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AN EXPERIMENTAL STUDY IN OLIGOPOLY

James W. Friedman

September 23, 1964

AN EXPERIMENTAL STUDY IN OLIGOPOLY[1]

James W. Friedman[2]

A very general model with which many theories of oligopolistic behavior might be tested would be a useful tool for economists. In the first part of this paper a model is proposed for this rôle. The model is not developed in the fullest degree of generality, but is presented in the way most relevant to experiments of the type presented in the second part of the paper. The experiments employ a simple market model in which each subject takes the role of a business firm, each firm has only one decision variable under its control (its own price) and the number of firms in an industry is two, three and four.

1. A General Behavioral Model for Oligopoly

In order to present the behavioral model in a form directly applicable to the experiments it is necessary to present first the market model utilized in them.

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1. Research undertaken by the Cowles Commission for Research in Economics under Contract Nonr-3055(00) with the Office of Naval Research.

2. I have incurred many debts in the course of this research. At one time or another probably all of my colleagues at the Cowles Foundation have been subjected to a model or results or some problem of mine. Any virtues of the experimental portion of this paper are due to helpful advice, encouragement and arguments with Professors William J. Fellner and Martin Shubik, and Dr. George J. Feeney. Professor John W. Hooper and Laurence E. Lynn offered helpful advice at early stages. The general model offered here owes its existence to the constructive needling of Professors James Lee Pierce and Donald D. Hester. They read a recent draft and made many additional suggestions which have improved both analysis and exposition. Mrs. Stephen Joseph was much help in carrying out the experiments and Charles A. Bakewell in carrying out computations for the analysis of results.

1.1 The Market Model

The market model used in the experiments is one in which demand is linear in the prices of the firms, marginal cost is a linear function of output, no balance sheet variables appear, and entry and exit are not allowed. The demand function for the i-th firm is:

$$q_i = a_1 - a_2 p_i + a_3 \bar{p} \qquad (i = 1, \ldots, n) \tag{1}$$

where q_i = the unit sales (and output) of the i-th firm

p_i = the price charged by the i-th firm

$$\bar{p} = \frac{1}{n} \sum_{i=1}^{n} p_i$$

$$a_1, a_2, a_3 > 0$$

$$a_2 > a_3$$

This demand function has the characteristics specified by Chamberlin [1] for a firm selling a differentiated product. The well known Chamberlin diagram (see [1], page 91) is shown at left. The dd' curve in Figure 1 results when the prices of all firms but the i-th are held constant at p^* (or when the mean of the other n-1 prices is p^*). The DD' curve is obtained by assuming all n-1 other prices equal p_i (or their mean equals p_i).

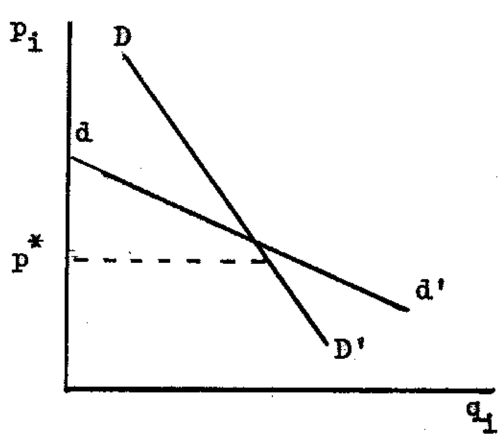

Figure 1

The cost function used in the experiments in

$$C_i = c_1 + c_2 q_i + c_3 q_i^2 \qquad (i = 1, \ldots, n) \tag{2}$$

where C_i = total cost for the i-th firm

$$c_1, c_2, c_3 \geq 0$$

This cost function gives rise to linear, increasing marginal cost for the firm.

The firms are symmetric. That is, the parameters of cost and demand function are the same for all i. There are two reasons for using a model which is structurally simple. Experiments using a carefully designed simple model may yield fundamental insights into behavior which carry over into more complex models. For example, from the experiments reported below, it appears the probability that a group of firms will make a jointly maximal decision decreases as the number of firms in the industry increases. It would be surprising if this result did not carry over into more complex models, although the particular probabilities may alter with the structure of the market.

The second reason for beginning with simple market models and moving toward the more complex is that this allows aspects of behavior to be attributed to their rightful sources. It can be seen how changes of structure are reflected in behavior.

1.2 The Behavioral Model

In the course of an experiment, subjects are divided into groups. Within each group, the subjects represent individual firms within the same industry. A group of subjects remain in the same industry for many "periods."

A period consists of the following steps: a) each subject chooses a price and, while so doing, is not informed of the choices being made by the others, b) the decisions are collected by personnel conducting the experiment, and c) each subject is told the prices chosen by the others and the profit earned by each firm. A "game" is defined as a set of periods during which the division of subjects into industries is unchanged, and the parameters of the market model are unchanged.

The behavioral model consists of treating the game as a Markov process.[1] It is assumed the industry can be in any one of N^* "states," $S_1, \ldots, S_{N^*}$ and there is a matrix of transition probabilities, $P = ||\pi_{ij}||$ $(i, j = 1, \ldots, N^*)$, where π_{ij} gives the conditional probability that the industry will be in state S_j in the next period given that it is in state S_i during the current period. And $\sum_{j=1}^{N^*} \pi_{ij} = 1, \pi_{ij} \geq 0$ for all i, j.

1.2.1 A Partial Theory of Behavior

The subject of this section is only a partial theory because a full-fledged, well defined theory would include a precise specification of how subjects should behave under given circumstances and the prescribed behavior would be derived from some prior principle(s) of behavior. It is hoped this defect will be remedied in the future.

From equations (1) and (2) in Section 1 a profit function may be written for each firm in which the profit of a firm depends upon all

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1. See Feller [3], Chapter XV

prices:

$$(3) \qquad \Pi_{it} = p_{it}\, q_{it} - C_{it} = \Pi_i\,(p_{1t}, \ldots, p_{nt}) \qquad (i = 1, \ldots, n)$$

The subscript, t, denotes a time period. It is assumed the firm wishes to maximize its profit stream over the course of the game, and that to do so it sets its price so as to maximize the following expression in the t-th period:

$$(4) \qquad \Pi_i(p_{1,\,t-1}, \ldots, p_{i-1,\,t-1}, p_{it}, p_{i+1,\,t-1}, \ldots, p_{n,\,t-1}) + \rho_{it} \sum_{j \neq i} \Pi_j(p_{1,\,t-1}, \ldots p_{i-1,\,t-1}, p_{it}, p_{i+1,\,t-1}, \ldots, p_{n,\,t-1})$$

The subject is assumed to act as if he expects all other subjects to repeat their prices of the previous period, and to maximize the sum of his (expected) profit plus a parameter (ρ_{it}) multiplied by the sum of the (expected) profits of the other subjects.[1] The questions raised here are: 1) Why assume the other firms will repeat their previous period prices? 2) Why maximize an expression like (4)? and 3) How is ρ_{it} chosen? These questions are related.

Taking last things first, ρ_{it} is assumed to be a function of the $\rho_{j,\,t-1}$ $(j \neq i)$. The subject is assumed to act as if he realizes that the ρ_i he chooses will affect subsequent choices of ρ_j by others. For

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1. This formulation is a generalization of some familiar solution concepts. Note that $\rho_1 = \ldots = \rho_n = 0$ is the noncooperative solution and $\rho_1 = \ldots = \rho_n = 1$ yields the joint maximum. See Mayberry, Nash and Shubik [4]. This generalization was shown to me by Dr. George Feeney of the General Electric Company.

the most part, he is assumed to think increasing his ρ will induce others to increase theirs; however, he knows this will not always occur and, if it does, he is not sure how big an increase to expect. Furthermore, if ρ values are high (near 1.0) the subject can reap temporary high profits by dropping his ρ to zero. He is tempted to do this and sometimes does. He is assumed to think other subjects are motivated in a somewhat similar fashion and are aware of the same sort of things.

The subject does not think explicitly in terms of ρ, but it is assumed his thoughts on how his price changes will translate into changes in profits are well represented by the ρ concept. He probably does think in terms of the extent to which he and others are cooperative, and ρ may be regarded as an index of cooperativeness. The higher is ρ, the more cooperative the subject is being.

As to the first question, the assumption on the part of the subject that the others will repeat their prices is a convenient simplification for both the experimenter and the subject. When the subject is making a decision, he is assumed to be thinking about how his current decisions will affect future decisions by the others, and his future profits. His mind is quite fully occupied with this, without regard to how his recent decisions are affecting the surrent decisions of the others. Past decisions of the others are known, the present is unknown but out of the subject's control, and the future can be affected by him and receives his attention.

It is therefore assumed that ρ_{it} has a probability distribution. The hypothesized probability distribution for ρ_{it} will now be specified. Divide the range of ρ_{it} into N mutually exclusive intervals. Denote the k-th interval E_k, and assume the range of ρ_{it} divided into the same N intervals for all i, t. If, in period t, the i-th firm has chosen a value for ρ_{it} in the k_i-th interval, then the state of the system in time t is specified by an n-tuple: $(E_{k_1}, \ldots, E_{k_n})$. There are N^n possible states. From the point of view of the firm, these N^n states may be divided into $m = \frac{(N+n-2)!}{(N-1)!(n-1)!}$ groups. The probability that the i-th firm will choose to be in the k-th interval when the state is presently one of the j-th group is ω^i_{jk}.

$$(5) \qquad \sum_{k=1}^{N} \omega^i_{jk} = 1 \qquad\qquad \omega^i_{jk} \geq 0 \quad \text{for all } i, j, k .$$

The grouping of N^n states into m categories is done on the following basis. The probability that the firm will choose a given interval in the next period is independent of the interval it is in currently. Furthermore, this probability depends only on the intervals chosen by the other firms and not upon which firm happened to choose a particular interval. Thus, if there are four firms in the industry who happen to have chosen (E_6, E_2, E_4, E_1), the first firm only cares about the numbers E_2, E_4, E_1. It could have chosen any interval other than E_6 and the probability distribution of ρ_1 would be unaffected. Also the firm does not care which firm chose E_2, which E_4 or E_1. These three numbers could be permuted in any way without affecting the distribution of ρ_1.

For each of the n firms there is a matrix of order $m \times N$

$$\Omega_i = ||\omega^i_{jk}|| .$$

From these $n \times m \times N$ probabilities one can construct a square matrix of order $N^n (= N^*)$:

$$P_\omega = ||\pi^\omega_{ij}||$$

P_ω is the matrix of transition probabilities governing the system if the distributions of the ρ_{it} are given by the matrices Ω_i . P_ω may be regarded as the matrix of theoretical probabilities, and P , calculated directly from the data, is the matrix of observed probabilities. The hypothesis Ho: $P_\omega = P$ may be tested by a Chi-square test with $N^n(N^n - 1) - nm(N-1)$ degrees of freedom.

An example might clarify this formulation. Assume N , the number of intervals, is two. E_1 corresponds to $\rho \geq .9$ and E_2 to $\rho < .9$. Assume the number of firms, n , is 2 . There are $N^n = 4$ possible states:

(E_1, E_1)	$\rho_1, \rho_2 \geq .9$
(E_1, E_2)	$\rho_1 \geq .9, \rho_2 < .9$
(E_2, E_1)	$\rho_1 < .9, \rho_2 \geq .9$
(E_2, E_2)	$\rho_1, \rho_2 < .9$

Ω_1 is given by:

Table of probabilities for firm 1

		firm 1 chooses E_1	E_2	
if firm 2 chose:	E_1	ω^1_{11}	ω^1_{12}	$\omega^1_{11} + \omega^1_{12} = 1$
	E_2	ω^1_{21}	ω^1_{22}	$\omega^1_{21} + \omega^1_{22} = 1$

Table of probabilities for firm 2.

		E_1	E_2	
firm 2 chooses	E_1	ω^2_{11}	ω^2_{12}	$\omega^2_{11} + \omega^2_{12} = 1$
if firm 1 chose	E_2	ω^2_{21}	ω^2_{22}	$\omega^2_{21} + \omega^2_{22} = 1$

The matrix, P, of transition probabilities is given below:

Table of Transition Probabilities

Next State

		(E_1, E_1)	(E_1, E_2)	(E_2, E_1)	(E_2, E_2)
	(E_1, E_1)	$\omega^1_{11}\ \omega^2_{11}$	$\omega^1_{11}\ \omega^2_{12}$	$\omega^1_{12}\ \omega^2_{11}$	$\omega^1_{12}\ \omega^2_{12}$
Present state	(E_1, E_2)	$\omega^1_{21}\ \omega^2_{11}$	$\omega^1_{21}\ \omega^2_{12}$	$\omega^1_{22}\ \omega^2_{11}$	$\omega^1_{22}\ \omega^2_{12}$
	(E_2, E_1)	$\omega^1_{11}\ \omega^2_{21}$	$\omega^1_{11}\ \omega^2_{22}$	$\omega^1_{12}\ \omega^2_{21}$	$\omega^1_{12}\ \omega^2_{22}$
	(E_2, E_2)	$\omega^1_{21}\ \omega^2_{21}$	$\omega^1_{21}\ \omega^2_{22}$	$\omega^1_{22}\ \omega^2_{21}$	$\omega^1_{22}\ \omega^2_{22}$

Consider $\pi_{23} = \omega^1_{22}\, \omega^2_{11}$. This is the transition probability from state 2 into state 3. This involves two events, one for each firm. The probability that firm 1 will choose E_2 in the coming period given that firm 2 has chosen E_2 in the current period is ω^1_{22}. The probability that firm 2 will choose E_1 given that firm 1 has chosen E_1 is ω^2_{11}. The probability that both these independent events will occur is equal to their product $\omega^1_{22}\, \omega^2_{11} = \pi_{23}$.

1.2.2 Other Hypotheses Testable Within This Model

Many additional hypotheses are testable within this framework. Those mentioned below are not intended as an exhaustive list.

It might be hypothesized that the interval chosen by the i-th subjects in the preceding period and on the interval he chose himself. In this case the Ω_i are replaced by Ω_i^* and the N^n states are divided into Nm groups. Each of the m earlier groups is subdivided into N groups, one for each interval the i-th subject may choose. The Ω_i^* may be used to generate a matrix of transition probabilities which may be tested for equality with the observed matrix, P.

One could also hypothesize that the interval chosen by the i-th firm depends only upon the interval into which the average of previous period ρ_j values falls. Or, a specific functional relationship between ρ_{it} and the $\rho_{j,\ t-1}$ may be hypothesized. Consider, for example, the hypothesis:

$$(6) \qquad H_o: \quad \rho_{it} = \alpha_i + \beta_i \frac{\sum_{j \neq i} \rho_{j,\ t-1}}{n-1} + u_{it} \qquad u_{it}: N(0, \sigma_i^2)$$

The α_i, β_i and σ_i^2 all could be estimated by least squares and the entries in the Ω_i could be specified with the aid of these estimates. From the Ω_i, P_ω is estimated and the test of equality between P_ω and P provides a very general test of the assumptions underlying the regression model.

The last hypotheses covered here relate to learning. Consider an experiment of n subjects in which the subjects play in many games, each game lasting many periods. Questions arise about learning both within and between games. The data may be divided into subsamples according to position within a game (e.g., first half, last half), or according to game (e.g., first five games, remaining games). Tests for equality of these subsamples may be made.

2. The Oligopoly Experiments

2.1 Background Information on the Experiments

Two experiments were conducted, each having its own group of subjects, its own payoff matrices and experimental design; however, certain procedures, which will be described below, were common to both. The subjects were Yale University undergraduates who were recruited through the Financial Aids Office. The students were hired to perform "clerical work" at $1.50 per hour. Each student was under the impression that he was the only person hired -- that only one was needed. The purpose in recruiting through the Financial Aids Office is to get subjects who need the money.

The payoffs in each series were chosen so that if all played strictly noncooperatively, profits would come out to about $1.70 per hour. It was hoped that this general level of payoffs would be sufficient to motivate subjects drawn in the manner described. Advertising for clerical workers rather than for people to work at someting which might sound interesting also had the purpose of attracting subjects for whom the money to be earned was of prime importance.

The experiments were designed so that subjects would not know or see one another. It was emphasized to the subjects that they should not discuss the games with anyone during the several days they took place, and preferably they should not discuss them later, in order to avoid biasing the results of these and future experiments.

The games varied in length from 22 to 26 periods. All periods were "regular" in that subjects were paid the profit they earned on them. Subjects were never told in advance how long games would last, and the game length was varied in the hope that they would always play under the assumption that there would be a number of periods to go -- except for the final period of the game. Just before the results of the next to last period were handed back, it was announced that the next period (for which decisions were about to be made) would be the last. This announcement was made, of course, to bring out "end effects." The subjects were paid their earnings on all games of the series after the last game was played. The reason for bringing out end effects will be discussed below.

The payoff matrices gave the subjects "complete information." A subject could determine the profit of each subject which corresponded to any set of prices the subjects might choose. The determination of which subjects would be in games together was arranged before the experiments took place.

The payoff matrices used in each experiment were chosen to correspond to particular values of two new variables: level of payoff, L, and relative payoff, R. L is defined as the noncooperative equilibrium profit to a firm in a game. This gives some indication of the level of payoffs in the game.

$$R = \frac{\Pi_J - L}{L}$$

where Π_J is the profit a firm would receive if the game were at the joint maximum. R is therefore the profit gained by moving from noncooperative equilibrium to the joint maximum (all firms making the move) divided by the noncooperative equilibrium profit. It gives a relative measure of the gains to joint maximization of profits. In the two experiments, L and R were varied in a regular manner to provide the payoff matrices used.

The coefficients used to generate the payoff matrices come, therefore, from a relatively small subset of possible coefficients. It is an open question whether (and how) the choice of these coefficients affects the results.

2.2 Experimental Design

There are six games in the first experiment. Table 1 gives the value of the coefficients used in generating the payoff matrices, and the noncooperative equilibrium and joint maximum prices (p_{nco} and p_{jm}). The six games were played in the order in which they appear in Tables 1 and 2; however, this order was randomly chosen. Table 2 shows the values of L and R which correspond to each of the six games of the experiment. There are three values of L, approximately 4¢, 10¢ and 20¢, and two values of R, aproximately .25 and .75. The six games exhaust the six possible combinations of L and R values.

TABLE 1

Game	a_1	a_2	a_3	c_1	c_2	c_3	p_{neo}	p_{jm}
1	9.09	.33	.12	107.58	15.0	1.65	51	63
2	.167	.00153	.000556	5.26	60.0	360.0	203	243
3	9.09	.66	.24	53.84	7.5	.825	25.5	31.5
4	.545	.00667	.00242	12.77	45.0	82.50	152	185
5	1.212	.0176	.0064	22.5	37.5	30.94	128	158
6	.809	.0165	.006	21.52	30.0	33.0	102	128

The original player pairings, which were so arranged that no subject was paired with the same person twice, could not be carried out, because one of the subjects, Player B, did not show up on the second day to play games 3 through 6. As a result, alternate player pairings had to be used. The pairings which were actually used appear in Table 3, below. Player C was paired with a pre-selected dummy strategy for the last four games and it was necessary to repeat two previously used player pairings involving E, F, G, and H.

TABLE 2

Game	L	R
1	21.0	.7762
2	4.1	.2683
3	10.5	.7714
4	9.9	.2727
5	20.5	.2585
6	4.2	.7857

TABLE 3

Game	Player Pairing			
1	AB	CD	EF	GH
2	AC	BD	EG	FH
3	AG	DF	EH	C[1]
4	AE	DH	FG	C[1]
5	AH	DE	FG[2]	C[1]
6	AF	DG	EH[2]	C[1]

1. Due to the absence of Player B, Player C had dummy players as competitors in these games.
2. These pairings appear for the second time; data from them are not used in some of the analysis.

The coefficients used to generate the nine payoff matrices which were employed in the second experiment appear in Table 4, along with the noncooperative equilibrium price and the joint maximum price of each game. Each game is identified by a letter and a number. The number tells how many firms are in the industry using the particular matrix and the letter tells to which "group" the game belongs. The significance of the "group" is explained below. Table 5 gives the actual values of L and R corresponding to each of the nine distinct games. Table 6 gives the player pairings for the whole series. The number under the heading "game" gives the chronological order of the games. Thus, in the first game played by the nine subjects, A, B, C, and D played in the four person game of the "A" group, E, H, and I played in the three person game of the "A" group, and F and G played in the two person game of the "A" group. From Table 8 it is apparent that each of the groups, A, B, and C, contain one two person game, one three person game, and one four person game, and that the first three games in which the subjects play are A games, the second three, B games, and the last three are C games. The Players A through H all play exactly once in each of the three games of a group; hence, after

TABLE 4

Game	a_1	a_2	a_3	c_1	c_2	c_3	P_{nco}	P_{jm}
2A	21.7	3.99	1.45	50.1	3.0	.138	10.1	12.5
3A	.855	.0125	.00304	8.56	37.5	43.8	122.0	157.0
4A	3.86	.283	.0514	21.3	7.5	1.95	23.9	30.9
2B	9.96	.236	.0858	185.5	22.5	2.33	77.0	95.0
3B	1.282	.0470	.0114	13.1	15.0	11.7	49.0	62.0
4B	.481	.0088	.0016	2.63	30.0	62.5	97.0	123.0
2C	1.45	.133	.0484	4.1	6.0	4.13	20.3	23.9
3C	1.78	.0218	.00529	60.0	45.0	25.3	145.0	187.0
4C	57.8	21.2	3.85	55.8	1.5	.026	4.8	6.3

TABLE 5

Game	L	R
2A	10.0	.760
3A	20.3	.261
4A	3.8	1.34
2B	19.2	1.19
3B	4.1	.708
4B	10.1	.237
2C	4.0	.250
3C	10.6	1.250
4C	19.7	.752

TABLE 6

Game						Group
1	ABCD	EHI	FG			A
2	EFGH	ACD	BI			A
3		BFG	CD	AE	HI	A
4	ABEG	DFI	CH			B
5	CDFH	ABE	GI			B
6		CGH	AB	DI	EF	B
7	ACFG	BDH	EI			C
8	BDEH	AGI	CF			C

the third, sixth, and ninth games had been finished, those eight players had played exactly once using each of the same three, six, and nine matrices. Table 7 shows which values of L, R, and n correspond to each of the three groups. The numbers of each group were chosen in such a way that each value of L and each value of R appear exactly once in each group. This, along with the requirement that each value of n appear exactly once, meant that the series was laid out in a Graeco-Latin square design with respect to the four factors, L, R, n, and group. The purpose of this layout is, of course, to balance the experimental design with respect to these four variables.[1]

TABLE 7

	Values of L		
R	4	10	20
.25	2C	4B	3A
.75	3B	2A	4C
1.25	4A	3C	2B

1. On experimental design in general and Latin squares in particular see Cochran and Cox [2].

2.3 Results and Analysis

The model is applied in two ways to the experiments. The first utilizes the regression formulation of equation (6) and a modified version. For the second experiment equation (7) is estimated for each subject in addition to equation (6).

$$(7) \qquad \rho_{it} = \beta_{i1} + \beta_{i2}D_{1t} + \beta_{i3}D_{2t} + (\beta_{i4} + \beta_{i5}D_{1t} + \beta_{i6}D_{2t}) \frac{\sum_{j \neq i} \rho_{j,\, t-1}}{n-1} + u_{it}$$

Where D_1 and D_2 are dummy variables. $D_{1t} = 1$ if, for the t-th observation, the game was 3 person, and $D_{1t} = 0$ otherwise. $D_{2t} = 1$ for 4 person games and zero otherwise. The second form of analysis utilizes the model in a more general form, but one in which the ρ_{it} are divided into only two intervals.

Unfortunately the data are not sufficiently plentiful to allow testing the linear form in the general way mentioned above. Only the traditional F and student "t" tests are possible. The second part of the analysis, utilizing the two interval formulation, does not permit of much conclusive hypothesis testing at all. This is in part because the general model had not been formulated when the experiments were designed. They were designed for analysis with the regression model, and only much later was the more general model, from which the regression may be derived, formulated.

2.3.1 Regression Analysis

In principle, if one has a T period game, there will be T-2 observations for each subject which may be had from it. These observations come from all periods except the first and second. None comes from the first because, to calculate ρ_{it} prices are needed for period $t-1$. Thus no values of ρ_{it} are computable for the first period. As an observation requires ρ_{it} and the $\rho_{j,\,t-1}$, no observation may be obtained earlier than period 3 of a game because the $\rho_{j,\,t-1}$ are available only from period 2 on. Of these $T-2$ observations, the last is not used in the regressions because the last reflects "end effects" (to be discussed below). Of the remaining observations, some of the last 10 are used. The observations from the early part of each game are not used because it is assumed much learning takes place in this part of each game, before the game settles down to a relatively stable pattern. Of the remaining 10 periods per game, all those were dropped out for which the subjects a) were at the joint maximum and b) had been at the joint maximum during the preceding period. Only when subjects were at the joint maximum were the same set of prices (and ρ values) repeated in successive periods. It is assumed that such repeating of prices leads to a lower variance for ρ_{it} than otherwise. As a result, a required assumption of the regression model would be violated by including these observations.

Table 8, below, gives the coefficients estimated for the behavioral equation of each subject in the first experiment. Below each coefficient is its Student "t" ratio. The column marked "N" gives the number of observations and the column marked "F pooled" gives the F-ratio testing the hypothesis H_o: $\beta_{1i} = \beta_{1p}$, $\beta_{2i} = \beta_{2p}$ where the β_{jp} are the coefficients estimated by pooling observations for all subjects. This is a way to test equality of coefficients for all subjects.

TABLE 8

Subject	$\hat{\beta}_1$	$\hat{\beta}_2$	N	F-pooled
A	.697 (4.08)**	-.392 (-1.21)	35	3.06
D	.216 (1.54)	.0501 (.35)	60	.87
E	.0973 (1.06)	.874 (5.80)**	30	12.54**
F	.141 (1.22)	.110 (.83)	27	1.23
G	.139 (1.39)	.049 (.47)	20	1.94
H	.276 (3.50)**	.402 (3.36)**	45	3.27*
ALL	.276 (5.17)**	.122 (1.81)	217	----

* significant at the 5% level
** significant at the 1% level

This analysis appears handicapped by the very small number of observations available, particularly for players F and G. While no coefficients are significant for player D, there were sixty observations available. Inspection of the games in which D played reveals a distinct pattern of play which is not captured by the regression model. He would often play a value of ρ higher than that of his competitor in an effort to induce the competitor to raise his ρ. As the competitor would raise ρ, D would raise his even higher until he reached $\rho = 1$, or even above. At this point D would switch to $\rho = 0$. He would maintain this until the competitor dropped his price (and ρ), then he would try to lure the competitor up again. This strategy met with varying degrees of success from game to game, and it is described above in an ideal form; however a questionnaire which D completed after the experiment was over claims this to be his strategy.

In sum, one would hope to find more significant coefficients than are present in Table 8. On the question of whether the subjects all can be considered to have the same coefficients, only two subjects' coefficients are significantly different from those estimated by pooling all observations; however these two are the only two with significant slope coefficients. The case for all subjects being the same appears poorly supported, but the evidence of Table 8 is really too scant to prove anything either way.

A very good result is that all the coefficients except one fall into the range to be expected. It is postulated a) that subjects would be more willing to cooperate (play a higher ρ), the more cooperative were other subjects in the same game. That is, β_2 is expected to be nonnegative; however, it should not be greater than one. As an upper limit $\beta_2 = 1$ implies a subject is willing to match any increases in cooperativeness shown by his competitors. Finally, it makes no sense for anyone to play a ρ greater than one. All these conditions will be met if, for any player, $\beta_1, \beta_2 > 0$ and $\beta_1 + \beta_2 \leq 1$. The only coefficient not meeting these conditions is β_2 for A, which is negative.

The regression analysis results for the second experiment are summarized below in Table 9. Two sets of coefficients are shown for each player. The second are directly comparable to those of the first experiment; however the first coefficients are for the form in equation (7). The column marked "F-pooled" compares the coefficients a given row with the corresponding pooled regression, and the column marked "F-players" tests for significant difference between the two regressions for one subject. (I.e., it tests the hypothesis Ho: $\beta_{2i} = \beta_{3i} = \beta_{5i} = \beta_{6i} = 0$). This six variable regression equation yields a separate intercept term and slope coefficient for two, three and four person games. The intercept and slope coefficient for two person games are β_{1i} and β_{4i}, for three person games, $\beta_{1i} + \beta_{2i}$ and $\beta_{4i} + \beta_{5i}$, etc.

Several tentative conclusions may be drawn from Table 9. The F ratios in the F-pooled column are ~~overwhelmingly~~ significant, indicating that the eight subjects cannot be considered to have equal coefficients. While many of the coefficients in the six variable regressions are not significant, suggesting that one should stand by the two variable regressions as being more reliable, still it is interesting to look at those coefficients for any persistent tendencies. Not that the β_{2i} and β_{3i} are almost always negative, while the β_{5i} and β_{6i} are nearly always positive. The positive values for the β_{5i} and β_{6i} imply that

TABLE 9

Player	β_1	β_2	β_3	β_4	β_5	β_6	N	F-player	F-pooled
A	.681 (5.31)** .378 (6.99)**	-.225 (-1.50)	-.592 (-3.91)**	.160 (1.32) .485 (6.57)**	.271 (1.35)	.712 (4.33)**	61	5.55**	6.77** 8.43**
B[1]	-.195 (-.75) .330 (2.04)*		1.018 (3.16)**	1.104 (1.25) .568 (2.10)*		-1.139 (-1.23)	40	7.47**	4.49** .94
C	.232 (1.45) .003 (.003)	-.357 (-1.38)	-.456 (-2.12)*	.485 (1.25) .663 (4.40)**	.452 (.96)	.207 (.46)	71	4.24**	7.04** 9.79**
D	.414 (5.80)** .480 (9.64)**	-.135 (-1.23)	.001 (.005)	-.339 (-4.03)** -.133 (-1.65)	.765 (4.06)**	.523 (1.85)	75	5.84**	7.19** 19.99**
E	.820 (.90) .056 (.64)	-.617 (-.67)	-1.013 (-1.09)	.167 (.16) .927 (7.47)**	.584 (.54)	1.034 (.96)	56	2.01	4.19** 11.82**
F	1.261 (2.25)* .229 (2.44)*	-1.093 (-1.85)	-1.085 (-1.90)	-.314 (-.39) .796 (5.38)**	1.073 (1.28)	1.213 (1.45)	47	2.08	3.11* 7.82**
G	.205 (.82) .248 (2.30)*	-.063 (-.18)	.085 (.29)	.507 (1.93) .425 (3.15)**	-.100 (-.25)	-.030 (-.09)	57	.82	1.23 .59
H	.807 (5.18)** .367 (4.23)**	-.615 (-3.25)**	-.437 (-1.66)	-.141 (-.48) .255 (1.61)	.637 (1.84)	.130 (.26)	75	4.28**	1.29 .36
ALL	.592 (10.71)** .339 (10.79)**	-.384 (-5.08)**	-.387 (-4.95)**	.063 (.86) .366 ([illegible])**	.476 (4.33)**	.526 (4.60)**	482	8.89**	---

Table 9 - Footnotes

* Significant at the 5% level.
** Significant at the 1% level.

1. A four variable regression is shown for B all observations from three person games are deleted.

subjects behave more independently of one another in two person games -- they respond less to changes in competitors' ρ values. The negative values of the β_{2i} and β_{3i} yield smaller intercept terms for three and four person games. Thus, while subjects show less response to changes in competitors' ρ values in two person games, they also show a strong degree of cooperativeness independently.

This behavior is plausible. The smaller response to changes in the ρ values of others is because the more subjects there are in a game, the less is spoiled when one drops from, say, the joint maximum. It may be profitable for all the others to continue cooperation. In a two subject case, the subject who remains at $\rho = 1$ after his competitor goes to $\rho = 0$ is purely benefitting the other at his own expense. The larger intercept may be due to a general awareness on the part of subjects that the larger is the number of subjects in a game, the more important it is for one to show some cooperative initiative; to raise ρ without waiting for someone else to lead the way.

One cannot generalize readily about difference between three and four person games. For some subjects, $\beta_{2i} > \beta_{3i}$ and for some the reverse holds. Similarly for β_{5i} and β_{6i}. It does seem clear, however, that n does affect individual behavior, and the subject warrants more investigation.

In checking the coefficients to see if they are within the bounds deemed reasonable, the coefficients should satisfy the following inequalities:

$$1 \geq \beta_{1i}, \beta_{1i} + \beta_{2i}, \beta_{1i} + \beta_{3i}, \beta_{4i}, \beta_{4i} + \beta_{5i}, \beta_{4i} + \beta_{6i} \geq 0$$
$$\beta_{1i} + \beta_{4i}, \beta_{1i} + \beta_{2i} + \beta_{4i} + \beta_{5i}, \beta_{1i} + \beta_{3i} + \beta_{4i} + \beta_{6i} \leq 1$$

The first inequality is satisfied, in the six variable regressions, about 80% of the time and the second, about 80% of the time. For the two variable regressions, only one coefficient is negative, and only one pair of coefficients sums to more than unity. These results are encouraging. One would expect relatively more violations from the six variable regressions because many of them have quite large variances. Finally, the results support quite firmly the hypothesis that the behavior of most subjects (as measured by ρ) is a function of the behavior of other subjects in the same game. Furthermore, there appears to be a positive correlation between the cooperativeness of a subject and that of his competitors.

The Durbin Watson test is used to test for positive serial correlation of error terms within each ~~regression.~~ Using the 1% level of significance, it is concluded that positive serial correlation is present in three of the six regressions of the first experiment (subjects D, F and G), and in seven of the second experiment (E is the only exception).

The observations for a subject are not a simple time series; hence a word is necessary on how the Durbin Watson statistic, d , is computed for these samples. The usual formula is:

$$(8) \qquad d_i = \frac{\sum_{t=2}^{T} (u_{it} - u_{i,\ t-1})^2}{\sum_{t=1}^{T} u_t^2} = \frac{\sum_{t=2}^{T} \delta_{it}^2}{\sum_{t=1}^{T} u_{it}^2} \qquad (i = 1, \ldots, n)$$

In these samples, consecutive observations sometimes come from consecutive observations within the same game, and sometimes they do not. Only consecutive observations which come from consecutive periods within a game do yield a term, δ_{it}, which may legitimately used in computing d_i.

Whenever a term, δ_{it}, is not appropriate in this sense, a value of zero is substituted for it. If T^* such zeros are introduced, the numerator of (8) is really a sum of $T-T^*-1$ terms instead of $T-1$ terms. Two courses are open to give numerator and denominator the relative weights they should have. They are to use:

(9) $$d_i = \frac{\frac{T-1}{T-T^*-1} \sum_{t=2}^{T} \delta_{it}^2}{\sum_{t=1}^{T} u_{it}^2}$$

or

(10) $$d_i = \frac{\sum_{t=2}^{T} \delta_{it}^2}{\frac{T-T^*}{T} \sum_{t=1}^{T} u_{it}^2}$$

In each case zeros are appropriately substituted. These two versions lead to numbers which are negligibly different because T^* is not very large compared to T. There is a question of whether T or $T-T^*$ should be considered the number of observations when using the tables. In no case does it matter to the conclusions which is used.

The positive serial correlation could be due to one of three sources) or some combination of them: a) the true relationship is non-linear,

b) some explanatory variables have been omitted, and c) ρ_{it} also depends upon $\rho_{i, t-1}$. No attempt is made to uncover which of these may be true. The reason is that the data are too-scant for any such attempt to lead to reliable results. It is enough to be aware of these possibilities in order that future experiments will be properly designed for testing these possibilities.

2.3.2. Analysis With a Two Interval Model

The model used in this section is a two interval model in which E_1 corresponds to $\rho_{it} < .85$ and E_2 to $\rho_{it} \geq .85$. Four separate questions will be taken up. They are a) What is the probability that the subjects in a game will choose a joint profit maximizing set of prices in a given period; b) Are the matrices Ω_i equal for all subjects; c) Can the matrices P and P_ω be considered equal; and d) Do subjects behave similarly in both the early and later periods of games. The items b) through d) are included primarily for illustrative purposes, for the data are not sufficiently rich to provide conclusive testing of the hypotheses involved.

Investigating the probability of all subjects making a joint profit maximizing decision must be done subject to some rather restrictive assumptions: The probability which will be estimated is really the probability that all subjects will choose to be in the second interval ($\rho \geq .85$). This interval is unbounded from above; hence it extends considerably beyond the region around $\rho = 1$. As the data permit only two intervals, this cannot be helped. Nearly all ρ values in the second interval are close to 1.0 . The second restrictive assumption is that all subjects behave identically. While the evidence is not conclusive, the writer believes this assumption to be false. Again, there is not enough data to rely upon individual estimates of behavior. In one respect, the violation of this assumption is probably not serious. While estimated probabilities may not be very reliable, the trends in those probabilities are likely to be reliable. I.e., the decreasing probability of being in the second interval as the number of firms increases is reliable. Of course, further experimental work is necessary to bear these conjectures out.

Table 10, below, gives transition matrices for four categories of games: first experiment (two person), second experiment two person, three person and four person. The row labels correspond to "current state," the column to "next state." Those matrices marked "late game" are estimated from the ten observations in each game prior to the last observation. Those marked "early game" are estimated from the observations prior to those used for late game matrices. The matrices are all estimated directly from the data -- not calculated from Ω matrices.

There are several interesting tendencies in comparing early game to late game matrices. The probability of everyone choosing E_1 when each has just chosen E_1 (first element, first row) generally is larger in the late game matrices. Similarly for the probability of all choosing E_2 when each has just chosen E_2 (last element last row). Finally, if the game is in an intermediate state (not first or last), the probability of dropping to a lower state is higher and the probability of going to a

TABLE 10

Table of Matrices of Transition Probabilities

First Experiment - Two Person

	Early Game			Late Game			Total Game		
	$E_1 E_1$	$E_1 E_2$	$E_2 E_2$	$E_1 E_1$	$E_1 E_2$	$E_2 E_2$	$E_1 E_1$	$E_1 E_2$	$E_2 E_2$
$E_1 E_1$	.789	.140	.071	.817	.171	.012	.801	.153	.046
$E_1 E_2$	.291	.527	.182	.400	.556	.044	.340	.540	.120
$E_2 E_2$	.032	.097	.871	.014	.096	.890	.022	.096	.882

Second Experiment - Two Person

	Early Game			Late Game			Total Game		
	$E_1 E_1$	$E_1 E_2$	$E_2 E_2$	$E_1 E_1$	$E_1 E_2$	$E_2 E_2$	$E_1 E_1$	$E_1 E_2$	$E_2 E_2$
$E_1 E_1$	.854	.134	.012	.928	.060	.012	.892	.096	.012
$E_1 E_2$	.280	.580	.140	.316	.526	.158	.290	.565	.145
$E_2 E_2$	.029	.114	.857	0	.085	.915	.012	.098	.890

TABLE 10

Table of Matrices of Transition Probabilities
Second Experiment - Three Person

Early Game

	$E_1 E_1 E_1$	$E_1 E_1 E_2$	$E_1 E_2 E_2$	$E_2 E_2 E_2$
$E_1 E_1 E_1$	.800	.200	0	0
$E_1 E_1 E_2$	.125	.333	.542	0
$E_1 E_2 E_2$	.031	.219	.563	.187
$E_2 E_2 E_2$	0	.037	.111	.852

Late Game

	$E_1 E_1 E_1$	$E_1 E_1 E_2$	$E_1 E_1 E_2$	$E_2 E_2 E_2$
$E_1 E_1 E_1$	.906	.063	.031	0
$E_1 E_1 E_2$	.267	.600	.133	0
$E_1 E_2 E_2$	.118	.235	.588	.059
$E_2 E_2 E_2$	0	.038	.038	.924

Total Game

	$E_1 E_1 E_1$	$E_1 E_1 E_2$	$E_1 E_2 E_2$	$E_2 E_2 E_2$
$E_1 E_1 E_1$	.872	.106	.022	0
$E_1 E_1 E_2$	.179	.436	.385	0
$E_1 E_2 E_2$	.061	.224	.571	.144
$E_2 E_2 E_2$	0	.038	.075	.887

TABLE 10

Table of Matrices of Transition Probabilities

Second Experiment - Four Person

Early Game

	$E_1 E_1 E_1 E_1$	$E_1 E_1 E_1 E_2$	$E_1 E_1 E_2 E_2$	$E_1 E_2 E_2 E_2$	$E_2 E_2 E_2 E_2$
$E_1 E_1 E_1 E_1$	.762	.238	0	0	0
$E_1 E_1 E_1 E_2$	.333	.500	.167	0	0
$E_1 E_1 E_2 E_2$	.167	.167	.166	0	.500
$E_1 E_2 E_2 E_2$	0	.071	.214	.429	.286
$E_2 E_2 E_2 E_2$	0	0	0	.556	.444

Late Game

	$E_1 E_1 E_1 E_1$	$E_1 E_1 E_1 E_2$	$E_1 E_1 E_2 E_2$	$E_1 E_2 E_2 E_2$	$E_2 E_2 E_2 E_2$
$E_1 E_1 E_1 E_1$	.893	.107	0	0	0
$E_1 E_1 E_1 E_2$	.286	.714	0	0	0
$E_1 E_1 E_2 E_2$	.167	.167	.333	.333	0
$E_1 E_2 E_2 E_2$	0	.143	.286	.428	.143
$E_2 E_2 E_2 E_2$	0	0	.200	.400	.400

Total Game

	$E_1 E_1 E_1 E_1$	$E_1 E_1 E_1 E_2$	$E_1 E_1 E_2 E_2$	$E_1 E_2 E_2 E_2$	$E_2 E_2 E_2 E_2$
$E_1 E_1 E_1 E_1$	.837	.163	0	0	0
$E_1 E_1 E_1 E_2$	.308	.615	.077	0	0
$E_1 E_1 E_2 E_2$	.167	.167	.250	.166	.250
$E_1 E_2 E_2 E_2$	0	.095	.238	.429	.238
$E_2 E_2 E_2 E_2$	0	0	.071	.500	.429

higher state is lower in the late game. Table 11 shows the probability of being in any given state T periods hence, as T goes to infinity (i.e., the asymptotic probability). The probability of being in a given state in T periods is conditional upon the current state; however, as T increases, the dependence on the current state decreases to insignificance. Looking

TABLE 11

Table of Asymptotic Probabilities for Each State

Early Game

First Experiment		Second Experiment				
2 person		2 person	3 person		4 person	
$E_1\ E_1$	.341	.497	$E_1\ E_1\ E_1$	.152	$E_1\ E_1\ E_1\ E_1$	.378
$E_1\ E_2$	.196	.232	$E_1\ E_1\ E_2$	.166	$E_1\ E_1\ E_1\ E_2$	.229
			$E_1\ E_2\ E_2$	.302	$E_1\ E_1\ E_2\ E_2$	.085
					$E_1\ E_2\ E_2\ E_2$	.152
$E_2\ E_2$	.463	.271	$E_2\ E_2\ E_2$	.380	$E_2\ E_2\ E_2\ E_2$	.156

Late Game

First Experiment		Second Experiment				
2 person		2 person	3 person		4 person	
$E_1\ E_1$	.574	.562	$E_1\ E_1\ E_1$	.624	$E_1\ E_1\ E_1\ E_1$	.729
$E_1\ E_2$	.257	.127	$E_1\ E_1\ E_2$	.172	$E_1\ E_1\ E_1\ E_2$	.271
			$E_1\ E_2\ E_2$	.111	$E_1\ E_1\ E_2\ E_2$	0
					$E_1\ E_2\ E_2\ E_2$	0
$E_2\ E_2$	.169	.311	$E_2\ E_2\ E_2$	.093	$E_2\ E_2\ E_2\ E_2$	0

Total Game

First Experiment		Second Experiment				
2 person		2 person	3 person		4 person	
$E_1\ E_1$	.408	.520	$E_1\ E_1\ E_1$	.334	$E_1\ E_1\ E_1\ E_1$	.574
$E_1\ E_2$	.214	.182	$E_1\ E_1\ E_2$	.167	$E_1\ E_1\ E_1\ E_2$	.277
			$E_1\ E_2\ E_2$	.216	$E_1\ E_1\ E_2\ E_2$	.050
					$E_1\ E_2\ E_2\ E_2$	.054
$E_2\ E_2$	.378	.298	$E_2\ E_2\ E_2$	.283	$E_2\ E_2\ E_2\ E_2$	.045

at the late game probabilities, it appears the probability of everyone choosing E_2 falls as the number of subjects in the game increases, and the probability of everyone choosing E_1 increases. These results are far from conclusive for reasons stated earlier, and they must be viewed with appropriate skepticism.

Testing the equality of Ω matrices is possible for the first experiment and for the duopolies of the second experiment, both on a total game basis. The hypothesis Ho: $\Omega_1 = \dots = \Omega_n$ is tested by χ^2 with 2n - 2 degrees of freedom. The theoretical probabilities are those obtained by pooling all subjects to estimate a pooled Ω matrix. For the first experiment $\chi^2 = 89$ (10 d.f.) and for the second, $\chi^2 = 85$ (14 d.f.). Both are significant at the 1% level, therefore the conclusion is that Ω matrices are not the same for all subjects. It is possible they are the same for late game and different for early game, in which case the above result is invalid. It is not possible to test on a late/early basis because of low theoretical frequencies in too many cells.

Testing the hypothesis Ho: $P = P_\infty$ where P is the transition matrix estimated directly from the data and P_∞ is calculated from a pooled Ω matrix is possible for all groups except the four person games. The values are:

first experiment	$\chi^2 = 102$	(4 d.f.)
second experiment (2 person)	$\chi^2 = 83$	(4 d.f.)
second experiment (3 person)	$\chi^2 = 57$	(4 d.f.)[1]

---

1. If several cells did not have to be consolidated, there would have been 9 degrees of freedom.

All are significant at the 1% level. This test is subject to the same criticisms and limitations as the preceding test.

The test of

$$Ho: \ P_1 = P_2$$

where P_1 is a transition matrix estimated from late game data and P_2 from early game is done for all groups except the four person games. The theoretical probabilities are taken to be the total game probabilities. The χ^2 values are:

first experiment	$\chi^2 = 9.2$ (6 d.f.)
second experiment (2 person)	$\chi^2 = 4.4$ (6 d.f.)
second experiment (3 person)	$\chi^2 = 18.7$ (6 d.f.)[1]

The critical value for the 1% level with 6 degrees of freedom is $\chi^2_{.01} = 16.8$. Only the three person game value is significant. Again, all previous warnings and limitations hold, however, this evidence gives limited support to the hypothesis that early and late game behavior are the same.

2.3.3 End Effects

The analysis for "end effects" is based on the hypothesis that subjects behave differently when the end of the game is known to be imminent than when the game may go on for an indefinite time. Data was generated by the practice of telling the subjects just before handing back results for the next to last period that the next period for which a decision was due would be the last period of the game. In Table 12, below, appear the number of rho values for the very last period of the

1. Again, pooling of cells reduces degrees of freedom from 12 to 6.

games which are in the ranges shown across the top of the table. The numbers in parentheses below these frequencies are the expected frequencies. The expected frequencies are computed from the frequency of rho values in each of the five cells in the five periods preceding the last period for each game from which a last period rho value was taken.

TABLE 12

Expected and Observed Final Period Frequencies: First Experiment

	< -.2	-.2 to 0	0 to .2	.2 to .6	> .6
Observed Frequencies	15	8	10	5	6
Expected Frequencies	(6.4)	(5.2)	(6.2)	(6.6)	(19.6)

Second Experiment

Frequencies	< 0	0 to .2	.2 to .4	.4 to .6	.6 to .8	.8 to 1	≥ 1
Actual	57	14	2	2	0	6	0
Expected	(8.2)	(11.8)	(6.2)	(12.0)	(6.6)	(26.6)	(9.6)

If final period play does not differ, in general, from the play of the immediately preceding periods, the same fraction of last period rho values would be expected in each cell as is found for the periods preceding the last.

The hypothesis may be tested by a chi-square test, with 4 degrees of freedom for the first experiment and 6 for the second. The value computed for the first experiment is 25.23, which exceeds the value from the χ^2 table (for a 1% significance level). The critical value is

13.28. Thus the hypothesis must be accepted that last period play differs from play in the immediately preceding periods. The computed chi-square value for the second experiment is 337.7, which exceeds the critical value (for the 99% level and 6 degrees of freedom) of 18.48. One must again reject the hypothesis that final period behavior does not differ from the behavior of the period just preceding the last one. The direction in which the final period data diverge from the earlier data is obvious from Table 12. Values of ρ have a distinct tendency to drop in the final period of a game.

3. Concluding Remarks

The model presented in the first part of this paper is suitable for the testing of a wide variety of hypotheses about behavior. Although it was never mentioned, the behavioral model need not have ρ as its basic variable. Theory may, in many circumstances, suggest another variable or variables. The Markov process could be defined in terms of them.

The experiments presented in the second part of the paper are, in retrospect, much more of an exploratory, pilot nature than was expected originally. They indicate the magnitude of data needed to carry out definitive tests of hypotheses. After looking at the experiments, the model may give a "pie-in-the-sky" appearance suggesting the model is fine in principle, but could never be applied in any except the most simple form. This writer does not believe so. Interesting use of the model requires much more data than was analyzed here, and it requires very careful experimental design.

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