1-1-1962

Inventory Investment

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COWLES FOUNDATION DISCUSSION PAPER NO. 131

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January 2, 1962

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1. Introduction

Investment in inventories is perverse in timing and magnitude, contributing to fluctuations in economic activity, to booms and unemployment. Although inventory investment over the years has constituted but a small percentage of aggregate Gross National Product, the extreme volatility of this component of effective demand means that it must be regarded as a prime culprit responsible in large part for short-run fluctuations in economic activity. The business recessions that have periodically interrupted the growth of the U.S. economy since World War II are at least partially explained by the rapid liquidation of inventory holdings by firms engaged in manufacturing and trade.[2/]

The crucial role of inventories in the generation of fluctuations in economic activity stands in marked contrast to the limited attention that economists have devoted in their empirical research to the study of inventory behavior. Of course, Jan Tinbergen [50], Lawrence Klein [28] and Colin Clark [9] included inventory equations in their econometric models. Such studies as those of Edwin Mills [37, 39, 40], P. G. Darling [11, 12], Michael Lovell [30, 31, 32], Franco Modigliani and Sauerlander [41], Nester E. Terleckyj[47], Jack Johnson [26] and Murray Brown [7] have involved somewhat more extended econometric analysis of the behavior of inventories. Nevertheless, relative to the voluminous literature on consumption and fixed investment behavior, the area of inventory investment has barely been touched in empirical investigations.

A convenient touchstone for appraising recent econometric investigations of aggregate inventory behavior is provided by the acceleration principle. In its

most elementary form, the accelerator principle involves the assumption that entrepreneurs succeed in maintaining their stocks at the equilibrium level, H_t^e , which is linearly related to sales X_t .

$$(1.1) \qquad H_t^e = \alpha + \beta X_t$$

This assumption concerning the behavior of the inventory stock implies that actual inventory investment, ΔH_t , is proportional to sales.

$$(1.2) \qquad \Delta H_t = \beta \, \Delta X_t$$

Estimates of the parameters of this elementary model have been derived by D. J. Smyth [45] from annual deflated national income data for the United States covering the years 1948 through 1958.

$$(1.3) \qquad \Delta H_t = - \, .86 + .30 \, \Delta Y_t + 0.07 \, t \qquad\qquad R^2 = .87$$
$$ (.06) \qquad (0.24)$$

The change in inventory is explained by ΔY_t , the change in gross national product; the coefficient of time is not significant.[3]/

A number of complications has been introduced into the basic accelerator concept in an attempt to obtain a more adequate framework for the econometric investigation of inventory behavior. The simple accelerator model does not explain the timing of inventory investment. Moses Abramowitz [1] pointed out in his monumental study that in contrast to the implications of the simple accelerator hypothesis, actual inventory investment is not proportional to changes in output. Complications that have been introduced into the basic accelerator model in order to provide an explanation of why inventory investment does not lead cyclical changes in Gross National Product are discussed in the second part of this paper. One class of problems arises from the possibility that other factors in addition to sales or

output affect the volume of inventories that firms desire to hold. The rate of interest and a possible influence of speculative inventory profits during periods of rising prices are but two of a host of candidates that require consideration. Errors made by firms in anticipating future sales volume constitute another problem that must be considered in the econometric investigation of inventory behavior. The buffer-stock versions of the accelerator principle of Eric Lundberg [34] and Lloyd Metzler [36] incorporate expectational errors in the analysis of the inventory cycle. Difficulties involved in introducing either actual sales anticipations or a suitable surrogate will be discussed in detail. Obviously, all of these complications of the accelerator principle are interrelated; expectational errors, for example, might be expected to influence the timing of inventory investment. Nevertheless, it greatly facilitates a review of recent econometric work on inventory investment to consider each complication in turn.

2. Equilibrium Inventory and Adjustment Lags

The lag of inventory investment behind changes in output might be taken into account by a slight change in the dating of variables. M. Kalecki [27] found that a closer fit was achieved with annual data for the United States for the period 1930 through 1940 by regressing inventory investment upon the change in output lagged six months; he reports a correlation of 0.913 for the lagged regression as opposed to 0.828 when the lag was not taken into account.[4/]

An alternative procedure, frequently employed in econometric investigations of plant and equipment as well as inventory investment, involves the flexible accelerator complication suggested in a theoretical paper by Richard Goodwin [20]. With this approach it is assumed that the typical firm attempts only a partial

adjustment of its inventory to the equilibrium level within a single period. It is assumed that actual inventory investment is only a fraction of the discrepancy between last period's stock and the current equilibrium level.

$$(2.1) \qquad \Delta H_t = \delta(H_t^e - H_{t-1}) \ .$$

Here H_t^e represents the equilibrium level of stocks, an unobserved variable possibly determined by sales according to equation (1.1), but more likely influenced by additional variables as explained later in this paper. Only if δ, the reaction coefficient, is exactly equal to unity is an attempt made to adjust inventories fully to the equilibrium level.

Several factors may account for the inertia of businessmen in adjusting inventories to equilibrium. Time may be required before orders placed to replenish stocks of purchased materials can be filled. Even if items are ordered promptly so as to maintain the sum of purchased materials inventory plus outstanding orders for additional items, what Ruth P. Mack calls "ownership position," adjusted to changes in sales volume, the physical magnitude of inventories actually on hand would still lag because of delays in delivery. Economies involved in large quantity orders may make it advisable for the cost conscious firm to preserve only an imprecise relationship between ownership position and sales volume. Because stocks are generally a conglomeration of heterogeneous items, the firm may find that considerable time is required in liquidating a surplus of a particular item, even though only a moderate excess in its aggregate inventory position is involved. When sales increase a concomitant expansion of inventories may require enlarged warehouse capacity, and this requires time. When sales of items produced to meet a seasonal pattern of demand prove disappointing, stocks may have to be carried over slack seasons before they can be liquidated. Such factors as these may

explain why firms are willing to suffice with a considerable departure of stocks from their equilibrium level.

Several alternative formulations of equation (2.1) have been employed in econometric studies of inventory behavior. Instead of utilizing inventory investment as the dependent variable, one may fit an expression for the total stock of inventory

$$(2.2) \qquad H_t = \delta \, H_t^e + (1-\delta) \, H_{t-1} + e_t$$

This is obtained by adding H_{t-1} to both sides of (2.1). With this procedure, the method of least squares yields precisely the same parameter estimates as before, although the correlation coefficient may be expected to be somewhat larger. Another possibility is to explain inventory investment by first differencing equation (2.2).

$$(2.3) \qquad \Delta H_t = \delta \, \Delta H_t^e + (1-\delta) \, \Delta H_{t-1}$$

This procedure has been followed by Mills [37, 39] in order to reduce the problem of auto-correlated error terms. A final alternative, appropriate in the study of finished goods inventory, is to utilize the definition of output $Q_t \equiv X_t + \Delta I_t$ in conjucture with (2.1) to obtain:

$$(2.4) \qquad Q_t = X_t + \delta(I_t^e - I_{t-1}) \ .$$

This approach has been employed by Modigliani and Sauerlander [41], by Edwin Mills [37, 39, 40], J. Johnson [26], and others in the analysis of the production decision.

Whatever the form chosen for the regression, a problem is created by the

fact that equilibrium inventory, H_t^e , is an unobserved variable.[5] If equilibrium inventory is regarded as a function of anticipated sales, as with equation (1.1), a simple process of substitution serves to eliminate the unobserved variable from the regression equation to obtain:

$$(2.5) \qquad H_t = \delta(\alpha + \beta) \hat{X}_t + (1 - \delta) H_{t-1} + e_t \ .$$

Since the coefficient of H_{t-1} provides an estimate of δ , the coefficient obtained for X_t may be unscrambled to obtain estimates of α and β , the parameters of the equilibrium inventory equation. In actual practice, of course, equilibrium inventory probably depends upon other variables in addition to sales, but this does not really introduce any new difficulties.

Lawrence Klein [28] pioneered the application of the flexible accelerator to inventory data. Least squares estimates derived from deflated annual data for the period 1921-40 are presented by Klein

$$(2.6) \qquad H_t = 1.06 + \underset{(1.15)}{4.66p_t} + \underset{(0.02)}{0.13X_t} + \underset{(0.08)}{0.48H_{t-1}} \ ,$$

where X_t represents final sales (GNP less inventory change) and p_t is a price index.[6] The reaction coefficient is approximately 1/2, rather than the value of unity implied by Smith's regression procedure, equation (1.1) above; firms attempt to adjust halfway towards equilibrium each year. Klein's regression implies that equilibrium inventories are determined by

$$(2.7) \qquad H_t^e = 2.03 + 8.96p_t + 0.25X_t \ ,$$

an equation suggesting that price changes, perhaps because of money illusion, have a pronounced influence on equilibrium stocks.

Paul G. Darling [11] considered the forecasting value of an equation explaining the behavior of book value quarterly manufacturing inventory investment.

(2.8) $\Delta H = -.387 + .415X_{-1} - .212H_{-2} + .324\Delta U_{-1}$
          $\qquad\qquad (.044) \quad\ (.022) \quad\ (.054)$

          $R = .945 \qquad\qquad \delta^2/s^2 = 1.85$.

The change in inventory, ΔH , is explained by lagged sales, X_{-1} , stocks lagged two periods, H_{-2} , and the previous quarter's change in unfilled orders, ΔU_{-1} . Data extending from the 3rd quarter, 1947, through the 3rd quarter, 1958, were utilized in the regression; the lag structure was empirically determined by trial and error. No attempt was made to incorporate explicitly within the regression the impact of errors in anticipating sales volume. In order to determine the equilibrium inventory equation implied by Darling's regression, it is necessary to inquire as to what values of the explanatory variables would have led to no attempt to change the level of inventories. Setting $\Delta H = 0$ and solving the implicit equation thus obtained for H yields:

(2.9) $H^e = -1.82 + 1.95X + 1.53 \Delta U$.

A dollar increase in quarterly sales generates almost twice as large an increase in equilibrium inventory; for every dollar increase in the change in unfilled orders, equilibrium inventory increases by $1.53. The reaction coefficient is 0.212 , implying that firms in manufacturing attempt to liquidate roughly one-fifth of the discrepancy between equilibrium and actual inventory each quarter.

Nester E. Terleckyj [47] has presented an interesting study focused upon the behavior of total inventory holdings in manufacturing and trade combined.

Although Terleckyj did not work with deflated data, he did subtract the inventory valuation adjustment in order to eliminate the effect of revaluating the existing inventory stock. The percentage quarterly change in the book value of trade and manufacturing inventory, less the inventory valuation adjustment, was explained by the lagged inventory sales ratio, I_{-1}/X_{-1}, the ratio of new orders to sales, N_{-1}/X_{-1}, and the unfilled orders/sales ratio, U_{-1}/X_{-1}.

$$(2.10) \qquad \frac{\Delta I}{I} = -14.59 - \underset{(2.23)}{11.26} \frac{I_{t-1}}{X_{t-1}} + \underset{(5.52)}{30.75} \frac{N_{-1}}{X_{-1}} + \underset{(.57)}{1.88} \frac{U_{-1}}{X_{-1}}$$

$$R^2 = .78$$

Although explicit consideration was not given to the problem of errors of anticipation, a number of alternative influences upon inventory was considered. In one regression the rate of price change was found significant, but perhaps this was a reflection of the tendency for inflation to contribute to a revaluation of existing inventory; the variable was only significant in the regression in which the percentage change in book value inventory was not adjusted by subtracting the inventory valuation adjustment. Terleckyj also considered a possible influence of monetary variables; the rate of interest on prime commercial paper had a negative but insignificant effect on inventory investment; corporate liquidity was not significantly correlated with the calculated residuals.

The adjustment mechanism implied by Terleckyj's analysis is not of the distributed lag type utilized in most studies of inventory investment. In order to see exactly what is involved, it is first necessary to determine the equation for equilibrium inventory. The level of inventory implying zero investment for

given levels of sales and new and unfilled orders is obtained by setting $\Delta I = 0$

in equation (2.10) and then solving the resulting implicit equation to obtain

(2.11) $I^e = -1.3X + 2.7N + .17U$

The coefficient of sales, $-1.3 = - \dfrac{14.59}{11.26}$, has an inappropriate sign; it is

unfortunate that in every one of Terleckyj's regression the intercept term is

negative rather than the positive value that is required if the equilibrium level

of inventory is to be positively associated with sales. In order to find the

nature of the delayed adjustment mechanism, it is only necessary to observe that

(2.10) may be rewritten in the form

(2.12) $\dfrac{\Delta I}{I} = \dfrac{11.26}{X_{-1}} (I^e_{-1} - I_{-1})$.

The speed of adjustment, $11.26/X_{t-1}$, thus depends upon the current volume of

sales. Over the period of the regression, sales per month ranged from $35 billion

to $60 billion; apparently, then, roughly 1/3 to 1/6 of the discrepancy between

equilibrium and current inventory is eliminated each period. This type of delayed

adjustment mechanism is reminiscent of the logistics curve.

As a final example consider the following regression derived from deflated

non-farm inventory investment data for the period extending from the second quarter

of 1947 through 1959. Non-farm business inventory investment is explained by Gross

National Product, the change in GNP, and the backlog of unfilled orders, all

measured in constant 1954 dollars.

(2.13) $\Delta H_t = \underset{(11.6)}{-96.2} + \underset{(.0405)}{.328} X_t - \underset{(.0485)}{.407} H_{t-1} - \underset{(.0925)}{.137} \Delta X_t + \underset{(.0277)}{.171} U_t + \epsilon$
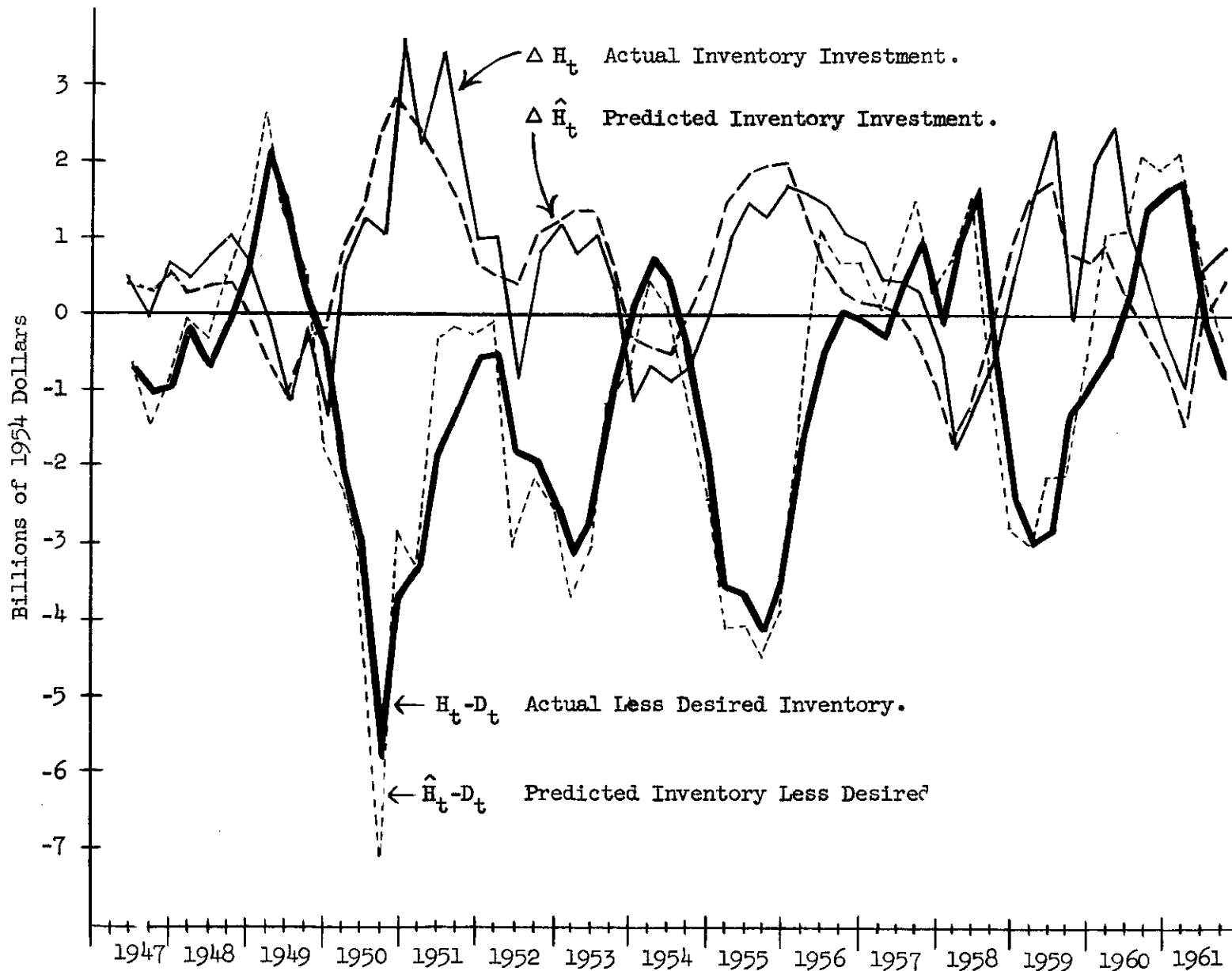
$$R^2 = 0.736$$

The change in GNP term, ΔX_t , is included in an attempt to take into account errors made by firms in anticipating sales volume; the details of this procedure are described later. The expression for equilibrium inventories is

$$(2.14) \qquad H_t^e = -236 + .806X_t + .419U_t \; .$$

The discrepancy between equilibrium and actual inventory is plotted on the attached chart, together with actual inventory investment. Two estimates are presented; the second includes the error of equation (2.13) in predicting actual inventory investment, as there seems to be no obvious basis on which to decide whether the stochastic disturbance should be included or suppressed in surrogative measurement of excess inventories.[7] In order to illustrate how the model performs outside the regression period, preliminary estimates of inventory change, predicted inventory investment and the inventory discrepancy are recorded for all of 1960 and three quarters of 1961. It is clear that there is a systematic tendency for predicted inventory investment to fall short of actual inventory accumulation; much better predictions could have been made by taking advantage of the tendency towards autocorrelated disturbances.[8]

Other variables may well belong in the equation determining the level of inventory investment. For one thing, it might be suspected that anticipations of rising prices might lead firms to accumulate additional inventory. Furthermore, a tightening of credit conditions would be expected to lead to a reduction in the equilibrium level of inventory. The first influence may be tested by including the percentage change of price in the regression; it should have a positive coefficient indicating that if firms speculate successfully they enlarge their inventories in advance of price increases, and conversely when price falls are in the offering.[9] The effects of credit conditions were introduced by including the bank

Δ H_t Actual Inventory Investment.

Δ $\hat{H}_t$ Predicted Inventory Investment.

$\leftarrow$ H_t-D_t Actual Less Desired Inventory.

$\leftarrow$ $\hat{H}_t$-D_t Predicted Inventory Less Desired

Billions of 1954 Dollars

Inventory Investment and Surplus Inventories
(Billions of 1954 Dollars)

Year and Quarter	Inventory Investment		Surplus Inventory	
	Actual ΔH_t	Predicted $\Delta \hat{H}_t$	Actual $H_t - D_t$	Predicted $\hat{H}_t - D_t$
1947 1				
2	.45	.40	.70-	.65-
3	.01-	.32	1.02-	1.42-
4	.72	.55	.98-	.80-
1948 1	.48	.32	.22-	.07-
2	.78	.38	.72-	.32-
3	1.02	.40	.10-	.52+
4	.75	.01	.62+	1.38+
1949 1	.10-	.60-	2.12+	2.62+
2	1.15-	1.05-	1.55+	1.42+
3	.20-	.38-	.30+	.48+
4	1.35-	.20-	.58-	1.72-
1950 1	.60	.80	2.12-	2.32-
2	1.20	1.40	3.00-	3.20-
3	1.02	2.35	5.85-	7.18-
4	3.62	2.80	3.68-	2.85-
1951 1	2.30	2.25	3.38-	3.32-
2	3.42	1.95	1.85-	.38-
3	2.30	1.22	1.22-	.15-
4	.98	.62	.60-	.25-
1952 1	1.00	.55	.55-	.10-
2	.82-	.40	1.80-	3.02-
3	.82	1.02	1.98-	2.18-
4	1.18	1.20	2.58-	2.60-
1953 1	.80	1.38	3.10-	3.68-
2	1.02	1.38	2.68-	3.02-
3	.38	.62	.92-	1.18-
4	1.08-	.28-	.12	.68-
1954 1	.65-	.42-	.68+	.45+
2	.85-	.50-	.50+	.15+
3	.68-	.18-	.48-	.98-
4	.02	.52	1.92-	2.42-

Inventory Investment and Surplus Inventories (Cont'd)
(Billions of 1954 Dollars)

Year and Quarter	Inventory Investment		Surplus Inventory	
	Actual ΔH_t	Predicted $\Delta \hat{H}_t$	Actual $H_t - D_t$	Predicted $\hat{H}_t - D_t$
1955 1	.98	1.48	3.62-	4.12-
2	1.45	1.85	3.70-	4.10-
3	1.35	1.98	4.15-	4.52-
4	1.65	2.00	3.55-	3.90-
1956 1	1.60	1.40	1.62-	1.42-
2	1.50	.72	.55-	1.08+
3	1.08	.38	.02+	.72+
4	.98	.18	.02-	.72+
1957 1	.45	.15	.25-	.05+
2	.42	.02	.32+	.72+
3	.32	.28-	.92+	1.52+
4	.50-	.92-	.12-	.30+
1958 1	1.75-	1.65-	.88+	.78+
2	1.28-	1.25-	1.60+	1.58+
3	.58-	.22-	.65-	1.00-
4	.48+	.82+	2.45-	2.80-
1959 1	1.52	1.55	2.98-	3.00-
2	2.42	1.75	2.82-	2.15
3	.02-	.80	1.30-	2.12-
4	2.00	.70	.92-	.62-
1960 1	2.48	.87	.55-	1.05+
2	1.18	.32	.22+	1.08+
3	.50	.23-	1.38+	2.10+
4	.32-	.70-	1.60+	1.98+
1961 1	.88-	1.24-	1.78+	2.15+
2	.60	.10-	.18-	.52+
3	.88	.44	.80-	.35-
4				

rate on short term business loans within the regression. When both these coefficients have the wrong sign, it suggests that the price speculation and interest rate hypotheses should be rejected.[10] In the Duesenberry, Eckstein and Fromm study both unfilled orders and their rate of change were included in their model.[11]

Although most recent econometric investigations have involved a flexible accelerator principle, no attempt has been made to examine empirically possible determinants of the speed of adjustment. In studies of fixed investment behavior it is sometimes argued that there exists a maximum rate at which capital can be liquidated.[12] It might well be asked whether the speed with which inventories are adjusted to the equilibrium level may not depend upon the sign as well as the magnitude of the discrepancy between actual and equilibrium inventory. The Goodwin formulation might be derived by assuming that the cost of adjusting inventories is related to the square of the discrepancy between equilibrium and actual inventories.[13] On the other hand, if costs of adjustment are simply proportional to the size of the discrepancy, firms may attempt an immediate adjustment to large departures from equilibrium but not respond at all when inventories are only slightly out of alignment.[14]

Closely related to the flexible accelerator explanation of the timing of finished goods inventory investment is the concept of production smoothing. This approach emphasizes the costs involved in changing output rather than inventory levels. A firm may systematically accumulate inventory of finished goods during periods of slack demand by having production exceed sales in order to run them off later during periods of peak demand. This practice serves to minimize costs involved in changing production levels and work force; it enables the firm to meet a larger peak demand with given plant capacity, thus economizing on capital. This approach has been employed in empirical studies by Franco Modigliani and Owen H. Sauerlander [41], by

Edwin Mills [37, 39, 40], and J. Johnson [26].

The production smoothing argument implies that the seasonal pattern in inventories (or production) cannot be explained entirely by concomitant seasonal movements in sales. The complication may be suppressed by employing seasonally corrected data, one of several approaches utilized by Modigliani and Sauerlander. An alternative to working with deseasonalized data is to include seasonal dummy variables within the regression equation. This procedure, utilized by J. Johnson, could facilitate a statistical test of the production smoothing hypothesis.[15/] At a cost of additional degrees of freedom, the regression may be fitted separately for each season; this procedure has been employed by Modigliani and Sauerlander and by J. Johnson. An advantage of this practice emphasized by Modigliani and Sauerlander, is provided by a theoretical argument that the extent to which changes in sales volume and other explanatory variables affect production levels and planned inventory depends upon whether the current quarter is typically one of seasonally high or low sales volume.

The production smoothing hypothesis would not be of direct interest from the point of view of understanding cyclical movements in inventory investment if it only provided an explanation of a divergency of the seasonal pattern of inventory from that of sales volume. But the production smoothing hypothesis may be invoked to explain the cyclical lag in inventory investment behind changes in sales volume that is to be observed in deseasonalized as well as uncorrected data. Edwin Mills introduced lagged production into equation (2.4); only a partial adjustment of production to the equilibrium level takes place within any one period. In this form, the production smoothing hypothesis explains a lag of production behind changes in sales volume; since sales differ from production by the change in inventory, this leads to a lag of inventory behind sales changes over and above that which results

from inventory smoothing considerations. The evidence with regard to the cyclical form of the hypothesis is not conclusive; while lagged production has proved statistically significant in some regressions, in other applications the coefficient of lagged output has consistently proved to have the wrong sign.[16]

There are, of course, alternative explanations that may be advanced to explain the timing of aggregate inventory investment. Systematic errors in anticipating future sales or production levels might lead to a lag of investment behind sales. Although this complication will be considered subsequently, it must be observed that the flexible accelerator production smoothing line of argument seems to suffice to explain, at least in rough measure, the cyclical timing of inventory movements.

3. Errors in Anticipating Sales Volume

Because production requires time, a firm selling its output in imperfect markets must have decided upon the current level of output on the basis of advanced estimates rather than precise knowledge of demand conditions. When sales exceed the anticipated level, the buffer of finished goods inventory carried in order to prevent runouts is depleted; on the other hand, when sales forecasts are unduly optimistic, unplanned inventory accumulation occurs. Only a firm fabricating goods to specific order escapes the problem. Two essential modifications of the basic accelerator model are required in order to take into account the complications created by anticipation errors in the analysis of finished goods inventory behavior. In the first place, anticipated sales X_t rather than actual sales must be inserted into the equilibrium inventory equation when the planned level of inventory is being determined. In addition, the impact of errors in anticipating sales volume upon the level of inventory must be taken into account.

These considerations suggest that finished goods inventory behavior is determined by the following equation

$$(3.1) \qquad I_t = \delta\alpha + \delta\beta_1 \hat{X}_t + \delta I_{t-1} + \lambda [\hat{X}_t - X_t] + \epsilon_{1t} .$$

This modification of the basic accelerator may be further complicated if equilibrium inventories depend on other variables in addition to sales or if production smoothing is introduced.[17]

The surprise element $\hat{X}_t - X_t$, the excess of anticipated over actual sales, is preceded by λ , the "production adaptation coefficient," in order to take into account a complication introduced by Modigliani and Sauerlander [1955]. If λ equals unity, the equation implies that the firm does not succeed in even partially compensating for errors made in anticipating sales during the period of observation; finished goods inventory falls below the planned level by the full extent of the forecast error. A λ less than unity implies that the firm manages at least partially to offset errors made in anticipating sales volume. At a possible cost of premium wage payments, or alternatively losses due to idle time, production schedules may be revised on the basis of current sales experience. If $\lambda = 0$, the revision of the production plan is drastic enough to keep inventory at the planned level, a magnitude that may no longer be appropriate for current sales experience. If $\lambda = -\delta\beta$, the firm succeeds completely in compensating for any errors made in anticipating sales volume.[18] It is apparent that the value of λ encountered in any empirical study would depend in part upon the length of the observational period involved in data collection relative to the duration of the production process. It must also be observed that the concept of adapting production so as to partially eliminate anticipation errors is the converse to the production smoothing conjecture. If inventories are carried in order to iron out short-term fluctuations in sales, we might well expect inventory rather than output levels to bear the brunt of the burden when sales

anticipations prove to be incorrect.

a. Utilizing Anticipations Data:

When suitable data on expectations are available the parameters of the inventory equation may be estimated directly. This approach has been followed by Modigliani and Sauerlander [41], by Lovell [30] and Edwin Mills [40] with Railroad Shippers Forecast data. Murray Brown [7] and Peter Pashigian [44] have presented reports on studies based on Fortune magazine forecast data and annual Commerce Department - Security Exchange Commission anticipation series. T. Thonstad and Jochems [49] have utilized Munich business test data. None of these sets of data is entirely appropriate for the purpose, either because the data are presented in a form that requires transformation or because of an inadequate number of observations. Furthermore, a controversy continues as to whether the tendency of observed anticipations to regress towards former levels should be interpreted as implying that the data are subject to systematic measurement error or as revealing an important characteristic of actual anticipations.[19] Consider the following regression derived from quarterly constant dollar manufacturing finished goods inventory data for the 1948-55 period:

$$(3.2) \qquad I_t = -903.1 + \underset{(.0195)}{.0746} \ X_t + \underset{(.0262)}{.1283} \ (\hat{X}_t - X_t) + \underset{(.0599)}{.7591} \ I_{t-1}$$

$$R^2 = .970$$

These estimates imply a marginal desired inventory coefficient, β_1, of 0.098; the production adaptation coefficient is 0.054, while the inventory reaction coefficient is 0.24. If these estimates could be taken at face value, they would suggest that although manufacturing firms are not prompt about adjusting inventories to their equilibrium level, they are extremely agile in adapting production schedules to

unanticipated changes in sales volume.

Unfortunately, the expectational data utilized in this regression are an inaccurate synthetic series constructed from suspect railroad shippers forecast data. A description of the procedure utilized in deriving the $\hat{X}_t$ series may be postponed until later in this paper. It is necessary to emphasize at this juncture that since the discrepancy between anticipated and actual sales volume, the term $\hat{X}_t - X_t$, is not observed with precision, the least squares procedure may be expected to yield a biased estimate of the production adaptation coefficient. This danger is enhanced if there is a systematic element in the measurement error. Suppose that the actual error in anticipating future sales is some definite fraction of the observed error

$$(3.3) \qquad \dot{X}_t^a - X_t = b (\hat{X}_t^o - X_t) + \epsilon_3 \, , \, 0 < b < 1 \, ,$$

where $\hat{X}_t^a$ is the actual level of anticipated sales and $\hat{X}_t^o$ observed anticipations. The conjecture is not unreasonable, for observed sales anticipation series often suggest that firms are unbelievably inaccurate forecasters of future sales. If this equation is substituted into (3.1), one obtains

$$(3.4) \qquad I_t = \delta\alpha + \delta\beta_1 X_t + \delta I_{t-1} + (\delta\beta_1 + \lambda) \, b \, (\hat{X}_t^o - X_t) + \epsilon_{1t} + (\delta\beta_1 + \lambda) \, \epsilon_3$$

an equation of the same form as (3.2). Clearly, if observed expectational data systematically overstate errors made by firms in anticipating future sales volume there will be a tendency for the regression to suggest an excessive degree of flexibility in production plans.[20]

Although considerable improvement in the availability and accuracy of expectational data is currently being made, a number of studies testify to the extreme inaccuracies present in the ex ante data currently available. In his recent

investigation of inventory investment utilizing expectational data compiled by the
Business Roundup staff of _Fortune_ magazine Murray Brown concluded with the comment
that

> ...the _Fortune_ ex ante variables provide only marginal gains to the prediction
> of inventory behavior. However, the anticipations data may become more useful
> in the future as observation error is reduced.

Undoubtedly, expectational data in time series form will prove of increasing useful-
ness as additional observations become available.[21/] While in principle additional
degrees of freedom might be obtained by utilizing observations on individual firms,
it has not yet proved possible to obtain data in the cross-sectional form most useful
for econometric investigation of inventory behavior.

One yardstick by which both the validity and the usefulness of actual antici-
pations data may be judged is provided by comparing the results obtained when the
data are utilized to explain inventory behavior with those provided when proxies for
actual anticipations data are utilized. It is necessary to review various procedures
that have been devised in order to circumvent the difficulties created by the meager-
ness of data on actual sales expectations.

b. _Structural Proxies for Anticipations:_

One alternative to the utilization of actual anticipations data is to make
some particular assumption about the _structure_ by which anticipations of future sales
volume are actually generated. It might be assumed, for example, that actual antici-
pations are a linear function of exogenous or predetermined variables.

(3.5) $$\hat{X}_t = \Sigma \, v_i \, E_{i,t-1} + \epsilon_{5t}$$

Here the v_i are unknown structural coefficients and the $E_{i,t-1}$ specified

predetermined or exogenous variables; ϵ_{2t} is a stochastic disturbance. If this equation is substituted into (3.1) we obtain

$$(3.6) \qquad I_t = \delta\alpha + \delta(\lambda+\beta_1)\, \Sigma v_i E_{i,t-1} + \delta I_{t-1} - \lambda X_t + (\delta\beta_1 + \lambda)\, \epsilon_{5t} + \epsilon_{1t} \; .$$

Once more there is an error of observation connected with an explanatory variable, raising the danger of biased estimates of the parameters of the equation. An additional difficulty with this technique is that its application does not yield an estimate of the marginal desired inventory coefficient, β_1 . Although the application of this procedure might provide some indication about the relative importance of various determinants of expectations, there is the danger that some of the variables thought to be determinants of expectations may actually have a direct influence upon equilibrium inventory. For example, the backlog of unfilled orders might influence sales anticipations; it might also have a direct influence upon desired inventory, thus belonging in equation (3.1) as well as (3.5). Since such influences cannot be disentangled when an expression relating to the structure of expectations is substituted into the inventory determination equation, the information which may be gained from this procedure concerning the structure of the inventory determining equation is not as great as might potentially be gained if good data on actual anticipations were available.

If one is willing to make more specific assumptions concerning the structure of the equation generating expectations, much more may be learned concerning the determinants of inventory investment. Alain Enthoven [14] attributed naive expectations to entrepreneurs in an interesting study of inventory behavior. If $\hat{X}_t = X_{t-1}$, and if it is assumed that both the reaction and production adaptation coefficients are unity, equation (3.1) may be written

$$(3.7) \qquad I_t + X_t - X_{t-1} = \alpha + \beta_1 X_{t-1} + \epsilon_{1t}$$

By making the total inventory stock plus the change in sales the dependent variable, Enthoven insured that the reaction coefficient would be unity.[22]

J. Johnson [26] has suggested that for undeseasonalized quarterly data the expectations function may take the more complicated form

$$(3.8) \qquad \hat{X}_t = X_{t-4} + v_1 X_{t-y} \left(\frac{X_{t-1} - X_{t-5}}{X_{t-5}} \right)$$

If this expression is substituted into (3.1), estimates of the marginal desired inventory coefficient may be obtained. Needless to say, valid results will be provided by this procedure only if the structure by which expectations are generated as well as the inventory equation has been correctly specified. Johnson himself has doubts about this particular formulation. For one thing, the parameter estimates are not too satisfactory. He considers an alternative, more flexible expectations generating equation due to Charles Holt [23] that is related to the "adaptive expectations" concept of Marc Nerlove [42] and MaGee [35], but more complicated in that both seasonal and trend terms are assumed to be determined by a distributed lag. He proceeds to compute artificial $\hat{X}_t$ series for alternative sets of possible values of the parameters of this second expectations generating equation, and then fits a production adjustment equation, contrasting the closeness of the fits obtained with alternative values of the expectation parameters.[23] Potentially, the use of such surrogative procedures may yield information concerning the way in which expectations are generated as well as an understanding of the production and inventory decision.[24]

c. Actual Sales as a Surrogative Measure of Anticipations:

Edwin Mills [37, 38, 39, 40] has argued that a second alternative to the utilization of anticipations data is to employ actual sales X_t as a proxy for the anticipated sales volume. This procedure was implicit in the pioneering Klein study [28] based on data for the interwar-period. Edwin Mills has spelt out in detail its rationale. It is not supposed that firms are clairvoyant. It is assumed that whatever the procedure utilized by the firm in predicting demand, it is not biased and that the errors of prediction are random;[25]/hence,

$$(3.9) \qquad \hat{X}_t = X_t + \epsilon_{5t} \ , \ \mathcal{E} \ (\epsilon_5) = 0$$

Substitution of this equation into (3.1) yields a stochastic equivalent of (2b).

$$(3.10) \qquad I_t = \delta\alpha + \delta\beta_1 X_t + \delta I_{t-1} + (\lambda + \delta\beta) \ \epsilon_{5t} + \epsilon_{1t} \ .$$

Klein calls the residuals "undesired inventory"; he presents numerical estimates of the disturbances for the sample period [28 p.   ]. Mills [39], who worked with a production determination equation, was interested in transforming the observed residuals in order to obtain an estimate of the actual error made in anticipating future sales; the analysis was based on the assumption that production schedules are completely inflexible, that $\lambda = 1$. Interpretation of the observed residuals of a regression of the form (3.10) in this way involves the implicit assumption that observational errors and the effects of variables omitted from the equation are small relative to the impact of erroneous sales anticipations. The straight-forward application of estimation procedures to an equation of the form of (3.10) under the assumptions embodied in (3.9) involves certain other difficulties. The limited information estimation procedure employed by Klein relies on the assumption

that the residuals (undesired inventory) of successive time periods are independent. When the procedure of least squares is applied, parameter estimates are necessarily inefficient if the residuals are autocorrelated; customary tests of significance are not valid. Furthermore, the sum over the sample period of excess inventories will necessarily be zero when the least squares procedure is employed. Edwin Mills [39] circumvented these difficulties by applying least squares to the first dif-firenced version of equation (2.4), a procedure that is appropriate if changes in errors made by firms in predicting sales are independent; in order to obtain estimates outside the sample period of errors made by firms in anticipating sales volume he substituted the parameter estimates obtained with the first differenced regression back into equation (3.10). It must be observed that a difficulty with this procedure is created by the fact that the presence of the stochastic element of (3.9) means that biased estimates will be yielded by the application of least squares to equation (3.10).[26]

d. Biased Expectations:

It is possible to examine empirically a more general assumption about the nature of expectations that includes as special cases both Edwin Mills' hypothesis, equation (3.9), and the alternative assumption of naive expectations employed by Alain Enthoven in his empirical work. Michael Lovell hypothesized [30, 31] that expectations, however formed, turn out to be a linear combination of lagged sales and actual developments:

$$(3.11) \qquad \hat{X}_t = \rho \, X_{t-1} + (1 - \rho) \, X_t + \epsilon_{7t}$$

If the "coefficient of anticipations" $\rho = 0$, we have the case considered by Mills.

On the other hand, $\rho = 1$ corresponds to the case of naive expectations. A value of ρ between these two extremes implies that on the average firms anticipate a definite fraction of actual changes in sales:

$$(3.12) \qquad \hat{X}_t - X_{t-1} = (1 - \rho)(X_t - X_{t-1}) + \epsilon_{7t}$$

Since the error made by the firm is

$$(3.13) \qquad \hat{X}_t - X_t = -\rho(X_t - X_{t-1}) + \epsilon_{7t} \; ,$$

the coefficient of anticipations is a measure of the bias of forecasts towards last period's sales. $\rho = 1$ implies that firms have no success in anticipating the direction of changes in sales volume; expected sales are randomly distributed about last period's sales. A negative ρ implies that firms have a systematic tendency to overstate changes; $\rho > 1$, on the other hand, corresponds to the perverse case in which the direction of change in sales is generally misjudged, an extreme form of regressive anticipations.[27]

The conjecture underlying equation (3.11) implies nothing about how expectations are actually formed; it says nothing about the structure of anticipations. In the study of inventory behavior, the conjecture does permit an investigation of a possible systematic tendency for firms to underestimate average changes in sales volume. Substitution of equation (3.11) into (3.1) yields

$$(3.14) \qquad I_t = \delta\alpha + \delta\beta X_t - (\delta\beta + \lambda)\rho \Delta X_t + \delta I_{t-1} + (\lambda + \delta\beta)\epsilon_{7t} + \epsilon_{1t} \; .$$

Lovell [31] reports the following estimates of the coefficients of this equation for quarterly deflated total manufacturing finished goods inventory data for the period

1948-55:

$$(3.15) \qquad I_t = -258.2 + \underset{(.0203)}{.0419} X_t - \underset{(.0417)}{.1315} \Delta X_t + \underset{(.0649)}{.8479} I_{t-1}$$

$$R^2 = .958$$

These coefficients imply that $\delta = .1521$ and $\beta = .2755$. It is not possible to unscramble the regression coefficients in order to obtain an estimate of ρ ; the effects of flexibility of production cannot be segregated from the measure of degree of bias of expectations. If it could be assumed that production plans are completely inflexible, $\lambda = 1$, then the estimates imply $\rho = .1262$; this figure may be interpreted as the _effective bias_ of expectations; although expectations may be much more strongly biased towards last period's sales than this figure suggests, the value of ρ obtained under the assumption of $\lambda = 1$ does indicate the net prediction bias after reductions for a possible partial readjustment of production plans.[28] It is to be noted that even if a fair degree of flexibility of the production plan is admitted, say $\lambda = 1/2$, then $\rho = .24$, a figure still implying that expectations are, on the average, quite precise. The estimates are consistent with an anticipation's coefficient greater than unity only if the flexibility coefficient is less than 9 percent.

Clearly, this procedure may again yield biased parameter estimates because of the presence of the stochastic term ϵ_{7t} . This in itself is reason for suspecting that even under the assumption of complete inflexibility, the effective bias should not be interpreted in terms of equation (3.9) as an accurate measure of any systematic tendency for firms to underestimate changes in demand. An additional reason is provided by an identification problem similar to that involved with one of the

surrogative procedures discussed earlier. This problem presents itself once we admit that other variables in addition to sales may influence the equilibrium level of inventories. Suppose, for illustrative purposes, that the structure explaining the generation of expectations takes the specific form

$$(3.5") \qquad \hat{X}_t = v_o + v_1 U_t + v_2 \Delta U_t + v_3 X_{t-1} + \epsilon''_{2t} \; ,$$

where U_t is the backlog of unfilled orders and $\Delta U_t = U_t - U_{t-1}$. If we substitute into equation (3.1) because X_t is unobserved, we obtain the equation

$$(3.1") \qquad I_t = \delta\alpha + \delta(\lambda+\beta)(v_o + v_1 U_t + v_2 \Delta U_t + v_3 X_{t-1}) + \delta I_{t-1} - \lambda X_t$$
$$+ (\delta\beta_1 + \lambda) e_{2t} + e_{1t} \; .$$

Comparing this equation with (3.14) we see that certain coefficients of the latter equation are identified only because the backlog of unfilled orders and its change were not included in the inventory determining equation. Consider next the following non durable manufacturing regression where total stocks, H_t , had to be utilized rather than just finished goods inventories because of restrictions on the availability of deflated data stratified by stage of fabrication.

$$(3.16) \qquad \Delta H_t = -.1885 - .0755 H_{-1} + .0362 X_t + .1950 U - .0922 \Delta X_t$$
$$(.5434) \; (.0522) \qquad (.0216) \qquad (.0595) \qquad (.0420)$$
$$R^2 = .378$$

In contrast, when the change in unfilled orders is added to the regression, we have

$$(3.16') \qquad \Delta H = -.5084 - .0823 H_{-1} + .0426 X_t + .2541 U - .0285 \Delta X_1 - .3557 \Delta U$$
$$(.4090) \; (.0390) \qquad (.0161) \qquad (.0456) \qquad (.0331) \qquad (.0581)$$
$$R^2 = .574$$

The order terms were included because of the conjecture suggested by earlier empirical work that they have a direct influence upon stocks of purchased materials and goods in process. If we wish to maintain the assumption that the change in orders does not influence stocks directly, the first of the reported regressions is, identified under the assumption that $v_2 \neq 0$ in (3.2"), the estimate of the effective bias is 0.089 . If the change in unfilled orders is regarded as having a direct influence upon stocks, certain coefficients of (3.14) are no longer identified because there is now no variable in (3.5") excluded from the structural stock equation. The second set of estimates, which implies a much lower effective bias of expectations of .032 would be identified only if we replaced (3.5") with an equation involving the maintained hypothesis that expectations are influenced by other variables in addition to current and lagged orders and sales. The choice between these two alternative estimates of expectational bias can be made only on the basis of a priori knowledge as to the actual structure generating expectations; it cannot be made on the basis of the statistical evidence summarized by equations (3.16) and (3.16').

The employment of surrogative procedures rather than actual data on expectations has not at this stage yielded positive results. On the one hand, Johnston's analysis has not yet established which of the alternative assumptions he considers concerning the structure by which expectations are actually generated is preferred. While Lovell found the ΔX_t term significant in his earlier regressions covering the period 1948-55, suggesting a definite bias in manufacturers' forecasts, subsequent regressions with more recent data reported elsewhere [32] as well as equations (2.13), (3.16), and (3.16') above do not yield such strong results. At this juncture there is insufficient evidence for rejecting Mills' simple conjecture

that actual sales provide a suitable proxy for anticipations.

e. A reconstitution of the Railroad Shippers Forecasts:

It is apparent that problems of interpretation arise with any attempt to circumvent the utilization of data on actual sales anticipations in the study of inventory behavior. Although this does not mean that such attempts are not worth while, it does emphasize the importance of a correct interpretation of what information we do have on actual expectations. Here only one body of expectational data will be considered, the Railroad Shippers Forecasts.

The data concerns anticipated quarterly shipments by rail broken down into 32 commodity groups. A sample of firms constituting a sizeable portion of the bulk of railway freight traffic has provided the data published since 1927 in the "National Forecast" of the Regional Shippers Advisory Boards under the auspicies of the Association of American Railroads.[29] The forecasts have proved to be quite inaccurate predictors of actual railroad carloadings, being frequently less accurate than simple naive projections of the previous quarter's shipments.[30] Nevertheless, they still constitute an important body of anticipations data which has been subjected to repeated analysis.

Albert G. Hart [21] attempted a reconstitution of the Railroad Shippers Forecast data for the inter-war period in order to obtain a series of more accurate carload anticipations in closer conformity with the type of expectations entre-preneurs might be expected to hold. Hart found it hard to believe that the actual anticipations held by business men could have the "regressive" property of the Shippers' Forecasts, a systematic tendency to predict a movement back toward earlier levels in the face of opposing trend. But arguments concerning the validity of a

revised anticipations series based upon their conformity with how expectations are expected to behave is inherently a most subjective process. Here a second attempt to reconstitute the Railroad Shippers' Forecasts based on post World War II data will be described.

Although the traffic manager generally completes the return utilized in preparing the Railroad Shippers' Forecasts of carload utilization, this does not imply that the estimate is derived independently of the firm's sales anticipations. The respondent is asked to state the anticipated percentage increase in carloadings over the corresponding quarter of the preceding year, actual shipments for that quarter of last year, and an anticipations figure in carload units. The respondent may simply assume that carloadings will increase by the same percentage as the increase anticipated by the firm for total sales by all modes of transportation. Firms frequently utilize comparisons with the same quarter of the preceding year as an implicit form of seasonal adjustment. Even if this procedure is not followed explicitly, it seems reasonable to assume that the traffic manager who completes the form must be aware of the sales forecast and be influenced both in his planning and in completing the questionnaire on carload shipments by that figure. If $\hat{X}_t$ represents anticipated total sales volume and X_{t-4} actual sales in the corresponding quarter of the preceding year, while C_{t-4} stands for actual shipments by rail in carload units for the same quarter of the preceding year, the hypothesis implies that anticipated carload shipments $\hat{C}_t$ were formulated by the respondents by utilizing the equation:

$$(3.17) \qquad \hat{C}_t = C_{t-4} \cdot \frac{\ddot{X}_t}{X_{t-4}} \quad .$$

This hypothesis cannot be tested directly for the variable $\hat{X}_t$ is not observed. Furthermore, the other variables are observed at best only in aggregative form. Inaccuracies may result not only from sampling errors but also because the reports of the various firms are weighted by the number of carloadings shipped by the firm in corresponding quarters of the preceding year; consequently, firms which ship a relatively large portion of their total output by rail have their anticipations overweighted when equation (3.17) is utilized to derive sales anticipations, the unobserved variable $\hat{X}_t$.

A possible test of the validity of the hypotheses is provided by the fact that the Railroad Shippers' Forecasts of carload shipments are not accurate predictors. An inspection of the above equation reveals that a sales anticipations series derived from the published rail forecast series could be either more or less accurate than the carload anticipations.[31] If the sales anticipations derived with equation (3.17) are in fact more accurate, it would offer support for the hypothesis that firms derive their carload anticipations on the basis of equation (3.17) and that the derived sales anticipations obtained by solving for the unobserved variable $\hat{X}_t$ is a valid representation of actual sales anticipations. Conversely, if the derived sales anticipations series are less accurate than the carload forecasts it would suggest that the derived sales anticipations are not as precise as might reasonably be expected of actual anticipations.

For the cement industry, data on sales in real terms as well as the forecast data in terms of carloadings are easily obtained. A pilot study testing the hypothesis of equation (3.17) was made. Although a relatively large portion of cement is shipped by rail, an inspection of the first column of correlation coefficients in the following table reveals that for the postwar period the relation-

ship between carloadings and sales is not too close and varies considerably for
different quarters of the year.

<u>Accuracy of Rail Forecasts and Derived Sales Anticipations</u>
<u>1947-56.</u> Cement Industry.

Period	Number of Observations	Correlation Coefficients		
		Sales and Carloadings	Forecast and Actual Carloadings	Anticipated and Actual Sales
all quarters	40	.8895	.9605	.9866
1st quarter	10	.9589	.4159	.9121
2nd quarter	10	.8345	.4568	.9688
3rd quarter	10	.8523	.6259	.9847
4th quarter	10	.0042	- .0753	.9538

The second and third columns of the table present correlation coefficients measuring
the closeness of the relationship between forecast and actual shipments and between
derived anticipations and actual sales. It is to be observed that for every quarter
of the year as well as for an overall comparison the sales anticipations series is
a much closer predictor than the rail forecasts. For the fourth quarter, the
correlation between actual sales and carloadings is extremely poor; the correspon-
dingly poor predictive power of the railroad forecasts for this quarter is to be
expected under the hypothesis formulated in equation (3.17).[32]

It seems safe to conclude that the raw railroad shippers forecast data
constitute a most tenuous form of evidence for judging the accuracy with which
business firms actually forecast demand. The conjecture summarized by equation (3.17)

offers an alternative explanation. While it cannot be concluded with great confidence that expectations are not regressive, the validity of the shippers forecast evidence seems open to serious question.

One test of the usefulness of the derived sales anticipation series is obtained by contrasting their ability to predict cement sales with a naive projection of the sales level realized in the preceeding period. The correlation between lagged and current sales is only 0.117; while the fit is improved to $r = .979$ when seasonal dummy variables are added, the derived sales anticipations series still provides a somewhat better prediction than that obtained by a naive projection of last quarter's experience.

Another test concerns the contribution that the anticipations series derived from the shippers' forecast can make in predicting the behavior of other operating variables. A preliminary test on the cement industry involved predicting output over the 1947-56 period in terms of anticipated sales and the lagged inventory stock; this is a special case of the model discussed in the preceeding section in which it is assumed that there is no production flexibility. With the assumption of static expectations, $X_t = X_{t-1}$, a multiple correlation of .646 was obtained; although the addition of seasonal dummies served to raise the multiple correlation coefficient to .914 , the inventory and lagged sales term were no longer significant, the dummies carrying the brunt of the explanatory burden. The derived anticipations sales series and the alternative provided by Edwin Mills suggestion that actual sales be utilized as a proxy did equally well, both yielding a multiple correlation coefficient of 0.933 ; when seasonal dummies were added, the correlation coefficient was raised to 0.951 for the Mills proxy procedure versus .946 for the derived sales anticipations series.

One of the regressions reported earlier in this study, equation (3.2), constituted a further attempt to utilize the reconstituted shippers' forecast anticipations data. Considerable effort was devoted in deriving an aggregative anticipations series.[33] It will be recalled that although the coefficient of the gap between anticipated and actual sales in that regression was significant, the low magnitude of the regression coefficient suggested that production plans are almost unreasonably flexible; an alternative interpretation is to suppose that there is a systematic tendency for the derived anticipations series to overstate the forecast error.

Problems of structural estimation may be regarded as of but secondary interest in many applications. For purposes of forecasting, the distinction between the degree of production flexibility versus bias of sales anticipations may be of little import. Even in the analysis of the implications for the stability of the economy of alternative patterns of inventory behavior, it may develop that the distinction is not crucial. Even if this issue cannot be resolved, it is necessary to face up to the problem of how expectations should be handled when ones chief interest centers on forecasting rather than questions of structure. One criterion involves goodness of fit. Since the inventory determination equation (3.1) involves the stochastic term ϵ_{1t}, a perfect fit could not be expected even if expectations were measured without error. Any approximation procedure may be expected to yield a still poorer fit in that it will generally involve an additional stochastic term of the form $(\delta\beta_1+\lambda)\,\epsilon_{jt}$, where $\epsilon_{.t}$ is the error involved in approximating expectations. If there is independence between the two error terms, closeness of fit will be proportional to the variance of $\epsilon_{.t}$, and a comparison of correlation coefficients obtained when alternative approaches are

applied to the same set of data may be regarded as indicative of the most appropriate assumption to make about the structure of expectations as well as pointing the way to the technique which will be most useful from the point of view of prediction.

In terms of this criterion the derived aggregate railroad forecast anticipations series appears to make a significant contribution in (3.2) in explaining the behavior of manufacturers' inventory holdings. Certainly, the regression offers an improvement in this application over what would have been obtained if actual sales were employed as a proxy for anticipations. A comparison of the closeness of fit with equation (3.15), however, reveals that it offers at best only a marginal improvement over that obtained under the assumption that expectations are biased towards last period's sales. This emphasizes the importance of considering surrogative alternatives in appraising the contribution that actual anticipations data can make towards predicting inventory behavior.[34]

4. Conclusions

While the direct forecasting value of sales anticipations data has frequently been questioned in such studies as that of Franco Modigliani and O. H. Sauerlander [41] and most recently by Peter Pashigian [44], it has been argued at the same time that the chief usefulness of ex ante sales observations may be in the assistance they provide in explaining changes in such other variables as inventory investment. The review presented here of alternative procedures for analyzing the impact of sales expectations upon inventory behavior suggests that the problem of making correct inferences concerning the structural determinants of inventories is extremely difficult. If data purporting to measure actual expectations have a

systematic tendency to overstate forecast errors, production plans will appear

excessively flexible. Procedures derived by Alain Enthoven and Jack Johnston for

circumventing the use of actual anticipations data require strong a priori judg-

ments concerning the structure by which anticipations are actually generated. On

the surface, both Edwin Mills' suggestion that actual sales provide a good surroga-

tive measure of anticipations and Lovell's generalization that the change in sales

may be proportional to the error made by firms in anticipating sales appear to

circumvent the problem of specifying the structure by which anticipations are

actually generated. On closer inspection, however, it becomes apparent that the

issue is clouded unless it is assumed that production plans are completely inflexible;

furthermore, the unspecified structure of the equation explaining the actual genera-

tion of expectations might conceivably be such as to imply that other parameters of

the inventory equation are unidentified.

It seems clear from all this that only a limited amount of information

about the structure of anticipations may be gleaned from the study of inventories.

In particular, the two sets of regressions summarized by equation (3.2) and (3.15)

are both compatible with either (a) quite inaccurate perhaps regressive anticipa-

tions but extremely flexible production plans or (b) rather accurate expectations

but not much flexibility in production scheduling. While the reconsideration of

the Railroad Shippers' Forecast data with regard to the cement industry offers some

support for the conjecture that expectations are not as inaccurate as ex ante data

sometimes imply, the investigation of inventory behavior has not served to establish

that the expectations of future sales held by individual firms do not have a

"regressive" tendency to forecast a reversion back towards former sales levels.

Research on inventories and anticipations are clearly complementary rather

than competing efforts. If, indeed, current investigations should establish that

expectations are actually regressive, this would also imply that production plans
are quite flexible and that expectational errors have only a limited impact upon
inventory. Furthermore, it would be very much worthwhile to test the various
alternative assumptions developed by Johnston concerning the structure of antici-
pations upon actual ex ante data as well as to see how well actual anticipations
are approximated by the conjectures of Mills and Lovell, equations (3.9) or
(3.12). In addition to providing a potential check for determining whether data
on anticipations actually help in describing inventory behavior, surrogative
procedures facilitate the study of inventory and production movements when con-
comitant series on sales anticipations are, unfortunately, unavailable.

The basic accelerator model has certain weaknesses which prevent it from
Although the literature reporting on econometric studies of inventory
behavior is quite small relative to the numerous studies on the determinants of
other components of effective demand, this neglect may be at least partially
explained by the difficulties of the subject. The distinction between actual versus
desired inventory and the problem of measuring anticipated sales are but two of a
host of hurdles confronting the investigator.

The basic accelerator model has certain weaknesses which prevent it from
serving as too effective a vehicle for investigation. The model assumes that the
impact of erroneous anticipations falls either upon output or inventory, making no
allowance for the possibility that adjustments in either price or advertising
expenditures may shoulder part of the burden. Of course, price adjustment models
are available, but the choice is between one extreme or the other as typified by
the two alternative approaches compared by Edwin Mills [40] and by Shozaburo
Fujino [18] rather than a blend on the two extremes; either a price or a quantity
adjustment model rather than a blend of the polar extremes must do the work.

Another difficulty is created by the problem of obtaining adequate a priori

restrictions upon the range of alternative models that are to be subjected to
empirical investigation. The assumption of profit maximizing behavior, emphasized
by Edwin Mills [37, 38, 39, 40], by Modigliani and Sauerlander [41], and by Holt and
Modigliani [24] still leaves a wide range of models for the empiricist to choose
from. A second source of a priori knowledge, the possibility that the assumption
that the economy is stable may place restrictions on the form of the equations
has been investigated by Lovell [32].

A final and perhaps most serious difficulty is created by the current unavaila-
bility of suitable cross section data reporting at monthly or quarterly intervals
on the movement of inventories, sales, and other related variables at the level of
the individual firm. The understanding of the structure of inventory behavior
which constitutes an essential prerequisite for successful prediction will only be
obtained when suitable cross section data become available on a confidential basis
for research purposes.

FOOTNOTES

1/   I am indebted to James Keaten and Fredrick Demming for valuable suggestions and assistance with the computations. Research time for this paper was provided through the generosity of the Cowles Foundation for Research in Economics at Yale University.

2/   Abramovitz [1, Ch. 21] presented a detailed analysis of the contribution of inventory investment to cyclical fluctuations during the interwar period; for example, he contrasted peak to trough movements of GNP with the magnitude of inventory disinvestment during the downward half of the reference cycle; Thomas M. Stanback [46] presents a similar analysis of inventory movements during the post World War II period. Lovell [30] contrasts the behavior of actual GNP with a hypothetical series derived by subtracting an estimate of the gross contribution of inventory investment to cyclical fluctuations, using the multiplier in order to compute the volume of consumption generated by inventory accumulation.

3/   The contrast between the small trend coefficient in the inventory investment regression with a trend parameter estimate of 1.48 reported by Smyth for fixed investment suggests that plant and equipment expenditures may be much more important than changes in business inventory as a generator of secular expansion and growth. On the other hand, the fixed investment accelerator coefficient is only 0.17, little more than half the estimated value of the inventory relation.

4/   This evidence is not conclusive, however, for Smyth [45] reports that he achieved a closer fit with the unlagged rather than with the lagged regression.

5/   Of course, the Munich business test surveys, the new Office of Business

Economics survey of manufacturers' inventory and sales expectations survey, and

certain other surveys do provide some information on equilibrium inventory. But

the data are often reported only in terms of the proportion of respondents reporting

inventory "high," "low," or "about right." Even here, the validity of the response

may be open to question. Thus Foss [17, p. 29] reports that"over the three year

period covered by the Survey relatively few firms have classified their stocks as

'low,' despite some sizeable increases in inventories. At the moment it is too

early to say whether the comparative absence of 'low' designations is an accurate

portrayal of business sentiment regarding inventory conditions over this period,

or whether it is the inevitable result of business thinking which always attempts

to keep stocks as small as possible and thus classifies stocks as 'about right'

so long as they are obviously not 'high.'" Foss also found it necessary to

transform the raw anticipated series in order to obtain a relatively good predictor

of actual inventory movements. Conceivably, an application of the "realization

function" procedure, such as has been attempted by Murray Brown on other data [7]

would prove helpful here.

6/   Klein also estimated the same equation by the method of limited information

within the context of a simultaneous equation model. It is interesting to note that

the two sets of parameter estimates are practically identical, differing less than

alternative parameter estimates of the same equation calculated by Carl Christ [8],

from data covering a longer sampling period; although there remains some question

concerning the accuracy of Christ's data, it may well be that parameter estimates

are more sensitive to the particular years utilized in the regression than to the

choice between a simultaneous equation vs. a single equation least squares

approach. Klein presented a third set of parameter estimates based on quarterly rather than annual data; a transformation procedure revealed that these coefficients were quite consistent with those derived from annual data.

7/   In an earlier study [31] I presented estimates of surplus inventory for durable manufacturing.

8/   Terleckyj has reported that his model did not perform too satisfactorily as a predictor of inventory investment during this same period. In the 1960-61 recession his equations indicated small amounts of inventory accumulation rather than the substantial disinvestment that actually took place [47, p. 161]. Of course, a test of the predictive ability of a model in this form is difficult at the current time because of the preliminary nature of data currently available on the 1960-61 recession. Judging by past experience, considerable revision in inventory data must be expected. An alternative test is to refit the equation over a subperiod and observe either the stability of the regression coefficients, a test reported by Terleckyj [47, p. 161], or to examine the ability of the regression fitted to the subperiod to "predict" the observations excluded from the regression, a procedure applied in another connection by Lovell [32, p. 131].

9/   In Lovell [31] an attempt was made to test the speculative hypothesis on both purchased materials inventory and on aggregate inventory for individual manufacturing industries; the results were again negative. Perhaps this is not too surprising, for literature describing current inventory practice contains reports that in the majority of firms purchasing agents are not actively encouraged to engage in speculative buying. For example, Baumes [3, p. 22] reports that

"While most companies say that they do not speculate in the commodity markets, some companies have a policy of allowing forward buying when the price is right. Companies that allow forward buying usually stipulate that purchases above normal requirements be approved by top management."

10/ Detailed discussions of inconclusive attempts to explore the influence of credit conditions upon inventory investment are presented by Paul F. McGouldrick [35b] and Lovell [32]. Darling [12] in a preliminary analysis of new data found that the coefficients generally had the expected sign only for trade regressions, although the coefficients were not significant.

11/ The model considered by Duesenberry, Eckstein, and Fromm [13, p. 798] is a complicated equation containing a number of lagged variables. But their empirical results might be the consequence of a much simpler structure of the form

$$I_t = \alpha + \beta_1 X_t + \beta_2 I_{t-1} + \beta_3 U_{t-1} + \epsilon_t \, ,$$

for simple calculations yield for arbitrary α

$$I_t = (1+\lambda) \alpha + (1-\lambda b_1) X_t + \lambda \beta_1 \Delta S_t + [\beta_2 (1-\lambda) - \lambda] I_{t-1} + \lambda \beta_2 \Delta I_{t-1} + \beta_3 (1-\lambda) U_{t-1}$$

$$+ \lambda \beta_3 \Delta U_{-1} + \epsilon_t - \lambda \epsilon_{t-1} \, .$$

the equation considered by Duesenberry, Eckstein, and Fromm. If the residuals of the first equation are autocorrelated, the second equation will yield a closer fit and perhaps be more satisfactory for prediction purposes; on the other hand, it will

not necessarily give a more accurate representation of the determinants of inventory investment.

12/   Hicks [22] made the one-way accelerator play a prominent role in his model of the trade cycle. Leontief [29] also employed the construct in his generalization of the Hawkins multi-sector dynamic input-output model.

13/   In an interesting review by Charles Holt and Modigliani [24] of the contribution that the Carnegie quadratic decision rule approach can make to our understanding of inventory investment, the relationships between several alternative cost structures and the implied decision rule are considered. It is pointed out that a flexible accelerator type of behavior may be derived from the assumption that marginal costs are an increasing function of output.

14/   Martin Beckmann [4] has worked out the details of this process.

15/   The appropriate F ratio for determining whether the addition of the set of seasonal dummies led to a significant improvement in fit was not made by Johnston. For one model [26, p. 255] 14 seasonal dummies out of 32 computed for eight industries were significant at the five percent level; 8 of these were significant at the .01 level. For another model [p.25], tested on the same data, 12 out of 32 were significant at the .05 level; four of these at the .01 level.

16/   When Edwin Mills ran his earlier tests [37] he analyzed his data in first-differenced form in order to avoid autocorrelated error terms. The production smoothing coefficient inevitably had the wrong sign. In a more recent study [40] based on other data Mills presents the results of regressions on

non-first differenced observations for four separate industries. In these regressions the lagged production terms generally have the right sign and are significant in terms of the customary tests; although the Durbin-Watson statistic indicates positive autocorrelation of residuals in two cases, it must be remembered that while inefficiency rather than bias is implied by autocorrelation, the standard tests of significance are not valid. Lovell [30, pp. 111-117] considered a flexibility of production term in his regressions, where inventories were treated as the dependent variable. The results were disappointing, perhaps because seasonally corrected data had to be employed. Approximately half the time the production smoothing coefficient had the wrong sign; the coefficients were generally small relative to their standard errors.

17/ Furthermore, the anticipated sales variable determining equilibrium inventory, the $\beta_1 \hat{X}_t$ term in [3.1], may most appropriately refer to moderately long term expectations; in contrast, the error of expectations term, $\lambda (\hat{X}_t - X_t)$, involves short term anticipations of sales for the current period. This distinction, emphasized by Holt, Modigliani, Muth, and Simon [23] may be of but secondary importance for econometric studies if firms generally regard sales as having a stable seasonal pattern, so that short and intermediate range expectations are more or less proportional, particularly if seasonal dummy variables, deseasonalized data, or separate regressions for each season are employed.

18/ As with δ , there is some question as to whether λ should be regarded as a parameter of the system unaffected by the magnitude or direction of the forecast error. Under the assumption of profit maximization the answer depends upon the costs involved in adjusting work force and in sufficing with an inventory that

is not at the equilibrium level; only if such costs are symmetric would we expect λ to be independent of the sign of the forecast error. If the costs are proportional to the square of the discrepancy, λ might be independent of the magnitude of the error.

19/   At one extreme, there is the argument of Albert Hart [21] that the expectations data must be reconstituted. On the other hand, Bossons [5] argues on the basis of cross-sectional evidence that expectations are actually regressive, and rightfully so!

20/   When, in the summer of 1958, I originally computed the coefficients of equation (3.2) I was laboring under the assumption that production plans are completely inflexible so that the error of anticipations, $\hat{X}_t - X_t$, should enter the equation with a coefficient of unity. When the regression was run with the coefficient of the prediction error forced equal to unity the fit was grossly unsatisfactory. I was then led, as a result of learning of Albert Hart's [21] attempts to "reconstitute" the basic railroad shippers' forecast data, to the conjecture summarized by equation (3.3). Some time later I was reminded by Arthur Okun of the Modigliani-Sauerlander [41] point that production plans might have an element of flexibility permitting their revision when actual sales proved to be developing in a different direction from that anticipated.

21/   The new Office of Business Economics Anticipation Survey [17] should prove particularly useful.

22/   When this equation was fitted to GNP data and to manufacturing and trade figures, an extreme problem of serially correlated disturbances was encountered.

Although Enthoven ingeniously applied a correction procedure of H. Wold [51] in order to test the significance of the marginal desired inventory coefficient, the fact remains that the highly autocorrelated disturbances imply that the lag of inventory investment behind changes in sales is not adequately explained by the assumption of naive anticipations. A delayed pattern of response rather than a reaction coefficient of unity may be more appropriate.

23/ Johnston cites Ferber's study of the Shippers' Forecasts [15] as a partial support for the assumptions he makes concerning the structure of the equation generating expectations; one interesting use of what expectations data is available would be in exploring the most fruitful assumptions to make concerning the structure of expectations for purposes of studying inventory behavior.

24/ Johnston's evidence is not decisive as five out of eight regressions yielded a closer fit as measured by the multiple correlation coefficient function of the form (3.8); this may be seen by comparing tables IV, and V in Johnston's study; on the other hand, the values of the estimated values of the parameters appear somewhat more reasonable with the more complicated regression function. Johnston also makes comparisons in terms of predictive ability, and here again the evidence is not decisive.

25/ It must be noted that this assumption does not involve any particular restriction upon the actual structure by which expectations are generated, equation [3.5]. It does imply, however, certain similarities between the structure generating actual sales volume and the way in which expectations are formed.

26/ If $\sigma_{\epsilon_1} = 0$, it might be appropriate to utilize X_t as the dependent variable rather than I_t , and then translate the equation back into the form of (3.10).

27/   If $X_{t-1} = 10$ and $X_t = 15$, then $\rho = -1/2$ implies $\mathcal{E}\left(\hat{X}_t\right) = 17.5$, $\rho = 0$ yields 15, $\rho = 1/2$ yields 12.5 , $\rho = 1$ yields 10, and $\rho = 2$ yields 5 .

28/   Lovell also reports regressions obtained over the same period with inventory data for a number of durable goods industries; purchased materials and goods in process inventories are not published separately from finished goods inventory with the industry breakdown. The "effective bias" coefficients range from a low of .0283 for transportation equipment to a high of 0.2114 for the stone, clay and glass industry.

29/   For a detailed discussion of the sampling procedures and other aspects of the survey see [15].

30/   Thor Hultgren, [25], pp. 364-371, 374-378.

31/   This is the essential difference between the conversion procedure proposed here and that utilized by Modigliani and Sauerland in a study of the value of the Shippers' Forecasts in the prediction of output in the cment industry. In their study, which covered only the output of firms in the first two quarters of each year, they converted carload into barrel figures by first assuming that firms correctly estimated the number of barrels of cement loaded into a freight car for the particular quarter, then deciding this was unrealistic and in effect averaged the figure given by the above formula with the one obtained by their original assumption. They did not discuss the effects of this procedure upon the accuracy of the anticipatory series. If it is assumed that entrepreneurs derive the carload anticipations by correctly forecasting the ratio of barrels to freight cars, the carload and sales anticipations will be of equal accuracy measured in terms of the variance of the percentage error in the forecast. [Cf. 41, p. 335].

32/   The same results are apparent when the accuracy of the forecasts
is measured in terms of variance of the percentage error.

33/   The prime difficulty arises from classification complications. Except
for the cement industry, the commodity classifications utilized in the preparation
of the railroad shippers' forecasts had to be reconciled with the grosser categories
of sales data published in the Survey of Current Business. This was accomplished
by constructing indicies, combining the various categories of the rail data with
weights in proportion to the value of sales; the ratio of the current figure in the
expected shipments index to the value of the index of weighted actual shipments in
the corresponding quarter of the preceding year was utilized as the estimate of antic-
ipated change in sales in accordance with equation (3.16). Needless to say, the
usual problems encountered in index number construction are involved.

34/   Although Modigliani and Sauerlander [41] observed that the Railroad
forecasts assisted in predicting cement inventories, they failed to consider possible
surrogative measures of anticipations as alternatives. For a more elementary
model with output as the dependent variable, Mills found that under the assumption
of production inflexibility current sales provided a much better fit than the
shippers' forecasts. [40   , p. 12a].

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