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Some Experimental Non-Constant-Sum Games Revisited. Part II

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1. Introduction

In a previous paper[1/] we reported our analysis of a series of 2x2 payoff matrix two-person non-constant-sum game experiments in which subjects knew only their own payoff matrix entries and not those of their opponents. In contrast with the earlier paper, which analyzed the effectiveness of five standard game-theoretic solutions as predictors of observed outcomes in such situations, the present paper suggests new theory and measurements of behavior that are better suited to game environments in which players do not know their opponent's payoffs. This paper examines the indirect effects of the information conditions present during the play of the games; we put off discussion of the direct consequences until Part III. Although the papers are highly related, inasmuch as they deal with the same set of experiments, they may be read completely separately as they analyze different aspects of the games.

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1.1 Form of the Experiments[2/]

Figure 1 shows the six games used in the experiments. In each game the subject labeled "Player 1" chose the row and the subject labeled "Player 2" the column; the labeling of the players was arbitrary. In each cell of a matrix is first the payoff to player 1 and second the payoff to player 2 resulting from the two players' corresponding strategy choices.

Game 1

6, 3	6, 7
10, 3	10, 7

Game 2

1, 3	2, 3
1, 1	2, 1

Game 3

2, 1	-1, -1
-1, -1	1, 2

Game 4

3, 3	-1, -1
-1, -1	2, 2

Game 5

3, 3	-2, 7
7, -2	-1, -1

Game 6

5, 2	-10,-13
4, 1	-20,-23

Figure 1

In our experiments subjects did not see the matrices shown in Figure 1 but instead say only their own payoffs, shown as the first two columns of matrices in Figure 2, where Player 1 chooses a row and Player 2 a column.

For certain purposes in this discussion it will be convenient to think of both players as choosing rows; for Player 1 this involves no change, but for Player 2 it implies transposition of the matrix (reflection about the main diagonal), as shown in Column (3) of Figure 2. When a player's payoff matrix is displayed so that he chooses rows we say the matrix is in its <u>standard aspect</u>; when it is displayed in the manner in which it is actually played, we say it is in <u>game aspect</u>. For Player 1 standard and game aspects coincide; for Player 2 they generally do not.

	Player 1's Payoff Matrix (1)		Player 2's Payoff Matrix in Game Aspect (2)		Player 2's Payoff Matrix in Standard Aspect (3)	
Game 1	5	5	3	7	3	3
	10	10	3	7	7	7
Game 2	1	2	3	3	3	1
	1	2	1	1	3	1
Game 3	2	-1	1	-1	1	-1
	-1	1	-1	2	-1	2
Game 4	3	-1	3	-1	3	-1
	-1	2	-1	2	-1	2
Game 5	3	-2	3	7	3	-2
	7	-1	-2	-1	7	-2
Game 6	5	-10	2	-13	2	1
	4	-20	1	-23	-13	-23

Figure 2

1.2 Motivation of Subjects to Maximize Their Payoffs[3/]

No monetary rewards were used in any of these experiments to motivate the players to equate maximization of their scores with maximization of individual utility. However, most of the subjects were students of game theory in classes taught by one of the authors, who advised that these experiments would teach non-constant-sum game phenomena in an easily assimilable way. Furthermore the students were specifically instructed to maximize their scores. These students, eager to learn and eager to please their instructor, should have been well motivated subjects.

The experimenters may, however, have introduced a distracting element themselves by requesting that each subject after playing each game draw a 2x2 matrix and enter in it what he believed to be the ranking of his opponent's payoffs. This interference with the score-maximization motive resulted if a subject altered his strategy choices in order to gain information about his opponent's matrix. Further discussion of this is deferred to Part III.

2. Games With the Opponent's Matrix Not Known

We shall approach a theory of playing matrix games in which the opponent's payoff matrix is not known by considering from the point of view of the player various stages in the iterated play of such games.

2.1 The Initial Move: Theory

Initially such a player has only his own matrix, and his task is to choose a move in period one: strategy 1 or strategy 2. In the one period subgame where players move simultaneously, move and strategy are equivalent. Although it is possible even at this point for considerations of the effect of his own move on his opponent's thinking to enter his calculations, we shall, as a first approximation, assume that his first move is based entirely on efforts to maximize his own payoff.

Several sequential decision rules can be imagined which will determine a first move, depending on the structure of the player's payoff matrix. For example:

α Dominant Strategy Subrule: If the matrix contains a dominant strategy (in the strong sense), choose it.

β Bayesian Subrule: If α fails to select a strategy, choose the strategy with the highest expected value.[4]

γ Security Maximization Subrule: If β fails to select a strategy, choose the strategy with the highest minimum value; choose the strategy which maximizes the "security level."

δ Randomization Subrule: If γ fails to select a unique strategy, randomize.

A few examples are presented here to illustrate the use of the above Initial Move Decision Rule. In the following four matrices row 1 would be chosen by subrule α :

4	3
2	1

4	2
3	1

4	2
3	2

4	3
4	2

In the two matrices below subrule α does not select a strategy, but subrule β specifies row 2:

3	2
2	4

1	3
4	2

In this matrix no decision results until subrule γ is invoked, picking row 1:

2	3
1	4

Finally, the following matrices require the application of subrule δ :

3	4
3	4

3	4
4	3

3	3
3	3

The sequence $\alpha\beta\gamma\delta$ is a reasonable first-move rule but not the only one. A player might reasonably reverse the order of subrules β and γ , which would make a difference in selecting an initial row in the matrix

5	1
2	3

,

for example. An optimist might insert at some point in the decision sequence the rule:

ϵ Maxmax Subrule: Choose the row containing the highest entry.

This would dictate the choice of row 1 in the matrix

4	0
3	2

,

whereas subrules β and γ would both select row 2. Still another possibility is

ζ Minimax Mixed Strategy Subrule: Use a probabilistic device to choose the starting move, with probabilities of playing each row determined by treating the matrix as though it represented the payoff in a zero-sum game.

2.2 The Initial Move: Hypotheses and Results

Concerning the initial move we have two hypotheses:

Hyp. 1: In games in which the decision sequence $\alpha\beta\gamma\delta$ uniquely selects an initial strategy, players will use it.

Hyp. 2: In games in which subrule α uniquely selects an initial strategy, players are more likely to use the strategy picked by the decision sequence $\alpha\beta\gamma\delta$ than in games in which rules β or γ must be invoked to select an initial strategy.

Table 1 presents the essential information relevant to these hypotheses. Column (3) indicates for each game matrix of Figure 2 the initial strategy selected by the decision sequence $\alpha\beta\gamma\delta$; for Player 1 it is a row, for Player 2 a column. Column (4) shows which subrule in the sequence must be invoked to select an initial move uniquely; it is seen that the sequence fails to pick a preferred first move in Game 2.

In the six replications of the experiment were 33 teams or pairs of players, so that there were generally 33 observations of initial moves for each player-position in each game. Due to an experimental error it was necessary to eliminate the data from one of the teams in Game 2, and a team failed to play Game 6; thus for these games $N = 32$ (see Column (5)).

Column (6) shows the number of subjects that failed to play the strategy picked by the decision rule. Against Hypothesis 1 we place the null hypothesis that subjects are equally likely to pick either of the two available strategies in period 1. Out of 33 observations, H_0 says we should expect 16 or 17 to depart from the one picked by the decision sequence. The appropriate test is the Binomial Test;[5/] for this size sample we use the normal distribution to approximate the binomial distribution in determining the probabilities, shown in Column (7), that H_0 is true. It is seen that the null hypothesis can be rejected at the .01 significance level in nine out of ten cases, the exception being that of Player 2, Game 3.

Table 1 - Initial Move Results

Game	Player	Strategy Selected by Decision Sequence αβγδ	Subrule Responsible for Choice	Number of Observations	Number of Departures from Strategy Selected by Decision Sequence αβγδ	Hyp. 1: Probability that Null Hyp. is True	Hyp. 2: Probability that Null Hyp. is True
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	1	2	α	33	1	<.00001	.0034
	2	2	α	33	5	.00006	.1335
2	1	1 or 2	δ	32	-	-	-
	2	1 or 2	δ	32	-	-	-
3	1	1	β	33	8	.0037	-
	2	2	β	33	12	.0901	-
4	1	1	β	33	6	.00026	-
	2	1	β	33	7	.0009	-
5	1	2	α	33	4	.00001	.0655
	2	2	α	33	2	<.00001	.0104
6	1	1	α	32	2	<.00001	.0104
	2	1	α	32	0	<.00001	.0009

To test Hypothesis 2 we put forth the null hypothesis that the likelihood of observing departures from the strategy selected by the decision sequence in Games 1, 5 and 6 is not significantly less than in Games 3 and 4. The probability of observing a departure in Games 3 and 4 is $(8 + 12 + 6 + 7) \div (4)(33) = .25$. Column (8) shows, for example, that the likelihood of observing 1 departure out of 33 observations if H_0 is true (as with Game 1, Player 1) is .0034.

It is seen that in two instances the null hypothesis cannot be rejected at the .05 significance level, while in four instances it can.

The evidence is very strongly in favor of accepting Hypothesis 1 and fairly strongly in favor of accepting Hypothesis 2.

We are hardly surprised. In fact it is rather astonishing that there should be as many departures as there are from such a sensible decision rule; it is the existence of any departures that seems to call for explanation. Fortunately we can gain some insight from the subjects themselves. After the last replication of the experiment each subject was asked to report his rationale for his mode of play. There were in this replication four instances of departure -- two in Game 3 and two in Game 4.

One of the Game 3 departers confirmed a possibility mentioned earlier, that players might use subrules other than α , β , γ and δ ; in fact he used subrule ζ . He wrote:

> The fact that we tended toward using our second strategies rather than our firsts stems from my initial playing of a mixed strategy. It started with the philosophy, "I'll play as if this were a zero-sum game just to see what he'll do." This, unfortunately, gave him the opportunity to assert himself right from the beginning.

While one of the Game 4 departers simply called his move "careless," the other Game 3 departer, a Player 2, wrote a long explanation which we have edited for the sake of brevity and clarity:

"My first choice was irrational because I didn't pick the move that would maximize my gain and minimize my loss. But it is interesting to conjecture whether this move didn't help in the long run, [because it misled my opponent concerning my payoff matrix]. In this game, for both of us to make a positive gain, one of us would have to give up his best choice. My opponent succumbed to the second best choice more often than I did. If my opponent believed that I liked Column 1 best, as might be indicated by my first move, but then switched to Column 2 as my less preferred strategy, he may have believed that I was in the same situation has was in. Thus while I was actually maximizing, he may have believed I was taking second best [i.e., was satisficing; and therefore he may have found it easier to acquiesce to a second-best position for himself]."

The above reasoning suggests that our first approximation assumption, that players do not consider the effect on the opponent of their initial moves, must eventually be dropped.

3. More Theory: Toward A Taxonomy of 2x2 Matrix Games[6/]

3.1 Inferences About the Opponent's Payoff Matrix

We return again to the point of view of a player of one of these games. The second event of the game, after the initial move, is the receipt of information concerning which strategy the opponent picked at his first move. If the player assumes his opponent is acting according to the decision sequence $\alpha\beta\gamma\delta$, the player can make certain inferences about the matrix his opponent must be facing.

In particular, if $A > B > C > D$ and the opponent chooses Strategy 1, his matrix (in standard form as defined in Section 1.1) must have one of the following formats:

AA	AA	AA	AB	AB	AB	or any of these with
BC	BB	AB	CD	DC	CC	columns interchanged,
						if opponent used sub-
AB	AB	AB	AC	AB	AC	rule α to determine
BC	CB	BB	BD	AC	BC	his initial move.
AC	AB	AC				or any of these with
DB	CA	CB				columns interchanged,
						if subrule β was used.
AC	AB	BC	CB	BB		or any of these with
DB	CA	AD	AD	AC		columns interchanged,
						if subrule γ was used.
AB	AB	AA				or any of these with
BA	AB	AA				columns interchanged,
						if subrule δ was used.

Figure 3

What may be surprising at first is the great variety that still exists -- 40 distinct matrix format possibilities, using only ordinal entries (forty because two of the above matrices appear twice and two remain unchanged when rows are switched). But, as we shall see, this is a reduction from an initial set of 75 -- a noticeable improvement.

The subject begins to form a picture of what his opponent's payoff matrix looks like. During the course of play, as a result of the particular sequence of moves made by himself and his opponent and in consequence of their reactions to each other's moves, these pictures change. Upon request, at any point in the experiment, a subject could write down what he believes at that moment to be true about his opponent's matrix.

In our experiments, we asked for such a statement at the end of the play of each game. Thus we have no record of the manner in which the beliefs of subject concerning his opponent's matrix developed during the course of play. But we have his final beliefs, those based on the largest body of experience. The analysis of these "perceived opponent's matrices" will occupy us in Part III.

3.2 Motivations to Switch from Initial Strategy Choice

After the subject's initial move and the report of his opponent's move, what kinds of behavior are reasonable? Will he stick with his initial strategy or will he want to switch? The answer depends not only on his opponent's moves, but on the structure of his own payoff matrix. There are three distinguishible "long-run" reasons why one might switch strategies during the course of play, and the structure of a given payoff matrix may imply all three, two, one, or more of them. The three incentives to switch during play we call the maximizing incentive, the investment incentive and the signalling incentive. They are "long-run" incentives because their aim is to effect a stable solution which will endure throughout the play of the game.

Consider the matrix

A	C
D	B

with $A > B > C > D$.

By either the Bayesian Subrule β or the Security Maximization Subrule γ we choose row 1 initially, insuring a payoff no smaller than C and possibly as large as A . Suppose the opponent plays column 1. We recieve A and are perfectly satisfied; we have no incentive to switch. But if the opponent plays column 2 we receive only C . If the opponent will continue to play column 2 we can improve our payoff by switching to row 2; we will then receive B . Of course we take the risk that our own switch will induce him to switch to column 1, in which case after receiving B for one period we will receive D henceforth. But our focus at the moment is only on the fact that the structure of the matrix provides a possible incentive to switch rows, depending on the opponent's initial move. Contrast this matrix with

A	B
D	C

, which provides no such incentive. Row 1 maximizes with respect to either column the opponent chooses. We say the first matrix provides a "maximizing incentive" to switch, while the second does not.

Next, examine the matrix

A	C
B	D

. Row 1 dominates row 2, hence we pick it initially by subrule α , and we have no maximizing incentive to switch, even if the opponent picks column 2. But we do then have an "investment incentive," which we define as a switch which we hope <u>will</u> induce the opponent to switch. This is the "opposite" of the maximizing incentive, which we hope will result in the opponent's <u>not</u> switching. If an investment succeeds we will recieve B ; if it fails, D . The following matrix contains both maximizing and investment incentives to switch from row 1, which is selected as the initial strategy by the Maximizing Security Subrule, γ :

B	C
A	D

. If the opponent picks column 1 we maximize by switching to row 2. If he picks column 2 we can invest in row 2, and hope our investment will induce him to switch to column 1.

Finally, there can be a more complex incentive to switch strategies, or, more accurately, not to stick with the initial strategy always. In the matrix

A	B
C	D

row 1 dominates, so we pick it initially by subrule α . If the opponent chooses column 2 we have neither a maximizing nor an investment incentive to switch. Yet we are not altogether pleased to receive B . Rapoport and Guyer[7] use the term "aggrieved" to describe a player of a 2x2 matrix game who gets less than the maximum entry in the matrix, i.e., less than A, and call the player who does receive A "satisfied." Somehow we would like to communicate to the opponent that we would prefer him to play column 1. Presumably he chose column 2 initially for a good reason; yet, the structure of his matrix may be such that he can be induced to switch. Examples might be

A	A
B	B

,

A	B
D	C

,

A	C
D	B

,

A	B
C	B

. In each of these instances the opponent might reasonably choose column 2 initially (by the Randomizing Subrule δ in the first case, and by subrule γ or possibly β (depending on the numerical entries) in the other three cases); yet the opponent can be induced to switch. In the last three cases he has his own maximizing incentive to switch; in the first, he is presumably indifferent. Our own motivation in switching to row 2 is not to stick with it, but to disturb the <u>status quo</u>, to let the opponent know all is less than well at the present outcome (1,2) . Whether the message will get through will depend on the relative advantage or disadvantage of outcome (2,2) to the opponent, as well as on his own perceptiveness and understanding of a relatively recondite and sophisticated game tactic. Of course, if his matrix looks, for example, like this:

C	A
D	B

, no amount of signalling, no matter how well received and understood, will move him to switch to column 1.

The signalling motivation is itself ambiguous, because it can be either an attempt to induce cooperation by indicating joint interest or an attempt to lead the opponent astray. This problem has been considered from the viewpoint of a noncooperative solution combined with an _a priori_ distribution on what might be the structure of the payoff matrix of the opponent by Aumann, Harsanyi and Maschler.[8]

A rough-and-ready comparison of the three motivations to switch suggests the notion that maximizing is a stronger incentive than investment, which in turn is stronger than signalling. This judgment is based simply on degree of obviousness of the tactic and on the number of periods required before the desired outcome can be experienced. The maximizing outcome will be experienced immediately, though only for one period if opponent is thereby induced to switch. The investment outcome will not be experienced for at least one period -- if at all -- for it will take that long for opponent to react to the switch. The signalling outcome may not be experienced for one or two periods -- at least one, so that opponent can switch, and one more if we wait to see whether opponent will switch before we switch back.

If a switch fails to accomplish its desired end, it may or may not result in a lowered payoff. Consider these two matrices:

A	C
D	B

,

A	C
C	B

. In the first we choose row 1 by the Bayesian Subrule, β or by the Security Maximization Subrule, γ ; in the second we choose row 1 by Subrule β . In either case if opponent plays column 2 we have an incentive to maximize by switching to row 2. But in the first case, if opponent switches to column 1 in consequence of our switch, we are worse off then if we had not switched; while in the second case we are not. We say in the second case that maximizing is costless, while in the first case it is not. Clearly a costless tactic is more attractive than a costly one; thus the second matrix provides a greater switching incentive than the first. Signalling we define as costless if neither of the intermediate outcomes is less than the initial outcome.

3.3 A Table Classifying 2x2 Matrices

Table 2 presents a taxonomy of 2x2 matrices based on the above discussion. Matrix entries are specified ordinally, with $A > B > C > D$. Twenty-one structurally distinct matrices are displayed in Column (2). Two matrices are considered structurally identical or indistinct if one can be transformed into the other by interchanging rows and/or columns. Generally a matrix is associated with three structurally identical ones, making altogether four forms of each matrix; although four of the 21 matrices shown have fewer distinct forms, as indicated in Column (14), so that there are altogether 75 matrix forms instead of 84. Since any of the 21 structurally distinct matrices may be paired with any of the 75 distinct matrix forms to form a distinct game, we may conclude that there are 1575 distinct 2x2 games.

In the table the player is assumed to choose initially row 1 of the matrix shown in Column (2) by the decision rule indicated in Column (3). The preferred payoff of the preferred initial strategy is always outcome (1,1). Columns (4)-(9) show for each matrix which incentives to switch from the initial strategy choice exist and which are costless. A "Yes" in parentheses means that even if the switch is successful the player will not be "satisfied," but will still be "aggrieved" -- that is, he will still not be receiving the payoff A .

On the basis of Columns (3)-(9) the 21 matrices are divided into twelve numbered types, as indicated in Column (1). In Column (10) of the table is a number M which attempts to quantify the degree to which a player may be tempted, because of the structure of his own payoff matrix, to switch away from his initial strategy. The measure is unashamedly *ad hoc*, but though based upon intuition it does not altogether outrage reason. M is computed as follows:

Source of Contribution	Increment to M
Initial Strategy selected by α	0
Initial Strategy selected by β or γ	1
Initial Strategy selected by δ	2
Matrix offers maximizing incentive to switch	3
Matrix offers investment incentive to switch	2
Matrix offers signalling incentive to switch	1
Added for each costless incentive	1

It will be seen that the matrices are listed in order of increasing M . The maximum possible M is 12.

In Columns (11) and (12) are shown the maximizing and investment tactics. The payoff above is the one received before the switch to row 2. The payoff in parentheses is received if the tactic is unsuccessful; the remaining payoff is the one received if the tactic succeeds.

Column (13) provides a quick look at the number of distinct entries in each matrix. Six of the matrices have four distinct entries, nine have three, five have two and one has one.

Table 2 - Taxonomy of 2x2 Ordinal Matrices

Type Number Identification Letter	Matrix	Subrule ⊠ Selecting Row 1 as Initial Strategy Choice	Incentive to Switch from Initial Strategy Choice: Maximizing — Exists	Maximizing — Is Costless	Investment — Exists	Investment — Is Costless	Signalling — Exists	Signalling — Is Costless	Overall Measure of Incentive to ⊕ Switch	Tactics: Maximizing	Tactics: Investment	Number of Distinct Entries in Matrix	Number of Distinct Forms of Matrix
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1a	AA BC	α	No	No	No	No	No	No	0			3	4
1b	AA BB	α	No	No	No	No	No	No	0			2	2
1c	AA AB	α	No	No	No	No	No	No	0			2	4
2a	AB CD	α	No	No	No	No	Yes	No	1			4	4
2b	AB DC	α	No	No	No	No	Yes	No	1			4	4
2c	AB CC	α	No	No	No	No	Yes	No	1			3	4
2d	AB BC	α	No	No	No	No	Yes	No	1			3	4
2e	AB CB	α	No	No	No	No	Yes	No	1			3	4
3	AB BB	α	No	No	No	No	Yes	Yes	2			2	4
4a	AC BD	α	No	No	(Yes)	No	Yes	No	3		C B (D)	4	4
4b	AB AC	α	No	No	Yes	No	Yes	No	3		B A (D)	3	4

Table 2 (Continued)

Type Number Identification Letter (1)	Matrix (2)	Subrule ⊛ Selecting Row 1 as Initial Strategy Choice (3)	Incentive to Switch from Initial Strategy Choice: Maximizing: Exists (4)	Maximizing: Is Costless (5)	Investment: Exists (6)	Investment: Is Costless (7)	Signalling: Exists (8)	Signalling: Is Costless (9)	Overall Measure of Incentive to ⊕ Switch (10)	Tactics: Maximizing (11)	Tactics: Investment (12)	Number of Distinct Entries in Matrix (13)	Number of Distinct Forms of Matrix (14)
5	AC BC	α	No	No	(Yes)	Yes	Yes	Yes	5		C B (C)	3	4
6a	AC DB	β & γ	(Yes)	No	No	No	Yes	No	5	C (D)B		4	4
6b	AB CA	β & γ	Yes	No	No	No	Yes	No	5	B (C)A		3	4
7	BB AC	γ⊛	Yes	No	Yes	No	No	No	6 or 7⊛	B A(C)	B A (C)	3	4
8	AC CB	β	(Yes)	Yes	No	No	Yes	Yes	7	C (C)B		3	4
9a	BC AD	γ⊛	Yes	No	Yes	No	(Yes)	No	7 or 8⊛	B A(D)	C A (D)	4	4
9b	CB AD	γ⊛	Yes	No	Yes	No	(Yes)	No	7 or 8⊛	C A(D)	B A (D)	4	4
10	AB AB	δ	No	No	Yes☆	Yes	Yes☆	Yes	8		B☆ A (B)	2	2
11	AB BA	δ	Yes☆	Yes	No	No	Yes☆	Yes	9	B☆ (B)A		2	2
[illegible]	AA AA	δ	Yes*☆	Yes	Yes*☆	Yes	Yes*☆	Yes	12 or 6☆	A☆ (A)A	A☆ A(A)	1	1

Notes to Table 2

⊠ Subrules are:

α Row 1 is a dominant strategy.

β Row 1 maximizes expected value on the Bayesian assumption that opponent is equally likely to play either column.

γ Row 1 maximizes security level.

δ Rows 1 and 2 are equally likely to be chosen by a random device (neither row is preferred _a priori_).

(#) This is explained in the text.

⊛ With ordinal matrix entries subrule β is inconclusive. One point is added to the measure of incentive to switch when β selects a different initial strategy than γ .

☆ The starred incentives and tactics exist in both rows of the matrix.

* The incentives exist, technically, but they are not really attractive because nothing is to be gained by switching (nothing is lost either). The measure of incentive to switch is 12 if the three asterisked yeses are counted, 6 if they are not.

It will be convenient to have a system for referring to the matrices classified by Table 2. Using type 2a matrices for the example, Figure 4 illustrates the four forms a standard aspect matrix can take and the way we shall label them henceforth.

MATRIX	A B C D	B A D C	C D A B	D C B A
FORM	Type 2a matrix as shown in Table 2	Columns interchanged	Rows interchanged	Rows and Columns interchanged
CODE	2a-11	2a-12	2a-21	2a-22

Figure 4

Thus the first number and the letter (if any) refer to the matrix type according to Table 2, the digit after the hyphen indicates which row is chosen initially, and the final digit tells whether the highest entry of the initially chosen row is in column 1 or in column 2. The matrices with less than four forms (see Column (14) of Table 2) have x's in place of one or both digits following the hyphen. In the cases of matrices of types 7, 9a and 9b, the symbols ! and ? are used as mnemonics instead of the row digits 1 and 2, in order to remind the reader that in these matrices Subrules β and γ may select different initial strategies depending on which cardinal values appear as matrix entries (see **note** to Table 10 and text in Section 3.1.1). A few more examples should provide sufficient clarification:

B	B
A	A

1b-2x

B	A
B	A

11-x2

A	A
A	A

12-xx

D	A
C	B

9a-?2

B	B
A	C

7-!1

3.4 Characteristics of Games Formed by Pairing Two 2x2 Matrices

The title of this part of the paper promises more than will be delivered. One of the authors is currently attempting to develop ways of classifying the structural characteristics of the 1575 distinct games that can be formed by pairing 2x2 matrices.

Among the questions being investigated is, "Which games promote steady states and which do not?" While Rapoport and Guyer[9] can answer in terms of threats, inducements and forcing moves, these tactics are hard to apply when the opponent's payoffs are not known. Our analysis is founded on maximizing, investment and signalling tactics and their costs. The distinction may be seen as "other-directed" versus inner-directed motivations.

While we are not yet prepared to present a complete taxonomy of all 1575 games, the remarks that we make in the next section about our six games not only provide conceptual ballast for the discussion of our data but also suggest the kinds of approaches that can be followed in developing the more complete classification scheme.[10]

4. Dynamics of Play: Theory and Results

Our method in this section is to select a particular phenomenon that emerges from the iterated play of 2x2 matrix games and attempt to gain insight into it by examining the structures of those games that illustrate it best, after which we look at the relevant data. To aid in this purpose we present Table 3, which summarizes from Figure 2 and Table 2 what we know **thus far** about our six games.

Table 3 - Structure of Six Games

Game	Player	Cardinal Matrix in Standard Aspect	Ordinal Matrix in Standard Aspect	Matrix	Initial Move		Overall Measure of Incentive to Switch	Switching Motivations					
								Maximizing		Investment		Signalling	
					Sub-rule	Strategy		Exists	Costless	Exists	Costless	Exists	Costless
1	1	6 6 10 10	B B A A	1b-2x	α	2	0	No	No	No	No	No	No
	2	3 3 7 7	B B A A	1b-2x	α	2	0	No	No	No	No	No	No
2	1	1 2 1 2	B A B A	10-x2	δ	1 or 2	8	No	No	Yes	Yes	Yes	Yes
	2	3 1 3 1	A B A B	10-x1	δ	1 or 2	8	No	No	Yes	Yes	Yes	Yes
3	1	2 -1 -1 1	A C C B	8-11	β	1	7	(Yes)	Yes	No	No	Yes	Yes
	2	1 -1 -1 2	B C C A	8-22	β	2	7	(Yes)	Yes	No	No	Yes	Yes
4	1	3 -1 -1 2	A C C B	8-11	β	1	7	(Yes)	Yes	No	No	Yes	Yes
	2	3 -1 -1 2	A C C B	8-11	β	1	7	(Yes)	Yes	No	No	Yes	Yes

Table 3 (Continued)

Game	Player	Cardinal Matrix in Standard Aspect	Ordinal Matrix in Standard Aspect	Matrix	Initial Move		Overall Measure of Incentive to Switch	Switching Motivations					
								Maximizing		Investment		Signalling	
					Subrule	Strategy		Exists	Costless	Exists	Costless	Exists	Costless
5	1	3 -2 7 -1	B D A C	4a-21	α	2	3	No	No	(Yes)	No	Yes	No
	2	3 -2 7 -1	B D A C	4a-21	α	2	3	No	No	(Yes)	No	Yes	No
6	1	5 -10 4 -20	A C B D	4a-11	α	1	3	No	No	(Yes)	No	Yes	No
	2	2 1 -13 -23	A B C D	2a-11	α	1	1	No	No	No	No	Yes	No

4.1 Degree of Involvement

It has already been observed that Game 1 is inessential, meaning that there is no structural reason why either player's move should be affected by the other's. The game's structure fosters total non-involvement.

Can this notion be generalized? Can we characterize some games as more "involving" than others? We are not yet prepared to map the entire array of 2x2 games into a measure of degree of involvement, but we feel the topic worth exploring and do so in this section.

4.1.1

Table 3 reveals that Game 1 is the only one in which both players are faced by a matrix with $M = 0$, M being the overall measure of incentive to switch. Is incentive to switch the same as degree of involvement? No; the former measures the motivation to switch provided by a player's own matrix "a priori," that is, independently of which opponent's matrix it is paired with to form a game. That the degree to which a player will be involved depends on more than M is illustrated by comparing Games 3 and 4.

In these games both players are faced by type 8 matrices, for which $M = 7$; so that to the players the game structures are initially indistinguishable. But in Game 3 the matrices are paired in a way that promotes instability far more than in Game 4; for in the latter, outcome (1, 1) is the unique Joint Maximum and Non-Cooperative Solution, and both players are "satisfied" with this outcome in the Rapoport-Guyer sense; whereas in Game 3 the JM and NCE solutions specify the two outcomes (1, 1) and (2, 2), and at both of them one of the players is motivated to switch because he is "aggrieved."

For this reason the strong incentives to switch provided initially by the type 8 matrices have entirely different consequences in play. For example, in both games Player 1 initially chooses row 1 by the Bayesian Subrule, β. Player 1 will have both maximizing and signalling incentives to switch from his initial choice, and both tactics are costless. But if Player 2 follows the initial choice decision rule $\alpha\beta\gamma\delta$, Player 1 will not be moved by these incentives in Game 4 because Player 2 will choose strategy 1, which will yield Player 1 a payoff of A.

In Game 3, on the other hand, these incentives will be present because Player 2 will choose strategy 2, yielding Player 1 a payoff of only C . Similar reasoning applies to Player 2. Therefore we should expect outcome (1, 1) to be a steady state in Game 4. We note, however, that if for some reason one of the players uses his second strategy, the other player will be motivated to switch to his second strategy too. Since this is not the case in Game 1; thus we may say Game 4 fosters a higher degree of involvement in both players than does Game 1.

In Game 3 at best only one player can be satisfied at a time; the other always has an incentive to improve his payoff. At worst both are moved to switch -- and if both do neither improves his payoff, for outcomes (1, 2) and (2, 1) alike give the players their worst payoff, C . In iterated play of such a game unless one player gives in to the other, accepting B instead of A , we can expect to observe chronic instability as the players jockey for position, attempting to influence each other to give in.

This game is known in the literature of game theory at the Battle of the Sexes. Luce and Raiffa,[11] for example, examine it at some length, but their discussion is predicted on the players' knowing each other's matrices. They also note a third possibility, that of co-ordinated mixed strategies.[12] If the players know each other's matrices and can communicate freely they may agree to switch strategies in a prearranged way, resulting in a sequence of outcomes such as (1, 1) , (2, 2) , (1, 1) , (2, 2) , ..., which would guarantee each an average long-run payoff per period of $\frac{A + B}{2}$ $(= \frac{3}{2}$ in Game 3 for both players). When communication is barred formidable obstacles are placed in front of achieving this happy resolution; and when the players are ignorant of their opponents' matrices it would appear obvious that they are very unlikely to hit upon a mutually satisfactory way of coordinating their moves. Even if this dictum does not seem so obvious, we can say at least that none of our player teams was able to do it.

The upshot of the discussion is that we expect more instability in the play of Game 3 than Game 4, because the former is "more involving" than the latter.

4.1.2

The ideal measure of degree of involvement induced by a game's structure ought not to depend on observed behavior at all. But as a first approximation, when it is not evident precisely how structure influences involvement, we will design a behavioral or empirical measure.

We begin by defining the degree of involvement evidenced by a particular pair j of players playing Game Γ as

$$I_{\Gamma}^{j} = 1 - \frac{|c_{1j} - c_{2j}|}{c_{1j} + c_{2j}}$$

where c_{ij} is the number of strategy changes made by player i $(i = 1, 2)$ of pair j. The rationale becomes clear when we remind ourselves that involvement means that if one player switches strategies the other will tend to be induced to switch also. Thus if player 1 of pair j shifts once in a game that fosters a high degree of involvement, Player 2 will be likely to shift also. In this case, $c_{1j} = 1$, $c_{2j} = 1$, and $I_\Gamma^j = 1 - \frac{|1 - 1|}{1 + 1} = 1 - \frac{0}{2} = 1$. If Player 1 shifts once in a game that fosters a low degree of involvement, Player 2 will probably not shift. In that case $c_{1j} = 1$, $c_{2j} = 0$, and $I_\Gamma^j = 1 - \frac{|1 - 0|}{1 + 0} = 1 - \frac{1}{1} = 0$.

Note that I_Γ^j is independent of the frequency or number of shifts: $I_\Gamma^j = \frac{1}{3}$ could result from $c_{1j} = 10$, $c_{2j} = 5$ or from $c_{1j} = 2$, $c_{2j} = 1$. For the point of interest is not the absolute number of shifts but the relative distribution of shifts between the two players. If $c_{1j} = c_{2j} = 0$, I_Γ^j is defined neither mathematically nor conceptually -- mathematically because this case requires division by zero, and conceptually because neither player has had an opportunity to demonstrate whether or not he is drawn into strategy changes when his opponent switches.

It is an obvious generalization to define the empirical measure of degree of involvement fostered by Game Γ as the average degree of involvement observed in all pairs playing it in which at least one strategy shift is observed:

$$I_\Gamma = \frac{1}{w} \sum_{j \text{ in } \tilde{S}} I_\Gamma^j ,$$

where S is the set of pairs achieving a steady state from the very beginning and $\tilde{S}$ the set of pairs not in S , there being W such pairs.

The chosen measures of degree of involvement can be objected to on grounds that they do not actually form on the reaction of one player to the other's moves: if a player switches in period t-1 , the measure of degree of involvement should be concerned only with whether his opponent switches in period t . But this approach is open to the criticism that a player may wait a while before reacting -- and that any definition of "a while" in terms of a certain number of periods must be arbitrary.

Another objection points out that a game may seemingly foster a high degree of involvement during a transient phase in which the players feel each other out, but that they may then settle down to a steady state. The longer the steady state (that is, the longer the game is played), the more closely I_{Γ}^{j} approaches zero. To have I_{Γ}^{j} possibly depend on the experimenter's whim as to how many periods the game will be played is an unsatisfactory feature, but it is less likely to be an important matter in our experiments; where no game was played more than 23 periods.

4.1.3

We have already given reasons why we should expect $I_1 < I_4 < I_3$.

We also conjecture that $I_1 < I_6 < I_5$ and leave it to the reader to examine Table 3 and supply his own arguments. Our conjectures do not include Game 2, and they do not lead to a complete ordering for the remaining five games.

Table 4 - Degree of Involvement Fostered by Six Games

Game (1)	Degree of Involvement (I_Γ) (2)	Rank of I_Γ (3)
1	.170	5
2	.620	2
3	.632	1
4	.450	4
5	.565	3
6	.133	6

Table 4 gives the results. All our conjectures are confirmed, except that it turns out that $I_6 < I_1$ -- though not by much.

4.2 Disappointment, Disillusionment, and End Effects in the Prisoner's Dilemma.

Game 5 is the most analyzed of 2x2 matrix games. Luce and Raiffa[13/] allot the Prisoner's Dilemma 9 pages, and Rapoport and Chammah[14/] have written an entire book about it. One would think its structure by now fully explored, but once again the constraint that the opponent's matrix is not known brings in new elements.

In this game both players face type 4a-21 matrices. If the players follow rule $\alpha\beta\gamma\delta$ they choose strategy 2 initially by the Dominant Strategy Subrule, α . But instead of receiving the hoped-for A payoff each receives C , with outcome (2, 2) being observed.

Some players we expect to deal with their disappointment by switching to strategy 1, either as an investment, in the hope of achieving outcome (1, 1) as a steady state with its per period payoff of B , or as a signal to the opponent that outcome (2, 2) is an unsatisfactory steady state. Other players will stick with strategy 2, either as conservative security insurers, guaranteeing themselves a payoff no lower than C , or as optimists hoping the opponent will switch and allow them to receive A .

Therefore in period 2 and the next few periods we expect the frequency of outcome (2, 2) to drop. But the structure of the game is such that the switchers will in all likelihood not succeed: investments will fail and signals, even if received, will not be responded to; the reader examining Table 3 will see why this is so. Thus after several periods in which outcomes (1, 2) and (2, 1) are common and even (1, 1) occasionally seen, the frequency of outcome (2, 2) should begin to drift upward, reflecting the players' disillusionment.

When it is announced that the next period will be the last, the players can infer that no possible good can come from efforts at signalling or investment; therefore we hypothesize that the last-period frequency of outcome (2, 2) will be significantly higher than in the other periods of the game.

Table 5 and Figure 5 show the frequency of outcome (2,2) during periods 1-22 of the play of Game 5. The "last period" varied from period 10 to period 23 in the several replications.

The evidence is consistent with the reasoning offered. Outcome (2, 2) accounted for 82% of the observed outcomes in period 1, but only 39% in periods 2 and 3, 42% in period 4 and 45% in period 5. From period 6 on, the percentage varied between 54 and 80, except for a 44% in period 18 and a 93% in period 21.

The last-period frequency was 88%. If we assume that the probability of observing outcome (2, 2) in periods 6-22 is a random variable distributed normally, we can test whether it is likely to last-period frequency comes from this population. Figure 6 shows that this assumption does not appear unreasonable. It does, however, conflict with the serial correlation implied by the notion of a stage of gradual disillusionment. The mean of the 17 observations from periods 6 through 22 is 66.5%, the standard deviation 11.5. For our hypothesis we calculate $z = \frac{88 - 66.5}{11.5} = 1.87$. Using a one-tailed test we find that the probability is .0307 that our last-period frequency comes from the same population as the other observations in periods 6-22: H_0 is rejected at the 5% significance level.

Table 5 - Frequency of Outcome (2, 2) Observed in the Play of Game 5

Period	Number of Pairs Playing Through This Period (Excluding those for which this period was the last)	Number of Pairs With Outcome (2, 2)	Column (3) as a Percent of Column (2)	Number of Pairs for which this was the "Last Period"
(1)	(2)	(3)	(4)	(5)
1	33	27	82	0
2	33	13	89	0
3	33	13	39	0
4	33	14	42	0
5	33	15	45	0
6	33	22	67	0
7	33	21	64	0
8	33	18	54	0
9	33	19	58	0
10	31	23	74	0
11	31	20	65	0
12	30	19	63	1
13	30	24	80	0
14	30	22	73	0
15	28	19	68	2
16	26	14	54	2
17	21	12	57	5
18	16	7	44	5
19	16	12	75	0
20	16	10	63	0
21	15	14	93	1
22	9	7	78	6
23	0	0	--	9
Last	33	29	88	-

Figure 5

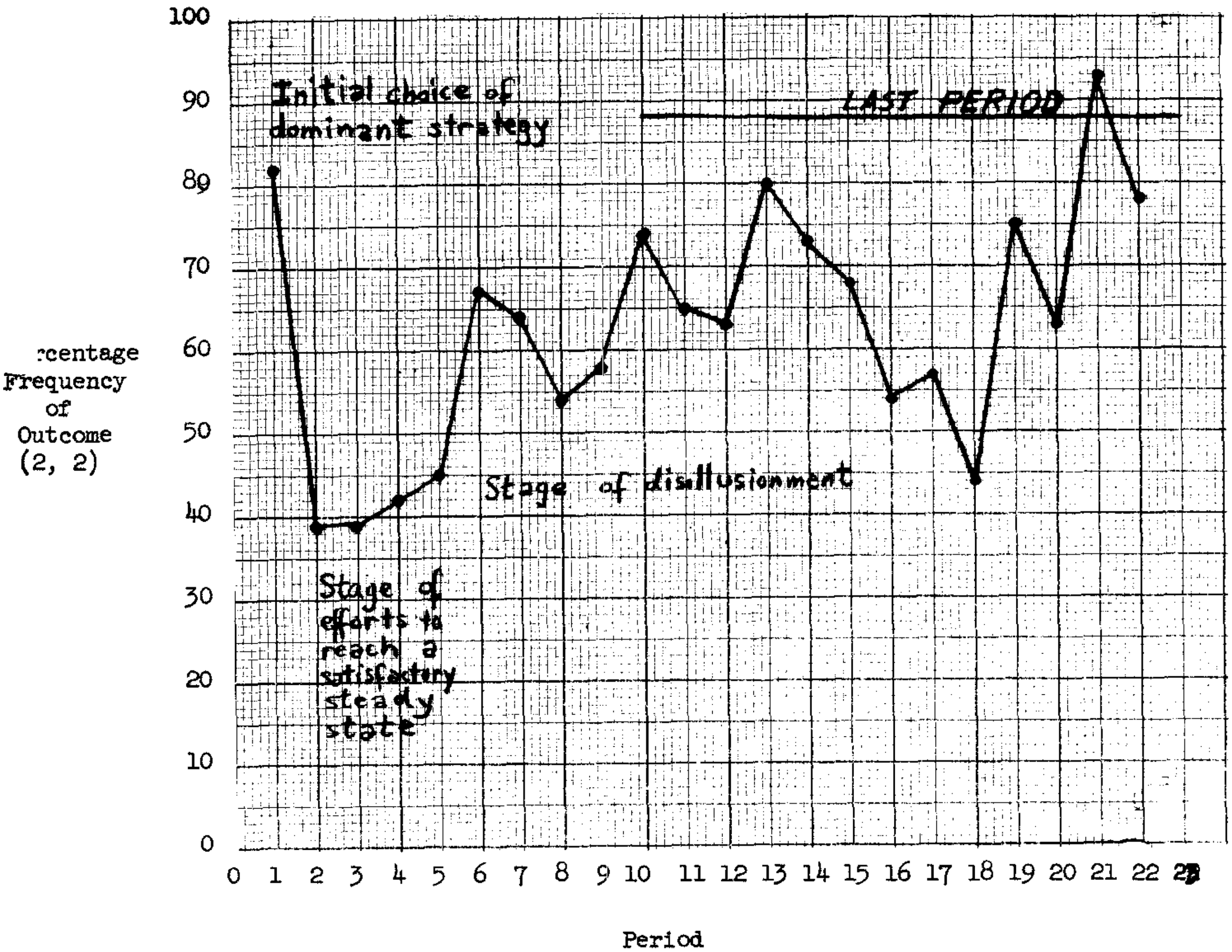
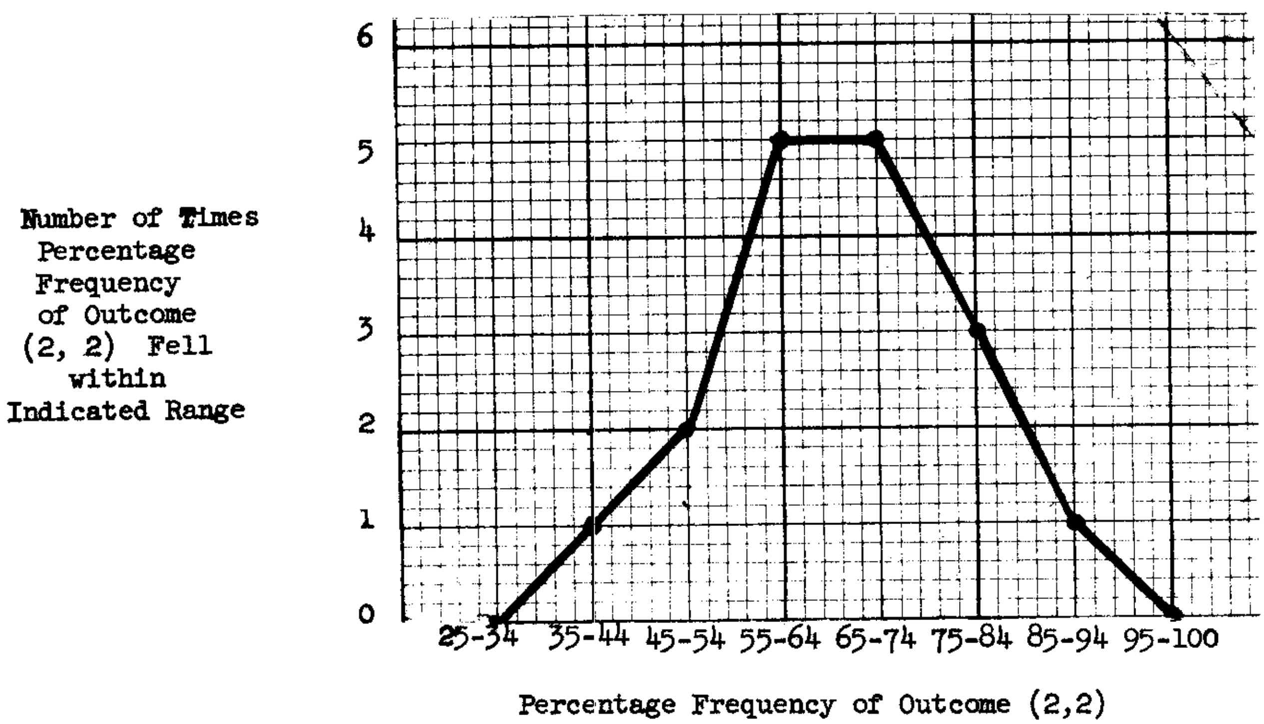

100
90
80
70
60
50
40
30
20
10
0
rcentage
Frequency
of
Outcome
(2, 2)
Initial choice of
dominant strategy
LAST PERIOD
Stage of disillusionment
Stage of
efforts to
reach a
satisfactory
steady
state
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Period

Figure 6


Number of Times
Percentage
Frequency
of Outcome
(2, 2) Fell
within
Indicated Range
6
5
4
3
2
1
0
25-34
35-44
45-54
55-64
65-74
75-84
85-94
95-100
Percentage Frequency of Outcome (2,2)

4.3 Learning to Control the No-Control Game

One of the subjects in our experiments summarized the learning problem of these games succinctly in his comments:

> "There are two types of information one gets from the successive plays of the game:
> 1. What the opponent's matrix looks like
> 2. How the opponent plays
>
> These two are not separately determinable but must be 'solved simultaneously.'"

Game 2 affords the best example of the difficulties the players experience in attempting to "solve simultaneously" the problem of learning to play well. In this game Player 1's matrix is type 10-x2 and Player 2's type 10x1 . This means that although Player 1 is indifferent as to which row he uses himself, he would prefer to see his opponent use strategy 2; similarly Player 2 would like Player 1 to choose row 1. These two facts imply that the jointly preferred outcome is (1, 2) , which is of course the Joint Maximum Solution. The problem is that neither knows, at least initially, that this is the case -- and if one succeeds in learning what the situation is he still has the task of communicating what he has learned to his opponent. In a sense it is fortunate that the investment and signalling tactics are costless,, but this fact also tends to increase the instability of the game. For suppose by accident the outcome (1, 2) is observed initially. While neither is motivated to switch, neither has any incentive <u>not</u> to switch. If a player switches, he does not diminish his own payoff, and he does not know that he is hurting his opponent. Under such unfavorable circumstances do subjects make any progress toward achieving joint maximization?

One might first inquire into whether the players achieve outcome (1, 2) as a steady state during the penultimate five periods more frequently than other outcomes, and the jointly minimal outcome (2, 1) less frequently. Table 6 of part I[15] showed that 4 pairs achieved outcome (1, 2) as a steady state, while none settled on (2, 1) ; and 3 pairs landed at the other two outcomes. This is suggestive, but it says nothing about the 25 pairs who did not reach a steady state.

Yet we may ask whether these had not learned something, even though not enough to commit themselves to a steady state. We conjecture that the average number of periods an outcome will be observed without either player switching is positively related to the payoffs obtained therefrom. Thus we expect longer "runs" of outcome (1, 2) than of outcomes (1, 1) and (2, 2) , and we expect runs of outcome (2, 1) to be shortest.

Table 6 presents the results, which are consistent with our conjecture. Evidently the players made some slight progress toward achieving better than random payoffs, but their success was not striking.

Table 6 - Average Length of Runs Observed at the Four Outcomes of Game 2

Outcome	Average Length of Run (in Periods)	Rank	Expected Rank
1, 1	1.58	3	2 or 3
1, 2	2.71	1	1
2, 1	1.44	4	4
2, 2	1.93	2	2 or 3

FOOTNOTES

1/ Shubik, M. and D.H. Stern, "Some Experimental Non-Constant Sum Games Revisited," Part I, CFDP 236, Yale University, October 2, 1967.

2/ To make this paper self-contained we are including in this and the following subsection much material already presented in Part I (footnote 1 above). The only new elements are the terms, "standard aspect" and "game aspect," explained in the text.

3/ A description of some of the characteristics of the subjects and of the times and circumstances of the experiment replications presented in Part I (footnote 1 above) is deleted from this paper.

4/ See reference to L. Hurwicz in J. Milnor, "Games Against Nature," pp. 49-59 in R.M. Thrall et al (Eds.), Decision Processes, John Wiley and Sons, Inc., New York, 1954.

5/ Siegel, Sidney, Nonparametric Statistics for the Behavioral Sciences, McGraw-Hill Book Company, New York, 1956, pp. 36-42, 247.

6/ The only other attempt to classify such games known to the authors is A. Rapoport and M. Guyer, "A Taxonomy of 2x2 Games," paper supported by PHS Grant NIH-MH-04238-06, Mental Health Research Institute, University of Michigan, Ann Arbor. Their schema is founded on the assumption that the players do know each other's payoff matrices and is more or less inapplicable to situations in which this condition fails to hold.

7/ Ibid, p. 4.

8/ Harsanyi, J.C., "A Game-Theoretical Analysis of Arms Control and Disarmament Problems," pp. IV, in Development of Utility Theory for Arms Control and Disarmament, MATHEMATICA, Princeton, New Jersey, June 1966. Aumann, R.J. and M. Maschler, "Game Theoretic Aspects of Gradual Disarmament," pp. V in same reference.

9/ Rapoport, A. and M. Guyer, op. cit.

10/ The work of Thomas Schelling, as exemplified in his book, The Strategy of Conflict, Harvard University Press, Cambridge, 1960, is also relevant to these problems.

11/ Luce, R.D. and H. Raiffa, Games and Decisions, John Wiley and Sons, Inc. New York, 1957.

[12] *Ibid.*, pp. 115-118, 119.

[13] *Ibid.*, pp. 94-102.

[14] Rapoport, A., and A.M. Chammah, Prisoner's Dilemma, The University of Michigan Press, Ann Arbor, 1965.

[15] Shubik, M., and D.H. Stern, *op. cit.*