =$              | 1.0              | $\rho = 0.0$                    | $\rho = 0.15$                   | $\rho = 0.30$                   | $\rho = 0.45$                   | $\rho = 0.60$                   | $\rho = 0.75$                   | $\rho = 0.90$                   | $\rho = 0.95$                   |
|----------------|------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\alpha = 0.1$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0654<br>$1.08 \times 10^{-5}$ | 0.0675<br>$8.53 \times 10^{-6}$ | 0.0656<br>$7.47 \times 10^{-6}$ | 0.0648<br>$8.74 \times 10^{-6}$ | 0.0624<br>$1.15 \times 10^{-5}$ | 0.0575<br>$9.20 \times 10^{-6}$ | 0.0472<br>$8.70 \times 10^{-6}$ | 0.0360<br>$3.35 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1184<br>$4.88 \times 10^{-5}$ | 0.1135<br>$4.64 \times 10^{-5}$ | 0.1055<br>$3.10 \times 10^{-5}$ | 0.0988<br>$2.42 \times 10^{-5}$ | 0.0883<br>$2.55 \times 10^{-5}$ | 0.0761<br>$1.71 \times 10^{-5}$ | 0.0565<br>$1.16 \times 10^{-5}$ | 0.0417<br>$4.25 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2917<br>$1.18 \times 10^{-4}$ | 0.2857<br>$1.74 \times 10^{-4}$ | 0.2706<br>$1.89 \times 10^{-4}$ | 0.2544<br>$2.23 \times 10^{-4}$ | 0.2408<br>$1.66 \times 10^{-4}$ | 0.2280<br>$5.65 \times 10^{-4}$ | 0.2051<br>$6.51 \times 10^{-4}$ | 0.2012<br>$7.16 \times 10^{-4}$ |
| $\alpha = 0.3$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0643<br>$7.27 \times 10^{-6}$ | 0.0681<br>$1.68 \times 10^{-5}$ | 0.0676<br>$2.43 \times 10^{-5}$ | 0.0649<br>$1.08 \times 10^{-5}$ | 0.0615<br>$1.17 \times 10^{-5}$ | 0.0552<br>$8.23 \times 10^{-6}$ | 0.0413<br>$4.49 \times 10^{-6}$ | 0.0318<br>$2.74 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1278<br>$4.98 \times 10^{-5}$ | 0.1252<br>$4.33 \times 10^{-5}$ | 0.1203<br>$9.06 \times 10^{-5}$ | 0.1103<br>$3.42 \times 10^{-5}$ | 0.0999<br>$3.69 \times 10^{-5}$ | 0.0854<br>$2.08 \times 10^{-5}$ | 0.0609<br>$9.43 \times 10^{-6}$ | 0.0458<br>$6.06 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2908<br>$1.17 \times 10^{-4}$ | 0.2821<br>$1.78 \times 10^{-4}$ | 0.2740<br>$1.65 \times 10^{-4}$ | 0.2606<br>$2.23 \times 10^{-4}$ | 0.2409<br>$2.93 \times 10^{-4}$ | 0.2252<br>$2.81 \times 10^{-4}$ | 0.2065<br>$7.04 \times 10^{-3}$ | 0.2012<br>$8.37 \times 10^{-4}$ |
| $\alpha = 0.5$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0608<br>$1.50 \times 10^{-5}$ | 0.0659<br>$5.06 \times 10^{-5}$ | 0.0684<br>$8.58 \times 10^{-5}$ | 0.0643<br>$4.80 \times 10^{-5}$ | 0.0604<br>$5.17 \times 10^{-5}$ | 0.0486<br>$1.59 \times 10^{-5}$ | 0.0338<br>$4.14 \times 10^{-6}$ | 0.0251<br>$1.82 \times 10^{-6}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1409<br>$3.60 \times 10^{-5}$ | 0.1448<br>$1.80 \times 10^{-4}$ | 0.1439<br>$2.69 \times 10^{-4}$ | 0.1324<br>$9.81 \times 10^{-5}$ | 0.1209<br>$1.56 \times 10^{-4}$ | 0.0965<br>$5.64 \times 10^{-5}$ | 0.0653<br>$1.39 \times 10^{-5}$ | 0.0480<br>$6.32 \times 10^{-6}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2905<br>$1.61 \times 10^{-4}$ | 0.2839<br>$2.38 \times 10^{-4}$ | 0.2728<br>$2.20 \times 10^{-4}$ | 0.2553<br>$1.60 \times 10^{-4}$ | 0.2429<br>$2.59 \times 10^{-4}$ | 0.2262<br>$4.57 \times 10^{-4}$ | 0.2036<br>$5.48 \times 10^{-4}$ | 0.2038<br>$7.08 \times 10^{-4}$ |
| $\alpha = 0.7$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0510<br>$1.70 \times 10^{-5}$ | 0.0666<br>$2.47 \times 10^{-4}$ | 0.0646<br>$2.21 \times 10^{-4}$ | 0.0698<br>$8.53 \times 10^{-4}$ | 0.0727<br>$2.74 \times 10^{-3}$ | 0.0511<br>$5.10 \times 10^{-4}$ | 0.0256<br>$3.93 \times 10^{-6}$ | 0.0175<br>$9.30 \times 10^{-7}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1594<br>$1.66 \times 10^{-4}$ | 0.1868<br>$1.52 \times 10^{-3}$ | 0.1800<br>$1.14 \times 10^{-3}$ | 0.1907<br>$4.90 \times 10^{-3}$ | 0.1912<br>$1.60 \times 10^2$    | 0.1369<br>$2.85 \times 10^3$    | 0.0709<br>$2.40 \times 10^5$    | 0.0487<br>$6.07 \times 10^6$    |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.2897<br>$2.23 \times 10^{-4}$ | 0.2859<br>$2.83 \times 10^{-4}$ | 0.2783<br>$2.45 \times 10^{-4}$ | 0.2530<br>$3.21 \times 10^{-4}$ | 0.2472<br>$2.39 \times 10^{-3}$ | 0.2211<br>$5.65 \times 10^{-4}$ | 0.2027<br>$1.03 \times 10^3$    | 0.1909<br>$4.60 \times 10^4$    |
| $\alpha = 0.9$ | S.E. of $\hat{\alpha}$ | Mean<br>Variance | 0.0199<br>$8.92 \times 10^{-6}$ | 0.0198<br>$3.38 \times 10^{-5}$ | 0.0165<br>$1.75 \times 10^{-5}$ | 0.0163<br>$2.29 \times 10^{-5}$ | 0.0156<br>$3.77 \times 10^{-5}$ | 0.0178<br>$5.69 \times 10^{-5}$ | 0.0134<br>$4.15 \times 10^{-6}$ | 0.0088<br>$8.47 \times 10^{-7}$ |
|                | $\hat{\beta}$          | Mean<br>Variance | 0.1281<br>$1.14 \times 10^{-4}$ | 0.1253<br>$2.68 \times 10^{-4}$ | 0.1107<br>$1.38 \times 10^{-4}$ | 0.1043<br>$1.82 \times 10^{-4}$ | 0.0953<br>$3.43 \times 10^{-4}$ | 0.0932<br>$6.67 \times 10^{-4}$ | 0.0637<br>$5.64 \times 10^{-5}$ | 0.0423<br>$1.05 \times 10^{-5}$ |
|                | $\hat{\gamma}$         | Mean<br>Variance | 0.3151<br>$4.47 \times 10^{-4}$ | 0.3106<br>$8.36 \times 10^{-4}$ | 0.2822<br>$4.56 \times 10^{-4}$ | 0.2603<br>$4.20 \times 10^{-4}$ | 0.2399<br>$1.16 \times 10^{-3}$ | 0.2237<br>$1.75 \times 10^{-3}$ | 0.2120<br>$9.60 \times 10^{-4}$ | 0.2046<br>$8.15 \times 10^{-4}$ |

Estimates of standard errors computed only when none of coefficients fell on boundary. Means and variances are based on available estimates.