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A GENERAL PANEL MODEL WITH RANDOM AND FIXED EFFECTS: A STRUCTURAL EQUATIONS APPROACH

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Abstract

Fixed and random effects models for longitudinal data are common in sociology. Their primary advantage is that they control for time-invariant omitted variables. However, analysts face several issues when they employ these models. One is the uncertainty of whether to apply the fixed effects (FEM) versus the random effects (REM) models. Another less discussed issue is that the FEM and REM models as usually implemented might be insufficiently flexible. For instance, the effects of variables, including the latent time-invariant variable, might change over time rather than be constant as in the usual FEM and REM. The latent time-invariant variable might correlate with some variables and not others. Lagged endogenous variables might be necessary. Alternatives that move beyond the classic FEM and REM models are known, but they involve different estimators and software that make these extended models difficult to implement and to compare. This paper presents a general panel model that includes the standard FEM and REM as special cases. In addition, it provides a sequence of nested models that provide a richer range of models that researchers can easily compare with likelihood ratio tests and fit statistics. Furthermore, researchers can implement our general panel model and its special cases in widely available structural equation models (SEMs) software. The paper is oriented towards applied researchers with most technical details given in the appendix and footnotes. An extended empirical example illustrates our results.

Introduction

Longitudinal data are more available today than ever before. The National Longitudinal Study of Youth (NLSY), National Longitudinal Study of Adolescent Health (Add Health), Panel Study of Income Dynamics (PSID), and National Education Longitudinal Study (NELS) are just a few of the more frequently analyzed panel data sets in sociology. The relative advantages of longitudinal data compared to cross-sectional are well-known (Baltagi, 2005:4–9; Halaby 2004) and panel data are permitting more sophisticated analyses than were previously available.

In sociology two common models for such data are referred to as the random effects model (REM) and fixed effects model (FEM) (Allison 1994; Guo and Hipp 2004). Indeed, a number of articles have made use of the FEM or REM in sociology (e.g., Nielsen and Hannan, 1977; Nielsen, 1980; Kilbourne, England, Farkas, Beron, and Weir, 1994;

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Alderson, 1999; Alderson and Nielsen, 1999; Conley and Bennet 2000; Mouw, 2000; VanLaningham, Johnson, and Amato, 2001; Budig and England, 2001; Wheaton and Clarke, 2003; Teachman, 2004; Yakubovich, 2005; Beckfield 2006; Brand, 2006; Matsueda, Kreager, and Huizinga, 2006; Shauman and Noonan, 2007). A major attraction of these models is that they provide a way to control for all time-invariant unmeasured (or latent) variables that influence the dependent variable whether these variables are known or unknown to the researcher. Given the likely presence of such omitted variables, this is a major advantage. The REM assumes that the omitted time-invariant variables are uncorrelated with the included time-varying covariates while the FEM allows these variables to freely correlate (Mundlak, 1978). The REM has the advantage of greater efficiency relative to the FEM leading to smaller standard errors of coefficients and higher statistical power to detect effects (Hsiao 2003). A Hausman (1978) test enables researchers to distinguish between the REM and FEM. Statistical software for REM and FEM is readily available (e.g., xtreg in Stata and Proc GLM, Proc Mixed in SAS).

Despite the many desirable features of the REM and FEM for longitudinal data, they are limited by the way sociologists typically use them. First, there appears to be confusion over when to use the REM versus FEM. Halaby (2004) reviews a number of panel studies from sociology and concludes that many studies "ignore the issue of unobserved unit effects altogether, or they recognize such effects but fail to assess and take steps to deal with their correlation with measured covariates" (p.520).

Researchers sometimes take false comfort in the use of the REM in that it does include a latent time-invariant variable ("individual heterogeneity") without realizing that biased coefficients might result if the observed covariates are associated with the latent time-invariant variable. Second, there are restrictions imposed in the usual estimation of FEM and REM that might not make substantive sense in sociological applications. For instance, both the usually implemented FEM and REM assume that the coefficients of the same covariate remain equal and the error variances of equations are equal across all waves of data.[1] If individuals pass through major life course transitions during the time period of the study (e.g., actively employed to retired or adolescence to adulthood), these assumptions of stable effects could be invalid. So in an analysis of say, income's effect on conservatism, a standard model forces the impact of income on conservatism to remain the same across all waves of data. Similarly, the impact of the latent time-invariant variable on the outcome variable is assumed to be stable across all waves of data.

Another constraint in the standard models is that the latent time-invariant variables are either free to correlate with all time-varying covariates as in the FEM or they must be uncorrelated with all covariates as in the REM. Even with strong prior evidence that some correlations are zero and some are not, the usual models result in just two choices, freely correlated or all uncorrelated. Incorrectly assuming that the latent time-invariant variable is uncorrelated with the observed covariates is likely to bias estimates. Unnecessarily estimating zero correlations as in the usual FEM uses up degrees of freedom and can increase asymptotic standard errors.

Yet another implicit constraint in the usual models is that the lagged dependent variables have no effects. In some areas, prior values of the dependent variable influence current values even net of other variables. Last year's income, for example, might influence this year's income net of control variables, so it would be helpful to include lagged income as an

[1]As we mention below, there are methods of handling time-specific coefficients for the time-varying covariates, but this issue is seldom examined in practice.

explanatory variable. Furthermore, in the usual FEM and REM, the observed covariates in the models are not permitted to influence the latent time-invariant variable.

There is a vast literature in econometrics that suggests models and estimators to overcome many but not all of these limitations (for reviews see Baltagi and Raj, 1992; Wooldridge, 2002; Hsiao, 2003; Halaby, 2004; Baltagi, 2005). For instance, interaction terms of the time period with a covariate permits the observed time-invariant or time-varying variables' effects to differ by wave (Chamberlain, 1984). While lagged values of time-varying covariates are straightforward to include, lagged dependent (endogenous) variables raise other complications in the FEM and REM that can lead to biased estimates with several authors suggesting solutions (e.g., Arellano and Bond, 1991; Arellano and Bover, 1995). Hausman and Taylor (1981) proposed a method by which observed time-invariant variables such as sex, race, place of birth, etc. can be included in FEM.

In the political science literature, Beck (2001) and Wilson and Butler (2007) review panel methods that work well when the number of waves of data are large relative to the number of cases. Beck and Katz (1995) recommend a cross-sectional time-series models that assumes that all cases have a common intercept and common slopes over time. Wilson and Butler (2007) argue that researchers should test these assumptions as well as whether a lagged dependent variable should be included.

A variety of estimators for these alternative models are proposed. This raises two issues. One is that the use of different models and different estimators complicates the ability to compare the relative fit of alternative models to the same data. This makes it hard to determine what improvements to the model are necessary and which are not. A practical related issue is that not all of these extensions are readily accessible in software packages which tend to inhibit their use. Indeed, Halaby's (2004) review of the sociological literature suggests that most panel analyses are limited to the standard FEM or REM with little consideration of alternatives.

Another limitation of the usual application of FEM and REM is that researchers make very limited assessments of the fit of their model to the data. When statistical tests of the model are applied it is the Hausman (1978) test comparing the usual FEM and REM that is typical. The Hausman test might lend support to one of these models even if the selected model is an inadequate description of the data. Additional tests are available that can provide evidence of model adequacy. As we will explain, the standard FEM and REM are *overidentified* models that imply overidentifying constraints. These overidentifying constraints are testable and provide evidence on the validity of the FEM, REM, or alternative specifications of models. We also will explain how it is possible to test not only whether the observed covariates have different effects for different waves, but how to test this same possibility for the latent time-invariant variables. This latter possibility is neglected in the panel model literature.

These tasks are facilitated by having a general panel model that encompasses not only the usual FEM and REM, but includes a wide variety of extensions. Furthermore, having a common estimator that permits comparisons of nested models and assesses the plausibility of the overidentifying constraints is a real asset to this work. Our purpose is to present a general panel model that includes the usual FEM and REM as well as a variety of other nonstandard models as special cases. We will explain how the general equation as well as its special cases can be estimated, tested, and compared using structural equation models (SEMs) and SEM software. More specifically, we will illustrate how to test the overidentifying restrictions of the random effects, fixed effects, and alternative models to help assess their relative fit; we will show how to test whether coefficients or error variances are equal across waves; we will illustrate the estimation of the covariance of the latent time-invariant variable and the time-varying observed covariates; we will develop models that permit lagged dependent variables or lagged covariates and create models that permit time-invariant observed variables in fixed effects like models. We also will take advantage of alternative estimators and fit statistics from the SEM literature to assess the correspondence of the models to the data and to compare different models. We will illustrate our results using a model and data from an excellent empirical application of the usual FEM and its estimation to demonstrate that we can gain new insights with this approach even in a well-executed study.

A few prior papers have drawn connections between SEMs and random or fixed effects panel models. An unpublished convention paper by Allison & Bollen (1997) and a SAS publication by Allison (2005) discuss SEM set-ups of the standard FEM and REM. Teachman, Duncan, Yeung, and Levy (2001) look at the FEM in SEM, but concentrate their discussion and example on cross sectional data with clusters of families rather than panel data. Finally, Ejrnaes & Holm (2006) look at different types of fixed effects estimators in panel data models, but do not cover the REM, lagged dependent variable models, or some of the other variants that we include here. None of these papers presents our general panel model, they do not discuss testing the equality of parameters over time or treat supplemental fit indices, and they do not provide the justification for the maximum likelihood estimator for models with lagged dependent variables.

Our paper is oriented to readers who have had experience with the standard FEM or REM. Our citations above to recent substantive literature reveal their presence in a broad range of fields in sociology and suggest that these techniques have wide appeal. Our reading of the literature also suggests that there is some confusion among practitioners regarding when to use one model versus the other. We do *not* intend our results only for specialists in quantitative methods. Despite the generality and flexibility of our models, they can be implemented with any of the numerous SEM software programs (e.g., LISREL, AMOS, Mplus, EQS, etc.) that are widely available. Because of our intended audience, full technical details are not provided in the text, but are reserved for footnotes, the appendix, and in the cited works.

The next section will present the notation, a general panel model, and its assumptions. Following this are subsections on restricted forms of the general panel model, the FEM, the REM, and the general panel model with lagged variables. Sections on estimation and testing model fit follow. Then an extended empirical example illustrates many of the results. The conclusion reviews the capabilities and limitations of our approach. An appendix provides more technical details including a justification of the maximum likelihood (ML) system estimator in SEMs with lagged dependent variables.

General Panel Model

In this section, we represent a general panel model that enables us to consider individual heterogeneity (latent time-invariant variables) as in the usual FEM and REM, but permits additional structures for comparison. There are two versions of this model, one with a lagged dependent variable as a covariate and another without it. We present the latter first and show how we can derive the well-known random effects, fixed effects, and alternative models by imposing restrictions on this general panel model. In a separate subsection we introduce lagged endogenous variables into this general panel model.

Consider the following equation

$$y_{it}=\mathbf{B}_{y\mathbf{x}t}\mathbf{x}_{it}+\mathbf{B}_{y\mathbf{z}t}\mathbf{z}_i+\lambda_t\eta_i+\varepsilon_{it}$$

where y_{it} is the value of the dependent variable for the i th case in the sample at the t th time period, $\mathbf{x}_{it}$ is the vector of time-varying covariates for the i th case at the t th time period, $\mathbf{B}_{y\mathbf{x}t}$ is the row vector of coefficients that give the impact of $\mathbf{x}_{it}$ on y_{it} at time t, $\mathbf{z}_i$ is the vector of observed time-invariant covariates for the i th case with $\mathbf{B}_{y\mathbf{z}t}$ a row vector of coefficients at time t that give the impact of $\mathbf{z}_i$ on y_{it}. The η_i is a scalar of all other latent time-invariant variables that influence y_{it} and λ_t is the coefficient of the latent time-invariant variable (η_i) at time t and at least one of these λ_t is set to one to provide the units in which the latent variable is measured (e.g., set $\lambda_1 = 1$). The ε_{it} is the random disturbance for the i th case at the t th time period with $E(\varepsilon_{it})=0$ and $E(\varepsilon_{it}^2)=\sigma_{\varepsilon_t}^2$. It also is assumed that ε_{it} is uncorrelated with $\mathbf{x}_{it}$, $\mathbf{z}_i$, and η_i and that $COV(\varepsilon_{it}, \varepsilon_{is}) = 0$ for $t \neq s$. As an example, y_{it} might be the infant mortality rate in county i at time t, $\mathbf{x}_{it}$ might consist of time-varying variables such as unemployment rate, physicians per capita, medical expenditures per capita, etc. all for county i at time t, $\mathbf{z}_i$ might be time-invariant variables such as region and founding date of county, and η_i would contain all other time-invariant variables that influence infant mortality, but that are not explicitly measured in the model. The η_i represents individual heterogeneity that affects the outcome variable.[2] We assume that η_i is uncorrelated with $\mathbf{z}_i$ if both are included in the same model.

Note that i always indexes the cases in the sample while t indexes the wave or time period. If either subscript is missing from a variable or coefficient, then the variable or coefficient does not change either over individuals or over time. For instance, $\mathbf{z}_i$ and η_i have no t subscript, but do have an i subscript. This means that these variables vary across different individuals, but do not change over time for that individual[3] and are time-invariant variables. In a similar fashion, the absence of an *í* subscript means that the coefficients in the model do not change over individuals. If a time period or wave of data were distinct, then we could include a dummy variable, say D_t, that is the same value across all individuals but could differ over time.

General Panel Model and Restrictive Forms

The general panel model in equation (general model) incorporates the usual REM and FEM as special cases and goes considerably beyond these options. Indeed, there are numerous variants of the general panel model and a large number of models that a researcher could choose. For instance, suppose researchers are studying conservatism among individuals. Assume that the relationships between authoritarian personality and other latent time-invariant variables and conservatism (y_{it}) increases over time. Allowing λ_t to increase rather than always being equal to one would permit us to accommodate the changing relations. Or the effects of education, race, and income on conservatism might vary with age and these differences in coefficients for the same variable over time are permitted in the general panel model. On the other hand, if the coefficients for the same variable are erratic and without discernable patterns at different waves, this raises questions about the specification of the model. In brief, the general panel model enables us to vary the coefficients of any of the

[2]Other SEM approaches use special cases of this model. Teachman, Duncan, Yeung, and Levy (2001) and Ejrnaes & Holm (2006) focus on the fixed effects model of $y_{it} = \mathbf{B}_{y\mathbf{x}}\, \mathbf{x}_{it} + \eta_i + \varepsilon_{it}$. An unpublished paper by Allison and Bollen (1997) present a model equivalent to what we call the REM/FEM Hybrid Model of $y_{it} = \mathbf{B}_{y\mathbf{x}}\, \mathbf{x}_{it} + \mathbf{B}_{y\mathbf{z}}\, \mathbf{z}_i + \eta_i + \varepsilon_{it}$ where ε_i can correlate with $\mathbf{x}_{it}$ as long as $\mathbf{z}_i$ does not. Allison's (2005) SAS book on fixed effects models also includes the Allison and Bollen (1997) Hybrid REM/FEM. No papers of which we are aware introduce the full general panel model that we have here.
[3]Though we use the term "individual" to refer to a case, the cases do not have to be individual people. They could be groups, organizations, nations, etc.

observed or latent variables while also serving as a sensitivity test of the model specification.

Given our general panel model ($y_{it} = \mathbf{B}_{y\mathbf{x}t}\mathbf{x}_{it} + \mathbf{B}_{y\mathbf{z}t}\mathbf{z}_i + \lambda_t\eta_i + \varepsilon_{it}$), we need a method to select more restrictive models to which we can compare its fit to the data. If a researcher is in the fortunate situation to have prior studies or theories suggest a formulation, then this hypothesized model is a desirable starting point. More commonly knowledge in an area is insufficiently developed to provide such specific guidance. A "forward model" search strategy begins with a model much more restrictive than the general panel model and lifts restrictions until the researcher judges the fit of the model to be adequate. So the standard random effects model (REM) could be the starting point and if its fit is not good, the researcher could test whether it significantly improves by, say, allowing the error variances to differ over time. If the model fit is still lacking, then the researcher could try a hybrid model that lets $\mathbf{x}_{it}$ and η_i correlate. This process could continue until the model fit is acceptable. So the modeling process is one of moving from more restrictive to less restrictive models and stopping when the fit passes some standard.

A "backwards model" search strategy begins by fitting the general panel model of $y_{it} = \mathbf{B}_{y\mathbf{x}t}\mathbf{x}_{it} + \mathbf{B}_{y\mathbf{z}t}\mathbf{z}_i + \lambda_t\eta_i + \varepsilon_{it}$. To identify this model we constrain the correlations of $\mathbf{z}_i$ and η_i to zero[4] and set one of the λ_t s to one (e.g., $\lambda_1 = 1$). This is the least restrictive model. If it does not fit the panel data, it is doubtful that more restrictive forms of the general panel model will. If it does fit, then we could impose further restrictions until we judge that the fit to the data is inadequate. Table 1 outlines a general strategy of fitting models from less to more restrictive when prior knowledge does not point to a single, specific model.

Assuming that the general panel model at the top of Table 1 fits the data, then researchers can turn to a model that constrains the effects of the latent time-invariant variable (η_i) to be equal over time as shown in option (1) in Table 1. Option (1) is nested in the general panel model, so as we explain in the section on Model Fit, we can test whether there is a statistically significant decline in fit by imposing the restriction of $\lambda_t = 1$ for all waves. If option (1) has an adequate fit, then researchers can impose an additional restriction that the coefficients for the impact of each observed time-invariant variable is equal over time (option (2) in Table 1). With this restriction, we have both the latent and observed time-invariant variables having stable influences over each wave of data. If option (2) proves to be a good fit, then we can introduce the additional constraint that the coefficients for the time-varying variables maintain the same values over time ($\mathbf{B}_{y\mathbf{x}t} = \mathbf{B}_{y\mathbf{x}}$) (option (3) in Table 1).

If the model represented by option (3) in Table 1 fits, then we introduce different types of restrictions in the other options. Option (4) keeps all coefficients equal over time, but also constrains the correlations of the time-varying variables ($\mathbf{x}_{it}$) with the latent time-invariant variable (η_i) set to zero. Table 1 points to two different options depending on the fit of the model represented in option (4). If we fail to reject option (4), then option (5)(a) introduces the additional constraint of the error variances being equal for all waves of data. If we do reject option (4), then option (5)(b) returns to the model in option (3) where the time-varying and latent time-invariant variables correlate and constrains the error variances to be equal over time. Table 1 does not show all possible options; it is meant more as an illustration rather than the only way to proceed. For instance, if the restriction of $\lambda_t = 1$ for all waves does not hold we could remove this restriction but still check whether $\mathbf{B}_{y\mathbf{x}t} = \mathbf{B}_{y\mathbf{x}}$ holds. In such a series of specifications, we would not recover the standard REM and SEM. We

[4]Attempting to estimate this correlation would lead to perfect collinearity between $\mathbf{z}_i$ and η_i. This would make it impossible to estimate the effects of these variables.

encourage readers to view Table 1 as providing guidelines rather than a rigid sequence of steps. The REM and FEM are such important restrictive forms of the general panel model that the next two subsections will discuss them more fully.

Random Effects Model (REM)

The Random Effects Model (REM) is one of the most popular models for panel data. Table 1 option (6) indicates that the REM is equivalent to the model of option (5)(a). If we assume that the coefficients of the time-varying variables ($\mathbf{x}_{it}$), of the observed ($\mathbf{z}_i$) and latent (η_i) time-invariant variables do not change over time (i.e., $\mathbf{B}_{y\mathbf{x}t} = \mathbf{B}_{y\mathbf{x}}$; $\mathbf{B}_{y\mathbf{z}t} = \mathbf{B}_{y\mathbf{z}}$, and $\lambda_t = 1$, respectively for all t), that the equation error variances are equal ($\sigma^2_{\varepsilon t}=\sigma^2_{\varepsilon}$), and that λ_i is uncorrelated with $\mathbf{x}_{it}$ and $\mathbf{z}_i$, then equation (general model) becomes

$$y_{it}=\mathbf{B}_{y\mathbf{x}}\mathbf{x}_{it}+\mathbf{B}_{y\mathbf{z}}\mathbf{z}_i+\eta_i+\varepsilon_{it}$$

which is the usual REM. Comparing this to the general panel model we can see that the REM assumes that all explanatory variables (i.e., $\mathbf{x}_{it}$, $\mathbf{z}_i$, η_i) have effects on y_{it} that are the same over all time periods. Furthermore, the REM allows the time-varying observed variables in $\mathbf{x}_{it}$ and the time-invariant observed variables in $\mathbf{z}_i$ to correlate, but none of these observed variables is permitted to correlate with the latent time-invariant variable, η_i. The assumption that η_i is uncorrelated with $\mathbf{x}_{it}$ and $\mathbf{z}_i$ is similar to the assumption that the disturbance in a cross-section is uncorrelated with the observed explanatory variables. The main difference is that with panel data there are circumstances when we can partially test this assumption as we will describe in out treatment of the FEM.

Figure 1 is a path diagram representation of a REM that is kept simple with a single time-varying variable (x) for four waves of data and a single time-invariant variable (z_1). A path diagram is a graph that represents a multiequation system and its assumptions. By convention, boxes represent observed variables, ovals represent latent variables, single-headed straight arrows represent the direct effect of the variable at the base of the arrow on the variable at the head of the arrow, and two-headed arrows such as those connecting the x s and z_1 stand for possible associations between the connected variables where that association is taken account of, but not explained within the model.[5] To simplify the notation the i subscript is excluded for the variables. It is noteworthy that the latent time-invariant variable (η) is part of the model, but it is shown to be uncorrelated with the time-varying variables ($\mathbf{x}_t$) and the time-invariant variable (z_1) since there are no two-headed arrows linking it to the observed variables. The direct impact of the latent time-invariant variable (η) on the repeated measures (y s) is equal to 1 as is implicit in the equation for the REM.

Fixed Effects Model (FEM)

Returning to the general panel model in equation (general model), suppose that we keep the coefficients for the time-varying variables equal for all waves ($\mathbf{B}_{y\mathbf{x}t} = \mathbf{B}_{y\mathbf{x}}$, $\lambda_t = 1$), we drop $\mathbf{B}_{y\mathbf{z}t}\mathbf{z}_i$, we allow the latent time-invariant variables (η_i) to correlate with, and we set the equation error variances equal ($\sigma^2_{\varepsilon t}=\sigma^2_{\varepsilon}$) This is option (7) in Table 1 and leads equation (general model) to become

[5] A possible source of confusion is that path analysis is sometimes used to refer just to recursive or nonrecursive models of only observed variables where measurement error and latent variables are not considered. This is an inaccurate restriction on the usage of the term. In fact, Sewall Wright, the inventor of path analysis, included latent variables as part of path analysis.

$$y_{it}=\mathbf{B}_{y\mathbf{x}}\mathbf{x}_{it}+\eta_i+\varepsilon_{it}$$

which is the equation for the usual fixed effects model (FEM).

The most obvious difference between the FEM and the REM one is the absence of $\mathbf{B}_{y\mathbf{z}}\mathbf{z}_i$. These are the time-invariant *observed* variables and their coefficients. The usual FEM does not explicitly include these variables, but rather folds them into η_i, the latent time-invariant variable. The reason is that the FEM allows η_i to correlate with $\mathbf{x}_{it}$ and if we were also to include time-invariant *observed* variables ($\mathbf{z}_i$), these would be perfectly collinear with η_i and we could not get separate estimates of the effects of η_i and $\mathbf{z}_i$. Hence, we allow η_i to include $\mathbf{z}_i$ as well as latent time-invariant variables. Though losing the ability to estimate the impact of time-invariant variables such as race, sex, etc. is a disadvantage, we still are controlling for their effects by including η_i in the model. In addition, we have a potentially more realistic assumption that allows the η_i variable to correlate with the time-varying covariates in $\mathbf{x}_{it}$. In the hypothetical example of infant mortality rates in counties presented above, we could not include the time-invariant variables of region and founding date explicitly in the model. But these and all other time-invariant variables would be part of η_i and hence controlled. If a researcher is not explicitly interested in the specific effects of the time-invariant variables, then this is not a serious disadvantage since the potentially confounding effects of all time-invariant variables would be controlled. In addition, we allow these time-invariant variables to correlate with the time-varying variables such as unemployment, physicians per capita, and so on.

Figure 2 is a path diagram representation of a FEM with a single time-varying variable. We drop $\mathbf{z}_i$ given its perfect collinearity with η_i. Easily visible within the diagram is the covariance of the time-varying x_{1t} and η that is part of the FEM specification. But one difference from the usual implementation of FEM is that the covariances of the time-varying variables with η are an explicit part of the model. This can provide the researcher a better sense of the properties of these latent time-invariant variables and their pattern of associations in that a researcher can estimate the correlation with observed covariates. The equality of the coefficients from x_{1t} to y_t is shown by using the same coefficient for each path as is the coefficient of 1 from η to y_t. Below we will present an empirical application where the repeated measure (y_t) is wages, the number of children is a time-varying covariate (x_{1t}), and all omitted time-invariant variables (e.g., intelligence, motivation, other stable personality traits) are combined in η with this latent variable permitted to correlate with x_{1t}.

Note that *if* the REM does *not* include $\mathbf{z}_i$, then the REM and FEM are nested and only differ in that the FEM allows $\mathbf{x}_{it}$ and η_i to freely correlate where the REM restricts them to be uncorrelated. If the REM does include $\mathbf{z}_i$ and the FEM does not, then the models are not nested. But if we include $\mathbf{z}_i$ in the FEM while keeping it uncorrelated with η_i, we are led to the model 5 (b) which is nested in the REM 5 (a). These two differ only in that the REM assumes that the covariances of the time-varying $\mathbf{x}_{it}$ and η_i are zero whereas model 5 (b) does not.[6]

Viewing the REM and FEM from the perspective of the more general panel model in equation (general model), these models are two important special cases, but there are other models which might better correspond to our theoretical and substantive ideas and which

[6]A reviewer points out that a classic approach views the FEM as a within-unit estimator and views the REM as a weighted average of the within and between-unit model. As we demonstrate later with our empirical example, if we use the ML estimator we will get the same estimates whether obtained via a classic FEM and REM procedure in Stata or using SEM software. This suggests that the classic interpretations hold regardless of software used. However, as we depart from these classic FEM and REM, this no longer will be true.

might better explain our panel data. We can enrich these models further by considering lagged effects as we do in the next subsection.

General Panel Model with Lagged Effects

Researchers can add lagged values of the time-varying covariates to the vector, $\mathbf{x}_{it}$. The cost of doing so is the loss of the first wave of data since the lagged value of the time-varying covariate is not available for the first wave of data and thus cannot be included. Lagged endogenous variables for autoregressive effects are also straightforward to include. We modify the general panel model in equation (general model) to include a lagged endogenous variable:

$$y_{it}=\rho_t y_{it-1}+\mathbf{B}_{y\mathbf{x}t}\mathbf{x}_{it}+\mathbf{B}_{y\mathbf{z}t}\mathbf{z}_i+\lambda_t\eta_i+\varepsilon_{it}$$

where the new symbol is the autoregressive coefficient ρ_t of the effect of y_{it-1} on y_{it}. We can get to more familiar models by introducing restrictions. A modified version of the FEM with equal autoregressive parameters would be

$$y_{it}=\mathbf{B}_{y\mathbf{x}}\mathbf{x}_{it}+\rho y_{it-1}+\eta_i+\varepsilon_{it}$$

The first wave y_{i1} should be treated as predetermined and correlated with the time-varying ($\mathbf{x}_{it}$) and latent time-invariant variables (η_i). Figure 3 is a modified version of Figure 2 that includes the lagged endogenous variable. Allison (2005: 135) mentions a similar, simpler FEM with a single *x* variable, but states that it has not been investigated analytically. The Appendix of this paper presents a general version of this model and treats its formulation and estimation.

An added complication to check with lagged endogenous variables is whether there is an autoregressive disturbance. This is particularly problematic if present with a lagged dependent variable since it creates a correlation between the disturbance and explanatory variable. We can treat this by adding an autoregressive relation between the disturbance term. We could further modify Figure 3 to include an autoregressive disturbance provided we allow an association between ε_{i2} and y_{i1} that would be created by the autocorrelation. It is also possible to include additional lagged values of the covariates or the endogenous variable. Furthermore, we could create a variety of special cases of equation (lagged general) in an analogous way to what we did in Table 1 but adding the autoregressive term ρ_t. To conserve space we do not present these extensions, but instead turn to the estimation and assessment of model fit using tools from structural equation models.

Estimation

Equation (general model) is the general panel model that incorporates the standard FEM, REM, and other models that we have presented except for those with the lagged endogenous variables which is in equation (lagged general). The latter we will discuss later. The literature on panel methods has proposed a variety of estimators for different special cases of this model. For instance, the Least Squares Dummy Variable (LSDV) estimator is popular for the usual FEM in (fixed effects) and generalized least squares (GLS) is a common choice for the usual REM in equation (random effects). To enhance the comparison of models we will use the same maximum likelihood estimator for both of these models as well as the model extensions proposed (see appendix). Structural equation model (SEM) software is well-suited to estimate these models in that it has a variety of estimators and it allows latent variables such as η_i. The default and dominant estimator for continuous dependent variables in SEM is the maximum likelihood estimator (MLE). The MLE is derived under the assumption that y_{it}, conditional on $\mathbf{x}_{it}$ and $\mathbf{z}_i$, $[y_{it}|\mathbf{x}_{it}, \mathbf{z}_i]$ comes from a multivariate normal distribution (Jö reskog 1973; Bollen 1989:126–28). Under these conditions, the coefficients and parameter estimates of the model have the desirable properties of MLE.[7] The appendix gives a more formal presentation of the model and MLE fitting function for SEMs. There is much work in the SEM literature that examines the robustness of the ML estimator to the normality assumption and it finds conditions where it is not required for accurate significance tests (e.g., Satorra, 1990).

Furthermore, there are other readily available estimators for SEMs that either do not require normal disturbances or that correct for nonnormality (e.g., Bollen and Stine 1990 e.g., Bollen and Stine 1992; Satorra and Bentler 1994). This means that when required we have alternative estimators that permit disturbances from nonnormal distributions.[8]

A practical matter in using the SEM software is preparing the data set. Panel data commonly appears in one of two forms. One is the long form where observations of the same individual are stacked on top of each other. Each row of the data set in a sample of individuals over several years is a "person-year." In the wide form of data, by contrast, each row refers to a different individual. The variables give the variable values for a particular individual in a particular wave of the data. The wide form is most suitable for the SEM approach. Statistical software have routines that enable easy movement between the long and wide form of panel data (e.g., in Stata, reshape)

Missing Data

Attrition or other sources of missing values on variables in panel analysis is common. In panel data "balanced" and "unbalanced" data are terms that capture the possibility that a different number of waves of data are available for different cases. The unbalanced design implies missing data. In a SEM there are two options for treating data that are Missing Completely at Random (MCAR) or Missing at Random (MAR) (Little and Rubin, 1987; Schafer, 2000). One is the direct MLE approach that allows the variables available for a case to differ across individuals and that estimates the parameters with all of the nonmissing variable information (Arbuckle, 1996). The second option is multiple imputation where multiple data sets are imputed, estimated, and their estimates combined. We apply the direct MLE in our application. Direct MLE forms the likelihood for each case in the sample using all variables that are not missing for that case. No data are imputed. Rather the contribution of a case to the total likelihood will depend on the number of observed variables with complete information for a case (Arbuckle, 1996; Wothke, 2000). Most SEM software now have the direct MLE capability to handle missing data.[9] Either of these approaches requires the analysis of the raw data rather than the covariance matrix of the observed variables.

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[7]MLEs are consistent, asymptotically unbiased, asymptotically normally distributed, asymptotically efficient among asymptotically unbiased estimators, and the inverse of the expected information matrix is available to estimate the asymptotic covariance matrix of the parameter estimator that we use for significance testing.

[8]A reviewer asked whether estimation with SEM requires any additional assumptions beyond the ones in classic FEM and REM. A variety of estimators are possible with the classic FEM and REM. However, given the variety of estimators and the generality of the models we present, the assumptions are less restrictive than the classic FEM and REM. Exceptions to this are discussed in the conclusions.

[9]Allison (2005:129) suggests that the SEM approach requires listwise deletion, though given the context of his comments they probably refer to Proc Calis in SAS. However, direct MLE or multiple imputation are both options when using most other SEM software for panel data.

Tests of Model Fit

Borrowing from the SEM literature, we can form tests of overall model fit for our panel data models. If a model is exactly correct, the null hypotheses (H_o) of

$$\boldsymbol{\mu}=\boldsymbol{\mu}(\boldsymbol{\theta})$$
$$\Sigma=\Sigma(\boldsymbol{\theta})$$

are true where **μ** and **Σ** are the means and covariance matrix of the observed variables and **μ**(**θ**) and **Σ**(**θ**) are the model-implied means and covariance matrix of the observed variables. The **θ** that is part of the model-implied means and covariance consists of the free parameters (e.g., coefficients, error variances, etc.) of a model. Each model implies a particular form of **μ**(**θ**) and **Σ**(**θ**) that predicts the means and covariance matrix. See the Appendix for these implied moment matrices for our models. In light of this, the null hypothesis in equation (moment Ho) is a test of the validity of the model. Rejection suggests that the model is incorrect while failure to reject suggests consistency of the model with the data.

The MLE provides a readily available test statistic, say *T*, that is a likelihood ratio (LR) test that asymptotically follows a chi-square distribution with degrees of freedom of $df=\left(\frac{1}{2}\right)P(P+3)-t$ where *P* is the number of observed variables and *t* is the number of free parameters estimated in the model. The $\left(\frac{1}{2}\right)P(P+3)$ is the number of variances, covariances, and means of the observed variables that provide information on the model parameters. Comparing *T* to a chi-square distribution with *df* at a given Type I error rate leads us to reject or fail to reject H_o.

The LR test of H_o : **μ** = **μ**(**θ**) & **Σ** = **Σ**(**θ**) can have considerable statistical power when the sample is large. Even minor misspecifications in the model can lead to its rejection. In practice, this means that nearly all models will be rejected in a sufficiently large sample and this might be due to errors in specification that most would consider trivial. See Satorra and Saris (1985) or Matsueda and Bielby (1986) for methods to estimate the statistical power of the chi square test of equation (moment Ho).

Alternative measures of fit have emerged in the SEM literature. The literature on these fit indices is vast (e.g., Bollen and Long, 1993; Hu and Bentler, 1998) and we do not have the space to fully review these. However, Table 2 lists several fit indices that we have found useful. Current guidelines would describe a model's fit to the data as inadequate if the Tucker and Lewis (1973) Index [*TLI*], the Bollen (1989) Incremental Fit Index [*IFI*], or the McDonald and Marsh (1990) - Bentler (1990) Relative Noncentrality Index [*RNI*] is less than 0.9. If the Steiger and Lind (1980) Root Mean Square Error of Approximation (*RMSEA*) is greater than 0.1 or if the Schwarz (1978) Bayesian Information Criterion (*BIC*) is positive (see Raftery, 1993; 1995), then the model is generally not acceptable.[10] In general it is good practice to report several fit indices along with the chi-square test statistic (*T*), degrees of freedom, and p-value. The Hausman Test provides another way to compare FEM and REM, when these are among the models estimated. Since these indices and tests measure model fit in different ways, they will not always lead to an unambiguous best model. This means that the researcher must take these fit statistics in conjunction with prior studies and knowledge of the substantive area, and perhaps further guided empirical

[10]These cutoffs are meant as rough guidelines. There are circumstances where different cutoffs might make sense. Nested and sometimes nonnested models are compared with these fit indices.

exploration of the data in coming to an assessment of which model appears to best represent the social world. We illustrate these ideas in our empirical example.

Comparisons of Models

In our presentation we described a number of models as having relations where the parameters of one model were a restricted form of another. Table 1 provides a number of examples. We can use the likelihood ratio (LR) test to compare such nested models. The LR chi-square and degrees of freedom associated with the least restricted model is subtracted from the model with fewer restrictions. A new LR chi-square test statistic and degrees of freedom results which tests whether the most restrictive model fits as well as the more restricted one. A nonsignificant chi-square is evidence in support of the more restricted model whereas a significant chi-square supports the less restricted model assuming that the less restricted model fits. This LR chi-square difference test allows us to test such hypotheses as whether the error variances or the variable coefficients are the same over time.

The fit indices described above are also a tool to compare different model structures. We already have mentioned how the BIC with the lowest value indicates the best fit. Differences in the other fit indices might also provide useful information, though in our experience, the differences in these other fit indices can be more difficult to interpret than the BIC, because the differences in the former are small in magnitude.

Wage Penalty Empirical Example

We illustrate a variety of the preceding models by examining the wage penalty for motherhood using data from the National Longitudinal Survey of Youth (NLSY). The NLSY is a national probability sample of 12,686 young men and women who were 14 to 22 years old when they were first interviewed in 1979; blacks and Hispanics are oversampled. These individuals were interviewed annually through 1994 and biannually thereafter. We begin by generally replicating the results from an earlier study by Budig and England (2001), who examined data from the 1982–93 waves of the NLSY; we differ in that, for simplicity, we only analyze every other year, i.e. 1983, 1985, 1987, 1989, 1991, and 1993. The coefficients are quite similar to those for all 12 years of data (not shown). Budig and England were interested in whether the relationship between number of children and women’s earnings is spurious or causal, and use the FEM to address this question. This study builds on a still earlier study by Waldfogel (1997), which also uses a common FEM to examine the wage penalty for motherhood. Budig and England’s study is an excellent empirical application of the usual FEM. Nevertheless, we will show how we gain new insights using our approach.

We limit our sample to women employed part-time or full-time during at least two of the years from 1982–93, to replicate Budig and England’s sample selection. Out of a total of 6,283 women in the 1979 NLSY, we have a final sample size of 5,285 women.[11] The dependent variable is log hourly wages in the respondent’s current job, where person-years whose hourly wages appear to be outliers (i.e., less than $1 or above $200 per hour) are eliminated. The main independent variable of interest is the total number of children that a respondent reported by the interview date.[12] Our first model, Model 1, includes only number of children as a covariate. In Model 2, we control for marital status using dichotomous measures to indicate married and divorced (including separated and widowed),

[11]Budig and England’s (2001) analysis resulted in a final sample of 5,287 women.

[12]Budig and England also examined the wage penalty with three dichotomous measures indicating one child, two children, and three or more children. They find that the effects are monotonic, although not perfectly linear, and prefer the continuous indicator of number of children for all other analyses.

where never married is the reference category. In Model 3, we further control for measures of human capital including years of educational attainment, current school enrollment, years of full-time and part-time work experience, years of full-time and part-time job seniority, and the total number of breaks in employment.[13] Budig and England also included a fourth model with a range of job characteristics. However, they find that these additional variables do little to change their estimates of the wage penalty for motherhood. We therefore only replicate the first three models. For all their models, Budig and England conduct a Hausman test to assess whether REMs were adequate, and in all cases, they find that the test supports the FEM. Therefore, they do not present the estimates for REM, only for FEM and the OLS estimator.

REM and FEM as SEMs

We first demonstrate how we can estimate the standard REM and FEM, as well as a hybrid of these models in a SEM framework. To estimate models in the SEM framework, we use data in wide format and estimate all models using Mplus 4.0. We apply the direct MLE (Arbuckle 1996) that estimates the parameters with all of the nonmissing variable information. Table 3 provides estimates for several model specifications in the SEM framework. The first three columns correspond to the standard REM and next three columns to the standard FEM, but estimated in the SEM framework. We compared these estimates to those obtained in Stata for the usual REM and FEM and the estimates were virtually identical except for rounding. We would expect this since we have programmed the SEM formulations to match the specifications of the REM and FEM. Hence we can reproduce the results of the REM and FEM using SEM. However, the SEM results provide additional information by way of the measures of model fit.

The model fit statistics that we include are the Likelihood Ratio (LR) test statistic (T_m), degrees of freedom (*df*), IFI/RNI, RMSEA, and BIC. We described the calculation of these fit indices in Table 2. In Table 3, the chi-square LR test statistic that compares the hypothesized FEM or REM to the saturated model leads to a highly statistically significant result, suggesting that these hypothesized models do not exactly reproduce the means and covariance matrix of the observed variables. With over 5,000 cases in this sample, the LR chi-square test has considerable statistical power to detect even small departures of the model from the data. In light of this statistical power, it is not surprising that the null hypotheses are rejected for these models ($p < 0.001$).

The supplemental fit indices provide an additional means by which to assess model fit. Both the REM and FEM for the model with no controls (Model 1) and the model that controls for marital status (Model 2) have values of IFI and RNI exceeding 0.90, a common cutoff value. However, values of RMSEA are often greater than 0.05 and values of BIC are positive, indicating problems with model fit for Models 1 and 2. In contrast, Model 3 that further controls for human capital variables has values of RNI and IFI close to 1, values less than 0.05 of RMSEA, and large negative values of BIC, all indicating good model fit. Thus, the fit statistics from the SEM results support the choice of Model 3 over Models 1 or 2 whether we use the REM or FEM versions.

As explained previously, when we do not include any observed time-invariant variables, the REM is a restricted form of the FEM where in the former, the latent time-invariant variables (η_i) are uncorrelated with all other covariates. Correlations are allowed in the FEM. Given this nesting, we can form a LR chi-square difference test to compare the FEM and REM by subtracting T_m test statistics for the REM versus FEM, taking a difference in their respective

[13]We thank Michelle Budig for sharing with us the experience and seniority variables from the Budig and England (2001) analysis.

degrees of freedom, and comparing the results to a chi-square distribution. A statistically significant LR chi-square is evidence that favors the FEM while an insignificant chi-square favors the REM. Performing these LR chi-square difference tests consistently leads to a statistically significant result lending support to the FEM versions of Models 1, 2, and 3 in Table 3. These results are consistent with the Hausman test in favoring the FEM. However, the large sample size combined with the large degrees of freedom for these models complicates the result in that the statistical power of all these tests is high and does not tell us the magnitude of the differences. The RNI and IFI (i.e. the baseline fit indices) differ in the third decimal place and values as close as these are generally treated as essentially equivalent. The RMSEA has slightly larger differences in the pairwise comparisons with a tendency to favor the REM. The greatest separation in the pairwise comparisons for Models 1, 2, and 3 occur for the BIC and the BIC favors the REM versions of the models.

These results are interesting in that they imply that the REM and FEM are closer in fit than the Hausman test or the LR chi-square tests suggest. One reason is that the REM has considerably more degrees of freedom than the FEM since the REM is forcing to zero all of the covariances of the latent time-invariant variable with the time-varying observed covariates. The BIC gives considerable weight to the degrees of freedom of the model in large samples and the greater degrees of freedom contributes to making the BIC more favorable towards the REM. A second related reason for the REM appearing more competitive is that the magnitudes of the estimated covariances between the time-varying covariates and the latent variable are not always large in the FEM. Table 4 provides a sample of the estimated covariances between the full set of time-varying covariates and η_i in the FEM version of Model 3. We provide only three years of covariances and correlations for simplicity, but note that we actually estimate all six years of covariances. These results show that many of the covariances of the latent time-invariant variable (η_i) and the time-varying *x* s are not statistically significantly different from zero even though the significance tests are based on an *N* greater than 5,000. Interestingly, number of children, the key explanatory variable, is essentially uncorrelated with η_i as is marital status, part-time seniority, and experience. The other statistically significant correlations are often modest in magnitude (e.g., currently in school and part-time experience correlates less than 0.01). Thus our results suggest a more nuanced picture of the association between η_i and the *x* s than suggested by the REM or FEM. The latent time-invariant variable has a statistically significant association with some *x* s, but not with others. Even the statistically significant ones are modest in magnitude (e.g., ≤ 0.2).[14] So the modest and sometimes not statistically significant covariances of η_i and the *x* s seem to be the reason that the REM and FEM have fits that are so close as gauged by our fit indices. The low correlation of η_i with our focal independent variable, number of children, likely explains why the coefficients for this variable do not differ even more between the REM and FEM.

As we noted above, these fit indices and the estimates of the covariances of the latent time-invariant variables and the observed time-varying covariates are not available with most random and fixed effects statistical routines; the Hausman test indicated, as is often the case, that the FEM are unambiguously superior to the REM. Our results present a more subtle view of their relative fit.

The results above show that Model 3's fit is superior to Models 1 and 2. While Model 3 fits the data fairly well, the LR chi-square test suggests the potential for improvement in fit. One

[14]A researcher could also use BIC tests on these covariances rather than conventional statistical significance tests. These would tend to be more conservative in finding noteworthy covariances. Both this approach and the one we take do not take account of the multiple tests of significance performed when examining these individual parameters one-at-a-time. For this reason, the simultaneous tests of overall model fit reported in Table 3 are more useful in that they permit comparisons of a model where all covariances are zero versus not.

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possibility is to estimate a FEM/REM hybrid model. In the final column of Table 3, we present results from the hybrid model, where the latent time invariant variable (η_i) is correlated only with the consistently significant covariances, i.e. educational attainment, currently in school, full-time experience, and employment breaks, and is uncorrelated with the remaining covariates. When we compare the FEM/REM hybrid model to the REM and FEM, we find that the chi-square difference tests support the FEM. However, the differences in the IFI, RNI, and RMSEA are small. Finally, the BIC supports the REM. If a researcher judges the correlations between the time-varying covariates and the latent time-invariant variable to be small, then it seems reasonable that BIC supports the REM. The coefficient estimate for number of children increases in absolute magnitude from −0.034 to −0.043 as we move from the REM to the FEM/REM hybrid and FEM. As a greater than 25% increase this is noteworthy, though substantively the change might *not* appear significant.

General Panel Model and Restrictive Forms

As we discussed in section 2, we can estimate a general panel model that enables us to consider individual heterogeneity (latent time-invariant variables) as in the usual FEM and REM, but also permits time-differing coefficients and additional structures for comparison. In these models, we can also include time-invariant observed variables, and thus we include indicators for race in these models (with one indicator for black, one for Hispanic, and where the omitted category is non-black, non-Hispanic). This addition provides estimates for the time-invariant variables, useful additional information in many applications. As we do not have prior knowledge as to a single, specific model, we adopt the strategy of fitting models outlined in Table 1. Table 5 contains the fit statistics for the models we estimate, beginning with the general panel model and followed by the more restrictive forms as described in Table 1.

As we see from Table 5, the general panel model fits quite well. Constraining the latent time-invariant effect to be invariant over time significantly reduces model fit from the general panel model, and indeed in all the specifications that follow we find that allowing this effect to vary over time is consequential to the fit of the model. Substantively, this implies that the latent time-invariant factors, such as stable personality traits, impact wages differently over a woman’s life course. Conversely, constraining the effect of race to be constant over time improves model fit, as we see from specification 2b.

Figures 4 plots the effects of number of children on log wages for several alternative specifications, including specification 2b. The x-axis indicates year and the y-axis indicates the child effect on women’s wages. The y-axis is in reverse order such that higher values indicate a larger wage penalty for motherhood.

Specification 2b allows the time-varying coefficients to vary over time, and we observe a generally increasing effect of number of children over time. This result makes substantive sense, suggesting a form of cumulative disadvantage associated with motherhood on women’s wages.

Constraining all the time-varying variables to be constant over time (specification 3) does not significantly change the fit indices. If we had specific hypotheses concerning which variables are the most likely to vary over time, then we could estimate the model freeing only those coefficients and compare the fit of this new model to the fixed and random effects versions of the same model where the coefficients of the same variable are set equal over time. In our case, we do not have specific hypotheses on which variable’s coefficients might differ over time. Therefore, we constrain *all* the coefficients of the time-varying variables to be fixed over time. The IFI, RNI, and RMSEA suggest only slight differences between models that do and do not allow the coefficients to vary over time. We report the results for specification 3b in Table 6.

Our specifications that set the covariance between the time-varying variables and the latent time-invariant variables to zero also improve model fit, and we report the results from specification 4b in Table 6. This is again not surprising, as most of the covariances of the latent time-invariant variable with the observed covariates are substantively near zero. The effect of number of children on wages for specification 4b is the smallest of those we have estimated, −0.028. Given the high level of model fit for specification 4b, we try some slight modifications at this point, allowing just the child coefficient to vary over time (4b*) and constraining some of the covariances of the observed time-varying and latent time-invariant variables to be zero (4b**). The fit is comparable between 4b and 4b*/4b**, except according to BIC which favors 4b. We report the results of the child coefficients for 4b* in Figure 4, where we again see a generally increasing negative effect of number of children over time suggesting a form of cumulative disadvantage. We next allow the error variances to vary over time (specification 5). The chi-square difference tests are statistically significant supporting the models where the error variances are allowed to differ, while the IFI, RNI, and RMSEA suggest only slight differences between models that do and do not allow the error variances to vary over time. The BIC comparisons support the conclusion that we prefer the models that do not allow the coefficients and error variances to change over time. If we continue to follow our series of specifications outlined in Table 1, we next estimate the classic REM (specification 6) and FEM (specification 7). The fit indices do not support these as preferred models. The weight of evidence tends to favor the models with a latent time-invariant variable whose effects vary over time, observed time-invariant and time-varying effects that are constrained to be equal over time (although perhaps the child coefficient is best left to vary over time), and error variances that are constrained to be equal over time. This alternative specification leads to some interesting substantive results, as we note above.

General Panel Model and Restrictive Forms with Lagged Effects

One possibility to further improve our models is having the lagged value of wages as a determinant of current wages. Substantively, including such an effect makes sense in that there is inertia in wages where last year’s wages are likely to be a good predictor of this year’s. Though raises typically occur, there is a high degree of stability in relative wages across individuals from year to year. As we described in section 2.4, lagged endogenous variables for autoregressive effects are also straightforward to include. Our appendix provides a more formal presentation of the SEM setup and assumptions for estimating such a model. We lose one wave of data for each lag by specifying such a model; thus, our hypothesized models will be compared to different saturated and baseline models than those above. Table 5 provides fit statistics for the general panel model and restrictive forms with a lagged endogenous variable. We plot the values for the specifications where the child coefficient is freed in Figure 4 and report results from some of the better fitting specifications in Table 6.

The overall fit statistics of the model in Table 5 enable us to compare the models. The large sample size and accompanying high statistical power lead all the LR chi-square tests to be statistically significant. We present fewer results than those above, generally omitting those specifications sequentially that do not improve model fit. We find once again that the equality constraint on the coefficients for the latent time-invariant variable is not supported. The RNI and IFI are consistently close to 1.00 and the RMSEA is considerably lower than the usual cutoff of 0.05. The BIC always takes large negative values supporting the selection of any of these models over the saturated model.

Specification 2b has particularly good fit according to IFI, RNI, and RMSEA, and we report the time-varying child coefficient of this specification in Figure 4. The child coefficients are quite similar to those in specification 4b* above. Specification 3b, which differs from 2b in that we constrain the observed time-varying effects to be constant over time is an improved fit according to BIC, but not according to the IFI, RNI, or RMSEA. According to BIC, the best fitting specification is 5b2, and we report the results for this model in Table 6. The coefficients are quite similar to specification 3b, and to the classic REM. We test a few alternative specifications to 5b2 (5b* and 5b**), and the fit is quite comparable. We report the time-varying child coefficient from specification 5b*, which is relatively flatter than the other specifications.

Taken together we reach the following conclusions on overall fit of the models. First, we significantly improve model fit when the coefficient on the latent time-invariant variable is allowed to vary over time. Allowing the coefficients of the latent time-invariant variable to vary is not an option in the usual FEM and REM that are typical in sociology. Our results provide strong evidence that these effects do vary with time in this case. Second, the IFI, RNI, and RMSEA do not reveal large differences among the different versions of these models, but tend to favor a less restrictive model, where only observed time-varying effects are allowed to vary over time. Third, the lagged endogenous variable models have very good fit.[15] These general panel model specifications and more restrictive forms suggest a smaller wage penalty for motherhood than the classic REM and FEM, and provide some additional substantive information. Moreover, models with lagged effects likewise suggest a smaller penalty for motherhood's *direct* effect, particularly in later years, than that suggested by REM and FEM without lagged endogenous variables. However, given the lagged endogenous variable there are additional lagged effects of motherhood on wages. For instance, the number of children in say, 1983, has a direct effect on wages in 1983, but it also has an indirect effect on wages in 1985 given the impact of 1983 wages on 1985 wages. Thus the effect of number of children in one period extends beyond that period through its indirect effect on wages through the lagged dependent variable. The same is true for the other covariates with significant effects on wages. Their effects are not only direct, but indirect. This useful distinction between direct, indirect, and total effects is well-known in the SEM literature (e.g., Sobel 1982; Bollen 1987) and most SEM software permits its exploration.

SEM models have helped us to uncover evidence that the standard assumptions of fixed coefficients, fixed error variances, and no lagged endogenous variables were not always supported when tested in our empirical example. We present this series of specifications to demonstrate the flexibility of our approach. Still, we could have estimated various other alternatives, or combined many of the elements we present separately. The SEM formulation also allows investigation of indirect effects, as we mentioned above. Another realm that we have not explored, but which is easily implemented, is to include latent covariates with multiple indicators.

Conclusion

REM and FEM panel model applications are becoming more common in *Social Forces* and throughout sociological research. However, too often researchers apply FEM or REM without careful consideration as to why they should prefer one model over another. In this paper, we show that these models are a restrictive form of a more general panel model that

[15]This statement recognizes the role that the large sample size plays in elevating statistical power for the LR chi-square tests and hence downplays the statistical significance of these tests. However, the chi-square tests mean that there is still room for improvement despite the favorable results with all other fit indices for these models.

permits a wider range of alternatives and that is estimable using widely available SEM software. With this general panel model, a researcher does not need to maintain these constraints but can test them and only impose those supported by the data. In addition, a wide variety of additional models and formulations are possible. For instance, a researcher can test whether a covariate's impact on the repeated measure stays the same across all waves of data; test whether the error variances should be allowed to vary over time; include lagged covariates or lagged dependent variables; free factor loadings on the latent time-invariant variable; include observed time-invariant variables in a FEM either as uncorrelated with the latent time-invariant variable or as a determinant of the latent variable; estimate the magnitude of the covariance of the latent time-invariant variables with the observed time-varying covariates, and estimate a hybrid FEM/REM model with the information gleaned as to the magnitudes of these covariances. For these additional formulations, we have useful tests of model fit and fit indices that are not part of the standard REM and FEM analysis. We also have a likelihood ratio test of the FEM and REM to a hybrid model and a variety of fit indices as supplements to the Hausman Test. Indeed it would be interesting to know how many of the FEM and REM that have appeared in the literature would have adequate model fit if some of these tools were applied.

Our empirical example of the impact of the number of children on women's wages illustrated some of the advantages that flow by casting FEM and REM panel models as part of this general panel model. For one thing, we had access to a more complete set of model fit statistics that revealed flaws in both the standard FEM and REM that were not evident in the usual approaches and the publication based on them. Specifically, neither model fully reproduced the covariance matrix and means of the observed variables as they should if the models were correct. Furthermore, we found evidence that the REM were more competitive than the Hausman test and likelihood ratio (LR) test alone revealed. In fact, the Hausman and LR tests from the study upon which our example was based unambiguously supported the FEM over the REM, as it generally does. The primary distinction between the FEM and REM is whether the covariates correlate with the latent time-invariant variable. Using the SEM approach we saw that many of the correlations of the covariates with the latent time-invariant variable were close to zero -- information unavailable with usual methods, and thus we fit a FEM/REM hybrid model in which only a subset of the covariates were correlated with the latent time-invariant variable. Furthermore, the SEM approach suggested that the impact of the latent time-invariant variable on wages was not the same across all years and that the unexplained variances were not constant over time in all models. Most applications of FEM and REM assume constant effects regardless of the year of the panel data. A further departure from the published models for these data was that we looked at whether lagged wages impacted current wages net of the other determinants. Given the degree to which current salary is closely tied to past salary, this is a substantively plausible effect and it was easy to explore with our model. We found strong evidence that the lagged endogenous variable models were superior to the models without them. A related substantive point is that these models show the importance of prior wages on current wages and this implies that any variables that impact wages in a given year have an indirect effect on later years as well. Thus, the number of children has direct as well as indirect effects on mothers' wages. We also incorporated an observed time-invariant variable, race, to the FEM, providing the typically unavailable coefficients of a potential variable of interest.

Still the empirical example did not exhaust the types of models for panel data that could be applied with our approach. For instance, it would be straightforward to develop a model that permits the dependent variable to be latent with several indicators and to have a fixed or random effects-like model for it. We could allow for measurement error in the time-varying or time-invariant covariates and include them in the model. In addition, latent curve models or Autoregressive Latent Trajectory (ALT) models might be applied (Bollen and Curran, 2006). In brief, researchers can build a broader range of models than is commonly applied, some of which might better capture the theory that they wish to test.

Although the SEM approach offers considerable flexibility, it does not adequately handle all situations that researchers might encounter. For instance, if the latent time-invariant variable has a different correlation with the covariates for *different* individuals, these models will not work. Ejrnaes & Holm (2006) show how a difference model or case mean deviation (fixed effects) model would work in this situation where our SEM approach would not unless difference scores were modeled. They also suggest a Hausman (1978) test that could test for this possibility for the FEM. Similarly, the models we treat permit the covariate's effect on the repeated measure to differ over time, but assume that these coefficients are constant over individuals. It is possible to estimate models where these coefficients differ over individuals (e.g., Beck & Katz, 2007). Also there are some inherently nonlinear relationships between variables that might be difficult or impossible to capture with the classic FEM and REM or with SEM. Finally, models with numerous parameters, a great deal of missing data, and many waves might exceed the computational capabilities of some desktops or SEM software. Nevertheless, our paper allows considerable flexibility in the variants of the FEM and REM that researchers can apply to panel data. By providing this SEM framework, researchers will be able to test a richer variety of theoretical models and explore flexible alternative models that could help test and shape new theories.

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Appendix

Fixed and Random Effects Models as Structural Equation Models

Classic Fixed and Random Effects Models

We represent the standard FEM and REM in the following matrix equation:

$$\mathbf{y}_i=\boldsymbol{\alpha}+\boldsymbol{\Gamma}\mathbf{w}_i+\boldsymbol{\varepsilon}_i$$

where

$$\mathbf{y}_i=\begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \mathbf{w}_i=\begin{bmatrix} \mathbf{x}_{i1} \\ \mathbf{x}_{i2} \\ \vdots \\ \mathbf{x}_{iT} \\ \mathbf{z}_i \\ \eta_i \end{bmatrix} \quad \boldsymbol{\varepsilon}_i=\begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$\boldsymbol{\alpha}=\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_T \end{bmatrix} \quad \boldsymbol{\Gamma}=\begin{bmatrix} \mathbf{B}_{y_1\mathbf{x}_1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{B}_{y_1\mathbf{z}} & 1 \\ \mathbf{0} & \mathbf{B}_{y_2\mathbf{x}_2} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{B}_{y_2\mathbf{z}} & 1 \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{y_3\mathbf{x}_3} & \cdots & \mathbf{0} & \mathbf{B}_{y_3\mathbf{z}} & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{B}_{y_T\mathbf{x}_T} & \mathbf{B}_{y_T\mathbf{z}} & 1 \end{bmatrix}$$

The $\mathbf{x}_{it}$ vector contains the values of the time-varying covariates for the *i* th case at the *t* th time, $\mathbf{z}_i$ is the vector of observed time-invariant variables for the *i* th case, and η_i is the latent time-invariant variable for the *i* th case. We assume that the mean of the disturbance is zero [$E(\boldsymbol{\varepsilon}_i) = \mathbf{0}$ for all *i*], that they are not autorcorrelated over cases[$COV(\boldsymbol{\varepsilon}_i, \boldsymbol{\varepsilon}'_j)=\mathbf{0}$ for $i \neq j$], and that the covariance of the disturbance with the covariates in $\mathbf{w}_i$ is zero [$COV(\mathbf{w}_i, \boldsymbol{\varepsilon}'_j)=\mathbf{0}$ for all i, j]. In SEMs the vector of means ($\boldsymbol{\mu}$) and the covariance matrix ($\boldsymbol{\Sigma}$) of the observed variables are functions of the parameters of the researcher's model. If we place all model parameters (coefficients, intercepts, variances, covariances) in a vector $\boldsymbol{\theta}$, then these implied functions are the model *implied covariance matrix* ($\boldsymbol{\Sigma}(\boldsymbol{\theta})$) and *implied mean vector* [($\boldsymbol{\mu}$)]. When the model is valid, then

$$H_o{:}\boldsymbol{\mu}=\boldsymbol{\mu}(\boldsymbol{\theta})\&\sum=\sum(\boldsymbol{\theta})$$

That is, we will exactly reproduce the means and covariance matrix of the observed variables by knowing the model parameter values and substituting them into the implied mean vector and implied covariance matrix. For equation (A1), the implied mean vector [($\boldsymbol{\mu}$)] is

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$$\boldsymbol{\mu}(\boldsymbol{\theta})=\begin{bmatrix} \boldsymbol{\alpha}+\boldsymbol{\Gamma}\boldsymbol{\mu}_{\mathbf{w}} \\ \boldsymbol{\mu}_{\mathbf{w}} \end{bmatrix}$$

and the implied covariance matrix [$\boldsymbol{\Sigma}(\boldsymbol{\theta})$] is

$$\sum(\boldsymbol{\theta})=\begin{bmatrix} \boldsymbol{\Gamma}\sum_{\mathbf{ww}}\boldsymbol{\Gamma}'+\sum_{\varepsilon\varepsilon} & \sum_{\mathbf{ww}}\boldsymbol{\Gamma}' \\ \boldsymbol{\Gamma}\sum_{\mathbf{ww}} & \sum_{\mathbf{ww}} \end{bmatrix}$$

where $\boldsymbol{\Sigma}_{\mathbf{ww}}$ is the covariance matrix of the covariates in $\mathbf{w}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon\varepsilon}}$ is the covariance matrix of the disturbances ($\boldsymbol{\varepsilon}$).

In the usual FEM, we would drop $\mathbf{z}_i$ from $\mathbf{w}_i$ and the corresponding coefficients from $\boldsymbol{\Gamma}$, set $\mathbf{B}_{y_1\mathbf{x}_1} = \mathbf{B}_{y_2\mathbf{x}_2} = \cdots = \mathbf{B}_{y_T\mathbf{x}_T}$, and make $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon\varepsilon}}$ a diagonal matrix with all elements of the main diagonal equal. The $\boldsymbol{\Sigma}_{\mathbf{ww}}$ covariance matrix allows all covariates to correlate, including the latent time-invariant variable. For the usual REM, we can return $\mathbf{z}_i$ to $\mathbf{w}_i$, but now we must constrain $\boldsymbol{\Sigma}_{\mathbf{ww}}$ so that all covariances of η with $\mathbf{x}_t$ and $\mathbf{z}$ are zero and we maintain the equality constraints on the coefficients so that $\mathbf{B}_{y_1\mathbf{x}_1} = \mathbf{B}_{y_2\mathbf{x}_2} = \cdots = \mathbf{B}_{y_T\mathbf{x}_T}$ and $\mathbf{B}_{y_1\mathbf{z}} = \mathbf{B}_{y_2\mathbf{z}} = \mathbf{B}_{y_T\mathbf{z}}$. As explained in the text, we can easily test these restrictions in SEMs.

The Maximum Likelihood Estimator (MLE) is the most widely used estimator in SEM software. The fitting function that incorporates the MLE is

$$F_{ML}=\ln|\sum(\boldsymbol{\theta})| - \ln|\mathbf{S}|+\mathrm{tr}[\sum{}^{-1}(\boldsymbol{\theta})\mathbf{S}] - p+(\bar{\mathbf{z}}-\boldsymbol{\mu}(\boldsymbol{\theta}))'\sum{}^{-1}(\boldsymbol{\theta})(\bar{\mathbf{z}}-\boldsymbol{\mu}(\boldsymbol{\theta}))$$

where $\mathbf{S}$ is the sample covariance matrix, $\bar{\mathbf{z}}$ is the vector of the sample means of the observed variables, p is the number of observed variables, ln is the natural log, |·| is the determinant, and tr is the trace of a matrix. The MLE estimator, $\hat{\boldsymbol{\theta}}$, is chosen so as to minimize F_{ML}. Like all MLEs, $\hat{\boldsymbol{\theta}}$, has several desirable properties. It is consistent, asymptotically unbiased, asymptotically efficient, asymptotically normally distributed, and the asymptotic covariance matrix of $\hat{\boldsymbol{\theta}}$ is the inverse of the expected information matrix.

The MLE estimator as implemented in F_{ML} leads to a consistent estimator of all intercepts, means, coefficients, variances, and covariances in the model under a broad range of conditions. This means that in larger samples, the estimator will converge on the true parameters for valid models. However, if we wish to develop appropriate significance tests, then we need to make assumptions about the distributions of the observed variables. The usual assumption is that the observed variables come from a multivariate normal distribution. A slightly less restrictive distributional assumption that maintains the properties of the MLE and its significance tests is that the observed variables come from a multivariate distribution with no excess multivariate kurtosis (Browne 1984). Multivariate skewness is permitted as long as the multivariate kurtosis does not differ from that of a normal distribution.

Fortunately, even when there is excess multivariate kurtosis there are a variety of alternative ways to obtain asymptotically accurate signficance tests including bootstrapping techniques (e.g., Bollen and Stine 1990 e.g., Bollen and Stine 1992), corrected standard errors and chi-squares (e.g., Satorra and Bentler, 1994), or arbitrary distribution estimators (e.g., Browne 1984). See Bollen and Curran (2006:55–57) for further discussion and references. These options provide a broader range of choices than is true in the usual implementation of the standard FEM and REM.

Dynamic Fixed and Random Effects Models

In the econometric literature, "dynamic" models refers to the FEM and REM with lagged dependent variables included among the covariates. In the usual implementations, the lagged dependent variable model creates considerable difficulties and is the source of much discussion (see, e.g., Hsiao 2003, Ch.4). Fortunately, these models are relatively straightforward in the SEM approach. A modification of equation (A1) permits lagged endogenous variables,

$$\mathbf{y}_i=\boldsymbol{\alpha}+\mathbf{R}\mathbf{y}_i+\boldsymbol{\Gamma}\mathbf{w}_i+\boldsymbol{\varepsilon}_i$$

where because of using a lagged dependent variable we need to redefine vectors to take account of treating the first time wave variable, y_{i1}, as predetermined and included among the other covariates and the presence of lagged y influences, so that

$$\mathbf{y}_i=\begin{bmatrix} y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \mathbf{w}_i=\begin{bmatrix} y_{i1} \\ \mathbf{x}_{i2} \\ \vdots \\ \mathbf{x}_{iT} \\ \mathbf{z}_i \\ \eta_i \end{bmatrix} \quad \boldsymbol{\varepsilon}_i=\begin{bmatrix} \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT} \end{bmatrix}$$

$$\boldsymbol{\alpha}=\begin{bmatrix} \alpha_2 \\ \vdots \\ \alpha_T \end{bmatrix} \quad \boldsymbol{\Gamma}=\begin{bmatrix} \boldsymbol{\rho}_{21} & \mathbf{B}_{y_2\mathbf{x}_2} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{B}_{y_2\mathbf{z}} & 1 \\ 0 & \mathbf{0} & \mathbf{B}_{y_3\mathbf{x}_3} & \cdots & \mathbf{0} & \mathbf{B}_{y_2\mathbf{z}} & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{B}_{y_T\mathbf{x}_T} & \mathbf{B}_{y_T\mathbf{z}} & 1 \end{bmatrix}$$

$$\mathbf{R}=\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ \rho_{32} & 0 & \cdots & 0 & 0 \\ 0 & \rho_{43} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \rho_{T,T-1} & 0 \end{bmatrix}$$

In this model, y_{i1} is predetermined and uncorrelated with $\boldsymbol{\varepsilon}_i$ as are the other covariates. However, there is a correlation between $y_{i2}\ldots y_{iT}$ and at least some elements of $\boldsymbol{\varepsilon}_i$ (e.g., y_{i2} correlates with $\boldsymbol{\varepsilon}_{i2}$) so we need to consider all but the first wave (y_{i1}) as endogenous.

For this model, the implied mean and covariance matrices become,

$$\boldsymbol{\mu}(\boldsymbol{\theta})=\begin{bmatrix} (\mathbf{I}-\mathbf{R})^{-1}(\boldsymbol{\alpha}+\boldsymbol{\Gamma}\boldsymbol{\mu}_{\mathbf{w}}) \\ \boldsymbol{\mu}_{\mathbf{w}} \end{bmatrix}$$

and the implied covariance matrix [$\boldsymbol{\Sigma}(\boldsymbol{\theta})$] is

$$\sum(\boldsymbol{\theta})=\begin{bmatrix} (\mathbf{I}-\mathbf{R})^{-1}[\boldsymbol{\Gamma}\sum_{\mathbf{ww}}\boldsymbol{\Gamma}'+\sum_{\boldsymbol{\varepsilon\varepsilon}}](\mathbf{I}-\mathbf{R})^{-1} & \sum_{\mathbf{ww}}\boldsymbol{\Gamma}'(\mathbf{I}-\mathbf{R})^{-1} \\ (\mathbf{I}-\mathbf{R})^{-1}\boldsymbol{\Gamma}\sum_{\mathbf{ww}} & \sum_{\mathbf{ww}} \end{bmatrix}$$

Fortunately, we can continue to use the ML fitting function in equation (Fml) and the resulting estimator maintains the properties of an MLE under the precedingly described distributional assumptions and the corrected test statistics are also available when needed (see above). Autoregressive relations among the $\boldsymbol{\varepsilon}_i$ disturbances combined with the autoregression of the y_i s would complicate the situation, but are not discussed here.

$$y_{it} = \mathbf{B}_{y\mathbf{x}}\mathbf{x}_{it} + \mathbf{B}_{y\mathbf{z}}\mathbf{z}_i + \eta_i + \varepsilon_{it}$$
$$E(\varepsilon_{it}^2) = \sigma_\varepsilon^2$$

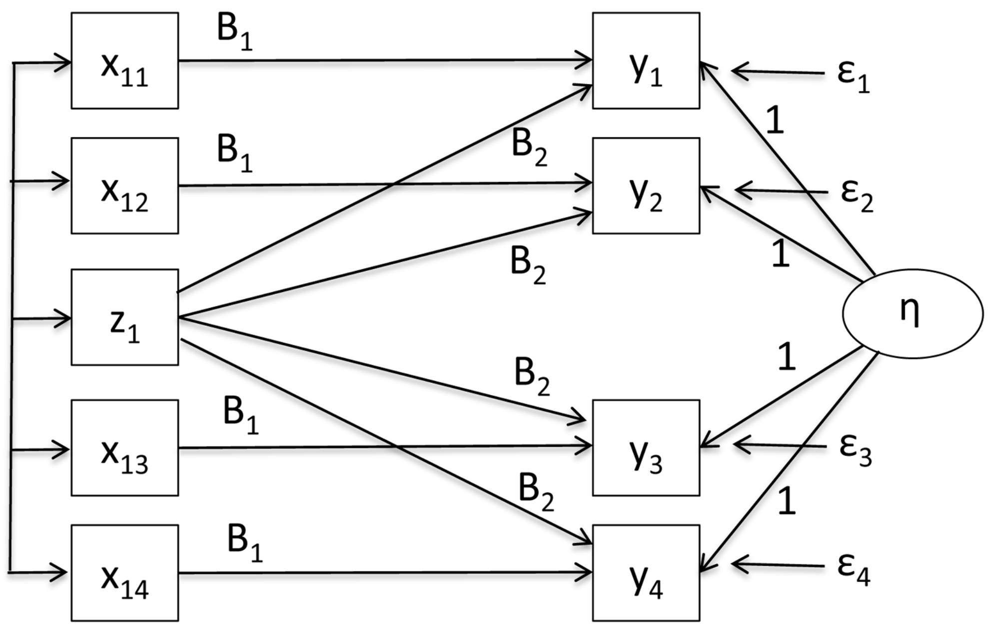

Figure 1.
Classic Random Effects Model in Path Diagram

$$y_{it} = \mathbf{B}_{y\mathbf{x}}\mathbf{x}_{it} + \eta_i + \varepsilon_{it}$$

$$E(\varepsilon_{it}^2) = \sigma_\varepsilon^2$$

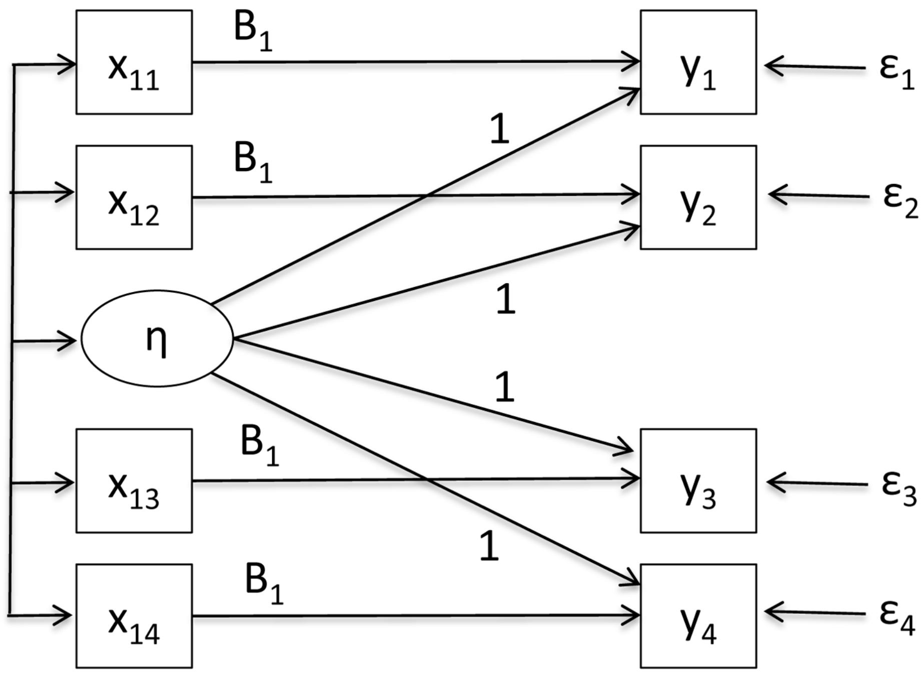

Figure 2.
Classic Fixed Effects Model in Path Diagram

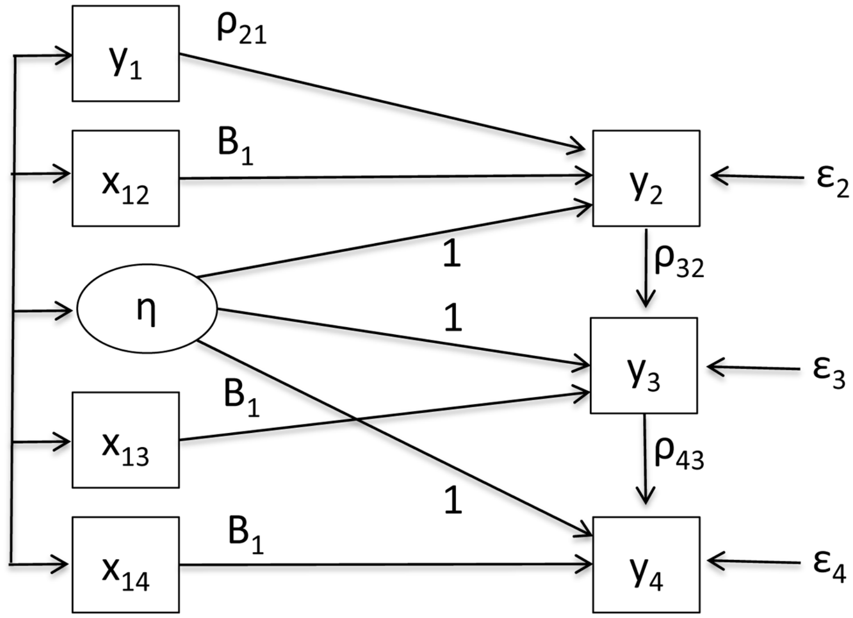

Figure 3.
Fixed Effects Model with Lagged Dependent Variables

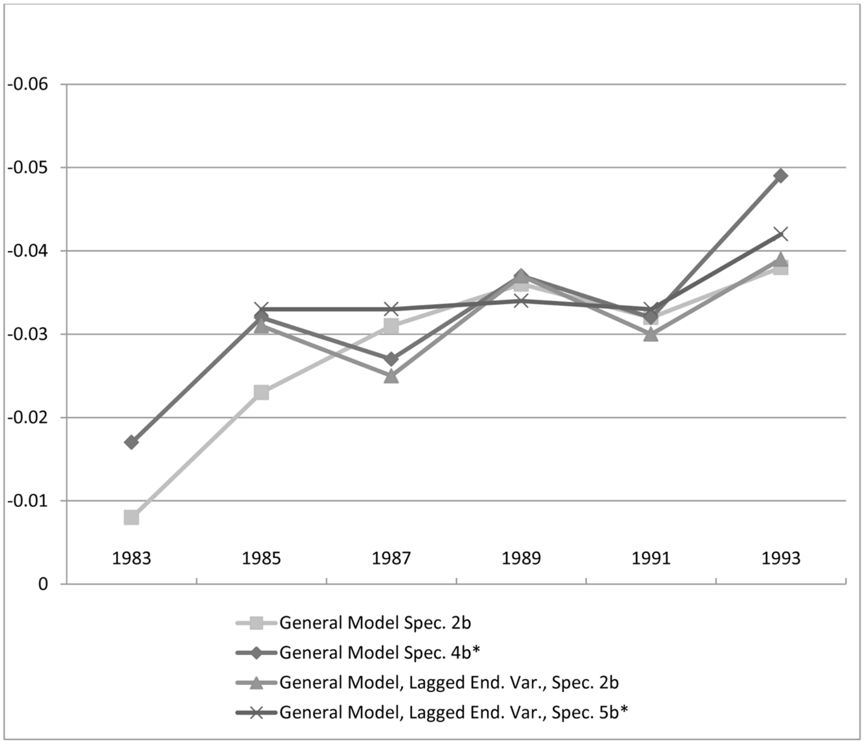

Figure 4.
SEM Coefficients for the Effect of Total Number of Children on Women's Log Hourly Wage: Model 3, Marital Status and Human Capital Variables

Table 1

General Panel Model and Special Cases

General Panel Model
$y_{it} = \mathbf{B}_{\mathbf{yx}t}\mathbf{x}_{it} + \mathbf{B}_{y\mathbf{z}t}\mathbf{z}_i + \lambda_t\eta_i + \varepsilon_{it}$
(1) time -invariant λ
$\lambda_t = 1$ for all t
(2) time-invariant λ & $\mathbf{B}_{y\mathbf{x}}$
$\lambda_t = 1$, $\mathbf{B}_{y\mathbf{z}t} = \mathbf{B}_{y\mathbf{z}}$ for all t
(3) time-invariant λ, $\mathbf{B}_{y\mathbf{z}}$, & $\mathbf{B}_{y\mathbf{z}}$
$\lambda_t = 1$, $\mathbf{B}_{\mathbf{yx}t} = \mathbf{B}_{y\mathbf{x}}$, $\mathbf{B}_{y\mathbf{z}t} = \mathbf{B}_{y\mathbf{z}}$ for all t
(4) time-invariant λ, $\mathbf{B}_{y\mathbf{z}}$, & $\mathbf{B}_{y\mathbf{z}}$, $COV(x_{it}, \eta_i) = \mathbf{0}$
$\lambda_t = 1$, $\mathbf{B}_{\mathbf{yx}t} = \mathbf{B}_{y\mathbf{x}}$, $\mathbf{B}_{y\mathbf{z}t} = \mathbf{B}_{y\mathbf{z}}$, $COV(x_{it}, \eta_i) = \mathbf{0}$ for all t
(5)(a) Fail to reject (4) time-invariant λ, $\mathbf{B}_{y\mathbf{z}}$, & $\mathbf{B}_{y\mathbf{z}}$, $COV(x_{it}, \eta_i) = \mathbf{0}$, σ_ε
$\lambda_t = 1$, $\mathbf{B}_{\mathbf{yx}t} = \mathbf{B}_{y\mathbf{x}}$, $\mathbf{B}_{y\mathbf{z}t} = \mathbf{B}_{y\mathbf{z}}$, $COV(x_{it}, \eta_i) = \mathbf{0}$, $\sigma_{\varepsilon t} = \sigma_\varepsilon$ for all t
(b) Reject (4) time-invariant λ, $\mathbf{B}_{y\mathbf{z}}$, & $\mathbf{B}_{y\mathbf{z}}$, σ_ε $\lambda_t = 1$, $\mathbf{B}_{\mathbf{yx}t} = \mathbf{B}_{y\mathbf{x}}$, $\mathbf{B}_{y\mathbf{z}t} = \mathbf{B}_{y\mathbf{z}}$, $\sigma_{\varepsilon t} = \sigma_\varepsilon$ for all t
(6) Classic Random Effects Model (REM)
Equivalent to (5) (a)
(7) Classic Fixed Effects Model (FEM)
Equivalent to (5) (b) with $\mathbf{B}_{y\mathbf{z}} = \mathbf{0}$ (no $\mathbf{z}_i$ in equation)

Table 2

Model fit indices and their definitions

Model Fit Index	**Definition**
Tucker-Lewis Index (TLI)	$TLI=\frac{T_b/df_b-T_m/df_m}{T_b/df_b-1}$
Incremental Fit Index (IFI)	$IFI=\frac{T_b-T_m}{T_b-df_m}$
Relative Noncentrality Index (RNI)	$RNI=\frac{(T_b-df_b)-(T_m-df_m)}{T_b-df_m}$
Root Mean Square Error of Approximation (RMSEA)	$RMSEA=\sqrt{\frac{T_m-df_m}{(N-1)df_m}}$
Bayesian Information Critierion (BIC)	$BIC = T_m - df_m \ln(N)$

where

T_m = chi-square test statistic for *m* (*m* vs. saturated)

m = hypothesized model

df_m degrees of freedom for *m*

N = # cases

T_b = chi-square test statistic for *b* (*b* vs. saturated)

b = baseline (uncorrelated obs.vars.)

df_b degrees of freedom for *b*

Table 3

SEM Coefficients for the Effects of Total Number of Children and Controls (Constrained to be Equal over Time) on Women's Log Hourly Wages: NLSY 1983, 1985, 1987, 1989, 1991, 1993

	Random Effects Models			Fixed Effects Models			Hybrid
Variables	**Model 1**	**Model 2**	**Model 3**	**Model 1**	**Model 2**	**Model 3**	**Model 3**
Number of Children	−0.088 *** (21.990)	−0.096 *** (23.575)	−0.034 *** (8.771)	−0.066 *** (11.836)	−0.070 *** (12.606)	−0.043 *** (7.775)	−0.043 *** (9.661)
Married	---	0.078 *** (9.162)	0.038 *** (4.703)	---	0.083 *** (7.888)	0.041 *** (4.191)	0.041 *** (4.996)
Divorced	---	0.027 * (2.315)	0.043 *** (3.981)	---	0.057 *** (3.926)	0.043 *** (3.067)	0.041 *** (3.704)
Educational Attain.	---	---	0.075 *** (35.665)	---	---	0.064 *** (12.380)	0.069 *** (15.272)
Currently in School	---	---	−0.119 *** (12.612)	---	---	−0.143 *** (13.361)	−0.142 *** (13.679)
Part-time Seniority	---	---	0.012 *** (3.687)	---	---	0.010 *** (2.587)	0.011 *** (3.220)
Full-time Seniority	---	---	0.021 *** (12.977)	---	---	0.021 *** (11.378)	0.021 *** (11.553)
Part-time Experience	---	---	0.018 *** (8.055)	---	---	0.019 *** (5.385)	0.017 *** (6.476)
Full-time Experience	---	---	0.037 *** (21.343)	---	---	0.026 *** (9.173)	0.028 *** (10.716)
Employment Breaks	---	---	−0.012 *** (5.247)	---	---	−0.011 ** (2.698)	−0.011 ** (2.768)
T_m *(LR chi-square)*	1942.91	2038.52	2545.34	1885.05	1941.30	2209.55	2357.28
df	54	124	369	48	106	309	339
IFI/RNI^	0.9392	0.9693	0.9923	0.9409	0.9706	0.9933	0.9929
RMSEA	0.0818	0.0543	0.0334	0.0855	0.0575	0.0341	0.0336
BIC	1480.55	976.79	−617.96	1474.05	1033.69	−439.40	−545.36

Notes: Numbers in parentheses are z-ratios. Number of individuals = 5285;

* p<.05

** p<.01

*** p<.001 (two-tailed tests)

^ IFI and RNI were identical up to 3 significant digits, so we do not report them separately here.

Table 4

Subset of Covariances and Correlations Between Time-Varying and Latent Time-Invariant Variables, Fixed Effects Model 3

Variables	1983 Covariance with η	1983 Correlation with η	1985 Covariance with η	1985 Correlation with η	…	1993 Covariance with η	1993 Correlation with η
Number of Children	0.000 (0.006)	0.000	−0.005 (0.007)	−0.021	…	−0.002 (0.009)	−0.006
Married	−0.001 (0.003)	−0.008	−0.004 (0.003)	−0.030	…	0.003 (0.003)	0.022
Divorced	0.002 (0.002)	0.028	0.000 (0.002)	0.000	…	−0.001 (0.002)	−0.009
Educational Attain.	0.057 * (0.022)	0.117	0.057 * (0.025)	0.105	…	0.073 * (0.028)	0.118
Currently in School	0.008 * (0.003)	0.071	0.006 ** (0.002)	0.062	…	0.003 * (0.001)	0.052
Part-time Seniority	−0.006 (0.004)	−0.029	−0.004 (0.005)	−0.017	…	0.007 (0.008)	0.018
Full-time Seniority	0.027 ** (0.009)	0.076	0.042 *** (0.010)	0.088	…	0.137 *** (0.022)	0.138
Part-time Experience	0.017 (0.010)	0.049	0.006 (0.012)	0.014	…	−0.041 * (0.020)	−0.055
Full-time Experience	0.046 *** (0.010)	0.108	0.081 *** (0.014)	0.144	…	0.207 *** (0.030)	0.191
Employment Breaks	0.027 ** (0.009)	0.081	0.006 (0.012)	0.015	…	−0.092 *** (0.017)	−0.147

Notes: Numbers in parentheses are standard errors.

* p<.05

** p<.01

*** p<.001 (two-tailed tests)

Table 5

Model 3 (Marital Status and Human Capital Variables as Controls) Fit Statistics (N=5285)

Model Specification	Log Lik.	T_m (LR chi-square)	*df*	IFI/RNI^	RMSEA	BIC
General Model	−188075.45	936.44	249	0.9976	0.0229	−1198.14
Restricted Models						
(1) time-invariant λ	−188432.66	1650.86	254	0.9951	0.0323	−526.59
(2) (a) time invariant λ & β_{yz}	−188443.6	1672.75	264	0.9951	0.0318	−590.42
(2) (b) time invariant β_{yz}	−188082.65	950.84	259	0.9976	0.0225	−1269.47
(3) (a) time invariant λ, β_{yz}, & β_{yx}	−188685.24	2156.02	314	0.9936	0.0333	−535.79
(3) (b) time invariant β_{yz} & β_{yx}	−188299.6	1384.75	309	0.9962	0.0257	−1264.19
(4) (a) time invariant λ, β_{yz}, & β_{yx}, *COV* (x_{it}, η_i) = 0	−188850.51	2486.57	374	0.9926	0.0327	−719.59
(4) (b) time invariant β_{yz} & β_{yx}, *COV* (x_{it}, η_i) = 0	−188476.71	1738.96	369	0.9952	0.0265	−1424.34
(4) (b*) time invariant βyz & βyx (except child), *COV* (xit, ηi) = 0	−188471.47	1728.48	364	0.9952	0.0266	−1391.96
(4) (b**) time invariant β_{yz} & β_{yx}; hybrid	−188420.97	1627.48	351	0.9955	0.0262	−1381.51
(5) (a.1) time invariant λ, β_{yz} & β_{yx}, *COV* (x_{it}, η_i) = 0, σε	−188881.59	2548.72	379	0.9924	0.0329	−700.31
(5) (a.2) time invariant β_{yz} & β_{yx}, *COV* (x_{it}, η_i) = 0, σε	−188530.19	1845.92	374	0.9949	0.0273	−1360.24
(5) (b.1) time invariant λ, β_{yz} & β_{yx}, σε	−188718.89	2223.32	319	0.9933	0.0336	−511.35
(5) (b.2) time invariant β_{yz} & β_{yx}, σε	−188352.47	1490.48	314	0.9959	0.0266	−1201.33
(6) Classic random effect model, with *z* (5a1)	−188881.59	2548.72	379	0.9924	0.0329	−700.31
(7) Classic fixed effects model (5b1, with no z)	−184387.90	2209.55	309	0.9933	0.0343	−436.22
General Model (lagged dep. var.)	−165671.81	355.63	154	0.9992	0.0157	−964.56
***Restricted Models* (lagged dep. var.)**						
(1) time-invariant λ	−165713.00	438.01	158	0.9988	0.0183	−916.46
(2) (a) time invariant lagged dep. var.	−165686.96	385.92	158	0.9990	0.0165	−968.55
(2) (b) time-invariant lagged dep. var., & β_{yz}	−165689.47	390.96	166	0.9991	0.0160	−1032.10
(3) (b) time invariant lagged dep. var., β_{yz}, & β_{yx}	−165797.31	606.63	206	0.9983	0.0192	−1159.33
(4) (b) time invariant lagged dep.var., β_{yz} & β_{yx}, *COV* (x_{it}, η_i) = 0	−166373.77	1759.55	308	0.9939	0.0299	−880.82
(5) (b.2) time invariant lagged dep.var., β_{yz} & β_{yx}, σε	−165804.96	621.93	210	0.9983	0.0193	−1178.32
(5) (b*) time invariant lagged dep.var., β_{yz} & β_{yx} (except child), σε	−165804.00	620.01	206	0.9983	0.0195	−1145.95
(5) (b**) time invariant lagged dep.var., β_{yz} & β_{yx}, σε; hybrid	−165991.76	995.52	246	0.9969	0.0240	−1113.34

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^IFI and RNI were identical up to 3 significant digits, so we do not report them separately here.

Table 6

SEM Coefficients for the Effects of Total Number of Children and Controls on Women's Log Hourly Wages: NLSY 1983, 1985, 1987, 1989, 1991, 1993

Variables	General Model Spec. 3b	General Model Spec. 4b	General Model Spec. 5a.2	Lagged Dep. Var. Model Spec. 3b	Lagged Dep. Var. Model Spec. 5b.2
Number of Children	−0.031 *** (5.349)	−0.028 *** (7.038)	−0.029 *** (7.121)	−0.034 *** (5.078)	−0.035 *** (5.126)
Married	0.036 *** (3.730)	0.030 *** (3.853)	0.031 *** (3.978)	0.025 * (2.025)	0.028 * (2.269)
Divorced	0.049 *** (3.643)	0.043 *** (3.976)	0.043 *** (4.007)	0.044 ** (2.679)	0.047 ** (2.855)
Educational Attain.	0.046 *** (11.516)	0.072 *** (33.769)	0.071 *** (33.735)	0.037 *** (5.896)	0.037 *** (5.785)
Currently in School	−0.120 *** (12.449)	−0.109 *** (11.976)	−0.112 *** (12.621)	−0.108 *** (8.754)	−0.110 *** (8.969)
Part-time Seniority	0.014 *** (3.550)	0.013 *** (3.897)	0.013 *** (3.904)	0.010 * (2.102)	0.010 * (2.134)
Full-time Seniority	0.019 *** (8.757)	0.021 *** (11.930)	0.021 *** (12.236)	0.018 *** (8.064)	0.018 *** (8.200)
Part-time Experience	0.020 *** (4.648)	0.017 *** (7.056)	0.017 *** (7.153)	0.017 *** (3.350)	0.016 ** (3.194)
Full-time Experience	0.049 *** (11.950)	0.043 *** (22.171)	0.042 *** (22.016)	0.017 *** (3.692)	0.016 *** (3.595)
Employment Breaks	−0.002 (0.414)	−0.011 *** (4.669)	−0.011 *** (4.628)	0.014 * (2.450)	0.014 * (2.453)
Black	−0.030 ** (2.724)	−0.035 *** (3.427)	−0.034 *** (3.305)	−0.046 *** (3.840)	−0.047 *** (3.990)
Hispanic	0.042 *** (3.350)	0.057 *** (4.841)	0.057 *** (4.870)	0.031 * (2.348)	0.030 * (2.255)
t' Wages	---	---	---	0.272 *** (20.283)	0.271 *** (20.914)
T_m *(LR chi-square)*	1384.75	1738.96	1845.92	606.63	621.93
df	309	369	374	206	210
IFI/RNI^	0.9962	0.9952	0.9949	0.9983	0.9983
RMSEA	0.0257	0.0265	0.0273	0.0192	0.0193
BIC	−1264.19	−1424.34	−1360.24	−1159.33	−1178.32

Notes: Numbers in parentheses are z-ratios. Number of individuals = 5285;

* p<.05

** p<.01

*** p<.001 (two-tailed tests)

^ IFI and RNI were identical up to 3 significant digits, so we do not report them separately here.