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Vincent M. Woolf
University of Nebraska at Omaha, vwoolf@unomaha.edu
Andrew A. West
Boston University, aawest@bu.edu

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# The M dwarf problem in the Galaxy 

Vincent M. Woolf ${ }^{1 \star}$ and Andrew A. West ${ }^{2 \star}$<br>${ }^{1}$ Department of Physics, University of Nebraska at Omaha, 6001 Dodge Street, Omaha, NE 68182, USA<br>${ }^{2}$ Department of Astronomy, Boston University, 725 Commonwealth Avenue, Boston, MA 02215, USA

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#### Abstract

We present evidence that there is an $\mathbf{M}$ dwarf problem similar to the previously identified G dwarf and K dwarf problems: the number of low-metallicity M dwarfs is not sufficient to match simple closed-box models of local Galactic chemical evolution. We estimated the metallicity of 4141 M dwarf stars with spectra from the Sloan Digital Sky Survey (SDSS) using a molecular band strength versus metallicity calibration developed using high resolution spectra of nearby M dwarfs. Using a sample of M dwarfs with measured magnitudes, parallaxes and metallicities, we derived a relation that describes the absolute magnitude variation as a function of metallicity. When we examined the metallicity distribution of SDSS stars, after correcting for the different volumes sampled by the magnitude-limited survey, we found that there is an $M$ dwarf problem, with the number of $M$ dwarfs at $[\mathrm{Fe} / \mathrm{H}] \sim-0.5$ less than 1 per cent the number at $[\mathrm{Fe} / \mathrm{H}]=0$, where a Simple model of Galactic chemical evolution predicts a more gradual drop in star numbers with decreasing metallicity.


Key words: stars: abundances - stars: late-type - stars: statistics - Galaxy: abundances Galaxy: evolution - Galaxy: stellar content.

## 1 INTRODUCTION

The compositions of stars provide a test for models of Galactic chemical evolution, with the general assumption being that the photospheric chemical composition of most stars represents the local Galactic chemical composition where they formed. van den Bergh (1962) and Schmidt (1963) first noted that the ratio of lowmetallicity G dwarfs to solar-metallicity G dwarfs in the local neighbourhood is too small to be explained by the 'Simple model' of Galactic chemical evolution. The Simple model, as summarized by Tinsley (1980), is a model assuming (1) the solar neighbourhood can be modelled as a closed system; (2) it started as 100 per cent metal-free gas; (3) the initial stellar mass function (IMF) is constant and (4) the gas is chemically homogeneous at all times. Each of the assumptions in the Simple model are demonstrably false, but it is an important starting point for the development of more complex models. The process of solving the G dwarf problem and explaining the Galaxy's chemical enrichment history has been underway for nearly five decades. The solution will require discarding one or more the Simple model's assumptions. The introduction of a variable IMF (e.g. Schmidt 1963; Carigi 1996; Martinelli \& Matteucci 2000; Romano et al. 2005), variable star formation rates with intermittent mixing (e.g. Malinie et al. 1993; Caimmi 2008), and/or inflow or outflow of material (e.g. Wyse \& Gilmore 1995; Pagel 2001) into models of Galactic chemical evolution produces metallicity distributions in better agreement with observations.

[^0]The G dwarf problem does not apply only to the local neighbourhood: the G dwarf problem exists in other galaxies as well (Worthey, Dorman \& Jones 1996). While G dwarf lifetimes would suggest that some G dwarfs may have left the main sequence during the Galaxy's lifetime, the paucity of low-metallicity stars locally also extends to the longer-lived K dwarfs (Casuso \& Beckman 2004). Mould (1978) suggested that there may be an M dwarf problem: using spectra of six M dwarfs and infrared spectroscopy of 16 old disc M dwarfs, he estimated the abundance dispersion for M dwarfs and found evidence that suggested a common chemical history of $G$ and $M$ dwarfs.

We have directly tested the existence of an M dwarf problem by estimating the metallicity of 4141 M dwarf stars with temperatures in the range where our analysis method is valid, $3500 \leq T_{\text {eff }} \leq$ 4000 K , using CaH and TiO molecular band strengths measured from Sloan Digital Sky Survey (SDSS) spectra (York et al. 2000) and find that there is an M dwarf problem in the Galactic disc similar to the previously known G and K dwarf problems. This does not suggest a new solution to the $\mathrm{M}, \mathrm{K}$ and G dwarf problems, but rather shows that for all stars with main sequence lifetimes comparable or larger than the age of the Galaxy, and where chemical abundance surveys have been performed, the number of low-metallicity stars is insufficient to match the Simple model.

## 2 ESTIMATING METALLICITIES

M dwarfs are inherently faint objects, and therefore must be nearby to appear bright enough to obtain high-resolution spectra, $\lambda / \Delta \lambda \gtrsim$ 30000 , of sufficient quality to measure atomic absorption lines for
abundance analyses, while using a reasonable amount of telescope time.

Molecular band indices $\mathrm{CaH} 2, \mathrm{CaH} 3$ and TiO5 (Reid, Hawley \& Gizis 1995) measured from low-resolution spectra, $\lambda / \Delta \lambda \sim 1800$, have been shown to be useful in classifying cool dwarfs. CaH 2 and CaH 3 correlate well with spectral type and the combination of CaH 2 or CaH 3 with TiO 5 separates cool dwarfs into rough metallicity classes: dwarfs, subdwarfs and extreme subdwarfs (Gizis 1997; Lépine, Rich \& Shara 2003). Woolf, Lépine \& Wallerstein (2009) used metallicities of 88 M dwarfs, estimated using atomic lines measured from high-resolution spectra (Woolf \& Wallerstein 2005, 2006), to develop and calibrate a method to use $\mathrm{CaH} 2, \mathrm{CaH} 3$ and TiO5 molecular band indices to estimate M dwarf metallicities more precisely. $\mathrm{CaH} 2, \mathrm{CaH} 3$ and their sum $(\mathrm{CaH} 2+\mathrm{CaH} 3)$ were found to correlate well with effective temperature ( $T_{\text {eff }}$ ) and TiO 5 was found to vary with both $T_{\text {eff }}$ and metallicity (Woolf \& Wallerstein 2006). The three indices were used with the calculated $[\mathrm{Fe} / \mathrm{H}]^{1}$ metallicites of the 88 M dwarfs to derive the relation
$[\mathrm{Fe} / \mathrm{H}]=a+b\left(\zeta_{\mathrm{TiO} / \mathrm{CaH}}\right)$,
where $a=-1.685 \pm 0.079, b=1.632 \pm 0.096$ and the metallicity index $\zeta_{\mathrm{TiO} / \mathrm{CaH}}$ is defined as described in Lépine, Rich \& Shara (2007). This calibration of the molecular band index versus metallicity relation is valid for stars with $3500 \leq T_{\text {eff }} \leq 4000 \mathrm{~K}$ and -1.5 $\leq[\mathrm{Fe} / \mathrm{H}] \leq+0.05$. As described in Woolf et al. (2009), the $[\mathrm{Fe} / \mathrm{H}]$ versus $\zeta_{\mathrm{TiO} / \mathrm{CaH}}$ fit, should be accurate to $\pm 0.3$ dex for stars with $[\mathrm{Fe} / \mathrm{H}]>-1.0$ and to $\pm 0.5$ dex for stars with $[\mathrm{Fe} / \mathrm{H}]<-1.0$. based on the maximum errors the empirical data appear to allow, as opposed to standard statistical errors based on repeated measurements of the same quantity. Most errors should be less than half of those maxima.

We used $\mathrm{CaH} 2, \mathrm{CaH} 3$ and TiO5 indices measured from SDSS spectra to estimate metallicities for the 4141 M dwarfs in our sample. The stars were placed in 0.1 dex $[\mathrm{Fe} / \mathrm{H}]$ bins from -1.5 to 0.1 . This extrapolates slightly beyond the limits for which our method was calibrated. The number of stars in bins with $[\mathrm{Fe} / \mathrm{H}]$ less than -0.50 is very small, so extrapolating to $[\mathrm{Fe} / \mathrm{H}]=-1.55$ at the low metallicity end makes little difference. The number of stars in the $[\mathrm{Fe} / \mathrm{H}] \approx+0.1 \mathrm{bin}$ is less certain because the entire bin is outside our calibration, but the bin is included to show that, if our calibration continues to be correct a little beyond where testing stopped because of a lack of stars for the calculation, it appears that the trend in number of stars versus metallicity drops for $[\mathrm{Fe} / \mathrm{H}]>0$.

## 3 STELLAR DATA SET

Our spectroscopic sample contains 4141 low-mass stars selected from the SDSS (York et al. 2000) Munn 'special plates', which were made public as part of the SDSS Data Release 4 (AdelmanMcCarthy et al. 2006). The Munn plates were targeted to be a magnitude-limited sample of M and late-K dwarfs with $i<18.26$, $i-z>0.2$ and be located near the $l=123^{\circ}, b=-63^{\circ}$ Galactic sight line. This sample was previously used to study the kinematics of the local Milky Way thin and thick discs (Bochanski et al. 2007a) to roughly 2 kpc above the Galactic plane and are contained in the larger SDSS M dwarf spectroscopic samples (West et al. 2004, 2008, 2011). We analysed the 8859 spectra that were observed using the Hammer spectral typing facility (Covey et al. 2007) and rejected

[^1]stars that had poor quality (SNR $<3$ near $\mathrm{H} \alpha$ ) and spectral types earlier than K5 (resulting in 7714 stars).

As part of our analysis, we computed radial velocities (RVs) by cross-correlating each spectrum with the appropriate (Bochanski et al. 2007b) M dwarf template. In the case of K5 and K7 dwarfs, we used the M0 template for our RV determinations. The computed velocities were used to correct the stellar spectra to zero RV. We then computed the $\mathrm{TiO5}, \mathrm{CaH} 2$ and CaH 3 molecular band indices (Reid et al. 1995; Gizis 1997) from the velocity-corrected spectra. Additional stars were removed from the sample if their molecular band indices, temperature or $[\mathrm{Fe} / \mathrm{H}]$ fell outside the range covered by the Woolf et al. (2009) metallicity calibration, leaving the 4141 stars for our study. All stars with temperature and $[\mathrm{Fe} / \mathrm{H}]$ within the metallicity calibration have $i-z$ colours larger the 0.2 value used in the initial selection, so the colour limit should introduce no selection effects for our metallicity statistics.

## 4 VOLUME AND STELLAR NUMBER DENSITY CORRECTIONS

### 4.1 Volume correction for $\boldsymbol{V}$ and $\boldsymbol{R}$ magnitudes

Low metallicity main sequence stars (subdwarfs) are less luminous than higher metallicity stars with the same temperature or spectral type (Reiz 1954; Mould \& McElroy 1978; Ake \& Greenstein 1980; Bochanski, Hawley \& West 2011). Because the SDSS M dwarf sample was selected to fall within a magnitude range, the subdwarf M stars must be closer on average than the higher metallicity stars, and thus sample a smaller volume of our Galactic neighbourhood. We must correct for this to avoid introducing a Malmquist Bias-like effect and overcounting the number of more luminous stars.
We used stars from Woolf et al. (2009) for which parallax data were available and thus absolute magnitudes could be calculated to find the luminosity variation with metallicity for M dwarfs in our temperature range. An absolute magnitude difference function was found by fitting a three-dimensional 'surface' to the $[\mathrm{Fe} / \mathrm{H}], \mathrm{CaH} 2+\mathrm{CaH} 3$, and $M_{V}$ data points calculated for the stars. $\mathrm{CaH} 2+\mathrm{CaH} 3$ acts as a temperature proxy in the calculated fit. We found that for the best-fitting surface, the magnitude change with metallicity did not depend on $\mathrm{CaH} 2+\mathrm{CaH} 3$ (temperature) within the range of our metallicity calibration. For example, two 3500 K stars with $[\mathrm{Fe} / \mathrm{H}]=0.0$ and -1.0 differ in magnitude by the same amount as two 4000 K stars with $[\mathrm{Fe} / \mathrm{H}]=0.0$ and -1.0 . The fit to the data points is

$$
\begin{align*}
M_{V}= & a+b[\mathrm{Fe} / \mathrm{H}]+c[\mathrm{Fe} / \mathrm{H}]^{2}+d(\mathrm{CaH} 2+\mathrm{CaH} 3) \\
& +e(\mathrm{CaH} 2+\mathrm{CaH} 3)^{2}, \tag{2}
\end{align*}
$$

where $a=25.356, b=-0.6394, c=0.8455, d=-15.778$ and $e=3.250$. The root mean squared deviation between the absolute $V$ magnitudes calculated using parallax and apparent $V$ magnitudes and those calculated using the above formula is 0.450 . The $M_{V}$ values calculated from equation (2), $M_{V}$ (model), are compared to those calculated from observed $V$ magnitudes and trignometric paralax, $M_{V}$ (observed), in Fig. 1. We did a similar fit using the absolute $R$ magnitudes and found coefficients $a=24.334, b=$ $-0.8258, c=0.6964, d=-16.379$ and $e=3.696$, with the root mean squared deviation between the $M_{R}$ values calculated from paralax and $R$ measurements and from the formula being 0.485 .

Because there is no temperature dependence in the difference in magnitude (or luminosity) for M dwarfs of the same temperature but with different metallicities there is also no temperature dependence


Figure 1. Absolute $V$ magnitudes for early M dwarfs calculated using equation (2) versus those calculated using observed $V$ magnitudes and parallax measurements. The diagonal line indicates where points would fall if the model and observed values were identical.
in the volume correction due to metallicity. We can apply the same volume sampling correction for all stars in our sample without considering temperature.

The magnitude-limited volume sampled for each metallicity bin is proportional to the distance cubed. The distance to a star is proportional to the square root of its luminosity for a given apparent magnitude. So for a given stellar luminosity, $L$, the sampled volume is proportional to $L^{3 / 2}$. Subdwarfs with smaller luminosities are sampled from a smaller volume, so we multiply the numbers of stars in the metallicity bins by the inverse of the volume factor, $L^{-3 / 2}$, to correct for the volume difference. The $V$ and $R$ luminosity and absolute magnitude differences and the volume correction factor for different $[\mathrm{Fe} / \mathrm{H}]$ values as calculated using Woolf et al. (2009) stars are shown in Table 1.

### 4.2 Volume correction for SDSS $\boldsymbol{i}$ magnitude

The SDSS stars in our sample were selected for observation using an SDSS $i$ magnitude cut-off. We must therefore use $\Delta M_{i}$, the difference in $i$ absolute magnitude caused by varying metallicity, in our volume correction.

None of the stars used for our volume correction due to metallicity differences have SDSS photometry available, so they cannot be used to calculate $\Delta M_{i}$ directly. Instead, we first use $\Delta M_{V}$ and $\Delta M_{R}$ to estimate $\Delta M_{r}$, the difference in SDSS $r$ magnitude due to metallicity differences. We then use $\Delta(r-i)$, the difference in $r-i$ colour due to metallicity differences, to calculate $\Delta M_{i}$ to use for our volume correction.

The SDSS $r$ photometric band overlaps the $V$ and $R$ bands. Differences in metallicity should therefore cause differences in $r$ absolute magnitude similar to the differences found for $V$ and $R$. The corrections for $V$ and $R$ differ at most by 0.06 magnitudes. Assuming that $\Delta M_{r}$ is equal to the mean of $\Delta M_{V}$ and $\Delta M_{R}$ should therefore introduce errors smaller than a few per cent.

We have therefore estimated the necessary corrections using updated synthetic spectra (Hauschildt, private communication) based on phoenix calculations (Hauschildt, Allard \& Baron 1999), finding the mean differences in $r-i$ colours in the spectra relative to the $[\mathrm{Fe} / \mathrm{H}]=0.0$ spectrum for temperatures between 3500 and 4000 K . We then calculated the difference in $i$ magnitude due to metallicity differences: $\Delta M_{i}=\Delta M_{r}-\Delta(r-i)$. The corresponding $i$ luminosity ratio and volume corrections were calculated in the manner described previously for $V$ and $R$ and are reported in Table 1.

An alternate method to calculate $\Delta(r-i)$ would use a published empirical colour-temperature calibration to estimage $B-V, V$ $-R$ and $V-I$ colours for stars of M dwarf temperatures then use an existing empirically derived formula to transform these to an $r-i$ colour. Worthey \& Lee (2011) provide such a colourtemperature calibration. The M dwarf metallicities used in their calibration are based on kinematics, with young disc stars assigned $[\mathrm{Fe} / \mathrm{H}]=-0.1$, old disc stars -0.5 , and halo stars -1.5 , unless the

Table 1. M dwarf absolute magnitude differences and luminosity, volume, and stellar number density correction factors for sampling differences compared to Solar $[\mathrm{Fe} / \mathrm{H}]$. Values in the volume ${ }^{-1}(i)$ and $\left(\frac{\rho}{\rho_{0}}\right)^{-1}$ columns are the factors by which the numbers of stars in each $[\mathrm{Fe} / \mathrm{H}]$ bin should be multiplied to correct for volume sampling and stellar number density differences. $\Delta(r-i)$ is the colour difference relative to $[\mathrm{Fe} / \mathrm{H}]=0.0$ due to metallicity difference.

| $[\mathrm{Fe} / \mathrm{H}]$ | $\Delta M_{V}$ | $\Delta M_{R}$ | $L(V)$ | $L(R)$ | Volume $^{-1}(\mathrm{~V})$ | $\Delta M_{r}$ | $\Delta(r-i)$ | $\Delta M_{i}$ | $L(i)$ | Volume $^{-1}(i)$ | $\left(\frac{\rho}{\rho_{0}}\right)^{-1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| +0.1 | -0.056 | -0.075 | 1.056 | 1.072 | 0.925 | -0.066 | +0.050 | -0.116 | 1.112 | 0.852 | 1.038 |
| 0.0 | 0.000 | 0.000 | 1.000 | 1.000 | 1.00 | 0.00 | 0.000 | 0.00 | 1.00 | 1.00 | 1.00 |
| -0.1 | 0.072 | 0.090 | 0.936 | 0.920 | 1.10 | 0.081 | -0.037 | 0.118 | 0.897 | 1.18 | 0.957 |
| -0.2 | 0.161 | 0.193 | 0.862 | 0.837 | 1.25 | 0.177 | -0.074 | 0.251 | 0.794 | 1.41 | 0.909 |
| -0.3 | 0.268 | 0.311 | 0.781 | 0.751 | 1.45 | 0.289 | -0.111 | 0.400 | 0.692 | 1.74 | 0.859 |
| -0.4 | 0.391 | 0.442 | 0.698 | 0.656 | 1.72 | 0.416 | -0.148 | 0.564 | 0.595 | 2.18 | 0.807 |
| -0.5 | 0.531 | 0.587 | 0.613 | 0.582 | 2.08 | 0.559 | -0.184 | 0.743 | 0.504 | 2.79 | 0.756 |
| -0.6 | 0.688 | 0.746 | 0.531 | 0.503 | 2.59 | 0.717 | -0.203 | 0.920 | 0.429 | 3.56 | 0.707 |
| -0.7 | 0.862 | 0.919 | 0.452 | 0.429 | 3.29 | 0.890 | -0.222 | 1.112 | 0.359 | 4.65 | 0.659 |
| -0.8 | 1.052 | 1.105 | 0.380 | 0.361 | 4.28 | 1.079 | -0.240 | 1.319 | 0.297 | 6.18 | 0.615 |
| -0.9 | 1.260 | 1.306 | 0.313 | 0.300 | 5.70 | 1.283 | -0.258 | 1.541 | 0.242 | 8.41 | 0.573 |
| -1.0 | 1.485 | 1.521 | 0.255 | 0.246 | 7.78 | 1.503 | -0.277 | 1.780 | 0.194 | 11.7 | 0.535 |
| -1.1 | 1.726 | 1.749 | 0.204 | 0.200 | 10.9 | 1.738 | -0.284 | 2.022 | 0.155 | 16.3 | 0.501 |
| -1.2 | 1.985 | 1.991 | 0.161 | 0.160 | 15.5 | 1.988 | -0.291 | 2.279 | 0.123 | 23.3 | 0.470 |
| -1.3 | 2.260 | 2.247 | 0.125 | 0.126 | 22.7 | 2.253 | -0.299 | 2.552 | 0.953 | 34.0 | 0.442 |
| -1.4 | 2.552 | 2.518 | 0.0953 | 0.0984 | 34.0 | 2.535 | -0.306 | 2.841 | 0.0730 | 50.7 | 0.418 |
| -1.5 | 2.861 | 2.802 | 0.0717 | 0.0757 | 52.1 | 2.831 | -0.313 | 3.144 | 0.0552 | 77.0 | 0.397 |

star is in a cluster, in which case the cluster metallicity was used. If we use their empirical calibration rather than the colours from synthetic spectra, we find much smaller corrections for $\Delta(r-i)$. For example, the $r-i$ correction for $[\mathrm{Fe} / \mathrm{H}]=-1.5$ stars is -0.3 if calculated from synthetic spectra, but is about 0.02 if calculated from the empirical calibration data.

Our goal is to compare the trend for M dwarf stars with a model which has predicted more low-metallicity stars than have been observed. We will therefore avoid the possibility of undercounting low-metallicity stars by using the larger correction based on synthetic spectra which tends to increase the corrected number of lowmetallicity stars. In the end, because the number of observed stars drops off so quickly with decreasing metallicity, our conclusions do not depend on which of these corrections we use. We correct by multiplying the number of stars in each metallicity bin by the values in the volume ${ }^{-1}(i)$ column in Table 1. The empirically based difference between $\Delta M_{V}$ and $\Delta M_{i}$ is very small, so if someone prefers not to use the correction based on synthetic spectra, one can multiply by the value in the volume ${ }^{-1}(V)$ column instead.
We note that the SDSS sample has both a faint $i$ magnitude limit and a bright $i$ magnitude limit, meaning that the volumes sampled are effectively spherical shells. The ratio of the distances to the faint magnitude cut-off is the same as the ratio of the distances to the bright magnitude cut-off however, so the volume ratio of the unsampled inner spheres is the same as the volume ratio of the spheres defined by the faint limit: no additional correction is required to account for the bright magnitude cut-off.

### 4.3 Correction for stellar number density differences

The observational direction for our SDSS M dwarf sample, near $l=$ $123^{\circ}, b=-63^{\circ}$, means that the more distant stars in the sample are farther from the plane of the Galactic disc and thus in a region with a lower stellar number density. Without correcting for this effect we would overestimate the fraction of fainter, low-metallicity subdwarf stars, which are found in a denser region of the disc. We use the difference in distances due to metallicity and a model of the Milky Way stellar number density distribution to correct for this effect.
Bochanski et al. (2011) report absolute $r$ magnitudes, $M_{r}$, for M dwarfs based on photometric parallax. Fig. 2 from their paper plots $M_{r}$ versus spectral type. From this figure we find that the typical $M_{r}$ for stars in our data set, mostly M0-M2 dwarfs, is about 9.0.


Figure 2. Fraction of stars found in each 0.1 dex $[\mathrm{Fe} / \mathrm{H}]$ bin. The dashed line represents the raw data. The solid line represents the volume and stellar number density corrected data. The dotted curve is the distribution predicted by the Simple model.

This is consistent with the absolute magnitude of the few warmer M dwarfs with reported SDSS photometry and trigonometric parallax (Davenport et al. 2006). A $M_{r}=9.0$ star with $r=17.5$, the mean observed $r$ magnitude for our sample, is at a distance $d=501 \mathrm{pc}$.
Jurić et al. (2008) report the stellar number density distribution for the Milky Way, providing an exponential equation to calculate estimates of changes in density with radial distance from Galactic Centre and perpendicular distance from the plane of the disc (Juric et al. equation 23). Stellar number density variation due to differences in radial distance from Galactic Centre is minimal for our stars, given the direction and distance to the sample. We assume the radial terms from the Juric et al. equation are constant and recognize that $\mathrm{Z}_{\odot}$ is above the plane of the Galaxy and the direction to the stars in our sample places them below the plane. The equation thus becomes $\rho=C \exp \left(\frac{\mathrm{Z}_{\odot}-Z}{H}\right)$, where C is the stellar number density at the vertical centre of the disc at the Solar radial distance, $\mathrm{Z}_{\odot}$ is the Solar distance above the Galactic plane, $Z$ is the distance to the star measured in the direction perpendicular below the plane, and $H$ is the scale height for the disc, thin or thick. Note that this form of the equation is not valid for stars above the plane or closer to the centre of the plane than the Sun. We use this modified equation to find the ratio of the stellar number density $\rho$ for a star in our sample with a distance $Z$ compared to the local density $\rho \odot$
$\frac{\rho}{\rho_{\odot}}=\exp \left(\frac{-Z}{H}\right)$.
Jurić et al. (2008) report $H=300 \mathrm{pc}$ for the thin disc, $H=900 \mathrm{pc}$ for the thick disc, and $\mathrm{Z}_{\odot}=20 \mathrm{pc}$.

To make a rough correction for differences in stellar number density caused by metallicity we first use $\Delta M_{r}$ from Table 1 to find the luminosity ratio relative to a $[\mathrm{Fe} / \mathrm{H}]=0$ star for each metallicity bin: $l / l_{0}=100^{-\Delta M_{r} / 5}$. The ratio of the distances for stars with the same apparent magnitude is then $d / d_{0}=\sqrt{l / l_{0}}$. If we then assume that the mean distance to a $[\mathrm{Fe} / \mathrm{H}]=0$ star in our sample is 501 pc we can use the distance ratio to find the mean distance to stars in the other metallicity bins. Because $b \approx-63^{\circ}$ for our sample, the distance perpendicular to the Galactic plane is $Z=d \sin 61^{\circ}-\mathrm{Z}_{\odot}$.
We calculate the ratio of the stellar number density at the typical perpendicular distance, $Z$, for stars in each metallicity bin relative to the local Solar neighbourhood density using equation (3). We calculate the ratio for both thin disc stars, $\left(\frac{\rho}{\rho}\right)_{\text {thin }}$, and thick disc stars, $\left(\frac{\rho}{\rho}\right)_{\text {thick }}$, and use these and the local 12 per cent thick disc fraction (Jurić et al. 2008) to calculate the density ratio $\frac{\rho}{\rho}=$ $0.88\left(\frac{\rho}{\rho} \odot\right)_{\text {thin }}+0.12\left(\frac{\rho}{\rho} \odot\right)_{\text {thick }}$. For each metallicity bin we divide this density ratio $\frac{\rho}{\rho}$ ) by the value found for the $[\mathrm{Fe} / \mathrm{H}]=0$ bin to get $\frac{\rho}{\rho_{0}}$, where $\rho_{0}$ is the stellar number density at the typical distance to solar metallicity stars in our sample. To correct for differences in stellar number density we multiply the number of stars in each metallicity bin by the inverse of this density ratio, $\left(\frac{\rho}{\rho_{0}}\right)^{-1}$, as reported in Table 1. We note that the volume correction and the stellar number density correction act in opposite directions: fainter, lower metallicity stars are sampled in a smaller volume, but in a region with a higher density of stars.

## 5 M DWARF METALLICITY TREND

The raw and corrected fractions of M dwarfs in our sample in each metallicity bin are shown in Table 2 and in Fig. 2. The distribution is centred at about $[\mathrm{Fe} / \mathrm{H}]=0.0$ and has a Gaussian full width

Table 2. Numbers and sample fraction of $M$ dwarfs at given metallicities. The last two columns are corrected for different volumes, volume ${ }^{-1}(i)$, and stellar number densities, $\left(\frac{\rho}{\rho_{0}}\right)^{-1}$, being sampled for different metallicities.

| $[\mathrm{Fe} / \mathrm{H}]$ | Number | Fraction | Corrected number | Corrected fraction |
| :---: | :---: | :---: | :---: | :---: |
| +0.1 | 1318 | 0.3183 | 1166.3 | 0.2685 |
| 0.0 | 1491 | 0.3601 | 1491.0 | 0.3433 |
| -0.1 | 868 | 0.2096 | 977.7 | 0.2251 |
| -0.2 | 312 | 0.0753 | 401.0 | 0.0923 |
| -0.3 | 99 | 0.0239 | 147.9 | 0.0340 |
| -0.4 | 33 | 0.0080 | 58.1 | 0.0134 |
| -0.5 | 8 | 0.0019 | 16.9 | 0.0039 |
| -0.6 | 3 | 0.0007 | 7.75 | 0.0018 |
| -0.7 | 1 | 0.0002 | 3.07 | 0.0007 |
| -0.8 | 2 | 0.0005 | 7.60 | 0.0018 |
| -0.9 | 1 | 0.0002 | 4.82 | 0.0011 |
| -1.0 | 3 | 0.0007 | 18.8 | 0.0043 |
| -1.1 | 0 | 0 | 0 | 0 |
| -1.2 | 0 | 0 | 0 | 0 |
| -1.3 | 0 | 0 | 0 | 0 |
| -1.4 | 2 | 0.0005 | 42.3 | 0.0097 |
| -1.5 | 0 | 0 | 0 | 0 |

half-maximum (FWHM) of 0.30 dex and a standard deviation (dispersion) of 0.13 dex.

Other studies have found that the number of stars peaks at a metallicity centred in the range $-0.25 \lesssim[\mathrm{Fe} / \mathrm{H}] \lesssim 0.0$ for G and K dwarfs (e.g. Wyse \& Gilmore 1995; Rocha-Pinto \& Maciel 1996; Favata, Micela \& Sciortino 1997; Jørgensen 2000; RochaPinto et al. 2000; Allende Prieto et al. 2004; Casuso \& Beckman 2004; Nordström et al. 2004; Luck \& Heiter 2005). We find that the peak for M dwarfs is at the high end of this range and note that our peak is more narrow than that typically found. There has been some 'narrowing' of the reported metallicity distribution over time since the first reports of the G dwarf problem in the 1960s, probably as a result of a reduction of the uncertainties as the methods for calculating elemental abundances have improved.

Our results for M dwarfs most closely match those found by Favata et al. (1997) for K dwarfs, however. In their study of G and K dwarfs, they separated stars into two groups with effective temperatures hotter and cooler than 5100 K and found very different metallicity distributions. They did not report mean and standard deviations for their corrected distributions, but we estimate from the values on their plots that the warmer stars showed a distribution similar to those previously reported for G dwarfs, centred at $[\mathrm{Fe} / \mathrm{H}]$ $\approx-0.28$ with $\sigma \approx 0.29$, and that the cooler star distribution dropped off more sharply towards lower metallicities and was more sharply peaked with a centre at $[\mathrm{Fe} / \mathrm{H}] \approx-0.03$ and $\sigma \approx 0.17$. Luck \& Heiter (2005), report elemental abundances for stars within 15 pc . Their sample contains mostly G and K dwarfs. When we separate their stars by temperature we find that the metallicity distribution for the cool stars, $T_{\text {eff }} \leq 5100 K$, has mean $[\mathrm{Fe} / \mathrm{H}]=0.0$ and $\sigma=$ 0.17 , while the warmer stars have mean $[\mathrm{Fe} / \mathrm{H}]=-0.11$ and $\sigma=$ 0.28 , again showing an identifiable difference with temperature. It appears that the trend of higher mean metallicity and narrower peaks seen for the metallicity distributions for cooler stars seen in the Favata et al. and Luck \& Heiter data applies to the even cooler M dwarfs in our sample.
'Simple' models of Galactic chemical evolution produce higher numbers of long-lived low metallicity stars than we find for M dwarfs and others have found for G and K dwarfs. Audouze \&


Figure 3. Theoretical (dashed curve) and observed (solid line) cumulative stellar metallicity distributions. The theoretical values are calculated using the Simple model. $S / S_{1}$ represents the fraction of stars that have metallicites less than $Z$.

Tinsley (1976) use the equation
$\frac{S}{S_{1}}=\frac{1-\mu_{1}^{Z / Z_{1}}}{1-\mu_{1}}$
to calculate $S / S_{1}$, the present day cumulative stellar metallicity distribution, or the fraction of stars that have metalliciies less than $Z$. In equation (4), $Z_{1}$ is the present day metallicity and $\mu_{1}$ is the fraction of the local baryonic matter that is now interstellar matter (as opposed to that contained in stars and stellar remnants). The smooth dotted curve in Fig. 2 is derived from the cumulative distribution described by equation 4 , given that the increase in the cumulative distribution must be caused by stars in each successive metallicity bin.

In Fig. 3 we compare $S / S_{1}$ to the value calculated from our M dwarf metallicities. We calculated $S / S_{1}$ from equation (4) where we assume that $\mu_{1}=0.27$ (Holmberg \& Flynn 2000) and let $\log \left(Z_{1} / Z_{\odot}\right)=+0.1$. The theoretical $S / S_{1}$ calculated for the Simple model will be somewhat different for other assumed $Z_{1}$ and $\mu_{1}$, and a fairly large range of values for $\mu_{1}$ are possible, given the 50 per cent uncertainty reported by Holmberg \& Flynn (2000). However for all reasonable values the theoretical curve drops off much more slowly towards low metallicity than the observed curve does.

## 6 CONCLUSIONS

Our data show that there is an M dwarf problem similar to the previously known $G$ and $K$ dwarf problems. The number of $M$ dwarfs peaks at $[\mathrm{Fe} / \mathrm{H}]=0.0$ and drops off quickly at lower metallicities.

We must be careful in comparing our M dwarf numbers to models of Galactic chemical evolution because our M dwarfs are not necessarily representative of the solar neighbourhood. Galactic chemical evolution models normally include a region about 1 kpc wide in the Galactic plane and about 1 kpc perpendicular to the plane to include stars that formed in the thin disc but which now leave the local neighbourhood (Tinsley 1980). The SDSS magnitude limits mean that our sample reaches out to about 2 kpc (Adelman-McCarthy et al. 2006) and that no nearby M dwarfs are included. The part of the survey which includes our M dwarf spectra is centred near the southern Galactic cap (Bochanski et al. 2007a). Much of the sampled region is more distant than the thin disc scale height, so we expect a larger fraction of thick disc stars to be included in the sample than would be found locally.

Including a larger fraction of stars outside the thin disc would presumably mean including more low-metallicity stars than would be found in a sample from the local neighbourhood. The fact that even with the probable inclusion of more thick disc stars we still find a paucity of low metallicity stars strengthens the case for the existence of an M dwarf problem.

In addition to our sample being made up of non-local stars, the fraction of thin disc versus thick disc stars should vary with metallicity because metallicity is correlated with luminosity and distance for late main sequence stars. We corrected for differences in sampling volume caused by low-metallicity stars being less luminous and thus being observed at smaller distances for similar apparent magnitudes. We did not correct for the difference in population sampling caused by the variation of brightness with metallicity. Nor did we correct for differences in average stellar metallicity with distance perpendicular to the disc. Solar metallicity stars in our sample are at a larger average distance from the Galactic disc than low-metallicity stars with the same temperature. We therefore expect the solar metallicity stars to be found in a region with a larger fraction of thick disc stars and a smaller average metallicity. Ivezić et al. (2008) found no correlation between metallicity and kinematics in the Galactic disc, but did measure a 0.2 dex variation in metallicity from 500 pc to several kpc above the disc. Because the metallicity variation with distance in our sample should be small, less than the 0.2 dex variation found by Ivezić et al. (2008), our lack of a correction for metallicity or population differences in our sample should not alter our results appreciably.

We find evidence that the small fraction of low-metallicity stars seen for G dwarfs is observed for M dwarfs as well, providing another strong constraint on models of Galactic chemical evolution. The solution to the $\mathrm{G}, \mathrm{K}$ and M dwarf problem will require using models which abandon the assumptions of the 'Simple model' as discussed in Section 1.

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[^0]:    *E-mail: vwoolf@unomaha.edu (VMW); aawest@bu.edu (AAW)

[^1]:    ${ }^{1}$ Here $[\mathrm{X}]=\log _{10}(\mathrm{X})_{\text {star }}-\log _{10}(\mathrm{X})_{\text {Sun }}$.

