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ANTICIPATED INFLATION AND ECONOMIC WELFARE

IN A NEOCLASSICAL MODEL

Edmund S. Phelps

April 21, 1964

ANTICIPATED INFLATION AND ECONOMIC WELFARE
IN A NEOCLASSICAL MODEL

Edmund S. Phelps[1]

It was once believed that a fully anticipated inflation has no effect upon the allocation of resources. While this proposition was reiterated as late as 1949 by Lerner, it met with an objection by Friedman[2]: If holders of money receive no interest on their deposits (or if the rate of interest on money is fixed) then the expectation of inflation -- by leading to a higher nominal rate of interest on other (earning) assets -- will widen the spread between the rate of interest on money and that on other assets and hence increase the incentive to "economize" on money. People will be driven to reallocate their resources -- e.g., take more trips to the bank --in an inefficient way.

Friedman's argument was later formalized by Bailey[3] who proposed to measure the "cost" of inflation (per unit time) by the area under the demand curve for real money balances (displayed as a function of the nominal rate of interest) between the initial and the new level of real money holdings, the new equilibrium level being smaller than the initial as a consequence of the rise in the equilibrium nominal rate of interest. Bailey joined Friedman in condemning inflation in view of this cost.

There the matter lay until Robert Mundell[4] demonstrated, in the context of the Metzler flexible wage-price model[5], that the expectation of inflation does not raise the nominal rate of interest on earning assets by the full amount of

the expected rate of inflation. Rather, as the nominal interest rate rises and people therefore seek to reduce their money holdings, prices move to a higher equilibrium level, reducing the real value of these holdings and thus stimulating greater saving at given levels of the real rate of interest and real income (the "Pigou effect"). The increased desire to save reduces the equilibrium real rate of interest and thereby prevents the nominal rate of interest from rising above its initial level by the full amount of the expected rate of inflation.

By implication the cost of inflation in Mundell's model cannot be measured as Bailey proposed because ceteris are not paribus: the real rate of interest has fallen. Mundell concludes that the expectation of inflation confers "benefits" or "evils" depending apparently upon whether the rise of saving and investment associated with the fall of the real interest rate is good or bad. The upshot is that the welfare analysis of inflation appears to lack definite conclusions.

Nevertheless definite welfare conclusions can be drawn. It will be shown in this paper that Mundell's analysis refers only to an exogenous expectation of inflation to which the government makes no fiscal or monetary response. Upon building a Metzler-like model having fiscal and monetary controls over consumption and investment demand, we show that whether the real interest rate falls or rises when inflation becomes expected depends entirely upon the government's use of these fiscal and monetary controls. Should the expectation

of inflation be induced by inflationary government policies the real rate of
interest still need not fall: the government has the latitude to induce
inflationary expectations by (fiscal) means which raise the real rate of interest
or by (monetary) means which reduce the real rate of interest. Thus, the
expectation of inflation, even when that expectation is induced and sustained
by the government, will depress the real rate of interest, as Mundell concluded,
only if the government desires that to happen.

Since there is no necessary connection between the expected rate of inflation
and the real rate of interest -- an infinity of anticipated inflation rates are
possible, all with the same real rate of interest -- it can be shown that, as
Friedman and Bailey argued, the expectation of inflation in excess of a certain
rate (which may be negative) has an unambiguously ill effect upon "feasible
welfare" provided the government finds it infeasible to pay interest on money at
a suitable rate. But it will also be shown that the expectation of inflation
need have no effect upon welfare if the government is able to pay the appropriate
rate of interest on money.

Finally, we reconcile our results with the well-known paper by Vickrey in
which he argued that in some circumstances an anticipated inflation would be
desirable.[6] A summary of our principal results concludes the paper.

II. CONSEQUENCES OF THE EXPECTATION OF INFLATION WITHOUT GOVERNMENT ACTION

We suppose that wages and prices are perfectly flexible and assume that
the economy is able in each period to achieve a full employment equilibrium

(implying nonnegative equilibrium price level and rate of interest). We suppose
that the supply of labor is perfectly inelastic so that equilibrium employment is
fixed and independent of the other variables. In what follows, therefore, the
labor market is ignored.

There is either one bank, the central bank, or else only banks which keep
100 per cent reserves at the central bank. In either case, "money" is equal to
central bank liabilities. Henceforth we refer to "the bank".

The economy divides its wealth between two assets, noninterest bearing
"money" and equity "shares" (claims upon capital goods). For every share there
is a unit of capital and conversely. Later we shall allow the bank to "monetize"
some of these securities but for the moment we suppose that the government owns
no shares. Hence the equilibrium real value of the privately held shares, $\underline{E}$,
in any period equals the real value of the economy's capital after the investment
of the current period.[7] Real private wealth is therefore $W = E + \dfrac{M}{p}$, where
$\underline{M}$ is the amount of money held and $\underline{p}$ is the price level.

We suppose that there is no uncertainity concerning the prospective rise of
the price level and the prospective real rate of return on shares. Further we
suppose that there is no speculative demand for money or shares. The motive
for holding money is its convenience as a medium of exchange. The transactions
models of Baumol and Tobin demonstrate the existence of a well-determined "interest
elastic" demand for money in a certainty world.

The demand for real money holdings will be supposed a function of the nominal

rate of interest, i (this is the prospective nominal yield on shares), and the aggregate volume of planned transactions during the period. Presumably the greater the yield on shares the smaller is the amount of money that people are willing to hold, given transactions. Since currently planned transactions depend mainly upon the volume of current production and the latter is a datum if resources are fully employed, we can suppress production from the demand function, writing $M = p \, L \, (i)$, $L' \, (i) < 0$.

The supply of money, which is determined by the fiscal and monetary authorities, we shall take as given for the moment, hence $M = M_o$.

The market in which these two assets (money and shares) are traded for one another will be in equilibrium -- a "portfolio" equilibrium -- when the price level and prospective nominal yield on shares equate the demand and supply of money:

$$(1) \qquad \frac{M_o}{p} = L \, (i)$$

Now if ρ , the expected relative rate of price inflation, is zero then the nominal rate of interest and the real rate of interest, r , are equal. Therefore the LM curve labelled $\rho = 0$ in Figure 1 is the locus of pairs of the real (as well as nominal) interest rate and real money holdings which satisfy this equilibrium requirement (equation 1) when the price level expected next period is equal to the current price level, i.e., when there is a zero rate of expected inflation.

The amount of capital invested, I , in any period is equal to private saving

(unconsumed production), $\underline{S}$, plus public saving (the government budgetary surplus). The latter is simply net government taxes $\underline{T}$, since we omit government expenditures from the model. Hence $I = S + T$. All taxes are lump sum.

We suppose that private saving depends upon the real rate of interest (i.e., the real prospective yield on shares), real private wealth, and taxes. Real pre-tax income is suppressed since we consider it constant as before. Hence $S = S(r, W, T)$. It is further supposed that an increase of wealth will decrease desired private saving, $S_W < 0$; that a rise of taxes reduces saving but by less than the amount of the tax, $-1 < S_T < 0$ and that an increase of the real return on shares will not decrease saving (if it decreases it at all) by as much as it decreases investment, $S_r > I'(r)$. Concerning public saving, for the present we take net taxes as given: $T = T_o$.

The prospective real rate of return on investment (shares) will be considered inversely related to the volume of investment, which gives us the relation $I = I(r), I'(r) < 0$.

Equilibrium of the whole system requires in addition to (1) that the real rate of interest and real wealth be such as to equate the corresponding level of investment to the corresponding desired level of saving:

$$(2) \qquad I(r) = S(r, E + \frac{M_o}{p}, T_o) + T_o$$

The IS curve in Figure 1 is the locus of pairs of the real interest rate and real money holdings which satify this equilibrium requirement. The slope of the curve

Figure 1
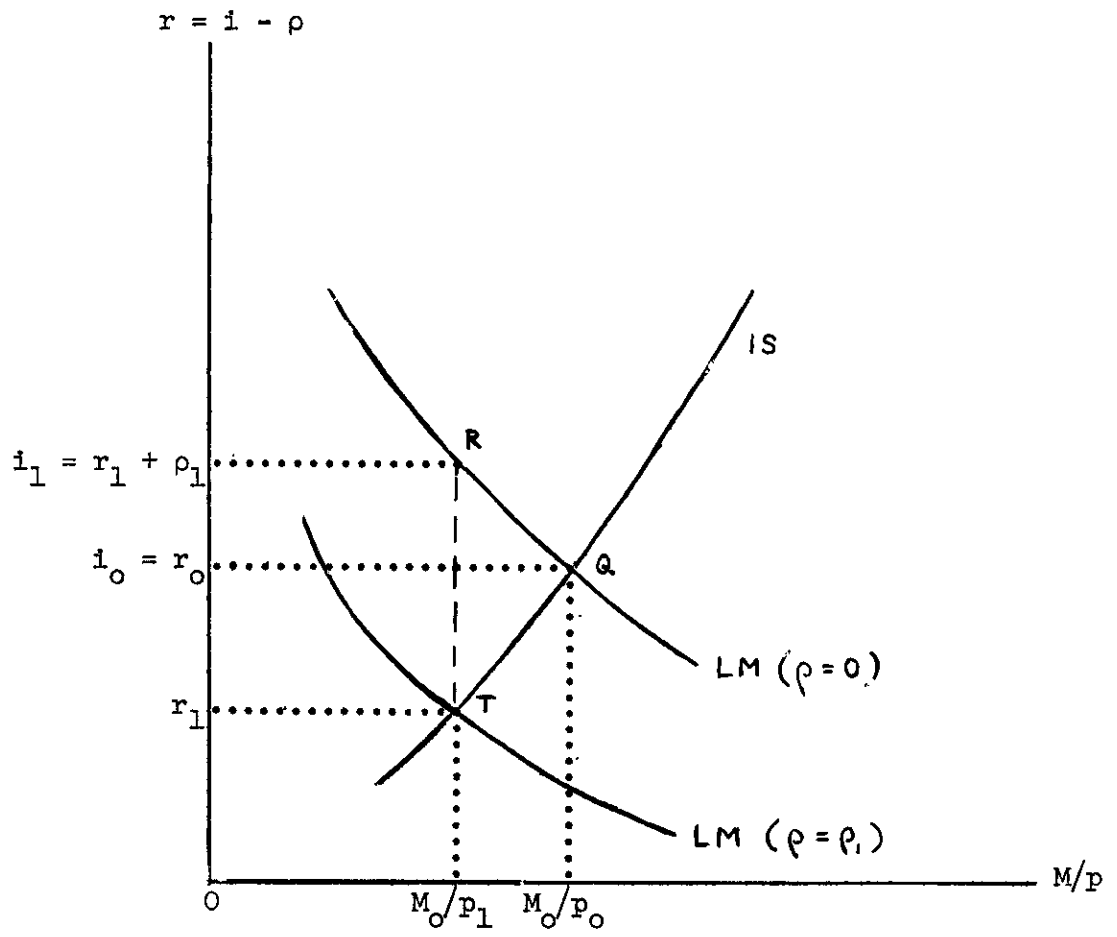
is positive because a rise of the real interest rate (due say to a decline in investment demand, i.e., an increase of liquidity preference) requires an increase of the real value of money holdings (inducing a decline of desired saving) in order to maintain equality of desired saving and investment (at the new and lower level).

If no inflation or deflation is expected then the intersection at Q of this IS curve with the LM curve for $\rho = 0$ determines the equilibrium nominal and real interest rate, $i_o = r_o$, and the equilibrium real value of money holdings, M_o/p_o (i.e., the equilibrium price level, p_o).

In general, however, the expected nominal rate will exceed the real rate (to a first approximation) by the expected relative rate of change of the price level between next period and the present one. Letting ρ denote the expected rate of inflation we have

$$(3) \qquad\qquad r = i - \rho$$

The expectation of a rising price level makes shares more attractive relative to money at every real rate of interest (real yield on shares) since the price of shares will keep pace with the price level. This increase of the expected nominal yield on shares decreases the quantity of real money balanced demanded, increases the quantity of shares (hence capital) demanded and thereby causes the price level to be bid higher. To the extent that the resulting decline of the real value of money holdings stimulates additional saving, additional investment will take place, driving down the real yield on shares and thus lessening the ultimate rise of the

nominal rate of interest. Equilibrium is reached when the real value of cash balances has declined sufficiently that money holdings are no longer in excess supply.

With reference to Figure 1, let ρ be positive and equal to $\rho_1 = RT$ (the distance between R and T). Then the LM curve for $\rho = \rho_1$ will lie below the LM curve for $\rho = 0$ by the amount ρ; because $\rho > 0$, it takes a lower r to keep $L(i) = L(r + \rho)$ constant at any specified M/p. The new equilibrium is at T where the real interest rate is r_1 and the real value of the money supply is M_0/p_1. The point R indicates the new nominal interest rate $i_1 = r_1 + p_1$.[8]

III. CONSEQUENCES FOR POLICY OPPORTUNITIES OF THE EXPECTATION OF INFLATION

The foregoing results, obtained by Mundell, describe the effects of a spontaneous change of the expected rate of price change. The tax level and money supply were held constant. The government made no response to the change of expectations nor was it responsible for the change of expectations.

Should there occur a spontaneous change of expectation of the price change the monetary and fiscal authorities need not acquiesce to the new equilibrium shown above. Investment and the real interest rate could be restored to their "original" levels by open-market sales of securities by the central bank or by a tax reduction; alternatively the authorities could restore real money holdings and the nominal interest rate to their original levels by the opposite steps; or the authorities could bring about some different equilibrium which lies "between" or "outside" these two possible equilibria.

Should the government induce the expectation of inflation by the way it employs one of its two tools (say, taxes) the government still need not accept the particular change of equilibrium shown above; the government's other tool (say, open-market purchases) gives it latitude in choosing what the new real interest rate and investment level shall be.

We shall now analyze the effects of these two government tools and go on to deduce the consequences for policy opportunities of the expectation (spontaneous or induced) of inflation.

Tax policy. A change of taxes changes by an equal amount the algebraic budgetary surplus (which may be positive or negative) of the government. Since there are just two assets, money and shares, any change of the surplus must be "financed" by an equal and opposite change of the money supply. A tax reduction, $\Delta T < 0$, will cause the money supply to be greater by the amount of the tax reduction, $\Delta M = -\Delta T$. (We assume for the moment that the tax reduction is not accompanied by any change in central bank operations vis a vis the private sector; the effect of open-market operations will be discussed shortly.)

A change of taxes can now be seen to have three points of impact upon the model: A change of T has an "income effect" upon consumption demand, hence available total saving, as shown in the right hand side of (2); a change of T changes M and thus has an "asset effect" (or a "Lerner effect") upon private saving, again as shown in (2); and finally, a change of M has monetary effects through (1).

Consider first the impacts of the change of M, holding T constant. The

reader will note that $\underline{M}$ appears in (1) and (2) only as a ratio to $\underline{p}$; in other words, it is only the real value of money holdings, not their nominal size, which affects behavior. Therefore, if there existed initially an equilibrium with real money balances M_o/p_o , then, when the reduction of the algebraic surplus increases the money supply from M_o to M' , there must exist a new equilibrium with the same $\underline{r}$, $\underline{E}$ and so on but with a price level, $\underline{p}'$, which makes the real value of money holdings the same as initially:

$$\frac{M_o + \Delta M}{p'} = \frac{M_o}{p_o} \; ; \quad \text{that is,} \quad p' = \left(\frac{M_o + \Delta M}{M_o} \right) p_o \quad \text{must be an equilibrium price}$$

level in the new situation. Moreover, since for every given $\underline{M}$ there is a unique equilibrium, $\underline{p}'$ must be the only possible equilibrium price level. It follows that the new equilibrium, holding $\underline{T}$ fixed, does not differ from the original equilibrium, except for the equiproportionate changes in M and p , since only a change of M/p can change the behavior of the economy.

Hence, taking as given the expected rate of inflation, any effects of the tax cut are due solely to its "income effect" upon consumption demand. This is a well-known story: The tax cut stimulates consumption and hence requires a reduction of real balances (to stimulate private saving) if any given real rate of interest is to continue to satisfy the investment-saving equality in (2). Hence the IS curve shifts to the left. The LM curve remains fixed.

Therefore, given the rate of expected inflation, a tax reduction results in a higher rate of interest (both real and nominal so as to satisfy (3)) and therefore a smaller level of real money holdings and a smaller level of investment. It

follows that, by suitable tax policy (shifts of IS) the government can bring about an equilibrium anywhere on the given LM curve.

Monetary actions. In the present model, in which "money" is equal to central bank liabilities, it is natural to confine one's attention, as Metzler did, to open-market operations -- the exchange of money (the bank's liabilities) for shares.

Open-market transactions can be viewed as having two impacts: the impact of the change in E , interpreted now as privately held shares (total shares issued less the bank's holdings), and the impact of the equal and opposite change of M .

Consider first the effect of a change of M , keeping the bank's holdings of shares constant. As we saw earlier such a ceteris paribus change of M has no effect on the interest rate (real or nominal, given ρ) nor any other real magnitude (real money holdings, etc.) -- only an effect upon the price level which must change in the same proportion as M .

Therefore any "real" effect of open-market operations must arise from the impact of the change in privately held shares. Indeed, as Metzler explained, open-market purchases of shares have the same effect as a capital levy payable only in shares.

A reduction of privately held shares (due to an open-market purchase) evidently has no effect upon the demand for real money holdings at any given interest rate since that demand is based upon transactions needs. But the loss of securities does increase the quantity of real money holdings needed at any interest rate to produce the private wealth total (E + $\frac{M}{p}$) which will equate desired saving to investment.

Hence the IS curve shifts to the right. Since $\frac{M}{p}$ must increase by the amount of
the decrease of E , the shift at any real interest rate is equal to the amount of
shares purchased.[9] Hence the intersection of the IS curve with the LM curve
moves downward and rightward along LM .

An open-market purchase therefore results in a lower rate of interest (nominal
and real), therefore greater real money holdings (as well as greater nominal holdings)
and a greater rate of investment. In these respects it is similar to a tax increase.

Note that both open-market action and tax variation work by shifting the IS
curve and leave unchanged the LM curve, given the expected rate of inflation. The
government can employ either a tax reduction or an open-market sale to raise the real
rate of interest; it can employ either a tax increase or an open-market purchase to
reduce the real rate of interest. Note, however, that these actions have different
price effects. A tax reduction raises the price level; in fact, since M/p falls,
p must rise proportionately more than M . An open-market sale, while also reducing
M/p , reduces the price level; since M/p falls by an amount less than $\frac{1}{p_o} \triangle M$,
the amount of money withdrawn by the bank, p must fall.

It is immediately clear that if the expected rate of inflation is completely
exogeneous then the government can use either the fiscal or the monetary tool to
raise or lower the real interest rate. Any point on the LM curve for $\rho = \rho_1$ can
be chosen by the government. Mundell's result that the real interest rate must fall
assumes the absence of any government response to the shift of the LM curve.

The conclusion is no different if the expectation of inflation is, rather than

exogeneous, induced by inflationary government actions. Just as the government can maintain a stationary price level from period to period and accomplish this by fiscal-monetary policies which produce a high or a low real rate of interest, the government can also engineer a rising price trend and a high or low real rate of interest.

Suppose (without loss of generality) that to induce a certain expected rate of inflation the government desires to raise the price level a certain amount (above what the price level would be if the government took no action). To raise the price level the government must either reduce taxes or engage in open-market purchases. The former action shifts the IS curve to the left while the latter shifts IS to the right. Hence the tax reduction will accomplish the desired inflation with a higher real rate of interest than will the open-market purchase. But the difference between these two resulting real interest rates does not define the range of choice. If the government should desire a higher real rate than would result by tax reduction alone it should engage in open-market sales (which reduce the price level) together with additional tax reduction (to keep the price level constant at the desired level) until the real rate of interest reaches the desired level. In short, the government can choose among the points on the LM curve which prevails in an anticipated inflation as easily and in just the same way as it chooses among the points on the (different) LM curve which prevails under stationary price expectations.

Thus the government can choose what kind of inflation to have: an inflation with high saving and a low real rate of interest -- brought about by open-market purchases -- or an inflation with low saving and a high real rate of interest -- brought about

by low taxes.[10] This proposition is the analogue for a flexible price model of the familiar proposition for inflexible price models that the government, by its use of fiscal and monetary controls, can control both the level of employment and the rate of interest; in a flexible price model, it is the rate of inflation and the (real or nominal) rate of interest which can be independently controlled.

We conclude that the expectation of inflation -- even when that expectation is deliberately induced and sustained by the government -- will depress the real rate of interest, as Mundell concluded, only if the government allows that to happen.

As we have shown, the pairs of values of the real interest rate and real money holdings which the government may feasibly bring about by taxation and open-market operations are represented by the LM curve. Provided that fiscal and monetary policies are effective in controlling the IS curve, the LM curve constitutes the "opportunity locus" for these policy instruments. The effect of anticipated inflation upon "feasible welfare" -- the maximum welfare attainable through tax and open-market policies under the given monetary arrangements and institutions -- depends therefore only upon the resulting downward shift of this opportunity locus in relation to community preferences. We are ready now to analyze the welfare effects of anticipated inflation.

IV. CONSEQUENCES FOR FEASIBLE WELFARE OF ANTICIPATED INFLATION (WITH AND WITHOUT INTEREST ON MONEY)

In the present model, welfare can be considered a function of the amount of time spent economizing on money holdings and the investment-consumption mix (i.e., the degree to which the economy's investment approximates to the "optimum" level).

Clearly the economy can invest so much of its output as to leave too little available for present consumption; or it can invest so little as to leave too little income available for consumption in the future. Therefore welfare depends importantly upon the level of investment, given the present capital stock, employment, and time spent in husbanding money holdings.

Given time spent in production and the amounts of output invested and consumed respectively, welfare will depend upon the amount of time left for leisure. The greater the number of deposits and withdrawals at the bank made by individuals in order to keep a greater share of wealth in the forms of earning assets (shares) the smaller is the amount of time left for leisure; hence, given the public's distaste for this activity, the greater the number of these trips to the bank the smaller will be the community's welfare.

We can represent this two-fold dependence of welfare in terms of the real interest rate and real money holdings on our simplifying -- but not crucial -- supposition that the community is determined to spend a fixed amount of time employed in producing. Then, as we have recognized already, the amount of output invested is an inverse function of the real rate of interest. The greater the equilibrium real rate of interest the smaller must be investment since the real rate of interest or yield on capital (shares) is a decreasing function of investment, given employment. Therefore, given employment and given time spent economizing on money (hence given leisure), welfare can be represented as a function of the real rate of interest: As the real rate is decreased (investment increased), welfare first increases and then eventually decreases

when investment becomes excessive. Thus, there is an optimal real rate of interest, say r^*, corresponding to the optimal rate of investment.

Also, the amount of time spent in economizing on money instead of spent in leisure can be represented as a function of real money holdings. Time spent thus economizing will be smaller the smaller the nominal rate of interest -- the smaller the incentive to economize instead of taking more leisure. But the nominal rate of interest is an inverse function of real money holdings. The greater equilibrium real money holdings the smaller must be the nominal rate of interest and therefore the smaller the incentive to economize on money. Therefore, given employment and given the real rate of interest (hence investment), welfare can be represented as a function of the real value of money holdings: As real money holdings are increased (the nominal interest rate decreased) welfare increases until the nominal rate of interest has fallen to a point near zero where the incentive to economize our money is nil. When the incentive to economize on money is nil, meaning that all transactions balances are held in the form of money, we shall say that there is "full liquidity". Assuming that it costs the government nothing to increase real balances (at least in the neighborhood of full liquidity), the optimal level of liquidity is at full liquidity. We denote this liquidity optimum m^*.

To determine m^* consider the LM curve corresponding to a zero rate of expected inflation ($\rho = 0$) in Figure 2. When $\rho = 0$ the nominal rate of interest is the same as the real interest rate. The curve shows that as the nominal (real) interest rate becomes small the quantity of real balances demanded reaches some upper limit;

This value of M/p corresponds to full liquidity and it is the value of m^* .

The optimal real rate of interest, r^* , may not be as small as the nominal interest rate required to produce full liquidity. Suppose that it is not. Then the optimum is a point (r^*, m^*) which lies above the LM curve for $\rho = 0$. Any point $(r, M/p)$ different from this optimum entails a below-optimal level of welfare.

Suppose now that $\rho = \rho_0 = 0$ and that the government is committed to sustaining the expectation of a stationary price level by keeping the price level stationary. Then the LM curve for ρ_0 is the opportunity locus of points from which the government must be content to choose if it is committed to sustain stationary price expectations. Let (r_*, m_*) be the best point on this opportunity locus. Then the government, if it seeks the greatest level of welfare which is feasible, will choose those fiscal and monetary actions (among all actions which keep the price level stationary) which cause the IS curve to intersect the LM curve at this point. Trivally, this point is inferior to (r^*, m^*) .

Suppose now that the community expects rising prices $(\rho = \rho_1 > 0)$ and that the authorities commit themselves to sustaining this expectation. Then the new LM curve for ρ_1 becomes the locus of policy opportunities. This locus is clearly inferior to the previous one because, being below the first (negatively sloped) locus, it must lie to the left of the first locus. Thus any real interest rate that can be achieved on the first locus can be achieved (if at all) on the second locus only at a smaller level of liquidity -- with a greater shortfall of liquidity from optimal liquidity. Hence, (r_{**}, m_{**}) , the best feasible equilibrium on the ρ_1 locus, is inferior to (r_*, m_*) . The expectation of

Figure 2
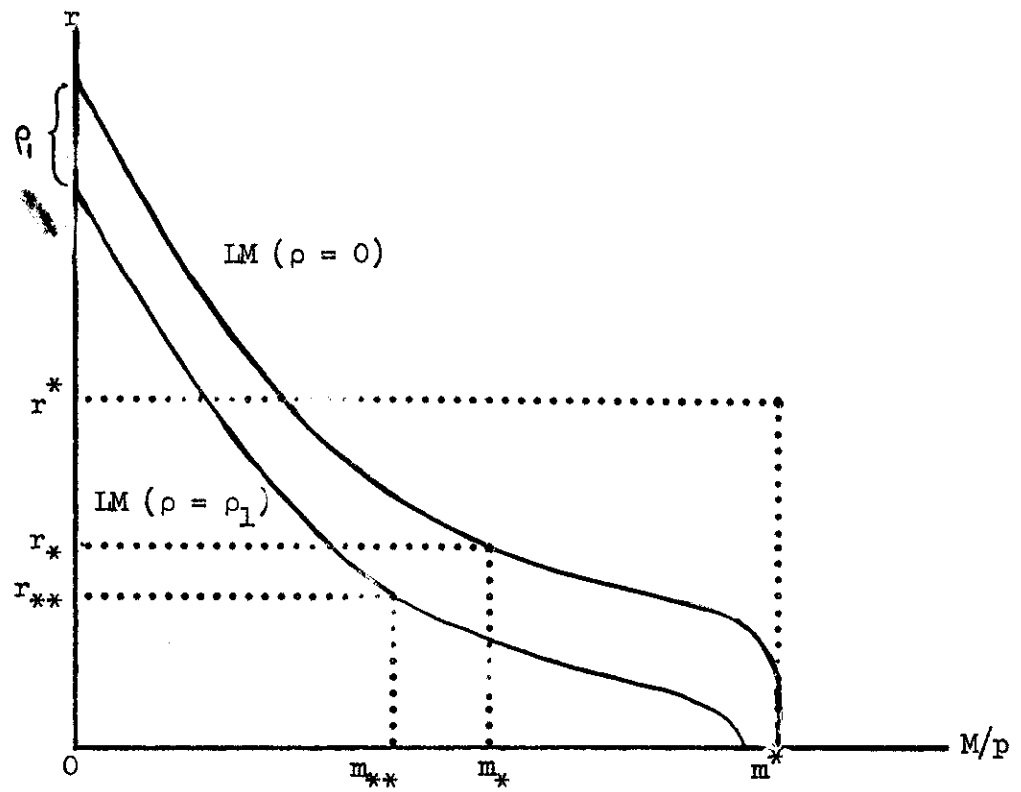
inflation reduces feasible welfare below that attainable when a stationary price trend is expected.[11]

We have finally arrived at a familiar proposition: Anticipated inflation is unambiguously bad. From this proposition it is sometimes concluded that, should the expectation of inflation develop, anti-inflationary measures ought to be introduced in order to induce the expectation of "stable prices". I shall make three remarks on this conclusion; perhaps all of them are well known.

First, should the government rely solely upon monetary tightening to curb the price trend (in the hope that people will thereby learn not to expect inflation) while sticking to the same fiscal course it is possible that the cure will be worse than the disease. Surrender of the equilibrium at (r_{**}, m_{**}) in favor of tighter money will, until the community revises its price expectations, place the economy higher up on the LM curve: Real money holdings will fall below m_{**} and, while the real interest rate will rise about r_{**}, the net effect of this alternation is to place society in a worse position since the former position was the best feasible on the LM curve. This initial loss of welfare must be weighed against the future gain which is possible when the LM curve eventually shifts upward.

Should the government rely solely upon fiscal tightening (increased surplus) to arrest the price rise then (given an unchanged course of action by the monetary authority) the equilibrium at (r_{**}, m_{**}) will give way to one with a lower real rate of interest and, despite greater real money holdings, welfare will here also fall below the feasible level -- until the desired effect upon the LM locus finally develops.

The lesson of this, of course, is that monetary and fiscal policies must be used jointly to bring about a moderation of the price trend together with maintenance of feasible welfare. We require both monetary and fiscal tightening -- i.e., open-market sales (which lowers the price level and raises the nominal interest rate) and an increase of the surplus (which lowers and slows the price level while reducing the nominal interest rate) in such proportions that the real rate of interest remains at r_{**} until the LM curve begins to shift rightward. In this way the economy can be guided along some optimal expansion path from (r_{**}, m_{**}) to (r_*, m_*) . (This power of the fiscal and monetary authorities, working together, to choose both the desired rate of change of prices and the desired point on the LM curve which is eventually associated with the planned price trend was already emphasized in the previous section.)

Our second remark is that there is nothing optimal in this model about stationary price expectations. The same logic which demands the elimination of inflation in favor of "price stability" also demands the abandonment of price stationarity in favor of an appropriate rate of price deflation. The optimal expansion path does not end at (r_*, m_*) but continues over to (r^*, m^*) , the grand optimum. To realize any point on this stretch of the expansion path it is necessary to bring about a certain rate of expected deflation, that is, a certain value of $\rho < 0$.

What rate of expected deflation is necessary to bring about equilibrium at the optimum, (r^*, m^*) ? Here there is "full liquidity". Hence the spread between the nominal (and real) yields of money and shares must be approximately zero. Since the

nominal yield on money is zero, the nominal rate of interest, therefore, must be approximately equal to zero.[12] This means that the expected rate of deflation must approximate to the real rate of interest r^* , the real rate which is optimal when there is full liquidity. In short $\rho \cong -r^*$ is required to cause the LM curve to pass through the point (r^*, m^*) .

In this optimal equilibrium there is no incentive to exchange money for shares in order to economize on money. Transactions balances consist only of money; there would be no purpose in increasing liquidity if that were possible. Moreover, the investment level is chosen solely with regard to intertemporal consumption preferences rather than with an eye also to the consequences for liquidity of the implied nominal rate of interest. By manipulating ρ the government would be able to break the link between the nominal interest rate (liquidity) and the real rate of interest (investment) and therefore to achieve simultaneously both full liquidity and the investment optimum.

This route to the optimum seems rather awkward, however. It might be argued that the process of teaching the community to expect deflation is uncertain and slow at best. The deflationary method of attaining the grand optimum depends upon a tendency of the community to extrapolate recent price trends into the future. But it is conceivable that price expectations are "adaptive" in too stabilizing a fashion -- that, the expected price level next period is a positively weighted average of the current and each past price level so that the expectation of a price fall could not be induced by the policy act of making prices fall.

Fortunately there is another method of attaining the optimum, one suggested by

Friedman and others,[13] and this constitutes our third remark. We supposed from the

start that money bore no interest. This placed money at a greater disadvantage

relative to shares when the nominal interest rate (yield on shares) rose. Planned

deflation which people will eventually anticipate is a device for making interestless

money bear a positive _real_ rate of return. If $\rho \cong - r = - r^*$ both money and shares

will bear approximately the same real (and same nominal) rate of return. The trouble

with any expected price trend which is less deflationary -- especially any inflationary

trend -- is that it creates an excessive spread between the nominal (and hence also

the real) rates of return on money and shares; it is clearly this spread between the

nominal interest rate on shares and that on money (until now supposed to be zero) on

which the demand for real money holdings depends. The greater this spread the greater

will be the community's socially inefficient efforts to economize on money.

Suppose now that the government pays interest (in money) to holders of money

(deposits at "the bank"). Let μ denote the (own) rate of interest on money. The

spread between the nominal rates of interest on shares and money is now

$i - \mu = r + \rho - \mu$. Therefore the real money demand function is:

$$(1') \qquad \frac{M}{p} = L\,(r + \rho - \mu)$$

Money is no longer at a potential disadvantage relative to shares. If $\mu > 0$,

money has an inherent attraction beyond its utility in making transactions. If ρ

should rise, the government can prevent the effect of that rise upon the LM curve,

hence its effect upon real money balances and the real interest rate, by raising μ

by an equal amount. Indeed, in a laissez-faire banking system in which it is legal to pay interest on deposits, the advent of inflationary expectations would presumably lead profit-maximizing banks to raise the interest rates they pay to depositors enough to maintain the spread between the yields on money and shares and thus maintain their deposits and earnings in real terms. This seems a significant qualification to the usual welfare analysis of inflation and to the analysis by Mundell: Such analysis has little relevance to economies in which the rate of interest on a large portion of the money supply fluctuates with other yields so as to keep most interest differentials roughly invariant to the expected rate of inflation.[14]

More importantly, the government can now achieve the optimum through control of μ rather than by efforts to control ρ . It can narrow the "spread" between the nominal (or real) rate of interest on shares and that on money by raising the own rate of interest on money rather than by attempting to lower the expected nominal rate of interest on shares (the real rate plus the expected rate of change of prices). Taxation and open-market operations can be used to bring about the desired price level and the desired real rate of interest on shares while the bank raises the own rate of interest on money toward the nominal rate of interest on shares, reducing the "spread" sufficiently to eliminate the incentive to economize on money.

By this method, then, the government can achieve the optimum without having to attempt to influence the rate of expected change of the price level. The economy is not restricted to maximizing the welfare level which is feasible under any given rate of expected algebraic inflation. The economy's full potential welfare can be achieved

through the device of paying interest on money.

Our analysis has led to the conclusion that anticipated inflation and even insufficient deflation is bad unless its ill-effects are neutralized by the payment of a suitable interest rate on money. But there is a qualification: We analyzed only the case in which the optimum point (r^*, m^*) lay above the LM curve for $\rho = 0$; we supposed, in other words, that r^* was so high that if $\rho = 0$ and $\mu = 0$ then, when $r = r^*$, the cost of holding money, $i - \mu = r + \rho - \mu = r^*$, would have been too high to permit full liquidity. It was for this reason that we required either a positive μ or a negative ρ to realize both the optimal r and a $i - \mu$ small enough to induce full liquidity.

But it is possible that the optimum point lies on the LM curve for $\rho = 0$, that is, somewhere on the vertical stretch of LM where $M/p = m^*$. The meaning of this case is that r^* is so small that, when $\rho = 0, \mu = 0$, the cost of holding money, $i - \mu = r + \rho - \mu = r^*$, is small enough to insure full liquidity.

In this second case there is no need to introduce a negative ρ or positive μ to shift upwards the LM curve; that curve already intersects the optimum point. Hence, to realize the optimum, it suffices to adopt a fiscal-monetary policy which is neither inflationary nor deflationary (in order to keep ρ equal to zero) and which causes the IS curve to intersect the LM curve for $\rho = 0$ at the point (r^*, m^*) . There is no need here for deflation or interest on money.

Further, just as a stationary price trend is consistent with optimality in this case without need for interest on money, a moderate rate of anticipated inflation may

be harmless. While a positive ρ will produce an LM curve below that for $\rho = 0$, this new LM curve will still intersect the optimum point if ρ and r^* are small enough. This is because the LM curve for $\rho = 0$ is vertical for some distance over the point $M/p = m^*$ (meaning that there is a finite range of smaller nominal interest rates consistent with full liquidity). Hence an LM curve slightly below that for $\rho = 0$ will also have a vertical stretch over the point $M/p = m^*$; the two curves will coincide near the horizontal axis and hence may coincide at the optimum point if the latter is close to the horizontal axis.

Our conclusion, therefore, is that anticipated inflation above a certain critical rate is bad unless offset by the payment of interest on money. The critical rate may be negative (deflation required in the absence of interest on money) or it may be positive. The critical rate is negative if and only if the optimal real interest rate is sufficiently high that equality of the nominal interest rate with that real rate (implying $\rho = 0$) would prevent full liquidity; that is, if and only if the optimum point (r^*, m^*) lies above the LM curve for $\rho = 0$.

The reader may feel that in fact the optimum point lies well above the LM curve for $\rho = 0$, that is, that r^* is so high that we require for full liquidity either anticipated deflation or interest on money. But this is less certain if we recognize that there may be a speculative as well as a transaction demand for money.

Suppose there is a speculative demand for money. Optimal liquidity occurs when the cost of holding money $(i - \mu)$ is small enough to induce "full liquidity" -- i.e., to cause all individuals to hold all their transactions balances in the form of money.

Let m^* denote the level of real money holdings when the cost of holding money is just small enough to induce full liquidity. A "difficulty" in this case, and it is not a real one, is that if the cost of holding money is reduced further there may be a larger speculative demand (the transactions demand is already satiated) so that the level of real money holdings corresponding to this reduced cost of holding money may be larger than m^*. Hence there may be a whole range of values of real money holdings, $M/p \geq m^*$, all of which correspond to equilibria of full liquidity. This is illustrated in Figure 3 of the next section where, rather than a single optimum point, there is an "optimum line": Any point on the horizontal line stretching rightward from the point (r^*, m^*) is a point of optimal investment and full liquidity.

The important difference which a speculative demand makes pertains to the position of (r^*, m^*) relative to the LM curve for $\rho = 0$. When there is only a transaction demand for money m^* is equal to the value of M/p at which the LM curve reaches the horizontal axis; hence the point (r^*, m^*) is either on the LM curve (if r^* is very small) or above the LM curve. But when there is a speculative demand for money as well, then full liquidity -- all transactions balances held in the form of money -- can occur without there also occuring "liquidity satiation" -- the demand for money equal to total wealth. In other words, the value of M/p at which the cost of holding money is just small enough to produce full liquidity may be smaller than the value of M/p corresponding to liquidity satiation. Hence m^* need not be the horizontal intercept of the LM curve for $\rho = 0$; rather, m^* may be to the left of this intercept. As a consequence, if r^* is rather small, the point (r^*, m^*) may be

below the LM curve for $\rho = 0$. (This is shown in Figure 3.) There are, therefore, three cases to be considered.

If the point (r^*, m^*) lies above the LM curve for $\rho = 0$ then either anticipated deflation or positive interest on money is needed to shift up the LM curve so that it will intersect (r^*, m^*) or any other point on the optimum line.

If the LM curve for $\rho = 0$ passes through (r^*, m^*) then, to realize optimal investment and liquidity, it suffices to adopt a fiscal-monetary policy which is neither inflationary nor deflationary (in order to keep ρ equal to zero) and which causes the IS curve to intersect the LM curve for $\rho = 0$ at the point (r^*, m^*).

If the point (r^*, m^*) lies below the LM curve for $\rho = 0$ then the optimum line must stretch rightward from that point, reaching the LM curve at some $M/p > m^*$, $r = r^*$. As has been explained, this point is also an optimum. Hence, for an optimum. it suffices to adopt a non-inflationary, non-deflationary, fiscal-monetary policy which makes the IS curve intersect the LM curve at this point, where $r = r^*$.

In the latter two cases, therefore, neither deflation nor interest on money is required for an optimum. Further, in the last case, a moderate rate of anticipated inflation is harmless. In that case there is a range of anticipated inflation rates which, while they produce LM curves below the one for $\rho = 0$, still allow the LM curve to pass through or above (r^*, m^*) and hence still allow an optimum to be achieved without payment of interest on money.

We are not sure which is the empirically relevant case. But we can say that the

presence of a speculative demand for money increases the likelihood that the last case is the relevant one, hence increases the likelihood that optimal investment and liquidity can be achieved without deflation or interest on money and can be achieved even with a moderate rate of inflation (without paying interest on money).

We have concluded that anticipated inflation, at least if it is excessive, reduces feasible welfare if no interest is paid on money. But what of Vickrey's well-known paper in which he argued that anticipated inflation is desirable? No analysis of fiscal and monetary policies for optimal investment and liquidity would be complete without its taking account of Vickrey's paper. We therefore append a final section in which our respective analyses are reconciled. The analysis below differs from that above in that, in what follows, we suppose, with Vickrey, that the bank is restrained from buying the quantity of shares it would like to buy.

V. THE ANALYSIS UNDER A MONETARY RESTRAINT

Vickrey supposes that there is a speculative demand for money. Further, he supposes that, when $\rho = 0$, there is some rate of interest, say $\overset{\smile}{i}$, such that at any $i \leq \overset{\smile}{i}$, everyone expects a negative yield on shares (including the capital loss). Hence the LM curve exhibits a so-called "liquidity trap" as shown in Figure 3. At any $i \leq \overset{\smile}{i}$ everyone wishes to sell his shares for whatever money he can get for them; the demand for real money is equal to real total wealth; liquidity preference is "absolute".

However, such a phenomenon in no way impairs the power of the monetary authority to bid for shares and hence to drive down the nominal rate of interest to any desired

level short of zero. At the kink in the LM curve, shareholders will be happy to sell their remaining shares to the bank for whatever positive price the bank is willing to pay; the price paid by the bank (the support price) determines the equilibrium rate of interest.

Yet, in Vickrey's model, the bank is unable to establish a "low" real rate of interest, if that should be desired. The reason is that the monetary authority is supposed to be unable to purchase all or at least some private shares. If, for example, all open-market operations in private claims were illegal (and no other debt, e.g., government bonds, existed for purchase or sale) then, should fiscal policy be committed to price stationarity (or any other price trend), there would be just one possible equilibrium real rate of interest -- where the IS curve corresponding to the required fiscal policy intersects the associated LM curve. In Figure 3, $\overset{o}{r}$ denotes the smallest real rate of interest which the monetary authority can bring about when the fiscal authorities are committed to a stationary price trend; it corresponds to the lowest (right-most) IS curve attainable by the monetary authority, given the constraint on fiscal action.

Vickrey's next step is to suppose that, as is surely conceivable, the optimum entails a real rate of interest which is smaller than the lowest real rate of interest capable of realization by the monetary authorities in a level price economy.[15] Accordingly, in Figure 3, an optimum occurs at any point where $r = r^*$ and $M/p \geq m^*$; r^* is smaller than $\overset{o}{r}$. All points on this "optimum line" are points of full liquidity.

In this situation, the government seems unable to drive the real interest rate to

Figure 3
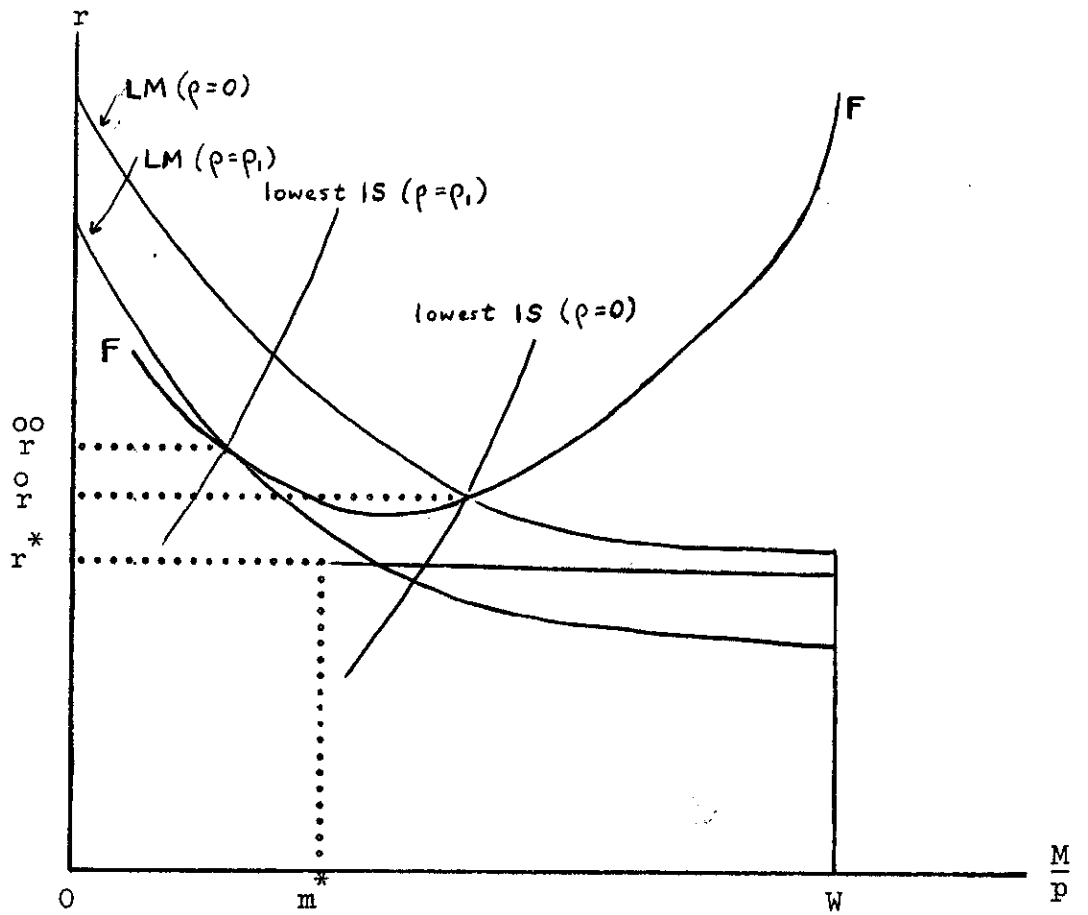
the optimal level. But Vickrey proposes a solution through inflation. He would have the government contrive a program of announced inflation in order to shift downwards the LM curve on the belief that a lower real rate of interest would then be feasible.

How is this inflation to be generated? The bank can cause a rising price trend only by repeated open-market purchases from period to period. But this route is evidently closed by Vickrey's assumption of a restraint on bank holdings of shares. What of fiscal policy?

To raise the price level and hence to induce the expectation of inflation by means of taxation the government must, on the usual stability condition, reduce taxes (increase the deficit). This shifts the IS curve in Figure 3 to the left for reasons earlier explained. The LM curve will shift downward by the amount of the expected rate of inflation which the new price trend induces. But notice that, as Figure 3 illustrates, the resulting real rate of interest, $\overset{oo}{r}$, is not necessarily lower than $\overset{o}{r}$. If and only if the downward shift of LM exceeds the upward shift of IS will the real rate fall. Thus there is no guarantee that this fiscal effort towards a lower real rate of interest will be successful.

In Figure 3, we have drawn the curve FF which is the locus of points of inter-section of every "lowest IS curve," each one corresponding to some rate of anticipated inflation, with the LM curve which corresponds to the same rate of anticipated inflation. This curve is the boundary of the policy opportunity set: Through monetary tightening it will be possible to achieve any point on or above this frontier; but the constraint on monetary ease makes it impossible to achieve any point below FF .

We have demonstrated that the government may be unable to reduce the real interest rate through fiscally induced inflation. The government's only tool for sustaining inflation, namely the deficit, may raise the real rate of interest on balance (and reduce liquidity in addition) and therefore reduce welfare. This is the result where FF is downward sloping as it may be in the lefthand range; it is the result shown in the diagram. But FF may be upward sloping -- the LM shift predominates -- in which case Vickrey's result occurs; the diagram shows FF to be upward sloping to the right where LM is very flat, as Vickrey supposed.

Even if the real rate does fall (FF upward sloping in the relevant range), however, there is no necessity that welfare will thereby be increased since liquidity also falls. If the new and smaller liquidity level still exceeds or equals "full liquidity" -- so that all transactions balances are held only in the form of money -- then the liquidity reduction is of no consequence for welfare; then, since the real rate was originally above the optimum level by assumption, welfare will be increased if the reduction of the real rate is of the "right" magnitude. But if the new liquidity level is less than "full", then the reduction of liquidity accompanying the fall of the real interest rate may reduce welfare. In any case it is always possible to go so far along FF that welfare begins to fall. Somewhere on the FF boundary there may be a constrained optimum. (This is the case in Figure 3 where the unconstrained optimum lies below FF .) The real interest rate associated with this constrained optimum may or may not be smaller than r^o , the lowest real rate consistent with noninflationary fiscal policy, so that a fall of the real rate below r^o may or may not increase welfare.

To summarize: inflation through fiscal means does not promise an increase of welfare. A fiscally induced inflation may fail to reduce the real rate, may reduce welfare even if it succeeds, and cannot -- if the restraint on the monetary authority is sufficiently binding -- make feasible the optimum.

To assure feasibility of the optimum or even a desirable reduction of the real rate of interest the government needs a third tool. A negative own rate of interest on money -- a tax on money holding -- is such a tool.

Suppose the government maximizes monetary ease subject to the restraint on the bank. Let the government choose that fiscal policy consistent with price stability and hence $\rho = 0$. The resulting IS curve -- the "lowest IS " for $\rho = 0$ -- may, as in Figure 3, lie to the right of the point (r^*, m^*) and hence intersect the "optimum line" connected to that point. If this IS curve lies to the left, let the government choose a sufficiently tighter fiscal policy which shifts the IS curve to the right so as to intersect somewhere with the optimum line. In either case the LM curve which corresponds to the IS curve chosen will intersect the IS curve somewhere on FF. By hypothesis, FF lies everywhere above the optimum line. The problem therefore is to make the LM curve intersect the chosen IS curve where the latter intersects the optimum line. This can be achieved by levying a tax on money. Let μ continue to denote the own rate of interest on money; then $-\mu$ is the tax rate on money. Just as the introduction of a positive own rate on money shifts the LM curve (corresponding to a given ρ) by the amount μ, the introduction of a tax on money will shift the LM curve downward by the absolute value of μ. Hence it is only

necessary to set the tax rate on money so as to shift the LM curve down from its intersection with the chosen IS at FF to an intersection with IS at the point where the chosen IS curve intersects the optimum line.

Hence, by establishing a negative own rate of interest on money and by adopting a sufficiently tight fiscal policy, the government can secure full liquidity and the optimal real rate of interest. Given the third tool of negative interest on money, the "price"of the monetary restraint, if binding, is only that a deflationary fiscal policy may be required to bring down the real rate of interest. In the present model there are no welfare effects of deflation, given that optimal levels of investment and liquidity are achieved; in particular, full employment is assumed to result what-ever the price trend.

The principal conclusions of this paper can now be summarized.

If fiscal and monetary policies are effective and unconstrained, there is no necessary connection between the rate of anticipated inflation and the real rate of interest (hence investment and growth). The government can engineer a high-investment or a low-investment inflation in the same way that it can engineer high or low investment and keep the price level stationary.

If the government has unconstrained power to buy and sell claims on wealth (shares in our model) and has the power to pay interest on money then considerations of investment and liquidity offer no basis for choosing among anticipated price trends: The desired levels of investment and liquidity can be achieved with any anticipated price trend. But if the government cannot pay interest on money then anticipated inflation in excess of some critical rate (which may be negative) will prevent attainment

of the optimum; attainment of the desired levels of investment and liquidity may require a deflationary mix of fiscal and monetary policies.

If, in addition to being unable to pay interest on money, the government faces a constraint on the quantity of private wealth it can monetize then the optimum may not be attainable; in that case, attainment of a constrained optimum (the best feasible investment-liquidity combination) may require a rising anticipated price trend, as Vickrey argued, or a falling anticipated price trend. But if the government can tax money holdings (thus making the own rate of interest on money negative) then the government can achieve the optimum whether or not there is a constraint on open-market operations; however, if this constraint is binding, attainment of the optimum by means of fiscal policy and the taxation of money holdings may entail a falling price level.

FOOTNOTES

1. I should like to acknowledge helpful discussions with William C. Brainard and Paul A. Samuelson on the subject of this paper.

2. Milton Friedman, "Discussion of the Inflationary Gap" (revised), in Friedman, Essays in Positive Economics (Chicago, 1953), 253-57.

3. Martin J. Bailey, "The Welfare Cost of Inflationary Finance", The Journal of Political Economy, LXIV (April, 1956), 93-110.

4. Robert A. Mundell, "Inflation and Real Interest", The Journal of Political Economy, LXXI (June, 1963), 280-83.

5. Lloyd G. Metzler, "Wealth, Saving and the Rate of Interest", The Journal of Political Economy, LIX (April, 1951), 93-116.

6. William S. Vickrey, "Stability Through Inflation", in K. K. Kurihara, editor, Post-Keynesian Economics (Rutgers, 1954), 89-122.

7. I have in mind a model in which the capital stock changes by discrete amounts in every "period" or planting season as it were.

8. Clearly one could alternatively represent the new equilibrium as the result of an upward shift of the IS curve considered as a function of the nominal rate of interest. A rise of ρ requires an equal rise of i if any given $\frac{M_o}{p}$ is to continue to satisfy equation (2).

9. Because T, which is held constant, signifies net government receipts, its constancy implies that the government reduces taxes by the amount of the earnings from the shares purchased by the bank. The Metzler model implies that tax reduction has no effect upon consumption demand if it is matched by an equal loss (to the private sector) of interest earnings due to a reduction of privately held shares, given total real wealth and the rate of interest. Mundell and Horwich have pointed out that this implies that the tax reduction must not be offered as a reduction on property tax rates or corporate tax rates otherwise it will be "capitalized" by the market and hence raise the market value of property (shares). A stronger point should be made: only if the tax reduction is considered "transitory" rather than "permanent" will the tax reduction fail to be "capitalized" to some degree and hence fail to stimulate consumption demand. Much confusion abounds from the practice of making consumption a function of wealth and total income rather than, say, a function of some kind of subjective net worth (which might include heavily discounted tax and wage anticipations) and possibly a liquidity indicator.

10. Note that a permanent change of tax rates has an effect upon the path of the price level from period to period which a once-for-all change in the supply of money due to an open-market transaction does not have. If the government sets the tax level so as to run a deficit $(T < 0)$ then, since the money supply will be increasing over time while the real value of money holdings will be constant from period to period (if the IS and LM curves remain the same over time) the price level will be increasing over time in proportion to the changing money supply. The monetary authority can cause inflation -- through continual increases of the money supply -- only if each period it engages in fresh open-market purchases. A permanent change of taxes causes a change in the trend of prices over time (as well as a change in the level of prices in the first period) while an open-market transaction causes a change of the price level only in the current period.

11. Another proof that (r_*, m_*) is better than (r_{**}, m_{**}) is the following: The first locus permits $(r_{**}, \hat{m})$ where $\hat{m} > m_{**}$ and this is better than (r_{**}, m_{**}). If (r_*, m_*) is preferred to $(r_{**}, \hat{m})$ it must also be better than (r_{**}, m_{**}). It is not possible, however, to measure how much better it is. Only if $r_{**} = r_*$ could we make use of Bailey's measure of the welfare cost of the inflation.

12. It is not necessary to drive the nominal interest rate all the way to zero in order to eliminate the incentive to switch transactions balances in and out of money. Provided even one such transaction has a finite disutility the LM curve will possess a vertical slope closed to the horizontal axis (i.e., for very small nominal interest rates).

13. See, for example, Milton Friedman, A Program for Monetary Stability (Fordham University Press, 1960).

14. For a fuller discussion of the significance of bank interest payments see Vickrey, op. cit., pp. 112-113.

15. "If a level trend of prices is considered a sine qua non, a high [nominal] interest rate also implies a high real rate of interest which in turn may discourage... capital formation to an undesirable extent." Vickrey, op. cit., p. 98.