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Polarization of Majorana Fermions in a Background Current

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Majorana Fermions

- Fermions are spin-1/2 particles with two definite spin orientations
- Majorana fermions are theoretical fermions with no distinction between the particle and antiparticle state [1]
- This means that Majorana fermions can self annihilate
- This annihilation rate is dependent on whether they are spin aligned (polarized) or anti-aligned (unpolarized), with polarized particles annihilating at a lower rate [2]
- Of the known Standard Model particles, neutrinos are the only possible Majorana fermion candidate

Anapole Moment

- Particles can interact electromagnetically with other particles through their charge, electric dipole moment, magnetic dipole moment, and anapole moment

EM moment	Distribution	Example	Interaction
Electric dipole	$\vec{\mathbf{p}}$	$\vec{\mathbf{p}}$	$\vec{\mathbf{N}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$ $U = -\vec{\mathbf{p}} \cdot \vec{\mathbf{E}}$
Magnetic dipole	$\vec{\mathbf{m}}$	$\vec{\mathbf{m}}$ spin	$\vec{\mathbf{N}} = \vec{\mathbf{m}} \times \vec{\mathbf{B}}$ $U = -\vec{\mathbf{m}} \cdot \vec{\mathbf{B}}$
Anapole	$\vec{\mathbf{a}}$	55 Cs Caesium 132.905	$\vec{\mathbf{N}} = \vec{\mathbf{a}} \times \vec{\mathbf{J}}$ $U = -\vec{\mathbf{a}} \cdot \vec{\mathbf{J}}$

Figure 1. Table showing three of the four different electro-magnetic interactions, their classical distribution, an example, and how they interact. All three will feel a torque in the presence of a field. Anapole moments feel a torque in the presence of a current, tending to align with the current.

- Majorana fermions have no charge, electric dipole moment, or magnetic dipole moment, only interacting through an anapole moment [3][4]
- Anapole moment interactions are generally weaker, meaning Majorana fermions interact weakly

Dark Matter

- Dark matter is a class of matter making up around 80% of the known matter in the universe [5]
- Its interactions with light are highly suppressed
- While we know some of its properties, we currently do not know what makes up dark matter
- Because Majorana fermions only have an anapole moment, their interactions with light are also highly suppressed, and are therefore good candidates for dark matter
- Understanding the annihilation rates of Majorana fermions is therefore important in understanding the relic density of dark matter
- Also, annihilation rates are needed to interpret results from indirect dark matter experiments

Research Question

The aim of my research is to investigate whether Majorana fermions can be polarized by a background current

- Because Majorana fermions interact solely through their anapole moment, they will tend to align with a background current
- This interaction, however, is generally weak and not guaranteed to happen in a timely manner

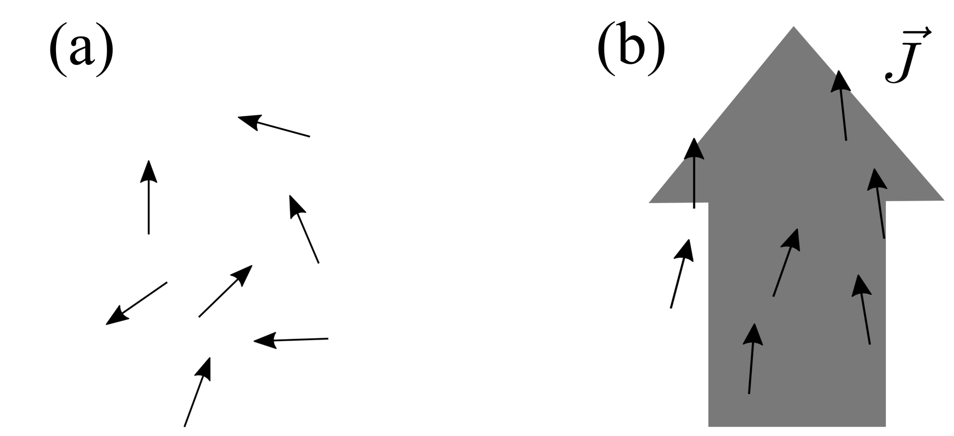

Figure 2. Majorana fermions represented as arrows pointing in the direction of their spin (a) unpolarized in the absence of a background current, and (b) in the presence of a background current, becoming polarized and aligning with the current.

Methods

- We used Feynman diagrams to set up our calculations
- Feynman diagrams are theoretical tools that help visualize quantum interactions and provide an organizational framework for the calculations (see *Fig. 3*)
- From our Feynman diagram, we can write down the amplitude for the interaction:

$$\mathcal{M} = e f_a \left[\bar{u}_\psi(k')\gamma^\mu u_\psi(k)\right] g_{\mu\nu} \left[\bar{u}_\chi(p') \left(\frac{q^2\gamma^\nu - q^\nu \not{q}}{q^2}\right)\gamma^5 u_\chi(p)\right]$$

To simplify this expression we assume:

- All particles are non-relativistic
- The Majorana fermion is initially at rest
- Momentum transfer is small (keep only leading order in q)
- The Majorana fermion flips spin

Shifting to spherical coordinates, the amplitude becomes:

$$\mathcal{M} = e f_a\left[\left(2M_\psi + \frac{|\mathbf{k}|^2}{2M_\psi} + 4M_\chi \frac{|\mathbf{k}|\cos\theta}{|\mathbf{q}|} - 2M_\chi\right)(|\mathbf{q}|\sin\theta\cos\phi + i|\mathbf{q}|\sin\theta\sin\phi)\right]$$

From this amplitude, we can then calculate the differential cross section, dσ:

$$d\sigma = |\mathcal{M}|^2 \frac{\hbar^2}{4\sqrt{(p_1\cdot p_2)^2 - (m_1 m_2)^2}} \left[\left(\frac{d^3\mathbf{p}_3}{(2\pi)^3 2E_3}\right)\left(\frac{d^3\mathbf{p}_4}{(2\pi)^3 2E_4}\right)\right](2\pi)^4 \times \delta^4(p_1 + p_2 - p_3 - p_4)$$

which can finally be integrated to yield the cross section.

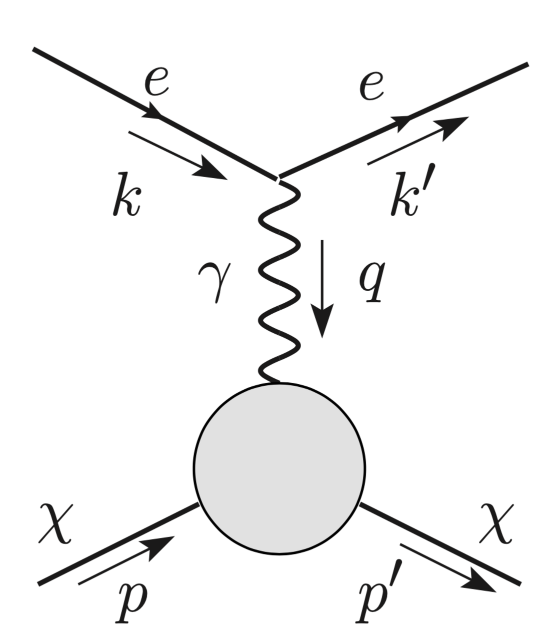

Figure 3. Feynman diagram for my project. Time moves left to right, and space up and down. Each line represents a particle, with the top lines representing the current, e, and the bottom lines representing the Majorana fermion, χ. The squiggly line represents a virtual photon which mediates the interaction. k, k', p, p', and q are the initial and final momenta of the current and fermion, and the momentum transferred by the virtual photon, respectively.

Results and Discussion

We calculated the final cross section to be

$$\sigma = \frac{\hbar e^2 f_a^2 |\mathbf{k}|^2 M_\chi^2 (|\mathbf{k}|^2 + 8M_\psi^2)^2}{96\pi M_\psi^2 (M_\psi + M_\chi)^4}$$

where $\hbar e^2 f_a^{\,2}$ is a constant, M_ψ and M_χ are the masses of the current and Majorana fermion respectively, and ***k*** is the incoming momentum of the current.

- We can see that the cross section is dependent only on the particle masses and incoming momentum of the current; however, this is unsurprising since we set the initial momentum of the Majorana fermion to 0
- When doing the cross section integral, we found that θ, the scattering angle with respect to the z-axis, is limited between 0 and π/2, meaning the Majorana fermion can't be scattered back in the direction the current came from

Future Work

- The interaction that I looked at is just one of a few ways for Majorana fermions to polarize in the presence of a background current
- My results will therefore need to be combined with others to get an overall spin flip interaction rate, which can then give us an idea of the time scale on which Majorana fermions will polarize in the presence of a background current
- Before being able to contribute to the overall spin flip interaction rate, I still need to calculate the interaction rate for my specific interaction, plugging in realistic numbers for the masses and momenta
- The momenta values for the current will come from measured values for high energy currents, like those in stars
- We estimate that the time scale on which polarization occurs will be large

Acknowledgments and References

Acknowledgments

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