8-1-1968

Money Illusion, Price Expectations, and the Aggregate Consumption Function

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COWLES FOUNDATION DISCUSSION PAPER NO. 253

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August 14, 1968

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I. Introduction and Background

A standard result of the theory of rational consumer behavior in a monetary economy is that a consumer's demand functions for commodities are homogeneous of degree zero in money prices, money income, and money wealth.[1] Rational utility-maximizing behavior in a static world leads to commodity demand functions which depend on the array of relative prices, real income, and real wealth so that an equiproportionate change in all money prices, money income, and money wealth would leave the quantities demanded by the consumer unchanged. Patinkin has defined this condition as the absence of money illusion. People whose demands for commodities would be altered by an equiproportionate change in all money prices, money income, and money wealth are said to "suffer" from money illusion.

* The research described in this paper was partially supported by grants from the National Science Foundation and the Mobil Foundation. Computation was done at the Princeton University Computer Center using the program "TSP" written by Robert E. Hall, with an Almon subroutine by Charles W. Bischoff. Research assistance was provided by Raymond Hill and Michael Murphy of Princeton University.

[1] See Patinkin [22, pp. 17-23, 403-405, and *passim*] and Samuelson [24, ch. V].

Aggregating over all commodities purchased by the consumer, this standard theorem leads to the conclusion that an individual's total real consumption demand is homogeneous of degree zero in money prices, money income, and money wealth. Finally, aggregating over all consumers, this result would imply that the economy's aggregate real consumption should be a function of aggregate real income and aggregate real wealth, but not the price level.

There are, however, at least two plausible reasons for expecting aggregate consumption behavior to differ from the behavior predicted in the static model of traditional consumer theory where rationality and perfect information prevail. The world we observe is a dynamic one. It is also one in which irrationality may exist and in which there are difficulties associated with the collection and interpretation of reliable information and lags in the processing of information. Suppose, for example, that prices have risen in the current quarter and, say, in the previous two or three quarters, with the value of money income and money wealth increasing proportionately. The pure theory of consumer behavior would predict no change in the aggregate level of real consumption. But if consumers extrapolate this price trend to the next quarter, and perhaps beyond, they may well increase their present real consumption in anticipation of the future price increase. Alternatively, if they believe that the rate of price increase cannot be sustained for another quarter and hence expect the price level to decline shortly, they may postpone real consumption.

On the other hand, real-world consumers may suffer from money illusion, being different from their theoretical counterparts in this respect. It may be that even in a static framework, consumers who generate the data we gather do not properly perceive their real income and real wealth. They may see their money incomes or stocks of nominal assets as convertible into more or fewer real goods and services than market prices actually allow. It is much easier for them to perceive their money income and wealth correctly than it is for them to aggregate a price index accurately for their consumption bundles, present and future.

In a recent paper, E. J. Kane and A. K. Klevorick [14] discussed the phenomenon of money illusion and its origin as a matter of price-level illusion. The price level consumers use in determining their consumption expenditures (P_I) is, in this case, something other than the "true" price level P. Consumers respond to their _perceptions_ of their real incomes and real wealths, Y/P_I and W/P_I, rather than to the _true_ values of these quantities.

Kane and Klevorick show that even if one suspects that consumers do suffer from such money illusion, it is not necessary to fear that the "invalid dichotomy" lurks in the background. So long as market participants' perception of the price level is sensitive to the true price level, the invalid dichotomy is destroyed. The fact that a determinate price level is observed in the real world does not, therefore, provide _prima facie_ evidence against the possibility that consumers suffer from money illusion.

Specifically, Kane and Klevorick investigate a parameterization of the misperceived price level in which $P_I = P^{\alpha}$ with $0 \leq \alpha < \infty$. This parameterization permits replacement of the dichotomous classification of complete absence of money illusion

($\alpha = 1$) and presence of money illusion ($\alpha = 0$) by a continuum of degrees of money illusion measured by $1-\alpha$. It is then shown, using this parameterization, that "it is not the complete absence of money illusion ($\alpha = 1$), but only the absence of extreme money illusion ($\alpha = 0$) which is necessary for prices to be determinate and money neutral."[2]

The proposition that consumers are free of money illusion has not been subjected to systematic testing. Consumption functions have generally been estimated using either money or real values of the variables. Rarely has any attempt been made to compare the results obtained when the same specification is used except for the substitution of real for money values or vice versa. Two exceptions are R. Ferber's [11] encyclopedic comparison of the main statistical studies of the assumption function that were completed prior to 1951 and J. J. Arena's [5], [6] study of the role of net worth and capital gains on that net worth in determining real consumption.

Neither of these studies yields any significant conclusions about the presence or absence of money illusion. Ferber re-estimates the parameters of all aggregate consumption function studies he selected using 1923-40 or some portion thereof as the period of observation and the postwar years as the period to be predicted, and compares the predictive power of the deflated and undeflated forms of the equations he estimates. He finds that, "Price deflation generally improves predictive accuracy, but its effect

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[2]Kane and Klevorick [14, p. 423].

seems to depend on the accuracy of the undeflated function. Those functions that were most accurate ... were less often improved by price deflation."[3]

Arena's results also fail to provide much insight into the question of the presence of money illusion. The results he obtains for the deflated-values equation "are about the same," he writes, as those obtained for the equation in money terms.[4] His specification assumes that the consumption-function equation has a constant term whether the variables are measured in real or in money values. The presence of the constant term in the latter case implies the existence of money illusion, but Arena's results are not conclusive in establishing that the constant term is significantly different from zero.[5]

These inconclusive glimpses at the question of money illusion's existence both come via an indirect route as side observations from studies that were concerned with other questions. There has been no systematic attempt to distinguish empirically between the complete absence of money illusion ($\alpha = 1$) and the presence of money illusion in the extreme ($\alpha = 0$).

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[3] Ferber [11, p. 60].

[4] Arena [5, p. 110].

[5] Arena's AER results are somewhat ambiguous as to whether or not the constant term is significantly different from zero. See his Table 1 [5, p. 108]. On the other hand, the results he presents in Table 1 of his YEE article [6, p. 274], as being closest to the model he was trying to estimate, seem to point to a significant constant in the money-values equation. For a rather detailed and damaging criticism of Arena's estimates, see Patinkin [22, p. 660].

There certainly has been no investigation of whether a "degree of money illusion" ($\alpha > 0$ but $\alpha \neq 1$) exists. Thus, Patinkin is led to conclude, at the end of his note on "Empirical Investigations of the Real-Balance Effect," that "There are other basic questions which have not been dealt with here. Thus it would be desirable to carry out a direct test of the hypothesis that consumers are free of money illusion ... "[6]

This paper attempts to provide the direct test suggested by Patinkin. More generally, it investigates whether the overall price level, as measured by the consumer price index (CPI), has an independent effect on aggregate real consumption. The next section presents some initial evidence on the existence of money illusion. This relatively crude test generated sufficient doubt that consumers are free of money illusion that a further investigation seemed warranted.

In Section III we formulate a consumption function, in the framework of the Ando-Modigliani-Brumberg (A-M-B) "life cycle" hypothesis, that allows the general price level to play an independent role in determining the level of aggregate real consumption. Fitting this function to United States quarterly data from 1955 I-1965 IV in Section IV, we find that the CPI does, indeed, play a significant role in determining the aggregate level of real consumption (of consumer nondurables and services plus the services of durables). In Section V we test whether this price effect can be imputed to price-expectations mechanisms in which consumers' expectations of future prices depend on the price levels or inflation rates of the recent past, or whether at least some

[6] Patinkin [22, p. 664].

degree of money illusion exists in the static Patinkin sense. We conclude that consumers do suffer from some degree of money illusion. Our results are shown to be consistent with a model which embodies a money-illusion effect via a distributed lag adjustment to the price level or with a model which has the money-illusion effect combined with a dynamic or static price-expectations mechanism. But the results are not consistent with a model which hypothesizes the complete absence of money illusion.

Section VI investigates the degree to which our conclusions themselves might be illusory because of statistical problems. Specifically, we examine the possibilities that our results might be spurious because (a) we use time-series data with all variables containing some elements of trend, (b) the price effect may simply be acting as a proxy for business-cycle effects such as an income-distribution effect that arises from changes in the unemployment rates of poorer people with higher-than-average marginal propensities to consume, or (c) there may be simultaneous-equations bias in the consumption-function estimates. Our consideration of these possible statistical pitfalls leads to the conclusion that a significant and substantial degree of money illusion does exist in the U. S. consumption function. The paper concludes with a brief exploration of the implications of our results for macroeconomic stability and for macro-labor-supply functions, and a brief indication of the further questions our study's results would suggest should be examined.

II. Some Initial Evidence on the Existence of Money Illusion

The "life-cycle" analysis of rational consumption behavior assumes a consumer who maximizes his utility function subject to the constraint imposed by his resources.[7] His utility function is taken to have as its arguments the level of his aggregate consumption in the present and in future periods, while his resources are the sum of his current net worth and his current and discounted future earnings over his lifetime. "As a result of this maximization, the current consumption of the individual can be expressed as a function of his resources and the rate of return on capital with parameters depending on age. The individual consumption functions thus obtained are then aggregated to arrive at the aggregate consumption function for the community."[8] Without making further assumptions about the shape of the individual utility functions, but using the same aggregation procedures as Ando and Modigliani use, one can obtain an aggregate real consumption function homogeneous of degree zero in money income, money wealth, and the overall price level.[9] Using the simplest assumption about the individual's "average annual expected nonproperty income" in the future, namely, that it equals the individual's current nonproperty income, the following type of aggregate real consumption function is obtained:

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[7] See Modigliani and Brumberg [19][20] and Ando and Modigliani [4] for the development of the "life-cycle" hypothesis of saving.

[8] Ando and Modigliani [4, p. 56].

[9] [4, p. 58]. Ando and Modigliani present a much more careful derivation, based upon carefully enunciated assumptions, of their linear "life cycle" hypothesis model. See pp. 56-62 therein.

$$(1) \qquad \frac{C}{P} = f\left(\frac{Y}{P}, \frac{W}{P}\right) \quad ,$$

where $\frac{C}{P}$ = real consumption (measured as the current outlays for nondurable goods and services -- net of changes in the stock of nondurables -- plus the rental value of the stock of service-yielding consumer durable goods), Y is nominal nonproperty income (measured by net labor income), W is nominal consumer net worth, and P is the price level of consumer goods. Patinkin also finds this form of the consumption function to be the one relevant to his general equilibrium model.[10]

Suppose that consumers suffer from money illusion so that instead of applying the correct deflator P to their money-value labor income and net worth in determining their real consumption, they apply some illusory price level P_I. In particular, assume that their illusory price level is related to the "true" price level by the following parameterization:

$$(2) \qquad (P_I)_t = P_t^{\alpha}, \quad 0 \leq \alpha < \infty \quad .$$

The aggregate real consumption function then becomes:

$$(3) \qquad \left(\frac{C}{P}\right)_t = f\left(\frac{Y_t}{P_t^{\alpha}}, \frac{W_t}{P_t^{\alpha}}\right) \quad .$$

[10] Patinkin [22, pp. 658-663].

As a linear approximation, one would have

$$(4) \qquad \left(\frac{C}{P}\right)_t = \beta_0 + \beta_1 \left(\frac{Y}{P^{\alpha}}\right)_t + \beta_2 \left(\frac{W}{P^{\alpha}}\right)_t ,$$

or if there is some direct interaction between income and wealth in determining real consumption demand so that the marginal as well as the average propensity to consume out of wealth (income) can be changed by a change in the level of income (wealth), we have, as the simplest approximation,

$$(5) \qquad \left(\frac{C}{P}\right)_t = \beta_0 + \beta_1 \left(\frac{Y}{P^{\alpha}}\right)_t + \beta_2 \left(\frac{W}{P^{\alpha}}\right)_t + \beta_3 \left(\frac{Y}{P^{\alpha}}\right)_t \left(\frac{W}{P^{\alpha}}\right)_t .$$

Equations (4) and (5) ignore, of course, many of the niceties of careful empirical work, such as distributed lag effects, simultaneous-equation relationships and the like, to which we shall return in the sections that follow. (That is the reason for describing this section's test as a "crude" one.) Nevertheless, estimates of equations (4) and (5) may provide some initial evidence on the presence or absence of money illusion and grounds for looking more carefully at the question of this deviation from the results theory implies for a community of "rational" consumers.

Equations (4) and (5) were each fitted by ordinary least squares, using various values of the money-illusion parameter α . The data used were real consumption, money net labor income, money consumer net worth, and the consumer price index. These data will be described at greater length in Section IV and appear in the Appendix. Table 1 reports the sums of squared residuals that resulted from the estimation of equations (4) and (5) with values of α ranging from 0.00 to 1.50.

If current real consumption is a function of current real net labor income and current real consumer net worth, then we would expect to find the highest R^2 and the smallest sum of squared residuals for the regression in which $\alpha = 1$. If the sum of squared residuals is smallest at some other value of α, then we may well be observing money illusion. This assumes, of course, that the <u>current</u>

TABLE 1

SUM OF SQUARED RESIDUALS FROM ESTIMATION OF THE "CRUDE" MONEY ILLUSION CONSUMPTION FUNCTIONS

	Sum of Squared Residuals ($\times 10^{-2}$) from Estimation of	
Value of α	(4) $\left(\frac{C}{P}\right)_t = \beta_0 + \beta_1\left(\frac{Y}{P^\alpha}\right)_t + \beta_2\left(\frac{W}{P^\alpha}\right)_t$	(5) $\left(\frac{C}{P}\right)_t = \beta_0 + \beta_1\left(\frac{Y}{P^\alpha}\right)_t + \beta_2\left(\frac{W}{P^\alpha}\right)_t + \beta_3\left(\frac{Y}{P^\alpha}\right)_t\left(\frac{W}{P^\alpha}\right)_t$
0.00	1.791	1.731
.25	1.758	1.711
.50	2.188	2.157
.75	3.378	3.365
1.00	5.819	5.819
1.25	10.340	10.310
1.50	18.380	18.120

level of real consumption depends only upon concurrent values of the independent variables, which may not be a correct specification of the aggregate consumption function. Indeed, the empirical work reported in later sections of this paper lends strong support to the position that this static hypothesis is ill-founded. Nevertheless Table 1, even with its very coarse grid, indicates that the maxima of the likelihood functions for equations (4) and (5) are far enough from the point $\alpha = 1$ to warrant further investigation. The results summarized in Table 1 suggest that a considerable degree of money illusion exists in the behavior of consumers in the United States. Only a more careful specification and examination of an aggregate real consumption function can provide further insight and enable a more reasoned judgment to be made about money illusion's presence. It is to such a specification and estimation that the next sections are devoted.

III. The Specification of a Money-Illusion Consumption Function

Beginning with the "life-cycle" model of consumers who perceive the price level as $P_I = P^{\alpha}$; $0 \leq \alpha < \infty$; rather than P, we obtain the aggregate real consumption function shown in equation (3) of the previous section. Without more detailed assumptions about the individual consumers' utility function -- as, for example, the ones made by A-M-B and mentioned earlier -- the only meaningful condition we can assume about (3) is that it is homogeneous of degree zero in Y, W, and P^{α}.

Therefore, we shall specify the aggregate real consumption function as a log-linear function,

$$(6) \qquad \left(\frac{C}{P} \right)_t = b_0' \left(\frac{Y}{P^{\alpha}} \right)_t^{b_1} \left(\frac{W}{P^{\alpha}} \right)_t^{b_2} \quad .$$

This specification can be justified by noting that beyond requiring that the form chosen maintain a certain set of characteristics, the choice of a particular form for a consumption function is an arbitrary process. One should ensure the proper signs of the relevant partial derivatives -- a positive marginal propensity to consume out of labor income and out of wealth -- and the proper signs of any possible substitution effects. It is also desirable to check that the estimated function implies marginal propensities to consume that are less than unity, and so on. In the case at hand, one should also ensure that the function is homogeneous of degree zero in money income, money wealth, and the perceived price level. The signs and magnitudes of the marginal propensities to consume and the relevant substitution effects can only be checked *ex post*, although we can assure the reader that the estimates we obtain do meet these prior specifications and equation (6) obviously does have the desired homogeneity property.

We should note briefly the main differences between our specification and that presented in equation (4),

$$(4) \qquad \left(\frac{C}{P} \right)_t = \beta_0 + \beta_1 \left(\frac{Y}{P^{\alpha}} \right)_t + \beta_2 \left(\frac{W}{P^{\alpha}} \right)_t \quad .$$

In (4), increases in perceived wealth (income) can only affect the average, not the marginal, propensity to consume out of perceived income (wealth). On the other hand, the specification in (6) permits more of an interaction between these two determinants of real consumption by allowing each to affect the marginal as well as the average propensity to consume out of the other, although the marginal propensities obviously cannot be affected *ceteris paribus* without altering the average propensities at the same time. Moreover, it will be recalled that the net worth variable is meant to represent the property-income component of the individual's income. That is the reason why, in fact, the series employed for Y is money net labor income. Hence, one can look upon either equation (4) or equation (6) as presenting real consumption as a function of total income computed as a weighted quasi-average of labor and nonlabor income. In equation (4), the weighted averaging process is arithmetic; in equation (6), it is geometric.

To develop our estimating version of (6), we will first isolate the price variable. The specification we will use will enable us to determine whether the overall price level has an independent effect on real consumption. After finding that it does have an independent effect, we will ascertain the cause of this effect -- money illusion and/or price expectations. Letting c denote real consumption $(= \frac{C}{P})$, y denote real net labor income $(= \frac{Y}{P})$, and w denote real consumer net worth $(= \frac{W}{P})$, we rewrite (6) in the form

(7) $$c_t = b_0'(y_t)^{b_1}(w_t)^{b_2}(P_t)^{b_3} \quad ,$$

or, in logarithmic form,

(8) $$\ln c_t = b_0 + b_1 \ln y_t + b_2 \ln w_t + b_3 \ln P_t \quad .$$

With the consumption function written in this form, if there is absence of money illusion in the Patinkin sense so that $\alpha = 1$, we have $b_3 = 0$ and real consumption depends only on real net labor income and real consumer net worth. On the other hand, if $\alpha \neq 1$ so that the price level is misperceived we have $b_3 \neq 0$ and, specifically, $b_3 > 0$ if $\alpha < 1$.[11] Thus, the presence of money illusion implies that $b_3 > 0$ and, as equation (8) clearly shows, a proportional increase in money net labor income, money consumer net worth, and the price level then leads to an increase in the level of aggregate real consumption.

The consumption function shown in (8) does lack realism in one regard. There is no reason to expect consumers to react "instantaneously," that is, within one quarter, to changes in real income, real wealth, or the price level. It is, in fact, much more plausible to suppose that consumers react with a lag to changes in

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[11] From this point on we restrict the meaning of the phrase "the presence of money illusion" to consumers' underestimation of the price level, that is, $\alpha < 1$. (In the literature, the term money illusion has traditionally been applied to just this case -- consumers overestimating the true value of their resources or $\alpha < 1$.)

these independent variables. Alternatively, one might think it plausible that in making their real consumption decisions in a particular quarter, consumers consider an "average" of recent experience with regard to the consumption-determining variables. Thus, we will rewrite (8) to allow for the possibility of distributed lag adjustments to income, wealth, and prices. The lag distributions will be estimated using the flexible interpolation technique developed by S. Almon [3]. Using this distributed lag model, the "basic equation" to be estimated is the following:

$$(9) \quad \ln c_t = \beta_0 + \sum_{i=0}^{I} \gamma_i \ln y_{t-i} + \sum_{j=0}^{J} \delta_j \ln w_{t-j} + \sum_{k=0}^{K} \eta_k \ln P_{t-k} + \epsilon_t .$$

Before going on to the estimation of the money-illusion consumption function (9), it will be useful to discuss briefly the interpretation of the price term, $\sum_{k=0}^{K} \eta_k \ln P_{t-k}$, and to make clear its role in testing the no-money-illusion hypothesis. The problem in interpreting the price term is to distinguish the effect of static money illusion from price-expectations effects, whether the latter be due to "static" or "dynamic" price expectations. There are at least three ways in which a price-level effect might make its appearance in our basic equation. Let us examine each of these effects, which are <u>not</u> mutually exclusive, in turn. It will be helpful to have the basic equation written in nonlogarithmic form before we begin:

$$(10) \quad \left(\frac{C}{P}\right)_t = e^{\beta_0} \left(\prod_{i=0}^{I} \left(\frac{Y}{P}\right)_{t-i}^{\gamma_i} \right) \left(\prod_{j=0}^{J} \left(\frac{W}{P}\right)_{t-j}^{\delta_j} \right) \left(\prod_{k=0}^{K} P_{t-k}^{\eta_k} \right) e^{\epsilon_t} .$$

The first type of possible price effect is the case of pure money illusion in the static Patinkin sense. Instead of basing their consumption decisions on real income and real wealth, consumers modify the deflating factors of income, $\prod_{i=0}^{I} P_{t-i}^{-\gamma_i}$, and wealth, $\prod_{j=0}^{J} P_{t-j}^{-\delta_j}$, by multiplying their product by $\prod_{k=0}^{K} P_{t-k}^{\eta_k} \neq 1$. When prices, money income, and money wealth all increase proportionately, consumers "notice" the income and wealth increases more than they do the price-level rise, and increase their real consumption. Hence, in the case of pure money illusion, we would have $\sum_{k=0}^{K} \eta_k > 0$: consumers exhibit money illusion via a distributed-lag adjustment to the price level.

Suppose, in contrast, that consumers do not suffer from money illusion, but that there is a price-expectations mechanism at work. That is, the real consumption function takes the form

$$(11) \qquad c_t = g(\vec{y}_{t-i}, \vec{w}_{t-j}, \vec{P}_t^{\,e}) \quad ,$$

where $\vec{y}_{t-i}$ is a vector of recent real income experience, $\vec{w}_{t-j}$ is a vector of recent real wealth experience, and $\vec{P}_t^e$ is a vector of the consumers' expectation of future price levels. The hypothesized behavior lying behind such a function is that if consumers expect prices to rise in the future, they will restructure the time pattern of their consumption by moving consumption from the future toward the present. Then, if their expectations are realized, they would reduce their consumption in the future. The $\vec{P}_t^e$ term thus acts as a proxy for an intertemporal substitution effect.[12]

The difference between what we have called "static" and "dynamic" price expectations rests with a difference in the assumption about how $\vec{P}_t^e$ is formed. By static expectations we mean that consumers derive their expectations of future prices from recent price-level experience so that

$$(12) \qquad \vec{P}_t^e = h_1(\vec{P}_{t-k}) \quad ,$$

where $\vec{P}_{t-k}$ is the vector of recent price experience. In terms of our log-linear consumption function $\vec{P}_t^e$ would have to enter into the consumption decision in the form of a product of the P_{t-k}'s, $\prod_{k=0}^{K} P_{t-k}^{\eta_k}$, with the weights summing to zero. The weights must sum to zero because in a steady state, with $P_{t-1} = P_t$ for all t, $\vec{P}_t^e$ must have no effect on the consumption decision as written

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[12]See Power [23] for a further discussion of the role of the intertemporal substitution effect, price expectations, and the real-balance effect.

in (11): prices will not be changing. Given a constant y and w, c should not depend on the constant level of P. In addition, we would expect that the more recent prices in the product $\prod_{k=0}^{K} P_{t-k}^{\eta_k}$ would have positive exponents and that the more distant prices would have negative exponents. This *a priori* expectation stems from the purely allocative role of the price-anticipations mechanism. A *ceteris paribus* rise in today's price will increase $\vec{P}_t^e$ and hence raise present consumption relative to consumption several periods hence. Thus P_t must enter with a positive coefficient in the sum $\sum_{k=0}^{K} \eta_k \ln P_{t-k}$. But as time passes and today's price becomes P_{t-k*}, with k* close to K, its sign must turn negative in accord with the allocative effect $\vec{P}_t^e$ has in (11). In short, in the case of a *pure static price-expectations mechanism and no money illusion, we would have $\sum_{k=0}^{K} \eta_k = 0$ and $\eta_k > 0$ for k small, $\eta_k < 0$ for k large.*

The term dynamic price expectations, as we have used it, is intended to convey the hypothesis that consumers form their beliefs about future prices from recent observations of the rates of change of the price level. That is,

$$(13) \qquad \vec{P}_t^e = h_2(\vec{\Delta P}_{t-k})$$

where $\vec{\Delta P}_{t-k}$ is the vector of recent price-change experience. In sum, under the assumption of static price expectations, consumers project the levels of recent prices to the future while under the assumption of dynamic price expectations, they extrapolate recent rates of change of prices. With a log-linear consumption function, (13) suggests that the function takes the following form

$$(14) \qquad c_t = e^{\beta_0}\left(\prod_{i=0}^{I} y_{t-i}^{\gamma_i}\right)\left(\prod_{j=0}^{J} w_{t-j}^{\delta_j}\right)\left(\prod_{k=0}^{K-1}\left(\frac{P_{t-k}}{P_{t-k-1}}\right)^{\theta_k}\right) e^{\epsilon_t} .$$

The price ratios represent the rates of change indicated in equation (13).

The assumption behind the dynamic price-expectations model is that if consumers have witnessed recent increases in the price level, they will extrapolate these increases to the future and will restructure their consumption stream. Real consumption today will be increased at the expense of real consumption in the future -- when their expectations of further rises in the price level will have been realized. This would lead us to expect that, similar to the case of the time distribution of the η_k's under the assumption of a static price-expectations mechanism, the more recent price ratios in the product will have positive exponents and the more distant price ratios will have negative exponents. As can be seen easily, a positive θ_k corresponds to a positive elasticity of expected future prices while a negative θ_k corresponds to a negative elasticity of expected future prices. The dynamic price-expectations mechanism tells us to expect a positive elasticity of expected future prices for the most recent price changes and a negative one for the price changes somewhat further back in the memory of consumers, although still part of the information they process in making their current consumption decision.

Note, moreover, that the consumption function in equation (14) is consistent with the behavior we should expect in a completely static or steady-state world. Specifically, if there exists a steady state so that prices do not change from one period to the next, the $\vec{P}_t^e$ argument should disappear from the function in (11), and c_t should depend on $\vec{y}_t$ and $\vec{w}_t$ alone. But this is precisely what equation (14) indicates would happen since with $P_{t-1} = P_t$ for all t, each $\frac{P_{t-k}}{P_{t-k-1}}$ ratio would equal unity and we would have

$$c_t = e^{\beta_0}\left(\prod_{i=0}^{I} y_{t-i}^{\gamma_i}\right)\left(\prod_{j=0}^{J} w_{t-j}^{\delta_j}\right).$$

The question remains as to what we should expect the price term in equation (9) to appear as if, in fact, there is no static money illusion but the dynamic price-expectations mechanism just described exists. The answer is really quite simple. Writing (14) in logarithmic form, we have

$$\text{(15)} \quad \ln c_t = \beta_0 + \sum_{i=0}^{I} \gamma_i \ln y_{t-i} + \sum_{j=0}^{J} \delta_j \ln w_{t-j} + \sum_{k=0}^{K-1} \theta_k (\ln P_{t-k} - \ln P_{t-k-1}) + \epsilon_t .$$

But the price term can be written as

$$\text{(16)} \quad \sum_{k=0}^{K-1} \theta_k (\ln P_{t-k} - \ln P_{t-k-1}) = \theta_0 \ln P_t + \sum_{k=1}^{K-1} (\theta_k - \theta_{k-1}) \ln P_{t-k} - \theta_{K-1} \ln P_{t-K} .$$

Therefore, if we estimate (9) and the true model is one of dynamic price expectations and absence of money illusion, that is (14), we would have

$$(17) \qquad \sum_{k=0}^{K} \eta_k = \theta_0 + \sum_{k=1}^{K-1} (\theta_k - \theta_{k-1}) - \theta_{K-1} = 0 \ .$$

Thus, in the case of a pure dynamic price-expectations model and no money illusion, we would have $\sum_{k=0}^{K} \eta_k = 0$, and $\theta_K > 0$ for K small and $\theta_K < 0$ for k large.

This discussion demonstrates that estimation of the basic equation (9) will yield an unambiguous test of the no-money-illusion hypothesis. If the true model has money illusion present to some degree, we should find $\sum_{k=0}^{K} \eta_k$ positive and significantly different from zero. If, on the other hand, the true model is one in which money illusion is absent we should find that $\sum_{k=0}^{K} \eta_k$ is not significantly different from zero. We cannot, however, distinguish between a model in which money illusion is present but there is neither a static nor a dynamic price-expectations mechanism at work, and a model in which money illusion is present but there is a static and/or a dynamic price-expectations mechanism in operation.

With this interpretation of the price term in (9), $\sum_{k=0}^{K} \eta_k \ln P_{t-k}$, as an unambiguous test of money illusion's presence, we now proceed to the estimation of the basic equation expressing the money-illusion consumption function.

IV. Estimation of the Money-Illusion Consumption Function

The consumption function developed in Section III will now be estimated using U. S. quarterly data from 1955-I to 1965-IV. Later sections will consider various potential problems of common trends, simultaneity, and so on.

The data used for real consumption, c, are essentially real consumption expenditures on nondurables and services plus depreciation and imputed interest on durables, with this last representing consumers' use of durables' services. Real net labor income, y, is employees' compensation plus an imputed proportion of proprietors' income plus transfer receipts less employees' social insurance contributions and state, local, and federal tax liabilities on labor income, deflated by the consumption deflator. The wealth term, w, is the net worth of households, including liquid assets, consumer durables, and housing. These three series, all in billions of 1958 dollars, quarterly at annual rates, are updated versions of the annual series used by Ando and Modigliani [4]. The consumption and income data were provided to us by Harold Shapiro, and the wealth data were provided by Albert Ando. The data are reproduced in the Appendix.

The consumer price index (1958 = 100) was chosen as the price variable, P, since it represents the set of prices most relevant to the consumer's buying decision. The data are quarterly averages of the monthly figures published in the Survey of Current Business. The CPI series is also listed in the Appendix.

These data will be used in estimating the basic equation

$$(9) \quad \ln c_t = \beta_0 + \sum_{i=0}^{I} \gamma_i \ln y_{t-i} + \sum_{j=0}^{J} \delta_j \ln w_{t-j} + \sum_{k=0}^{K} \eta_k \ln P_{t-k} + \epsilon_t .$$

An I quarter lag distribution assigns non-zero values to the coefficients of the variable lagged 0,1,...,I-1, quarters and a zero value to the coefficient of the variable lagged I,I+1,... quarters. Since the equation is linear in natural logarithms the estimated coefficients are, of course, estimates of the elasticity of real consumption demand with respect to changes in y, w, and P.

Each of the independent variables is entered in the form of a distributed lag with current real consumption dependent on current and past values of the independent variables. The distributions of the coefficients of these lagged independent variables, which show the time-shape of response of c to changes in y, w, and P, will be estimated using the flexible Almon interpolation technique.[13] This method takes the lagged values of each of the independent variables as a set and estimates a separate distribution of coefficients for each variable, subject only to the constraint that the coefficients be interpolated from Lagrange polynomials of a given degree.

Specification of a second-degree polynomial restrains the lag distribution to have at most one critical (maximum or minimum)

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[13] See Almon [3] for the basic theory. Almon, Bischoff [7] and Modigliani and Sutch [21] have all used the Almon technique extensively.

point except at the beginning and end. That is, the distribution can first rise, then fall to zero; first fall, then rise to zero; or begin with a maximum (minimum) and fall (rise) monotonically to zero. Use of a third degree polynomial, on the other hand, permits the distribution to assume two critical values, for example, rising to a peak, falling through zero to a minimum, and then finally rising to zero. Since this last sequence roughly describes the shape of the lag distribution on the price terms that would be expected if the purely allocative static-expectations hypothesis discussed in Section III were correct, we will use third degree polynomials in estimating the coefficients of (9). More generally, the additional freedom this accords to the shape of the distributions of the income, wealth, and price coefficients will ensure that our results do not come from the imposition of monotonic lag distributions on the coefficients.

Since changing the lag length on one variable in (9) will usually affect the coefficients of all terms, searching for the optimal lag length on all three variables is a fairly complex procedure. In addition, there can sometimes be a conflict between criteria for determining the best lag length: overall goodness of fit, significance of the last coefficient, shape of the lag distribution, and so on. We began by setting the lag lengths I, J, and K in (9) all at 4 quarters, and then experimented with changes in those lengths.

With I=J=K=4 in the initial estimate of (9), the price coefficients were all positive with a significant sum. Only the current wealth coefficient was at all significant, and the income lag was obviously too short -- the coefficient of y_{t-3} was significantly positive. We therefore lengthened the lag distribution on income and shortened that on wealth until we reached the first equation shown in Table 2, in which I = 7; J = 1; K = 4. Table 2 lists the coefficients of each equation horizontally, with the first number for each variable giving the lag length and the second the sum of the coefficients in the lag distribution for that variable, with the t-ratio of the sum in parentheses. Figure 1 shows the lag distribution of coefficients of ln y and ln P in Table 2 equations 2-1 through 2-4.

The regressions that led to equation 2-1 from the initial 4-4-4 specification showed that while the current wealth term was highly significant in all cases, lagged wealth terms were uniformly insignificant, and always quite near zero, whenever the wealth lag was extended beyond one quarter. This is to be expected because (a) the wealth series is highly autocorrelated, and (b) we would expect a weighted average of past labor incomes to be collinear with the wealth of very recent periods.

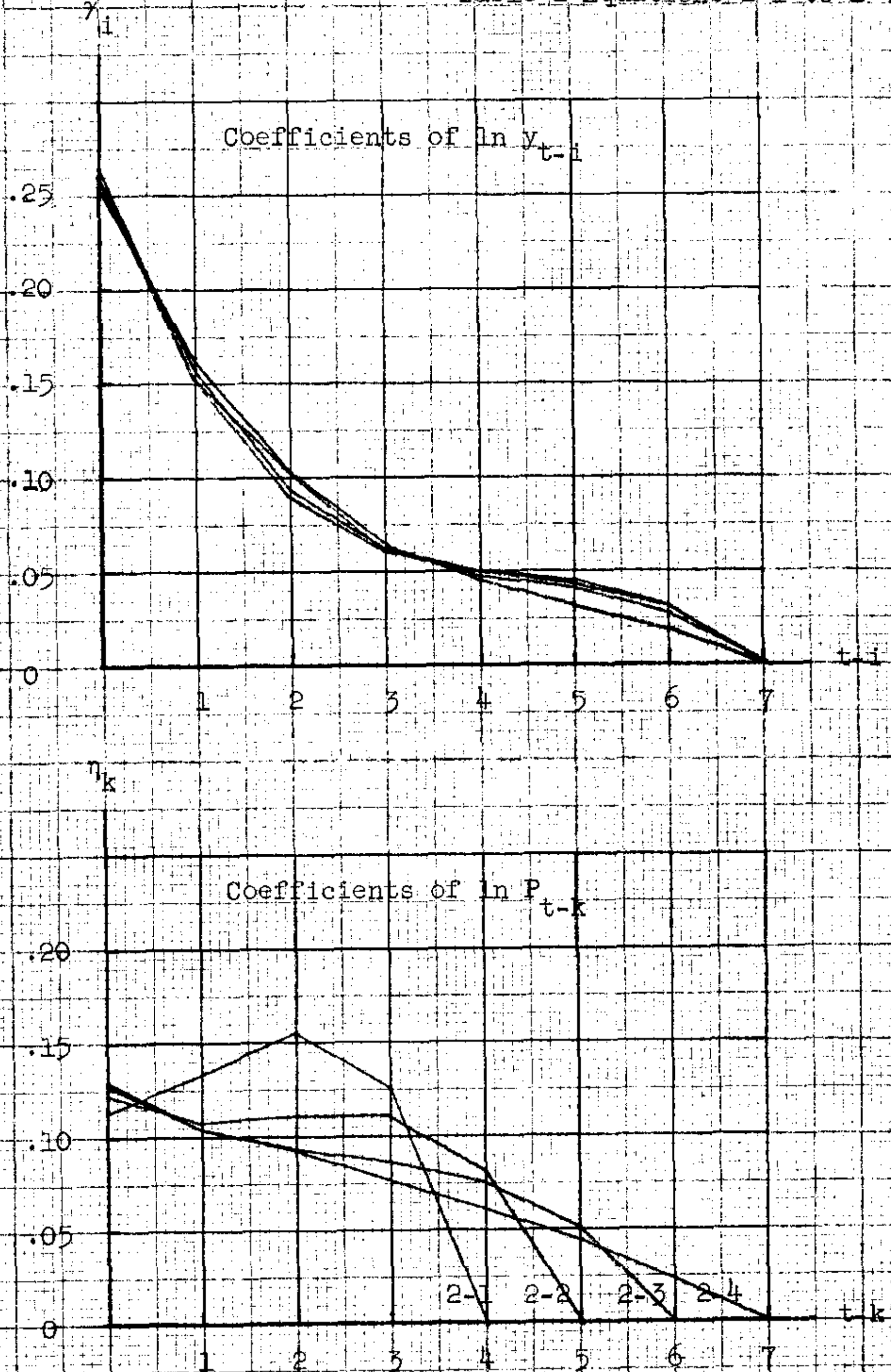
Equation 2-1 with the income lag at seven quarters and the price lag at four quarters shows that the coefficients of all three explanatory variables are highly significant. The income lag, in Figure 1, is positive and monotonically declining, while the price lag is positive in the shape of an inverted U. The long-run elasticity of real consumption demand with respect to changes in real net labor income is 0.678, corresponding to a marginal propensity to consume of about .73 at mean c of \$333 billion, and y of \$309 billion. The net wealth elasticity is 0.149 giving

TABLE 2

Estimation of the Basic Money-Illusion Consumption Function
Text Equation (9)

Equation	Independent Variables: Constant	ln y: I	ln y: $\Sigma\gamma_i$	ln w: J	ln w: δ_0	ln P: K	ln P: $\Sigma\eta_k$	R^2	$SE\times10^2$	D
2-1	-1.629 (17.04)	7	0.678 (16.58)	1	0.149 (4.08)	4	0.530 (10.58)	.9994	.3090	1.686
2-2	-1.648 (17.42)	7	0.691 (16.67)	1	0.138 (3.77)	5	0.536 (10.91)	.9994	.3031	1.739
2-3	-1.662 (17.62)	7	0.696 (16.97)	1	0.134 (3.69)	6	0.540 (11.11)	.9994	.2994	1.751
2-4	-1.667 (17.63)	7	0.694 (17.05)	1	0.135 (3.77)	7	0.542 (11.14)	.9994	.2985	1.766
2-5	-1.657 (17.28)	8	0.692 (16.15)	1	0.139 (3.92)	7	0.535 (10.49)	.9994	.3009	1.757
2-6	-1.665 (17.41)	8	0.688 (16.01)	1	0.142 (4.00)	8	0.539 (10.59)	.9994	.2989	1.767

Figure 1. Distribution of Coefficients of $\ln y_{t-i}$ and $\ln P_{t-k}$ in Table 2 Equations 2-1 to 2-4



a marginal propensity of .03 at mean w of $1693 billion. These marginal coefficients can be compared with Ando and Modigliani's representative values of .7 and .06 on income and on wealth.

The long-run elasticity of consumption with respect to changes in the CPI is 0.530, indicating that an increase in the CPI from 100 to 101 with y and w constant will raise c by 0.53%, or $1.76 billion at mean sample c of $333 billion. This coefficient is highly significant.

The fit of equation 2-1 is excellent, with an R^2 of .9994, and a standard error $1.122 billion. The Durbin-Watson statistic is a reasonably satisfactory 1.686, well above the range normally encountered in this sort of time series estimation when there are no lagged dependent variables.

Extending the length of the price lag gave us equations 2-2 to 2-4 of Table 2. The effect of lengthening the price lag, as can be seen in Figure 1, was to change its shape from an inverted U to a highly significant monotonically declining distribution. However, as is shown in Table 2, the sum of the price coefficients rises only slightly -- from 0.530 to 0.542, while the entire distribution becomes more significant. Lengthening the price lag from four to seven quarters leaves the income and wealth coefficients substantially unchanged. It does, however, reduce the standard error of the estimate of real consumption from $1.122 to $1.034 billion and raises the Durbin-Watson statistic from 1.69 to 1.77.

When the lag in the wealth variable was extended in any of the Table 2 equations, the coefficients of all wealth terms but the current one were completely insignificant and the t-ratio of their sum was less than that of w_t alone in the unlagged version. For example, with the income and price lags set at seven quarters as in equation 2-4, a four-quarter wealth lag gave the coefficients shown in Table 3. The income and price coefficients and the equation statistics for the regression on which Table 3 is based are not significantly different from those of equation 2-4 and the wealth lag t-ratio has fallen, with only the coefficient of w_t at all useful. Thus from now on the wealth term only enters our equations unlagged.

Equation 2-4 of Table 2 has been chosen as our best estimate of the money-illusion consumption function on the basis of two sets of considerations. First, while it is only marginally superior to equations 2-1 to 2-3 in a statistical sense, it does have more significant coefficients in both the income and price lags, a lower standard error, and a higher Durbin-Watson statistic. Second, if our basic theory is that real consumption depends on present and lagged income deflated by a misperceived price level, then the price lag and income lag should be roughly the same length since the variable really moving consumption is misperceived real income. This would argue that we should prefer the equation with the seven quarter lag on both income and price, all other things equal.

Our choice of 2-4 as the estimate of the money-illusion consumption function is buttressed by the fact that when we extend the lags on income and price beyond seven quarters in equations 2-5 and 2-6, the standard error rises again, and the t-ratios on both the income and price lags fall. Furthermore, equation 2-6 with an

TABLE 3

WEALTH COEFFICIENTS IN AN EQUATION WITH I=K=7; J=4

Lag length (j)	Coefficient of wealth term
0	0.129 (2.48)*
1	0.017 (0.33)
2	-0.021 (0.87)
3	-0.016 (0.32)
4	0
Sum	0.109
t-ratio of the Sum	(1.88)

* The numbers in parentheses are the t-ratios of the coefficients.

eight period lag on both price and income is better than equation 2-5 which has a seven period price lag and eight period income lag. This tends to confirm our judgment that the price and income lag should be the same length.

The final equation for the money-illusion consumption function is, then,

$$(18) \quad \ln c_t = \underset{(17.63)}{-1.667} + \sum_{0}^{7} \gamma_i \ln y_{t-i} + \underset{(3.77)}{0.135} \ln w_t + \sum_{0}^{7} \eta_k \ln P_{t-k} \, .$$

$$R^2 = .9994; \quad S.E. = .002985 \; ; \quad d = 1.766 \, ,$$

The standard error of predicted c implied by (2) is only \$1.034 billion, compared with a mean c of \$333 billion.

The coefficients of lagged income, γ_i, and lagged price, η_k, are plotted in Figure 1 and listed in Table 4. The coefficients of real net labor income lagged zero to six quarters sum to 0.694 in our final equation. This is the long-run elasticity of real consumption with respect to changes in real net labor income. The implied marginal propensity to consume is .75 at 1965-IV levels of $414 billion for consumption and $384 billion for income. Similarly, the real wealth coefficient and elasticity is 0.135, giving a marginal propensity to consume out of real net wealth of .025 at 1965-IV wealth of $2183 billion. It is interesting to note that addition

TABLE 4

COEFFICIENTS OF $\ln y_{t-i}$ and $\ln P_{t-k}$

IN EQUATION 2-4 OF TABLE 2, TEXT EQUATION (18)

Lag (i, k)	Coefficient of $\ln y_{t-i}$	Coefficient of $\ln P_{t-k}$
0	0.260 (5.65)*	0.130 (1.55)
1	0.153 (9.56)	0.110 (3.67)
2	0.091 (4.13)	0.093 (1.98)
3	0.061 (3.38)	0.077 (1.97)
4	0.050 (4.17)	0.062 (2.82)
5	0.045 (2.75)	0.045 (1.55)
6	0.032 (1.68)	0.025 (0.76)
7	0	0
Sum	0.694	0.542
t-ratio of Sum	(17.05)	(11.14)

* The numbers in parentheses are the t-ratios of the coefficients.

of the independent price term in the consumption function brings out the significance of the wealth term. In contrast, deLeeuw and Gramlich, reporting the FRB-MIT model, say that, "We have experimented with wealth effects on consumption but were unable to get usable results due to colinearity between wealth and income."[14]

The more interesting coefficients are those of the current and lagged values of the CPI. While the exact shape of the price lag is not completely clear, as a glance at Figure 1 can show, in all the versions of the money-illusion consumption function shown in Table 2, the sum of the price coefficients is between 0.530 and 0.542 -- the difference of .012 is completely insignificant. Furthermore, the sum of the price coefficients is highly significant in all versions of the equation -- the t-ratio of $\Sigma\, \eta_k$ is never less than ten. Thus real consumption rises when the CPI rises, real income and wealth being held constant, with an elasticity of 0.542 in our final equation.[15] If the CPI rises by 1 percent (not percentage point) consumption rises by 0.542%, or, at 1965-IV levels, $2.24 billion (in 1958 dollars).

Finally it is interesting to note that with monotonically declining lag distributions of equal length on both price and income, the money-illusion consumption function could reflect an adaptive adjustment process with real consumption following

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[14] deLeeuw and Gramlich [10, p. 37].

[15] This positive value for $\partial c/\partial p$ represents the presence of money illusion in the sense of the traditional Patinkin experiment -- double all prices, money income and money wealth and see if real consumption rises.

misperceived real income in the manner suggested by Koyck [15].[16] In such a model, desired consumption would depend on Y/P^{α} and actual consumption would adjust by a fraction of the difference between the actual and desired levels of real consumption.

Next we will examine the nature of the price effect on real consumption implied by our estimate of the money-illusion consumption function. In particular, we will consider the relationship between the price effect and the price expectations hypotheses. Then we will discuss the possibility that our conclusions are the spurious results of various econometric pitfalls.

V. The Price Effect in the Money-Illusion Consumption Function

The results presented in Section IV showed that the price level has a significant, independent, positive effect on the level of real consumption. When prices, money income, and money wealth rise in the same proportion, real consumption rises. This result conflicts with the implications of neoclassical rational utility-maximizing demand theory and we interpret it to mean that a significant degree of money illusion exists within the economy. When prices, money income, and wealth all rise, consumers "notice" the income increase more than the price increase and feeling richer, they increase real consumption.

As the discussion in Section III showed, though, the price level might also affect the level of real consumption through a price-expectations mechanism; there might exist a "static" price-

[16] See Griliches [12] for a review of this and similar models.

expectations mechanism or a "dynamic" price-expectations mechanism at work in consumer decisionmaking. But Section III also showed that the conclusive test for the presence or absence of money illusion in our specification of the consumption function was whether or not $\Sigma \eta_k$ -- the sum of the price coefficients -- was positive and significantly different from zero. The estimates in Section IV can lead to no other conclusion than that the sum of the price coefficients in (9) is significantly positive. Hence, our results are inconsistent with either a pure static price-expectations model or a pure dynamic price-expectations hypothesis. But let us look briefly beyond the test on the sum of the price coefficients in investigating the possibility that we are simply observing a pure expectations mechanism at work.

Under the static expectations hypothesis, when consumers' perceptions of the levels of recent prices lead to an increase in expected prices, real consumption gets moved from the future to the present. Present consumption increases above what it would have otherwise been while future consumption decreases below its _ceteris paribus_ level.[17] Thus if we had data on price expectations spanning one quarter, P_t^e, we would expect a regression of c_t on other variables and P_t^e and P_{t-1}^e to give P_t^e a positive coefficient and P_{t-1}^e a negative coefficient with their sum equal to zero, if the expectations effect is only to reallocate consumption among time periods. Since the static price expectations model has $\vec{P}_t^{\,e}$ as a function of the levels of recent prices, this reasoning would lead us to expect

[17] It seems reasonable to require that an expectations function -- if long maintained -- should yield usually correct predictions or else it would be discarded.

a regression of c_t on current and lagged values of real income, the current value of real consumer net worth, and P lagged zero to K periods to give, as indicated in Section III, a lag distribution with first positive and then negative coefficients, summing to zero.

As is obvious from Figure 1, the price coefficients in the estimated consumption function are all positive, no matter which of the variants in Table 2 one might choose to inspect. The sum of the price coefficients is at least ten times its standard error in each case and there is no tendency for the coefficients at the end of the distribution to become negative as the lag is lengthened. It will be recalled that one of the reasons for using a third-degree polynomial in estimating the Almon lag on prices was specifically to allow such an effect to appear if it existed. Thus our results are not consistent with the pure static price-expectations mechanism described in Section III.

Similarly, the pure dynamic price-expectations hypothesis cannot explain our results. As was shown in Section III, the specification of the dynamic price-expectations hypothesis equivalent to our logarithmic money-illusion consumption function is

$$(19) \quad \ln c_t = \beta_0 + \sum_0^I \gamma_i \ln y_{t-i} + \sum_0^J \delta_j \ln w_{t-j} + \sum_0^{K-1} \theta_k \ln \frac{P_{t-k}}{P_{t-k-1}} + \epsilon_t .$$

In that same section, we showed that if the correct specification were (19), then $\Sigma\, \eta_k$ in our estimated equation should be zero.

But this sum is eleven times its standard error in our estimated equation so that the dynamic-expectations hypothesis is rejected.

To further explore the dynamic expectations hypothesis, several estimates of (19) were prepared and are listed in Table 5, which follows the same format as Table 2. Equation (19) was first estimated with the same lag lengths as our final estimated equation (18). The lag on the logarithms of the price ratios is six quarters here since the price lagged six quarters in (9) gets absorbed in the sixth ratio in (19).

The first two equations of Table 5 essentially misspecify our true function (9) by constraining the sum of the price coefficients in (9) to be zero, as noted above. The result of this misspecification is obvious in equation 5-1. The standard error is more than twice that of our final equation (18) and the Durbin-Watson statistic is only 0.8, indicating that an explanatory variable has been left out. Furthermore, the price lag is not only insignificant, but the coefficients of all the price ratios are negative. This means that if (19) <u>were</u> the correct specification, when prices <u>rise</u>, with y and w constant, real consumption <u>falls</u> permanently, quite different from the effect assumed by the dynamic expectations hypothesis.

A search for the "best fitting" misspecified equation yielded 5-2, with an eleven-quarter income lag and a seven-quarter price lag. The equation fits better than 5-1 but still has a standard error higher than any of the equations in Table 2, a Durbin-Watson statistic less than unity, and negative coefficients on all the price ratios.

TABLE 5

Estimates of the Dynamic Expectations Consumption Function, Text Equation (19)

	Constant	ln y		ln w		$\ln \frac{P_{t-k}}{P_{t-k-1}}$			Statistics:		
		I	$\Sigma\ \gamma_i$	J	δ_0	K-1	$\Sigma\ \theta_k$	$\ln P_t$	R^2	$SEx10^2$	D
5-1	-0.681 (9.91)	7	0.652 (7.54)	1	0.373 (6.05)	6	-0.501 (0.83)		.9976	.6232	0.813
5-2	-0.901 (12.73)	11	0.914 (10.98)	1	0.203 (3.56)	7	-1.604 (2.91)		.9985	.4925	0.855
5-3	-1.661 (17.39)	7	0.691 (16.50)	1	0.139 (3.78)	6	-1.177 (3.97)	0.539 (10.93)	.9994	.3008	1.759

That equation 5-2 is significantly inferior to the money illusion consumption function, equation (18), can be seen from the following analysis of variance. The sum of the squared residuals in (18) is $.3209 \times 10^{-3}$; that in 5-2 is $.8732 \times 10^{-3}$. With 44 observations and eight regression variables in each equation, we have $F(1,36) = \frac{.8732-.3209}{.3209/36} = 61.9$ to test the significance of the effect of the added restriction that $\Sigma \eta_k$ is zero in 5-2. Since $F(1,36) = 7.39$ at the 1% level, it is clear that constraining $\Sigma \eta_k$ to equal zero significantly worsens the explanatory power of the equation.

In estimating equation 5-3 we removed the constraint on the sum of the $\ln P_{t-k}$ coefficients by entering $\ln P_t$ separately into equation 5-1. This equation yielded the following coefficients for $\ln (P_t/P_{t-1}),\ldots,\ln(P_{t-5}/P_{t-6})$: -.46, -.28, -.18, -.12, -.08, -.05. Combined with the $\ln P_t$ coefficient of .54, these give the following implied coefficients for $\ln P_t,\ldots,\ln P_{t-6}$: .08, .16, .10, .06, .04, .03, .05, with a sum of 0.54, which is similar to the sum of the coefficients in the price lags of the consumption functions of Table 2. In addition, equation 5-3 is similar to the Table 2 equations in all other respects: the income lag and wealth coefficients are the same as are the equation statistics.[18] Thus, by removing the constraint on the price coefficients we get back the money-illusion consumption function of Section IV, as expected. This confirms our earlier conclusion that the dynamic price expectations equation (19) is, indeed, misspecified, and that the correct function is the money-illusion consumption function,

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[18] Particularly note the improvement in the Durbin-Watson statistic between 5-1 and 5-3.

equation (18).

We have shown that our results are not consistent with what we have called the pure static price-expectations model or the pure dynamic price-expectations model. There may be a price-expectations mechanism at work in the determination of real consumption. But, if there is, it is operating in conjunction with the existence of money illusion in the static Patinkin sense. In any event, we conclude that consumers do suffer from money illusion, whether or not price expectations play a role in their purchasing decisions.

Next we will turn to some potential objections that would assert that our results are related to statistical problems of time and timing -- the problems of trend, cycle, and simultaneity.

VI. Statistical Problems of Trend, Cycle, and Simultaneity

This section reports several further tests of the money-illusion consumption function, equation (18), which were conducted to insure that our results are not seriously affected by trend interrelationships between variables, cyclical factors in the economy, or simultaneous-equations bias.

Trend Relationships Between Variables

In any time-series regression analysis there exists the possibility that a spurious fit may be obtained due to the fortuitous presence of trends in both dependent and independent variables. Our theory relating consumption to income, wealth, and the price level implies, however, that consumption should be related to income, wealth, and the CPI, both along trend and when the variables deviate from trend. While the high Durbin-Watson statistic of equation (18) -- 1.77 -- indicates that we have captured more than a trend relationship, there exists a more direct test of the role of time trend in our results. This test is obtained by regressing the log of consumption deviations from trend on similar transformations of the income, wealth and price variables. More precisely, we first extracted a logarithmic trend from each variable by dividing it by its log trend value (e.g., consumption deviation = c_t/consumption trend, ct_t), and then took the natural logarithms of these deviation-from-trend variables as our regression variables.

Equation 6-1, Table 6, shows the results of re-estimating the money-illusion consumption equation (18) using deviations from trend. The format of Table 6 is the same as that of Table 2 except that, since lag lengths are fixed at I=7, J=1, K=7, they are not shown. The coefficients of the income and price terms in equation 6-1 are shown in Figure 2.

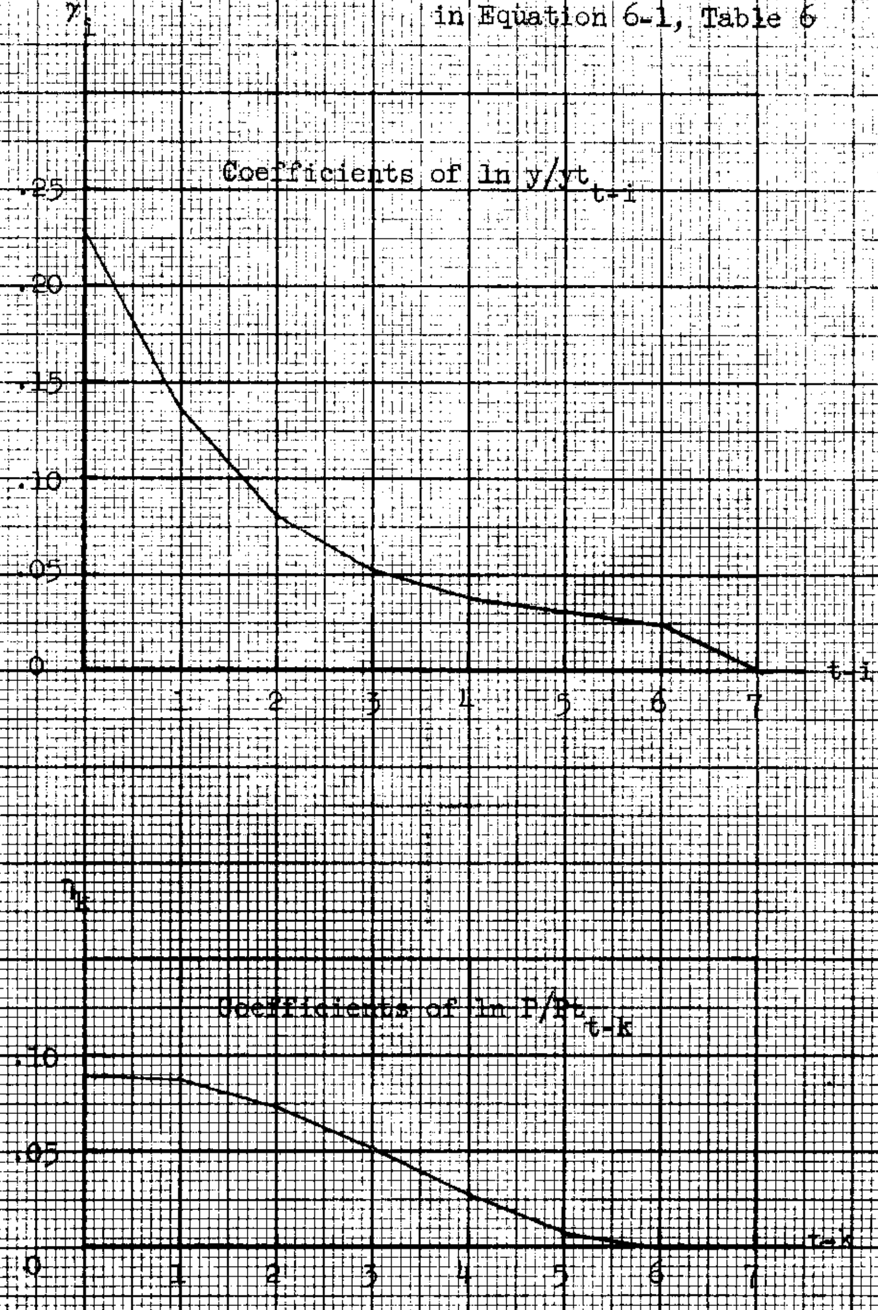
The deviations-from-trend version of the consumption function, equation 6-1, explains 95% of the variance of deviations of consumption from trend (in log form). All the independent variables are highly significant with t-ratios of 9.7 on the sum of the income coefficients, 4.9 on wealth, and 3.6 on price. Figure 2 shows that the income lag has the same shape as that of equation (18), shown in Figure 1, but that it has a slightly smaller sum -- 0.59 vs. 0.69. This indicates that consumption responds to income along trend a bit more strongly than in deviations from trend.

TABLE 6

Tests of Statistical Problems of Trend, Cycle, and Simultaneity

Equation	Dependent Variable	Independent Variables							Statistics		
		Constant		$\ln y/yt$	$\ln w/wt$	$\ln P/Pt$			R^2	$SE \times 10^2$	D
6-1	$\ln(c/ct)$	-0.675		0.595	0.148	0.333			.9491	.2901	1.863
		(4.82)		(9.70)	(4.87)	(3.62)					
				$\ln y$	$\ln w_t$	$\ln P$		$\ln y/y_p$			
6-2	$\ln c$	-1.742		0.674	0.140	0.574		0.032	.9994	.3007	1.786
		(12.06)		(13.48)	(3.80)	(8.53)		(0.70)			
			$\ln \hat{y}_t$	$\ln y_{t-i}$	$\ln w_t$	$\ln \hat{P}_t$	$\ln P_{t-k}$				
6-3	$\ln c$	-1.556	0.112	0.544	0.189	-0.105	0.582		.9993	.3406	1.735
		(13.28)	(1.07)	(5.48)	(4.50)	(0.69)	(4.56)				
			$\ln \hat{y}_t$	$\ln y_{t-i}$	$\ln w_t$		$\ln P_{t-k}$				
6-4	$\ln c$	-1.594	0.181	0.479	0.173		0.543		.9993	.3486	1.714
		(14.71)	(1.93)	(5.63)	(4.15)		(5.94)				

Figure 2. Distributions of Coefficients of $\ln(y/yt)_{t-i}$ and $\ln(P/Pt)_{t-k}$ in Equation 6-1, Table 6



The price lag in Figure 2 is shaped roughly similar to that of equation (18) except that now the coefficient of $\ln P_t$ is only slightly larger than that of $\ln P_{t-1}$, and the sum of the η_k coefficients is smaller -- 0.33 vs. 0.54. This decrease in the sum reflects both the large difference in the $\ln P_t$ coefficients and slightly smaller coefficients of the P_{t-1} to P_{t-6} variables in the deviations-from-trend version. This would seem to indicate that, due to trend, the current price term is more important in explaining consumption along trend, while the lagged price terms play a more substantial role in explaining consumption's deviations from, as well as its movements along, trend.

The deviations-from-trend version of the money-illusion consumption function thus strongly confirms the independent significance of the price level in the determination of real consumption. Prices are independently important in explaining both trend consumption and fluctuations of consumption from trend. Next we turn to the question of the influence of factors related to the business cycle.

Price Movements and the GNP Gap

Another potential source of significance of the price term in the consumption function, besides the existence of money illusion in the true function or price-expectations mechanisms, could be the presumed correlation of price movements with employment and distributional factors in the business cycle. It might be possible, for example, that as aggregate demand rises relative to potential output and unemployment falls, prices rise. The falling unemployment rate could increase the income of low-income families with higher-than-average consumption propensities, shifting the aggregate consumption function (of income and wealth alone) up. If this movement were generally associated with rising prices, and vice versa, we might find prices significant in the consumption function due only to this distributional effect associated with a diminishing GNP gap.

Two points can be made to counter this hypothesis. First, balancing the increased income at the lower end of the income distribution is the well-known tendency for the profit share to rise in a cyclical upswing, shifting income to families with presumably lower-than-average consumption propensities.[19] Thus the expected direction of the consumption function shift as GNP rises relative to potential is ambiguous, depending on the relative importance of these various shifts in income distribution.

Second, the correlation between price movements and the GNP gap (= actual/potential real GNP) is not all that clear in the period over which our consumption function was estimated. The period 1955 IV to 1958 II saw actual/potential GNP <u>fall</u> from 1.05 to 0.94 while the CPI rose from 93.5 to 100.0, a 7.0% increase. In the period 1961 I to 1965 IV, however, while actual/potential GNP <u>rose</u> from .94 to 1.03 the CPI rose from 103.9 to 111.1, still only 7%.[20]

To directly test the hypothesis that our price terms only reflect cyclical effects, we re-estimated the money-illusion consumption function (18) adding the natural log of the ratio of actual real GNP to potential real GNP.[21] If the hypothesis is

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[19] See Kuh [16] for evidence on the cyclical behavior of the profit share.

[20] See Kuh [17] for a recent criticism of the Phillips' curve explanation of price level determination.

[21] Potential GNP was computed following the Council of Economic Adviser's formulation. See [9, pp. 60-63].

correct, the price term is merely a proxy for the GNP gap -- as actual GNP rises relative to potential, the consumption function shifts up and prices rise simultaneously. Inserting the gap into the equation should then greatly reduce the price term's significance and assign a significant coefficient to the gap term.

Equation 6-2 of Table 6 gives the results of this test. The variable y/yp is the ratio of actual real GNP to potential real GNP. The rejection of the cyclical hypothesis should be clear. The gap term has an insignificant coefficient with a t-ratio of only 0.70. The t-ratios of the other variables are somewhat lower than they are in the money-illusion consumption function (18), and the standard error of the equation is somewhat higher. The coefficients on income, wealth, and price are, however, not significantly different from their equation (18) values. In fact, the price coefficient is slightly higher, rather than lower as the cyclical hypothesis would suggest.

One might conjecture that the cause of the insignificance of the GNP gap in equation 6-2 is its collinearity with the price term. The fact that r^2 between y/yt and the CPI is just 0.22 in our sample period does not lend much support to this conjecture.

It seems clear that our results cannot be explained by cyclical coincidence or trend correlation. The last test performed in this section will search for possible simultaneous-equations bias.

Simultaneity Between Consumption, Income, and the CPI

Our consumption function is, of course, in reality part of a simultaneous system explaining consumption, income, and price determination. The part of the system that interests us here may be written schematically as:

$$
\begin{aligned}
c_t &= f_c(y,w,P) + \epsilon_{1t} \;; \\
P_t &= f_p(y,w,z,x) + \epsilon_{2t} \;; \qquad (20) \\
y_t &= f_y(c,z) + \epsilon_{3t} \quad .
\end{aligned}
$$

Here the equation for c_t is our consumption function with y, w, and P representing current and lagged values of labor income, wealth, and price; the P_t equation makes price a function of demand, which in turn depends on y, w, and exogenous demand variables z, and capacity, x, with these symbols also representing current and lagged values of the variables; finally, y_t is a function of c_t and the set of exogenous demand variables, z, which includes investment, government expenditure, and so on. The error terms ϵ_{it} are independent random variables.[22] All first partial derivatives of the f functions are positive except for $\partial f_p/\partial x$ which is negative.

Given the simultaneous model (20) a positive value of ϵ_{1t} will generate a positive c_t residual. This in turn will increase y_t and, through y_t, increase P_t. Thus the error term in the c_t equation may be positively correlated with the contemporaneous values of y and P in that equation, biasing upward the estimates of the coefficients of these contemporaneous terms and downward the estimates of all the other coefficients. Roughly speaking, this bias appears because the concurrent y and P terms seem to be "explaining" movements in c which are actually due to the random error ϵ_1.[23]

[22]This model is written only to illustrate the source of simultaneous-equations bias and is obviously not meant to describe the economy perfectly.

[23]For a complete treatment of the simultaneity problem see Malinvaud [18, Ch. 16]. Here we are only trying to sketch briefly the problem for the non-specialist.

In our case, we would strongly suspect an upward bias in the coefficient of ln y_t in the consumption function (18) because of the close connection of c_t and y_t through the third equation of the simultaneous model, (20), presented above. The coefficient of ln P_t is, however, less suspect because it is likely that prices in this quarter are mainly determined nonsimultaneously by events in previous quarters, through mark-up pricing procedures and the like. This would lead us to consider ln P_t as a predetermined, rather than a simultaneously determined, variable. In other words, the coefficient of y_t in the P_t equation may well be close to zero, breaking the price-consumption simultaneity. Furthermore, there is no reason to expect past income, past prices, or current wealth to be determined simultaneously with current consumption. Thus, the only potential simultaneous elements in our model are those between c_t, y_t, and P_t, and probably the only serious element is that between c_t and y_t.[24]

To test the extent of simultaneous equations bias, we must break the simultaneity in model (20). One way to do this is by introducing into the consumption function regression extraneous estimates of the concurrent values of the y_t and P_t variables so that the links from the equations for y_t and P_t back to the consumption function are broken. This can be done by regressing y and P on a set of exogenous variables, called instrumental variables, and replacing y_t and P_t in the consumption function regression by the estimates $\hat{y}_t$

[24]If the ϵ_1 term were serially correlated we would be concerned about lagged simultaneity. But the Durbin-Watson statistic of consumption function (18), 1.76, shows that this is not the case.

and $\hat{P}_t$ from these instrumental-variable equations.[25] The instrumental-variable equations relating y to current and lagged money supply, government expenditure, investment, and net exports, and relating P to current and lagged wage and profit rates are shown in the Appendix. The actual form of these equations is irrelevant; their only purpose is to break the link of simultaneity in (20).

Using our estimates of y_t and P_t, we re-estimated the money-illusion consumption function first replacing $\ln y_t, \ldots, \ln y_{t-6}$ by $\ln \hat{y}_t, \ln y_{t-1}, \ldots, \ln y_{t-6}$, and similarly replacing $\ln P_t, \ldots, \ln P_{t-6}$ by $\ln \hat{P}_t, \ln P_{t-1}, \ldots, \ln P_{t-6}$. The coefficients of income and price lagged t-1 to t-6 were estimated using the Almon technique while $\ln \hat{y}_t$ and $\ln \hat{P}_t$ were entered separately into the regression. This estimate is equation 6-3 in Table 6. The coefficients listed under $\ln y_{t-i}$ and $\ln P_{t-k}$ in 6-3 are the sums of the coefficients of the variables lagged 1 to 6 quarters.

In equation 6-3 the sum of the coefficients of ln y is 0.656, slightly less than the sum 0.694 in the consumption function estimate (18). The $\ln \hat{y}_t$ coefficient alone is 0.112 as opposed to 0.260 in (18). This comparison is not strictly legitimate since the $\ln y_t$ coefficient in (18) is constrained by the Almon estimation procedure while the coefficient of $\ln y_t$ in 6-3 is not.

The sum of the price coefficients in 6-3 is 0.477, again slightly lower than the sum of 0.542 of equation (18). The coefficient of $\ln \hat{P}_t$ is insignificant and negative, while that of $\ln P_t$

---

[25] See Malinvaud [18, pp. 604-608] for a discussion of the instrumental-variable technique.

in (18) was insignificant and positive. The fit of 6-3 is worse than that of equation (18), and the Durbin-Watson statistic is slightly lower.[26]

Thus our first test of simultaneous-equations bias indicates that there may be a slight upward bias in the $\ln y_t$ and $\ln P_t$ coefficients and also in the coefficient sums in (18). But whatever bias exists is slight indeed with the coefficient sums in 6-3 both within one standard error of the sums in the basic equation (18).

Since we suggested earlier that the simultaneity between P_t and c_t may be quite weak, and since the coefficients of $\ln P_t$ in both 6-3 and (2) are insignificant, we performed a second test of simultaneous-equations bias. Equation 6-3 was reestimated with the $\ln \hat{P}_t$ variable excluded and the original price lag $\ln P_t, \ldots, \ln P_{t-6}$ included, testing only income simultaneity. The result, in equation 6-4, is similar to that of 6-3.

The sum of the income coefficients in 6-4 is 0.660, again insignificantly less than the 0.694 of the original (18), while the coefficient of $\ln \hat{y}_t$ is 0.181, somewhat less than the coefficient 0.260 of $\ln y_t$ in (18). The sum of the price coefficients is insignificantly larger in 6-4 than in (18), with a coefficient of $\ln P_t$ of 0.178 in 6-4 as opposed to 0.130 in (18). This is as might be expected, since if P and y are at all correlated, removal of the simultaneity between y_t and c_t would raise the P_t coefficient.

---

[26] Of course, the R^2 and D statistics and all the standard errors of 6-3 are equal to those of the true equation only asymptotically, since we are using y and P estimates which are only asymptotically equal to the true y and P values. But with R^2 of .99+ on the instrumental variable equations, our standard errors are probably quite close to correct.

This test of simultaneity bias has indicated that there may be a slight, but statistically insignificant, upward bias in our estimates of the coefficients of concurrent income and price in the money-illusion consumption function (18). But since the bias appears to be so small as to be insignificant it can probably be ignored quantitatively and certainly does not affect our principal qualitative conclusion: the price level has an independent effect on real consumption due to what is commonly called money illusion.

This conclusion drawn from the Section IV estimates of the consumption function has withstood the series of tests to which we have subjected it in Sections V and VI. We now turn to the implications of this conclusion for macroeconomic stability and labor market behavior.

VII. Conclusion: Some Implications of the Money-Illusion Consumption Function

The principal result of this paper -- that a significant degree of money illusion appears in the aggregate consumption function for the United States -- has a number of interesting implications for macroeconomic theory and policy. Two issues upon which it has an important bearing are the degree of stability of the economy and the nature of the aggregate labor supply function. In this section we examine the implications of our conclusion for these two questions.

The Implications for Macroeconomic Stability

It is intuitively clear that the presence of money illusion ($0 \leqq \alpha < 1$ in $P_I = P^{\alpha}$) makes the economy less stable than it would be in the absence of money illusion. Consider, for example, an autonomous increase in demand with output fixed. Suppose that consumers' behavior is marked by money illusion while investors' and government's demands are free of money illusion. The autonomous rise in demand will lead to a rise in prices. This price increase will reduce the level of all real demands <u>but</u> it will not decrease real consumption demand as much in the presence of money illusion as it would in the absence of money illusion. Prices will have to increase more than when there is no money illusion, and hence the economy may be more prone to enter an inflationary spiral in the presence of money illusion than in its absence.

To make the point somewhat clearer, consider a very simple model of an economy in an inflationary process. Suppose the economy is on a balanced-growth path with the full-employment level of output, investment, and government spending all growing at rate φ. It is assumed that government spending is entirely for the purpose of public capital formation. All nonconsumption demands, namely, the sum of government and investment demands, are met in real terms. By the assumed nature of government spending, the sum of government and investment demands -- denoted z -- is the net addition to the economy's capital stock. The consumption function is taken to be a linear life-cycle model with no constant term but with the money-illusion parameter α appearing as the exponent of the true price level. Individuals hold money only for transactions purposes while the government follows a completely passive monetary policy, issuing as much money as is needed to support the economy's demands for real output. The wealth entering the consumption function consists of securities issued in a one-to-one correspondence with additions to the capital stock by government or the private sector. The inflationary gap in the economy is equal to the demand for output in real terms <u>minus</u> the supply of output in real terms. Following the traditional gap model, we assume that the increase in the price level is directly proportional to the size of the inflationary gap.[27]

The "story" of the model then is as follows. Government and private investors decide on an amount of investment and government spending, z, which grows exponentially at the rate φ . Consumers observe their perceived income and perceived wealth, and they decide upon a level of real consumption demand, c.[28] The demands on output, c+z, however, exceed the available output. Someone must be disappointed, and it is assumed, in this model, that consumers' demands are not met fully. Investment and government demands are met in real terms and what remains of real output goes to real consumption. At the same time consumers receive securities equal to the nominal, or money, value of the net additions to the capital stock. That is, the increment to consumers' nominal wealth is the nominal value of <u>ex post</u> saving. The price level rises at a rate proportional to the gap.

In equation form, the model can be summarized by equations (21)-(25).

$$(21) \qquad Y_t \equiv P_t y_t = P_t y_0 e^{\varphi t} \quad .$$

---

[27] See, for example, Bronfenbrenner and Holzman [8, pp. 604-605].

[28] We are using the same notational convention as above, with upper-case letters denoting nominal values and lower-case letters denoting real values.

$$c_t = \beta_1 \left(\frac{Y}{P^{\alpha}} \right)_t + \beta_2 \left(\frac{W}{P^{\alpha}} \right)_t = \beta_1 y_0 e^{\varphi t} P_t^{1-\alpha} + \beta_2 W_t P_t^{-\alpha} . \tag{22}$$

$$z_t = z_0 e^{\varphi t} . \tag{23}$$

$$\frac{dP}{dt} = \lambda\{c_t + z_t - y_t\} = \lambda\{\beta_1 y_0 e^{\varphi t} P_t^{1-\alpha} + \beta_2 W_t P_t^{-\alpha} + z_0 e^{\varphi t} - y_0 e^{\varphi t}\} . \tag{24}$$

$$\frac{dW}{dt} = P_t z_t = P_t z_0 e^{\varphi t} . \tag{25}$$

The first question one would want to answer about such a model is whether or not there exists a constant equilibrium price P_e, that is, a P_e such that $\frac{dP}{dt} = 0$. The answer is that there exists a constant equilibrium price if and only if $\alpha \neq 1$ and it is then equal to

$$P_e = \left[\frac{y_0 - z_0}{\beta_1 y_0 + \beta_2 \frac{z_0}{\varphi}} \right]^{\frac{1}{1-\alpha}} . \tag{26}$$

Unless there is some misperception of the price level a constant equilibrium price level does not exist.

The next important question to be asked is under what conditions will the economy's price level P_t approach P_e as time approaches infinity, that is, under what conditions is the equilibrium price level a stable one? Using a) the expression for the equilibrium price in (26), b) the fact that the real wealth of the economy, accurately perceived, is

(27) $$w_t = \int_0^t z(\tau)d\tau = \frac{1}{\varphi} z_0 e^{\varphi t} \quad ,$$

and c) the fact that if the economy is in equilibrium from $\tau = 0$ to $\tau = t$ its aggregate nominal wealth is

(28) $$W_t = P_e \frac{1}{\varphi} z_0 e^{\varphi t} \quad ,$$

it can be shown that

(29) $$\text{sign} \frac{dP}{dt} = \text{sign} \, [P_t^{1-\alpha} - P_e^{1-\alpha}] \quad .$$

It is not difficult to show, using (29) that if $P_0 \neq P_e$ then P_t approaches P_e as $t \to \infty$ if and only if $\alpha > 1$.[29] That is, if and only if consumers overestimate the true price level, and hence understate the true value of their real income and real wealth, will the economy's equilibrium price level be a stable one in this very simple model.

For the relationship between the degree of money illusion, measured by $1-\alpha$, and stability, we have from (24),

(30) $$\frac{\partial \left(\frac{dP}{dt} \right)}{\partial \alpha} = -\lambda \ln P_t \{\beta_1 y_0 e^{\varphi t} P_t^{1-\alpha} + \beta_2 W_t P_t^{-\alpha}\} < 0 \; .$$

As α increases, the rate of change of prices decreases. Hence, the more consumers underestimate the price level and overestimate the

[29] Our results of Section III are interpreted, in the simple model, as showing that α is significantly <u>less</u> than one.

true real value of their income and wealth, that is, "suffer" from money illusion, the faster prices increase.

It should be clear that the model we have just discussed is a tremendously oversimplified version of any true model of the U. S. economy.[30] All stabilizing influences and cushioning features have been omitted. For example, an active monetary policy would not help to feed the inflation the way the monetary authority of the model does. In addition, all interest-rate effects and all influences of world trade and the like have been omitted. Hence, our conclusion is not that the money illusion consumption function we found to fit the U. S. data for 1955 I-1965 IV implies that the economy is unstable. Rather our conclusion only concerns the degree of instability in the economy, and our empirical results taken in conjunction with the theoretical results just presented suggest that the economy is less stable than a model with no money illusion in the consumption function would suggest.

Implications for the Aggregate Labor Supply Function

Here we will briefly discuss the implications of the existence of money illusion for the labor supply function and, in turn, for the question of whether monetary and fiscal policy will affect equilibrium employment in a theoretical, static, general equilibrium macroeconomic model. Tobin has commented,

> ... if wage-earners are victims of a "money illusion" when they act as sellers of labor, why should they be expected to become "rational" when they come into the

[30] The same qualitative results obtain, however, if the model used here is replaced by the type of model Ackley discusses [1,p.431] in which there is no unsatisfied demand, but spending lags behind the receipt of income.

> market as consumers? Most of the reasons which compel them to behave non-rationally in making money wage bargains would logically compel them to act "non-rationally" as consumers.[31]

If Tobin's observation can be inverted, it suggests that the presence of money illusion in the consumption function implies its presence in the labor supply function. In this case, the aggregate supply of labor would not be a function of the real wage, but would react differently to changes in wage rates and the price level.

In a perfectly competitive static model with flexible prices and wages, consistent savings and investment behavior, and no liquidity traps, a labor supply function with no money illusion fixes employment, the real wage, and output in the labor market, leaving the interest rate and price level to be determined by the commodity and money markets.[32]

If the labor supply function reflects money illusion, however, the level of employment and output is determined simultaneously with interest rates and the price level, eliminating the separability of the no-money-illusion model, and giving monetary and fiscal policy a role in determining the equilibrium level of employment.

To make these points more clear, consider a simple static model of equilibrium in the commodity, money, and labor markets.[33]

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[31]Tobin [25, p. 222].

[32]This formidable list of assumptions permits us to focus on the question of the nature of the labor supply function. It should be noted that nowhere in the analysis does any "Keynesian" wage or price rigidity appear. Throughout the analysis we will assume a stable static equilibrium solution exists.

[33]This model is a composite of those presented in Patinkin [22, Ch.XI] and Allen [2, Ch. 7].

Equilibrium in the commodity market is described by

$$(31) \qquad q = c(q) + i(r) + g \quad ,$$

where q is national income, c is consumption expenditure, i is investment expenditure, a function of r, the interest rate, and g is government expenditure, all in real terms.[34]

Money market equilibrium is described by

$$(32) \qquad M/P = L(r,q) \quad ,$$

where M is the exogenously determined money supply and P is the price level.[35] Real output and employment, N, are linked by a production function,

$$(33) \qquad q = q(N); \quad q'(N) > 0; \quad q''(N) < 0 \quad ,$$

which assumes a fixed technology and capital stock.

In a competitive economy, the demand for labor equates the wage rate to the marginal revenue product of labor, so that labor demand can be written as a function for the demand real wage (that offered by the employers),

$$(34) \qquad W/P = q'(N) \quad ,$$

---

[34] y was used above for labor income, so we use q here for national income in place of the customary y. As usual, $dc/dq > 0$ and $di/dr < 0$.

[35] Equation (32) is a condensed version of Patinkin's equation (1) [22, p. 252]. We assume $\partial L/\partial r < 0$ and $\partial L/\partial q > 0$.

where W is the money wage rate.

If the labor supply function exhibits no money illusion, then the supply real wage is given by

$$(35) \qquad W/P = f(N); \quad f'(N) > 0.$$

Then equating the labor demand and supply wages gives

$$(36) \qquad f(N) = q'(N) \quad ,$$

as the labor market equilibrium condition determining N and W/P quite independently of (31) and (32). This fixes q through (33), and (31) and (32) then determine the price level, the interest rate, and thus the level of investment. In this model, as Patinkin has shown, a monetary policy change in M will affect only the price level and a fiscal policy change in g will affect both the price level and interest rate.[36] But neither can change the level of employment, which is fixed in the labor market.

Now if the labor supply function is replaced by a money-illusion supply function giving the supply wage as

$$(37) \qquad W = h(N,P); \ \partial h/\partial N \text{ and } \partial h/\partial P > 0,$$

then the labor market equilibrium condition equating the supply W in (34), and the demand W in (36) is

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[36] See Patinkin [22, Ch. X] and Allen [2, Ch. 6].

(38) $g(N,P) = Pq'(N)$.

Here the labor market has been linked to the commodity and money markets (31) and (32) by the price variable, P. Substituting q(N) for q in (31) and (32), we see that (31), (32), and (38) now are a truly simultaneous set of equations in the variables r, N, and P. No longer is employment independently determined in the labor market, and now monetary policy increases in M will raise equilibrium N, as will fiscal policy increases in g.

Thus the introduction of money illusion into the labor supply function changes the standard macroeconomic static general equilibrium model from one with output fixed independently in the labor market to a completely simultaneous model with the level of output and employment affected by all the parameters of the system.

In closing, there are a number of lines of further investigation that our results suggest might be fruitful to pursue. First, it would be interesting to disaggregate consumption expenditure and investigate such subaggregates as real personal consumer expenditures on durables and real personal consumer expenditures on nondurables and services using a money-illusion specification of the respective demand functions. Second, it would be most useful to introduce the money-illusion consumption function into a complete simultaneous-equation model, observe its performance in such a model, and observe the implications for stability as viewed through simulation experiments. The results presented in this paper suggest that in constructing such macro-models greater attention should be paid to the link between the price-wage sector and the expenditure sector. Last, it would be interesting to investigate directly the presence or absence of money illusion in other aggregate functions as, for example, the labor supply function.

Appendix: Instrumental Variable Equations and Data Listing

In Section VI the money illusion consumption function was re-estimated with instrumental variable estimates of **the consumer price index, P_t,** and real net labor income, y_t, substituted for the actual series in the unlagged terms. While in principle the form of the instrumental equations used to construct the estimates should be irrelevant since the sole purpose of the technique is to break the simultaneity between c_t, y_t, and P_t (and clearly <u>not</u> to estimate a behavior function or structural equation for y_t or P_t), we will show here the estimated equations for y_t and P_t that were used to construct $\hat{y}_t$ and $\hat{P}_t$. Then we will conclude with a listing of the basic c, y, w, and P series used in the study.

I. Instrumental Variable Equations for y and P

The instrumental variable equation for real net labor income, y_t, has y_t as a function of current and lagged values of the money supply, real government expenditure, real gross private domestic investment and real net exports. In linear estimating form the equation is

$$\text{(A-1)} \qquad y_t = \alpha + \sum_{i=0}^{I} \beta_k G_{t-i} + \sum_{j=0}^{J} \gamma_j I_{t-j} + \sum_{k=0}^{K} \delta_k X_{t-k} + \sum_{m=0}^{M} \eta_m M_{t-m} + \epsilon_t \ .$$

where G is government expenditure, I is gross private domestic investment, and X is net exports, all in billions of 1958 dollars from the Survey of Current Business, and M is the money supply -- currency plus demand deposits in billions of dollars -- from the Federal Reserve Bulletin.

As it turns out, the estimated version of (A-1) used to construct $\hat{y}_t$ includes only current values of G and X, I lagged 0, 1, and 2 quarters, and M lagged 0-12 quarters with the coeff-cients estimated using a third degree Almon lag. The estimated equation is

$$\text{(A-2)} \quad y_t = \underset{(19.38)}{-238.53} + \underset{(4.59)}{0.423}\, G_t + \sum_{j=0}^{3} \gamma_j I_{t-j} + \underset{(5.23)}{0.838}\, X_t + \sum_{m=0}^{12} \eta_m M_{t-m} .$$

R^2 = .9976; S. E. = 1.78; Mean = \$309.27 billion; D = 1.13.
Period of fit: 1955 I-1965 IV.

The γ_i coefficients of lagged investment and the η_m coefficients of lagged money supply are shown in Table A-1. The coefficients of equation (A-2) were used to compute $\hat{y}_t$.

The instrumental variable equation for P_t, the CPI, has P_t as a function of lagged wage rates, W, the average hourly earnings of manufacturing workers in dollars from the Monthly Labor Review, and profit rate, R, the average rate of profit on stock-holders' equity from the Federal Trade Commission Quarterly Financial Reports on Manufacturing Corporations. The estimated instrumental variable equation is

Table A-1: Coefficients of I_{t-j} and M_{t-m} in Instrumental Variable Equation (A-2) for y

Lag (j,m)	Coefficient of I_{t-j}	Coefficient of M_{t-m}
0	0.423 (4.59)*	0.582 (4.01)
1	0.155 (1.41)	0.290 (4.33)
2	0.204 (2.16)	0.123 (2.20)
3	0	0.054 (0.79)
4	.	0.062 (0.91)
5	.	0.121 (2.16)
6	.	0.209 (5.22)
7	.	0.302 (7.95)
8	.	0.375 (7.21)
9	.	0.405 (6.23)
10	.	0.368 (5.56)
11	.	0.241 (5.12)
12	0	0

* The numbers in parentheses are the t-ratios of the coefficients.

(A-3)
$$P_t = \underset{(58.14)}{52.56} + \underset{(0.61)}{3.006}\, W_{t-1} + \underset{(2.31)}{9.860}\, W_{t-2} + \underset{(3.24)}{7.973}\, W_{t-3}$$
$$+ \underset{(0.77)}{2.851}\, W_{t-4} - \underset{(3.02)}{0.138}\, R_{t-1},$$

R^2 = .9929; S. E. = 0.47; Mean = 101.95; D = 0.83.
Period of fit: 1955 I-1965 IV.

The coefficients of equation (A-3) were used to compute the $\hat{P}_t$ series used in the text.

II. Data Listing

The following table lists the basic consumption, net labor income, net worth of households, and CPI data used in the study. The first column lists an identification number composed of the last two digits of the year and the quarter. Column 2 lists real consumption; column 3 lists real net labor income. These two series are unpublished data of Harold Shapiro. Column 4 lists the real net worth of households, from Albert Ando. Columns 2-4 are in billions of 1958 dollars. Column 5 lists the consumer price index (1958 = 100).

DATA

(1)	(2)	(3)	(4)	(5)
531	249	255	1221	93.1
532	252	257	1213	92.6
533	251	258	1215	93.5
534	249	257	1225	93.8
541	249	260	1246	93.8
542	249	262	1272	93.8
543	251	265	1298	93.7
544	257	269	1320	93.8
551	259	272	1349	93.3
552	264	276	1373	93.1
553	268	279	1406	93.1
554	273	284	1419	93.5
561	276	287	1444	93.7
562	280	289	1462	93.4
563	279	291	1473	94.1
564	282	294	1468	95.4
571	283	296	1466	96.0
572	284	298	1484	96.7
573	284	301	1482	97.6
574	282	302	1464	97.8
581	278	301	1474	99.0
582	278	304	1501	100.0
583	283	308	1531	100.7
584	287	310	1567	100.9
591	291	315	1599	100.9
592	297	318	1619	100.8
593	295	321	1626	101.2
594	297	323	1623	101.8
601	301	325	1629	102.3
602	303	330	1636	102.3
603	304	330	1642	103.0
604	302	331	1639	103.2
611	304	333	1674	103.8
612	307	336	1711	103.9
613	311	339	1719	103.9
614	316	343	1748	104.4
621	321	346	1754	104.6
622	323	349	1723	105.9
623	324	353	1702	105.7
624	325	357	1721	105.7
631	329	360	1761	104.8
632	330	362	1792	106.1
633	333	367	1809	106.3
634	337	369	1827	107.1
641	347	376	1878	107.4
642	351	380	1905	107.7
643	357	388	1929	107.9
644	362	392	1951	108.3
651	367	395	1973	108.7
652	371	402	1978	108.9
653	379	407	1981	109.7
654	384	414	2015	111.1

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