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An Informal Aggregative Socio-Economic Simulation of a Latin American Country, Part 1

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COWLES FOUNDATION DISCUSSION PAPER NO.170

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AN INFORMAL AGGREGATIVE SOCIO-ECONOMIC SIMULATION OF A
LATIN AMERICAN COUNTRY

PART I

James Friedman and Martin Shubik

April 22, 1964

AN INFORMAL AGGREGATIVE SOCIO-ECONOMIC SIMULATION
OF A LATIN AMERICAN COUNTRY *

PART I

James Friedman^{**} and Martin Shubik^{**}

1. INTRODUCTION

The simple model constructed here is built to serve as a first approximation for a socio-economic model to be used in connection with the investigation of development problems in parts of Latin America.

The technique of simulation serves as an excellent data organizing device and framework for the conceptual scheme behind a national economic accounts system and aggregate financial reporting system. This simulation utilizes in part the national economic accounts format developed in The Country Study Program of the Yale Economic Growth Center and the financial statistical reporting of the International Monetary Fund and World Bank^{1/}.

Although this first model is based primarily upon an aggregated national income system of a variety that is well known to economic studies, it contains several parameters and relations of a socio-economic variety, which are not commonly found in national income models.

* Research undertaken by Cowles Commission for Research in Economics under Task NR 047-006 with the Office of Naval Research.

** We are indebted to G. Blanksten, R. Holt, S. Mintz and R. and N. Ruggles for their comments and helpful discussions.

^{1/} International Monetary Fund, International Financial Statistics, Washington, D.C.

The simulation described below has been constructed in a manner that employs what can at best be called "casual empiricism". It has been used to provide a structure for joint discussion and as a possible basis for research between anthropologists, economists, political scientists, sociologists, and others. Part of the motivation behind this work is to show that relatively interesting and thought-provoking models can be constructed and examined quickly and relatively cheaply by means of computer simulation. Before validation problems must be faced, pre-validation problems must be overcome. These involve the sorting out of variables, the organization of common sense, uncommon sense, insights and "guesses", and the "playing" with models as a preliminary to well-defining structural relationships and devising methods for empirical verification of the chosen relationships.

Accuracy and validity should depend upon the question to be answered; models and questions which appear to be vague to the econometrician may thus have a place and use to the policy maker, political scientist, administrators, and others. These comments should not be interpreted as a defense of sloppy thinking. The gaps separating policy oriented studies, economic theory and econometrics as well as highly relevant sociological and political studies are large. A simulation is addressed to providing an interlinkage for the different approaches.

In particular, in this simulation the population of the country is broken down into four sociological groups which are broadly described as "commercial white", "agricultural white", mestizo and Indian. Although it is difficult to gather accurate statistics based upon a sociological breakdown,

we suggest that from the viewpoint of policy and for the development of behavioral models a disaggregation of national accounts in terms of different decision-making groups may be of more interest than a disaggregation of industrial categories.

2. Gross National Product or Expenditure

We will now give the traditional identities which relate the major expenditure components of gross national product. These show the basic components of GNP (consumption, investment, government, etc.), and the relationship of GNP to other important concepts such as personal income and disposable income. We will also set up functional relationships relating the major expenditure components to relevant variables.

2.1 Consumption

In this section we first give the expenditure identity for GNP. Next the identity relating GNP to Personal Income is given, and then the relationship of Personal to Disposable income. Finally, a simple consumption function is introduced for each socio-economic group.

- Let
- Y_t = Gross national product in period t in constant currency
 - I_t = Gross private investment in constant currency
 - C_t = Total consumption in constant currency
 - G_t = Government expenditures on goods and services in constant currency
 - X_t^1 = Exports of goods and services in constant currency
 - M_t^1 = Imports of goods and services in constant currency
 - M_t^2 = Imports of capital in current domestic currency
 - X_t^2 = Exports of capital in current domestic currency
 - $M_t^{2,1}$ = Repatriated earnings to the country in current domestic currency
 - $X_t^{2,1}$ = Repatriated earnings from the country in current domestic currency
 - P_t = Domestic price level

We have the (accounting) identity:

$$(1) \quad Y_t = I_t + C_t + G_t + X_t^1 - M_t^1 + \frac{M_t^{2,1}}{P_t} - \frac{X_t^{2,1}}{P_t}$$

$$(2) \quad Y_t^1 = Y_t - D_t - T_t^1 + R_t - T_t^2 - (\Pi_{5,t} - T_{5,t}^2) - \Pi_{6,t}$$

$$= Y_{1t}^1 + Y_{2t}^1 + Y_{3t}^1 + Y_{4t}^1$$

This equation (2) specifies that total personal income equals Gross National Product minus depreciation and indirect business taxes, and plus transfer payments. This may also be regarded as the sum of factor payments augmented by transfers. We are assuming that income from capital accrues to individuals. $(\Pi_{5,t} - T_{5,t}^2)$ is after tax profits of foreign nationals, and $\Pi_{6,t}$ is profits of government owned enterprise.

Y_t^1 = Total personal income

D_t = Depreciation (Capital Consumption Allowances)

T_t = Total taxes

T_t^1 = Indirect business taxes

T_t^2 = Profits taxes

R_t = Transfer payments to individuals

Π_t = Total profit

$\Pi_{i,t}$ = Profit of the i -th group ($i = 1, \dots, 6$)

We consider the population to be broken down into four socio-economic categories in the countries of our interest, (for example: Peru, Bolivia, Ecuador, Mexico and Guatemala). They are: "Commercial" Whites ($i=1$), "Agricultural" Whites ($i=2$), Mestizos ($i=3$), and Indians ($i=4$). The color divisions are not actually racial in South and Central America. An approximate criterion is that a detribalized Indian becomes a mestizo. A mestizo who attains an appropriate degree of wealth and education may become a white.

These divisions will be discussed in further detail in subsequent sections. The subscript 5 is used to denote foreign nationals in the country; while 6 refers to government.

$Y_{i,t}^1$ = Personal income of the i -th socio-economic category

$$(3) \quad Y_t^2 = Y_t^1 - T_t^3 = Y_{1t}^2 + Y_{2t}^2 + Y_{3t}^2 + Y_{4t}^2$$

In equation (3) disposable income (Y_t^2) is defined as personal income minus taxes levied on personal income.

Y_t^2 = disposable income

T_t^3 = personal income tax

Y_{it}^2 = disposable income of i -th socio-economic category

$$(4) \quad C_t = \sum_{i=1}^4 C_{it}$$

$$(5) \quad C_{it} = \alpha_{i,1} N_{i,t} + \alpha_{i,2} Y_{i,t}^2$$

In equation (5) we have assumed as a first approximation that per capita consumption can be described as a linear function of per capita disposable income.

C_t = total consumption

C_{it} = consumption of the i-th group

N_t = total population

$N_{i,t}$ = population of the i-th socio-economic group

2.2 Government

Government activities may be grouped, for national accounts purposes, into two broad categories: expenditures, which represent an allocation of part of gross national product, and taxes and transfers, which are, of course, means of redistributing income. The next section takes up government expenditures and, although taxes and transfers do not necessarily belong here, they are taken up in Section 2.2.2.

2.2.1 Government Expenditures

Government expenditures in our model are divided into three categories, which we believe are particularly relevant for a development study: gross investment; health, education and welfare; and a residual category. These are denoted:

G_t^1 = gross investment by government

G_t^2 = government expenditures on health, education and welfare

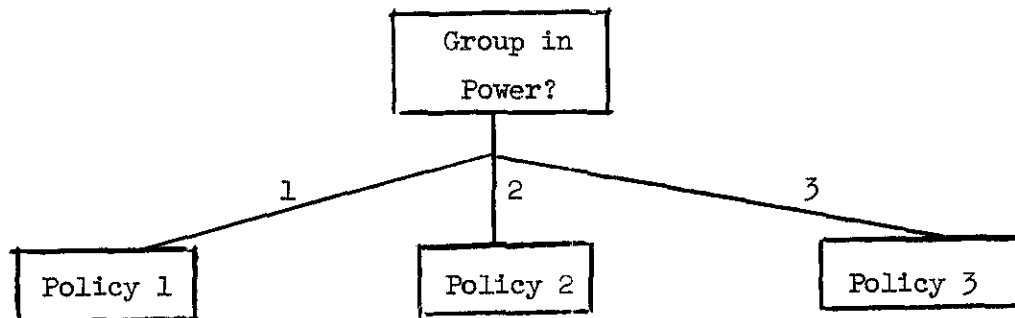
G_t^3 = other government expenditures

Equation (6), below, gives the relevant accounting identity:

$$(6) \quad G_t = G_t^1 + G_t^2 + G_t^3$$

The breakdown of government expenditures will be determined by the group in power and represents one of the politico-economic aspects of this model which needs to be investigated in detail. For example it appears to be worthwhile to distinguish amongst two types of "revolutionary" change in a Latin American state; the first a change in the "palace guard" which might result in a shift in the biases between landed and commercial or industrial interests; the second, a deep social revolution such as that of Mexico in 1911 or the recent Cuban revolution.

In the content of this model we eventually will wish to introduce political control alternatives explicitly. The method for doing this is indicated in the flow diagram below. However here we limit ourselves



by the assumption that governmental policy will remain constant over the period under consideration.

As a simple first approximation we assume that government investment is given by:

$$(7) \quad G_t^1 = \beta_1 + \beta_2 Y_{t-1}$$

Here government investment is a simple linear function of lagged gross national product.

Government expenditures on health, education and welfare are made to depend on gross national product and total population during the previous period. This allows for an increase in health, education and welfare expenditures as either population or GNP rises. This is shown below in equation (8).

$$(8) \quad G_t^2 = \beta_3 + \beta_4 Y_{t-1} + \beta_5 N_{t-1}$$

Other government expenditures are assumed constant:

$$(9) \quad G_t^3 = \beta_6$$

2.2.2 Government Revenues and Transfers

We allow for four types of taxes, indirect business taxes other than tariffs, tariffs, profits tax and personal income tax. Only two types of transfers are considered in this model. They are transfers to the government from abroad, and transfers from the government to people and/or institutions within the country.

The basic tax identities are given below:

$$(10) \quad T_t = T_t^1 + T_t^2 + T_t^3$$

$$(11) \quad T_t^1 = T_t^{1,1} + T_t^{1,2}$$

T_t = total tax revenue in constant currency

T_t^1 = indirect business taxes in constant currency

T_t^2 = taxes on profits in constant currency

T_t^3 = personal taxes in constant currency

$T_t^{1,1}$ = indirect business taxes, except tariffs in constant currency

$T_t^{1,2}$ = tariffs in constant currency

Equation (10) gives total tax revenue as the sum of indirect business taxes ($T_t^1 = T_t^{1,1} + T_t^{1,2}$) and income taxes ($T_t^2 + T_t^3$).

$T_t^{1,1}$ is assumed proportional to gross national product, and $T_t^{1,2}$ to imports of goods and services:

$$(12) \quad T_t^{1,1} = \beta_7 Y_t$$

$$(13) \quad T_t^{1,2} = \beta_8 M_t^1$$

The tax on profits is assumed proportional to profits earned in the private sector:

$$(14) \quad T_t^2 = \beta_9 (\Pi_t - \Pi_{6,t})$$

$\Pi_{6,t}$ = profits of government owned enterprise in constant currency

Personal income taxes are assumed proportional to personal income; however, the constant of proportionality need not be the same for all four socio-economic groups.

$$(15) \quad T_{i,t}^3 = \alpha_{i,3} Y_{i,t}^1$$

$$(16) \quad T_t^3 = \sum_{i=1}^4 T_{i,t}^3$$

It is clear from equation (2) that personal income includes the after tax profits of the four socio-economic groups. Thus profits are double-taxed.

Transfers to the government from abroad are assumed exogenous to the model, being determined by the foreign policy of other nations such as the United States. Transfers by the government to the four socio-economic groups is given below:

$$(17) \quad R_t = \sum R_{i,t}$$

$$(18) \quad R_{i,t} = \alpha_{i,4} Y_{t-1} - \alpha_{i,5} (Y_{t-1} - Y_{t-2})$$

$M_t^{2,3}$ = transfers to the government from abroad in current currency

$R_{i,t}$ = transfers by the government to the i-th socio-economic group in constant currency

Transfers are assumed to depend on past GNP and changes in past GNP. It is assumed they rise secularly as GNP rises, but are counter-cyclical.

2.3 Investment in Physical Capital

In this section we consider gross investment by the four socio-economic groups and by foreign nationals. Government investment was discussed above in Section 2.2.1.

2.3.1 Gross Investment by Commercial Whites

Gross investment by commercial whites is given below in equation (19):

$$(19) \quad I_{1,t} = \text{Max} \left[0, \left\{ \alpha_{1,6} (Y_{t-1} - Y_{3,t-1}^2 - Y_{t-2} + Y_{3,t-2}^2) + \alpha_{1,7} (Y_{3,t-1}^2 - Y_{3,t-2}^2) + \alpha_{1,8} K_{1,t-1} \right\} \right]$$

$K_{i,t}$ = the capital stock owned by members of the i-th socio-economic group

It is the sum of two accelerator terms and a depreciation term. The first accelerator term involves changes in GNP with changes in mestizo disposable income subtracted out, while the second accelerator term involves changes in mestizo disposable income. It is assumed that change in mestizo income is of particular import to investment in an economy as its growth represents the possible expansion of a lower middle class market. A serious drop in this income may also be an important factor in the growth of political instability.

2.3.2 Gross Investment by Agricultural Whites

Investment in this group is taken to depend on disposable income within the group. For these people investment is much more a matter of status than of rational calculation. Investment is given by:

$$(20) \quad \begin{aligned} I_{2,t} &= \alpha_{2,6} + \alpha_{2,7} Y_{2,t-1}^2 && \text{if } Y_{2,t-1}^2 \geq K \\ &= 0 && \text{if } Y_{2,t-1}^2 < K \end{aligned}$$

2.3.3 Gross Investment by Mestizos and Indians

In general, mestizos have little surplus for investment; nevertheless we assume that:

$$(21) \quad I_{3,t} = Y_{3,t-1}^2 - C_{3,t-1} + D_{3,t}$$

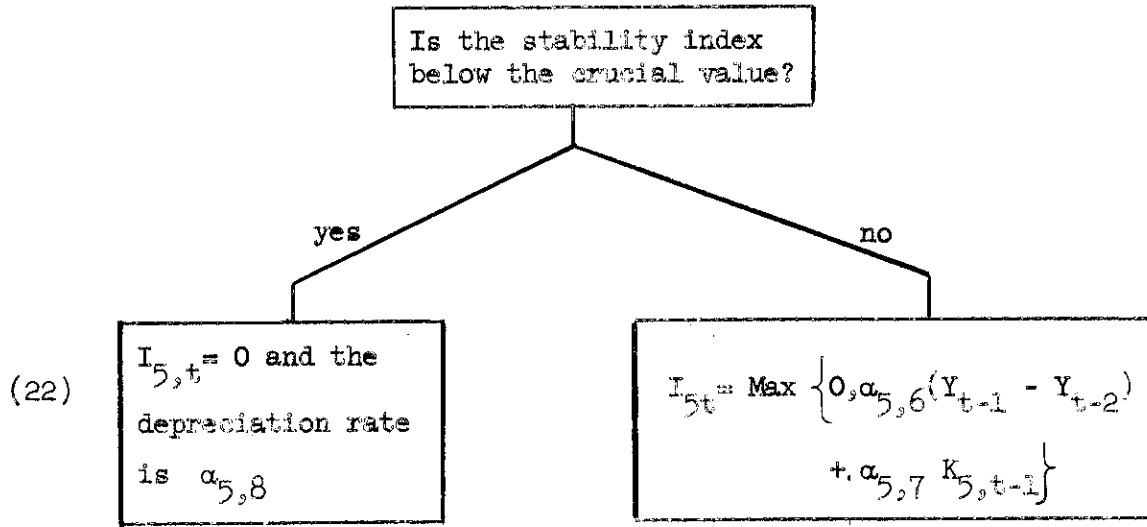
We note that the sum of consumption and net investment equal disposable income.

Our initial assumption is that investment by the Indian sector is negligible. In this model we omit further consideration of it.

2.3.4 Gross Investment by Foreign Nationals

The final category of investment is investment by foreign nationals. Foreign nationals are probably more ready, willing and able to pull out of a country when the country appears to be heading toward social or political instability than are the local citizens. One simple way to represent this is with a "domestic stability index". The index could be a function of the length of time the present government has been in power, the recent growth rate of gross national product and the growth rates of real wages for the various socio-economic groups. The index will be the

higher, and the country the more stable, the higher are these variables. If the index is below a crucial value, the foreign investors seek to pull out at the fastest possible rate, which is the maximum depreciation rate. The diagram below represents the investment decision for foreign investors:



$I_{5,t}$ = the rate of gross investment of foreign nationals

Thus if the index is below the crucial value, gross investment is zero and net investment is $-\alpha_{5,8} K_{5,t-1}$. On the other hand, if the index is sufficiently high investment is determined by depreciation and the change in gross national product between the previous period and two periods prior; it is the greater of zero or the amount calculated from these considerations. We note that there are two depreciation rates above, $\alpha_{5,7}$ and $\alpha_{5,8}$. Through hard driving and poor maintenance, capital can be made to depreciate faster than it would with careful use and maintenance. The actual rate of depreciation for a capital good will depend on the cost of

replacement, expected returns on it and the cost of various levels of maintenance. If a foreign investor wishes to pull out of a country due to the instability of the political situation, he may expect expropriation or wholesale destruction of his investment. He then wishes to work his capital as hard as possible until it has no value left, without spending anything on it. Such expenditures are a payment for a prolonged life of the equipment into a period when he expects to be unable to gain from it. $\alpha_{5,8}$ is this maximal rate of depreciation relevant to when the capital is used up as quickly as possible. $\alpha_{5,7}$ is the "normal" rate, applicable when the investor expects to stay in the country indefinitely. It will be seen later that the stability index has a bearing on international capital movements.

2.4 Exports and Imports of Goods and Services

Exports are assumed to be autonomous being determined by conditions abroad which are essentially unaffected by developments in the country. In some countries such as Ecuador or Guatemala four or five items account for 75 - 90% of exports. A reasonably satisfactory model for exports could be added by including some features of prices and demands for these commodities. This is not done at this time.

Imports depend on consumption, investment and foreign transfers. Some consumption goods and some investment goods are imported; and transfers from abroad stimulate imports.

$$(23) \quad X_0^1 = \beta_{10}$$

$$(24) \quad X_t^1 = \beta_{11} X_{t-1}^1$$

$$(25) \quad M_t^1 = \beta_{12} + \beta_{13} C_{t-1} + \beta_{14} (I_t + G_t^1) + \beta_{15} \frac{M_t^{2,3}}{P_t}$$

3. Human Capital

A very important ingredient in successful growth for an under-developed country, indeed for any country, is investment in a healthy, educated population. The importance of attempting to measure the effect of health, education and welfare (HEW) expenditures on productivity is obvious.

Below we sketch two models for the measurement of human capital. The first is an input-output model which we do not utilize at present. It is included because it is our hope this basic approach will prove both feasible and extremely fruitful. The second approach, which we are utilizing for the first approximation, includes a method for converting the labor force of each socio-economic group into standard labor units which are comparable between classes. The number of standard labor units into which a member of the labor force converts depends on the HEW expenditures made on members of his group. While this method leaves something to be desired, it does represent a beginning at the difficult task of measuring labor productivity as a function of HEW.

3.1 Input-Output Model

In Table I, below, appears a simple input-output scheme. Each column is an activity. The negative coefficients are outputs and the positive, inputs. Thus, looking at the first activity, it shows that if we put in one infant, $\gamma_{6,1}$ of social and medical resources and $\gamma_{7,1}$ of education resources, we will get as outputs one child and $\gamma_{8,1}$ of labor.

Infants	+1				
Children	-1	+1			
Young Adults		-1	+1		
Middle Aged			-1	+1	
Old				-1	+1
Costs: Social, Medical	$\gamma_{6,1}$	$\gamma_{6,2}$	$\gamma_{6,3}$	$\gamma_{6,4}$	$\gamma_{6,5}$
Costs: Education	$\gamma_{7,1}$	$\gamma_{7,2}$	$\gamma_{7,3}$	$\gamma_{7,4}$	$\gamma_{7,5}$
Labor Output	$-\gamma_{8,1}$	$-\gamma_{8,2}$	$-\gamma_{8,3}$	$-\gamma_{8,4}$	$-\gamma_{8,5}$

TABLE I

We have divided the life span into five stages here and have implicitly assumed that at each stage there will be a linear relationship linking the labor output (measured in some standard unit) with the HEW inputs.

In order to study the effectiveness of the HEW programs we might wish to expand the number of activities shown in Table I to account for different inputs in education, for example. As a good second approximation four social groups, each portrayed by five activities, should

provide sufficient information for the estimation of human capital.

If we are able to measure labor output in terms of a standardized dollar and if we are willing to specify a discount factor in the economy we can calculate a value for an individual in the t-th age group.

$$V_t = \sum_{i=t}^5 \rho^{i-t+1} (\gamma_{8,i} - \gamma_{6,i} - \gamma_{7,i})$$

where

V_t = The human capital value of an individual in age group t

ρ = discount rate

$\gamma_{k,i}$ = input output coefficients

3.2 The Current Model

The model which we look at in this section treats all members of a socio-economic group identically, regardless of age and sex. It would not be difficult to expand it to keep track of the population within each socio-economic group by age groups. The steps in the model are two:

- 1) The size of the population is computed, taking into account births, deaths and transfers from one class to another.
- 2) The labor force is converted into standard labor units, taking into account the per capita HEW expenditures made in recent years.

Let:

N_t = Total population in time t

$N_{i,t}$ = population of i -th group in time t

A_t = deaths in total population during period t

$A_{i,t}$ = deaths in i -th group

B_t = births in total population

$B_{i,t}$ = births in i -th group

$F_{1,t}$ = number of people who transfer from 3rd (mestizo) group to group 1 (Commercial White)

$F_{3,t}$ = number who transfer from 4th (Indian) group to 3rd

Some basic bookkeeping relationships which involve the variables defined above are:

$$(26) \quad N_t = N_{t-1} + B_{t-1} - A_{t-1}$$

$$(27) \quad N_{1,t} = N_{1,t-1} + B_{1,t-1} - A_{1,t-1} + F_{1,t-1}$$

$$(28) \quad N_{2,t} = N_{2,t-1} + B_{2,t-1} - A_{2,t-1}$$

$$(29) \quad N_{3,t} = N_{3,t-1} + B_{3,t-1} - A_{3,t-1} - F_{1,t-1} + F_{3,t-1}$$

$$(30) \quad N_{4,t} = N_{4,t-1} + B_{4,t-1} - A_{4,t-1} - F_{3,t-1}$$

Equation (26) gives total population in period t as population in period $t-1$ plus births and minus deaths during period $t-1$. Equations (27) through (30) are the analogous equations for each group. They allow for two avenues of inter-group transfer. The relationships governing

births and deaths are given below:

$$(31) \quad A_{i,t-1} = \alpha_{i,30} N_{i,t-1}$$

$$(32) \quad B_{i,t-1} = \alpha_{i,31} N_{i,t-1}$$

Transfers from one class to another are considered to occur when a mestizo acquires sufficient wealth and position to be, in fact, a member of the commercial white community; or when an Indian turns outward from his tradition-bound environment and tries to get along in the more market oriented part of society. The two transfer equations are:

$$(33) \quad F_{1,t-1} = \alpha_{1,9} N_{3,t-1} \left[\frac{N_{1,t-1} \cdot N_{3,t-1}}{(N_{1,t-1} + N_{3,t-1})^2} \right] \left[\alpha_{1,10} \frac{Y_{3,t-1}^2}{N_{3,t-1}} + \alpha_{1,11} H_{3,t-1} \right]$$

$$(34) \quad F_{3,t-1} = \alpha_{3,9} N_{4,t-1} \left[\frac{N_{3,t-1} \cdot N_{4,t-1}}{(N_{3,t-1} + N_{4,t-1})^2} \right] \left[\alpha_{3,10} \frac{Y_{4,t-1}^2}{N_{4,t-1}} + \alpha_{3,11} H_{4,t-1} \right]$$

In both these equations, the first expression in brackets is the proportion of the combined population of the two groups which is in one of the classes multiplied by the proportion in the other. This proportion is modified by a parameter, $\alpha_{i,27}$, and by an expression involving per capita disposable income for the group and HEW expenditures. The result is modified by the size of the group from which people will transfer. It is assumed, of course, that as HEW and per capita income rise, the numbers which will transfer rise.

The next step is to determine the labor force for each group and then convert into standard labor units.

$$(35) \quad N_{i,t}^1 = \alpha_{i,12} N_{i,t}$$

$$(36) \quad N_{i,t}^2 = e^{\left(\alpha_{i,13} - \frac{\alpha_{i,14}}{H_{i,t}^1 + \alpha_{i,15}} \right)} N_{i,t}^1$$

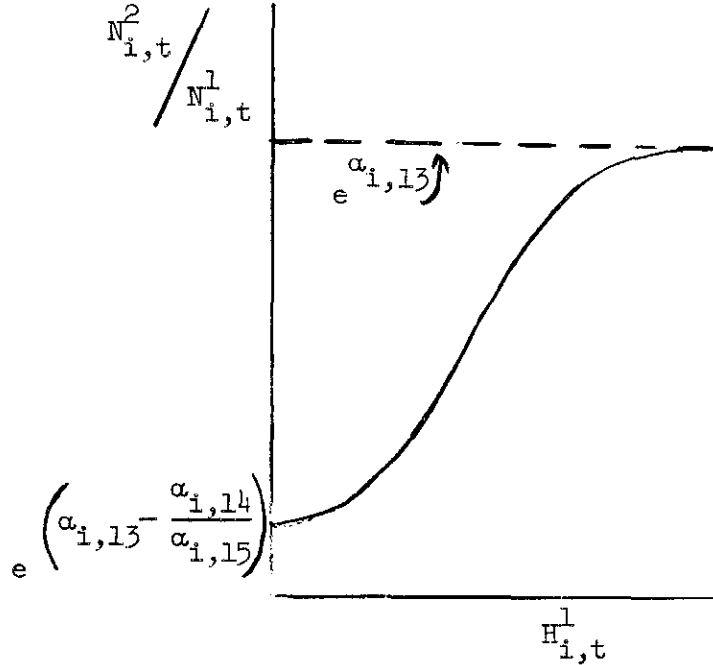


Figure 1

where:

$N_{i,t}^1$ = labor force of group i

$N_{i,t}^2$ = number of standard labor units represented by the labor force of group i

$H_{i,t}^1$ = per capita HEW on group i , summed for $\beta_{16} + 1$ years

$\alpha_{i,12}$ is the proportion of the i -th group which is in the labor force. The general shape and characteristics of equation (36) are illustrated in Figure 1. The minimum value of $N_{i,t}^2$, when $H_{i,t}^1$ is zero, is $N_{i,t}^1 \left(e^{\alpha_{i,13} - \alpha_{i,14}/\alpha_{i,15}} \right)$.

This is constrained to be non-negative.

HEW shows increasing, then decreasing returns. The marginal product of HEW in terms of labor units is always positive, although it is asymptotic to zero and total labor units is asymptotic to $N_{i,t}^1 e^{c_{i,13}}$.

The derivation of $H_{i,t}^1$ remains:

$$(37) \quad H_{i,t} = \alpha_{i,16} + \alpha_{i,17} C_{i,t} + \alpha_{i,18} + \alpha_{i,19} G_t^2$$

$$(38) \quad \sum_{i=1}^4 \alpha_{i,18} + \alpha_{i,19} G_t^2 = G_t^2$$

$$(39) \quad H_{i,t}^1 = \sum_{\theta=t-\beta_{16}}^t \frac{H_{i,\theta}}{N_{i,\theta}} = H_{i,t-1}^1 - \frac{H_{i,t-\beta_{16}-1}}{N_{i,t-\beta_{16}-1}} + \frac{H_{i,t}}{N_{i,t}}$$

According to equation (37), the HEW expenditures on the i -th group are a linear function of the consumption expenditures of the group and of government HEW expenditures. Equation (38) is merely a constraint to assure that the whole of G_t^2 is parcelled out among the four groups.

Equation (39) is the sum of per capita HEW in group i for the current year and the β_{16} preceding years.

4. Production and Distribution

In this section the treatment of production and distribution is outlined. The production side of the model utilizes an aggregate Cobb-Douglas production function. The assumptions about distribution are a variant of traditional marginal productivity theory.

4.1 Production

We assume the existence of three factors: capital, labor and the "institutional" factor. The third of these is not represented directly in the production function, and is discussed below.

Let:

Y_t^0 = gross domestic product in constant currency

$$(40) \quad Y_t^0 = Y_t - \frac{M^{2,1}}{P_t} + \frac{X^{2,1}}{P_t}$$

Equation (40) contains an accounting identity which defines gross domestic product as the sum of all goods and services produced within the country. It gives our assumption of how gross domestic product, as an output, relates to the labor force and capital stock as inputs. This "aggregate production function" we have assumed to be a Cobb-Douglas function.

$$(41) \quad Y_t^0 = \beta_{17} K_t^{\beta_{18}} N_t^{1-\beta_{18}} \quad (0 < \beta_{18} < 1)$$

4.2 The Consistency of Production and Consumption

In the preceding sections national income has been calculated both from the consumption and production sides of the economy. These calculations were based on technical and behavioral equations. It is likely that the two numbers arrived at will not be consistent although a complete model would insure equality. The reason for the possible mismatch in this model

is that the price system has not yet been introduced and it is via this mechanism that consistency will be obtained. In the program written from this paper, as a first approximation, when consumption and production figures do not match a scale factor correction is applied which represents an overall change in price level.

4.3 Distribution

We assume that generally labor and capital are paid in proportion to their marginal productivity with the exception of the Indian sector of the population where institutional rigidities have an influence. An allocation of income is also made to an "Institutional Administrative" factor. The appropriate licenses, administrative blessings, cooperation of local authorities, adequate protection against institutional delays

and so forth all are part of a production process and may be regarded as very valuable factors. They are in general resources with increasing returns to scale. Essentially it costs the mayor nothing extra to sign additional permissions to build, but each permission is nevertheless needed as a factor. There is possibly an upper bound or a capacity beyond which an individual cannot go without dipping too heavily into his "political capital" (a quantity we do not attempt to measure at this point); hence a given administrative system may be regarded as having a capacity in the same way as does a bridge or railway system.

4.3.1 Wages

The returns to labor in the several groups are given by:

$$(42) \quad W_{i,t} = \alpha_{i,20} \frac{\partial Y_t^0}{\partial N_t^2} N_{i,t}^2 \quad i = 1, 2, 3$$

and

$$(43) \quad W_{4,t} = (\alpha_{4,20} \frac{\partial Y_t^0}{\partial N_t^2} + \beta_{19} + \xi_{3,t}) N_{4,t}^2$$

Where the $\alpha_{i,20}$ for $i = 1, \dots, 4$ are proportionality factors which will be discussed further below.

$W_{i,t}$ = labor income of i -th social group in constant currency

β_{19} = a constant component of income to Indians

ξ_3 = a random variable

The equations (42) are merely the standard returns to labor in a competitive economy modified by the $\alpha_{1,20}$. The equation (43) is a modified version of (42) where we assume that the income to the Indians has three components, the first which is in proportion to marginal productivity, the second a constant determined from socio-economic considerations and the third a random variable determined by such things as weather and crop conditions.

4.3.2 Gross Profits

As we have written the production function for gross domestic product, equations (44) below include depreciation coverage as part of the returns to capital.

$$(44) \quad \Pi_{i,t} + D_{i,t} = \alpha_{i,21} \frac{\partial Y_t^0}{\partial K_t} K_{i,t} \quad i = 1, 2, 3, 5, 6$$

Where $\alpha_{i,21}$ = factors of proportionality in return to capital.

4.3.3 Income to the Institutional Factor

The third set of equations (45) given below represent the sources of payments to the institutional administrative factors. We assume that there are primarily eight sources, being respectively government investment and transfers, and private investment.

$$Z_{i,t} = \text{Income to the institutional factor supplied by the } i\text{-th social group.}$$

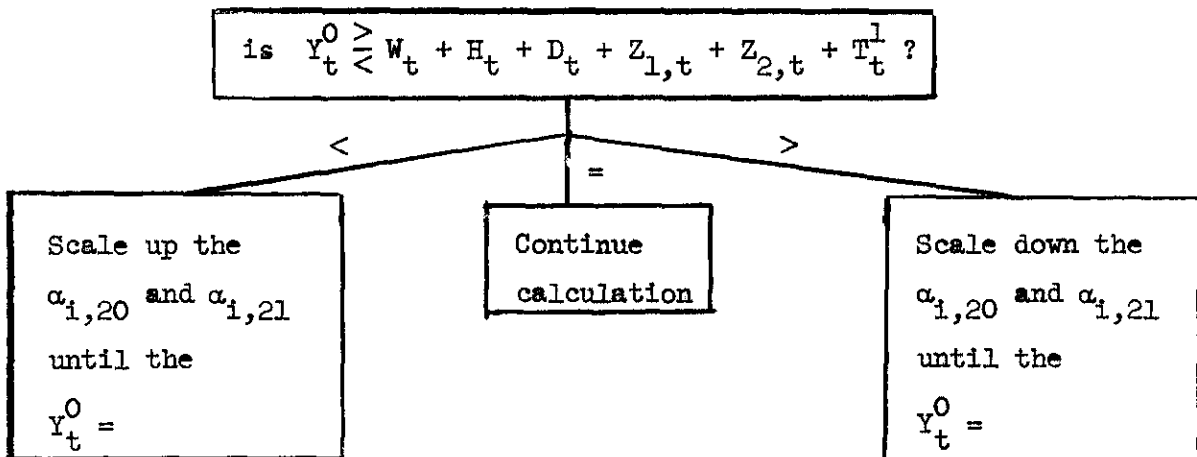
$$\begin{aligned}
 (45) \quad Z_{i,t} &= \alpha_{i,22} G^1 + \alpha_{i,23} (G_t^2 + G_t^3) \\
 &+ \alpha_{i,24} \left(R_t - \frac{M_t^{2,3}}{P_t} \right) + \alpha_{i,25} \frac{M_t^{2,3}}{P_t} + \alpha_{i,26} I_{1,t} \\
 &+ \alpha_{i,27} I_{2,t} + \alpha_{i,28} I_{3,t} + \alpha_{i,29} I_{5,t} \qquad i = 1,2
 \end{aligned}$$

We assume that income from this source only accrues to the commercial and agricultural whites.

There remains the problem of consistency checking as the identity

$$(46) \quad Y_t^0 = \sum_{i=1}^4 W_{i,t} + \sum_{i \neq 4}^6 \Pi_{i,t} + D_t + \sum Z_{i,t} + T_t^1$$

must hold. This is ensured in the manner shown in the flow diagram below.



In order to guarantee that (46) holds we adjust the scale factors to the returns to capital and labor. This implies that administrative costs and indirect business taxes have the priority in being satisfied. After they have been met then the residual is divided for the other factors.

5. International Capital Flows

Economic relations between a country and the rest of the world may be viewed in the context of imports and exports of goods and services, and imports and exports of capital. Imports and exports of goods and services have been discussed above; however international capital movements have received little attention. These flows are a barometer reflecting basic conditions and problems within the country. Thus, an adverse trade balance will show up here and perhaps give an indication of whether devaluation, stricter exchange control or other such measures are needed. A developing social condition inimical to the wealthier segment of the population may cause substantial short term capital movements out of the country which aggravate any balance of payments problems which might previously have existed.

5.1 Capital Imports

Capital inflows may be under any of three categories: transfers, long term flows and short term flows. They are denoted:

$M_t^{2,2}$ = long term inflows in constant currency

$M_t^{2,4}$ = net purchase of domestic currency by foreigners in constant currency

$L_t^{2,1}$ = the country's holdings of foreign cash in constant domestic currency

$L_t^{2,2}$ = the country's holdings of foreign long term assets in constant currency

$$(47) \quad M_t^{2,2} = \text{Max} [0, \text{policy}]$$

$$(48) \quad M_t^{2,1} = \beta_{20} L_t^{2,2} + \beta_{21} L_t^{2,1}$$

(49) $M_t^{2,3}$ is assumed to be policy determined hence exogenous

$$(50) \quad M_t^{2,4} = \beta_{22} [X_t^1 P_t - X_{t-1}^1 P_{t-1}]$$

Equation (50) gives net purchases of domestic currency by foreigners as an adjustment mechanism which keeps foreign stocks equal to $\beta_{22} X_t^1 P_t$.

This is the necessary transactions level for international trade.

5.2 Capital Exports

On the assumption that no transfer payments are made by the country abroad, capital exports consist of long and short term movements. Short term outflows are of three types, one of which is related to the transactions needs of the country and two of which are categories of flight capital:

$X_t^{2,2}$ = long term capital exports in current currency

$X_t^{2,3}$ = flight capital sent by foreign investors in current currency

$X_t^{2,4}$ = flight capital sent by citizens of the country in current currency

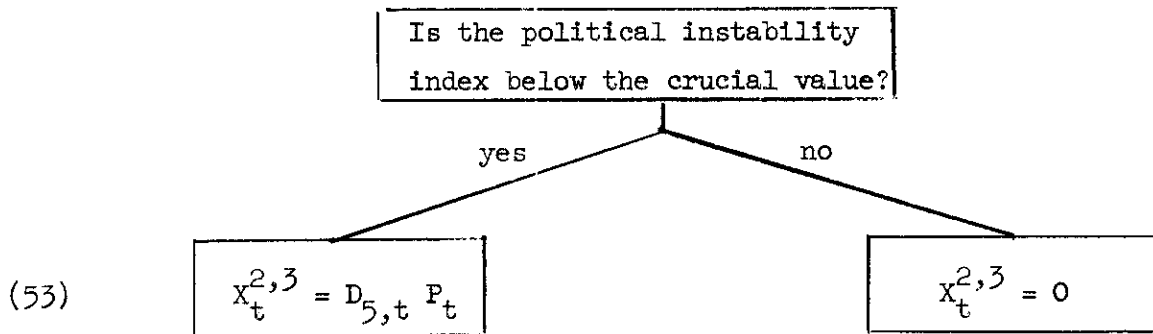
$X_t^{2,5}$ = balance of payments surplus in current currency

$$(51) \quad X_t^{2,1} = (\Pi_{5,t} - T_{5,t}^2) P_t$$

$$(52) \quad X_t^{2,2} = \beta_{24} + \beta_{25} (\Pi_{1,t} - T_{1,t}^2) P_t + \beta_{26} Z_{1,t} P_t$$

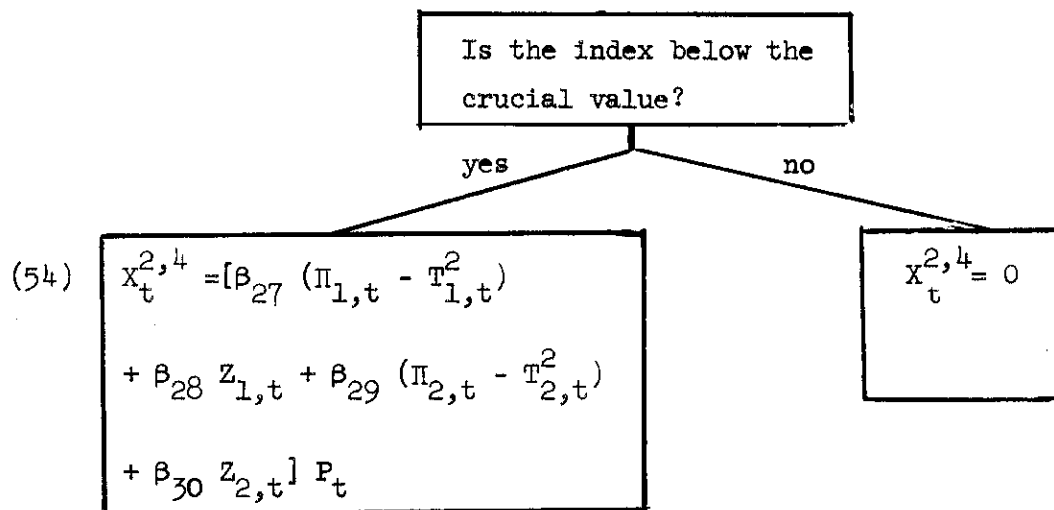
As a first approximation it is assumed that long term capital exports are a linear function of the profits earned by the commercial whites and the earnings in the institutional-administrative overhead category.

The diagram below describes the level of $X_t^{2,3}$, flight capital sent abroad by foreign nationals:



If political unrest is too great, depreciation charges are sent abroad.

A similar scheme is utilized for $X_t^{2,4}$:



(55) $X_t^{2,5} = X_t^1 P_t + \sum_{i=1}^4 M_t^{2,i} - P_t M_t^1 - \sum_{i=1}^4 X_t^{2,i}$

Equation (55) is an accounting identity which gives short term outflows as a residual which just balances the international accounts for the country. Transfers were discussed above in the sections on government receipts and expenditures.

6. The Monetary System

Consistent with our practice in most other aspects of this model, the monetary system is included in a rather rough and ready way. Most of the fundamental relationships and accounting identities appear; however, some are omitted. For example, we have not yet modelled borrowing and lending.

In several preceding sections the functional relationships involving the money supply, price level and international monetary transactions have been stated and explained. It remains only to give the accounting identities.

P_t^1 = the price of foreign exchange in terms of domestic currency

$L_t^{2,1}$ = our holdings of foreign exchange in current domestic currency

$L_t^{2,2}$ = our holdings of foreign long term assets in current domestic currency

L_t = total domestic nominal money supply

$L_{i,t}$ = domestic cash balances of i-th group

$$(56) \quad L_t^{1,1} = L_{t-1}^{1,1} + M_{t-1}^{2,4}$$

$$(57) \quad L_t^{1,2} = L_{t-1}^{1,2} + M_{t-1}^{2,2}$$

$$(58) \quad L_t^{2,1} = \frac{P_t^1}{P_{t-1}^1} L_{t-1}^{2,1} + X_{t-1}^{2,1} + X_{t-1}^{2,3} + X_{t-1}^{2,4} + X_{t-1}^{2,5} - M_{t-1}^{2,1} - M_{t-1}^{2,3}$$

$$(59) \quad L_t^{2,2} = \frac{P_t^1}{P_{t-1}^1} L_{t-1}^{2,2} + X_{t-1}^{2,2}$$

The first two equations above are nearly self explanatory. Foreign holdings of our money today equal yesterday's holdings plus the change during the intervening time. Similarly for $L_t^{1,2}$. The next equation requires a bit more explanation. The value of yesterday's foreign exchange

holdings in terms of the foreign currency is $\frac{L_{t-1}^{2,1}}{P_{t-1}^1}$ and the value of those

holdings in terms of local currency today is $P_t^1 \frac{L_{t-1}^{2,1}}{P_{t-1}^1}$. Similarly for

the last equation.

For any segment of the economy, its stock of money today equals yesterday's stock plus yesterday's disposable income, depreciation and factor income from abroad, and minus its consumption, investment purchases of long term foreign assets and flight capital sent abroad. For the economy as a whole, today's money supply equals yesterdays plus the government deficit and factor incomes from abroad and minus foreign transfers, foreign purchases of domestic currency, trade surplus (net increase in foreign exchange holdings) and factor incomes paid to foreigners. These are shown below.

$$(60) \quad L_{1,t} = L_{1,t-1} + P_{t-1} (Y_{1,t-1}^2 + D_{1,t-1} - C_{1,t-1} - I_{1,t-1}) - X_{1,t-1}^{2,4} \\ - X_{t-1}^{2,2} + M_{1,t-1}^{2,1}$$

$$(61) \quad L_{2,t} = L_{2,t-1} + P_{t-1} (Y_{2,t-1}^2 + D_{2,t-1} - C_{2,t-1} - I_{2,t-1}) - X_{2,t-1}^{2,4} \\ + M_{1,t-1}^{2,1}$$

$$(62) \quad L_{3,t} = L_{3,t-1} + P_{t-1} (Y_{3,t-1}^2 + D_{3,t-1} - C_{3,t-1} - I_{3,t-1})$$