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HOUSEHOLD PORTFOLIOS AND FINANCIAL PREPAREDNESS FOR RETIREMENT

By

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January 2020

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Household portfolios and financial preparedness for retirement*

Rowena Crawford[†] Cormac O’Dea[‡]

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Abstract

Using a lifecycle model of consumption, saving and portfolio choice combined with linked survey and administrative data on wealth and lifetime earnings we evaluate measures of retirement preparedness. We estimate heterogeneous discount factors for households and compare the estimates of their patience to their replacement rates – the simple measure often used to evaluate the adequacy of retirement savings. We find first that the specification of the model’s asset structure matters quantitatively for preference parameter estimates – households appear to be much more patient when they are assumed to have access only to a risk-free asset compared to when we account for the fact that much of their wealth is stored in higher-return tax-advantaged private pensions and in housing. Second we find that only the most patient households achieve the replacement rates out of *final* earnings that are often recommended by policy-makers and industry as sensible benchmarks for retirement preparedness. Notwithstanding this, we find that even quite impatient households in the population we study achieve high replacement rates out of lifetime *average* income – a more sensible summary measure of preparedness for retirement.

JEL Classification: D91, D31, E21, D14, H55

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1 Introduction

Many countries are implementing policies to encourage saving for retirement. Examples include: mandatory private pension saving in Australia (introduced in 1992), a compulsion for all employers to auto-enroll employees into private pension saving in the United Kingdom (introduced from 2012) and auto-enrollment in the US army in 2010, with a number of US states now implementing or piloting similar initiatives. The rationale for such policies is that individuals, left to their own devices, will not save enough for retirement. This paper examines the evidence for the presumption, implicit or explicit, that there is systematic undersaving.

We have two distinct approaches. The first develops a lifecycle model of consumption and saving and combines this with micro-data on the wealth holdings (from household survey data) and lifetime earnings histories (from linked administrative data) for a sample of English households. The model is a rich one: households can save in each of pension and non-pension wealth; they face risks over earnings, pension fund returns and longevity; and they are heterogeneous in their earnings processes, their fertility and their patience. We use this model to estimate the implied level of patience, for each household in our sample, that would rationalize observed wealth holdings.

The data which, together with our lifetime earnings data and model, is used to identify the discount factor is wealth close to retirement. Our estimates of discount factors, therefore, summarize how households trade-off consumption during working life and saving for retirement and, as such, the estimated discount factor gives a summary measure of the extent to which households have planned for their retirement. We complement this approach by estimating a more conventional summary measure of how households trade-off working life consumption and saving – replacement rates out of lifetime earnings (see, for example, Munnell and Soto (2005) and Banks et al. (2005)). Comparisons of observed or projected replacement rates (out of either career average or of final earnings) to target levels are often used by policy-makers to assess adequacy of savings among cohorts approaching retirement (e.g. Bridges & Choudhury (2005), Pension Commission (2004)) and by financial advisers in recommendations to clients (e.g. TIAA (2018)). While such approach has the advantage of simplicity, any threshold selected as the appropriate one for saving is necessarily ad-hoc. Furthermore, assuming a single replacement rate for all households, regardless of their personal circumstance (trajectory of income, number

of children, life expectancy etc.) is ‘conceptually flawed’ (Scholz and Seshadri (2009), building on earlier work by Gokhale et al. (1999)). We show the relationship between our estimated replacement rates and the estimated discount factors from our first approach and highlight some implications for users of the simple replacement rate approach.

We have four principle findings. First, we can match the distribution of wealth with modest heterogeneity in the degree of patience. The interquartile range for estimates of the discount factor from our preferred model stretches from 0.92 to 0.97. The median discount factor is 0.95, close to conventional values estimated (or assumed) by papers using lifecycle models that assume homogeneous discounting. We validate these estimates of discount factors in a number of ways. In particular, we show that those with higher education are estimated to be more patient (as has been previously found in the literature), the estimates are associated with other known correlates of patience (e.g. smoking behavior) and that our estimated discount factors correlate with self-reported measures of patience elicited from the same survey respondents.

Second, we highlight the importance of capturing the richness of the savings environment available to households for the estimation of discount factors. The median discount factor estimated using a simple one-risk-free-asset model would be 1.04, with an interquartile range that stretches from 1.01 to 1.08. When we take account in the model of the fact that households, in reality, have access to a tax-advantaged pension fund which incentivizes saving for retirement, the median estimate falls to 0.99. Neither of the models underlying these estimates accounts, though, for the particular features of housing as a store of wealth (for example that it returns a flow of services and that it tends to be purchased using leverage). Once we account for these features in the return on non-pension wealth, the median estimated discount rate factor falls to 0.95 – our preferred estimate.

Third, few of the cohorts that we study (those born in England in the 1940s) will have substantially lower income in retirement than their average earnings over the course of their working life. The median replacement rate is 84.5%, and only around one third would have pension income at age 65 of less than 70% of their average lifetime earnings. However, replacement rates are lower when pension income is compared to a proxy for final pre-retirement earnings (which is common practice among policy makers and industry). In particular, the median replacement of the average of the best five years of earnings is 47%, and almost nine out of ten households are found to replace less than 70% of this measure of earnings in retirement.

Finally, bringing together our two approaches, we show that these estimated replacement rates out of a measure of *final* earnings are low – compared to thresholds for ‘adequacy’ commonly

used – even when wealth holdings can be justified without appealing to high levels of individual impatience. This suggests that those using replacement rates to assess retirement preparedness must be careful with their choice of replacement rate and the associated threshold against which adequacy is assessed. While the ease of communicating simple replacement rate benchmarks is clear, those recommending them need to a) be clear that suggesting high replacement rates out of *final* earnings represents an extremely demanding threshold and b) be cognizant of the importance of individual circumstances and preferences when recommending savings choices.

Our paper contributes to two distinct literatures. The first is that which has used lifecycle models to assess household retirement saving (most notably Engen, Gale & Uccello (1999) and Scholz, Seshadri & Khitatrakun (2006)). Our paper is most similar to Scholz et al. (2006), who showed that levels of wealth accumulated by the vast majority of households in a now-retired cohort in the US were more than an ‘optimal’ level of wealth that the authors derive. ‘Optimal’ wealth in that paper is pinned down using a lifecycle model in which a homogeneous discount rate is assumed, which is set approximately equal to the rate of return on the model’s (single) asset. We add to their paper by a) enriching the asset structure, primarily by including a risky, higher expected return and tax-advantaged pension asset; b) estimating the distribution of discount rates rather than assuming a value; and c) showing the relationship between those discount rates and the instrument central to notions of optimal saving used by policy-makers – replacement rates.

The second literature to which we contribute is that concerning the estimation of discount factors. Lifecycle models have typically assumed homogeneity of discount rates (or homogeneity conditional a set of observable characteristics – e.g. education). Early estimated structural models include Attanasio et al. (1999), Gourinchas and Parker (2002) and Cagetti (2003); more recently (and focusing on those papers most relevant to the current paper), papers studying Medicaid (De Nardi et al. (2016)), old-age means-tested transfers (Braun et al. (2016)) and intergenerational transfers (Lockwood (2018)) have assumed that agents all have the same degree of patience.

This assumption of a homogeneous discount factor is a strong one. Experiments have provided evidence in support of heterogeneous discount rates (Andersen et al. (2008), Andreoni and Sprenger (2012)), and such heterogeneity has been proposed as a candidate explanation for observed wealth inequality (Krusell and Smith (1998), Hendricks (2007), Hubmer et al. (2019)). Using a ‘semi-structural approach’, Alan and Browning (2010) and Alan et al. (2018), use, respectively, data on the consumption growth and on the co-movements of income and con-

sumption to estimate distributions of discount rates, finding substantial dispersion in measures of patience.

Samwick (1998) and Gustman and Steinmeier (2005) both solve lifecycle models and identify, as we do, a discount factor for each household by matching wealth levels in the data. Samwick (1998) solves a model with income uncertainty and estimates a discount factor for each household in a cross-sectional data set as that which rationalizes wealth to *current* income ratios; Gustman & Steinmeier (2005) solve a lifecycle model with deterministic income and estimate a discount factor for each household as that which rationalizes wealth stocks conditional on life-history of earnings. Computational advances, combined with our rich data, allow us to relax many of the assumptions made in those papers. We broaden the asset structure beyond the ‘single-safe-asset’ workhorse model to make it more reflective of the assets in which households save for retirement, and we incorporate uncertainty in multiple dimensions (earnings, returns and survival).

The rest of this paper proceeds as follows. Section 2 introduces the linked survey and administrative data that we use. Section 3 outlines the model, while Section 4 discusses the estimation and parameterization of features of the model. Section 5 discusses our results. Section 6 concludes.

2 Data

Our data is formed by linking survey and administrative data. The survey data come from the the English Longitudinal Study of Ageing (ELSA) - a biennial longitudinal survey of the household population of England aged 50 and over.¹ ELSA contains detailed data on demographics, labor market circumstances, income and, most importantly for our purposes, the level and composition of wealth holdings.

ELSA respondents were also asked for their National Insurance number (the equivalent of a US Social Security number) and permission to link to their history of National Insurance contributions. Almost 80% of ELSA respondents agreed to the linking of their survey records with their administrative data. Data on these contributions allow us to calculate individuals’ state pension entitlements, and (subject to some top-coding) to obtain a detailed history of their earnings. Appendix D.1 discusses how we convert our administrative data on National Insurance

¹ELSA is part of a family of international aging surveys, including the Health and Retirement Study (HRS) in the US and the Survey of Health, Ageing and Retirement in Europe (SHARE), that collect a similar set data using broadly comparable methodologies.

contributions into a panel of earnings.

We use the 2002/03 ELSA data (the year for which the sample was linked) together with the linked National Insurance data.

2.1 Sample

We restrict our attention to couple households that contain a man born between 1940 and 1949. There are 1,615 couples of this type in the 2002/03 ELSA data. These individuals would be aged between 52 and 63 (and therefore approaching the UK public pension age of 65) when observed in 2002/03. We exclude households where either partner refused permission to link to their administrative data, since for these households we cannot obtain lifetime earnings. We also exclude households where the man is observed in the NI data for fewer than five years and/or where households have more than five years of self-employment activity (since the NI data are less well suited to calculating the earnings histories of the self-employed – as is discussed in more detail in Appendix D.1). After applying these restrictions, 995 couples, or approximately 62% of couples, remain.

Table I provides descriptive statistics for our sample of households and for all ELSA couples in the relevant cohort. The average age of men in our sample when they are observed in 2002/03 is just under 57, with women on average being 54. Nearly 70% of men in our sample reported still being in work, and only 15% defined themselves as retired. Home ownership was the norm for this cohort – in our sample, nearly 90% of households own their home (either outright or still mortgaged).

A comparison of the descriptive statistics from the survey data for our sample and the descriptives for all couples in ELSA in the relevant cohort suggests a close match along most observables between our sample and the full sample of couples in the cohort of interest. An exception to this is with regard to self-employed individuals who are under-represented in our sample. This is unsurprising given that we drop those with significant histories of self-employment though this is accentuated by the self-employed being less likely to grant permission to link to their administrative records (see Bozio et al. (2010a)).

2.2 Wealth measures in ELSA

Our model contains each of private pension wealth and non-pension wealth (which, in turn, comprises each of housing and more liquid wealth). Therefore we must define an empirical

Table I. Descriptive statistics for couples in ELSA of cohort 1940-1949

	Our sample		All	
<i>Individual characteristics</i>	<i>Male</i>	<i>Female</i>	<i>Male</i>	<i>Female</i>
Mean age	56.8	54.1	56.9	54.0
Low education	35.3%	43.5%	36.5%	41.9%
Mid education	27.3%	32.2%	26.6%	31.1%
High education	37.4%	24.3%	36.8%	22.4%
Employee	60.5%	59.5%	55.4%	52.9%
Self-employed	11.0%	3.6%	15.2%	4.6%
Retired	14.9%	11.1%	14.3%	10.8%
Other	13.6%	25.8%	15.0%	27.2%
<i>Household characteristics</i>				
Owner occupier	89.6%		87.6%	
Median total income	£22,544		£22,166	
Median employment income	£17,870		£17,189	
Median asset income	£185		£182	
Median private pension wealth	£95,593		£72,384	
Median housing wealth	£120,000		£121,000	
Median non-pension, non-housing wealth	£32,200		£30,625	
Sample size	995		1,615	

analogue for each of these. The sum of these components is referred to as ‘private wealth’.

Private pension wealth includes both Defined Benefit (DB) and Defined Contribution (DC) pensions. Wealth is calculated differently for each of these. DC pension wealth is the reported value of funds held. We calculate DB pension wealth as the capital sum that would be required in our survey year to purchase the projected stream of DB income to which the household is entitled given their reported accrual of rights to date and the rules of the pension scheme. To calculate this, we first calculate the capital sum required at age 65, given annuity rates at that age.² The capital sum required in 2002/03 is then calculated by assuming that households would achieve a real rate of return equal to the model’s mean return on DC wealth between 2002/03 and the year in which they reach the age of 65.

Housing wealth is the gross value of owner-occupied housing less any mortgage debt. Liquid wealth is the sum of all other non-pension wealth less any outstanding non-mortgage debt. The largest component of this is cash (and we refer to this form of wealth as ‘cash’ below), but it also includes other liquid financial wealth, net non-primary housing wealth and other wealth (business wealth and physical assets such as land, antiques and collectibles).

Table II summarizes, for our sample of couples, the distribution of wealth held in each of these components. The mean level of total wealth is £407,400, of which around 36.9% is held in housing, 34.3% in private pension wealth, and 28.8% in cash and other liquid assets. This compares to mean lifetime earnings of £985,400. Holdings of wealth and each of its components rise with lifetime earnings, and ratios of mean total wealth to mean lifetime earnings are highest among those at the bottom and top of the lifetime earnings distribution (see also Venti and Wise (1999); Gustman and Steinmeier (1999); Bozio et al. (2017)), although the very large difference between mean and median wealth in the bottom decile highlights the fact that wealth is particularly skewed at that part of the distribution and that many households who have had low lifetime earnings hold little private wealth.

3 A model of saving for retirement

We solve and estimate a lifecycle model of consumption, saving and portfolio choice. Details are given in this section. Briefly, its key features include: decisions made (collectively) by households that differ in their patience, uncertainty over employment, earnings, returns on a pension fund

²The annuity rate is calculated using the model’s risk free interest rate and survival probabilities (details are given below).

Table II. Observed private net wealth (£,000s), by lifetime earnings

	Lifetime earnings		Total		Pension		Cash		Housing	
	Median	Mean	Median	Mean	Median	Mean	Median	Mean	Median	Mean
All	888.9	985.4	280.7	407.4	95.6	139.8	32.2	117.2	120.0	150.5
<i>Lifetime earnings decile:</i>										
Lowest	299.8	287.6	64.3	251.9	14.0	35.5	2.0	152.5	23.5	63.9
2	517.4	510.3	146.2	216.7	35.7	65.6	10.2	53.4	75.0	97.7
3	663.3	658.3	176.7	302.9	52.0	80.5	19.3	109.2	89.5	113.2
4	768.0	765.4	208.7	274.6	62.5	94.0	19.4	71.9	95.0	108.7
5	851.7	849.5	242.2	331.0	72.3	109.6	27.2	88.5	100.5	132.9
6	932.6	935.8	252.5	336.6	98.1	118.8	30.0	82.0	115.0	135.8
7	1051.7	1049.1	348.1	376.5	109.5	132.3	46.9	94.1	132.5	150.1
8	1202.2	1201.4	319.7	413.4	125.2	148.7	34.5	76.7	140.0	188.0
9	1440.9	1447.6	546.6	619.3	196.2	241.3	66.5	144.7	221.3	233.4
Highest	1894.0	2155.7	683.7	952.7	287.4	372.3	103.7	298.8	250.0	281.6

and mortality; exogenous heterogeneity over the earnings process and fertility; and a careful specification of the tax and benefit system.

3.1 Preferences and the economic environment

3.1.1 Preferences

Household utility in each period (one year in the model) is assumed to exhibit constant relative risk aversion in equivalized consumption, multiplied by the number of equivalized adults in the household:

$$U(c_t) = n_t \frac{\left(\frac{c_t}{n_t}\right)^{1-\gamma}}{1-\gamma}$$

where $\left(\frac{c_t}{n_t}\right)$ is household equivalized consumption and n_t is the number of equivalized adults in the household. The subscript t refers to the age of the household (taken to be the age of the male).

3.1.2 Employment and earnings

Employment starts at age 20 and can continue until age 64, at which point employment is no longer possible. In each period, households get an employment offer with probability π_j , where j represents education group. Households are divided into three education groups: high-school drop outs, high school graduates and those with at least some college.³

Earnings (e_{it}) are therefore given by:

$$e_{it} = \begin{matrix} \tilde{e}_{it} & w.p. & \pi_j \\ 0 & w.p. & 1 - \pi_j \end{matrix}$$

where (\tilde{e}) is household productivity, defined below. $1-\pi_j$ can therefore be interpreted as the probability of a spell of long-term unemployment (lasting a year) for both members of the couple.

The log productivity of a household (the sum of earnings of both members of the couple) is given by the sum of a fixed effect, a quadratic in age (t) and a stochastic process which contains

³In the UK context, these are defined as having compulsory education only, compulsory but no post-secondary education and those with some post-secondary education. We use the (better known) US terminology though in this paper.

a first order autoregressive process with normally distributed innovation. Households receive this value unless they receive an unemployment shock:

$$\begin{aligned} \ln \tilde{e}_{it} &= \alpha_i + \delta_1^j t + \delta_2^j t^2 + u_{it} \\ u_{it} &= \rho^j u_{it-1} + \xi_{it} \\ \xi &\sim N\left(0, \sigma_j^2\right) \end{aligned} \tag{1}$$

The coefficients of the earnings process depend on education.

3.1.3 Household heterogeneity

Households are ex-ante heterogeneous. They differ by their education. Let \bar{j} index the combination of education levels of each member of a couple (and so takes one of nine values).⁴ They also differ in the fixed effect in their earnings process (α_i) which they are assumed to know from the start of life. We also include in the vector of fixed effects the number of children in the household at each age ($\{k_{it}\}_{t=20}^{100}$). The implication of this assumption is that couples know with certainty from the age of 20 exactly how many children they will have and when those children will be born.⁵ In what follows, we summarize the ‘type’ of household i by $\theta_i = (\bar{j}_i, \alpha_i, \{k_{it}\}_{t=20}^{100})$. Each household has its own type (θ_i) which will be part of the state space, and so each household faces a different optimization problem which must be individually solved.

3.1.4 Assets

Private pensions Households can, each period, pay a proportion of their *pre-tax* income into a Defined Contribution (DC, i.e., 401(k)-style) pension. These funds earn a risky rate of return. The stock of wealth held in DC funds is modeled as illiquid in two senses. First, the wealth cannot be accessed until age 65. Second, there is a compulsion in the model, as there was in reality for decades in the UK (until 2014), to use three quarters of the fund to purchase an

⁴While we assume that the education of the husband only determines the earnings process, the education of both spouses will be relevant for survival probabilities (and therefore also for annuity pricing).

⁵An alternative assumption – that children arrive probabilistically according to a process that depends on age (see Hong and Ríos-Rull (2012) & Jørgensen (2017)) – would add substantially to the computational cost (as we would have to add the number of children as a state variable and include an additional dimension of integration) and would, we conjecture, be no closer to reality than assuming that households know how many children they will have.

annuity. We assume that annuitization happens at the age of 65. While the flow of income from an annuity is taxed, the non-annuitized quarter of the DC fund can be taken in cash tax free (both in the model and in reality), giving this form of saving a degree of tax advantage.⁶

The evolution of the stock of wealth in the DC fund (DC) depends on flows into the fund (dc) and the return on the fund in each year (ϕ).

$$DC_{t+1} = (1 + \phi_{t+1})(DC_t + dc_t)$$

The return on DC funds is assumed to be normally distributed with mean $\bar{\phi}$ and variance σ_ϕ^2 . Returns are truncated at -100% and each household in a given year earns the same return.

Annuity rates ($q^{\bar{j}}$) are assumed to be actuarially fair after a proportion (z) has been deducted to meet the administrative costs (including profits) associated with the provision of annuities. Survival probabilities will vary by education, therefore, the annuity rate depends on the education of each member of the couple (\bar{j}). Private pension income at each age post-retirement (pp_t) is the annuity rate at age 64 (which yields an income flow starting from the age of 65) multiplied by the three quarters of the stock of wealth accrued by the end of age 64:

$$pp_t = q_{64}^{\bar{j}}(0.75)(DC_{64} + dc_{64})$$

This reflects a timing convention that contributions to the pension are made at the end of the period and therefore those in the final year of potential employment (age 64) do not accrue a return and are not subject to risk.

Non-pension wealth Household resources that are neither used for consumption nor paid into a private pension are saved in non-pension wealth. The challenge here is to specify an asset that reflects the fact that many households hold a mix of cash and housing, while not taking these to be two separate state variables over the whole of the lifecycle.⁷ The return on these two

⁶Pension saving is advantaged by the tax system to a greater extent in the UK than in the US. In both countries contributions are exempt from income tax, but in the US, contributions are subject to Social Security and Medicare taxes, while in the UK, contributions can be made exempt of the equivalent levies (National Insurance Contributions). As a further advantage to pension saving in the UK one quarter of withdrawals are exempt from income tax – while all withdrawals in the US are subject to federal income tax.

⁷Computational constraints preclude us from incorporating housing wealth and cash separately early in life when the annual pension saving decision is being made – solving and estimating the model household-by-household with three different assets: cash, housing and pension wealth would not be possible. Our secure access data can only be used on desktop computers and, cannot, therefore be ported to High Powered Computers.

forms of non-pension wealth are very different. Cash has a low rate of return, while the return on housing has tended to be higher for many decades, and those higher returns have typically been magnified by leverage. Failing to take into account that much of wealth accumulation has been due to these returns by assuming that non-pension wealth evolves according to a standard intertemporal budget constraint with a risk-free interest rate would mean that estimates of patience would be biased substantially upwards.

Our treatment of non-pension wealth is different pre- and post-retirement. Taking pre-retirement first, we take non-pension wealth to comprise cash and housing. We do not model the choice of how much households save in each of cash and housing but specify that the shares held in each form (and therefore the overall return on non-pension wealth) is an exogenous function of age and total wealth holdings.⁸ We do this by defining two functions. The first is $s(a)$ – the share of non-pension wealth held in *net* housing – which we assume is a function of non-pension wealth. The second is $lev(t)$ – the leverage ratio (the ratio of mortgage debt to gross housing wealth) – which we assume is a function of age. These functions will be estimated outside the model – details will be given in Section 4.

Using $s(a)$, we can decompose non-pension wealth into cash (a_t^c) and net housing (nh):

$$\begin{aligned} a_t^c &= (1 - s(a_t))a_t \\ nh_t &= s(a_t)a_t \end{aligned}$$

and using $lev(t)$ we can further decompose net housing into gross housing (gh) and mortgage debt ($mort$) and therefore calculate the overall return on net housing ($r^h(t)$):

$$r^h(t)nh_t = (r^{hcg} + r^{hr}) \underbrace{\frac{1}{(1 - lev(t))}nh_t}_{gh_t} - r^{mort} \underbrace{\frac{lev(t)}{(1 - lev(t))}nh_t}_{mort_t} \quad (2)$$

where, following Kaplan and Violante (2014) the return on gross housing wealth comprises a capital gain (r^{hcg}) and a service flow (r^{hr}). r^{mort} is the mortgage interest rate.

Cash assets earn a return r^c and therefore the overall return on non-pension wealth pre-retirement is:

$$r(a_t, t) = s(a_t)r^h(t) + (1 - s(a_t))r^c \quad (3)$$

⁸Kaplan et al. (2018) adopt a similar approach in splitting wealth into equities and housing, although they make the additional assumption that the proportion held in each form of wealth does not vary with wealth.

The implication of these assumptions is that prior to the age of 65, we keep track of just one non-pension wealth state variable (a_t). The intertemporal budget constraint for the composite non-housing wealth variable before the age of 65 is given by:

$$a_{t+1} = \left(1 + r(a_t, t)\right) \left(a_t + y_t - c_t - dc_t\right) \quad \forall t \leq 64 \quad (4)$$

Non-pension wealth is assumed to be liquid pre-retirement.

Turning to the post-retirement period, from the age of 65, we treat cash and housing as separate state variables. Here, we first give the initial post-retirement (age 65) values of these state as a function of composite non-pension wealth holdings at age 64 (a_{64}), the last period before we split the assets, followed by the intertemporal budget constraints for the rest of retirement. Cash assets at the age of 65 are given by:

$$a_{65}^c = (1 + r^c) \left((1 - s(a_{64}))a_{64} + y_{64} - c_{64} \right) + (0.25) (DC_{64} + dc_{64}) \quad (5)$$

where $(1 - s(a_{64}))a_{64}$ is cash at the age of 64, to which we must add savings ($y_{64} - c_{64}$), interest and the tax-free lump sum from the pension saving. In keeping track of housing wealth, we can choose to keep track of gross or net housing wealth as each is a deterministic function of the former and age (through the leverage ratio). We take the state variable here to be gross housing wealth at the age of 65:

$$gh_{65} = (1 + r^{hcg}) \underbrace{\frac{1}{(1 - lev(64))}}_{gh_{64}} \underbrace{s(a_{64})a_{64}}_{nh_{64}} \quad (6)$$

where $s(a_{64})a_{64}$ is net housing at the age of 64, which must be converted into gross housing wealth, which in turn accrues a capital gain r^{hcg} .

Through retirement, households continue to accrue a service flow from their owner-occupied housing, and must continue to pay their mortgage (until it is paid off at time t *s.t.* $lev(t) = 0$). That is they get income from housing of⁹ :

$$r^{hr} gh_t - r^{mort} \underbrace{lev(t) gh_t}_{mort_t}$$

⁹This income on net housing is rolled into the return on net housing pre retirement (see equation (2)). In the model exposition we treat it as income post-retirement – see equation (9).

We add one more assumption – the capital value of housing cannot be consumed.¹⁰

Cash and gross housing evolve as follows from the age of 66 onwards:

$$a_{t+1}^c = (1 + r^c)(a_t^c + y_t - c_t) \quad \forall t \geq 65 \quad (7)$$

$$gh_{t+1} = (1 + r^{hcg})gh_t \quad (8)$$

The approach described above, while our preferred approach, makes a number of assumptions which are admittedly strong (for example, the fact that housing wealth accumulates continuously rather than discretely, and the change in the liquidity of housing wealth on retirement). Therefore, in addition to the results from this model, we also show results of two alternative methods for treating housing wealth. First, we make no account for housing wealth in the calculation of rates of return and in the formulation of the inter-temporal budget constraint, but still include housing wealth in the data used for estimation. This approach makes no allowances for the different rates of return on housing and cash, nor of the fact that housing tends to be leveraged; nevertheless the approach is relatively common (Scholz et al. (2006), French (2005), De Nardi et al. (2010)). We show our results under this specification (referring to it as a specification with ‘no housing treatment’). The second method is to deduct an estimate of housing costs (mortgage interest and capital) from income, and then to estimate parameters with non-housing wealth (following Cagetti (2003)). We give more details on this and show results in Appendix B.

3.1.5 Public pensions

The UK public pension system is described in Appendix A. Briefly, payments are substantially smaller than Social Security is in the US, and payments are only weakly related to earnings – as most of the payment is proportional only to the number of years in work, with earnings when in work not relevant.¹¹

¹⁰In reality, the vast majority of households do not decumulate their housing wealth in the UK (Blundell et al. (2016b)). In place of our simplifying assumption the observed behavior could be generated by a combination of a bequest motive, transaction costs associated with liquidating housing wealth and the service flow households receive by staying in one’s home.

¹¹To give a sense of the scale and variation in entitlements, the median public pension entitlement in our sample is approximately £7,800 with an the interquartile range stretching from £6,330 – £9,100 in 2002. Adjusting for prices to 2014 and converting to US dollars using the average exchange rate for that year yields \$15,100, \$18,600

We model households as receiving a public pension from the age of 65. Its level depends on the fixed effect in the earnings process, final pre-retirement productivity and education.¹² Parameterization and estimation of that function is described in Section 4.1.

3.1.6 Taxes, transfers and net income

Taxes and other transfers are based on the UK tax system in 2002/2003 - the year represented by our main data source. The modeled components of the tax and benefit system are income tax, National Insurance (the UK equivalent of the US Payroll Tax), Jobseekers' Allowance (a payment to the unemployed), Child Benefit and the Minimum Income Guarantee for pensioners. These are described in more detail in Appendix F. Household net income is a function (τ):

$$y_t = \tau(e_t, a_t, a_t^c, gh_t, pp_t, sp_t, h_t, k_t, dc_t, t) \quad (9)$$

that depends on household earnings (e_t), non-pension wealth pre-retirement (a_t), cash and gross housing post-retirement (a_t^c, gh_t), private pension payments (pp_t), public (or 'state') pension payments (sp_t), number of adults still alive (h_t), number of dependent children (k_t), chosen contributions to the pension fund (dc_t) (since those attract tax relief) and finally, on the age of the household (t) (the UK tax system taxes the elderly to a lesser extent than those of working age).

3.1.7 Uncertainty

Four features of the household's problem are uncertain. These are mortality, productivity, employment and the return on the pension fund. The probability distribution over mortality is summarized by the sequence of probabilities $\{s_t^{j,m}, s_t^{j,f}\}_{t=20}^{100}$ which gives the probability of surviving to age t , conditional on being alive at age $t - 1$. Survival probabilities vary by gender and education. We assume no mortality before the age of 65. The joint distribution over earnings (summarizing realizations of both productivity and employment) and pension fund returns is

and \$21,700 for the 25th, 50th and 75th percentiles. This compares to values (calculated using the HRS) of household Social Security Society income for a similarly selected sample in the US of \$18,800, \$26,360 and \$33,400.

¹²It is worth noting that, given the weak relationship between lifetime average earnings and pension entitlement, we do not include in the state space a measure of lifetime earnings (such as Average Indexed Monthly Earnings) – included in many models that incorporate Social Security.

given by $F(\phi_{t+1}, e_{t+1}|e_t)$. The stochastic processes underlying the two of these are mutually independent, though the distribution of e in period $t + 1$ depends on its value in period t .

3.2 Household maximization problem and value functions

We can now outline the household maximization problem and value functions. The period of life where decisions are modeled starts at age 20, and all individuals are assumed to die by the age of 100 at the latest. We do not model marriage or divorce – couples are assumed to start their productive life already married and stay together until death. We now discuss in turn the optimization problem facing retired households and working age households.

Retired household's problem Households in the model enter retirement at the age of 65. Retirement here is associated with two distinct events. The first is withdrawal from the labor market. The second is the conversion of three quarters of the stock of wealth held in the DC fund into a life annuity, with the remaining quarter taken as a tax-free lump sum. The state variables that summarize the household's problem in retirement are age (t), cash (a^c), gross housing (gh), private pension income (pp), final pre-retirement productivity (\tilde{e}_{64}) and household composition (h). This last variable takes a value of 1, 2, or 3 indicating, respectively, that both spouses are still alive, only the male is alive and only the female is alive. Households in retirement make a single choice in each period - their level of consumption (c).

The problem facing retired households with both spouses alive at time t is therefore (Appendix G.1 gives the corresponding problem for households in which one spouse has died.):

$$\begin{aligned}
 V_t(a_t^c, gh_t, pp_t, \tilde{e}_{64}, h_t = 1; \theta_i) = \max_{c_t} & \left(u(c_t) + \beta_i s_{t+1}^{j,m} s_{t+1}^{j,f} V_{t+1}(a_{t+1}^c, gh_{t+1}, pp_{t+1}, \tilde{e}_{64}, h_{t+1} = 1; \theta_i) \right. \\
 & + \beta_i s_{t+1}^{j,m} (1 - s_{t+1}^{j,f}) V_{t+1}(a_{t+1}^c, gh_{t+1}, pp_{t+1}, \tilde{e}_{64}, h_{t+1} = 2; \theta_i) \\
 & \left. + \beta_i (1 - s_{t+1}^{j,m}) s_{t+1}^{j,f} V_{t+1}(a_{t+1}^c, gh_{t+1}, pp_{t+1}, \tilde{e}_{64}, h_{t+1} = 3; \theta_i) \right) \\
 s.t. \quad y_t & = \tau(e_t, a_t, a_t^c, gh_t, pp_t, sp_t, h_t, k_t, dc_t, t) \\
 & \text{and intertemporal budget constraints (5) to (8)}
 \end{aligned} \tag{10}$$

where $V_t()$ is the value function in period t which is a function of the state variables and the type of the household (θ_i), β_i is a household-specific geometric discount factor and the rest of the variables have been defined earlier in this section. That the value function, and therefore

the maximization problem and decision rules, faced by each household differ is indicated by the inclusion of θ_i as an argument.

Working age household's problem The state variables that summarize the household's problem during working life are age (t), non-pension wealth (a), pension wealth (DC) and productivity (\tilde{e}). At each age during working life households make two choices: their level of consumption (c_t) and their payments into a DC fund (dc_t). The balance of their resources is saved in a risk-free asset. The problem facing working households (we assume no mortality during working life, so h – household composition is equal to 1 for all periods) is:

$$\begin{aligned}
 V_t(a_t, DC_t, \tilde{e}_t; \theta_i) = \max_{c_t, dc_t} & \left(u(c_t) + \beta_i \int V_{t+1}(a_{t+1}, DC_{t+1}, \tilde{e}_{t+1}; \theta_i) dF(\phi_{t+1}, e_{t+1} | e_t) \right) \\
 \text{s.t.} \quad y_t & = \tau(e_t, a_t, a_t^c, gh_t, pp_t, sp_t, h_t, k_t, dc_t, t) \\
 a_{t+1} & = (1 + r(a_t, t))(a_t + y_t - c_t - dc_t) \\
 DC_{t+1} & = (1 + \phi_{t+1})(DC_t + dc_t)
 \end{aligned}$$

3.3 Model solution and the simulation of consumption and savings behavior

The model outlined contains two choice variables (consumption and pension saving), six state variables post-retirement (age, cash, housing wealth, pension income, final productivity and household composition) and four state variables pre-retirement (age, non-pension wealth, pension wealth and productivity). There is no analytical solution to the maximization problem. Full details on the methods used in solving the households' problem and simulating their behavior are given in Appendix G. In short, we obtain decision rules numerically by solving the households' problem, first for the final period of life, storing the value functions for this period and then iterating backwards. After having obtained decision rules (the pension saving function and consumption function), we simulate behavior (consumption and saving) using these and realizations for the stochastic variables. Following Scholz et al. (2006), instead of using draws from a pseudo-random number generator, we use actual realizations for the stochastic components drawn from the data on these households. That is, in each year in which we simulate behavior using the calculated decision rules, we use households' actual employment status, realized earnings and the average return observed on DC assets in that year.

4 Estimation and parameterization

Estimation follows a two step process. In the first step, we set features of the economic environment and a sub-set of preference parameters with reference to the institutional structure or existing literature. In the second step, we estimate the earnings process and the distribution of discount rates. We discuss these steps in turn.

4.1 Step 1: Parameterization

This section discusses the model parameterization – first, the parameters that set the economic conditions (rates of return, policy environment) faced by our cohort of interest, and second, demographic and preference parameters.

4.1.1 Economic environment

Return on cash The rate of return on cash is set at the average real return on cash balances, which was 1.6% between 1952 and 2012 (see Table 1 of Barclays Capital (2012)).

Share of non-pension wealth in housing To calculate $s(a_t)$ – the share of non-pension wealth held in housing – we use data from the Wealth and Assets Survey which is a dedicated wealth survey of the British population that started in 2006. We regress the proportion of non-pension wealth held in housing wealth on a quadratic in total wealth. We also include cohort dummies and a set of time dummies constrained to sum to zero (a normalization suggested by Deaton and Paxson (1994) given the identification problem implied by the collinearity of age, period and cohort). We do this only for those who own a house (recall that 90% of the households in our sample are home-owners). The modeled relationship is shown in Figure 16 in Appendix E – the share of wealth held in housing is high for those with the lowest wealth levels (approximately 0.9), falling to 0.65 for those with £1m of non-pension wealth.

Return on housing To specify the return on housing (given in equation (2)) we need to specify two rates of return (r^{hcg} – the housing capital gain and r^{hr} – the rental income from owner occupied housing), the mortgage interest rate (r^{mort}) and the leverage ratio ($lev(t)$).

In setting rates of return we follow the broad approach outlined by Kaplan and Violante (2014). The average real capital gain over the period 1976 to 2002 – the first year that we have data to the year in which our sample of households are observed – was 3.2% (Nationwide Building

Society (2014)). Our model abstracts from housing risk, so we follow Kaplan and Violante (2014) by subtracting the variance of returns, giving a real capital gain of 2.28%. The service flow from housing is calculated using the ratio of the estimate of aggregate housing consumption to the estimate of the value of the housing stock in the National Income and Product Accounts. This yields an average of 8.4%. We risk-adjust by subtracting the variance of returns and also subtract depreciation (1%, using data from the National Income and Product Accounts) and insurance costs (0.35%, taken from Kaplan and Violante (2014)). This yields a return of 7.02%.¹³

The mortgage rate is set at 5.9% and is calculated using data from the Bank of England (Bank of England (2017)). This is the average rate over time and over all products where historical data is available. Finally, the leverage ratio ($lev(t)$) is set in a manner similar to the housing share. Using the Wealth and Asset Survey we regress the observed leverage ratio on a quadratic in age, as well as cohort and time dummies. The estimated function is given in Figure 17 in Appendix E.

Pension fund returns We base the mean and standard deviation of pension fund returns on an index known as the “DCisions index”. This is an index of total fund returns that reflects the asset allocation decisions made by leading DC pension plans in their default investment strategies. This index provides information on returns stretching back to 1994. For years prior to 1994 when the DCisions index is not available, we estimate ϕ_t using the FTSE all-share index (on which data is available back to the early 1960s) and the ratio between the FTSE all-share index and the DCisions index over the period where both are available (1994 - 2010). We discuss how this is estimated in Appendix H. We use the mean and standard deviation of this time series in our model. These parameters are, respectively, $\bar{\phi} = 3.97\%$ and $\sigma_\phi = 13.8\%$.

Unemployment rate The period in our model is a year. We consider a household in the data to be unemployed for a year if they have total earnings of less than £4,402 - the level of unemployment benefit payable to an unemployed couple in 2002/03. In our data, the incidence of unemployment, so defined, between the ages of 25 and 50 is 8.4%, 6.5% and 6.4% for our low-, medium- and high-education groups respectively. These are the unemployment probabilities

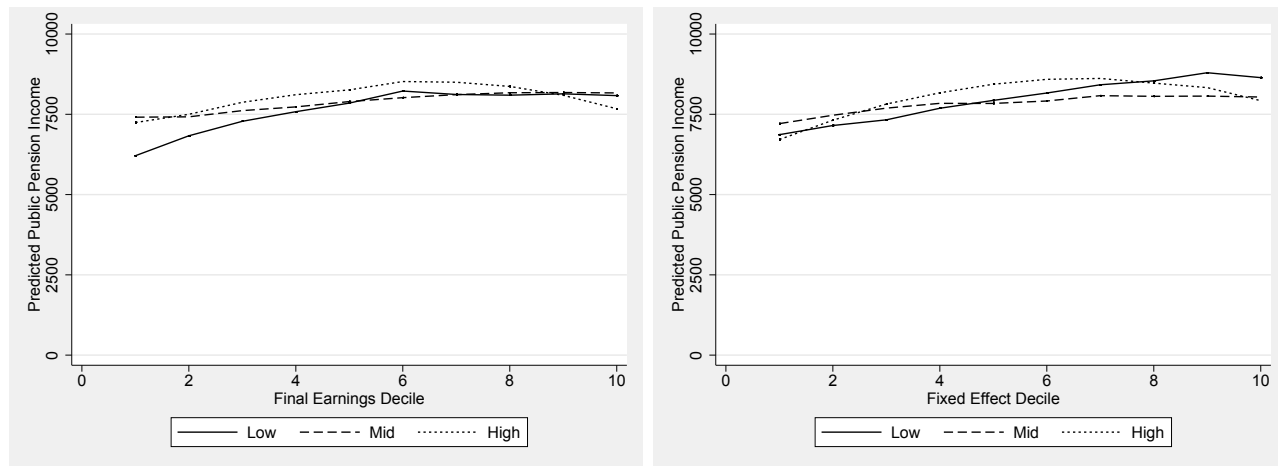
¹³This is the consumption value from having access to owner-occupied housing. We assume that this value does not increase as the value of a given quantity of housing increases. Therefore, households earn this 7.02% return on the value of their current stock of gross housing in 2002 housing prices (all other quantities in the model are also expressed in prices of this year). We convert housing wealth in other years to these prices by deflating (inflating) values in years after (before) that year by r^{hcg} per year.

used in the model.

Public Pension We model public pension entitlements as a quadratic in decile of fixed effect and decile of ‘final earnings’ where the coefficients are allowed to vary by education. Final earnings in the data are measured as the average decile of the last five years of observed earnings. For estimating this process we only use data on those aged over 60 (for whom public pension entitlements are largely determined).

Figure 1 illustrates the relationship between modeled public pension entitlements and each of final earnings and earnings fixed effect. In the left hand panel we give predicted public pension as final earnings decile varies (where for each earnings decile we set the fixed effect decile equal to its average for that group). In the right hand panel we give public pension as fixed effect decile increases (with final earnings similarly set at its group-specific mean). In the case of both state variables, there is only a slight gradient with respect to decile.

Figure 1. Public pension process



Annuity rates The actuarially fair annuity rates are calculated using survival probabilities (the estimation of which are described below) and the risk-free interest rate. The administrative load is assumed to be 10% of the value of the DC fund to be annuitized. This is taken from Murthi et al. (1999) who apply the methodology of Mitchell et al. (1999) to the UK. We give the annuity rates in Appendix I (with the formula given in Section 3 of the online supplemental material accompanying this paper).

4.1.2 Demographic and preference parameters

Survival probabilities Our objective is to have survival curves for each gender and education group for our cohort of interest. The UK’s Office for National Statistics (ONS) produces *period* and *cohort* survival curves by gender but not by education. Period survival curves give survival probabilities by age given the age-specific mortality rates at a particular point in time – they therefore make no allowance for any later actual or projected changes in mortality. Cohort survival curves give survival probabilities by age for a given cohort and allow age-specific mortality rates to vary for known or projected changes in mortality over time. Cohort survival curves are thus more appropriate for our purposes.

We use observed deaths in the ELSA panel between 2002/03 and 2012/13 to estimate a set of *period* survival curves by gender and education. Using these we calculate the difference between these education-age-gender period survival probabilities and the ONS’ age-gender period survival probabilities. We then adjust the ONS’ age-gender cohort survival curves (for the 1945 cohort) using these differences to obtain education-age-gender cohort survival probabilities.

Equivalence scale The number of equivalent adults in a household n is set using the ‘modified OECD equivalence scale’ (see Anyaegbu (2010) for a discussion). The first adult in a household counts for one equivalent adult, subsequent adults and children aged 14 and over count for half an equivalent adult, while children aged 13 or younger account for 30% of an equivalent adult.

Coefficient of relative risk aversion The coefficient of relative risk aversion has proved a difficult coefficient to identify.¹⁴ Our approach is to set the coefficient of relative risk aversion to 3, following the most closely-related work (Scholz et al. (2006)).

Summary These parameters of the model and values assigned are summarized in Table III.

4.2 Step 2: Estimation

4.2.1 Estimation of earnings processes

To estimate the parameters of the earnings process we aggregate individual earnings histories into household earnings histories. We then divide households into three groups according to

¹⁴See Chiappori and Paiella (2011) for a review and recent empirical contribution.

Table III. Parameterization

Parameter	Symbol	Value/Source
Unemployment rate	$\{\pi\}_{j=1}^3$	8.6%, 6.5%, 6.4%
Return on cash	r^c	1.6%
Housing capital gain	r^{hcg}	2.3%
Housing rental yield	r^{hhr}	7.0%
Mortgage interest rate	r^{mort}	5.9%
Mean return on DC fund	$\bar{\phi}$	4.0%
Variance of return on DC fund	σ_ϕ^2	13.8%
Survival probabilities	$s_t^{j,m}, s_t^{j,f}$	ONS Life Tables adjusted for education using survival differences by education observed in ELSA
Administrative load on annuities	z	10%
Coefficient of relative risk aversion	γ	3
Equivalence scale	n	Modified OECD scale

the education of the man in the couple (indexed by j). The parameters to be estimated are $\{\alpha_i\}_{i=1}^N$ and $\{\delta_1^j, \delta_2^j, \rho^j, \sigma_j^2\}_{j=1}^3$.

To allow for measurement error in earnings, we augment the earnings process given in equation (1) with an *iid* measurement error term m_{it} . The assumed data generating process for our earnings data is given in equations (11) to (13):

$$\ln \tilde{e}_{it}^{data} = \alpha_i + \delta_1^j t_i + \delta_2^j t_i^2 + v_{it} \quad (11)$$

$$v_{it} = u_{it} + m_{it} \quad (12)$$

$$u_{it} = \rho^j u_{it-1} + \xi_{it} \quad (13)$$

The approach to estimation is a standard one (see, for example, Low et al. (2010)). It involves first running a fixed effects regression and estimating the household fixed effect and quadratic in age. Residuals (r) are then obtained:

$$r_{it} = \ln \tilde{e}_{it}^{data} - \hat{\alpha}_i - \hat{\delta}_1^j t_i - \hat{\delta}_2^j t_i^2$$

The parameters of the wage process are obtained by choosing those that minimize the distance

between the empirical covariance matrix of differences in these residuals and the theoretical covariance matrix implied by equations (11) to (13). Estimates of the parameters of the earnings process for each of three education groups are given in Table XV in Appendix I.

4.3 Estimation of discount factor

Conditional on a discount factor, the model solution gives decision rules for each household. We can use these rules, combined with realized earnings and investment returns that they receive to simulate behavior (consumption, savings in each asset and therefore total wealth ($A = a + DC$)) at each age:

$$A_{it}^{sim} = f_i\left(\beta_i, \theta_i, \{e_{i,\tau}\}_{\tau=1}^t\right) \quad (14)$$

where β_i is patience, θ_i is household type and $\{e_{i,\tau}\}_{\tau=1}^t$ contains earnings shocks up to t . The discount factor estimate is that for which simulated wealth and observed wealth coincide in 2002 (when wealth is observed).

$$A_{i,2002}^{data} = A_{i,2002}^{sim} = f_i\left(\hat{\beta}_i, \theta_i, \{e_{i,\tau}\}_{\tau=1}^{2002}\right) \quad (15)$$

We calculate a discount factor for all households whose wealth can be rationalized with a discount factor in the range 0.5 to 1.5. 97.3% of households in our sample have wealth levels that can be rationalized with patience in this range (and 96.0% in the narrower range of 0.8 to 1.2).

5 Results

The model laid out in the previous section has three purposes. First, it will be used to estimate the distribution of discount factors which would rationalize observed household wealth holdings. Second, it will be used to understand the importance of accounting for household portfolios in models of saving for retirement. Third, it will be used to document a link between measures of preparedness for retirement used by policy-makers (replacement rates) and discount factors, the preference parameter which bears most heavily on the preparedness for retirement of agents in lifecycle models.

Table IV. Distribution of discount factors - baseline models

Model	Mean	p10	p25	Median	p75	p90
one asset	1.052	0.980	1.011	1.042	1.080	1.125
no housing treatment	1.002	0.949	0.976	0.999	1.021	1.052
modeled housing	0.947	0.894	0.923	0.950	0.975	1.002

5.1 Estimates of discount factors

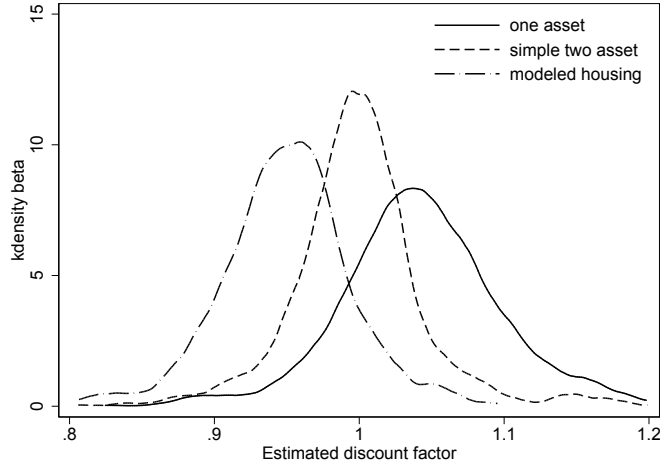
Table IV summarizes the distribution of discount factors for each of three versions of our model. The first row, which we give as a benchmark, gives estimates derived from a simple one-asset lifecycle model where all wealth accrues the risk-free rate (1.6%) as a return. The median discount factor is 1.042 with an interquartile range stretching from 1.011 to 1.080. These rates are extremely high relative to those which have been found in the literature. The second row adds the capacity for agents to save in an illiquid tax-advantaged pension asset, which earns an expected return that is higher than the safe return but is risky. All of non-pension wealth still earns the safe rate of return. The median estimated discount factor falls to 0.999. Additionally the distribution shows less variation, with an interquartile range of 0.976 to 1.021, as part of heterogeneity in wealth is explained by differential (across households of different income and with different fertility profiles) incentives to avail of the tax breaks associated with pension saving.

Neither model takes into account that much of non-pension wealth is held in housing. The third row gives estimates from our preferred version of our model that, in addition to including the pension asset, also calibrates the return on non-pension wealth to take into account housing (as discussed in Section 3.1.4). The median discount factor is 0.950, with an interquartile range of 0.923 to 0.975. Figure 2 illustrates the three distributions summarized in Table IV.

5.1.1 Validation of discount factor estimates

The discount factors we estimate are identified by fitting *total* private wealth simulated by our model to observed *total* private wealth holdings. One test of the model is therefore how holdings of individual components of wealth fit the data. Figure 3 illustrates how modeled non-pension wealth (left-hand panel) and modeled pension wealth (right-hand panel) compare to that observed in the data for each household. Non-pension wealth (which includes housing and non-housing assets) is strongly clustered around the 45-degree line - the correlation coefficient

Figure 2. Distribution of discount factors



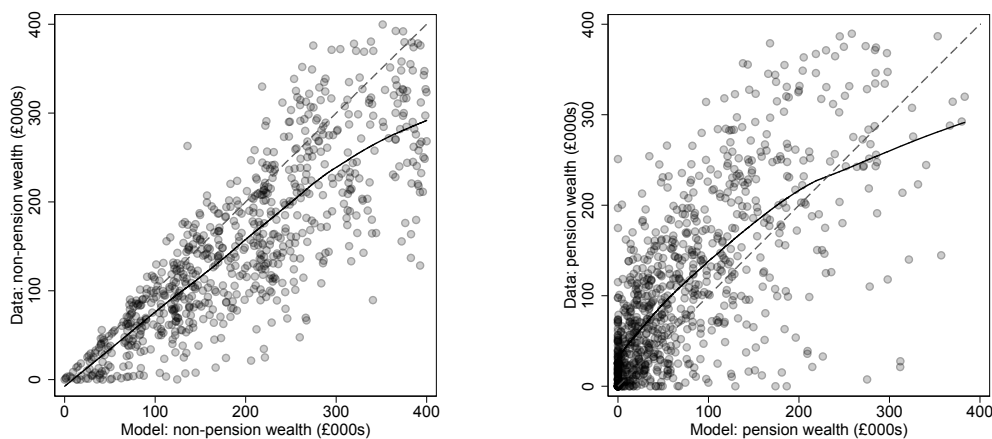
between modeled and observed non-pension wealth is 0.8577. Pension wealth is more clustered around low levels, but is also strongly correlated, with a correlation coefficient of 0.7282. Table V shows the mean holdings of each of pension and non-pension wealth in the data and the mean holdings implied by the model by decile of lifetime earnings. Figure 9c in Appendix B uses the quantities in this table to graph the ratio of mean pension wealth to mean total wealth – showing that, in addition to matching the level, the model also replicates the increase in the pension share by lifetime wealth. The model therefore performs well in terms of fitting household portfolio choices.

An additional test of the validity of our estimates is that we can use the breadth of data collected in ELSA to validate the estimated discount factors using information that is not used at all in the modeling process. More specifically, we can examine how the estimated discount factors vary with individual characteristics thought to correlate with, or be indicative of, individuals’ time preference. We examine three particular characteristics: education, self-reported financial planning horizon, and current and former smoking behavior.

The left-hand panel of Figure 4 illustrates how the distribution of estimated discount factors varies for our three education groups. The distribution of those with lower education lies to the left of those with more education. This is consistent with the bulk of existing literature on education and discount factors (for example, Lawrance (1991); Cagetti (2003); Dohmen et al. (2010)).

The right-hand panel of Figure 4 illustrates how the distribution of estimated discount factors varies according to individuals’ answers to the question “In deciding how much of your income to

Figure 3. Portfolio composition – preferred model (housing modeled)



Notes: Each dot represents a household. Solid line is a lowess plot. Dashed line is the 45-degree line.

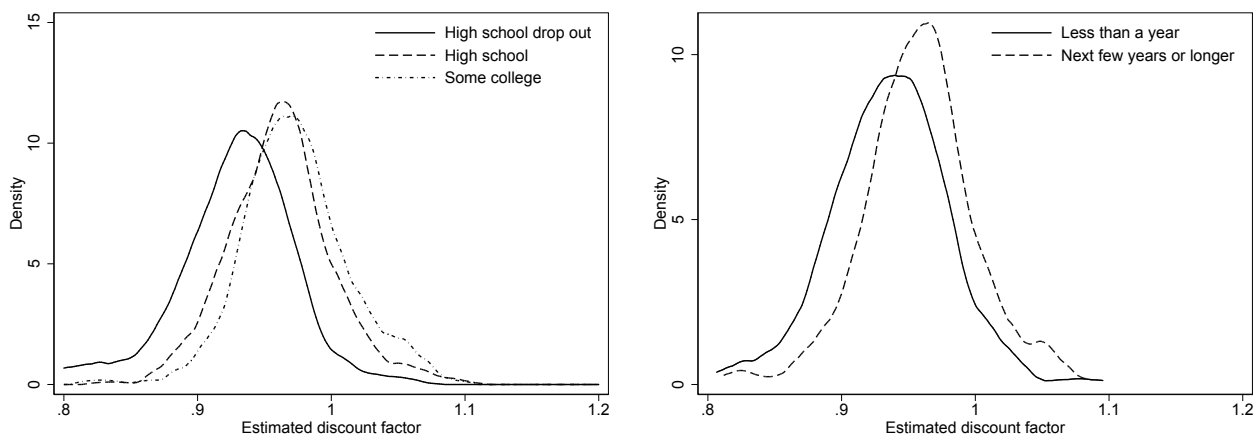
Table V. Observed and modeled wealth, Means (£,000s)

	Total		Pension		Non-pension	
	Data	Modeled	Data	Modeled	Data	Modeled
All	387.4	387.3	142.9	120.8	244.5	266.5
<i>Lifetime earnings decile:</i>						
Lowest	172.8	172.9	41.2	22.6	131.6	150.2
2	218.9	218.6	66.3	45.3	152.6	173.3
3	240.1	240.0	78.5	50.3	161.6	189.8
4	277.4	277.0	94.9	72.7	182.5	204.3
5	296.5	296.2	110.6	75.2	185.9	221.0
6	343.7	343.3	121.2	95.1	222.5	248.2
7	376.5	376.6	132.3	104.5	244.3	272.1
8	413.4	413.3	148.7	126.3	264.8	287.0
9	619.3	619.2	241.3	216.8	378.1	402.4
Highest	881.7	882.6	378.3	384.6	503.4	498.0

The totals in the data columns differ slightly from those in Table II as the sample used here are only those for whom we can rationalize observed wealth holdings with a discount factor between 0.5 and 1.5.

spend or save, people are likely to think about different financial planning periods. In planning your family’s saving and spending, which of the following time periods is more important to you?” The options given to respondents are: the next few weeks, the next few months, the next year, the next few years, the next 5-10 years, longer than 10 years. For ease of illustration we group individuals into those who respond with a period up to ‘the next year’ and those who respond with a period ‘the next few years’ or longer (a finer categorization is used later in Table VI). Those reporting longer planning horizons are estimated to have higher discount factors. This is strong validation that at least some of the heterogeneity in discount factors estimated through the modeling process is indeed reflective of heterogeneity in individuals’ preferences. Samwick (1998) shows the relationship between a similar question and his estimates of discount rates and also finds that the estimated and stated measures of patience correlate.

Figure 4. Validation of discount factors - distributions by education



In Table VI, we report the results of median regression analysis examining the correlation between estimated discount factors and individuals’ self-reported planning horizon, education level, smoking behavior and these three characteristics simultaneously. These confirm statistically significant differences in median discount rates between the groups illustrated in Figure 4 and also show that estimated discount factors are significantly negatively correlated with current smoking behavior (see Khwaja et al. (2007) which documents, in a different setting, a correlation between some measures of time preference and smoking behavior). We also find that the correlation between discount factors and each characteristic holds up even when the others are being controlled for. We take this as evidence that the distribution of discount factors we have estimated is indeed reflective of heterogeneity in time preferences.

Table VI. Median regressions of estimated discount factor on household characteristics

	(1)	(2)	(3)	(4)
Planning (weeks)	—			—
	(—)			(—)
Planning (up to a year)	0.017***			0.011**
	(0.005)			(0.005)
Planning (a few years)	0.033***			0.019***
	(0.005)			(0.005)
Planning (5+ years)	0.036***			0.023***
	(0.005)			(0.004)
Low educ.		—		—
		(—)		(—)
Mid educ.		0.030***		0.027***
		(0.003)		(0.003)
High educ.		0.040***		0.033***
		(0.004)		(0.004)
Never smoked			—	—
			(—)	(—)
Former smoker			-0.007	-0.004
			(0.004)	(0.003)
Smoker			-0.018***	-0.007*
			(0.005)	(0.004)
Constant	0.923***	0.932***	0.956***	0.921***
	(0.004)	(0.002)	(0.003)	(0.004)
Observations	954	943	964	930

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

5.2 Household portfolios and saving for retirement

5.2.1 The role of pensions

The previous section illustrated the importance of the asset structure of our lifecycle model for the estimated distribution of discount factors. To examine the role of household portfolios in more detail, we can use our model to evaluate what would happen, at our estimated discount factors, if some of the features of the asset structure we have added were removed. Table VII shows how the distribution of each of total private wealth and its two components, pension wealth and non-pension wealth, vary with the characteristics of the model's asset structure. We show four scenarios:

1. Our baseline
2. Removing the pension tax advantage. Pension saving is now done out of net income, and yields a non-taxable annuity in retirement (rather than being done out of gross income and yielding a taxable annuity). The pension remains risky and earns a higher expected return than cash.
3. Removing the pension asset entirely (and so retirement funds are all saved in the composite non-pension wealth asset).
4. Additionally, removing access to housing (and so all wealth is held in cash)

Comparing the baseline with the third row of the top panel shows that removal of the pension would, holding the distribution of discount factors constant, reduce mean accumulated wealth from approximately £451,000 to approximately £353,000. The proportionate reduction is greater at the top of the wealth distribution (where the fall was approximately 20% at the 90th percentile) than at the middle and bottom (where the fall was just over 10% at the median and just under 10% at the 10th percentile). The result that, conditional on a set of discount factors, wealth accumulation would be less in the absence of access to the pension is isomorphic to the result presented in Section 5.1 that estimated discount factors would be higher if no access to the pension was assumed.

Pension wealth differs from non-pension wealth in two respects that are quantitatively important for increasing saving. First it is treated advantageously by the tax system. Second,

it has a higher expected return than the cash portion of non-pension wealth.¹⁵ The second row of each panel in Table 7 allows the effects of these two aspects to be disentangled. Of the approximately 22% fall in mean wealth with the removal of the pension asset (from £450,900 to £353,600), two-fifths is due to the favorable tax treatment (removal of which would lead to a fall from in mean wealth from £450,900 to £411,200); with the remaining three-fifth due to the returns available from the risky asset.

Finally, removing access to housing (which forces all of non-pension wealth to be held in cash and so substantially reduces the rate of return on offer) dramatically reduces wealth further. Mean wealth would fall to £120,700 and private wealth in the bottom 10 percent is very close to zero. Those at the bottom of the lifetime earnings distribution have reasonable earnings replacement from the public pension as well as an asset-tested transfer that essentially taxes wealth accumulation over a certain range. Both of these factors mean that they have limited incentives to save for retirement; these incentives are further reduced in the counterfactuals studied here.

We can use these counterfactual estimates to place our results in the context of the literature which estimates the extent to which household responses to pension tax incentives represent new saving or whether households respond to them by saving more in pension wealth at the expense of other forms of wealth. Poterba et al. (1995) and, more recently, Gelber (2011) argued that 401(k) saving is largely ‘new’ saving that would not have been saved in the absence of the savings vehicle being offered; while Engen et al. (1994) and, more recently, Chetty et al. (2014) argued that subsidies largely increase pension wealth at the expense of other forms of wealth. Our results are intermediate to these sets of studies – the approximately £75,000 fall in pension wealth as a result of removing tax advantages (from £192,100 to £117,200) is offset by an increase in non-pension wealth of approximately £35,000). Additionally, the fall in pension income will be (proportionately) less than the fall in pension wealth due to the fact that the figure after the removal of the tax advantages represents already-taxed income. Therefore, our estimated model implies that tax incentives toward pension saving incentivize households to switch from non-pension wealth to pension wealth to a greater extent than incentivizing them to do new saving (Engen et al. (1994), Chetty et al. (2014)).

¹⁵In our setting the pension asset also gives access to an annuity though the effect of this on wealth accumulation is small.

Table VII. Distribution of modeled wealth: pension tax advantage counterfactual analysis at age 64

	p10	p25	Median	p75	p90	Mean
<i>Total wealth</i>						
Baseline	85.0	185.6	334.6	578.5	883.6	450.9
No pension tax adv.	81.2	172.9	315.8	522.7	797.9	411.2
No pension asset	77.8	164.5	295.3	480.1	711.3	353.6
No pension and no housing	0.5	25.4	80.2	168.0	279.1	120.7
<i>Pension wealth</i>						
Baseline	14.7	49.9	118.0	240.2	394.7	192.1
No pension tax adv.	0.0	19.6	60.7	137.0	237.4	117.2
No pension asset	0.0	0.0	0.0	0.0	0.0	0.0
No pension and no housing	0.0	0.0	0.0	0.0	0.0	0.0
<i>Non-pension wealth</i>						
Baseline	59.1	126.8	223.8	350.7	508.2	258.8
No pension tax adv.	78.9	148.1	243.6	394.7	568.6	294.0
No pension asset	77.8	164.5	295.3	480.1	711.3	353.6
No pension and no housing	0.5	25.4	80.2	168.0	279.1	120.7

5.2.2 The potential role of portfolios in previous estimates of ‘oversaving’

Scholz et al. (2006) used microdata similar to that which we use to compare the retirement saving of a now retired cohort to a benchmark (which they refer to as ‘optimal’ wealth) which is pinned down by a single-asset lifecycle model with a homogeneous discount rate assumed to be approximately equal to the interest rate.¹⁶ They find that that a large majority of households (almost 85%) had accumulated more than this benchmark – that is that there was little evidence of households having undersaved for retirement.

In this section, we implement a similar exercise to theirs. Rather than allow an estimated discount factor to explain the heterogeneity in wealth accumulation as we do in the rest of this paper, we set the discount rate for all households equal to the risk-free interest rate (1.6%, and so $\beta = 1.016^{-1}$), and evaluate observed wealth relative to a benchmark level produced by the resulting parameterized model.

¹⁶The interest rate assumed was 4%, with a discount factor of 0.96.

We first do this using a one-asset model in which households can only save in cash. Notwithstanding the fact that our cohort is a different one and our setting is the UK (where the public pension system differs – especially for higher earners) we get a very similar result: 87.8% of households have ‘oversaved’ relative to a benchmark where the discount rate is set equal to interest rate.¹⁷ Additionally, as Scholz et al. found, the deficits among the undersavers are much smaller than the surpluses among the oversavers – among the 12% of households who have undersaved, the median deficit is £31,500, while the median surplus among the oversavers is about £185,000.

In generating this result, we have followed Scholz et al. (2006) in setting the discount rate equal to the (risk-free) interest rate in determining the benchmark against which accumulated wealth will be compared. Adding any feature to the model which increases the return on saving (either by introducing an asset that tends to have a higher return, or by introducing an asset which has tax breaks associated with it) will, all else equal, increase the benchmark required to have greater than ‘optimal’ saving, and so reduce the proportion of households found to be oversaving. Scholz et al. anticipate that portfolio structure is a contributory factor in generating their result – when they increase the rate of return to 7%, the rate of oversaving falls to approximately 64.1%, and when they (separately) exclude half of property wealth from their data, the rate of oversaving falls to 61.2%. We showed in the previous section the effect that adding our additional assets would have on saving, conditional on estimated discount factors; here, we compare here how adding our additional features to the portfolio structure affects adjudications of over-saving relative to a model that is parameterized with a fixed homogeneous discount factor.

Figure 5 and Table VIII summarize saving, relative to the benchmark, for three models: i) the one-asset model, in which all wealth accrues a return equal to the discount rate, ii) a two-asset model, which adds the tax-advantaged private pension, and iii) our preferred model, in which the return on non-pension wealth is set to take into account the high (and typically leveraged) returns that housing has accrued. Figure 5 shows a scatter of observed and ‘optimal’ wealth for each model. Table VIII shows median wealth (data and optimal), the percentage

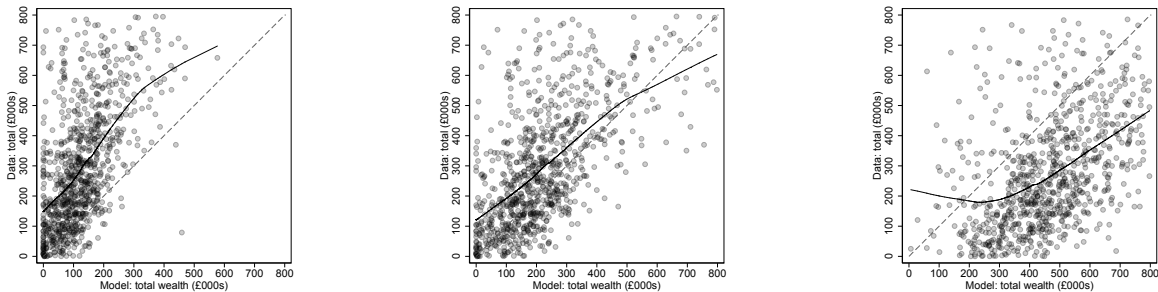
¹⁷There are some other differences between our model and that of Scholz et al. (2006). First they assume Defined Benefit wealth is exogenous – we treat it as if it has accumulated as DC wealth – though we show in Appendix C that our conclusions are essentially unchanged if we treated it as exogenous. Second, reflecting institutional differences between the US and the UK, they additionally model the risk of out-of-pocket medical expenses in retirement.

oversaving, conditional deficits and surpluses and the R^2 from a regression of observed wealth on predicted wealth and a constant.¹⁸ The proportion oversaving falls from 87.8% to 65.4% when the private pension is added (as each of higher expected returns and the tax incentives increase the benchmark against which we compare optimal wealth). Taking account of potential housing returns reduces this to 18.1%, with the median deficit now larger than the median surplus.

Table VIII. Comparison to Scholz et al. (2006)

Spec	Median wealth		Percent oversaving	Median surplus	Median deficit	R^2
	Observed	Optimal				
One asset	280.7	117.9	87.8	185.0	31.5	0.366
Simple two asset	280.7	203.8	65.4	133.2	65.6	0.398
Modeled housing	280.7	482.9	18.1	198.0	238.2	0.306

Figure 5. Comparison of Modeled and Optimal Wealth with Homogeneous Discount Rate set equal to safe rate of return



From this we take that the modeled portfolio structure matters substantially in exercises that use lifecycle models to derive benchmarks against which to evaluate household saving. For two reasons, we do not, however, take these results to challenge the central contention of Scholz et al. (2006) – that their cohort of interest have, for the most part, prepared well for retirement. First, housing wealth has appreciated to a much greater extent in the UK than in the US and so the effect of adding housing might be more modest in their context. Second, while enriching the model’s asset structure to account for incentives to save more for retirement raises the bar against which realized wealth will be assessed, the fact that they set the discount rate equal to the model’s interest rate (reflected in our setting by setting the discount rate equal to the risk-free interest rate) makes the bar a very demanding one – households are evaluated against

¹⁸Our models explain less of the variation compared to Scholz et al. (2006), who report an R^2 of 86%.

a benchmark in which they are no more impatient than they are compensated for by risk free returns. We show in the next section that most households who are more impatient than this still replace a reasonable share of their lifetime average earnings in retirement.

The fact that most of our households are found, in our baseline, to have undersaved relative to the modeled benchmark is isomorphic to the fact that we estimate that most households have an estimated discount factor of less than 0.984 (that used to derive the benchmark). In the next section we compare those estimated discount factors to another benchmark used to evaluate whether households have saved appropriately for retirement – replacement rates.

5.3 Household preparedness for retirement: comparing replacement rates and patience estimates

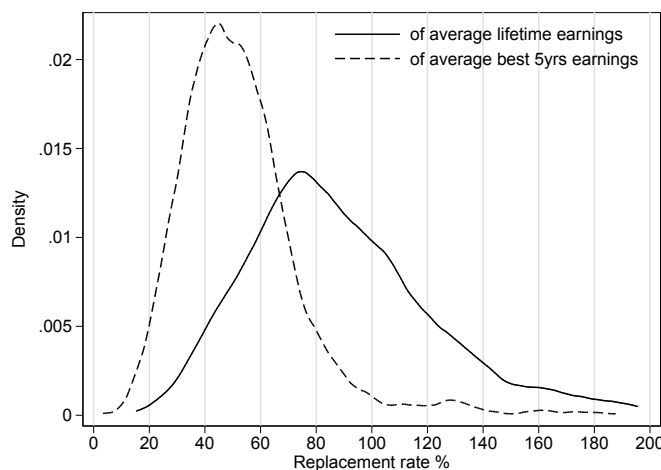
The rate of replacement of lifetime average or final earnings by pension income in retirement is a metric often used by policy-makers and private pension providers to make assessments of individuals' preparedness for retirement. There are strong theoretical arguments why optimal saving behavior would typically lead to replacement rates of less than 100%: one no longer needs to save for retirement, children have become more self-sufficient and average tax rates on retirement income tend to be lower than on income during working life. However, the actual replacement rate desired varies across individuals, depending on their circumstances and preferences. This means that the benchmark target thresholds used to assess savings adequacy by policy makers and the industry are at best necessarily ad hoc and at worst 'conceptually flawed' (Scholz and Seshadri (2009)). However, because of their simplicity and a lack of viable alternatives, replacement rates continue to be extensively used as a basis for which to organize retirement planning advice (see TIAA (2018) and online calculators by many other providers of retirement saving products) and as an official basis for evaluating the retirement preparedness of those approaching retirement (Bridges and Choudhury (2005), Pension Commission (2004)).

The data we use in this paper, on lifetime earnings and late life wealth holdings, is ideally suited to examining the likely retirement income replacement rates of our cohort of couples. To do so we project forward current public and private pension entitlements, assuming that individuals maintain their current labor market activity and private pension savings behavior from when they are observed in 2002/03 to when they reach age 65. We can then compare gross pension income (we consider broader measures of retirement income below) at age 65 with measures of average gross working life earnings.

The distribution of two definitions of replacement rates is shown in Figure 6. The solid line shows the distribution of pension income at age 65 relative to average (real) working life earnings between ages 20 and 64. Replacement rates, even on this narrow measure (which only include public and private *pension* income in the numerator), tend to be high, with a median of 84.5% and only about a third of households having pension income in retirement of less than 70% of their average lifetime earnings. This indicates that households in our sample appear well prepared for retirement, in the sense that they will be little worse off in retirement compared to the average of their working life.

How can we rationalize this with the popular policy concern about systematic undersaving? At least in part this is likely to be caused by industry focusing on households being able to have similar levels of net income in retirement to what they had towards the end of working life. The dashed line in Figure 6 shows the distribution of total pension income at age 65 relative to average earnings in the best five years of working life. We use this denominator to proxy for ‘final earnings’ since we do not observe hours worked in our data, and we do not want a measure of final earnings that is biased downwards by individuals reducing their hours as they move into retirement. Since this denominator is obviously higher than average working life earnings, the resulting replacement rates are lower, with a median of just below 50%. On this definition over four-fifths of households would have a replacement rate of less than 70%, more consistent with the general consensus of concern.

Figure 6. Distribution of estimated retirement replacement rates



It is questionable though, whether encouraging households to aim to replace such proportions out of their *final* earnings is sensible – this is essentially encouraging all households to act

in the manner that only those who are the most patient act. Table IX illustrates this. It shows median replacement rates, by decile of estimated discount factor on three definitions of retirement income: pension income only, pension income plus the annuitized value of liquid wealth (that measure of wealth that was described as ‘cash’ in the model), and pension income plus the annuitized value of all wealth (‘cash’ plus housing). The panel on the left shows median replacement rates out of average lifetime earnings; that on the right shows replacement rates out of the best five years earnings.

Three points are apparent from the table. The first, which is simply an implication of the manner in which the discount factors are estimated, is that replacement rate rise with decile of estimated discount factor. Second, the replacement rates out of lifetime earnings are reasonable for even for those who are estimated as impatient. Even those in the lowest decile of patience ($\beta < 0.89$) have median replacement rates of 61%, 63% and 73% on, respectively, the narrowest, middle and broadest measure of retirement income. On the middle definition, by the second decile the median replacement rate is above 70%; by the sixth decile it is above 100% – emphasizing once again that this is a cohort that will not have systematically lower living standards in retirement than over the course of their working life.

Third, most of the distribution of replacement rates out of the best five years earnings would not meet the thresholds suggested by policy-makers and industry even among households who are very patient. On the measure of pension income which includes annuitized liquid wealth but not housing, the median replacement rate is less than 50% for the bottom three deciles, and it does not reach 70% until the ninth decile. This suggests that policymakers and industry should be cautious of their common use of standard replacement rates applied to measures of later-life or final earnings. Revealed behavior indicates that households, across most of the distribution of patience, aim for much lower replacement rates. Attempting to shift most households towards providing a replacement of the order of 70% or so applied to measures of later-life or final earnings, essentially involves attempting to make them act as if they are much more patient than they actually are and even if successful, risks imposing welfare costs early in life.

6 Conclusions

In this paper, we have examined the presumption that households are ‘undersaving’ for retirement that has, implicitly or explicitly, motivated a significant quantity of international policy reforms to encourage private saving.

Table IX. Median income replacement rates by estimated discount factors

	Replacement of average lifetime earnings			Replacement of average best 5yrs earnings		
	(1)	(2)	(3)	(1)	(2)	(3)
	Pension	...+ 'cash'	...+ housing	Pension	...+ 'cash'	...+ housing
All	0.845	0.978	1.373	0.498	0.582	0.786
<i>Decile of estimated discount factor:</i>						
1 [$\beta \leq 0.89$]	0.610	0.630	0.732	0.367	0.385	0.424
2 [$0.89 < \beta \leq 0.92$]	0.701	0.730	0.924	0.422	0.446	0.572
3 [$0.92 < \beta \leq 0.93$]	0.720	0.767	1.042	0.451	0.483	0.667
4 [$0.93 < \beta \leq 0.94$]	0.776	0.882	1.172	0.485	0.548	0.711
5 [$0.94 < \beta \leq 0.95$]	0.811	0.915	1.293	0.488	0.539	0.791
6 [$0.95 < \beta \leq 0.96$]	0.910	1.020	1.422	0.542	0.602	0.822
7 [$0.96 < \beta \leq 0.97$]	0.945	1.167	1.585	0.567	0.656	0.965
8 [$0.97 < \beta \leq 0.98$]	0.977	1.264	1.714	0.551	0.677	0.962
9 [$0.98 < \beta \leq 1.00$]	1.121	1.528	2.075	0.579	0.821	1.188
10 [$\beta > 1.00$]	1.307	2.452	3.712	0.602	1.153	1.675

Using a lifecycle model of consumption, non-pension and pension saving, combined with rich micro-data from survey and linked administrative sources, we have estimated the distribution of household discount factors that would be required to rationalize observed late-life wealth holdings, given lifetime earnings histories, for a sample of English households. Using our preferred specification of the model we find a median discount factor of 0.95 and an interquartile range of 0.92-0.97 – close to conventional values estimated or assumed in the literature. We show how heterogeneity in discount factors relates to heterogeneity in replacement rates and find that even those with low estimated patience have modest replacement rates out of lifetime income, while even among those who we estimate to be the most patient many fail to have accumulated sufficient funds to replace much of their peak (or final) earnings. This calls into question the rationale for the focus by the pension industry and by some policymakers on the ability of households to replace the level of income enjoyed towards the end of working life, rather than

over the whole of working life.

While we interpret our results as suggesting that concerns about undersaving relative to conventional benchmarks might be exaggerated, we do not mean to suggest that there is no one for whom undersaving for retirement is a concern. We conclude with three notes of caution. First, for reasons to do with data availability, our sample only includes couples and we do not model divorce risk. There may be particular at-risk groups outside of this population, such as those who find themselves single late in life (see Munnell et al. (2018) for a recent discussion of the association of divorce and retirement security).

Second, we do not account for the risk of needing to pay for long-term care in retirement in our model (while there is universal care of hospital and out-patient health costs in the UK, nursing home care is only paid for those with the lowest assets). De Nardi et al. (2010) have shown that the risk of large expenses of nursing home stays explains the fact that households do not decumulate wealth in retirement. However, Kopecky and Koreshkova (2014) show that the effect of nursing home care risks on the wealth distribution is modest – explaining about 6.7% of aggregate wealth holdings – with approximately half of that accounted for by wealth accumulated pre-retirement.¹⁹ This latter fact reassures us that our abstraction from these risks will have only minor impacts on our estimated measures of patience. However, it remains an open question whether those risks *should* bear more heavily on saving given that medical costs and the time spent in care might both be higher for future cohorts.

Finally, our sample represents those born in 1940s. Hood and Joyce (2013) document lower levels of wealth accumulation by younger cohorts in the UK relative to the cohort we study. Younger cohorts might differ in the extent to which they discount the future, the rates of return that they will expect and experience and the government policies that support incomes in retirement (public pension benefits, tax incentives for private pension saving). Any of these could lead to those cohorts having lower living standards in retirement than those which our cohort of interest will have.

¹⁹Supporting this evidence of a modest impact of this risk on wealth accumulation, we note that ELSA respondents in 2016 were asked “Have you thought about how to pay for your care and support needs?”. Of those aged 52 to 62 (the ages of those in our sample), 63% say that they have not thought about it at all, and 31% say that they have thought about it a little. Fewer than 6% say that they have thought about it a lot.

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A UK Public Pension System

The UK public pension is complicated and has been reformed many times over the past fifty years. For the cohort we are concerned with, those born in the 1940s, the public pension consists of two components:

1. Entitlement to the first and largest component, known as the ‘Basic State Pension’ depends only on the number of years an individual has been in work (or been doing other ‘creditable’ activities – such as caring for a young child). Those with a full working life between ages 16 and 64 (or those with 30 or more years for those born after 1945) would be entitled to a fixed amount worth around £3,800 per year in 2002 while those with a less complete work history would receive an amount that was pro-rated by their period in work.
2. The second component, introduced in 1978 and reformed slightly on two occasions subsequently, provided an earnings related aspect. This was known variously as Graduated Retirement Benefit, the State Earnings Related Pension and the Second State Pension. This gave individuals approximately 20% of earnings between a lower threshold (at approximately the 8th percentile of positive earnings) and an upper threshold (at approximately the 80th percentile of positive earnings).

The second (earnings-related) component accounts for approximately 20% of entitlements. The first (closer to flat-rate) components accounts for approximately 80% of entitlements.

The median entitlement for our sample of couples is £7,800 with an the interquartile range stretching from £6,330 – £9,100 in 2002. To compare to Social Security in the US, we can adjusting for prices to 2014 and converting to US dollars using the average exchange rate for that year yields for these percentiles (\$15,100 – \$18,600 – \$21,700). This compares to values (calculated using the Health and Retirement Study) of household Security Society income for a similarly selected sample in the US of (\$18,800 – \$26,360 – \$33,400). Payments under the UK public pension system are lower on average, and dispersed to a lesser extent that are those in the US.

For further information on the UK public pension system see Bozio et al. (2010b).

B Sensitivity to alternative treatment of housing wealth

In this section we compare our results with an alternative treatment of housing wealth. Following Cagetti (2003), we deduct all housing wealth from the measure of wealth which we use for estimation. We also deduct from net income an age-specific share which we estimate as being spent on mortgage interest and principal. We estimate this share using data on couples in the same cohort as that studies in this paper from the UK’s Household Budget Survey: The Living Costs and Food Survey. Figure 7 shows this estimated share.

Figure 7. Mortgage share of income

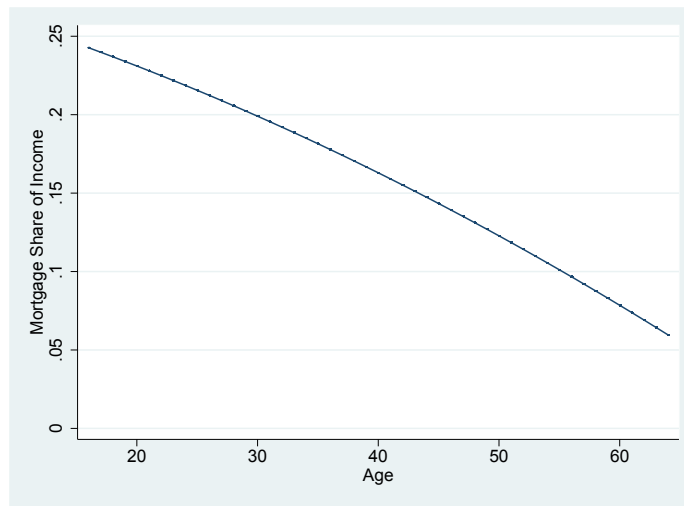


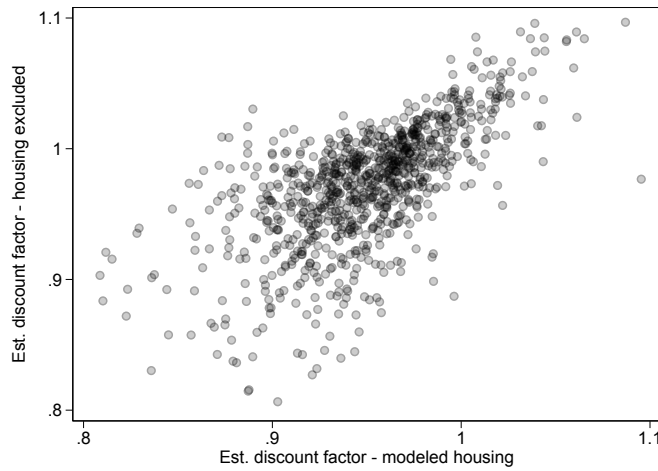
Table X summarizes the distribution of discount factors under our baseline and with the alternative treatment of housing; Figure 8 illustrates how the estimated household discount factors correlate with each other at a household level – these two distributions have a correlation coefficient of 0.65.

Table X. Distribution of discount factors - comparison to alternative housing treatment

Model	Mean	p10	p25	Median	p75	p90
modeled housing	0.947	0.894	0.923	0.950	0.975	1.002
data excludes housing	0.969	0.901	0.944	0.977	1.002	1.026

Figure 9 below compares the share of wealth held in pension wealth in three different versions of the estimated model. Each graph shows, for each decile, the ratio of mean wealth held in private pensions to the mean wealth. Graph a) shows this ratio for a model where the returns on housing are ignored in the calibration of the return on non-pension wealth (this is the model

Figure 8. Comparison of estimated discount factors – baseline (modeled housing) and housing excluded

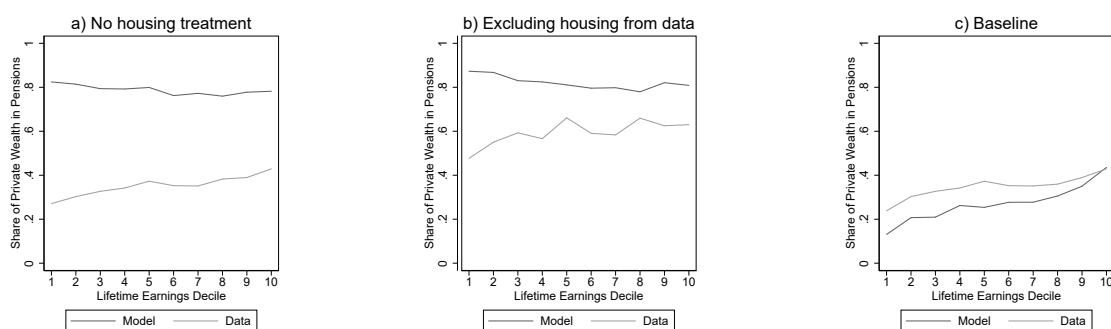


that is the basis for the ‘no housing treatment’ estimates of discount factors given in the second row of Table IV). In this case the model suggests that households will hold most of their wealth in pensions, while in the data most of wealth held in non-pension wealth. Graph b) shows this ratio for our model with the alternative specification outlined in this Appendix where we deduct housing wealth in the data. The model here is the same as that in graph a) (though the estimated discount factors will differ given the difference in the measure of wealth used for estimation). In this version, the model continues to over-predict the share of wealth held in pensions, but to a lesser extent (as in removing housing wealth from non-pension wealth, the latter’s share in the data falls).

Graph c) shows the share for our baseline. The data figure here is the same as that for graph a). However, by accounting for the returns on the wealth held in that part of non-pension wealth assumed to be held in housing, the model much more closely matches the share of wealth held in pensions. This is our preferred approach to housing and is the basis for the main results in the paper.

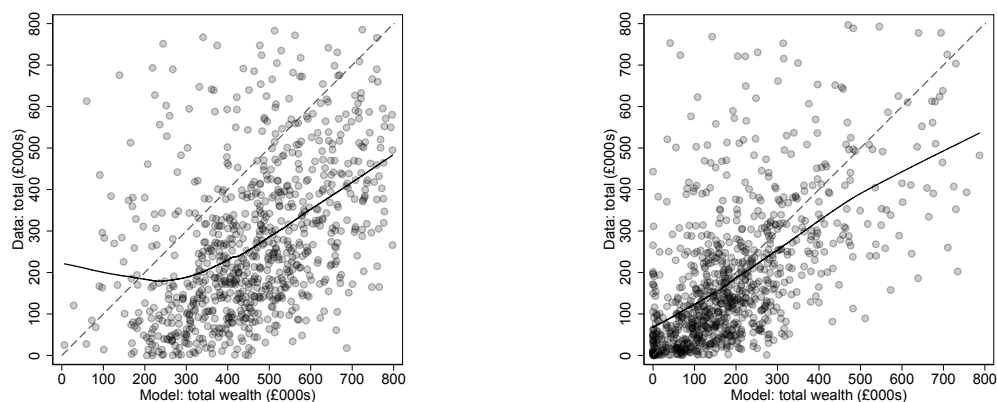
We turn now to the implications of this alternative model for our comparison of our results of those of Scholz et al. (2006) in Section 5.2.2. The left panel of Figure 10 reproduces the graph in Figure 5 which illustrates the fact that, once housing is modeled, fewer than 20% of households have ‘oversaved’ relative to a benchmark where the discount rate is set equal to the interest rate. The right hand panel shows the equivalent figure when, instead of modeling the return to housing, housing is deducted. Wealth levels are lower in the data here (due to the

Figure 9. Portfolio Shares - Comparing Models



exclusion of housing); they are also lower in the model due to the return on non-pension wealth being considerably lower and due to our deduction from income of mortgage expenses. In this case, the proportion oversaving is 41.7%, not as low in the case of the housing return being modeled, but (as in our baseline) substantially lower than then the 65.4% who oversave when no adjustment is made for housing. Table XI adds to Table VIII from the paper an additional row which summarizes the oversaving/undersaving results for the alternative housing model.

Figure 10. Comparison of Scholz et al. (2006) style results when housing return is modeled and when housing wealth is excluded



C Sensitivity to alternative treatment of DB wealth

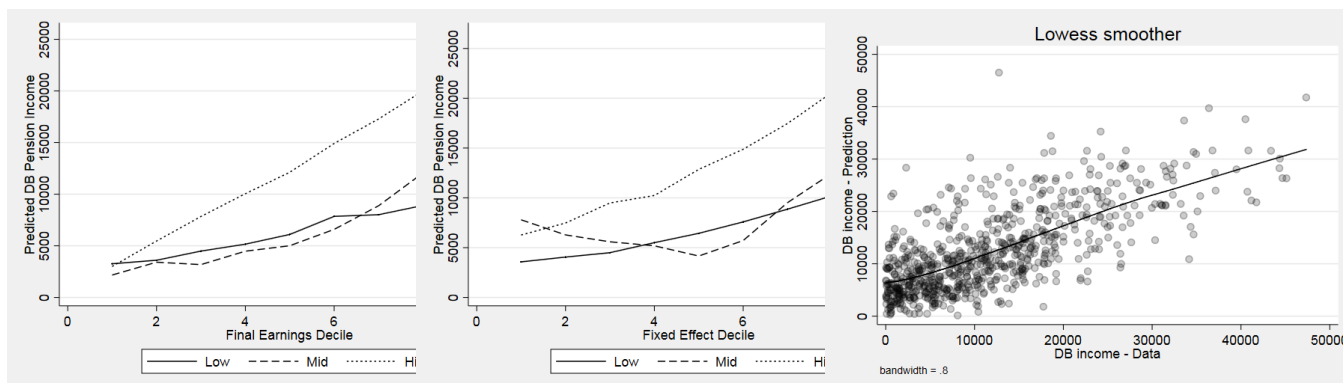
Our model does not include a Defined Benefit pension wealth. Where DB wealth is observed in the data, we model them as having being accumulated as Defined Contribution wealth. In this section, we assess the sensitivity of our results to an alternative treatment of Defined Contribution wealth.

Table XI. Comparison to Scholz et al. (2006)

Spec	Median wealth		Percent oversaving	Median surplus	Median deficit	R^2
	Observed	Optimal				
One asset	280.7	117.9	87.8	185.0	31.5	0.366
Simple two asset	280.7	203.8	65.4	133.2	65.6	0.398
Alternative housing treatment	148.3	171.6	41.7	77.0	73.3	0.346
Modeled housing (baseline)	280.7	482.9	18.1	198.0	238.2	0.306

We let Defined Benefit income be a function of a subset of our state variables. We let Defined Benefit pension income be a quadratic function of the earnings fixed effect and ‘final earnings’ (where our measure of final earnings in the data is taken to be the average decile in the final five years in which the individuals are observed). Figure 11 shows the relationship between DB pension income and each of fixed effect (conditional on mean ‘final earnings’), final earnings (conditional on mean fixed effect), as well as a scatter of data and predicted Defined Benefit pension income.

Figure 11. Defined Benefit Process



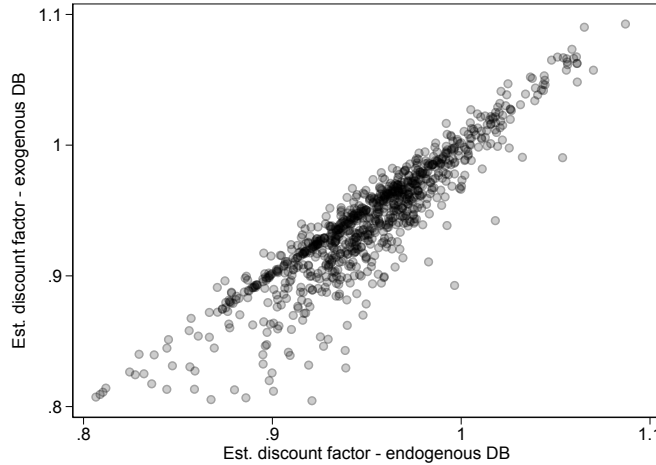
We summarize the differences between our baseline specification and this suggested approach below. In Table XII we compare the distribution of discount factors under our baseline and under the exogenous-DB treatment showing a close correspondence between the two distributions. Figure 12 shows the comparison of discount rates at an individual level – and shows that there is a tight link between the estimated degree of patience of each household – the correlation coefficient between these is 0.9.

Turning to the implications of this alternative model for our comparison with Scholz et al. (2006) in Section 5.2.2. Figure 13 below shows a version of Figure 5 from the paper, which

Table XII. Distribution of discount factors – comparison to exogenous DB specification

Model	Mean	p10	p25	Median	p75	p90
Baseline	0.947	0.894	0.923	0.950	0.975	1.002
... with exogenous DB	0.939	0.881	0.912	0.942	0.969	0.999

Figure 12. Comparison of estimated discount factors – baseline and exogenous DB



shows the scatter of wealth in the data and ‘optimal’ wealth, where the latter is determined by setting the discount rate equal to the interest rate. The three figures show, from left to right: i) the one-asset model in which all wealth accrues a return equal to the discount rate, ii) a two-asset model which adds the tax-advantaged private pension, and iii) our preferred model in which the return on non-pension wealth is set to take into account the high (and typically leveraged) returns that housing has accrued. Unlike the versions in the body of the paper, in these versions of the graph, Defined Benefit wealth is paid out as income in retirement according to the function discussed above.

The figures are very similar to those in our baseline – as are the proportions ‘over-saving’. For each of these three models (with those from the baseline model reported in Table VIII in the paper given in parentheses) are 82.0% (87.8%), 65.5% (65.4%), 16.5% (18.1%) for, respectively the one-asset model, the two-asset model not accounting for housing wealth and the the two-asset model which does account for housing wealth.

Figure 13. Comparison of Modeled and Optimal Wealth with Homogeneous Discount Rate set equal to safe rate of return

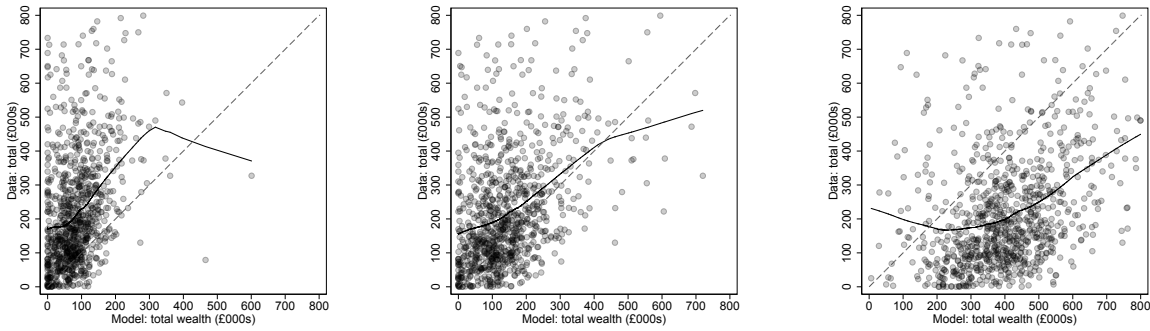
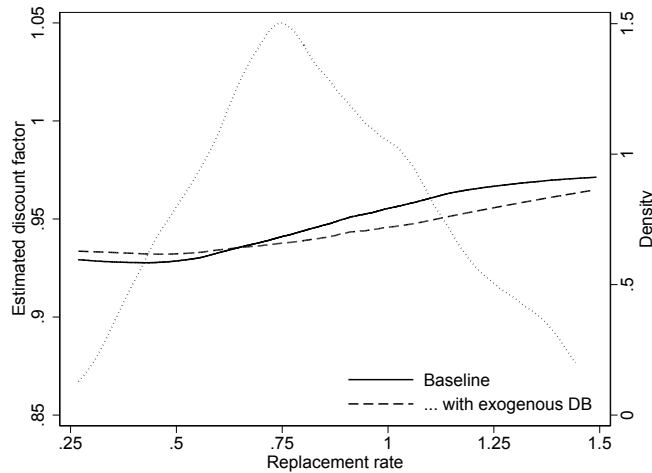


Figure 14. Comparison of relationship between Replacement Rates and Discount Factor – baseline and exogenous DB



D Data Appendix

D.1 Earnings data

The national insurance (NI) data are the administrative record of individuals’ national insurance contributions, and are the data that is used by the UK government to establish individuals’ rights to claim contributory benefits such as the state pension. We use this data to estimate ELSA respondents history of earnings. The NI records cover the years 1948 to 2003, though there are different levels of of information for each of three sub-periods: 1948-1974, 1975-1996 and 1997 to 2003.

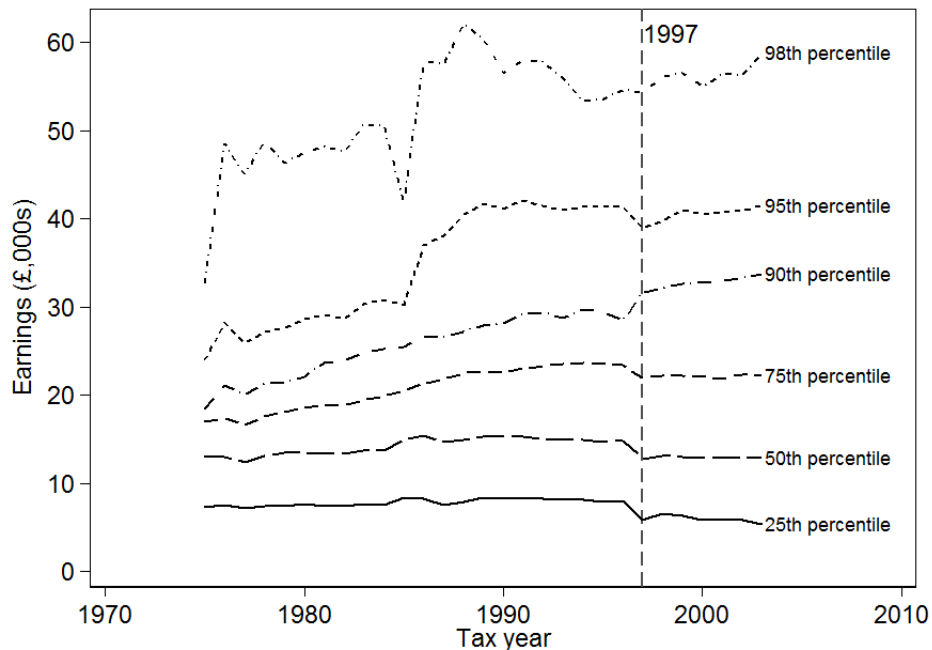
Taking the most recent period first, the NI records contain uncensored data on annual earn-

ings as, in these years, employers were required to report the total earnings of their employees. For the middle period - years between 1975 and 1996 - the NI records contain data on employee National Insurance contributions. National Insurance contributions in that interval were levied as a proportion of earnings between two values which are known as the Lower Earnings Limit (LEL) and the Upper Earnings Limit (UEL). For the period under consideration these values have been located at approximately the 8th and 80th percentile of the distribution of (positive) earnings. This data on NI contributions therefore allow us to calculate earnings, subject to right-censoring at the UEL and conditional on there being some earnings above the LEL. Prior to 1975 the NI records contain only data on the number of weeks that an individual earned above the LEL (and therefore paid NI contributions) and not the level of earnings. (This is because during this period the level of earnings was not relevant to the accrual of rights to state benefits or the state pension.)

To predict censored earnings in the years 1975 to 1996, we estimate the coefficients of a fixed-effect Tobit on earnings from 1975 to 2003 with the censoring point in each year up to 1996 equal to UEL (from 1997 there is no censoring). We use these coefficients to predict earnings for those who are affected by the censoring. The fixed-effect Tobit, with a fixed panel length, yields inconsistent results due to the incidental parameters problem (see Neyman and Scott (1948)). However Greene (2004) investigates its properties, using a Monte Carlo approach, and finds that parameters of the fixed effects Tobit model are little affected by this problem even with panel of lengths substantially shorter than our panel (which has length 29). Further, Figure 15 shows a plot of selected quantiles of earnings through time using the censored and imputed data prior to 1997 and the uncensored data from 1997 onwards. This shows only a very small discontinuity in 1997.

To simulate earnings before 1975 we follow broadly the methodology used by Bozio et al. (2017). Using the NI data, we calculate an individual's mean earnings over the years 1975 to 2003 in which they are observed working, and then estimate potential previous years' earnings by adjusting for average economy-wide earnings growth and individual level earnings growth given their age, sex and education level. Having obtained this measure of potential earnings in each year, we then need to predict the years in which the individuals were working. The NI data records how many weeks the individual made NI contributions between 1948 and 1975. For men we assume they worked those weeks immediately prior to 1975 (therefore any periods not working were at the start of working life). To take account of the diminished propensity for women to work after having children, we assume that they worked those weeks from the point of

Figure 15. Selected quantiles of earnings



leaving full-time education (therefore any periods not working were immediately prior to 1975). The combination of the estimates of potential earnings in a particular year for each individual and the years in which they were working yields our earnings estimates for years prior to 1975.

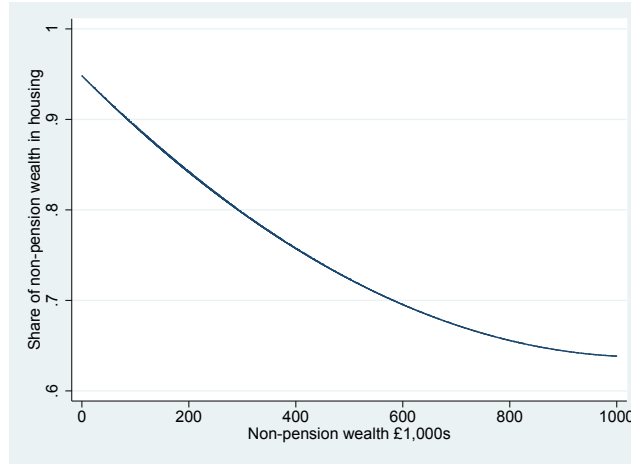
Household earnings are calculated by summing in each year the earnings for each individual in the household.

The discussion above relates only to earnings in employment and not income earned in self-employment. National insurance payments are levied on self-employment income- but in a different manner than on earnings. As a result, the NI records enable us to identify years in which self-employment income was earned, but not the level of that income. Our measure of earnings therefore excludes income from self-employment. However, we have confirmed that our results are not affected by the exclusion of the 13% of households with more than 5 years of self-employment income.

E Additional Details on Parameterization/Estimation

Figure 16 shows the estimated share of non-pension wealth held in net housing wealth: $(s(a))$. Figure 17 shows the estimated leverage ration: $lev(t)$.

Figure 16. Share of wealth in housing



Notes: Coefficients that generate liquidity share (i.e. one minus the quantity shown in this figure) on the constant and the linear and quadratic terms on assets (in £000s) are respectively 0.0517, .000589, -2.79e-07.

F Taxes and Transfer Function

Net of tax income (y) is given by a function τ :

$$y_t = \tau(e_t, a_t, a_t^c, pp_t, sp_t, h_t, k_t, dc_t, t)$$

that depends on household earnings (e_t), non-pension wealth pre-retirement (a_t), cash post-retirement (a_t^c), private pension payments (pp), public (or ‘state’) pension payments (sp), number of adults still alive (h_t), number of dependent children (k_t), chosen contributions to the pension fund (dc_t) (since those attract tax relief) and finally, on the age of the household (t) (the UK tax system taxes the elderly to a lesser extent than those of working age).

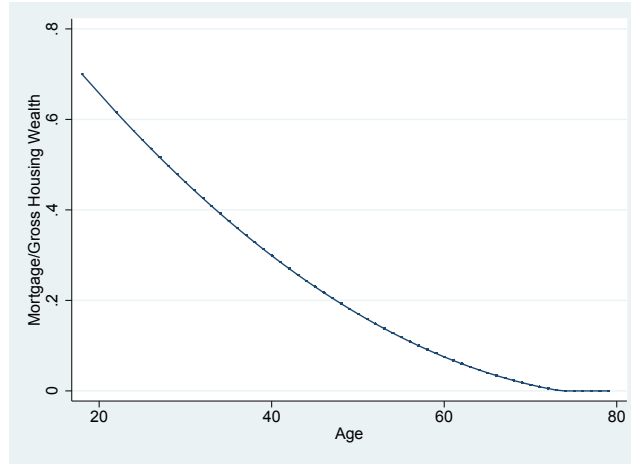
Income tax

Income tax is levied in the UK on a definition of income which includes earnings, private and state pensions and income from capital (excluding the return on home-ownership). income tax system used in the model.

$$y^{gr} = e + pp + sp + r^c a^c$$

In 2002/03, income was taxed in four bands, the smallest was exempt from tax, the second attracted tax at 10%, the third at 22% and the largest at 40%. The thresholds that define the

Figure 17. Leverage Ratio



Notes: Coefficients that generate the leverage ratio on the constant and the linear and quadratic terms on assets (in £000s) are respectively 1.16, -.028 and .00017.

bands vary with age, with a more generous treatment of older individuals. The equations below, together with Table XIII, give the income tax function:

$$\begin{aligned}
 it(e, pp, sp, a^c, t) &= 0 && \text{if } y^{gr} \leq \kappa_1 \\
 &= 0.1(y^{gr} - \kappa_1) && \text{if } \kappa_1 < y^{gr} \leq \kappa_2 \\
 &= 0.1(\kappa_2 - \kappa_1) + 0.22(y^{gr} - \kappa_2) && \text{if } \kappa_2 < y^{gr} \leq \kappa_3 \\
 &= 0.1(\kappa_2 - \kappa_1) + 0.22(\kappa_3 - \kappa_2) + 0.4(y^{gr} - \kappa_3) && \text{if } \kappa_3 > y^{gr}
 \end{aligned}$$

Table XIII. Income tax thresholds

	Age		
	< 65	≥ 65, < 75	≥ 75
κ_1	4,615	6,100	6,370
κ_2	6,535	8,020	8,290
κ_3	36,435	37,920	38,190

National Insurance

National Insurance contributions are levied on earnings (not on pension income, capital income or other forms of income) and only on those aged less than the state pension age (65). In 2002/03 it was levied at a rate of 10% of income between the ‘Lower Earnings Limit’ (LEL - £3,900) and the ‘Upper Earnings Limit’ (UEL £30,420).

$$\begin{aligned}ni(e, t) &= 0.1(\max(0, (\min(uel, e) - lel))) \text{ if } t < 65 \\ &= 0 \text{ if } t \geq 65\end{aligned}$$

Jobseekers’ Allowance

Jobseekers’ Allowance is paid to unemployed households under the age of 60 at a rate which depends on the number of adults and children in the household. In 2002/03 an unemployed couple were entitled to £4,401.80 with an additional payment of £1,924 for each dependent child.

$$\begin{aligned}jsa(e, h, k, t) &= 4401.8 + 1,924k \text{ if } t < 60 \text{ and } e = 0 \\ &= 0 \text{ if } t \geq 60 \text{ or } e > 0\end{aligned}$$

Child Benefit

Child Benefit is paid on the basis of the number of dependent children that a household has. It is paid at a more generous rate (£834.6 per year) for first children than subsequent children (£559).

$$childben(k) = 834(1(k \geq 0)) + 559(\max((k - 1), 0))$$

Minimum Income Guarantee

Households aged over 60 are entitled to a means-tested transfer (the Minimum Income Guarantee) that aims to ensure no older household faces destitution. Entitlement to the MIG is based

on current circumstances only and does not depend on a household's history of tax or national insurance contributions. The MIG simply tops *net* income up to a minimum level (f , which was £5,184 per year for singles and £7,790 for couples in 2002/03). Define net income (before payment of any MIG) as:

$$y^{preMig} = e + r^c a^c + sp + pp - it - ni + childben + jsa$$

MIG is then:

$$\begin{aligned} mig(e, ra, sp, pp, h, k, t) &= \max(0, f - y^{preMig}) \text{ if } t \geq 60 \\ &= 0 \text{ if } t < 60 \end{aligned}$$

Net income

We can now summarize the model's net income function. Income before taxes and transfers is given by:

$$\begin{aligned} y^{pre} &= e && \text{if } t \leq 64 \\ &= sp + pp + r^{hr} gh_t - r^{mort} lev(t) gh_t && \text{if } t \geq 65 \end{aligned}$$

Pre-retirement y^{pre} is simply earnings, post-retirement this is state and private pension income plus return on gross housing less mortgage interest²⁰

$$\begin{aligned} \tau(e, ra, pp, sp, h, k, dc, t) &= y^{pre} - it(e, pp, sp, a^c, t) - ni(e, t) \\ &\quad + jsa(e, h, k, t) + childben(k) + mig(e, ra, sp, pp, h, k, t) \end{aligned}$$

G Computational Appendix

G.1 Value functions

Section 3.2 gives the optimization problems in both retirement and working life faced by households in which neither spouse has died. Here we give the corresponding value function for

²⁰Pre-retirement, these last two terms are included in the rate of return on the overall return on housing rather than explicitly measured as in income.

households where the male has died (the only two differences between these and the case where the wife has died are that the value function in the latter case are conditional on $h_t = 2$ and the survival probabilities are those relating to the male $s_{t+1}^{j,m}$).

Retired household's problem

$$V_t(a_t^c, gh_t, pp_t, \tilde{e}_{64}, h_t = 3; \theta_i) = \max_{c_t} \left(u(c_t) + \beta_i s_{t+1}^{j,f} V_{t+1}(a_{t+1}^c, gh_{t+1}, pp_{t+1}, \tilde{e}_{64}, h_{t+1} = 3; \theta_i) \right)$$

$$s.t. \quad y_t = \tau(e_t, a_t, a_t^c, gh_t, pp_t, sp_t, h_t, k_t, dc_t, t)$$

and intertemporal budget constraints (5) to (8)

G.2 Model solution and simulation of optimal behavior

In this section we outline how we a) solve the households' maximization problem to obtain decision rules (functions which give, as a function of the state variables, optimal consumption and optimal pension saving) and b) use these decision rules, along with our data to simulate the optimal saving behavior of the households in our sample.

a) Solution

There is no analytical solution to the maximization problem outlined. Decision rules are obtained numerically by iterating on the value function from the final period of life. Let us rewrite the value function in (10) as:

$$V_{100}(\mathbf{X}_{100}; \theta_i) = \max_{c_{100}} u(c_{100}) + \beta E[V_{101}(\mathbf{X}_{101}; \theta_i) | \mathbf{X}_{100}] \quad (16)$$

where the vector \mathbf{X} contains the state variables of the problem and the expectation operator is over survival past the age of 100. For years before retirement, the expectation will additionally be over employment offers, earnings draws and returns on the DC fund. Our assumption that death in the next period is certain for those still alive at the age of 100 ($s_{101}^{j,m} = s_{101}^{j,f} = 0$), combined with the assumption on the absence of bequest motives means that the expectation in equation (16) evaluates to 0. At any particular point in \mathbf{X} , the maximization is therefore possible and we can obtain $c_{100}(\mathbf{X}_{100}; \theta_i)$, the consumption function, and $V_{100}(\mathbf{X}_{100}; \theta_i)$, the associated value function at those points (we discuss below our procedure for maximization). The knowledge of $V_{100}(\mathbf{X}_{100}; \theta_i)$ at a subset of points in \mathbf{X} , combined with approximation methods (also discussed below), yields an approximation of $V_{100}(\mathbf{X}_{100}; \theta_i)$ ($\hat{V}_{100}(\mathbf{X}_{100}; \theta_i)$) at each point in \mathbf{X} .

With an approximation to $V_{100}(\mathbf{X}_{100})$ so obtained, we can solve for approximations to the true consumption function ($\hat{c}_{100}(\mathbf{X}_{100}; \theta_i)$) and value function ($\hat{V}_{100}(\mathbf{X}_{100}; \theta_i)$) for the particular household i at age 99, again at a subset of points in the state space in that period, by solving the following functional equation:

$$\hat{V}_{99}(\mathbf{X}_{99}; \theta_i) = \max_{c_{99}} u(c_{99}) + \beta E[\hat{V}_{100}(\mathbf{X}_{100}; \theta_i) | \mathbf{X}_{99}] \quad (17)$$

and obtain $\hat{c}_{99}(\mathbf{X}_{99}; \theta_i)$ and ($\hat{V}_{99}(\mathbf{X}_{99}; \theta_i)$). This iterative process is repeated until we get to the beginning of working life (at age 20). For periods before retirement, a second decision rule - the quantity paid into the pension fund ($\hat{d}c_t(\mathbf{X}_t; \theta_i)$) is also calculated and stored.

Four particular features of the solution procedure will be detailed in the following discussion. These are the i) the discretization of the continuous variables, ii) the process by which the integral in the functional equation is evaluated, iii) the manner in which the value function is approximated at points outside the discretized state space and iv) how the optimization is carried out.

Discretization of state and control variables We have four continuous state variables that need to be discretized. These are earnings, cash assets, pension wealth and pension income. Earnings are placed on a grid (that has 33 elements) using a procedure suggested by Tauchen (1986). Assets, DC stocks and pensions are discretized in a manner that gives smaller gaps between successive entries on the grid at lower levels. This is as the curvature of the value function (with respect to those state variables) will be greater at lower realizations of these states. 15 discrete points for each of cash assets (and other continuous state variables). Our method for approximating the value function at points not on this grid is discussed in the next sub-section.

There are two choice variables in the model - consumption and the contribution to the DC pension fund. Consumption is not placed on a grid - households can choose any feasible consumption value. To avoid the computational burdens associated with having two continuous control variables, the proportion of earnings that is contributed to the DC pension fund is restricted to take on one of 8 values. That is, households can contribute 0%, 5%, 10%, 15%, 25%, 40%, 75% or 90% of their earnings to the pension fund.

Approximation It is required to evaluate the functions $V_t(\cdot)$ at points in the state space other than those in the discrete sub-set of points in the discretized state space. To approximate

we implement a method in Blundell et al. (2016a), in the spirit of Carroll (2006). From the perspective of solving for period t consumption, we know marginal utility at every point in the state space of $t + 1$: $u'(\hat{c}_{t+1}(a_{t+1}, \cdot))$. Of the state variables we only make explicit non-pension wealth a_{t+1} here, but the consumption choice does, of course, depend on the values of the rest of the state variables (indicated by a period). We also know expected marginal utility, conditional on the point in the state space at time t : $E_{\mathbf{X}_t}[u'(\hat{c}_{t+1}(a_{t+1}, \cdot))]$. Our objective is to find the root of the Euler equation by finding $\hat{c}_t(a_t, \cdot)$ such that:

$$u'(\hat{c}_t(a_t, \cdot)) = E_{\mathbf{X}_t} \left[u' \left(\hat{c}_{t+1}(a_{t+1}(\hat{c}_t(a_t)), \cdot) \right) \right] \quad (18)$$

where we omit, for expositional purposes the discount factor and the interest rate. Note that assets tomorrow ($a_{t+1}(\hat{c}_t(a_t))$) depend on consumption chosen today (through the intertemporal budget constraint), and so we will need to evaluate the right hand side of equation (18) at points off our grid. We could use linear interpolation (we indicate the object that is approximated with an overbar):

$$u'(\hat{c}_t(a_t, \cdot)) = \overline{E_{\mathbf{X}_t} \left[u' \left(\hat{c}_{t+1}(a_{t+1}(\hat{c}_t(a_t)), \cdot) \right) \right]}$$

However, following Carroll (2006) and Blundell et al. (2016a), we can improve the quality of the approximation by, before interpolating applying the analytical inverse of the marginal utility function, applying the approximation, and then applying the marginal utility function once again.

$$u'(\hat{c}_t(a_t, \cdot)) = u' \left(\overline{u'^{-1} \left(E_{\mathbf{X}_t} \left[u' \left(\hat{c}_{t+1}(a_{t+1}(\hat{c}_t(a_t)), \cdot) \right) \right] \right)} \right)$$

The ‘quasi-linearized’ expected marginal utility function is closer to linear than the marginal utility function, this allows a better approximation, and the use of a lower number of grid points. For a comprehensive exposition of this method see Blundell et al. (2016a) Appendix pages 7 and 8.

Integration Evaluation of the expectations in the households’ problem involves integration of the value function over four stochastic variables. These are unemployment, productivity, survival and the return on DC funds. Realizations of survival and earnings take one of a number of discrete outcomes – the former as it is naturally discrete, the latter as the procedure we apply (Tauchen (1986)) allows earnings to take only a discrete subset of outcomes. Integration over

the possible realizations of earnings and survival is therefore carried out by taking a weighted average of the value function realized at each possible outcome with the weights equal to the probability of that outcome. Realizations of the return on the DC fund are not restricted to a discrete subset. Integration over the distribution of possible outcomes is carried out using Gauss-Hermite quadrature with 10 nodes.

Optimization In retirement households face a single choice each year - how much to consume (with the rest of their resources saved in a safe asset). Each optimization is carried out by finding the root the Euler equation. This will successfully find a maximum as our approximated Value Function in retirement is quasiconcave. In working life households face two choices - how much to consume and how much to pay into a pension (again, with the rest of their resources saved in the safe asset). Here we solve (again by finding the root of the Euler equation) for optimal consumption at each of the permitted rates of contribution to the DC fund. The optimal rate of contribution to the DC fund is that which, of these, maximizes utility.

The discretization of the pension contribution choice implies that the approximated value function may be not be quasiconcave and therefore a local optimization routine, like finding the root of the Euler equation, may not find the global optimum. The reasons for this are discussed in Appendix A of Low et al. (2009) where a similar issue arises. Those authors suggest that in problems where there is a lot of uncertainty (as there is in ours) local optimizers are likely to obtain the global solution. Their approach is to use the local estimator while estimating their parameters (which involves many solutions of the value function), and then at the set of parameters to check their result using a global optimizer. Our approach is similar. The results presented in the paper (on both the model set out and the sensitivity analyses) use the local optimizer. We then check at our baseline estimate of discount factors that the predicted level of wealth does not materially change when we take a different approach to optimization - one that is robust to departures from quasiconcavity of the value function. This involves restricting consumption to be on a grid of 100 values (so that, in each period, households can choose to consume 1% of their resources, 2%, 3% etc.) and selecting (from the discrete subset of permissible selections) the levels of consumption and pension contribution that maximize utility. The results from this check support the use of the local optimizer - the distribution of wealth is very similar in both cases.

b) Simulation

Once decision rules for pension saving ($\hat{dc}_t(\mathbf{X}_t; \theta_i)$) and consumption ($\hat{c}_t(\mathbf{X}_t; \theta_i)$) are obtained we can simulate the behavior that a household member would exhibit if they followed those rules. The procedure is as follows:

1. Set initial values for state variables at the beginning of working life (age 20). The state variables that are relevant at the start of working life are cash, pension fund value, earnings and household composition. We set cash and pension fund value to zero. We set earnings to the value on the grid that is closest to actual observed earnings at the age of 20. For household composition we assume both members of the couple are alive and in a couple at that age.
2. Using these values for the state variables and our knowledge of the household's type (θ_i), and the decision rules ($\hat{c}_t(\mathbf{X}_t; \theta_i)$ and $\hat{dc}_t(\mathbf{X}_t; \theta_i)$) we obtain optimal consumption and optimal payments into the pension fund in period 20 (c_{20}^i, dc_{20}^i).
3. Obtain the new state variables for period 21. These are obtained as follows:
 - (a) Non-pension wealth in period 21 will follow from the consumption and saving decisions in period 20 along with equation (4) - the intertemporal budget constraint of the working age household.
 - (b) Pension wealth in period 21 will be the sum of the stock of pension wealth in period 20, the flow into the pension wealth and the assumed growth rate of pension funds between ages 20 and 21 (equation (19)). That growth rate is assumed to be equal to the growth rate (from our time series of pension fund growth rates) in the year that this household turns 21.

$$DC_{21}^i = (1 + \phi_{21})(DC_{20}^i + dc_{20}^i) \quad (19)$$

- (c) Earnings in period 21 will be that point on the earnings grid that is closest to actual earnings observed at the age of 21.
- (d) Household composition will remain set equal to $h_{t+1} = 1$, that is both members of the couple are still alive. This is as we only retain sample members where nobody has died by the time they are observed in the data.

4. Repeat steps 2 and 3 to obtain optimal consumption and pension saving at each age up to the age at which the (male in the household) is observed in the data in 2002 (we call this age τ). None of these men will have reached their state pension age before this period and therefore the decision rules of retired households are not needed in the simulations.²¹ This will allow a time series of the value held in both assets from the age of 20 to age τ : $\{(x_t^i, DC_t^i)\}_{t=20}^\tau$. Our central results involve used simulated optimal wealth at age τ (x_τ^i, DC_τ^i) and that observed in the data at that age for estimation.

H Estimating DC fund return

The DCisions index is an index of total fund return that reflects the asset allocation decisions made by leading DC pension plans in their default investment strategies. Over the period 1994 - 2010 the DCisions index exhibited slightly greater growth than that of the FTSE all-share index (an index representing the performance of the majority of companies listed on the London Stock Exchange). Across financial years where the FTSE all-share index grew in nominal terms, the median ratio of the growth in the DCisions index to the growth in the FTSE all-share index was 1.17, while across financial years where the FTSE all-share index fell in nominal terms, the median ratio was 0.89. This is the result of including re-investment of dividends (the DCisions index is a total return index while the FTSE all-share is an asset price index), slightly offset by the average DC pension plan being diversified into a portfolio with slightly lower return (but also lower risk) than the equities included in the FTSE all-share.

For years 1994 to 2010, therefore, ϕ_t (the model's rate of growth of funds in pension wealth) is assumed to be the real growth in the annualized DCisions index. For years prior to 1994 in which the FTSE all-share index increased in nominal terms, ϕ_t is assumed to be 1.17 times the growth in the FTSE all-share index; for years prior to 1994 in which the FTSE all-share index fell in nominal terms, ϕ_t is assumed to be 0.89 times the decline in the FTSE all-share index. The FTSE index is assumed to have grown by 4% per year in nominal terms in years before data on the FTSE all-share index are available.

²¹Though of course the decision rules for working age households could not have been calculated without first solving the retired households' problem.

I Supplementary Tables and Figures

Section 4.1 gives our method for estimating education-specific survival curves, as well as our values for the interest rate and administrative load. Table XIV gives the modeled annuity rates for each of the nine couple types. These are actuarially fair up to the administrative load. The formula is given in Section 3 of the Supplementary online material accompanying this paper.

The annuity rates vary from the highest rate of 5.47% for a couple where both members of the couple are in a low-educated group to 4.73% where both members are in a high-educated group.

Table XIV. Annuity Rates, by education level

		Husband's Education Level		
		Low	Middle	High
Wife's Education	Low	5.47%	5.34%	5.02%
	Middle	5.33%	5.22%	4.91%
	High	5.13%	5.02%	4.73%

Table XV give the estimates of the parameters of the earnings process for each of three education groups.

Table XV. Estimates of earnings process parameters

	Education group		
	Low	Middle	High
ρ	0.914 (0.0149)	0.897 (0.0023)	0.875 (0.0831)
σ_ξ^2	0.0496 (0.0035)	0.0453 (0.0022)	0.0531 (0.0021)
σ_m^2	0.0063 (0.0021)	0.0060 (0.0019)	0.0066 (0.0020)