2-1-1959

A Capital-Intensive Approach to the Small Sample Properties of Various Simultaneous Equation Estimators

Robert Summers

COWLES FOUNDATION DISCUSSION PAPER NO. 64

A Capital-Intensive Approach to the Small Sample Properties of Various Simultaneous Equation Estimators

February 2, 1959

Robert Summers

A Capital-Intensive Approach to the Small Sample Properties of Various Simultaneous Equation Estimators*,**

Robert Summers

A. The Problem

1. Haavelmo's celebrated article in 1943 on the statistical implications of an economic model consisting of a set of simultaneous equations led to an almost exhaustive investigation of these implications in the following decade [8], [9]. Relatively recently, important new contributions have been made [11], [1]. Quite naturally, most of the interest so far demonstrated in simultaneous equation estimation has been centered on the sampling statistic aspects of the problem rather than on empirical applications.*** Though a number

*** The most obvious exceptions, perhaps, are [5], [6], and [4].

of consistent estimation procedures have been made available to empirical workers, without exception the computation required for carrying them out up to now has been extremely burdensome, and the difficulty of just understanding them has been enough to discourage their use. The question of which of the proposed methods is "best" has been decided virtually solely on the basis of computational rather than sampling considerations. Since the electronic revolution of the middle 1950's will surely make the computation problem almost

* Prepared for presentation at a session of the Econometric Society meetings in Chicago, Illinois, on December 28, 1958.

** The author finds it a pleasure to acknowledge the help he received in prosecuting the research reported on in this paper. First, and foremost, the National Science Foundation made the project possible by a generous grant of its funds. At commercial rates, IBM 704 computing time would have swallowed the lion's share of the NSF grant however, so without the grant of machine time by the Massachusetts Institute of Technology Computation Center, the scope of the project would have had to be substantially curtailed. The staff of the Center is particularly to be commended for its helpfulness and patient tolerance during the long period of devising and debugging the IBM 704 program developed for this project. Kazuo Sato provided invaluable research assistance that could be a model for graduate students everywhere; his work convinces me that constructing a large special-purpose program is not best done by a professional programmer who lacks a deep understanding of the applied problem.

a push-button matter, it is time to return to some of the unresolved sampling problems. This paper is a progress report on a project in which the small-sample properties of various alternative estimators are being explored.

2. The estimators to be considered in this paper are: (1) Full Information Maximum Likelihood (FIML), (2) Ordinary Least Squares (OLS), (3) Limited Information, Single Equation (LISE), and (4) Two-stage Least Squares (TSLS). It may be worthwhile to review briefly what is known about these estimators. The Method of Maximum Likelihood is, of course, the standard estimating technique of statisticians. Both FIML and LISE are maximum likelihood methods and so they have the usual optimal asymptotic properties of maximum likelihood estimators. In FIML, all parameters of the simultaneous equation system are estimated at the same time, and the likelihood maximization process takes into account all of the a priori information available. LISE is applied one equation at a time, and in estimating the parameters of a particular equation, only the a priori information relating to that equation is utilized. Both estimators are biased for small samples but have the important property of consistency. FIML is extremely difficult to apply computationally, and in fact it has not yet been used for a system consisting of more than three equations. Of the four methods here considered, LISE is the second most difficult to apply. OLS is the most straightforward procedure, but in most sophisticated econometric circles it is in disrepute because it produces biased estimates even for large samples.* The essential point of Haavelmo's article was a demonstration of

* But see [2] and [3].

this fact. TSLS is the newest of the methods. The rationale behind it is as follows: The bias of OLS is a consequence of the fact that, quite arbitrarily, one or more jointly determined variables is treated in each of the individual equations as predetermined. The lack of independence between these variables and the structural disturbances leads to the OLS bias. In the first stage of TSLS, the "contamination" of these improperly specified predetermined variables is eliminated, at least partially, by estimating the reduced form disturbances of the equations containing the variables and then subtracting the disturbances from the corresponding variables. These new "purified" jointly dependent variables are then used in the OLS manner as though they were really predetermined variables. It has been shown that TSLS yields consistent estimates and that indeed the estimates are asymptotically as efficient as LISE estimates.

3. If, indeed, computer improvements and increased availability of machine time are likely to make the computation burden trivial in the near future, it might be conjectured that soon FIML will become the sole estimating procedure. In the case of investigations where firmly held convictions leave no doubt about the *a priori* restrictions, perhaps FIML will, in fact, automatically be the procedure of choice. It must be remembered, however, that rounding errors in the course of calculation build up more rapidly in FIML than in the single-equation methods, so the limited accuracy of economic data may impose a restraint on how large a system can be handled with FIML.

Ordinarily, economic theory serves as a guide in selecting variables to consider in a simultaneous equation model. Rarely is the guidance so specific, however, that it can be translated directly into *a priori* restrictions without empirical testing. This searching (or less euphemistically, fishing) is better carried on by means of single equation methods because they allow a very weak specification of the model as a whole while individual equations are being examined. Furthermore, the remark above that we can look forward to the end of the computation burden even in FIML was not meant to imply that we can expect to dismiss lightly the burden of using FIML to try dozens of possible formulations of, say, a 15 equation system.

For each of these reasons then (the limitation on how large a system can be handled with FIML; and the problem of finding *a priori* restrictions) an interest in LISE, TSLS, and OLS is not merely academic.

B. The Approach

4. An analytical derivation of the sampling properties of a statistic is usually preferred to a derivation synthesized in a sampling experiment. In part, this is because an analytical derivation has a generality about it that is almost impossible to attain with a sampling experiment. Fully as important, however, is a non-rational reason. We are always attracted by the aesthetic appeal of the triumph of the unaided intellect. From a pragmatic point of view, however, in terms of just getting the job done, this latter reason must be given no weight when one has to decide on how to attack a sampling distribution problem. Should a computer be used? Elementary production theory will be drawn upon to illustrate the decision process. Consider a production function in which there are two factors of production, capital and labor. Output in this technological process is statistical knowledge, and the isoquants of the production contour map have the usual convexity properties.

Diagram 1 depicts the familiar graph. The horizontal axis is calibrated in units of labor inputs, and the vertical axis in units of capital services. The well-known slogan of our most prominent computer manufacturer pairs nicely with the relevant aspect of labor input to provide labels for the axes. If we ignore the capital services of paper and pencils (and erasers!) the isoquants representing elementary knowledge will cut the horizontal axis; on the

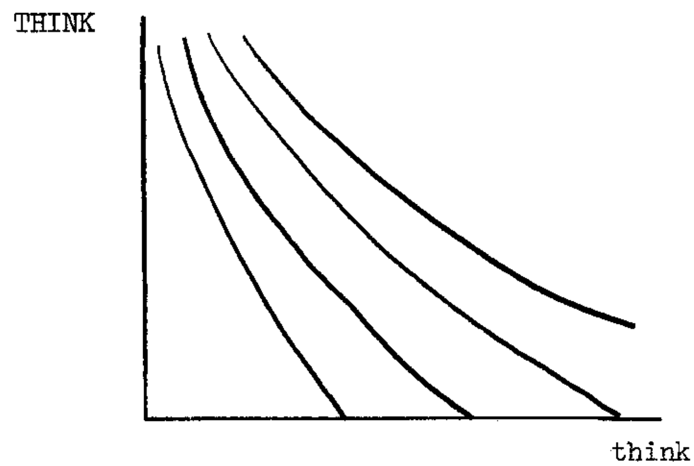

Diagram 1

other hand, in intractable problems, the isoquants are likely to become asymptotic to the axis. It is surely true, however, that even in the most trivial problem, the isoquants are always asymptotic to the vertical axis -- even apart from the overhead thought necessary to make possible the THINKing. It should be clear that the relative prices of computer services and brain-power ought to determine the proper input mix. Without doubt, Anderson and Rubin could find the small sample properties of their LISE if they were willing to devote sufficient time to the project. Rumor has it that a member of Professor Theil's organization in the Netherlands already has worked out some aspects of the small-sample properties of TSLS (and of the more general k-class estimators). Perhaps Chernoff, the Sir Edmund Hillary of the likelihood slopes, could be enticed back to this area long enough to solve the problems of FIND. But for better or worse, Koopmans' team of the late Forties has spread to the winds and is no longer concerned with simultaneous equation estimation. I suggest, however, that since IBM now generally provides university researchers with free, if limited quantities of machine time on some of its larger computers, the current relative prices (taking into account the costs of travelling to and from the computation centers) favor the use of a capital intensive approach to the small-sample properties of simultaneous equation estimators.

C. The Model

5. With the exception of the discussion of Section 14, the model to be considered in this report is given by Equations (1) and (2).

$$(1) \quad y_{1t} + \beta_{12} y_{2t} + \gamma_{11} z_{1t} + \gamma_{12} z_{2t} + \gamma_{10} = u_{1t}$$

$$(2) \quad y_{1t} + \beta_{22} y_{2t} + \gamma_{23} z_{3t} + \gamma_{24} z_{4t} + \gamma_{20} = u_{2t}$$

The y's are jointly determined variables while the z's are predetermined. The u's are bivariate normal variables with zero means and a variance covariance matrix denoted by Σ_u. This particular model was selected because it was the simplest one conceivable which embodied the essential character of a simultaneous system, and for which the differences between the estimating procedures were interesting. It is easy to confirm that each of the equations is overidentified. The reduced-form of the model is given by Equations (3) and (4).

$$(3) \quad y_{1t} = \pi_{11} z_{1t} + \pi_{12} z_{2t} + \pi_{13} z_{3t} + \pi_{14} z_{4t} + \pi_{10} + v_{1t}$$

$$(4) \quad y_{2t} = \pi_{21} z_{1t} + \pi_{22} z_{2t} + \pi_{23} z_{3t} + \pi_{24} z_{4t} + \pi_{20} + v_{2t}$$

The π's are functions of the β's and γ's and the v's are linear combinations of the u's. Of crucial importance is the fact that the absence of z_3 and z_4 from Equation (1) and the absence of z_1 and z_2 from Equation (2) imply certain interdependencies among the π's in Equations (3) and (4).

D. The Sampling Experiment

6. Only since the advent of the high-speed computer has it been feasible to investigate in detail the small-sample properties of simultaneous equation estimators numerically by performing sampling experiments [13], [7]. The sampling experiment approach, known as distribution sampling, is a variation of the so-called Monte Carlo technique.*

* An eminent practitioner of the art of Monte Carlo suggests, however, that straight distribution sampling without sampling tricks should not be dignified by that name [12].

The following sampling procedure was used in obtaining the empirical results given in Sections 8-14 of this paper.

(a) Equations (1) and (2) were used as the basic model of a hypothetical economy. In any particular experiment, the values of all of the parameters of the model, both the regression coefficients and the variance-covariance matrix of structural disturbances, were specified. In addition, the values taken on by all of the predetermined variables were specified for each of the T observation periods considered.

(b) Using a random sampling method,* T observations were generated on

* A random normal deviate generator [10] was used to produce structural disturbances which were then transformed into reduced-form disturbances. (In the actual computing, these two processes were collapsed into one step.)

both y_1 and y_2, conditioned by the set of values of the predetermined variables. The T sets of observations on y_1, y_2, z_1, z_2, z_3, z_4 constituted a sample.

(c) Each of the four estimating methods was then applied to the sample to estimate the parameters, (i.e., the β's, the γ's, and the σ_{ij}'s) of the system.

(d) Steps (b) and (c) were then repeated N times to get N different estimates of each of the parameters by each estimating method.

(e) The N different estimates of each parameter obtained for each estimating method were then organized into a relative frequency distribution and summary measures were computed for each distribution.

(f) Then a new specification of the values of the parameters was made and Steps (b), (c), (d), and (e) were repeated. A variety of specifications was made in order to observe the sensitivity of the relative frequency distributions to the specification of the characteristics of the model.

No discussion will be given here of the optimum strategy that should be followed in selecting parameter combinations to be studied. It is sufficient to say that some trimming down of the dimensions of the problem must be resorted to. Even the simplest possible grid, where each parameter can assume either of two values, would require over two thousand runs, even if one were content to hold unchanged the values taken on by the z's. When the production runs reported on here were set up, the choices of model specifications were quite arbitrary.

In addition to a sensitivity analysis, an analysis of the consequences of mis-specification is contemplated for the future. The first step in this direction has been taken already, and the results are described in Section 14.

7. A limited effort has been made to explore the small-sample properties of estimates of reduced-form coefficients based upon the four estimating methods. Least squares estimates of the reduced-form coefficients taking into account none of the implications of the over-identifying restrictions were also computed. Such estimates are identified by the abbreviation, LSNR. The results of this investigation of reduced form estimation are given in Section 13.

E. Empirical Results

8. The same set of z's were used in each of the sampling experiments. Diagrams 2 and 3 depict the values of $(z_{it}-\bar{z}_i)$ that were used. The values are index numbers, deflated by population size, selected from the data presented in Economic Fluctuations in the United States, 1921-1941 [5], Model III. The variables were: z_1 , labor income originating in government; z_2, residential construction (non-farm); z_3, stock of non-farm housing; and z_4, excise taxes. The deflation was a halfhearted effort to remove the trend from the series, and it was hardly successful. Since economic data have this trend character, perhaps it is as well to leave some trend in.

Table 1 lists the parameter combinations which are reported on here. In all but one experiment, the small-sample properties considered were for samples of size 20. For Experiment 1, a specification of the β's and γ's was made which made the equation system resemble a supply and demand model. In this case, 100 samples were drawn. Experiment 2 differs from Experiment 1 only in that the variances and covariance of the structural disturbances were quadrupled. In Experiment 2 and all subsequent experiments, only 50 samples were drawn. Experiment 3 was the only one in which the basic sample size was 40; in all other respects, Experiment 3 was the same as Experiment 2. In Experiment 4, β_{12} was changed slightly and the signs of the γ's were reversed; apart from these changes, Experiment 4 was just like Experiment 2. Experiment 5 was an initial effort to see how much difference specification error makes in the use of FIML. The data to which the estimating methods were applied were ordinarily generated by a process which corresponded exactly to the model given in Equations (1) and (2). For Experiment 5, however, the data were actually generated by Equation (1) and Equation (2'):

$$(2') \qquad y_1 + .4\, y_2 - .5\, z_1 + .6\, z_3 - .4\, z_4 - 149.6 = u_2$$

The parameters were estimated, however, on the assumption that γ_{21} (i.e., -.5 in Equation 2') was really equal to zero.

Diagram 4 is designed to convey some notion of the portion of the variance of the jointly determined variables which is explained by the predetermined variables. y_{it}^c is the non-stochastic part of y_{it} . In generating values of y_{it} , reduced form disturbances are added to y_{it}^c . The graphs of $(y_{it}^c - \bar{y}_i)$ are given for the parameters of Experiment 1. For different sets of parameters, the graphs would be different, of course.

9. Tables 2, 3, 3A, 4, 5, 5A, 6, and 6A give the empirical results of the individual experiments. The meaning of the labels on the tables is self-evident. Corresponding to each estimating method and each parameter estimated, there is a cell containing three entries. The upper one is the observed mean of $(\tilde{\alpha}-\alpha)$, where α is the true value of the parameter and $\tilde{\alpha}$ is the estimate

of the parameter produced by the estimating method. The entry in the box on the right is the root-mean-square-error of $\tilde{\alpha}$. The entry in the lower left corner is the standard deviation of the observed $\tilde{\alpha}$'s. If the upper left-hand corner of a cell is shaded, then the observed mean for that cell is more than 1.96 standard errors (as estimated from the empirical sampling distribution) from zero. This means that the estimating procedure associated with the cell has been revealed to be biased. Of course, all of the methods are in fact biased, so the shaded corners only should be interpreted as signals that the associated estimating method is likely to be particularly bad.

For reasons not important to this discussion, FIML was not used in Experiment 4. Otherwise, all estimating methods were used in all experiments. All of the structural coefficients of Equations (1) and (2) were estimated in each experiment. Only the coefficients of the first reduced-form equation, Equation (3), were estimated.

10. It will be noted that one table each is presented for Experiments 1 and 3. Two tables each are presented for Experiments 2, 4, and 5. A close examination of the results of the estimates derived from individual samples revealed that in Experiments 2 and 5, there were three very "bad" (i.e., outlandish) samples, and in Experiment 4 there were two. These samples produced outliers far out in the tails of the empirical distributions of either LISE or TSLS. The same three samples were bad in Experiments 2 and 5, and additional investigation confirmed that the random normal deviate generating procedure was indeed producing sufficiently "unusual" normal deviates in those samples to give estimates for either LISE or TSLS or both which deviated wildly from

the patterns exhibited by the other samples. The random normal deviate generator had been checked in some detail, so it is not clear why these samples should have been so peculiar. It may be conjectured that the generator gives "patchy" numbers. The various tests administered to the generator showed, among other things, that the sequence it produced had serial correlation coefficients -- of order zero, one, two, three, and four -- all equal virtually to zero for the first 8000 numbers in the sequence. This does not guarantee, however, that in some sub-sequences of forty numbers there is not a non-random pattern. The results of the experiments on the basis of all 50 samples are presented in Tables 3, 5, and 6. The results on the basis of all the samples but the ones producing the outliers are given in Tables 3A, 5A, and 6A. In the case of Experiments 2 and 5, this second tabulation is not available for the reduced-form estimates. It is felt that the data in which the outliers are excluded are more representative than the data including the outliers.

11. It should be remembered that the five experiments described here are the forerunners of many more. Furthermore, it is expected that by the use of appropriate variance-reducing techniques, it will be possible to get the equivalent of results of even more experiments by appropriately combining the results of ones already done. At this fumbling, exploratory stage, great precision and sophistication in the analysis of the tables would be misplaced effort. In any tests of significance one might want to perform to evaluate the relative performances of the different estimators, the use of the covariances -- between different methods -- of the estimates of any particular

parameter would be essential. These covariances are not zero because the same random normal deviates were used when the various estimating methods were used. The data necessary for computing these covariances were obtained, but the covariances are not yet available.

The conclusions arrived at about the different methods are based upon a casual comparison of root-mean-square-errors. No attempt will be made here to justify the use of the quadratic loss function which underlies the root-mean-square-error criterion.

Tables 7 and 8 summarize the various two-way comparisons that can be made between the different estimating methods. The entries in the tables give the frequencies of comparisons in which the outcome was as indicated by the column heading.

Table 7 consolidates the data on structural parameter estimation. Comparisons of root-mean-square-errors based upon all the observations, including outliers, of each experiment are given first. Then, the same comparisons are made again, but now based upon data which exclude the outliers.

Table 8 presents the results of similar comparisons made for estimates of reduced form coefficients. Experiments 2, 4, and 5 had outliers, but the reduced form root-mean-square-errors of only Experiment 4 were recomputed for the data after dropping the outliers.

These tables are based upon ordinal comparisons. No attempt was made to weight these comparisons by cardinal differences. Casual inspection of the differences does not suggest this would change the qualitative character of the results given below. It should be remembered that the fact that the parameters used in the different experiments were similar rules out the possibility of using a non-parometric significance test to see if the well-defined patterns of numbers in Tables 7 and 8 could have been formed by chance.

As indicated in Section 10 above, it is felt that the data which do not include the outliers provide more reliable guides to the underlying relationships between the various estimating methods than the data including the outliers. Therefore, it is felt that in principle the sections of Tables 7 and 8 relating to such data are the ones most relevant to an appraisal of the methods. Actually, in the case of estimates of structural coefficients, dropping the outliers made very little difference in the ordinal comparisons, though the absolute differences in root-mean-square-errors changed substantially. One is wise to exercise great restraint in tampering with data before performing tests of significance. The "tests" performed here are informal, judgment ones, of course, with no statistical foundation, but the restraint is still appropriate. For this reason, the results of the comparisons are displayed for the data before the outliers were dropped and the data after.

Unfortunately, the case for judging the abilities of the different methods to estimate reduced form coefficients on the basis of the outlier-free data is weakened by the fact that the results of some experiments are not available for that data. The differences between the left and right sections of Table 8 arose either from the dropping of outliers or the dropping of experiments. Examination of the comparisons underlying Table 8 reveal that only the OLS-TSLS comparisons were affected by dropping outliers, and even there the difference was slight. As a consequence, in Table 8, the section to consider in appraising the different methods is the one based upon data including the outliers, the one on the left.

From the standpoint of ordinal comparisons of root-mean-square errors then, including or excluding the outliers seems to make almost no difference. This would not be the case for cardinal comparisons.

12. The outstanding conclusions about structural parameter estimation, based upon Tables 2 to 7, are:

(1) OLS clearly does poorly relative to the other estimating methods. The minimum variance feature of OLS seems to be preserved for small samples, but the substantial discrepancies between the mean estimates and the parameters estimated make the root-mean-square-errors very large. In most respects the models presented here are all the same, so a conservative person may still think it is premature to make the categorical statement that OLS is an unacceptable estimating method.

(2) FIML performs better than any of the other estimating methods.

(3) TSLS is distinctly better than LISE. Except in the case of Experiment 2 when the bad samples are counted, the root-mean-square-errors of TSLS are never larger than those of LISE and are usually smaller. The fact that in Tables 2 to 6A there are more shaded corners in TSLS cells than in LISE ones is of no significance.

(4) The absolute biases of FIML, LISE, and TSLS all tend to be quite small, usually under ten per cent. (Inexplicably, when the bad samples are dropped from Experiment 4, the biases go up and are substantial.)

Findings (1), (2), and (4) were to be expected. Finding (3) probably could not have been predicted (except, perhaps, by interested parties!).

13. The outstanding conclusions about reduced form parameter estimation is the following:

(1) FIML is distinctly better than any of the other estimating methods.

(2) LSNR, ordinary least squares applied to the reduced form, ignoring the over-identifying restrictions of the structural equations, is distinctly worse than the others.

(3) TSLS is slightly superior to LISE and LISE is slightly superior to OLS. The differences are quite small, however, so this ranking is a very tentative judgment.

The first two findings were not unexpected. It is surprising, though, that OLS did so well relative to LISE and TSLS.

14. In Experiment 5, data on the y's were generated from a model other than that assumed for estimation purposes. The specification error was designed to throw FIML "off the scent" in estimating the parameters of the first structural equation. In fact, it did not actually do so: FIML remained the best of the estimating methods, despite the specification error. Surprisingly, however, OLS proved better than FIML in estimating reduced form coefficients. TSLS was a poor third, LISE a poor fourth, and as before LSNR was far behind. Further analysis will be necessary to determine what the "force" is of the specification error used (i.e., how much real influence inclusion of z_1 in Equation (2) had).

F. Conclusion

15. The conclusion of this paper will be a mild disclaimer. This paper is only a progress report, and the conclusions stated here should be regarded as extremely tentative. They are surely a useful basis for further investigation, but five experiments which are mutually interdependent by no means prove a case. A variety of different parameter combinations will have to be tried before a final verdict can be reached. It is hoped that by means of regression analysis, possibly cast in a response surface format, some definitive statement may be made about the relative merits of the different estimating methods considered.

REFERENCES

[1] Basman, R. L., "A Generalized Classical Method of Linear Estimation of Coefficients in a Structural Equation," Econometrica, Vol. 25 (1957), pp. 77-83.

[2] Christ, C., "Aggregate Econometric Models," American Economic Review, Vol. 46(1956), p. 400.

[3] Fox, K. A., "Econometric Models of the United States," Journal of Political Economy, Vol. 64(1956), pp. 130-132.

[4] Hildreth, C. and Jarrett, F. G., A Statistical Study of Livestock Production and Marketing, Cowles Commission Monograph No. 15 (New York: John Wiley, 1955).

[5] Klein, L. R., Economic Fluctuations in the United States 1921-1941, Cowles Commission Monograph No. 11 (New York: John Wiley, 1950).

[6] __________ and Goldberger, A. S., An Econometric Model of the United States, 1929-1952 (Amsterdam: North-Holland, 1955)

[7] Ladd, G. W., "Effects of Shocks and Errors in Estimation: An Empirical Comparison," Journal of Farm Economics, Vol. 38 (1956), pp. 485-495.

[8] Statistical Inference in Dynamic Economic Models, Cowles Commission Monograph No. 10, T. C. Koopmans, ed. (New York: John Wiley, 1950)

[9] Studies in Econometric Method, Cowles Commission Monograph No. 14, W. C. Hood and T. C. Koopmans, ed. (New York: John Wiley, 1953).

[10] Teichroew, D. "Distribution Sampling with High Speed Computers" unpublished Ph.D. dissertation, North Carolina State College, 1953, Chapters 3 and 5.

[11] Theil, H., Economic Forecasts and Policy (Amsterdam: North-Holland, 1958).

[12] Trotter, H. F. and Tukey, J. W., "Conditional Monte Carlo for Normal Samples" in Symposium on Monte Carlo Methods, H. A. Meyer, ed. (New York: John Wiley, 1956), p. 64.

[13] Wagner, H. M., "A Monte Carlo Study of Estimates of Simultaneous Linear Structural Equations," Econometrica, Vol. 26(1958), pp. 117-133.

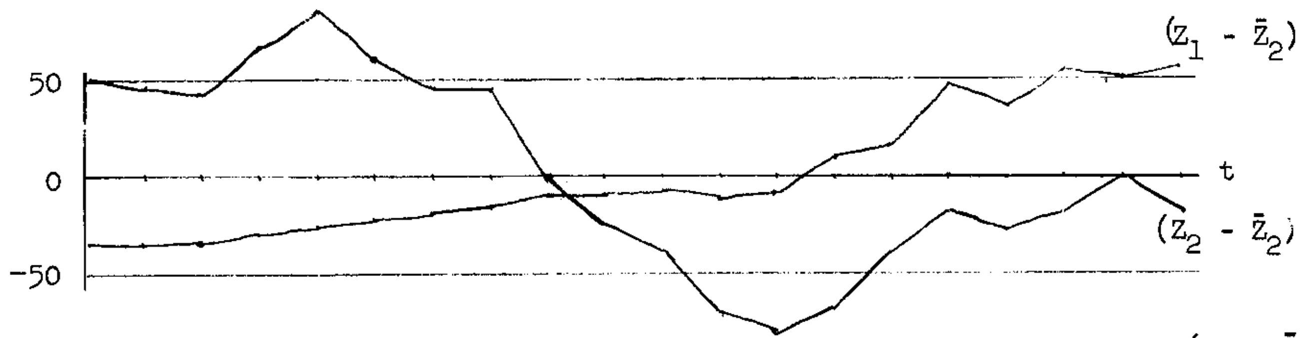

Diagram 2

Timepath of the variables $(Z_1-\bar{Z}_1)$ and $(Z_2-\bar{Z}_2)$

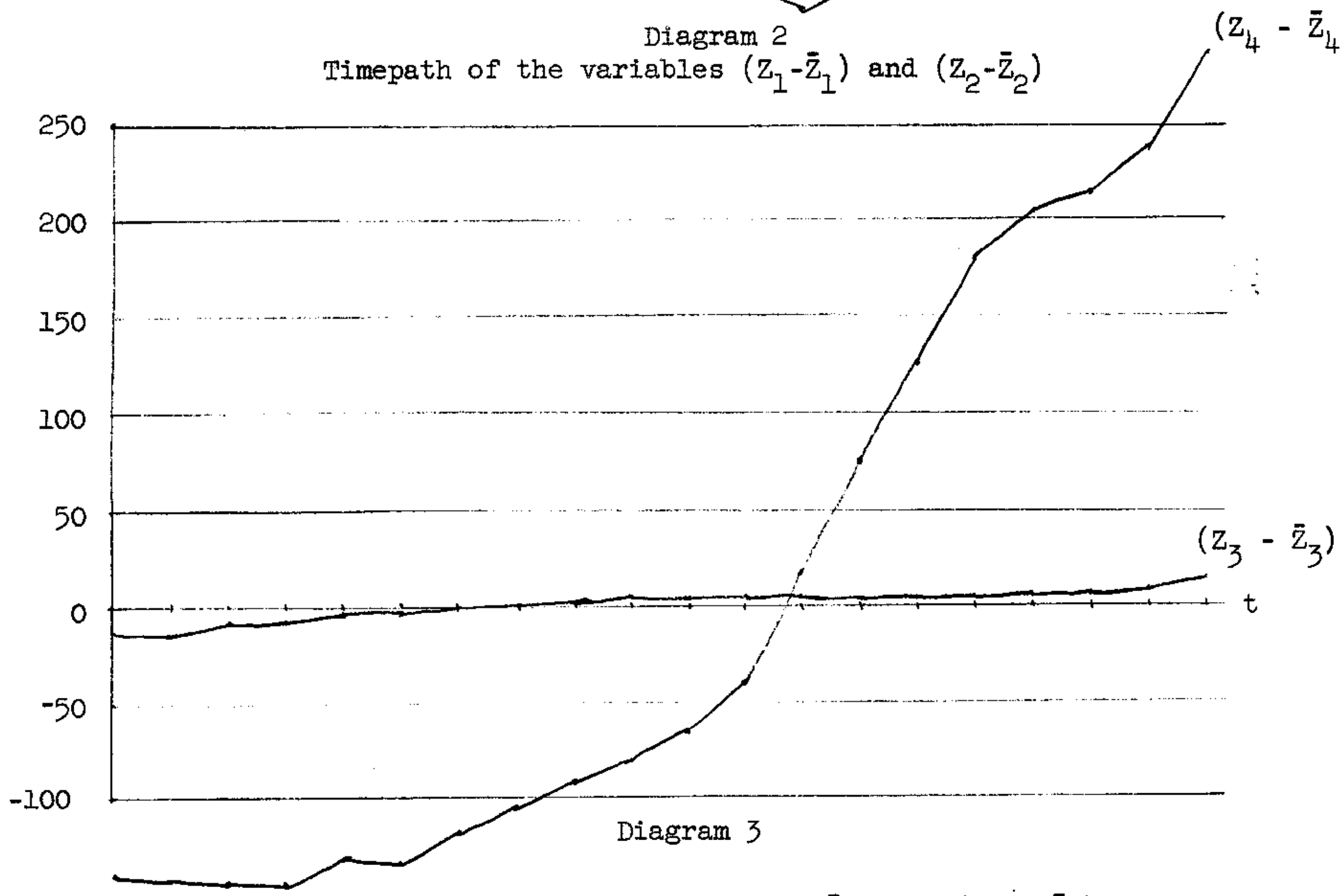

Diagram 3

Timepath of the variables $(Z_3 - \bar{Z}_3)$ and $(Z_4 - \bar{Z}_4)$

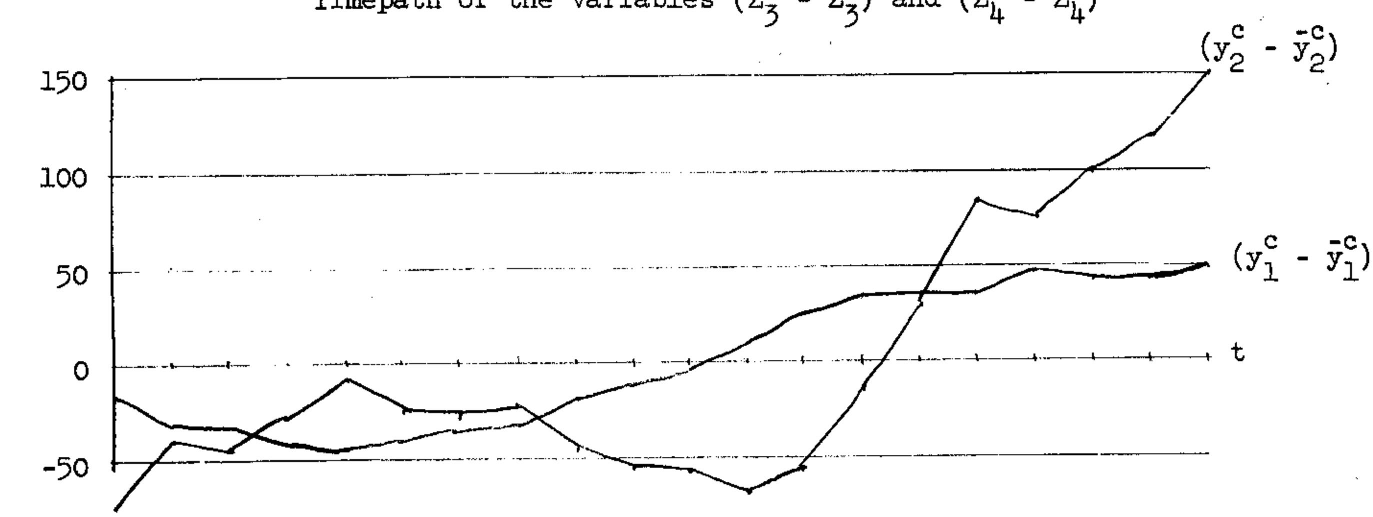

Diagram 4

Timepath of the variables $(y_1^c - \bar{y}_1)$ and $(y_2^c - \bar{y}_2)$ for Experiment 1

TABLE 1

PARAMETER COMBINATIONS USED IN SAMPLING EXPERIMENTS

	β_{12}	γ_{11}	γ_{12}	γ_{10}	β_{22}	γ_{23}	γ_{24}	γ_{20}	σ_{11}	σ_{12}	σ_{22}	T	N
EXPERIMENT 1	- .7	.8	.7	-149.5	.4	.6	-.4	-149.6	100.	50.	100.	20	100
EXPERIMENT 2	- .7	.8	.7	-149.5	.4	.6	-.4	-149.6	400.	200.	400.	20	50
EXPERIMENT 3	- .7	.8	.7	-149.5	.4	.6	-.4	-149.6	400.	200.	400.	40	50
EXPERIMENT 4	-1.3	-.8	-.7	+149.5	.4	-.6	+.4	+149.6	400.	200.	400.	20	50
EXPERIMENT 5*	- .7	.8	.7	-149.5	.4	.6	-.4	-149.6	400.	200.	400.	20	50

T: Number of observations in each sample in the experiment

N: Number of observations in the sampling experiment

* In EXPERIMENT 5, $\gamma_{21} = -.5$ instead of zero as assumed in all of the other experiments.

TABLE 2

EXPERIMENT 1

Structural Coefficients

PARAMETERS	β_{12}	γ_{11}	γ_{12}	γ_{10}	β_{22}	γ_{23}	γ_{24}	γ_{20}
TRUE VALUES	-.7	+.8	+.7	-149.50	.4	+.6	-.4	-149.6
LISE	.0013 .208 .208	-.0010 .480 .480	-.0031 .123 .123	-.0288 38. 38.	.0053 .078 .078	-.0839 .617 .622	.0016 .046 .046	7.464 66. 66.
TSLS	.0268 .200 .200	-.0597 .460 .462	-.0172 .118 .118	4.458 36. 36.	.0025 .078 .078	-.0902 .612 .619	.0030 .045 .046	8.389 65. 66.
OLS	.2080 .148 .255	-.4742 .350 .589	-.118 .089 .152	35.902 28 46	.0407 .073 .083	-.1840 .598 .624	.0242 .042 .049	22.44 63. 67.
FIML	-.0049 .187 .187	-.0140 .428 .428	-.0002 .110 .110	-1.186 34 34	.0043 .078 .078	-.0685 .569 .572	.0014 .045 .045	5.938 61. 61.

Reduced Form Coefficients

PARAMETERS	π_{11}	π_{12}	π_{13}	π_{14}	π_{10}
TRUE VALUES	-.291	-.254	-.382	+.255	149.56
LISE	.020 .134 .134	-.002 .043 .043	+.052 .408 .410	-.0072 .037 .038	-6.14 49. 49.
TSLS	.091 .138 .165	-.007 .043 .043	+.046 .405 .407	-.0099 .038 .039	-5.74 49. 49.
OLS	.129 .179 .222	-.071 .039 .081	-.060 .336 .342	-.0357 .045 .057	+6.77 46. 47.
IML	.012 .114 .114	-.001 .043 .043	+.0043 .376 .376	-.0050 .033 .033	-4.63 45. 45.
LSNR	.028 .380 .381	-.002 .051 .051	+.058 .672 .675	-.0089 .069 .070	-7.43 61. 61.

T = 20
N = 100

TABLE 3

EXPERIMENT 2

Structural Coefficients

PARAMETERS	β_{12}	γ_{11}	γ_{12}	γ_{10}	β_{22}	γ_{23}	γ_{24}	γ_{20}
TRUE VALUES	-.7	+.8	+.7	-149.50	.4	.6	-.4	-149.6
LISE	.0177 .723 .723	-.1109 1.635 1.638	-.0515 .402 .405	18. 122. 123.	-.0180 .169 .169	-.0689 1.332 1.334	.0097 .107 .107	8. 145. 145.
TSLS	-.1113 1.383 1.387	.1706 2.872 2.877	.0054 .645 .645	0. 178. 178.	-.0323 .156 .159	-.1056 1.272 1.277	.0171 .097 .098	13. 138. 138.
OLS	.4057 .223 .463	-.9589 .537 1.099	-.2584 .152 .300	78. 46. 91.	-.1662 .131 .211	-.3741 1.182 1.239	.0815 .0816 .116	55. 127. 138.
LIML	-.0870 .424 .433	.1608 .942 .955	.0090 .245 .245	-5. 72. 72.	-.0214 .165 .168	-.1188 1.211 1.217	.0131 .100 .101	13. 133. 134.

Reduced Form Coefficients

PARAMETERS	π_{11}	π_{12}	π_{13}	π_{14}	π_{10}
TRUE VALUES	-.29091	-.25455	-.38182	+.25455	+149.56363
LISE	-.2075 .765 .813	.0110 .087 .087	-.0018 .926 .926	-.0405 .167 .172	-18. 119. 120.
TSLS	.1600 .462 .489	.0015 .080 .080	.0148 .875 .875	-.0332 .110 .115	-14. 111. 112.
OLS	.4640 .482 .669	-.1168 .106 .158	-.0862 .521 .528	-.1111 .102 .151	-11. 92. 98.
FIML	.03928 .236 .240	.0245 .084 .087	.0729 .841 .844	-.0075 .068 .068	-14. 102. 103.
LSNR	.1448 .745 .759	.0149 .088 .089	-.0589 1.164 1.164	-.0248 .146 .148	-8. 118. 119.

T = 20

N = 50

Table 3A

EXPERIMENT 2, without outliers

Structural Coefficients

PARAMETERS	β_{12}		γ_{11}		γ_{12}		γ_{10}		β_{22}		γ_{23}		γ_{24}		γ_{20}	
TRUE VALUES	-.7		.8		.7		-149.5		.4		.6		-.4		-149.6	
LISE	.0405		-.1249		-.0556		15.52		-.0170		-.0689		.0073		3.99	
	.432	.434	.983	.991	.261	.267	77.3	78.9	.170	.171	1.32	1.32	.110	.110	145.	145.
TSLS	.0958		-.2503		.0856		24.94		-.0311		-.0682		.0148		9.38	
	.398	.409	.910	.944	.241	.256	71.7	76.0	.158	.161	1.25	1.26	.099	.100	137.	137.
OLS	.4076		-.9621		-.2593		78.38		-.1629		-.3683		.0789		52.07	
	.228	.471	.549	1.12	.153	.303	47.0	92.1	.131	.210	1.12	1.22	.084	.116	125.	136.
FIML	-.0314		.0469		-.0169		1.95		-.0201		-.1714		.0108		9.95	
	.360	.362	.839	.840	.226	.227	68.0	68.1	.168	.169	1.22	1.23	.102	.103	132.	133.

T = 20
N = 47

TABLE 4

EXPERIMENT 3

Structural Coefficients

PARAMETERS	β_{12}	γ_{11}	γ_{12}	γ_{10}	β_{22}	γ_{23}	γ_{24}	γ_{20}
TRUE VALUES	-.7	.8	.7	-149.5	.4	.6	-.4	-149.6
LISE	-.0229 .325 .326	.0507 .731 .733	.0048 .185 .185	-3.41 57.2 57.3	.0101 .116 .117	-.1715 .854 .871	.0037 .0657 .0657	15.32 91.1 92.4
TSLS	.0410 .275 .278	-.0956 .622 .630	-.0308 .163 .166	7.74 49.3 50.0	.0055 .117 .117	-.1827 .852 .872	.0060 .0660 .0662	16.93 91.0 92.6
OLS	.4064 .155 .439	-.9269 .352 1.001	-.2277 .107 .254	69.59 30.7 76.0	-.1464 .0936 .175	-.5099 .789 .942	.0802 .0541 .0974	66.17 83.0 106.6
FIML	-.0101 .261 .262	.0239 .578 .578	-.0022 .147 .147	-1.5816 44.5 44.5	.0109 .117 .117	-.1087 .773 .781	.0009 .0653 .0653	8.6876 82.9 83.3

Reduced Form Coefficients

PARAMETERS	π_{11}	π_{12}	π_{13}	π_{14}	π_{10}
TRUE VALUES	-.2909	-.2546	-.3818	.2546	149.56
LISE	.0285 .206 .209	.0021 .0635 .0635	.1037 .570 .579	-.0117 .0526 .0539	-11.48 67.76 68.74
TSLS	.0491 .208 .214	-.0118 .0592 .0604	.0911 .547 .555	-.0170 .0514 .0541	-10.43 65.30 66.15
OLS	.4168 .316 .526	-.1412 .0711 .158	-.0890 .373 .385	-.1065 .0713 .129	-2.69 68.20 68.25
FIML	.0206 .155 .156	-.0016 .0652 .0652	.0761 .501 .507	-.0091 .0415 .0425	-7.98 57.90 58.45
LSNR	.0550 .521 .524	-.0029 .0634 .0634	.1159 .950 .957	-.0179 .0904 .0922	-14.86 83.07 84.41

T = 40

N = 50

TABLE 5

EXPERIMENT 4

Structural Coefficients

PARAMETERS	β_{12}	γ_{11}	γ_{12}	γ_{10}	β_{22}	γ_{23}	γ_{24}	γ_{20}
TRUE VALUES	-1.3	-.8	-.7	+149.5	+.4	-.6	+.4	+149.6
LISE	.0285 3.14 [3.14]	-.0026 5.42 [5.42]	-.0105 1.36 [1.36]	1.989 350. [350.]	.0720 .244 [.254]	-.1591 1.26 [1.27]	.0243 .098 [.101]	20.245 137. [139.]
TSLS	-.1659 1.01 [1.02]	-.2640 1.46 [1.48]	-.0924 .412 [.422]	24.220 113. [116.]	.0583 .243 [.249]	-.1399 1.26 [1.27]	.0200 .976 [.996]	17.365 137. [138.]
OLS	.5666 .322 [.652]	+.8023 .896 [.951]	-.1258 .140 [.178]	-56.173 44. [72.]	-.1656 .210 [.269]	.2148 1.18 [1.28]	-.0523 .087 [.101]	-33.808 128. [132.]
FIML (omitted)								

Reduced Form Coefficients

PARAMETERS	π_{11}	π_{12}	π_{13}	π_{14}	π_{10}
TRUE VALUES	+.18824	+.16471	+.45882	-.30588	-149.576
LISE	.04651 .253 [.258]	.02430 .0908 [.0940]	.09731 1.014 [1.019]	-.00985 .0733 [.0739]	-16.81634 116.2 [117.4]
TSLS	-.08586 .867 [.872]	.04451 .139 [.146]	.33567 1.115 [1.116]	.01012 .152 [.152]	-33.49209 110.8 [115.9]
OLS	-.20126 .517 [.556]	.28932 .143 [.612]	.57274 .750 [.947]	.06289 .115 [.131]	-85.69522 131.3 [159.5]
FIML (omitted)					
LSNR	.13404 .768 [.780]	.01431 .0910 [.0922]	-.03238 1.174 [1.175]	-.02376 .153 [.155]	-9.19033 121.4 [121.7]

T = 20
N = 50

Table 5A

EXPERIMENT 4, without outliers

Structural Coefficients

PARAMETERS	β_{12}	γ_{11}	γ_{12}	γ_{10}	β_{22}	γ_{23}	γ_{24}	γ_{20}
TRUE VALUES	-1.3	-.8	-.7	+149.5	+.4	-.6	+.4	+149.6
LISE	-.3179 1.59 1.62	-.4935 1.42 1.51	-.1471 .409 .435	41.77 112. 119.	.0747 .183 .198	-.2656 1.11 1.14	.0287 .093 .098	31.75 122. 126.
TSLS	-.2287 .978 1.01	-.3623 1.39 1.44	-.1147 .403 .419	31.94 109. 113.	.0623 .188 .198	-.2473 1.10 1.13	.0247 .092 .095	28.96 121. 125.
OLS	.5461 .207 .589	.7652 .485 .913	.1703 .140 .222	-53.14 42.6 68.6	-.1668 .167 .237	.1216 1.04 1.05	.0497 .083 .097	-23.92 44.4 50.6
FIML (omitted)								

Reduced Form Coefficients

PARAMETERS	π_{11}	π_{12}	π_{13}	π_{14}	π_{10}
TRUE VALUES	.1882	.1647	.4588	-.3059	-149.58
LISE	.0359 .252 .255	.0255 .092 .096	.1966 .822 .846	-.0106 .071 .072	-26.73 99.5 103.
TSLS	-.0764 .297 .307	.0451 .141 .148	.2268 .843 .873	-.0106 .078 .079	-35.01 112. 117.
OLS	-.1683 .500 .528	.2940 .148 .331	.6163 .733 .962	.0563 .112 .126	-94.02 127. 159.
FIML (omitted)					
LSNR	.0860 .743 .748	.0098 .081 .091	-.0038 1.14 1.14	-.0366 .138 .143	-16.76 110. 111.

T = 20
N = 48

Table 6

EXPERIMENT 5

Structural Coefficients

PARAMETERS	β_{12}	γ_{11}	γ_{12}	γ_{10}	β_{22}	γ_{23}	γ_{24}	γ_{20}
TRUE VALUES	-.7	.8	.7	-149.5	.4	.6	-.4	-149.6
LISE	.0256 .699 / .699	-.1153 1.27 / 1.28	-.0551 .392 / .396	18.61 120. / 121.	.0518 .197 / .204	.4107 1.46 / 1.51	.0683 .111 / .130	-4.73 157. / 157.
TSLS	.0241 .572 / .572	-.0993 .966 / .971	-.0547 .307 / .312	16.97 87. / 87.	.0247 .166 / .168	.3290 1.29 / 1.33	.0805 .088 / .120	5.32 137. / 138
OLS	.4057 .223 / .559	-.7745 .441 / .898	-.2584 .152 / .302	78.14 46. / 91.	-.1353 .141 / .196	-.0360 1.19 / 1.19	.1445 .075 / .164	53.22 126. / 137.
FIML	-.0097 .380 / .380	-.0367 .698 / .699	-.0558 .258 / .264	12.78 70. / 71.	.5320 1.45 / 1.54	1.5799 4.39 / 4.67	-.1428 .689 / .704	-151.61 524. / 546.

Reduced Form Coefficients

PARAMETERS	π_{11}	π_{12}	π_{13}	π_{14}	π_{10}
TRUE VALUES	-.6091	-.2546	-.3818	+.2546	+149.56
LISE	.4067 .419 / .587	-.0065 .083 / .084	-.2145 .899 / .925	-.0747 .106 / .130	-10.30 116. / 116.
TSLS	.4060 .369 / .551	-.0241 .085 / .088	-.2245 .881 / .910	-.0770 .095 / .122	- 6.50 114. / 114.
OLS	.6135 .386 / .730	-.1193 .109 / .162	-.1862 .571 / .601	-.1378 .085 / .070	-13.15 97. / 98.
FIML	.3220 .316 / .453	-.0152 .097 / .097	-.2264 .795 / .827	-.0574 .087 / .105	- 0.79 105. / 105.
LSNR	.1448 .745 / .759	.0149 .089 / .090	-.0588 1.164 / 1.166	-.0249 .146 / .148	- 7.53 119. / 119.

T = 20
N = 50

Table 6A

EXPERIMENT 5, without outliers

Structural Coefficients

PARAMETERS	β_{12}	γ_{11}	γ_{12}	γ_{10}	β_{22}	γ_{23}	γ_{24}	γ_{20}
TRUE VALUES	-.7	+.8	+.7	-149.5	+.4	+.6	-.4	-149.6
LISE	.0412 .429 [.431]	-.1077 .786 [.794]	-.0560 .259 [.265]	15.62 76.8 [78.4]	.0523 .200 [.207]	.4590 1.45 [1.52]	.0657 .111 [.133]	-9.37 157. [158.]
TSLS	.0970 .394 [.406]	-.2088 .727 [.757]	-.0864 .239 [.255]	25.13 72.1 [75.5]	.0240 .167 [.169]	.3720 1.27 [1.33]	.0785 .090 [.120]	1.27 137. [137.]
OLS	.4077 .228 [.471]	-.7768 .452 [.905]	-.2592 .156 [.304]	74.00 39.4 [84.5]	-.1329 .140 [.194]	-.0457 1.18 [1.18]	.1436 .075 [.163]	50.65 125. [135.]
FIML	.0266 .361 [.362]	-.0976 .666 [.673]	-.0730 .254 [.265]	17.96 67.5 [69.9]	.5511 1.49 [1.59]	1.5535 4.54 [4.80]	-.1544 .709 [.726]	-161.26 537. [561]

T = 20
N = 47

Table 7

Comparison of Root-Mean-Square-Errors of Structural Coefficient Estimates Obtained by FIML, LISE, TSLS, and OLS

Method		Root-Mean-Square-Error Comparisons for Experiments 1,2,3,4,5*							
		Including outliers: Tables 2,3,4,5,6*				Excluding outliers: Tables 2,3A, 4,5A,6A*			
A	B	A better	Ties***	B better	Total	A better	Ties***	B better	Total
FIML	- OLS	27	0	1	28**	28	0	0	28**
FIML	- LISE	25	3	0	28**	26	2	0	28**
FIML	- TSLS	24	2	2	28**	23	2	3	28**
OLS	- LISE	13	0	23	36	11	1	24	36
OLS	- TSLS	11	0	25	36	10	1	25	36
LISE	- TSLS	2	6	28	36	6	6	24	36

* The maximum number of comparisons possible is 40 (8 coefficients estimated times 5 experiments). Since the misspecification of the second equation in Experiment 5 makes estimates of the coefficients of the second equation meaningless in that experiment, only the coefficients of the first equation were used as a basis for comparing the methods. As a consequence, the maximum number of comparisons made was 36.

** FIML estimators were not computed in Experiment 4, so only 28 comparisons were made (8 coefficients times 3 experiments plus 4 coefficients times 1 experiment).

*** To three decimal places for slope coefficients; to the nearest unit for intercept coefficients.