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# Factor-Price-Frontier Estimation of a 'Vintage' Production Model of the Post-war U.S. Nonfarm Business Sector

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Edmund and Charlotte Phelps [1]

We report here on the construction and application of a method of estimating the parameters of an aggregative "vintage" model of "potential" output in the postwar U. S. nonfarm business sector. The model admits capital-embodied as well as unembodied technical progress. It posits that *ex post* and *ex ante* substitution possibilities between capital (of any vintage) and cooperating labor are alike. There are constant returns to scale. Ultimately we resort for convenience to the Cobb-Douglas function, although certain estimates, such as that of the marginal productivity of investment, do not hinge on this specification. Theoretical aspects of the model have been studied by Solow [10] and Phelps [6]. The model has been estimated by a conventional method most recently by Berglas [1].

The principal novelty of our method of estimation is the use of Samuelson's factor price frontier construct [9]. This approach permits us to dispense with capital stock data. The virtue of this is that, frequently, capital stock estimates are unavailable because of the absence of early investment

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data. Moreover, as Fisher has recently proved [3], an aggregate production function relating aggregate output to aggregate "effective capital" exists, even in a one-product model, only if _ex post_ and _ex ante_ substitution possibilities are alike and any capital-embodied technical progress can be represented as solely capital-augmenting (i.e., as if all capital-embodied change could be expressed by a capital "improvement factor"). Since capital stock data for the U. S. business sector are available and since, in our model, there is _ex post_ substitutability and embodied progress can be described as capital-augmenting, our method of estimation can be regarded as unnecessary. We present it, however, in the hope that other investigators will be stimulated to apply our method to more general models in which an aggregate production function does not exist and to countries for which no capital stock data exist. Further, we hope that our results will be welcomed for purposes of comparison with those from other techniques of estimating the same model.

Turning to our results, we should like to emphasize that our statistical procedure is somewhat amateurish, that the data may be insufficiently accurate and that there is serious mis-specification in the model (especially with respect to the _ex post_ substitutability of capital and labor). Nevertheless we cite the following findings for whatever they are worth: (1) the rate of technical progress increased significantly over the postwar period (accelerating technology); (2) the capital elasticity of output is much smaller than capital's relative share, which suggests that there is a large element of monopoly rent in capital's income; (3) embodied progress was negligible in the early postwar years but eventually exceeded unembodied progress by the end of the period; (4) the marginal productivity of investment began at about 30 percent in the late forties, fell to 17 percent in the middle fifties and recovered to 25 percent in the late fifties; (5) what we call the rate of obsolescence (the proportionate rate at which the price of new capital goods in efficiency units declines relative to the price of consumption goods) was about 5 percent in the middle fifties and 9 percent in the late fifties; (6) so that, if the rate of physical depreciation was about 4 percent, the one-period social rate of return was about 8 percent in the middle fifties and about 12 percent in the late fifties. (See Tables 3, 6, 7, 8 and 9.)

## THE MODEL AND ESTIMATION METHOD

### Aggregating a Two-Sector Model

We imagine an economy in which there are two products, consumption goods and investment goods ("machines"), and two kinds of inputs, labor and capital goods (machines of various vintages). The output of consumption goods, $C(v, t)$, produced by machines of vintage $\underline{v}$ at time $\underline{t}$, $\bar{M}_C(v, t)$, and cooperating labor, $L_C(v, t)$, is given by the production function

$$(1) \qquad C(v, t) = F(\bar{M}_C(v, t), L_C(v, t) ; v, t)$$

which is linear homogeneous (constant returns to scale), twice differentiable (smooth marginal productivities) and strictly concave in both arguments (diminishing marginal productivities).

The rate of output of machinery, $M(v, t)$, produced by machines of vintage $\underline{v}$ at time $\underline{t}$, $\bar{M}_M(v, t)$, and cooperating labor, $L_M(v, t)$, is given by an almost identical production function

$$M(v, t) = R(t)\, F(\overline{M}_M(v, t)\ ,\ L_M(v, t)\ ;\ v, t)\ , \qquad R(t) > 0 \tag{2}$$

(Note that $\overline{M}(v, t)$ is input and $M(v, t)$ is output.) If $R'(t) > 0$ then unembodied progress is proceeding faster in the machinery sector than in the consumption-good sector, and vice-versa if $R'(t) < 0$ . This differential rate of unembodied technical progress is the same for all vintages.

We suppose that machines of any vintage can be shifted from one sector to another. As a consequence of this, it can be shown that the marginal rate of transformation between the consumption good and machinery at any moment of time is a constant, $R(t)$; that is, the cost in consumption goods of producing an extra machine is $R(t)^{-1}$ . This linearity of the production-possibility curve is required for the aggregation of our vintage production functions, (1) and (2), into an "aggregate" vintage production function. Aggregation will be discussed shortly.

The value of total output, $Q(v, t)$ , from total machinery of vintage $\underline{v}$ at time $\underline{t}$ , $\overline{M}(v, t)$ , and total cooperating labor, $L(v, t)$ , is defined as follows:

$$Q(v, t) = C(v, t) + q(t)\, M(v, t) \tag{3}$$

when $q(t)$ is the price of new machinery in terms of consumption goods. As already suggested and as will be shown, the competitive price of new machinery will be $R(t)^{-1}$ .

To close the model we have the additional relations:

$$L_C(v, t) + L_M(v, t) = L(v, t) \tag{4}$$

$$\overline{M}_C(v, t) + \overline{M}_M(v, t) = \overline{M}(v, t) \tag{5}$$

$$\int_{-\infty}^{t} L(v, t)dv = L(t) \tag{6}$$

and

$$\overline{M}(v, t) = \Phi(t-v)\ M(v) \qquad \Phi'(t-v) \leq 0\ . \tag{7}$$

The last relation indicates how machinery of vintage $\underline{v}$ has deteriorated up to time $\underline{t}$ ; $M(v)$ is the rate of total machinery output at time $\underline{v}$ .

There are also the aggregate relations

$$C(t) = \int_{-\infty}^{t} C(v, t)dv \tag{8}$$

$$M(t) = \int_{-\infty}^{t} M(v, t)dv \tag{9}$$

which give total consumption and machinery output. The value of aggregate output is given by

$$Q(t) = \int_{-\infty}^{t} Q(v, t)dv = C(t) + q(t)\ M(t) = C(t) + I(t) \tag{10}$$

where $I(t)$ is the value, in terms of the consumption good, of total gross investment in new machines.

Finally, for given $C(t)$, we have to allocate inputs between the two sectors and allocate labor among the vintages of machinery. We suppose pure competition so that labor allocation is determined by the relations:

$$(11)\qquad \begin{aligned} &F_{L_C}(\overline{M}_C(v,t)\,,\,L_C(v,t)\,;\,v,\,t) \begin{cases} = w(t) & \text{if} \quad L_C(v,t) > 0 \\ \le w(t) & \text{if} \quad L_C(v,t) = 0 \end{cases} \\ &q(t)\,R(t)F_{L_M}(\overline{M}_M(v,t)\,,\,L_M(v,t)\,;\,v,\,t) \begin{cases} = w(t) & \text{if} \quad L_M(v,t) > 0 \\ \le w(t) & \text{if} \quad L_M(v,t) = 0 \end{cases} \end{aligned}$$

Thus the marginal value product of labor in each sector equals the real wage rate, $w(t)$ , for all those vintages being operated and is less than or equal to the wage rate for other vintages. Hence the marginal value product of labor will be equalized wherever labor is being used.

For machinery of given vintage, its marginal value product in each sector is equalized, being equal to the rental on machinery of vintage $\underline{v}$ at time $\underline{t}$ , $r(v, t)$:

$$(12)\qquad F_{\overline{M}_C}(\overline{M}_C(v,t)\,,\,L_C(v,t)\,;\,v,\,t) = q(t)\,R(t)F_{\overline{M}_M}(\overline{M}_M(v,t)\,,\,L_M(v,t);\,v,\,t) = r(v,t)$$

As a consequence of (11) and (12), the following least-cost conditions are satisfied for any utilized vintage:

$$(13)\qquad \frac{r(v,t)}{w(t)} = \frac{F_{\overline{M}_C}}{F_{L_C}} = \frac{R(t)F_{\overline{M}_M}}{R(t)F_{L_M}} = \frac{F_{\overline{M}_M}}{F_{L_M}}$$

Since the marginal rate of substitution between labor and machines of any given vintage is the same in the two sectors and the isoquants for the vintage production functions have identical shapes, it follows that factor intensities will be the same for any utilized vintage in the two-sectors:

$$(14) \qquad \frac{\bar{M}_M(v, t)}{L_M(v, t)} = \frac{\bar{M}_C(v, t)}{L_C(v, t)}$$

It can now be seen that $q(t) = R(t)^{-1}$ . For, by (14) and the linear homogeneity of the function $F$ ,

$$(15) \qquad F_{L_C} = F_{L_M} \quad \text{and} \quad F_{\bar{M}_C} = F_{\bar{M}_M}$$

for all utilized vintages of machinery. And the relative price of new machinery is equal to the ratio of the marginal physical productivities of labor (or, equivalently, of machinery):

$$(16) \qquad q(t) = \frac{F_{L_C}}{R(t)F_{L_M}} = \frac{F_{\bar{M}_C}}{R(t)F_{\bar{M}_M}} = R(t)^{-1}$$

Hence the price of new machinery is the reciprocal of $R(t)$. And the behavior of the model, given $C(t)$ , is determined.

It will be shown now, following a line of argument due to Green [4], that it is possible to construct an aggregate <u>vintage</u> production function. First, we have, by (1), (2) and (3)

$$(17) \quad Q(v, t) = F(\bar{M}_C(v, t), L_C(v, t) ; v, t) + q(t) R(t) F(\bar{M}_M(v, t) , L_M(v, t); v, t),$$

or, by (16) ,

$$(18) \quad Q(v, t) = F(\bar{M}_C(v, t) , L_C(v, t) ; v, t) + F(\bar{M}_M(v, t) , L_M(v, t) ; v, t) .$$

Now, by (14), there is a number $c(v, t)$ , $0 \leq c \leq 1$ , such that

$$(19) \qquad \begin{aligned} \bar{M}_C(v, t) &= c(v, t) \bar{M}(v, t) , \quad L_C(v, t) = c(v, t) L(v, t) , \\ \bar{M}_M(v, t) &= [1-c(v, t)] \bar{M}(v, t) , \quad L_M(v, t) = [1-c(v, t)] L(v, t) . \end{aligned}$$

Hence

$$(20) \qquad Q(v, t) = F(c(v, t)\,\overline{M}(v, t)\,, c(v, t)\,L(v, t)\,;\,v, t) + F([1-c(v, t)]\,\overline{M}(v, t)\,,\,[1-c(v, t)]\,L(v, t)\,;\,v, t)\,.$$

By virtue of constant returns to scale we have:

$$(21) \qquad Q(v, t) = c(v, t)\,F(\overline{M}(v, t)\,,\,L(v, t)\,;\,v, t) + [1-c(v, t)]\,F(\overline{M}(v, t)\,,\,L(v, t)\,;\,v, t)\,.$$

Hence we obtain the aggregate vintage function:

$$(22) \qquad Q(v, t) = F(\overline{M}(v, t)\,,\,L(v, t)\,;\,v, t)\,.$$

This result and (10) yield the following equation for aggregate output:

$$(23) \qquad Q(t) = \int_{-\infty}^{t} F(\overline{M}(v, t)\,,\,L(v, t)\,;\,v, t)dv$$

By (7) and (10) we also have

$$(24) \quad \overline{M}(v, t) = \Phi(t-v)\,M(v) = \Phi(t-v)\,I(v)\,q(v)^{-1} = \Phi(t-v)\,R(v)\,I(v)$$

Hence, in terms of the <u>value</u> of investment,

$$(25) \qquad Q(t) = \int_{-\infty}^{t} F(\Phi(t-v)\,R(v)\,I(v)\,,\,L(v, t)\,;\,v, t)dv\,.$$

We may regard the increase (with respect to $\underline{v}$) of $R(v)$ as quasi-capital-embodied technical progress at time $\underline{v}$ (of the capital-augmenting type), noting that $R'(v) < 0$ is just as possible as $R'(v) > 0$ ; that is, recent $I(v)$ may receive a larger or smaller weight, $R(v)$ , than old $I(v)$ . (We say "quasi" because $\Phi(t-v)\,I(v)$ is not <u>physical</u> capital; $R(v)$ merely translates the value of investment into a physical capital quantity.)

It will be convenient to express the above equation in the form:

$$Q(t) = \int_{-\infty}^{t} G(\Phi(t-v)\, I(v)\ ,\ L(v,\ t)\ ;\ v,\ t)dv \tag{26}$$

Here the argument $\underline{v}$ does double duty: it translates $I(v)$ into physical machinery units and then allows for differences in the character of machinery of different vintages. It should be recognized that because quasi-embodied progress may be negative, there can be no strong presumption that $\frac{\partial G}{\partial v} \geq 0$ , that is, that true embodied plus quasi-embodied progress is nonnegative.

Finally, we adopt the notational convention

$$K(v,\ t) = \Phi(t-v)\ I(v)\ ,\quad \Phi(0) = 1\ ,\ \Phi' \leq 0\ . \tag{27}$$

where $K(v,\ t)$ is the value of the investment at time $\underline{v}$ , depreciated up to time $\underline{t}$ . Then

$$Q(t) = \int_{-\infty}^{t} G(K(v,\ t)\ ,\ L(v,\ t)\ ;\ v,\ t)dv \tag{28}$$

<u>The Marginal Productivity of Investment</u>

Our method of estimation involves the marginal productivity of investment (in value terms). This concept indicates by how much the value of output tomorrow will be increased by a one unit reduction of consumption today. From (28) we have

$$\dot{Q}(t) = G(K(t,\ t)\ ,\ L(t,\ t)\ ;\ t,\ t) + \int_{-\infty}^{t} \frac{dG}{dt}\, dv \tag{29}$$

By constant returns to scale we obtain

$$(30)\qquad \dot{Q}(t) = G_K\, K(t, t) + G_L\, L(t, t) + \int_{-\infty}^{t} G_L \frac{\partial L(v, t)}{\partial t}\, dv + \int_{-\infty}^{t} G_t\, dv + \int_{-\infty}^{t} G_K \frac{\partial K(v, t)}{\partial t}\, dv$$

Since the marginal productivity of labor is equalized on all utilized vintages,

$$(31)\qquad \dot{Q}(t) = G_K\, K(t, t) + G_L \left[ L(t, t) + \int_{-\infty}^{t} \frac{\partial L(v, t)}{\partial t}\, dv \right] + \int_{-\infty}^{t} G_t\, dv + \int_{-\infty}^{t} G_K \frac{\partial K(v, t)}{\partial t}\, dv$$

$$= G_K\, K(t, t) + G_L\, \dot{L}(t) + \int_{-\infty}^{t} G_t\, dv + \int_{-\infty}^{t} G_K \frac{\partial K(v, t)}{\partial t}\, dv$$

The third term represents unembodied progress and the last term physical depreciation. Suppose that unembodied progress is Hicks neutral and occurs at the constant exponential rate $\mu$ . And suppose that depreciation also occurs at the constant exponential rate $\delta$ , so that

$\frac{\partial K(v, t)}{\partial t} = -\delta\, K(v, t)$ . Then

$$(32)\qquad \dot{Q}(t) = G_K\, K(t, t) + G_L\, \dot{L}(t) + \mu\, Q(t) - \delta \int_{-\infty}^{t} G_K\, K(v, t)\, dv$$

The integral in (32) is total quasi-rents accruing to capital. Hence, if we let $a(t)$ denote capital's relative share in gross income, and, further, let $p(t)$ denote $G_K$ , the marginal productivity of current investment, and substitute $w(t)$ , the real wage rate, for $G_L$ , the marginal productivity of labor, we finally obtain

(33) $\dot{Q}(t) = p(t)\ I(t) + w(t)\ \dot{L}(t) + [\mu - a(t)\ \delta]\ Q(t)$

This is a result of considerable generality. We first saw it derived in unpublished work by M. E. Yaari. (See [7] for use of this relationship.) It has been derived, on a certain continuity assumption ensuring that the wage rate measures the average product of labor on marginal machines, for "putty-clay" or no-*ex-post*-substitutability models by G. Pyatt in unpublished work. In an important special case of the above, rather general model, in which special case all embodied progress is purely capital-augmenting so that there exists an aggregate production function containing an aggregate effective capital stock, the result is easy to derive.[2]

In principle, $p(t)$, the marginal productivity of investment, is directly observable since it is equal to the rental "rate" on new machinery, $r(t, t)/q(t)$ under pure competition. In practice, we need another method of calculating $p(t)$.

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2. Suppose

$$Q(t) = e^{\mu t} F(J(t),\ L(t))$$

where

$$J(t) = \int_{-\infty}^{t} e^{-\delta(t-v)}\ B(v)\ I(v) dv\ .$$

Then

$$\begin{aligned} \dot{Q}(t) &= \mu\ Q(t) + e^{\mu t}\ F_J\ \dot{J}(t) + e^{\mu t}\ F_L\ \dot{L}(t) \\ &= \mu\ \dot{Q}(t) + e^{\mu t}\ F_J[B(t)\ I(t) - \delta\ J(t)] + e^{\mu t}\ F_L\ \dot{L}(t) \\ &= [\mu - \frac{e^{\mu t}\ F_J J(t)}{Q(t)}\ \delta]\ Q(t) + [\ e^{\mu t}\ F_J B(t)]\ I(t) + e^{\mu t}\ F_L\ \dot{L}(t) \\ &= [\mu - a(t)\ \delta]\ Q(t) + p(t)\ I(t) + w(t)\ \dot{L}(t) \end{aligned}$$

We use (33) for this purpose. Letting $\rho(t) = \mu - a(t)\delta$ we obtain from (33):

$$p(t) = \frac{\dot{Q}(t) - \rho(t)\, Q(t) - w(t)\, \dot{L}(t)}{I(t)} \tag{34}$$

Now ours is a full-employment model and $p(t)$ is a full-employment concept. If we can observe or synthesize observations of such a full-employment economy, everything on the right-hand side of (34) will be directly observable except $\rho(t)$ . In the next part we construct a full-employment picture of the economy. Our procedure, then, as will be seen, is to take alternative, constant values of $\rho$ and compute the $p(t)$ time series corresponding to each $\rho$ . These $p(t)$ series can be used to estimate the parameters of a specified vintage production function.

## The Factor Price Frontier

In order to estimate the parameters of the vintage production function, we have to specify the form of that function. We assume that this function is of the Cobb-Douglas form:

$$Q(v, t) = V_o\, e^{\mu t}\, (e^{-\delta(t-v)} B(v)\, I(v))^{\alpha}\, (L(v, t))^{1-\alpha} \tag{35}$$

where $B(0) = 1$ and restrictions upon $\dot{B}(t)/B(t)$ will be introduced shortly. $\alpha\dot{B}(t)/B(t)$ measures what was earlier called "true embodied plus quasi-embodied progress." Since $I(v)$ is the value of investment at time $\underline{v}$ in terms of consumption goods, i.e., money capital outlays deflated by the price of consumption goods, $B(v)$ could decrease with $\underline{v}$ rather than increase with $\underline{v}$ . (This point was made earlier in connection with (26).) The algebraic rate of increase of $B(t)$ is a measure of the rate at which the relative price of new capital goods is falling when "quality improvement" of new capital goods is taken into account.[3] We shall call $\dot{B}(t)/B(t)$ the rate of obsolescence,

3. This is not to suggest that the phenomenon of "capital-embodied" progress is <u>generally</u> just a matter of correcting capital-goods prices for quality change. This is true only if embodied change can be expressed as "capital-augmenting," as in (35). It is fallacious to think that all capital-embodied progress is necessarily capital-augmenting.

though it is due not only to quality improvement of new capital goods but also to differential unembodied progress in the capital-goods sector.

Our selection of the Cobb-Douglas function, with its property that embodied progress can be described as capital-augmenting, permits the construction of an aggregate production function. But we make no use of this fact in deriving the factor price frontier, which would exist for vintage production functions other than the Cobb-Douglas.

From (35) we have

$$(36) \qquad Q(t, t) = V_o e^{\mu t} (B(t) K(t, t))^{\alpha} (L(t, t))^{1-\alpha}$$

where $K(t, t) = e^{-\delta(t-t)} I(t) = I(t)$ . From this relation we can derive the factor price frontier relating the wage rate to the marginal productivity of investment.

From (36) we obtain

$$(37) \qquad p(t) = \frac{\partial Q(t, t)}{\partial K(t, t)} = \alpha V_o e^{\mu t} B(t)^{\alpha} \left(\frac{K(t, t)}{L(t, t)}\right)^{\alpha-1} .$$

$$(38) \qquad w(t) = \frac{\partial Q(t, t)}{\partial L(t, t)} = (1-\alpha) V_o e^{\mu t} B(t)^{\alpha} \left(\frac{K(t, t)}{L(t, t)}\right)^{\alpha}$$

To obtain the relationship between $w(t)$ and $p(t)$ we derive from (37) the equation

$$\frac{K(t,\ t)}{L(t,\ t)} = V_o^{\frac{1}{1-\alpha}}\ e^{\frac{\mu}{1-\alpha}t}\ \alpha^{\frac{1}{1-\alpha}}\ B(t)^{\frac{\alpha}{1-\alpha}}\ p(t)^{\frac{-1}{1-\alpha}} \tag{39}$$

and substitute this into (38) to obtain the factor-price frontier equation

$$w(t) = [\alpha^{\alpha}\ (1-\alpha)^{1-\alpha}\ V_o]^{\frac{1}{1-\alpha}}\ [e^{\mu t}\ B(t)^{\alpha}]^{\frac{1}{1-\alpha}}\ p(t)^{\frac{-\alpha}{1-\alpha}} \tag{40}$$

This has the inverse relation between $w(t)$ and $p(t)$ characteristic of all factor price frontiers.

A variant of (40) with which we shall work utilizes the fact that in our Cobb-Douglas world:

$$w(t) = (1-\alpha)\ \frac{Q(t)}{L(t)}\ . \tag{41}$$

From the last two equations we obtain

$$\frac{Q(t)}{L(t)} = [\alpha^{\alpha}\ V_o]^{\frac{1}{1-\alpha}}\ [e^{\mu t}\ B(t)^{\alpha}]^{\frac{1}{1-\alpha}}\ p(t)^{\frac{-\alpha}{1-\alpha}} \tag{42}$$

Concerning $B(t)$ , the simplest assumption is

$$B(t) = e^{\beta t} \tag{43}$$

Then $Q(t)/L(t)$ grows exponentially at rate $\frac{\mu+\alpha\beta}{1-\alpha}$ for a constant $p(t)$ (as would be the case in a golden age).

A weaker assumption allows for increasing or decreasing embodied progress:

$$B(t) = e^{\beta t + \nu t^2} \tag{44}$$

Either assumption leads to a convenient linear-logarithmic relationship between $Q(t)/L(t)$ and $p(t)$ for regression purposes. On the first assumption, (43), we have

$$(45) \qquad \ell n \frac{Q(t)}{L(t)} = \frac{1}{1-\alpha} \ell n(\alpha^{\alpha} V_o) + \left(\frac{\mu + \alpha\beta}{1-\alpha}\right) t - \frac{\alpha}{1-\alpha} \ell n\, p(t)$$

And, on the other assumption,

$$(46) \qquad \ell n \frac{Q(t)}{L(t)} = \frac{1}{1-\alpha} \ell n(\alpha^{\alpha} V_o) + \left(\frac{\mu + \alpha\beta}{1-\alpha}\right) t + \left(\frac{\alpha \nu}{1-\alpha}\right) t^2 - \frac{\alpha}{1-\alpha} \ell n\, p(t)$$

The estimation procedure can now be described. For each $\rho$ we calculate a $p(t)$ series. We then regress $\frac{Q(t)}{L(t)}$ on each $p(t)$ series. We choose that $\rho$ which gives the best fit. This optimal regression relationship yields direct estimates of $\alpha$, $\nu$ and $V_o$. Given the best $\rho = \mu - \alpha\delta$ and assuming that $\delta = .04$, we can infer $\mu$. Knowing $\mu$ and $\alpha$, we can infer $\beta$. Thus all parameters but $\delta$ are identified.

## THE ACTUAL AND 'POTENTIAL' DATA

The economy for our empirical purposes was the business sector excluding government enterprises, residential housing, and farming (including farm land rents to nonfarm landlords). The business sector is the total economy minus "general government" (which excludes public enterprises), "households and institutions" and the "rest of the world."

The pertinent data for this sector are shown in Table 1. The investment series includes the average annual amount of inventory investment, \$2.7 billion, in the period 1947-1962. The underlying investment expenditures in current

Table 1: Actual Data

| Year | t | Q | I | w | H | u |
|---|---|---|---|---|---|---|
| 1947 | -7 | 209.7 | 28.9 | 1.34 | 90.9 | 3.6 |
| 1948 | -6 | 216.0 | 29.6 | 1.36 | 92.2 | 3.8 |
| 1949 | -5 | 212.5 | 26.8 | 1.41 | 87.7 | 6.1 |
| 1950 | -4 | 233.2 | 29.9 | 1.49 | 90.7 | 5.3 |
| 1951 | -3 | 251.7 | 31.7 | 1.52 | 94.9 | 3.4 |
| 1952 | -2 | 258.3 | 32.1 | 1.58 | 96.4 | 3.0 |
| 1953 | -1 | 270.6 | 33.9 | 1.66 | 98.1 | 2.9 |
| 1954 | 0 | 262.6 | 32.5 | 1.70 | 95.1 | 5.6 |
| 1955 | 1 | 288.4 | 36.3 | 1.75 | 99.6 | 4.3 |
| 1956 | 2 | 294.3 | 40.7 | 1.79 | 102.0 | 4.0 |
| 1957 | 3 | 299.7 | 40.7 | 1.84 | 100.7 | 4.1 |
| 1958 | 4 | 289.7 | 32.8 | 1.87 | 95.8 | 6.5 |
| 1959 | 5 | 314.7 | 35.0 | 1.95 | 98.9 | 5.1 |
| 1960 | 6 | 321.4 | 37.8 | 2.03 | 98.8 | 5.1 |
| 1961 | 7 | 323.2 | 36.4 | 2.08 | 97.4 | 6.2 |
| 1962 | 8 | 349.8 | 39.0 | 2.14 | 100.3 | 5.0 |

t: time variable

Q: output in billions of 1954 dollars; see Appendix, Table 1.

I: investment in billions of 1954 dollars; see Appendix, Table 2.

w: hourly wage rate in 1954 dollars; see Appendix, Table 3.

H: billions of manhours; Council of Economic Advisers.

u: unemployment rate in percent, among experienced nonagricultural workers; B.L.S., Employment and Earnings, Annual Supplement, Table SA 30.

dollars have been deflated not by the price index for capital goods but rather by the implicit price deflator for the output of the sector as a whole. This is in the spirit of the model which evaluates investment in terms of consumption goods, i.e., which deflates investment outlays by the price of consumption goods.

Unfortunately these data cannot be used to estimate our model without seriously biasing downwards the capital elasticity estimate. The model predicts that a one percent increase in employment will produce only a $(1-\alpha)$ percent increase in output; the evidence shows that output will increase by some 3 percent if initially there was considerable unemployment. Students of this phenomenon have inferred that there is little or no ex post substitutability between capital and labor, that when there is sizeable unemployment of labor there is typically underutilization of capital goods, even new capital goods (which contradicts efficient competitive behavior in existing vintage models), and that corresponding to every kind of plant and equipment constructed there is a certain minimum, "overhead" amount of labor necessary if there is to be positive output at all. Our model fails to specify these phenomena: ex post substitutability is the same in our model as ex ante, which implies, given no ex ante overhead labor, the absence of overhead labor ex post and also, given pure competition, the full utilization of all new capital. (In our Cobb-Douglas case, efficiency demands that all vintages of capital be used.) We sought to minimize the biases resulting from these mis-specifications in the model by confining ourselves to "synthetic" observations of the economy at full capacity. But it should be emphasized that a superior procedure would have been to work with a correctly specified model.[4]

4. The present model does not correctly estimate the parameters of a "putty-clay" model lacking ex post substitutability because (36) implies equality of the real wage rate and the ex ante marginal productivity of labor on new machinery, unlike the situation in putty-clay models. See Phelps [8] for some economics of this kind of model.

Our task then was to synthesize observations of "potential" or full-capacity output, and also observations of potential man-hours and the wage rate.

We define potential output to be the output that would have occurred had there been a 3 percent unemployment rate. To estimate potential output we followed a procedure not unlike that used by Okun [5].

First, the ratio of actual to potential output, $Q(t)/\bar{Q}(t)$ , was assumed to be the following function of the unemployment rate:

$$\frac{Q(t)}{\bar{Q}(t)} = e^{g+hu(t) + ju(t)^2} \tag{47}$$

Where $\underline{g}$ was chosen so that $Q(t) = \bar{Q}(t)$ when $u = 3.0$ .

Second, for purposes of estimating $\underline{h}$ and $\underline{j}$ , it was assumed that the potential output path is the following function of time:

$$\bar{Q}(t) = e^{b+ct+dt^2+ft^3} \tag{48}$$

Hence

$$Q(t) = \frac{Q(t)}{\bar{Q}(t)} \bar{Q}(t) = e^{b+ct+dt^2+ft^3+g+hu(t)+ju(t)^2} \tag{49}$$

or

$$\ln Q(t) = b + g + ct + dt^2 + ft^3 + hu(t) + ju(t)^2 \tag{50}$$

Upon estimating this regression equation, we obtained

$$\ln Q(t) = 5.2336 + \underset{(.0067)}{.0595t} - \underset{(.0009)}{.0027t^2} + \underset{(.00003)}{.00008t^3} + \underset{(.0232)}{.0366u(t)} - \underset{(.0024)}{.0066u(t)^2} \tag{51}$$

$R = .9986$

where R is the correlation coefficient and the numbers in parentheses are the standard errors of the coefficients.

The coefficient g was obtained from the requirement that

(52) $$e^{g+h(3.0)+j(3.0)^2} = 1 ;$$

$g = -\ .0504$

$\overline{Q}(t)$ was then calculated from $Q(t)$ and (45), that is, from the formula

(53) $$\overline{Q}(t) = Q(t) \ [e^{-.0504 + .0366u(t) - .0066u(t)^2}]^{-1}$$

This calculated $\overline{Q}(t)$ series is shown in Table 2.

The potential wage rate, $\overline{w}(t)$, i.e., the wage rate that would have prevailed at an unemployment rate of 3 percent, was estimated in the same way. The regression equation is

(54) $$\ell n\ w(t) = (b+g) + ct + dt^2 + ft^3 + hu(t) + ju(t)^2 .$$

The estimated equation is

(55) $$\ell n\ w(t) = \underset{}{.2196} + \underset{(.0067)}{.0428t} - \underset{(.0009)}{.0011t^2} + \underset{(.00003)}{.00003t^3} + \underset{(.0231)}{.0107u(t)} - \underset{(.0024)}{.0014u(t)^2}$$

$R = .9985$

The coefficients of $u$ and $u^2$ are not significant but we carried out the calculation of $\overline{w}(t)$ anyway. From the requirement that $e^{g+h(3.0)+j(3.0)^2} = 1$ we calculated that $g = -\ .0195$ .

Then $\bar{w}(t)$ was calculated from the formula

(56) $$\bar{w}(t) = w(t)\,[e^{-.0195 + .0107u(t) - .0014u(t)^2}]^{-1}$$

This series, shown in Table 2, does not differ much from $w(t)$.

To compute $\bar{H}(t)$, the man-hours that would have been worked at a 3 percent unemployment rate, we followed an almost identical procedure. Here it was found that $u^3$ was significant and that $t^3$ was not. Our estimated regression equation was

(57) $$\ln H(t) = 3.952 \underset{(.0018)}{+ .0188t} \underset{(.0001)}{- .0006t^2} \underset{(.1120)}{+ .3710u(t)} \underset{(.0246)}{- .0813u(t)^2} \underset{(.0017)}{+ .0054u(t)^3}$$

$$R = .9895$$

From the requirement that $e^{g+h(3.0)+j(3.0)^2+k(3.0)^3} = 1$ we found $g = -.5271$. Then $\bar{H}(t)$ was calculated from the formula

(58) $$\bar{H}(t) = H(t)\,[e^{-.5271 + .3710u(t) - .0813u(t)^2 + .0054u(t)^3}]^{-1}$$

This series is also shown in Table 2.

Of course, since our object was to explain the growth of potential output in terms of time (technical progress) and actual investment expenditures, there was no adjustment of the $I(t)$ series.

We then computed the potential marginal productivity of investment, $\bar{p}(t)$. Even though our potential $\bar{Q}(t)$, $\bar{w}(t)$ and $\bar{H}(t)$ series are fairly smooth, computation of $\bar{p}(t)$ from (34) using annual changes of $\bar{Q}(t)$ and $\bar{H}(t)$ produces a very ragged series of $\bar{p}(t)$. We elected to compute $\bar{p}(t)$ from

Table 2: 'Potential' Data

| Year | $\bar{Q}$ | I | $\bar{W}$ | $\bar{H}$ |
|---|---|---|---|---|
| 1947 | 210.6 | 28.9 | 1.34 | 90.3 |
| 1948 | 217.4 | 29.6 | 1.36 | 91.7 |
| 1949 | 228.6 | 26.8 | 1.42 | 93.5 |
| 1950 | 243.2 | 29.9 | 1.49 | 94.5 |
| 1951 | 252.3 | 31.7 | 1.52 | 94.3 |
| 1952 | 258.3 | 32.1 | 1.58 | 96.4 |
| 1953 | 270.5 | 33.9 | 1.66 | 98.4 |
| 1954 | 276.8 | 32.5 | 1.71 | 100.1 |
| 1955 | 292.8 | 36.3 | 1.75 | 100.2 |
| 1956 | 297.2 | 40.7 | 1.79 | 101.8 |
| 1957 | 303.1 | 40.7 | 1.84 | 100.8 |
| 1958 | 317.5 | 32.8 | 1.89 | 102.5 |
| 1959 | 326.1 | 35.0 | 1.95 | 102.3 |
| 1960 | 333.0 | 37.8 | 2.03 | 102.2 |
| 1961 | 349.2 | 36.4 | 2.10 | 104.0 |
| 1962 | 361.4 | 39.0 | 2.14 | 103.3 |

three-year differences of $\bar{Q}(t)$ and $\bar{H}(t)$ . In particular, $\bar{p}(t)$ is computed from the formula

$$(59) \qquad \bar{p}(t) = \frac{\Delta \bar{Q}(t) - \rho \sum_{v=t}^{t+2} Q(v) - \bar{w}_{t+1} \Delta \bar{H}_t}{\sum_{v=t}^{t+2} I(v)}$$

where

$$\Delta \bar{Q}(t) = \frac{\bar{Q}(t+3) + \bar{Q}(t+4)}{2} - \frac{\bar{Q}(t) + \bar{Q}(t+1)}{2}$$

$$\Delta \bar{H}(t) = \frac{\bar{H}(t+3) + \bar{H}(t+4)}{2} - \frac{\bar{H}(t) + \bar{H}(t+1)}{2}$$

Note that $\bar{p}(t)$ is a kind of average of marginal productivities between $\underline{t}$ and $\underline{t} + 3$ . Because of the off-center character of $\Delta \bar{Q}$ and $\Delta \bar{H}$ here, our $\bar{p}(t)$ series may be biased upwards slightly. But it is hoped that they are accurate enough. Table 3 shows $\bar{p}(t)$ series for selected values of $\rho = \mu - \alpha \delta$ .

The table shows that if we assume rapid unembodied progress relative to depreciation, so that $\rho$ is large, then little of the growth is attributable to investment and $\bar{p}(t)$ is small. Our own a priori notions of $\bar{p}(t)$ and this table suggest to us that values of $\rho$ as low as -.02 or as high as .02 are implausible. The behavior of $\bar{p}(t)$ over time, for given $\rho$ , accords with our expectation that considerable capital-deepening occurred in the early postwar period and that the slackening of investment after 1957 led to a recovery of $\bar{p}(t)$ in the (last) interval, 1958-1961. These series do not "predict" investment behavior very well; possibly the degree of capacity utilization is a more important determinant of actual investment. But $\bar{p}(t)$ less depreciation and obsolescence rates (the competitive private and social rate of return) is presumably an important determinant of "potential" investment.

Table 3: 'Potential' Marginal Productivity of Investment

| Year | ρ = -.02 | ρ = -.01 | ρ = 0 | ρ = .005 | ρ = .01 | ρ = .02 |
|---|---|---|---|---|---|---|
| 1947 | .496 | .419 | .342 | .303 | .265 | .188 |
| 1948 | .489 | .409 | .329 | .289 | .249 | .169 |
| 1949 | .429 | .347 | .265 | .224 | .183 | .101 |
| 1950 | .359 | .278 | .198 | .157 | .117 | .037 |
| 1951 | .384 | .304 | .225 | .185 | .145 | .065 |
| 1952 | .413 | .331 | .250 | .209 | .168 | .086 |
| 1953 | .387 | .305 | .224 | .183 | .142 | .060 |
| 1954 | .367 | .288 | .209 | .169 | .130 | .051 |
| 1955 | .359 | .283 | .207 | .169 | .131 | .055 |
| 1956 | .404 | .323 | .243 | .203 | .162 | .082 |
| 1957 | .434 | .346 | .259 | .215 | .172 | .085 |
| 1958 | .479 | .386 | .294 | .248 | .202 | .109 |

## THE RESULTS

One procedure would have been to use (45) or (46), replacing $Q(t)$ by $\overline{Q}(t)$ , $L(t)$ by $\overline{H}(t)$ and using $\overline{p}(t)$ . But since $\overline{Q}(t)$ and $\overline{H}(t)$ may be somewhat unreliable, a superior procedure, we believe, was to assume that

$$(60) \qquad \frac{Q(t)/H(t)}{\overline{Q}(t)/\overline{H}(t)} = e^{g+hu(t)+ju(t)^2}$$

or

$$(61) \qquad \ell n\, Q(t)/H(t) = \ell n\, \overline{Q}(t)/\overline{H}(t) + g + hu(t) + ju(t)^2$$

Then, using (45), we have

$$(62) \qquad \ell n\, Q(t)/H(t) = g + \frac{1}{1-\alpha}\, \ell n(\alpha^{\alpha} V_o) + hu(t) + ju(t)^2 + \left(\frac{\mu + \alpha\beta}{1-\alpha}\right)t - \frac{\alpha}{1-\alpha}\, \ell n\, \overline{p}(t)$$

The remaining problem was that perhaps $\overline{p}(t)$ is related not to $\overline{Q}(t)/\overline{H}(t)$ but to $\overline{Q}(t+i)/\overline{H}(t+i)$ where $i$ , the lag, may be 1, 2, or even 3. This possibility occurs because our computed $\overline{p}(t)$ is an average of marginal productivities between <u>t</u> and $t + 3$ . So we explored this possiblility in our regressions; we also tried $i = -1$ in the (realized) hope that this would be inferior to $i \geq 0$ . (Of course, if $Q(t)/H(t)$ is lagged, so is $u(t)$.)

The results of using (62) for lags $i = -1$ , 0, 1, 2, 3 and $\rho = -.02$ , $-.01$, 0, .01, and .02 can be reported briefly. For every $\rho$ , the regression of $Q(t)/H(t)$ , rather than $Q(t+i)/H(t+i)$ , $i = -1$ , 1, 2, 3 on $\overline{p}(t)$ gives the best fit. Lags do not appear to be desirable. Second, the "best" $\rho$ , from the point of view of $R$ , the correlation coefficient, was $\rho = .02$ . The estimated regression equation for $i = 0$ , $\rho = .02$

is the following:

$$(63)\qquad \ell n\ Q(t)/H(t) = .967 \underset{(.023)}{-.038}\, u(t) + \underset{(.002)}{.003}\, u(t)^2 + \underset{(.001)}{.024}\, t \underset{(.006)}{-.032}\, \ell n\ \bar{p}(t)$$

$R = .9969$

While the coefficient of $\ell n\ \bar{p}(t)$, which is $-\frac{\alpha}{1-\alpha}$, is significantly less than zero, as it should be if $\alpha > 0$, no one would agree that $\alpha$ is on the order of .03. The coefficients of $u$ and $u^2$ are also unsatisfactory; they are insignificantly different from zero. The only credible estimate here is the time coefficient, $\frac{\mu + \alpha\beta}{1-\alpha}$. According to (61), productivity would have increased at 2.4 percent per annum had $u(t)$ and $\bar{p}(t)$ been constant; this is the "golden-age" growth rate of productivity.

Probably the explanation of the disappointing results in (63) is that, were technical progress really constant, the steadiness in the growth of potential productivity in the face of the sharp rise in $\bar{p}(t)$ toward the end of the period (due to the slackening of investment) could only be explained by attributing little importance to $\bar{p}(t)$, that is, little importance to capital.

Let us then drop the assumption that technical progress is constant and allow for the possibility of increasing or decreasing progress as shown in (46). Equations (46) and (61) yield the new regression equation

$$(64)\qquad \ell n\ Q(t)/H(t) = g + \frac{1}{1-\alpha}\ \ell n\ (\alpha^{\alpha} V_o) + hu(t) + ju(t)^2 + \left(\frac{\mu + \alpha\beta}{1-\alpha}\right) t + \left(\frac{\alpha\,\nu}{1-\alpha}\right) t^2 - \left(\frac{\alpha}{1-\alpha}\right) \ell n\ \bar{p}(t)$$

Once again we found that $Q(t)/H(t)$ (and $u(t)$ ) without lag or lead correlated best with $\bar{p}(t)$ . The estimates for these unlagged equations are shown in Table 4. We tried $\rho = .005$ thinking that the optimum might lie between $\rho = 0$ and $\rho = .01$ . As the F ratios in that table indicate, $\rho = 0$ gives the best fit.

A likelihood ratio test of the "alternative" hypothesis that $t^2$ adds significantly to the explained variation of the dependent variable and that $\rho = 0$ against the null hypothesis, (63), that $t^2$ does not add significantly and that $\rho = .02$ passed at the 95 percent confidence level, indicating that the model with $t^2$ is superior.

The chief effects of introducing $t^2$ are to increase the absolute value of the $\ell n\, \bar{p}(t)$ coefficient (thus increasing $\alpha$) for every $\rho$ ; to decrease the value of $\rho$ which gives the best fit (from .02 to 0); and to increase the significance of the $u(t)$ and $u(t)^2$ coefficients. The $t^2$ coefficient, which is statistically significant, indicates that the rate of technical progress of the embodied kind was increasing over the period of observation.

Our last step was to introduce $u(t)^3$ into the regression equation in the hope of obtaining a significantly tighter fit. Our new equation is

$$(65) \qquad \ell n\, Q(t)/H(t) = g + \frac{1}{1-\alpha}\, \ell n\, (\alpha^{\alpha} V_o) + hu(t) + ju(t)^2 + ku(t)^3 + \left(\frac{\mu + \alpha\beta}{1-\alpha}\right)t + \left(\frac{\alpha\nu}{1-\alpha}\right)t^2 - \frac{\alpha}{1-\alpha}\, \ell n\, \bar{p}(t)$$

Table 4: Estimation of Equation (64)

| $\rho$ | Constant term | h | j | $\frac{\mu + \alpha \beta}{1-\alpha}$ | $\frac{\alpha \nu}{1-\alpha}$ | $\frac{-\alpha}{1-\alpha}$ | R | F |
|---|---|---|---|---|---|---|---|---|
| -.02 | .918 | -.083 (.031) | .0081 (.0032) | .019 (.003) | .0014 (.0008) | -.247 (.074) | .9973 | 225.0 |
| -.01 | .894 | -.079 (.027) | .0076 (.0028) | .019 (.003) | .0014 (.0007) | -.212 (.055) | .9978 | 273.7 |
| 0 | .889 | -.073 (.025) | .0068 (.0025) | .019 (.002) | .0014 (.0006) | -.164 (.040) | .9980 | 301.6 |
| .005 | .896 | -.068 (.024) | .0063 (.0025) | .019 (.002) | .0012 (.0006) | -.134 (.033) | .9980 | 300.0 |
| .01 | .908 | -.062 (.024) | .0057 (.0024) | .019 (.002) | .0011 (.0006) | -.105 (.026) | .9979 | 288.3 |
| .02 | .963 | -.051 (.025) | .0046 (.0026) | .022 (.002) | .0006 (.0005) | -.044 (.013) | .9974 | 231.8 |

Table 5 records our estimates of the coefficients of this equation. The closeness of fit is rather insensitive to $\rho$ but the best fit occurs for $\rho = .005$ , which is about the same value of $\rho$ as was best for the previous model without $u(t)^3$ .

A likelihood ratio test of the hypothesis that $u(t)^3$ is significant and $\rho = .005$ against the null hypothesis that $u(t)^3$ has no influence and $\rho = 0$ passed at the 99 percent confidence level.

While Table 5 does not give much confidence that $\rho = .005$ , this is a reasonable value and we shall draw this conclusion for purposes of illustration. The Cobb-Douglas parameters are shown in Table 6 on the assumption that $\delta = .04$ ; $V_o$ is

Table 6: Estimates of the Cobb-Douglas Parameters

| $\rho$ | $\alpha$ | $\mu = \rho + \alpha\delta$ | $\alpha\beta$ | $\beta$ | $\nu$ | $V_o$ |
|---|---|---|---|---|---|---|
| .005 | .130 | .010 | .0065 | .050 | .006 | 2.42 |

computable from the convention that $g + hu + ju^2 + ku^3 = 0$ for $u = 3.0$ , from which we obtain $g = - .387$ .

There are two results that are suspect in Table 6. First $\alpha$ is only .13 while Denison [2] suggests that capital's relative share (excluding land) is .25. In our <u>ex post</u> substitutability model (but only there), $\alpha$ will equal capital's share under pure competition. Therefore, if our estimate of $\alpha$ is at all near the mark, there must be a very large element of monopoly rent in

Table 5: Estimation of Equation (65)

| $\rho$ | Constant term | h | j | k | $\frac{\mu + \alpha\beta}{1-\alpha}$ | $\frac{\alpha\nu}{1-\alpha}$ | $\frac{-\alpha}{1-\alpha}$ | R | F |
|---|---|---|---|---|---|---|---|---|---|
| -.02 | .535 | .180 (.251) | -.051 (.056) | .004 (.004) | .020 (.003) | .0011 (.0008) | -.259 (.074) | .9978 | 191.7 |
| -.01 | .446 | .227 (.213) | -.061 (.048) | .005 (.003) | .019 (.002) | .0011 (.0007) | -.226 (.051) | .9985 | 269.1 |
| 0 | .370 | .280 (.184) | -.073 (.041) | .006 (.003) | .019 (.002) | .0011 (.0005) | -.180 (.034) | .9989 | 366.4 |
| .005 | .323 | .321 (.171) | -.082 (.039) | .006 (.003) | .019 (.002) | .0009 (.0005) | -.150 (.026) | .9990 | 426.7 |
| .01 | .314 | .341 (.173) | -.086 (.039) | .007 (.003) | .020 (.002) | .0008 (.0005) | -.119 (.021) | .9990 | 420.0 |
| .02 | .259 | .432 (.188) | -.105 (.042) | .008 (.003) | .022 (.002) | .0002 (.0004) | -.053 (.010) | .9989 | 377.2 |

capital's earnings in the nonfarm business sector.[5] But probably our $\alpha$

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5. This suggests that $\bar{w}(t)$ underestimates the marginal productivity of labor so that, among other things, our estimate of $\bar{p}(t)$ is erroneous. But since $\Delta \bar{H}_t$ in (57) is small, as Table 2 shows, little bias results from taking $\bar{w}(t)$ as a measure of the marginal product of labor.

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estimate is too small.[6]

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6. On the other hand, Berglas also estimated $\alpha$ to be .13 when he assumed an "improvement rate," roughly our $\dot{B}(t)/B(t)$, equal to 3 percent. While Berglas rejects 3 percent in favor of zero, 3 percent is not unreasonable.

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Second, $\nu$ is very large. It implies that $\dot{B}(t)/B(t)$ has been increasing rapidly. From (44) we have

$$\frac{\dot{B}(t)}{B(t)} = \beta + 2\,\nu\,t \qquad (66)$$

$$= .050 + .012\,t$$

Recalling that $t = 0$ in 1954, we obtain the series, $\dot{B}(t)/B(t)$, in Table 7.

Table 7: The Rate of Obsolescence: $\dot{B}(t)/B(t)$

| 1947 | 1948 | 1949 | 1950 | 1951 | 1952 | 1953 | 1954 | 1955 | 1956 | 1957 | 1958 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| -.034 | -.022 | -.010 | .002 | .014 | .026 | .038 | .050 | .062 | .074 | .086 | .098 |

Now there is no reason why $\dot{B}(t)/B(t)$ could not be negative; this means merely that the price of capital goods, corrected for quality improvement, has been rising relative to consumption goods. But published price indices indicate that $\dot{B}(t)/B(t)$ should not be algebraically smaller than about -1 percent per annum. So $\nu$ seems too large.[7] If $\frac{\alpha}{1-\alpha}$ had been as large as relative

7. On the other hand, Taylor [11] shows that unembodied progress was much greater in the consumption-good sector than in the capital-good sector in the period 1947-1953 so that conceivably $\dot{B}(t)/B(t)$ was as large and negative as our results indicate in the early postwar period. (Taylor's analysis does not extend beyond 1953.)

shares suggest, $\nu$ would have been about half as large and this surprising result of negative obsolescence would not have occurred.

As equations (36) or (42) indicate, the rate of Hicks-neutral embodied technical progress, comparable to the Hicks-neutral $\mu$ , is $\alpha\,\dot{B}(t)/B(t)$ . This is given in Table 8.

Table 8: The Rate of Embodied Progress: $\alpha\,\dot{B}(t)/B(t)$

| 1947 | 1948 | 1949 | 1950 | 1951 | 1952 | 1953 | 1954 | 1955 | 1956 | 1957 | 1958 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| -.004 | -.003 | -.001 | .000 | .002 | .003 | .005 | .007 | .008 | .010 | .011 | .013 |

Table 8 shows that, initially, embodied progress was a negligible part of total technical progress, $\mu + \alpha\,\dot{B}(t)/B(t)$ , even slightly negative, and eventually came to exceed unembodied progress.

These explorations of the implications of our results would not be complete without computation of the social rate of return to investment.

This is the marginal productivity of investment less the rate of depreciation, $\delta$ , and the rate of obsolescence, $\dot{B}(t)/B(t)$ . Using the $\bar{p}(t)$ series for $\rho = .005$ we obtain the rate of return series shown in Table 9.

Table 9: The Social Rate of Return: $\bar{p}(t) - \delta - \dot{B}(t)/B(t)$

| 1947 | 1948 | 1949 | 1950 | 1951 | 1952 | 1953 | 1954 | 1955 | 1956 | 1957 | 1958 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| .297 | .271 | .194 | .115 | .131 | .143 | .105 | .079 | .067 | .089 | .089 | .110 |

The social rate of return was elsewhere estimated by Phelps [6]. It was found that if $\alpha = .15$ , then the 1954 rate of return was approximately .078 for all "improvement" or "obsolescence" rates between 0 and 3 percent. To this figure Phelps should have added one percent to reflect the average one percent rise in the measured price of capital goods relative to the price of consumption goods. Hence the correct estimate by that method is about .09. This compares rather well with the estimate in Table 9 for the interval 1953-56, which is .105.

## CONCLUDING REMARKS

We have acknowledged that some of our results are surprising, even suspect, although there is some precedent for them in the work of Berglas [1] and Taylor [11]. We shall cite here some of the reasons why no great confidence can be placed in these results and thus indicate some of the improvements that are needed.

First, alhtough it may not be of great quantitative importance, there is a statistical flaw in our method. This is the simultaneous-equation bias in our estimate of the capital elasticity. While our approach takes $\bar{p}(t)$ to be an independent variable, both $\bar{p}(t)$ and $\tilde{Q}(t)/\bar{H}(t)$ are really dependent variables in the underlying model; they are (presumably stochastic) functions of the ratio of effective capital to labor and of time. So, presumably, $\bar{p}(t)$ is correlated with the error term in our regression equation, contrary to the classical assumption in ordinary least-squares analysis. On certain reasonable assumptions, it can be shown that this correlation with the error term biases downward our estimate of the capital elasticity. Hence, there is a statistical reason for believing that our estimate of the capital elasticity is too small. But the extent of the bias is unknown. This requires investigation.

Second, inaccuracies in the data may have biased our results. It seems especially likely that there are random errors in the "observations" of $\bar{p}(t)$ . These will bias our estimate of the capital elasticity toward zero. Further, there may be nonrandom errors of observation. There is a suspicion, we understand, that the upward trend of the official series on producers' durable equipment expenditures since 1954 is too small because of failure to revise the sample of firms in the official survey. If so, this accounts for some of the rise of $\bar{p}(t)$ after 1954. As explained earlier, it is probably the maintenance of a high rate of growth of productivity in the latter half of our period of observation in the face of the sharp rise of $\bar{p}(t)$ that accounts for our estimate of accelerating technology. Hence our finding of an increasing rate of (embodied) technical progress may be due in part to an error in the trend of our investment series.

Lastly, the underlying economic model is not satisfactory. It would be preferable, though far more difficult, to estimate a "putty-clay" model -- one in which there is little or no ex post substitutability -- possibly of the Cobb-Douglas type analyzed by Phelps [8]. It would also be an improvement to allow the substitution elasticity to differ from unity; but factor-price-frontier estimation of a constant-elasticity-of-substitution model is substantially more difficult than estimation of a Cobb-Douglas model. Further, the assumption of identical factor intensities in the capital and consumption goods sectors -- on which the aggregation of a two-sector model into a one-sector model depends -- flies in the face of some evidence that the capital-goods sector is more labor-intensive. Use of a two-sector model would be a step in the right direction. Another improvement would be to make potential output a function of potential labor services and potential capital services, rather than of the stock of capital; our procedure would be correct only if there were no change in the full-employment work week and the number of shifts. Finally, although many more improvements could be listed, it would be desirable to make technical progress a function of research and of education since one is interested in the payoffs to alternative kinds of investment. Our results are unlikely to be invariant to these improvements of the model.

Despite these statistical and theoretical objections, and hence our reservations about our empirical results, it is hoped that our factor-price-frontier method will prove a useful addition to the arsenal of methods of estimating the parameters of vintage production models.

## REFERENCES

[1] Berglas, Eitan, "Investment and Technological Change," Journal of Political Economy, vol. 73, No. 2 (April, 1965).

[2] Denison, Edward F., "Capital Theory and the Rate of Return," American Economic Review, vol. 54, no. 5 (September, 1964).

[3] Fisher, Franklin M., "Embodied Technical Change and the Existence of an Aggregate Capital Stock," Review of Economic Studies,

[4] Green, H. A. John, Aggregation in Economic Analysis: An Introductory Survey (Princeton, 1964).

[5] Okun, Arthur M., "Potential GNP: The Measurement and Significance," 1962 Proceedings of the Business and Economic Statistics Section of the American Statistical Association.

[6] Phelps, Edmund S., "The New View of Investment: A Neoclassical Analysis," Quarterly Journal of Economics, vol. 73, no. 4 (November, 1962).

[7] ________ and M. E. Yaari, "'The New View of Investment': Reply," Quarterly Journal of Economics, vol. 75, no. 1 (February, 1965).

[8] ________, "Substitution, Fixed Proportions, Growth and Distribution," International Economic Review, vol. 4, no. 3 (September, 1963).

[9] Samuelson, Paul A., "Parable and Realism in Capital Theory: The Surrogate Production Function," Review of Economic Studies, vol. 29, no. 3 (June, 1962).

[10] Solow, Robert M., "Investment and Technical Progress," ed., K. J. Arrow, et al., Mathematical Methods in the Social Sciences, 1959 (Stanford, 1960).

[11] Taylor, Vincent D., "Technical Progress in the Capital Goods and Consumer Goods Sectors of the United States," RAND Corporation P-3012, November, 1964.

APPENDIX

TABLE 1

Q: Nonfarm Business Output

| Year | (1) Business Output Billions 1954 dollars | (2) FGP and OGE Billions 1954 dollars | (3) Rents to Nonfarm Landlords Billions 1954 dollars | (4) Residential Rents Current dollars | (5) GNP Deflator 1954=100 | (6) Residential Rents Billions 1954 dollars | (7) Q Billions 1954 dollars |
|---|---|---|---|---|---|---|---|
| 1947 | 250.2 | 20.2 | 1.5 | 15.6 | 83.0 | 18.8 | 209.7 |
| 48 | 260.4 | 23.1 | 1.4 | 17.6 | 88.5 | 19.9 | 216.0 |
| 49 | 258.3 | 22.3 | 1.6 | 19.3 | 88.2 | 21.9 | 212.5 |
| 1950 | 281.8 | 23.3 | 1.6 | 21.2 | 89.5 | 23.7 | 233.2 |
| 51 | 299.4 | 22.1 | 1.5 | 23.2 | 96.2 | 24.1 | 251.7 |
| 52 | 308.6 | 22.8 | 1.6 | 25.4 | 98.1 | 25.9 | 258.3 |
| 53 | 323.7 | 23.7 | 1.6 | 27.5 | 99.0 | 27.8 | 270.6 |
| 54 | 317.9 | 24.6 | 1.6 | 29.1 | 100.0 | 29.1 | 262.6 |
| 1955 | 346.2 | 25.8 | 1.7 | 30.7 | 101.2 | 30.3 | 288.4 |
| 56 | 352.7 | 25.4 | 1.7 | 32.7 | 104.6 | 31.3 | 294.3 |
| 57 | 359.1 | 25.3 | 1.6 | 35.2 | 108.4 | 32.5 | 299.7 |
| 58 | 351.2 | 25.6 | 1.9 | 37.7 | 110.8 | 34.0 | 289.7 |
| 59 | 377.4 | 25.7 | 1.8 | 39.6 | 112.6 | 35.2 | 314.7 |
| 1960 | 386.8 | 26.8 | 1.9 | 41.9 | 114.2 | 36.7 | 321.4 |
| 61 | 392.8 | 26.9 | 1.8 | 44.2 | 115.8 | 38.2 | 323.2 |
| 62 | 418.5 | 27.1 | 1.8 | 46.5 | 116.7 | 39.8 | 349.8 |

Sources and explanatory notes:

(1) Survey of Current Business (SCB), October 1962, Table 4, p. 12 for 1947-1960 data. SCB, July 1964, Table I-13 for 1961 and 1962 data. Gross national product less output from households and institutions, from general government and from the rest of the world.

(2) Farm Gross Product and Output of Government Enterprises SCB, October 1962, Table 4, p. 12 for 1947-1960 data. The data for 1961 and 1962 were estimated from data in Table I-15 (gross farm product) and in Table I-12 (government enterprises) in the SCB, July 1964. The Table I-12 data were deflated by the general government price deflator and linked to the 1960 government enterprise figure.

(continued on next page)

APPENDIX

Table 1 (Cont.)

Sources and explanatory notes:

(3) U.S. Income and Output (USIO) Table VII-10 line 8 for 1947-1955 data. SCB, July 1961 for 1956-1958 data. SCB, July 1964 for 1959-1962 data.

(4) USIO, Table II-4 line 21 for 1947-1955 data. SCB, July 1961 for 1956-58 data. SCB, July 1964 for 1959-1962 data.

(5) USIO, Table VII-2 for 1947-1955 data. SCB, July 1961 for 1956-1958 data. SCB, July 1964 for 1959-1962 data.

(6) = (4) ÷ (5)

(7) = (1)-(2)-(3)-(6)

TABLE 2

I: Nonfarm Business Investment (Including Average Inventory Investment)

Deflated By Business Sector Deflator

| Year | (1) Nonfarm Business Investment Billions 1954 dollars | (2) PDE Deflator 1954=100 | (3) RNFC 1954=100 | (4) Investment Deflator 1954=100 | (5) Nonfarm Business Investment Billions current dols. | (6) Business Deflator 1954=100 | (7) Nonfarm Business Investment Billions 1954 dols. | (8) Net Change in NFBI Billions current dollars | (9) Net Change in NFBI Billions 1954 dollars | (10) I Billions 1954 dollars |
|---|---|---|---|---|---|---|---|---|---|---|
| 1947 | 28.3 | 76.8 | 78.4 | 77.6 | 22.0 | 84.2 | 26.2 | 1.3 | 1.5 | 28.9 |
| 48 | 28.2 | 83.1 | 88.6 | 85.9 | 24.2 | 89.9 | 26.9 | 3.0 | 3.3 | 29.6 |
| 49 | 24.8 | 87.0 | 85.9 | 86.5 | 21.5 | 89.2 | 24.1 | -2.2 | -2.5 | 26.8 |
| 1950 | 27.0 | 89.0 | 90.9 | 90.9 | 24.5 | 90.2 | 27.2 | 6.0 | 6.7 | 29.9 |
| 51 | 29.0 | 96.8 | 97.5 | 97.2 | 28.2 | 97.2 | 29.0 | 9.1 | 9.4 | 31.7 |
| 52 | 29.3 | 97.5 | 100.3 | 98.9 | 29.0 | 98.8 | 29.4 | 2.1 | 2.1 | 32.1 |
| 53 | 30.9 | 99.0 | 101.3 | 100.2 | 31.0 | 99.3 | 31.2 | 1.1 | 1.1 | 33.9 |
| 54 | 29.8 | 100.0 | 100.0 | 100.0 | 29.8 | 100.0 | 29.8 | -2.1 | -2.1 | 32.5 |
| 1955 | 33.0 | 102.6 | 103.0 | 102.8 | 33.9 | 100.8 | 33.6 | 5.5 | 5.5 | 36.3 |
| 56 | 36.2 | 109.0 | 109.0 | 109.0 | 39.5 | 104.0 | 38.0 | 5.1 | 4.9 | 40.7 |
| 57 | 36.0 | 115.7 | 111.2 | 113.5 | 40.9 | 107.6 | 38.0 | .8 | .7 | 40.7 |
| 58 | 28.6 | 118.9 | 111.2 | 115.1 | 32.9 | 109.4 | 30.1 | -2.9 | -2.6 | 32.8 |
| 59 | 30.4 | 121.4 | 114.3 | 117.9 | 35.8 | 111.0 | 32.3 | 6.5 | 5.9 | 35.0 |
| 1960 | 33.2 | 121.6 | 115.5 | 118.6 | 39.4 | 112.1 | 35.1 | 3.2 | 2.9 | 37.8 |
| 61 | 32.1 | 121.3 | 115.9 | 118.6 | 38.1 | 113.2 | 33.7 | 1.5 | 1.3 | 36.4 |
| 62 | 34.7 | 120.9 | 117.2 | 119.1 | 41.3 | 113.9 | 36.3 | 5.3 | 4.7 | 39.0 |

Sources and explanatory notes:

(1) Council of Economic Advisers

(2) Producers Durable Equipment deflator, USIO, Table VII-2, line 10.

(continued on next page)

Table 2 (Cont.)

Sources and explanatory notes:

(3) Residential Non-Farm Construction deflator, USIO, Table VII-2, line 8. USIO source of deflator data for 1947-1955. SCB, July 1961 source for 1956-1958. SCB, July 1964 source for 1959-1962.

(4) (Col. (2) + Col. (3)) ÷ 2

(5) Col. (1) x Col. (4).

(6) USIO Table VII-8, line 2 for 1947-1955 data. SCB, July 1961 for 1956-1958 data. SCB, July 1964 for 1959-1962 data.

(7) Col. (5) ÷ Col. (6).

(8) Net Change In Non-Farm Business Inventories, USIO Table V-8, line 3 for 1947-55 data. SCB, July 1961 for 1956-1958 data. SCB, July 1964 for 1959-1962 data.

(9) Col. (8) ÷ Col. (6).

(10) Col. (7) + 2.7. 2.7 is the average of Col. (9).

TABLE 3

w: Real Hourly Wage Rate in the U. S. Nonfarm Business Sector

| Year | (1) Wage Bill Billions Current Dollars | (2) Manhours Billions | (3) Hourly Wage Rate Current Dollars | (4) Business Deflator 1954=100 | (5) w 1954 Dollars |
|---|---|---|---|---|---|
| 1947 | 102.3 | 90.9 | 1.13 | 84.2 | 1.34 |
| 48 | 112.7 | 92.2 | 1.22 | 89.9 | 1.36 |
| 49 | 110.2 | 87.7 | 1.26 | 89.2 | 1.41 |
| 1950 | 121.7 | 90.7 | 1.34 | 90.2 | 1.49 |
| 51 | 140.5 | 94.9 | 1.48 | 97.2 | 1.52 |
| 52 | 150.6 | 96.4 | 1.56 | 98.8 | 1.58 |
| 53 | 162.3 | 98.1 | 1.65 | 99.3 | 1.66 |
| 54 | 161.3 | 95.1 | 1.70 | 100.0 | 1.70 |
| 1955 | 174.8 | 99.6 | 1.76 | 100.8 | 1.75 |
| 56 | 190.1 | 102.0 | 1.86 | 104.0 | 1.79 |
| 57 | 199.4 | 100.7 | 1.98 | 107.6 | 1.84 |
| 58 | 196.7 | 95.8 | 2.05 | 109.4 | 1.87 |
| 59 | 214.8 | 98.9 | 2.17 | 111.0 | 1.95 |
| 1960 | 225.4 | 98.8 | 2.28 | 112.1 | 2.03 |
| 61 | 229.2 | 97.4 | 2.35 | 113.2 | 2.08 |
| 62 | 245.1 | 100.3 | 2.44 | 113.9 | 2.14 |

Sources and explanatory notes:

(1) SCB, October 1962, Table 6, pp. 14-15 for 1947-1960 data. Total employee compensation less employee compensation on farms, households and institutions, government, government enterprises and rest of world. Figures for 1961 and 1962 were estimated from data in SCB, July 1964, Table VI-1. The wages bill for the nonfarm business sector was approximated by subtracting compensation of employees in agriculture, forestry and fisheries (line 2), private households (line 69), medical and other

(continued on next page)

Table 3 (Continued)

Sources and explanatory notes:

(1) (cont.)

health services (line 75), educational services, n.e.c. (line 78), nonprofit membership organizations, n.e.c. (line 79), government and government enterprises (line 80), and rest of world (line 85) from all industries (line 1). The figures estimated from the July 1964 SCB were linked to the October 1962 SCB series at 1960.

(2) Council of Economic Advisers.

(3) Col. (1) ÷ Col. (2).

(4) See Appendix Table 2, footnote 6.

(5) Col. (3) ÷ Col. (4).