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Some Experimental Non-Constant-Sum Games Revisited. Part I

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by

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1. Introduction

In 1960 an experiment was carried out on six simple non-constant-sum two-by-two matrix games with five pairs of students (Yale seniors in a class on Industrial Organization). The theory, analysis and results were reported in a paper entitled: "Some Experimental Non-Zero Sum Games With Lack of Information About the Rules."[1/] The qualification, "with lack of information about the rules," in the title means that each player saw only his own payoff matrix and not that of his opponent.

In the present series of papers the authors make use of data from the original experiment and five later replications. This paper applies traditional game-theoretic analysis, while Parts II and III suggest new theoretical approaches and behavioral measures.

1.1 Form of the Experiments

Figure 1 shows the six games used in the experiments.

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In each game the subject labeled "Player 1" choose the row and the subject labeled "Player 2" the column; the labeling of the players was arbitrary. In each cell of a matrix first is the payoff to player 1 and second the payoff to player 2 resulting from the two players' corresponding strategy choices.

Game 1

6, 3	6, 7
10, 3	10, 7

Game 2

1, 3	2, 3
1, 1	2, 1

Game 3

2, 1	-1, -1
-1, -1	1, 2

Game 4

3, 3	-1, -1
-1, -1	2, 2

Game 5

3, 3	-2, 7
7, -2	-1, -1

Game 6

5, 2	-10, -13
4, 1	-20, -23

Figure 1

In our experiments subjects did not see the matrices shown in Figure 1 but instead saw only their own payoffs, shown as the first two columns of matrices in Figure 2, where player 1 chooses a row and player 2 a column.

	(1) Player 1's Payoff Matrix		(2) Player 2's Payoff Matrix	
Game 1	6	6	3	7
	10	10	3	7
Game 2	1	2	3	3
	1	2	1	1
Game 3	2	-1	1	-1
	-1	1	-1	2
Game 4	3	-1	3	-1
	-1	2	-1	2
Game 5	3	-2	3	7
	7	-1	-2	-1
Game 6	5	-10	2	-13
	4	-20	1	-23

Figure 2

Figure 3 shows a typical Decision Form; this one would be used by Player 1 in Game 3. At the top is the appropriate matrix from Figure 2. In any period the subject writes "1" or "2" in Column (2) of the form, corresponding to his strategy choice of Row 1 or Row 2 that period. A monitor reports the opponent's choice of Column (1)or(2)of the matrix in Column (3) of the form. The subject writes in his own payoff in Column (4) The procedure is repeated each period.

Your Choice		1	2
→	1	2	-1
→	2	-1	1

Game 3
Player 1
Team ______

(1) Period	(2) Your Decision	(3) Your Opponent's Decision	(4) Your Payoff
1			
2			
3			
4			
5			
6			

Figure 3

Subjects were not told how many periods would be played, but they were told at the approporiate point, "The next period will be the last." The purpose in withholding this information was to eliminate end effects except as might be induced during the final period. Although the Decision Forms had room for 25 periods of play, no game ran more than 23 periods; some ran as few as 6 in some replications. In these experiments a subject was paired with one particular opponent for all six games. Also the six games were always played in the same order. The latter tends to increase comparability between results of different subject pairs playing a given game, but the universality of applicability of the results is less than it would have been had the sequence of games been randomized for each team and still less than would have obtained had the player pairings("teams") been chosen separately and randomly for each game.

1.2 Character of Subjects and the Problem of Motivation to Perform

Table 1 presents some information about the circumstances of the six replications of the experiment.

Table 1 - Summary of Circumstances of Six Replications of the Experiment

Replication	Date	Place	Number of pairs of subjects	Characterization of subjects	Subjects' knowledge of Game Theory, as reported by them
1	1960	Yale University	5	Seniors in class on Industrial Organization	None
2	1963	"	6	Graduate students in class on microeconomic theory	Had been given several lectures on game theory and were somewhat acquainted with basic concepts of zero-sum game solutions.
3	1964	"	5	"	"
4	1965	Instituto de Economia, Santiago, Chile	5	Graduate students in Class and Faculty	"
5	1965	Yale University	6	Graduate students in class on Microeconomics	"
6	1967	"	6	Graduate students in class on Special Topics in Economic Analysis	"

No monetary rewards were used in any of these experiments to motivate the players to equate maximization of their scores with maximization of individual utility. However, it can be seen from Table 1 that the subjects were for the most part students of one of the authors, and it may reasonably be inferred that all were interested in learning more about game theory. The subjects were told that these experiments would provide a good and painless introduction to non-constant-sum game phenomena. Furthermore they were specifically instructed to maximize their scores. Between their desire to learn and their desire to please, these subjects should have been highly motivated to perform.

The experimenters may, however, have introduced a distracting element themselves by requesting that each subject after playing each game draw a 2x2 matrix and enter in it what he believed to be the ranking of his opponent's payoffs. This interference with the score-maximization motive resulted if a subject altered his strategy choices in order to gain information about his opponent's matrix. Further discussion of this is deferred to Part III in which the study of incomplete information is stressed.

2. Theories of Reasonable Behavior in Games

Analysis of the results of these experiments means comparison of actual behavior with reasonable behavior. What is reasonable behavior in repeated but finite play of two-person non-constant-sum 2x2 matrix games in which the players know only their own payoffs?

To provide a benchmark, we review in subsection 2.1 four well-known solution concepts which are applicable when players do know their opponents' payoff matrices. In subsection 2.2 we compare these theories with our experimental results. We shall present elements of a theory more appropriate to the present games in Part II.

While there is little doubt that in the development of a satisfactory theory for dynamic games, psychological and sociological factors must be taken into account, we have not done so formally here; although some informal discussion of these features is given with the interpretation of the data.

2.1 Four Game-Theoretic Solution Concepts

Let $P_1(s_1, s_2)$ stand for the payoff to Player 1 when he employs his strategy s_1 and Player 2 his strategy s_2 ; $P_2(s_1, s_2)$ is the payoff to Player 2. The pair (s_1, s_2) determines the outcome.

We must distinguish between strictly competitive and not strictly competitive games. A game is strictly competitive if an increase in the welfare of one player implies an equal decrease in the welfare of the other. That is,

$$P_1(s_1, s_2) = -P_2(s_1, s_2) + k ,$$

for all outcomes. This is equivalent to stating that the sum of the two payoffs is a constant (k) , no matter what strategies the players use. Strictly competitive games are called zero-sum when $k = 0$; otherwise they are called constant-sum. It will be observed that the games illustrated in Figure 1 are not constant sum games; i.e., the sum of the payoffs to the players may vary from one outcome to another. Hence there is room for cooperation which will yield rewards to both players.

Von Neumann and Morgenstern[2/] suggest that in a non-constant-sum game the players should jointly maximize the sum of their two payoffs and then work out some arbitrated division of the proceeds between them. This presupposes that they are in a position to communicate with each other and are also in a position to make side-payments. Von Neumann and Morgenstern do not explicitly include the bargaining and haggling over side-payments as part of their description of the play of the game, but as something which takes place outside of it. The description of the behavior of the players in the game is given mathematically by the condition:

$$[1] \qquad \underset{s_1}{\text{Max.}} \ \underset{s_2}{\text{Max.}} \ P_1(s_1, s_2) + P_2(s_1, s_2) \quad .$$

This merely states that each player should select his strategy in such a manner that the sum of their payoffs is maximized. By way of example, examine the matrix for Game 2 in Figure 1; the joint maximum solution yields $s_1 = 1$ and $s_2 = 2$; for at outcome $(1, 2)$, $P_1 + P_2 = 5$, which is maximal.

John Nash[3/] has suggested a theory of non-cooperative play which is a generalization of economic theories of equilibrium. His theory applies to situations where communication between the players is limited and they are not in a position to make side-payments.

Nash shows that in any finite game which can be described by a set of payoff matrices there will exist at least one pair of strategies s_1^* and s_2^* such that the two conditions

$$s_1^* = \underset{s_1}{\text{Max}}\ P_1(s_1,\ s_2^*)$$

[2]

$$\text{and} \quad s_2^* = \underset{s_2}{\text{Max}}\ P_2(s_1^*,\ s_2)$$

are simultaneously satisfied by the choice of s_1^* and s_2^* by the first and second players respectively. In words, if the first player believes that the second player will utilize s_2^* against him, his optimal reply (in the sense that it will maximize his own payoff against that strategy) is s_1^* and vice versa. In Game 5, for example, the Non-Cooperative Solution is the outcome (2, 2) .

It is possible that both players may strive to play in a manner that defends against the worst that can happen.[4/] Suppose a player believes or pretends that the entries in his opponent's payoff matrix are the negative of those in his own, and that his opponent will play to maximize his payoff. This implies that the opponent will be attempting to minimize the player's own payoff. The player attempts to maximize his own payoff on the basis of this pessimistic assumption concerning the pattern of his opponent's intended method of play. If each player defends against this sort of hostility on the part of his opponent, we will observe the Minimax Solution, which can be expressed as:

$$[3] \qquad \max_{s_1} \min_{s_2} P_1(s_1, s_2)$$

$$\text{and} \quad \max_{s_2} \min_{s_1} P_2(s_1, s_2) \text{ .}$$

While the Minimax Solution requires each player to play as if his own payoff matrix were the basis of a strictly competitive game, the Competitive Solution has both players actually playing a zero-sum game obtained by subtracting one payoff matrix from the other. The rationale is that the players adopt the attitude that it is more important to maximize the difference in gain between them than it is to maximize individual gain. This type of thinking is prevalent in tactical calculations of damage exchange rates. The Competitive Solution is given by:

$$[4] \qquad \max_{s_1} \min_{s_2} [P_1(s_1, s_2) - P_2(s_1, s_2)] \text{ .}$$

2.2 Application of Solution Concepts to the Six Experiment Games

In Table 2 all four solution concepts have been applied to the six games of Figure 1, and the resulting strategy pairs which are the solutions are noted. For example, the expression (1, 1) stands for the outcome where each player selects his first strategy, i.e., $s_1 = 1$ and $s_2 = 1$. This gives the pair of payoffs in the upper left-hand corner of the payoff matrix.

Table 2 - Four Solution Concepts Applied to the Six Games of Figure 1

	Joint Maximum Solution [1]	Non-Cooperative Solution [2]	Minimax Solution [3]	Competitive Solution [4]
Game 1	(2,2)	(2,2)	(2,2)	(2,2)
Game 2	(1,2)	(1,1) (1,2) (2,1) (2,2)	(1,1) (1,2) (2,1) (2,2)	(2,1)
Game 3	(1,1) or (2,2)	(1,1) or (2,2)	{2/5,3/5}* and {3/5,2/5}	(1,2)
Game 4	(1,1)	(1,1)✭ or (2,2)	{3/7,4/7}* and {3/7,4/7}	(1,1) (1,2) (2,1) (2,2)
Game 5	(1,1)	(2,2)	(2,2)	(2,2)
Game 6	(1,1)	(1,1)	(1,1)	(1,1) (1,2) (2,1) (2,2)

*These both involve mixed strategies. The probabilities employed by each player are indicated in the curled **brackets**.

✭Only (1,1) is a non-cooperative solution in the strict sense.5/

All four solution concepts when applied to Game 1 yield the same solution pair (2,2) . A closer examination of the game shows the structural reason why this is so. The players are strategically independent. Their fates are not interlinked. This game illustrates the atomistic isolation between any two competitors in a purely competitive market. Regardless of their motives, the structure makes their predicted behavior is the same in all instances.

A non-constant sum game in which the fates of the players are not interlinked is referred to as an inessential game. It is inessential in the same way as is a strictly competitive game. There is nothing to be gained by discussion, negotiation or collusion. In essence, collusion has no meaning in this context.

Wilson and Bixenstine[6/] suggest an interpretation of these types of two-person, two-choice games in terms of social control; thus it is noted that in Game 2, neither player has any control over his own payoff, but each has complete control over the payoff of his competitor. In Game 3 the amount of control is symmetric, as it is in Game 4. Game 5 is the classical Prisoners' Dilemma,[7/] and Game 6 has individual and joint interests completely correlated -- a visible example of the "invisible hand."

3. <u>Experimental Results Compared With Standard Game-Theoretic Solutions</u>

It was the belief of the authors that of these four concepts the Non-Cooperative Equilibrium Solution would best explain the results of our experiments, in which information concerning the payoffs of the other was scanty; communication was difficult, complex, expensive and hard to interpret; and sociological and personal knowledge of one's competitor was minimal--because the NCE Solution is an "inner-directed" theory, in contrast with the Joint-Maximum Solution which is "other-directed" and max-min of the difference in payoffs which is competitive or possibly "status" oriented.

We expected the NCE Solution to predict better in those games (1, 3, 4 and 6) in which it coincides with the JM Solution, for in them the game structure presents no disharmony between these two approaches to play -- the structure promotes implicit collusion. Finally, we expected the best performance from the NCE Solution in those games (1, 4 and 6) in which the NCE and JM Solutions together specify a unique outcome.

3.1 Test of Behavior During Penultimate Five Periods of Play

In formulating statistical hypotheses based on these surmises we were faced with a formidable problem: the solutions do not accommodate random error. The expected frequencies with which the four outcomes (1,1), (1,2), (2,1), and (2,2) will be observed in Game 1, according to the JM Solution, are 0, 0, 0, and 100%. A chi-square test is undefined in such a situation, as its statistic then is the sum of terms containing zero in the denominator. That is, a single instance other than the outcomes predicted by the solution being tested would suffice to reject the hypothesis that that solution explains the observed behavior, no matter how many instances occur in which the solution outcomes <u>are</u> observed. The reason is that the chi-square test, which is the most common way of determining whether observed correspond to expected frequencies, consists in summing the quantities $\frac{(Oij - Eij)^2}{Eij}$ over all outcomes, where E_{ij} is the expected frequency of outcome (i,j) and O_{ij} is the observed frequency. If $E_{ij} = 0$ for any outcome, the statistic χ^2 is undefined.

Columns (5)-(8) of Table 3 show the expected outcome frequencies for the four solutions in each game, and it can be seen that each of the four yields a zero expected frequency in several games.

Though a chi-square test is ruled out as a means of testing these four solutions, it can be used to test the null hypothesis, illustrated by Column (9), that there is <u>no</u> pattern to the observed outcomes. This null hypothesis, the "Random Solution," asserts that strategies will be chosen at random, so that all four outcomes will be observed with equal frequency. For our two-by-two games the expected frequency with which any outcome is observed in N trials is always 25% of N .

Table 3 - Outcomes Observed and Expected During Penultimate

Five Periods of Six Games

Game	Outcome	Observed Absolute Frequency (N=165 or 160*)	Observed Percentage Frequency	Expected Percentage Frequencies: Joint Max	Non-Coop	Minimax	Competitive	Random
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	1,1	2	1.2	0	0	0	0	25
	1,2	2	1.2	0	0	0	0	25
	2,1	11	6.7	0	0	0	0	25
	2,2	150	90.9	100	100	100	100	25
2	1,1	21	13.1	0	25	25	0	25
	1,2	45	28.2	100	25	25	0	25
	2,1	48	30.0	0	25	25	100	25
	2,2	46	28.7	0	25	25	0	25
3	1,1	49	29.1	50	50	24	0	25
	1,2	33	20.0	0	0	16	100	25
	2,1	10	6.1	0	0	36	0	25
	2,2	73	44.8	50	50	24	0	25
4	1,1	151	91.6	100	100	18.4	25	25
	1,2	6	3.6	0	0	24.5	25	25
	2,1	6	2.4	0	0	24.5	25	25
	2,2	4	2.4	0	0	32.6	25	25
5	1,1	5	3.0	100	0	0	0	25
	1,2	19	11.5	0	0	0	0	25
	2,1	20	12.0	0	0	0	0	25
	2,2	121	73.4	0	100	100	100	25
6	1,1	155	96.9	100	100	100	25	25
	1,2	3	1.9	0	0	0	25	25
	2,1	2	1.2	0	0	0	25	25
	2,2	0	0	0	0	0	25	25

*33 player pairs playing 5 periods yields N = 165 for Games 1, 3, 4 and 5. In Game 2 one set of data had to be rejected because of an error; in Game 6 one team did not play at all; thus for Games 2 and 6, N = 160 .

Columns (3) and (4) of Table 3 present the absolute and percentage frequencies of observed outcomes during the penultimate five periods of play for all six games. The last period is excluded because of possible end effects, while earlier periods are eliminated because solutions themselves are relevant only after learning has ceased; that is, the four solutions, regardless of their differences, are alike in predicting a stable outcome (stable over time), which cannot reasonably be expected during the early periods of play in this kind of experiment.

Table 4 shows the χ^2's calculated from observed frequencies and those expected if the Random Solution is true. With 3 degrees of freedom, a $\chi^2 \geqq 11.34$ means that the null hypothesis is true with probability $p < .01$, $\chi^2 \geqq 16.27$ implies $p < .001$, and $\chi^2 \geqq 27.63$ implies $p < .00001$.

Table 4 - Test of the Random Solution

Game	χ^2
(1)	(2)
1	383.5
2	12.15
3	51.25
4	389.7
5	208.6
6	441.9

Since the value of χ^2 in all but Game 2 are far larger than 27.63, we can conclude that our sample of outcomes is not from a population in which the frequencies of observing any outcome are equal. Even in Game 2 we reject the null hypothesis at a significance level of .01, but it is of interest that in this game, where the Random Solution coincides with the NCE Solution, the value of χ^2 is much lower than in the other games.

3.2 Effectiveness of Solutions as Predictors

We use the following measure of how well the solutions compare in predictive quality, "predictive quality" being defined by our measure, which is:

[5] $$Q = 1 - \frac{L}{2N}$$

$$\text{where } L = \sum_{i=1}^{2} \sum_{j=1}^{2} |Oij - Eij| .$$

$Q = 1$ when the observed frequencies and those predicted by a solution coincide; $Q = 0$ when none of the observed outcomes fall into the predicted categories. It varies continuously toward one as the frequencies observed approach those expected.

Table 5 shows Q for all six games for the four game-theoretic solutions and the Random Solution. The averages over the six games are shown also, and the solutions are ranked on that basis as predictors of behavior.

Table 5 - Quality of Outcome-Prediction for Five Solution Concepts

Line of the Table	Game	Joint Maximum	Non-Cooperative	Minimax	Competitive	Random (Null Hypothesis)
1	1	.909	.909	.909	.909	.341
2	2	.281	.881	.881	.300	.881
3	3	.740	.740	.701	.200	.761
4	4	.915	.915	.268	.335	.335
5	5	.030	.733	.733	.733	.517
6	6	.969	.969	.969	.281	.281
7	Average Q	.641	.858	.744	.460	.529
8	Rank	3	1	2	5	4
9	Average Q when solution coincides with Non-Cooperative Solution	.883	-	.873	.821	.881
10	Rank	1		3	4	2
11	Average Q when solution does not coincide with Non-Cooperative Solution	.156	-	.484	.279	.447
12	Rank	4		1	3	2

It is seen from lines 7 and 8 of the table that $\bar{Q} = .858$ for the Non-Cooperative Equilibrium Solution, which makes it the best predictor, as we expected. In games in which the JM and NCE Solutions coincide, $\bar{Q} = .883$, as seen in line 9. In games in which they differ, the JM Solution is the poorest predictor, with $\bar{Q} = .156$ (line 11), while in those games (2 and 5) the NCE Solution

Q averages .807 . This lends support to our belief that the NCE Solution is a better predictor when it coincides with the JM Solution.

In Games 1, 4 and 6, where the JM and NCE Solutions pick a unique outcome, $\bar{Q} = .931$, confirming our expectation once more. In those games (1 and 6) in which the JM, NCE and Minimax Solutions all coincide, $\bar{Q}$ rises still higher, to .939 .

Actually, it is clear from line 9 that any solution predicts well when it coincides with the NCE, though of the four, the JM is best. Line 11 makes it equally clear that when a solution does not coincide with the NCE it predicts poorly. It is of interest, however, that lines 11 and 12 show the Minimax and Random Solutions (which like the Non-Cooperative Solution are "inner-directed" in that they do not depend on correct perception of the opponent's payoff matrix) to be better independent predictors than the Joint Maximum and Competitive Solutions.

It should be re-emphasized that Q as a measure of prediction quality is arbitrary in the sense that our loss function L , the sum of the absolute differences between observed and expected frequencies, is arbitrary. Were it possible to show on economic grounds that some other loss function, as, for example, the more commonly used sum-of-squared-differences, should be minimized by a predictor, our judgement on these five solution concepts might have to be revised.

3.3 Steady States

Another way to determine whether any of the four game-theoretic solutions is a good predictor of observed behavior is to examine steady states. The question resolves itself into two parts: how frequently are steady states observed, and to what extent are the steady states observed the ones predicted by the four solutions.

Table 6 presents the necessary raw data in Columns (2)-(5), a steady state being defined as the observation of the same outcome during all five of the penultimate five periods.

The first question is answered in Columns (6)-(8), with Column (8) showing the percent of instances in which a steady state was observed.

Table 6 - Steady States

Game	Number of Steady States Observed at Outcome				Total Number of Steady States	Number of Obser-vations *	Steady States as a Percent of Number of Observations
	(1,1)	(1,2)	(2,1)	(2,2)			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	0	0	0	25	25	33	75.7
2	1	4	0	2	7	32	21.9
3	7	0	0	11	18	33	54.5
4	28	0	0	0	28	33	84.8
5	0	0	0	13	13	33	39.4
6	29	0	0	0	29	32	90.5

* See note to Table 3 for explanation of variation in N .

In Part II we will reconsider these data from a dynamic point of view, for our solution concepts cast no light on how frequently we should expect steady states. Nevertheless we may note here that the greatest stability frequencies (over 75%) were in games in which the NCE and JM Solutions coincide, while the lowest frequency was in the game in which the NCE has no resolving power. Intermediate frequencies were seen in Game 3, which has two coinciding JM-NCE outcomes, and in Game 5, in which JM and NCE Solutions differ.

Table 7 answers the second question by using once again the measure Q defined above; in its calculation from Formula [5] N is taken to be the total number of steady states as shown in Column (6) of Table 6, rather than the total number of observations. In other words, the question is taken to be, "Of those steady states observed, how well does Solution X predict the particular outcomes seen?"

Table 7 - Quality of Prediction of Steady States of Five Solutions

Game	Joint Maximum Solution	Non-Cooperative Solution	Minimax Solution	Competitive Solution	Random Solution
1	1.000	1.000	1.000	1.000	.250
2	.571	.643	*	0	.643
3	.889	.889	*	0	.500
4	1.000	1.000	*	.250	.250
5	0	1.000	1.000	1.000	.250
6	1.000	1.000	1.000	.250	.250
Average Q	.743	.922	*	.417	.357
Rank	2	1	*	3	4
Average Q when solution coincides with Non-Cooperative Solution	.972	-	*	1.000	.643
Average Q when solution does not coincide with Non-Cooperative Solution	.286	-	*	.063	.300

Table 7, continued

⊛Where the Minimax Solution requires mixed strategies, it implies by definition that no long-term steady state will occur. The probability of observing a run of five observations can be calculated for each outcome; in all cases here that probability is very small, the largest being for outcome (2,2) in Game 4, where it is $\left(\frac{16}{49}\right)^5 = .0037$. For this reason it seems inappropriate to adapt the Minimax Solution to the prediction of steady states except where it specifies a pure strategy equilibrium pair.

Once again the Non-Cooperative Solution emerges as the most accurate predictor, and again when a solution is deprived of its association with NCE it performs poorly. Comparison of Table 7 with Table 5 is suggested.

FOOTNOTES

1/ Martin Shubik, "Some Experimental Non-Zero Sum Games With Lack of Information About the Rules," Management Science, Vol. 8 No. 2 (January 1962), pp. 215-234.

2/ John Von Neumann and Oskar Morgenstern, The Theory of Games and Economic Behavior (Princeton University Press, Princeton N.J., 1944), Ch. VI; also R. Duncan Luce and Howard Raiffa, Games and Decisions (John Wiley and Sons, New York, 1957), pp. 115-119.

3/ John F. Nash Jr., "Non-Cooperative Games," Annals of Mathematics, Vol. 54 (September 1951), pp. 286-295; also Luce and Raiffa, op.cit., pp. 106-109.

4/ Von Neumann and Morgenstern, op.cit., Ch. III; also Luce and Raiffa, op.cit., pp. 65-73, 92.

5/ Luce and Raiffa, op.cit., p. 107.

6/ K. V. Wilson and E. Bixenstine, "Forms of Social Control in Two Person, Two Choice Games," Behavioral Science (January 1962), pp. 91-102.

7/ Anatol Rapoport and Albert M. Chammah, Prisoner's Dilemma (The University of Michigan Press, Ann Arbor, 1965).