A Fast Hybrid Transform Algorithm for Beam Digitization

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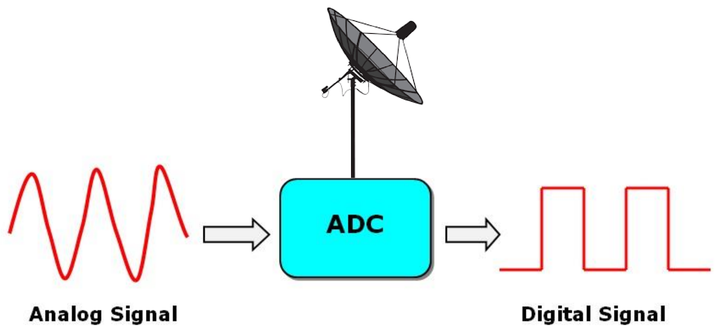
INTRODUCTION

The Fast Fourier Transform (FFT) is used to compute the Discrete Fourier Transform (DFT) and its inverse efficiently. It is sufficient to quote "The FFT is the most important numerical algorithm of our lifetime", by Gilbert Strang [1]. The DFT, Discrete Cosine Transform (DCT), and Discrete Sine Transform are the pillars of modern multimedia signal processing and play crucial roles in increasing throughput and decreasing circuit complexity, manufacturing cost, and power consumption [2, 3]. Based on the DFT and DCT, we define the Beam Digitization Transform (BDT) as follows;

$$A_n = [a_{ij}]_{i,j=0}^{n-1} = \left[\sum_{k=0}^{n-1} \epsilon_n(k)\, \omega_n^{ik} \cos\frac{k(2j+1)\pi}{2n}\right]_{i,j=0}^{n-1}$$

where $\epsilon_n(0) = \epsilon_n(n) = \frac{1}{\sqrt{2}}, \epsilon_n(k) = 1$ for $n \geq 2$ is an even integer and $\omega_n = e^{-2\pi i/n}$ is the primitive root of unity.



Many associate the Fourier transform with the transformation of a signal from the time domain to the frequency domain and vice-versa. The FFT simply extends the ability to discrete points without the need for the derivation of the wave equation of the original signal. The FFT must wait until enough sequential points in time have been gathered. Then it may begin the process of converting this. signal from the time domain into the frequency domain for further analysis. Based on the proposed algorithm, we present the signal flow graph for the 8- point case; however, these blocks are used and duplicated to realize very large-scale integrated circuits to reduce ADCs.

METHODOLOGY

The reduction and minimization of the number of analog-to-digital converters (ADCs) is of paramount importance in array processing receivers with applications in wireless communications, radar, microwave imaging, and radio astronomy [4]. We propose a fast, exact, and stable BDT algorithm via a novel matrix factorization technique in connection to Chebyshev-like polynomials and realize the proposed algorithm to minimize the number of ADCs.

RESULTS

The BDT is defined as $\boldsymbol{A_n = F_n C_n^{II}}$, where $\boldsymbol{F_n}$ is the DFT matrix and $\boldsymbol{C_n^{II}}$ is the type II DCT matrix. The matrix $\boldsymbol{A_n}$ can be factored into the product of sparse and orthogonal matrices for any $n = 2^t \ (t \geq 1)$;

$$\boldsymbol{A_n} = \widetilde{D}_n \begin{bmatrix} I_{\frac{n}{2}} & 0 \\ 0 & F_{\frac{n}{2}} \end{bmatrix} \begin{bmatrix} A_{\frac{n}{2}} & 0 \\ 0 & I_{\frac{n}{2}} \end{bmatrix} \begin{bmatrix} I_{\frac{n}{2}} & 0 \\ 0 & W_{\frac{n}{2}} \end{bmatrix} \begin{bmatrix} I_{\frac{n}{2}} & 0 \\ 0 & C_{\frac{n}{2}}^{III} \end{bmatrix} \begin{bmatrix} I_{\frac{n}{2}} & 0 \\ 0 & B_{\frac{n}{2}}^{T} \end{bmatrix} H_n,$$

where

$$\widetilde{D}_n = \begin{bmatrix} I_{\frac{n}{2}} & D_{\frac{n}{2}} \\ I_{\frac{n}{2}} & -D_{\frac{n}{2}} \end{bmatrix}, H_n = \begin{bmatrix} I_{\frac{n}{2}} & \tilde{I}_{\frac{n}{2}} \\ I_{\frac{n}{2}} & -\tilde{I}_{\frac{n}{2}} \end{bmatrix},$$

$$C_n^{III} = H_n^T \begin{bmatrix} I_{\frac{n}{2}} & 0 \\ 0 & W_{\frac{n}{2}} \end{bmatrix} \begin{bmatrix} C_{\frac{n}{2}}^{III} & 0 \\ 0 & C_{\frac{n}{2}}^{III} \end{bmatrix} \begin{bmatrix} I_{\frac{n}{2}} & 0 \\ 0 & B_{\frac{n}{2}}^{T} \end{bmatrix},$$

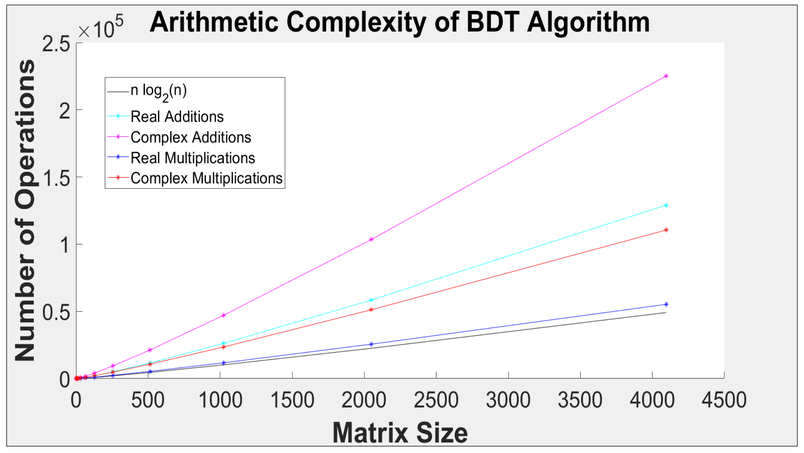
$$F_n = \frac{1}{\sqrt{n}} [\omega_n^{jk}]_{j,k=0}^{n-1}, \ B_{\frac{n}{2}}^T = \begin{bmatrix} \sqrt{2} & & & & \\ 1 & 1 & & & \\ & 1 & \ddots & & \\ & & \ddots & 1 & \\ & & & 1 & 1 \end{bmatrix},$$

$$D_{\frac{n}{2}} = diag[\omega_n^k]_{k=1}^{\frac{n}{2}}, W_{\frac{n}{2}} = \frac{1}{2}\, diag\left[\sec\left(\frac{(2k-1)\pi}{2n}\right)\right]_{k=1}^{\frac{n}{2}}.$$

Arithmetic Complexity

Real Input	**Complex Input**
$\mathbb{A}(\boldsymbol{A_n}, n) = 3nt - \frac{9}{2}n + 4t - 1$	$\mathbb{A}(\boldsymbol{A_n}, n) = \frac{9}{2}nt + n - 2t - 9$
$\mathbb{M}(\boldsymbol{A_n}, n) = nt + \frac{3}{2}n - 2t - 1$	$\mathbb{M}(\boldsymbol{A_n}, n) = 2nt + 3n - 4t - 2$

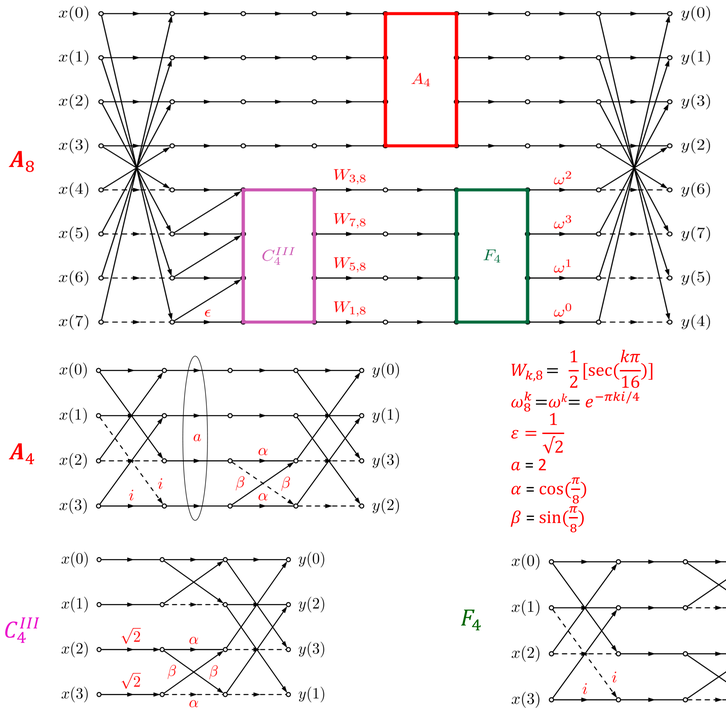
The BDT algorithm costs $O(n \log n)$ operations as opposed to the brute-force calculation with $O(n^2)$ operations.



74.5 ≤ Speed Improvement Factor ≤ 151.8

RESULTS

Signal Flow Graphs



CONCLUSION

Results on the proposed project will lead to:

- **Sparse and Orthogonal Factorization**
- **Recursive BDT Algorithm**
- **Reduction in Complexity from $O(n^2)$ to $O(n \log n)$**
- **Reduction in Number of ADCs**
- **Minimize Chip Area and Reduce Power Consumption**

REFERENCES

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[4] M. Zatman, "Digitization requirements for digital radar systems," Proceedings of the 2001 IEEE Radar Conference 163-168, 2001.