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Aerodynamic resistances to transfers of heat, mass and momentum

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Abstract The transfer of heat or mass to a vegetated surface encounters greater aerodynamic resistance than does the transfer of momentum. Causes for this difference are examined. Available information on the 'excess resistance' term and methods for approximating the aerodynamic resistance are discussed.

Les résistances aérodynamiques aux transferts de chaleur, de masse et de quantité de mouvement

Résumé Le transfert de chaleur ou de masse à une surface recouverte de végétation est soumis à plus de résistance aérodynamique que ne l'est le transfert de quantité de mouvement. On a étudié les raisons de cette différence. On a passé en revue l'information disponible sur le terme "surrésistance" ("excess resistance" en anglais) et les méthodes qui permettent d'établir une estimation approximative du résistance aérodynamique.

INTRODUCTION

Resistance models have been extensively employed in a variety of scientific investigations (e.g., evapotranspiration estimation via remote sensing techniques, dry deposition). These models are based on the consideration of the resistance to the flow of water vapour through the stomata of plant leaves and the aerodynamic resistance to transfer of heat (or mass). In this paper, the concepts involving the aerodynamic resistances to the transfer of heat, mass and momentum are reviewed and methods for their estimation are discussed.

THEORETICAL CONSIDERATIONS

Heat (or mass) transfer in the immediate vicinity of a vegetated surface is primarily controlled by molecular diffusion. Momentum, however, is transferred to a large extent by pressure forces, which have no analogue in heat (or mass) transfer. Transfer of momentum in aerodynamically rough flows (e.g., over a vegetated surface) is primarily independent of molecular viscosity. Therefore, the Reynolds' analogy between transfer of heat (or mass) and momentum may or may not apply above the canopy, but it can never apply right at the vegetation surface (Owen & Thomson, 1963; Chamberlain, 1966, 1968; Thom, 1972).

The transfer of heat (or mass) to or from a vegetated surface encounters greater aerodynamic resistance than does the transfer of momentum. Accordingly, an "excess resistance" or "quasi-laminar layer resistance" term (r_b) is defined:

$$r_{ah} = r_{av} = r_{am} + r_b \tag{1}$$

where r_{am}, r_{ah} and r_{av} are the aerodynamic resistances to transfer of momentum, sensible heat and mass, respectively.

The terms r_{am} and r_{ah} may be obtained from:

$$\tau = \rho_a \frac{U}{r_{am}} \tag{2}$$

and

$$H = \rho_a c_p \frac{T_m - T}{r_{ah}} \tag{3}$$

where τ and H are the fluxes of momentum and sensible heat, ρ_a and c_p are the density and specific heat of air at constant pressure, and U and T are the mean wind speed and potential temperature at a reference level (z). The term T_m represents an "effective surface temperature".

The level at which U becomes zero on extrapolation of the mean wind speed profile is given by $d + z_0$ (where d is the zero plane displacement and z_0 is the roughness length for momentum). Thom (1971) interpreted $d + z_0$ as the virtual momentum sink level. For sensible heat, an analogous virtual source can be defined[1] at $d + z_h$, when z_h is the roughness length for sensible heat. The effective surface temperature term (T_m) is the temperature obtained by extrapolating T(z) versus ln (z – d) to $z = d + z_h$ (see Fig. 1).

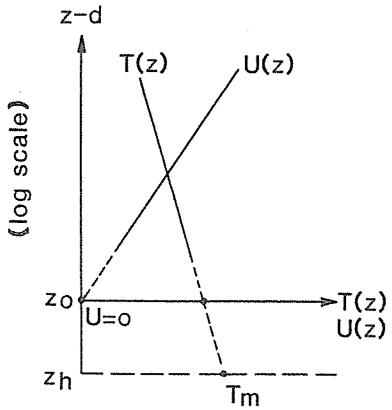

FIG. 1. Profiles of mean wind speed (U) and potential temperature (T).

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[1] It is assumed that d, the zero plane displacement, represents a reference level common to flux profile relations in general (Thom, 1972; Garratt, 1978).

As discussed above, the aerodynamic resistance to heat transfer to or from a vegetated surface is greater than the aerodynamic resistance to the momentum transfer. Owen and Thomson (1963) proposed a dimensionless parameter B^{-1} (sublayer Stanton number) to express the difference between these resistances:

$$B^{-1} = r_{ah}^{+} - r_{am}^{+} \tag{4}$$

where r_{ah}^{+} and r_{am}^{+} are nondimensional resistances given by:

$$r_{ah}^{+} = r_{ah}\, u_* \tag{5}$$

$$r_{am}^{+} = r_{am}\, u_* \tag{6}$$

In (5) and (6), u_* is the friction velocity. The gradient ($\partial T/\partial z$) of mean potential temperature can be expressed as:

$$\frac{\partial T}{\partial z} = -\frac{T_*}{kz}\phi_h \tag{7}$$

where $T_* = \dfrac{H}{\rho_a\, c_p\, u_*}$ (8)

and ϕ_h is the diabatic correction factor for heat transfer.

Substituting (8) into (7) and integrating between the limits of the roughness length and the height of interest yields:

$$T_m - T = \frac{H}{\rho_a\, c_p\, u_*\, k}\left(\ln\left(\frac{z-d}{z_h}\right) - \psi_h\right) \tag{9}$$

Similarly for mean wind profile, we have

$$U = \frac{u_*}{k}\left(\ln\left(\frac{z-d}{z_0}\right) - \psi_m\right) \tag{10}$$

where ψ_h and ψ_m are the integral diabatic correction factors for heat and momentum transfer (see e.g. Paulson, 1970, for approximate formulae).

Combining (9) and (10) with (3) yields:

$$r_{ah} = \frac{U}{u_*^2} + \frac{1}{k u_*}\left(\ln\left(\frac{z_0}{z_h}\right) + \psi_m - \psi_h\right) \tag{11}$$

Recalling (1) and (2), the aerodynamic resistance to momentum transfer (r_{am}) and the excess resistance term can be expressed as:

$$r_{am} = \frac{U}{u_*^2}\,, \tag{12}$$

and $$r_b = \frac{1}{k u_*}\left(\ln\left(\frac{z_0}{z_h}\right) + \psi_m - \psi_h\right) \tag{13}$$

Equation (13) would then yield:

$$k B^{-1} = k u_* r_b = \left(\ln\left(\frac{z_0}{z_h}\right) + \psi_m - \psi_h\right) \tag{14}$$

Under near neutral conditions (14) reduces to:

$$k\,B^{-1} = \ln \frac{z_0}{z_h} \tag{15}$$

(Chamberlain, 1966; Thom, 1972).

Traditionally, (15) has been employed to express $k\,B^{-1}$ and one may wish not to include stability correction terms in calculating r_b. In that case, (11) can be rewritten as:

$$r_{ah} = \frac{1}{ku_*}\left(\ln\left(\frac{z}{z_0}\right) - \psi_h\right) + \frac{1}{ku_*}\left(\ln \frac{z_0}{z_h}\right). \tag{11a}$$

Therefore, (12) and (13) can be modified accordingly:

$$r_{am} \approx \frac{1}{ku_*}\left(\ln\left(\frac{z}{z_0}\right) - \psi_h\right) \tag{12a}$$

or

$$r_b \approx \frac{1}{ku_*}\left(\ln \frac{z_0}{z_h}\right) \tag{13a}$$

$$\text{or} \quad k\,B^{-1} = \ln \frac{z_0}{z_h}$$

RESULTS FROM STUDIES OF B^{-1}

Through theoretical analysis and a series of wind tunnel experiments, Owen & Thomson (1963) evaluated the term B^{-1}. Their results can be presented in the following form (Garratt & Hicks, 1973):

$$k\,B^{-1} \approx 0.2\,(30\,Re_*)^{0.45}\,\sigma^{0.8} \tag{16}$$

where $Re_* = u_*\,z_0/\upsilon$, σ is the Prandtl number and υ is the kinematic viscosity.

Chamberlain (1966) conducted a series of experiments over surfaces of "fibrous character". His results, however, indicated a much smaller dependence of B^{-1} on u_* and z_0 than was suggested by the Owen-Thomson model (16).

Thom (1972) also suggested that B^{-1} was virtually independent of z_0 and varied slowly with u_*. For a bean crop he derived:

$$B^{-1} = 1.35\,u_*^{1/3} \tag{17}$$

for heat exchange and transpiration. Thom indicated that such equations "may well provide accurate enough first approximations to the values of B^{-1} for many vegetated surfaces - except perhaps where vegetation elements are unusually bluff, or fail to provide a complete and fairly uniform cover."

Garratt & Hicks (1973) presented a comprehensive analysis of available atmospheric and wind tunnel heat and mass transfer data for various kinds of surfaces; i.e. (a) water, snow, soil, grass, vineyard and pine forest (atmosphere) and (b) towelling, rough glass, artificial grass and several arrays of cylinders and spheres (wind tunnel). They found little difference in values of $k\,B^{-1}$ obtained for heat and water vapour transfer. For $Re_* < 100$, $k\,B^{-1}$ values obtained from various experiments were consistent and showed a gradual increase with Re_* (see Fig. 2), as expected from the Owen-Thomson model (1). Above $Re_* = 100$, the situation was different. While there was overall consistency within particular experiments, the values of $k\,B^{-1}$ deduced from various experiments differed by an order of magnitude. Garratt & Hicks interpret the separation of data at large Re_* in terms of different bluff body contributions because of differences in surface structure. The data contributing to the upper band of Fig. 2 were obtained over surfaces comprised of relatively solid geometrical arrays of well-spaced roughness elements (cylinders and spheres in a wind tunnel and a vineyard). Garratt & Hicks suggest that the vineyard may be considered to be similar to an array of horizontal cylinders in well-spaced parallel rows (Hicks, 1973), thus being similar in structure to the wind tunnel models. Also, the vineyard data suggested considerable enhancement of the bluff body effect when the wind was at right angles to the rows. The individual leaves of vines apparently played little part in the determination of the bulk aerodynamic resistance to heat and momentum transfer.

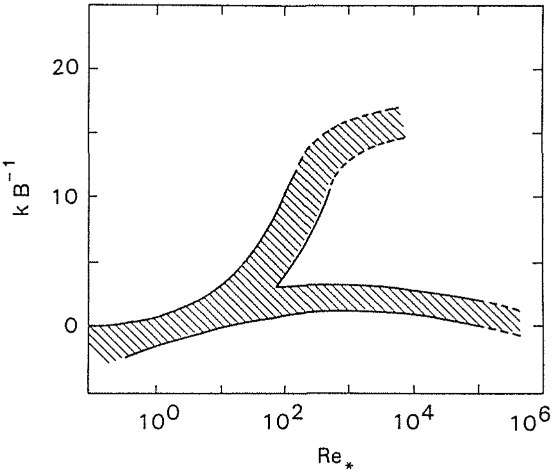

FIG. 2 *k* B^{-1} *as a function of the roughness Reynolds number,* $Re_* = \frac{u_* z_0}{\nu}$ *(adapted from Garratt & Hicks, 1973).*

The surfaces over which the data in the lower band in Fig. 2 were obtained, however, consisted of vegetation such as grass, beans and pine forests. It seems likely, Garratt & Hicks suggest, that for such surfaces the aerodynamic characteristics of individual elements (leaves, twigs, etc.) determine the bulk aerodynamic characteristics. The bluff body effect was considered to be substantially smaller. These surfaces were characterized by closely-packed, complex roughness elements of a "fibrous character".

Heilman & Kanemasu (1976) computed B^{-1} values from their field measurements over soybeans and sorghum. Their results compared well with the lower bands in Garratt & Hicks' Fig. 2 (for large Re_*). From measurements over a heterogeneous surface comprising 8 m tall trees with transpiring foliage, 1 m tall dry grass, and sandy soil, Garratt (1978) concluded that k B^{-1} for heat transfer is about 2.5 ± 0.5.

Brutsaert (1984) presented an excellent summary of theoretical and experimental results on B^{-1} (see Fig. 3). As in Fig. 2 (Garratt & Hicks, 1973), he segregated data into two categories of surfaces and discussed the pronounced dissimilarity between the bulk transfer properties for heat (or water vapour) at "bluff-rough surfaces" and those at "permeable-rough surfaces". For surfaces with bluff roughness elements, e.g., plowed field, rigid vegetation with very large leaves e.g., cabbage and beet plants, irregular ice surfaces and developing water waves, k B^{-1} rises steeply (up to $Re_* = 1000$). But for surfaces with permeable roughnesses e.g., vegetated surfaces consisting of roughness elements which are characteristically of a more porous and fibrous nature and which are often densely spaced, $k B^{-1}$ is relatively insensitive to Re_*. Results indicated that for these kinds of surfaces, $k B^{-1}$ depends only slightly on u_* and is practically independent of z_0 (as was also suggested by Thom, 1972).

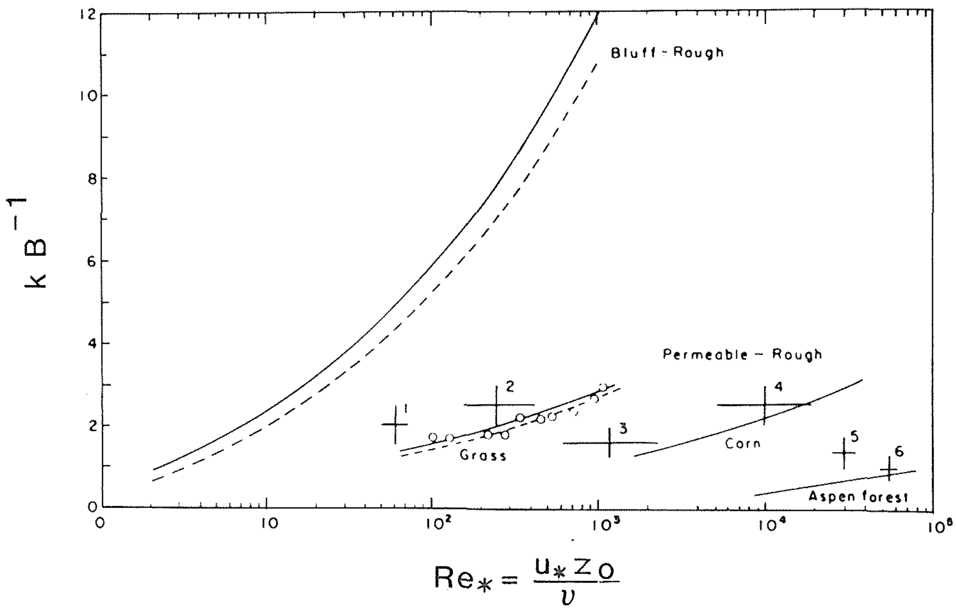

FIG. 3 *$k B^{-1}$ as a function of the roughness Reynolds number,* $Re_* = \frac{u_* z_0}{\nu}$ *(adapted from Brutsaert, 1984).*

ESTIMATING AERODYNAMIC RESISTANCE FOR MOMENTUM TRANSFER, r_{am}

Micrometeorological technique

The term r_{am} (see (12) or (12a)) can be evaluated if information on friction velocity (u_*), roughness length for momentum (z_0) and mean wind speed (at a reference level) is available. Friction velocity can be obtained by several methods. These include (a) direct measurement using a three-dimensional velocity sensor in an eddy correlation approach, and (b) estimation from measurements of mean wind speed profiles, e.g., using the Deacon-Swinbank low-level drag coefficient approach (Deacon & Swinbank, 1958; Bradley, 1972). Values of z_0 can be estimated from measured mean wind speed profiles.

Another approach

Based on the consideration that the standard deviation of wind direction (σ_θ) contains information on stability and surface roughness, Hicks *et al.* (1987) suggest using measurement of σ_θ in estimating r_{am}. The term σ_θ can be approximated as:

$$\sigma_\theta \approx \frac{\sigma_v}{U} \tag{18}$$

where σ_v is the standard deviation of the crosswind velocity component. Substituting (10) into (18) results in:

$$\sigma_\theta \approx \frac{\sigma_v}{u_*}\, k \left(\ln\left(\frac{z-d}{z_0}\right) - \psi_m \right)^{-1} \tag{19}$$

From (10), (12) and (19) we can write:

$$r_{am} = \left(\frac{\sigma_v}{u_*}\right)^2 \frac{1}{U\sigma_\theta{}^2} \tag{20}$$

In near-neutral and stable stratification, the ratio (σ_v/u_*) is about 2 and, therefore:

$$r_{am} \approx \frac{4}{U\sigma_\theta{}^2} \qquad \text{(near neutral \& stable conditions)} \tag{21}$$

The ratio σ_v/u_* increases rapidly after the onset of instability and asymptotically approaches a value of about 3 and so:

$$r_{am} \approx \frac{9}{U\sigma_\theta{}^2} \qquad \text{(unstable conditions: } z/L < -0.1\text{)} \tag{22}$$

If net radiation (R_n) is positive and σ_θ exceeds some "cardinal value" (A), then conditions can be assumed to be unstable and (22) can be used (Hicks *et al.*, 1987). Otherwise, (21) should be applicable (near-neutral and stable conditions). As can be inferred from (19), A is site specific because of the functional dependence of σ_θ on z_0 and d. Hicks *et al.* use the value $A \pm 0.17$ (corresponding to $\sigma_\theta \pm 10$ degrees) in their present calculations. Further research is needed to refine this value.

Estimating the excess resistance term, r_b

As discussed earlier:

$$r_b = \frac{B^{-1}}{u_*} \tag{23}$$

Brutsaert (1984) and many other investigators suggest that for surfaces with densely spaced permeable roughness elements[2], as a first approximation, $k\,B^{-1}$ can be assumed to be of the order of 2 for scalars whose Schmidt or Prandtl number is of the order of 0.6 to 0.8. The relationship:

$$r_b \approx 2\left(\frac{1}{k u_*}\right)\left(\frac{S_c}{P_r}\right)^{2/3} \tag{24}$$

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[2] A recent analysis of data (Dr William Kustas, personal communication) over sparse vegetation in Owens Valley, California suggests that $k\,B^{-1}$ can be significantly larger than 2.

is generally used to approximate r_b for transfer of a scalar. The factor $(S_c/P_r)^{2/3}$ is incorporated to account for the fact that the basic information was derived from heat transfer observations primarily (Wesely & Hicks, 1977).

Based on the estimated r_{am} (see above), an internally consistent value of u_* can be derived from (12). With information on u_*, r_b can now be estimated employing (24) (Hicks *et al.*, 1987).

SUGGESTIONS FOR FUTURE RESEARCH

Further work needs to be done to permit reasonably accurate evaluation of aerodynamic resistance (r_{am}) from meteorological variables which can be measured easily and routinely. The approach suggested by Hicks *et al.* (1987) is a good example, but thorough testing for a range of surfaces (partial canopy cover, patchy vegetation) and environmental conditions is needed. The excess resistance term (r_b, B^{-1}) needs to be evaluated over a variety of vegetated surfaces. The relationships between the roughness lengths (z_0, z_h) needs to be examined thoroughly. Research needs to be conducted on ways to accurately estimate surface temperature (T_m) for use in (3).

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