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The narrative of what it means to be teachers and learners of mathematics is changing, redefining what we consider our vision of high-quality mathematics instruction to be. Therefore, we must think about how to support prospective and practicing teachers in enactment of this evolving vision. Responsive teaching is one type of teaching that encompasses this vision—one that requires teachers to attend to the details of children's mathematical thinking and find ways to build on their ideas. Finding ways to elicit and build on children's mathematical thinking using teaching moves (i.e., questions, statements, or actions) is challenging, which suggests that teacher educators need to consider the perspectives of those enacting this vision. Only then, can we find effective ways to support its enactment. Eliciting the perspectives of prospective teachers is particularly important because they often carry feelings of anxiety about teaching mathematics and uncertainty about what teaching moves to use in the moment with children. Further, similar to research that describes the importance of teachers being responsive to children's mathematical thinking, this study is built on the assumption that it is important for teacher educators to be responsive to the thinking of prospective teachers.

The purpose of this study was to understand the teaching moves and rationales of prospective teachers as they engaged with children solving mathematical story problems prior to the start of their teacher education program. Using a monostrand conversion

mixed-methods design, I investigated the prospective teachers' teaching moves, rationales, and the relationship between them. Specifically, I observed prospective teachers engaging in one-on-one problem-solving interviews with children to capture the teaching moves they made. Through stimulated-recall interviews, I retrospectively elicited their rationales for making those teaching moves. Problem-solving interviews and stimulated-recall interviews were analyzed both qualitatively and quantitatively.

Examination of teaching moves enacted during problem-solving interviews showed prospective teachers used a range of teaching moves that fell into three main categories: (a) comprehending story problems, (b) exploring details of children's mathematical thinking, and (c) telling information to children. Further, when prospective teachers enacted teaching moves in each category, these teaching moves took a variety of forms. Findings also revealed unexpected strengths of prospective teachers as well as room to grow in their expertise.

Exploration of rationales shared during stimulated-recall interviews indicated that prospective teachers had specific rationales for enacting their teaching moves. Broadly these rationales sometimes focused on benefitting children and sometimes focused on benefitting the PSTs themselves. Findings revealed five categories of rationales. Four were parallel categories within these two broad types including rationales focused on increasing comfort (both for children and prospective teachers) and rationales focused on enhancing understanding (both of children and prospective teachers). The final rationale category focused on benefitting children by guiding their problem solving. An exploratory investigation of the relationship between categories of teaching moves rationales showed that prospective teachers' rationales were sometimes *aligned* and sometimes *misaligned* with the teaching moves they chose.

This study contributes to the research base on responsive teaching with children, in particular as it relates to prospective teachers working with children by reporting the range of teaching moves prior to engagement in a mathematics methods course. I also categorized the prospective teachers' rationales for their teaching moves into a framework that teacher educators can use to be responsive to the thinking of prospective teachers. In additions, suggestions for future research are provided. Finally, the findings have practical implications for working with prospective teachers on responsive teaching including: (a) increasing prospective teachers' access to research based frameworks of children's mathematical thinking (b) using artifacts of practice from prospective teachers' work with children, (c) expanding prospective teachers' repertoire of teaching moves for helping children comprehend story problems, and (d) asking prospective teachers to reflect on their practice in more specific ways.

TEACHING MOVES AND RATIONALES OF PROSPECTIVE ELEMENTARY SCHOOL TEACHERS IN ONE-ON-ONE MATHEMATICAL CONVERSATIONS WITH CHILDREN

by

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To my husband Joshua and my children Ellie, Penelope, William, and Jackson for supporting and encouraging me every step of the way.

AND

To the PSTs who braved this study along with me. Thank you for allowing me insight into your practice so that we may all learn from you.

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This dissertation, written by Montana L. Smithey, has been approved by the following committee of the Faculty of The Graduate School at The University of North Carolina at Greensboro.

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~ ~ *She believed she could, so she did* ~ ~

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CHAPTER I

INTRODUCTION

Policy documents support an evolving vision of change in mathematics education, one that calls for children to develop conceptual understanding through productive conversations and demonstrate problem-solving abilities including reasoning (National Council of Teachers of Mathematics [NCTM], 2014; National Research Council [NRC], 2001). Further, this vision emphasizes teachers as facilitators who elicit and build on children's mathematical thinking by listening closely in-the-moment and being responsive to children's needs.

Responsive Teaching

Consistent with the current vision, this study focused on ways to be responsive to children's mathematical thinking—known as responsive teaching. Robertson et al., (2016) described *responsive teaching* as a type of teaching that foregrounds children's ideas, makes conceptual connections within their ideas, and takes up and pursues children's ideas. Although I chose to ground this study in teaching that is responsive to children's mathematical thinking, it is important to recognize that there are other ways teachers need to be responsive to children in the classroom. For instance, research supports the importance of drawing from children's cultural backgrounds and using their assets to make instructional decisions (Gay, 2002; Ladson-Billings, 1995). Although this study does not focus on responsive teaching in this way, it does, however, draw from similar ideas that children are capable, diverse in their thinking, and have unique assets on which teachers can build. The increased interest in teachers' attention to children's mathematical thinking, and the competence children have, draw on an extensive research base about children's mathematical thinking (for overview, see e.g., Cai, 2017). Research across content areas in mathematics have included the ways children develop conceptual understanding and strategies children use to solve problems that often differ from teachers' thinking (Carpenter et al., 2015; Clements & Sarama, 2009; Fuson et al., 1997). Researchers have also studied the ways teachers acquire such knowledge and use it in the classroom to support children in teaching responsively.

For this study, I particularly drew from a long-standing and influential research and professional development project, *Cognitively Guided Instruction* (CGI). CGI gives teachers access to research-based knowledge of children's mathematical thinking and opportunities to develop instructional practices that build on this knowledge during instruction (Carpenter et al., 1996). Their work also highlights the power of using story problems as familiar and contextualized ways for children to engage in mathematics. I focus specifically on the instructional practices of eliciting and building on children's mathematical thinking in the context of story problems.

Eliciting and Building on Children's Mathematical Thinking

Teachers enact the instructional practices of eliciting and building on children's mathematical thinking through use of teaching moves. *Teaching moves* include questions, series of questions, statements, or even actions (Jacobs & Empson, 2016). The ways in which teachers enact these practices open space for certain types of responses from children, which can ultimately impact not only what children learn but also what teachers learn or understand about children's mathematical thinking. It is also important to recognize how difficult and complex responsive teaching is to enact, especially in-the-moment during instruction. Teachers must be knowledgeable about children's mathematical thinking, and then simultaneously attend to, interpret, and decide how to respond to children's ideas—all while keeping in mind their mathematical goals (Jacobs et al., 2010; van Es & Sherin, 2008).

Expertise in the core instructional practices of eliciting and building on children's mathematical thinking develops over time (Jacobs, et al., 2010; McDonald et al., 2013). To help teachers develop this expertise, teacher educators often decompose these practices or provide opportunities for teachers to engage in these practices in simplified ways that still seem authentic for teaching, but are not overwhelming (Grossman et al., 2009). In this study, I focus on one of these simplified experiences, problem-solving interviews that are one-on-one conversations with children around mathematical story problems. These interviews provide learning opportunities not only for children but also for teachers. Teachers can focus on learning about the mathematical details of children's strategies and eliciting and building on children's thinking, away from the complexities of the classroom (Ginsburg, 1997).

There is a growing research base about how practicing teachers elicit and build on children's mathematical thinking—including the teaching moves they use—as well as how this expertise develops. However, this research primarily includes practicing teachers, many of whom have already developed some knowledge of children's mathematical thinking. When prospective teachers (PSTs) are studied, their practices are often compared to what is known about practicing teachers (Jacobs et al., 2010; Sleep & Boerst, 2012) or evaluated for areas in which they are lacking and need support (Sun & van Es, 2015; Webel et al., 2018). These practices are challenging to develop for teachers who have had a range of experiences with children, and they present even greater challenges for PSTs, especially PSTs who are just beginning their journey as teachers of mathematics.

Current Study

In this study, I explored the teaching moves PSTs used to elicit and build on children's mathematical thinking because these practices are foundational to responsive teaching. Eliciting and building on children's mathematical thinking occur regularly throughout instruction—during whole class and small-group discussions, as well as in one-on-one conversations with children. Specifically, I wanted to understand the teaching moves PSTs used as they engaged with children solving mathematical story problems, prior to the start of their teacher education program. This study focused particularly on teaching moves that occurred when working with individual children to limit the complexities of a classroom setting while providing space for PSTs to focus on eliciting and building on children's mathematical thinking. Further, similar to research that describes the importance of teachers being responsive to children's mathematical thinking, I argue it is important for teacher educators to be responsive to the thinking of PSTs. Thus, the goal for this study was to identify not only the teaching moves PSTs used but also the rationales PSTs had for making those teaching moves and whether their rationales were aligned (or misaligned) with the teaching moves they chose to enact. The study addressed three research questions:

- Research Question 1 [RQ1]: What teaching moves do PSTs make when engaging in problem-solving interviews with children around story problems?
- Research Question 2 [RQ2]: What rationales do PSTs give as to why they make the teaching moves they do?
- Research Question 3 [RQ3]: What is the relationship between the PSTs' teaching moves and their rationales for making them?

To address these research questions, I observed PSTs engaging in one-on-one problem-solving interviews with children to capture the teaching moves they made. Through stimulated-recall interviews, I retrospectively elicited their rationales for making those teaching moves (Gass & Mackey, 2000). Problem-solving interviews and stimulated-recall interviews were analyzed both qualitatively and quantitatively in this mixed-methods study.

This study was designed to contribute to the literature on responsive teaching by articulating issues specifically relevant to PSTs. I also wanted to provide a framework that captured the rationales PSTs shared as to why they made the teaching moves they did in a way that was helpful for teacher educators. I was particularly interested in amplifying the PSTs' voices as they are not often foregrounded in this research. I argue there is much to learn from PSTs that can be used to support them in developing expertise to teach children responsively and this information should support teacher educators in teaching PSTs responsively.

Positionality

I position myself as a teacher educator and researcher. I believe people have varying perspectives and experiences—different lenses with which they see the world and that are important to consider in teaching. As a teacher educator who primarily works with PSTs, I am interested in recognizing and learning from the PSTs' perspectives and the experiences they have already had in mathematics education. In my own practice, I believe it is important to teach in ways that are responsive, and I try to find ways to build on what PSTs already know to further develop their expertise in teaching mathematics. In addition, when teaching elementary mathematics methods courses, I support PSTs in learning a variety of core practices—in particular, eliciting and building on children's mathematical thinking. In other words, I try to be responsive to PSTs' thinking in helping them learn to be responsive to children's thinking. Being responsive to PSTs' thinking was part of the motivation for this study, and thus it was important to elicit PSTs' voices and to determine their existing expertise upon entering the education program.

Overview of Chapters

I share this dissertation study in seven chapters. Building on this chapter's introduction, Chapter 2 reviews the literature on responsive teaching, specifically what we know about eliciting and building on children's thinking and ways the development of that expertise can be supported. Chapter 3 provides insight as to the methods used for the study. Chapters 4–6 present the findings, organized by research question—teaching moves, rationales, and the relationship between them. Finally, Chapter 7 summarizes the findings and shares theoretical, practical, and research implications as well as limitations of the study.

CHAPTER II

REVIEW OF THE LITERATURE

The narrative of what it means to be teachers and learners of mathematics is changing and continues to evolve. Grounded in research, policy documents provide recommendations that support this evolution (NCTM, 2014; NRC, 2001). The changing narrative extends how we typically think about mathematics teaching and learning, by redefining what we consider our vision of high-quality mathematics instruction to be. I adopt Munter's (2014) definition of instructional vision, which includes considerations of teachers' roles in the classroom, discussions that occur within classrooms, and mathematical tasks in which children engage. Further, his idea of vision also involves "ways of seeing the world that encompass horizons not yet reached" (Munter, 2014, p. 586). Therefore, we must think about how to support PSTs and practicing teachers in enactment of this evolving vision of mathematics teaching and learning—keeping in mind the narratives of the past.

Historically in mathematics education, teachers model for children how to solve problems and present series of steps to follow—down a smooth path to the correct answer. Children engage in repeated practice demonstrated by their teachers and often rely on memorized procedures—with limited opportunities to reason and engage in discussion with others (Weiss & Pasley, 2004). When opportunities for discussion arise, conversations typically appear as a pattern of initiate–respond–evaluate (IRE) (Mehan, 1979). Teachers *initiate* classroom discussion by posing a question, children *respond* to the question posed, and teachers *evaluate* whether the response is correct. Teachers direct this pattern of talk by selecting children to answer questions, and they maintain control of the conversation, including children's understanding of the topic (Lemke, 1990). This pattern of discourse supports the notion that teachers are the sole keepers of knowledge in the classroom—teachers take on the majority of the talking and mathematical work while students listen and follow (Freire, 1993; Wood, 1998).

In contrast, the narrative is changing toward a vision of mathematics instruction in which teachers facilitate children taking on the mathematical work and engaging in sensemaking. Further, they promote children's engagement in productive struggle and development of conceptual understanding (Hiebert & Grouws, 2007; NCTM, 2014; NRC, 2001). Children have extensive opportunities to engage in mathematics and as Lampert (1990) described, developing intellectual authority in the classroom so mathematics is not an isolated school activity for them. As children engage in mathematics, teachers can elicit and build on children's mathematical thinking by posing purposeful questions to advance their mathematical thinking (Chapin et al., 2009; Fraivillig et al., 1999). Franke et al., (2007) described these kinds of interactions as "not about receiving information" but as "sense-making as we participate together" (pp. 228–229). Overall, this vision describes children taking ownership of their ideas as they engage in meaningful conversations grounded in rich mathematical tasks as well as teachers facilitating and learning from children. Further, not only are the roles of teachers changing but so are the interactions between children and teachers. Responsive teaching is one type of teaching that encompasses this instructional vision. In the following sections, I describe responsive teaching as it relates to children's mathematical thinking and the various components that make up responsive teaching—knowledge of children's mathematical thinking, noticing of children's mathematical thinking, and the ways teachers respond to children's mathematical thinking. Finally, I describe common ways teacher educators support teachers in developing expertise in responsive teaching because my study methodology involved use of these activities, which has implications for future work with PSTs.

Responsive Teaching

Although there are many ways teachers can be responsive to children in the classroom (Gay, 2002; Ladson-Billings, 1995), I adopt Robertson's et al., (2016) conceptualization of responsive teaching—one that requires teachers to attend to the details of children's thinking, maintain the focus on the underlying concepts of children's thinking, and employ opportunities to take up children's ideas and follow them. Drawing from features of other ways to be responsive in the classroom, this type of teaching positions children as capable, possessing unique and creative ways of thinking, and having assets on which teachers can build. As this type of responsive teaching relates to mathematics education, teachers provide space for children to make sense of mathematical ideas. As children share their mathematical thinking, teachers not only attend to the mathematical details in the strategy but do so to try to better understand—not evaluate—and use what they learn to make informed instructional decisions, including in the moment (Jacobs & Empson, 2016). To be responsive to children's mathematical thinking, knowledge of how that thinking develops is essential.

Knowledge of Children's Mathematical Thinking

Some researchers have explored what children know and understand across mathematical content areas whereas others have focused on how children's mathematical thinking can be foundational for classroom conversations (Baroody & Wilkins, 1999; Cai, 2017; Carpenter & Moser, 1984; Kaput, 2008; NRC, 2001; Sarama et al., 2003) In this study, I primarily drew from CGI because it gives teachers access to research-based knowledge of children's mathematical thinking to highlight the innate capabilities of children and elevate the creative ways they solve problems, as well as the patterns in how their reasoning often differs from our own. Enhancing teachers' understanding of children's mathematical thinking—including the strategies children use to solve problems—supports teachers in learning how to teach responsively in the classroom (Carpenter et al., 1996; Fennema et al., 1993). CGI is one of the few projects that has regularly documented links between teacher learning and advances in children's achievement (Carpenter et al., 1989; Fennema et al., 1996; Jacobs, et al., 2007). Not only do children benefit from teachers' attention to children's thinking, but also teachers benefit as they acquire expertise in children's mathematical thinking and use that expertise in their instructional decisions (Jacobs & Spangler, 2017). Thus, knowledge of children's thinking is part of the foundation teachers need to be able to notice and respond to children's mathematical thinking (Jacobs et al., 2010).

Professional Noticing of Children's Mathematical Thinking

Teaching in ways that are responsive to children's mathematical thinking is complex and includes actions that we cannot see, including teacher noticing. The idea of *teacher noticing*, which captures the complex process in which teachers make sense of what they see and hear to make instructional decisions, has a strong research base (Schack et al., 2017; Sherin, et al., 2011; Van Es, & Sherin, 2008). For example, Jacobs et al., (2010) presented a framework of professional noticing of children's mathematical thinking that helps us think more about what it means to teach responsively. They articulated how teachers use a set of interconnected skills when deciding how to respond to children's mathematical thinking. Specifically, teachers *attend* to the mathematical details of children's strategies by gathering information as they observe and listen. Teachers then *interpret* what children understand based on what teachers have observed children say and do as well as their understanding of how children's mathematical thinking develops. Finally, teachers *decide* how to respond to children in ways that build on children's understandings. The enactment of the three interconnected skills happens quickly—almost simultaneously—and multiple times throughout a lesson. Further, teacher noticing is invisible, yet a critical precursor to the visible responding—the teaching moves that can support eliciting and building on children's thinking.

Responding to Children's Mathematical Thinking

To be responsive to children's mathematical thinking, teachers notice what children do and then take up and pursue children's ideas. Specifically, eliciting and building on children's mathematical thinking are core instructional practices, and expertise in these practices develops over time (Grossman et al., 2009; Jacobs et al., 2010; McDonald et al., 2013). These practices open spaces for children's thinking and how these practices are enacted influences what both children and teachers can learn. A large body of literature helps us understand the ways PSTs and practicing teachers elicit and build on children's mathematical thinking (Ellis et al., 2019; Fraivillig et al., 1999; Jacobs & Empson, 2016; Shaughnessy & Boerst, 2018).

Eliciting and Building on Children's Mathematical Thinking

Teachers elicit and build on children's mathematical thinking through use of teaching moves (i.e., questions, series of questions, statements, or actions), and they enact these teaching moves in different kinds of conversations—whole class, small group, and one-on-one conversations. I synthesized the literature on teaching moves to highlight five major categories in relation to children's mathematical thinking and their role in supporting or hindering responsive teaching: (a) comprehending story problems (b) exploring details of children's mathematical thinking, (c) telling information to children, (d) revoicing children's ideas, and (e) using wait time. Because this study focused on one-on-one conversations between a teacher and a child, I did not include teaching moves specific to orienting children to each other, even though those moves play a significant role in responsive teaching (Smith & Stein, 2018).

In the following sections, I describe the five categories of teaching moves and their various forms. Note that much of the research on teaching moves has been done with practicing teachers, but the findings about the major categories of teaching moves are generally similar in the research with PSTs, with PSTs demonstrating less expertise than practicing teachers. Therefore, I describe together the research on teaching moves for PSTs and practicing teachers, highlighting distinctions as needed. Further, although the attention to these categories of teaching moves was consistent across the literature, the frameworks created, and the terminology used were not. For example, researchers often used the terms "teaching moves" and "questions" interchangeably. For this study, I use the term "teaching moves" but in describing the literature, I used the terminology chosen by the authors to preserve their work.

Comprehending Story Problems. Comprehending story problems is a category of teaching moves that supports children in understanding the story situation, which includes not only the vocabulary and story context but also the mathematical question. By helping children comprehend story problems, teachers are ensuring children have access to the mathematics of the problem and are encouraging sense making (see, e.g., Lucangeli et al., 1998). Teaching moves in this category occur in many forms. For example, teachers might ask children to summarize story problems in their own words so that they can learn what the children do and do not understand (Jacobs & Ambrose, 2008). Teachers may also focus on unpacking an unfamiliar story context by, for instance, highlighting details about the context, providing background knowledge about the context, or rephrasing or elaborating the context in ways that connect it to children's lives (Ball, 1993; Jackson et al., 2013; Jacobs & Empson, 2016). In summary, comprehending story problems is a category of teaching moves that supports responsive teaching because it helps children use story situations as tools for sensemaking.

Exploring Details of Children's Mathematical Thinking. Exploring details of children's mathematical thinking is a category of teaching moves in which teachers focus on the mathematical details of what children say and do thereby opening space and showing appreciation for children's reasoning. The most prevalent form described in the literature is pressing (also called probing). *Pressing* is a teaching move in which teachers ask children to provide reasoning or support for a claim, strategy, or solution they have put forth to promote reflection and deeper understanding of mathematics (Cengiz et al., 2011; Franke et al., 2009; Hiebert & Wearne, 1993; Kazemi & Stipek, 2001). Teachers can use pressing to encourage children to articulate and clarify ideas shared, to extend what children already know to something new, and to learn about children's mathematical thinking (Ball, 1993; Boaler & Brodie, 2004; Brodie, 2010; Jacobs & Empson, 2016; Moyer & Milewicz, 2002; Smith et al., 2008).

Other forms of teaching moves also fall within the category of exploring details of children's mathematical thinking. For instance, teaching moves that open conversations by inviting children to share create space for mathematical details to surface so that teachers can explore them (Boaler & Brodie, 2004; Franke et al., 2015; Jacobs & Ambrose, 2008; Moyer & Milewicz, 2002; Shaughnessy & Boerst, 2018). Teachers may also use teaching moves to link the mathematical details of children's strategies back to the story context or discuss in depth the quantities children use in problem solving (Jacobs & Empson, 2016). Other times, teachers may ask children to solve problems using more than one strategy or write a symbolic representation of a strategy, idea, or context (Cengiz et al., 2011; Hiebert & Wearne, 1993; Jacobs & Empson, 2016).

In short, exploring details of children's mathematical thinking is a category of teaching moves central to responsive teaching because the moves focus on what children are saying and doing in-the-moment. Researchers have noted that although PSTs have demonstrated some ability to elicit or attend to children's strategy details, they have also generally shown an inability to use these details to gain a deep understanding of children's thinking (Sleep & Boerst, 2012) or build on those details in determining next instructional steps (Jacobs, et al., 2010). Thus, PSTs' explorations of children's mathematical thinking are often limited.

Telling Information to Children. Telling information to children is a category of teaching moves that focuses on providing children with pieces of knowledge teachers believe to be key for problem-solving or furthering discussions. Forms of teaching moves in this category may appear as labeling terminology or reminding children of mathematical goals (Boaler & Brodie, 2004; Chazan & Ball, 1999; Cengiz et al., 2011; Lobato et al., 2005) whereas other forms focus on describing concepts or demonstrating for children what to do and then expecting children to repeatedly practice that procedure (Moyer & Milewicz, 2002). Many of these teaching moves are evaluative, emphasizing correct or incorrect answers, or focused on sharing ideas or interpretations of teachers. Unfortunately, what teachers "tell" children during problem-solving may not be pertinent to children's understanding or development of their mathematical thinking (Chazan & Ball, 1999; Moyer & Milewicz, 2002), and thus children often no longer make sense of the mathematics and the teacher ends up doing most of the mathematical work (Wood, 1998). In brief, telling information to children is a category of teaching moves that provides children with knowledge or ideas and although some forms of telling can build on children's ideas, most hinder responsive teaching because they limit children's sense making and taking ownership of their problem solving. Researchers have found that teaching moves related to telling can be particularly prominent with PSTs (Moyer & Milewicz, 2002; Sleep & Boerst, 2012; Sun and van Es, 2015).

Revoicing Children's Ideas. Revoicing children's ideas is a category of teaching moves focused on teachers using the language of children's contributions to clarify or amplify children's ideas. Teachers may re-utter an idea that a child has shared by repeating or expanding on that idea verbally or through gestures, leaving room for children to comment on that re-utterance (O'Connor & Michaels, 1993; Shein, 2012). Specifically, Yifat and Zadunaisky-Ehrlich (2008) described two forms of teaching moves in this category, exact revoicing and reformulated revoicing. In *exact revoicing*, teachers repeat word for word what children have expressed whereas in *reformulated revoicing*, teachers repeat children's ideas but the words are either rephrased or new information is added. Other researchers have focused on how teachers can use revoicing to not only share children's ideas, but also to position children in positive ways in the classroom. *Positioning* in a classroom context refers to situating children in relation to each other (e.g., as knowledgeable or not) when publicly discussing shared ideas (see, e.g., Enyedy et al., 2008; Herbel-Eisenmann et al., 2015). In short, revoicing is a category of teaching moves that supports responsive teaching by showcasing children's ideas rather than relying on teachers' contributions to drive discussion.

Using Wait Time. Wait time is a category of teaching moves that provides moments of silence for children to give them space to think and reason before responding. Forms of teaching moves in this category sometimes highlight the timing of the moments of silence. For instance, Rowe (1986) distinguished pauses that occur after teachers have finished speaking but before children respond from pauses that occur after children have finished speaking but before teachers respond. Others have focused on the use of activities like quick writes and think-pair-shares to give children space to generate ideas before having to share aloud (Chapin et al., 2009; Staples & King, 2017). Overall, wait time supports responsive teaching because it gives children time to make sense of and take ownership of the mathematics.

The descriptions of these five categories of teaching moves provide a glimpse into the complexity of enacting responsive teaching and the variety of foci researchers have chosen to explore. Researchers have also identified ways to support teachers in developing the expertise needed to enact these teaching moves and teach in ways that are responsive to children's mathematical thinking.

Developing Expertise in Responsive Teaching

As the vision of high-quality mathematics instruction has shifted, teachers are no longer the sole keepers of knowledge but instead partners with children in learning. Enacting this new vision, and more specifically eliciting and building on children's mathematical thinking, presents new challenges for teachers. Teachers must focus on the mathematics and interact with children and their ideas using what they know about how children's understanding develops, all while abstaining from imposing their own thinking

(Ball, 1993). Not only do teachers have to move away from what may be familiar practices based on how they were taught, but they also have to find ways to navigate the complexity of these new practices. We must keep in mind how difficult it can be to embrace this new narrative as teachers "cannot simply shed their old ideas and practices like a shabby coat, and slip on something new" (Cohen, 1990, p. 323). This complexity suggests that teacher educators need to consider the perspectives on the changing narrative taken by PSTs and practicing teachers because only then can we find effective ways to support them in enacting this vision. Eliciting the perspectives of PSTs is particularly important because PSTs often carry feelings of anxiety about teaching mathematics and uncertainty about what teaching moves to use in the moment when interacting with children (Crespo & Nicol, 2003; Dunphy, 2010).

Teacher educators also need learning activities that support the development of expertise in responsive teaching for both PSTs and practicing teachers. Grossman et al., (2009) suggested use of representations, decompositions, and approximations of practice. *Representations of practice* refer to ways to make practices visible. Some examples include—but are not limited to—examining lesson plans, viewing model lessons, analyzing children's written work, or watching video recordings of classroom instruction (Aguirre & Zarala, 2013; Ball & Cohen, 1999; Little, 2004; Van Es, & Sherin, 2008). *Decomposition of practices* refers to breaking practices into smaller and simplified sets of skills that are less overwhelming so they can more easily be learned before being put back together (Ball & Forzani, 2011). Finally, *approximations of practice* refer to ways PSTs and practicing teachers can engage in simplified versions of teaching practices that still seem authentic and connected to teaching. Examples include engaging in rehearsals of teaching, participating in simulations, or working with children during one-on-one problem-solving interviews (Boerst et al., 2011; Crespo & Nicol, 2003; Moyer, & Milewicz, 2002; Webel et al., 2018). The following two sections highlight two of these activities—viewing videos of teaching and engaging in problem-solving interviews because these activities are particularly important to the methodology used and subsequent implications of this study.

Viewing Videos of Teaching. A representation of practice—videos of teaching—can be used to help teachers develop expertise in eliciting and building on children's mathematical thinking. Videos allow teachers to not only see children's written work but also hear children talk about their thinking, even as it unfolds. Videos have other advantages as well. Specifically, videos can be of varying lengths, and researchers have found that selecting shorter video clips can be helpful because the narrowed focus of the video makes it easier for learning (Schack et al., 2013). Further, video can be re-played as needed and discussed as a shared experience, especially when the video comes from teachers' own practice or a familiar practice (Sherin & van Es, 2005). For instance, Philipp et al., (2002) shared videos of teaching with PSTs, and when PSTs saw children in videos that had the same struggles as the children in their practicum experiences, they recognized the complexity of the mathematics and the video acted as a motivator for them to increase their mathematics content knowledge.

Engaging in Problem-Solving Interviews. An approximation of practice—problem-solving interviews—have proven to be beneficial for developing expertise in responsive teaching because they provide opportunities for teachers to engage directly with children's mathematical thinking and practice their questioning in a "low-risk" setting (Crespo & Nicol, 2003; Jenkins, 2010; McDonough et al., 2002; Moyer & Milewicz, 2002). Problem-solving interviews are considered low risk because the complexities of the classroom are removed, and teachers have the opportunity to listen and observe children's mathematical thinking without distraction. However, the setting is similar enough to the classroom that the interactive nature of teaching is preserved (Ginsburg, 1997). Limiting complexities of the classroom is especially critical for PSTs because they often feel overwhelmed with the responsibility of managing an entire classroom (Grossman et al., 2009), and they typically carry anxiety when it comes to learning to teach mathematics. Moreover, problem-solving interviews provide a narrowed focus on children's mathematical thinking in a way that increases awareness of how children solve problems and the kinds of questions that elicit particular responses (Jenkins, 2010; McDonough et al., 2002).

Purpose of Study

To support PSTs and practicing teachers in teaching in ways that are responsive to children's mathematical thinking, I argue that more research is needed in three areas. First, because much of the research on teaching moves has been done with practicing teachers (and comparing PSTs to practicing teachers), research is needed to capture the specific capabilities PSTs have in eliciting and building on children's mathematical thinking. Second, the field lacks information about a baseline for PSTs—how they use teaching moves with children at the start of an elementary preparation program, prior to explicit instruction (Shaughnessy & Boerst, 2018). Third, the voices of PSTs in terms of their rationales underlying the teaching moves they enact, are often not foregrounded in this research. Teacher educators cannot be responsive to PSTs' thinking if this thinking is not elicited. My study was designed to address these three gaps, and the next chapter describes the study which focused on three research questions:

- Research Question 1 [RQ1]: What teaching moves do PSTs make when engaging in problem-solving interviews with children around story problems?
- Research Question 2 [RQ2]: What rationales do PSTs give as to why they make the teaching moves they do?
- Research Question 3 [RQ3]: What is the relationship between PSTs' teaching moves and their rationales for making them?

CHAPTER III

METHODOLOGY

To understand the ways PSTs engage in eliciting and building on children's mathematical thinking, I used a monostrand conversion mixed methods design (Teddlie & Tashakkori, 2006). Specifically, I captured the teaching moves PSTs used prior to explicit instruction in teaching elementary mathematics. I also explored the rationales PSTs shared about the teaching moves they used, and I examined the relationship between the rationales and teaching moves. Qualitative methods elevated the voices and perspectives of PSTs (Creswell & Clark, 2011) whereas quantitative methods supported interpretation of the qualitative findings in terms of frequencies and additional patterns in the data (Tufte, 2006). In this chapter, I describe the participants, data sources, and data analysis for the study.

Participants

I selected PSTs using criterion sampling, selecting participants that met two predetermined criteria of importance to my study (Patton, 2001). First, PSTs needed to be enrolled in the first semester of coursework in an elementary education program. Second, PSTs could not yet have taken a mathematics methods course because this study aimed to gain a general sense of how PSTs naturally engaged with children prior to explicit instruction in teaching mathematics.

Using the selection criteria, I recruited PSTs from a university in the southeastern region of the United States that offered a degree in elementary education and served both traditional and non-traditional students. The elementary education program was a 2-year program in which undergraduates typically began as juniors, after completing a variety of education-related courses during their sophomore year. Two of these courses and the experiences embedded within them, connected with the work PSTs did in this study. The first course, an introduction to education, explored both traditional and contemporary perspectives on teaching and learning at the elementary and secondary levels. This course also required a practicum experience in which PSTs observed classroom instruction for an average of three hours per week. The second course was an elementary mathematics content course that developed subject-specific knowledge in the content areas of numbers and operations, algebra, data analysis, and probability. However, this course did not explicitly address methods for teaching. At the time of data collection, PSTs were enrolled in two courses focused on methods of teaching in content areas other than mathematics, and they engaged in a 10-hour per week internship at a local elementary school.

I recruited PSTs in-person, in a general education class, using a recruiting script on three occasions. In appreciation for their participation, I provided PSTs with a small gift card (see Appendix A for the recruitment script). Twelve PSTs volunteered to participate in the study, but I excluded one PST from the final sample due to incomplete data linked to technology difficulties. The 11 PSTs who participated in the study all self-identified as female. Most were traditional aged undergraduates (18-21 years old) with one PST being slightly older (22-25 years old). Six PSTs self-identified as White, three as Black or African American, one as Hispanic, and one as Asian. Most of the PSTs had prior experiences teaching children in a variety of settings—beyond university coursework—before admission into the elementary education program.

Data Sources

Data sources for this study included: (a) observation of a problem-solving interview, and (b) a stimulated-recall interview. First, I observed a problem-solving interview involving a one-on-one conversation between each PST and a child around a series of mathematical story problems. Second, I conducted a stimulated-recall interview, which engaged each PST to retrospectively elicit their decision-making during their problem-solving interview.

All data collection occurred in one setting and lasted about 1.5 hours—approximately 15 minutes for the problem-solving interview, 45 minutes for the stimulated-recall interview, and 15 minutes for an informal conversation about the PSTs' backgrounds (e.g., their experiences learning mathematics and working with children), which helped to build rapport. Table 3.1 connects the data sources to the research questions, and the following sections describe each data source in more depth.

Table 3.1

Connections Between the Data Sources and Research Questions

Research Questions	Observation of a Problem-Solving Interview	Stimulated-Recall Interview
1. What teaching moves do PSTs make when engaging in problem-solving interviews with children around story problems?	X	
2. What rationales do PSTs give as to why they make the teaching moves they do?		X
3. What is the relationship between PSTs' teaching moves and their rationales for making them?	X	X

Observation of the Problem-Solving Interview

The problem-solving interview was a one-on-one conversation between each PST and a second grader. My only participation in these interviews was as an observer (and videographer). The purpose of my observation of the problem-solving interview was to capture teaching moves (e.g., questions, gestures, comments, etc.) PSTs used when engaging with children in solving mathematical story problems. This interview lasted approximately 15 minutes and was audio and video recorded, with the video focused on the child and their work. Video not only captured the complex nature of teaching moves so that they could be viewed multiple times, but also because the video served as the foundation for the upcoming stimulated-recall interview. The following sections describe the school context in which the problem-solving interviews occurred, the procedure for the interview, and the story problems used (See Appendix B).

School Context for the Problem-Solving Interview

The interviews occurred in three elementary schools that were in the same school district in which the PSTs attended their weekly internship experiences. Therefore, PSTs were familiar with the school contexts, but the children recruited for this study were not in their classroom internship placements, with one exception. The district served a midsize city and had a student enrollment with approximately 45% White, 30% Black or African American, 20% Hispanic, and 5% Other. About half of the children in the district were eligible for free or reduced cost lunch and about 15% were classified as English Language Learners. The demographics of the three elementary schools' student populations were reflective of those of the district. I recruited children from one second grade classroom at each of the three schools, and the classrooms selected were based on principal recommendations. I chose second grade because the study's focus on place value and problem solving closely aligned with the curriculum for second grade. Children were selected based on those with consent and availability on the days of the interviews. Note that although I recruited children for problem-solving interviews, they were not the focus of this study.

Procedures for the Problem-Solving Interview

Before each problem-solving interview began, I told the PST that my intention was to learn what follow-up questions PSTs like themselves ask children. I explained that their goal was to understand the child's mathematical thinking, and I was not expecting particular questions. Instead, they should ask questions to understand the child's mathematical thinking in ways that were helpful for them. The term "questions" was used in place of the term "teaching moves*"* because it was a more familiar term for PSTs, even prior to the start of an elementary education program. I provided PSTs with the same set of seven story problems to review just before children arrived. The abbreviated time to review the problem set was intentional, as the focus of the study was on how PSTs naturally worked with children and their mathematical thinking, without having extensive time to plan.

After the child arrived for the problem-solving interview, I told the child that they could solve the problems any way they wished because the purpose of the interview was to teach PSTs how *children* solve problems. The child was then shown the materials available for use, which included unifix cubes, base-ten blocks, hundreds charts, and blank paper. Moreover, I arranged materials in a way that did not privilege one tool over the other so that children felt comfortable choosing a method that made sense for them. The PSTs then posed 3–7 story problems from the provided list, and any written work produced during the problem solving was collected (see Appendix B for the protocol for the directions provided to PSTs and children).

Story Problems for the Problem-Solving Interview

Guided by five principles drawn from a pilot study and the work of Cognitively Guided Instruction (Carpenter et al., 2015; Carpenter et al., 1989), I designed a set of seven story problems to be used in the problem-solving interviews. First, to increase children's access to the mathematics, I designed contexts for the story problems to be meaningful and make sense to children at that age. Second, story problems were designed to include place value concepts because place value plays an important role within the elementary school curriculum, and PSTs tend to feel more comfortable with place-value concepts than many other mathematical topics. Third, story problem structures were varied to include a range of problem difficulty so that PSTs would have an opportunity to respond to a variety of situations. Fourth, story problem numbers were strategically chosen to (a) be accessible to children in second grade, (b) make sense in the problem contexts, (c) allow children to directly model the problem situation by ones without needing excessive time (i.e., all numbers were less than 40), and (d) engage children's knowledge of tens. Fifth, I arranged story problems in a required group of three problems and an optional group of four problems. Specifically, I asked PSTs to pose the first three story problems, which reflected a range of difficulty. They could then choose from any of the remaining four problems, as time permitted. This grouping made sure that all PSTs posed story problems with a range of difficulty (even if they only posed a few story problems), but also gave PSTs choice to encourage more ownership over the conversation (see Table 3.2 for the set of seven story problems).

Table 3.2

Story Problems for the Problem-Solving Interview

Problem Name	Story Problem
Blueberries problem	Jackson had 20 blueberries. He ate 8 of them. How many blueberries does Jackson have left?
Books problem	Ebony had 18 books. Her dad gave her some more books for her birthday. Then she had 25 books. How many books did Ebony's dad give her for her birthday?
Toys problem	Marcos had 4 boxes of toys. There were 12 toys in each box. How many toys did Marcos have altogether?
Legos problem	Luke had 15 lego pieces. Sarah gave Luke 5 more lego pieces. How many lego pieces does Luke have now?
Candy problem	Gabriel had 20 pieces of candy to give to his friends. He gave 2 pieces of candy to each friend. How many friends were given candy?
Buttons problem	Deja had 33 buttons. She put the buttons into 3 bags with the same number of buttons in each bag. How many buttons did she put in each bag?
Balloons problem	Logan had 25 balloons. Sofia had 19 balloons. How many more balloons did Logan have than Sofia?

Note. The first three story problems were required and the last four story problems were optional, as time permitted.

The three required story problems were purposefully selected. The blueberries problem had a simple problem structure (20 – 8 = □) and gave children the chance to feel comfortable. The books problem had a problem structure (18 + □ = 25) which children typically approach with addition, although many textbooks encourage the use of subtraction. Therefore, this problem was likely to provide PSTs with a range of strategies to address. The toys problem offered an opportunity for PSTs to observe children in second grade engage with a multiplication problem—a problem structure that PSTs typically think is out of reach for young children. The last four story problems (legos, candy, buttons, and balloons problems) were posed when time permitted, and they were designed to ensure that a range of mathematical structures were included in the problem-solving interview as a whole. In sum, I intentionally designed the set of seven story problems with a common mathematical topic and strategic selection of contexts, problem structures, and number choices.

Stimulated-Recall Interview

The stimulated-recall interview was a one-on-one discussion I had with each PST to retrospectively elicit the PSTs' rationales for their teaching moves during the problem-solving interview. Specifically, immediately following the problem-solving interview, each PST watched the video-recording of that interview—one story problem at a time—so that the video could provide visual cues to support the PST in recalling their rationales (Bloom, 1953). I asked the PSTs to stop the video-recording at any time to share the rationales underlying their teaching moves. After discussing the PST-selected teaching moves, I returned to teaching moves not yet discussed and asked about their rationales. (In four cases, due to time constraints, I primarily selected what teaching moves were discussed throughout the interview.) The stimulated-recall interview lasted approximately 45 minutes and was audio and video recorded.

I took measures to improve the accuracy of PSTs' recall of their rationales for their teaching moves in four main ways. First, the stimulated-recall interview occurred immediately following the problem-solving interview because minimizing time between the recorded interaction and the time of playback increases validity of this data collection method (Gass & Mackey, 2000). Second, throughout the interview, I emphasized my interest in understanding why PSTs said and did the things they did when working with children. Narrowing the scope for PSTs was important to provide them with a focus for recall as opposed to an invitation to reflect about their experiences and feelings in general (Lyle, 2003). Third, after the initial prompt and responses of PSTs, I regularly posed questions to clarify PSTs' responses in relation to their decision making for the particular moves discussed. These follow-up questions were strategic because questions such as "You said ____, can you say a little more about that?" or, "I understand you did ____, but can you tell me why you asked that question?" re-directed PSTs' attention back to their decision-making when they began to instead share what they should have done or how they felt (O'Brien, 1993). Fourth, I asked PSTs to discuss each story problem—one at a time—during playback. The smaller segments made it more likely that the rationales for each story problem would be fully explored.

Not only did I take measures to improve the accuracy of PSTs' recall, but I also took three measures to lessen the anxiety they may have felt as they engaged in the stimulated-recall interview (Calderhead, 1981). First, I paid special attention when I video-recorded the problem-solving interviews to make sure PSTs were off-screen as much as possible. In this way, the video shared during the stimulated-recall interview focused on children so that PSTs would focus less on how they appeared on video and more on recalling their rationales. Second, to begin the stimulated-recall interview, PSTs were broadly asked what stood out to them about their problem-solving interview, which provided an opportunity for PSTs to become comfortable sharing their thoughts and reiterated that the purpose of the interview was to learn from them. Third, PSTs generally had some control over times the video stopped, and we engaged in discussion, which again reinforced the value placed on PSTs' ideas (See Appendix C for the protocol of the stimulated-recall interview).

Data Analysis

I analyzed data in three phases, which are summarized here and described more fully in the following sections. In *Phase 1*, I explored problem-solving interviews and stimulated-recall interviews separately and iteratively through qualitative analyses. The goal was to develop and apply coding schemes to capture the teaching moves PSTs made and their rationales for making those teaching moves. In *Phase 2* teaching moves and rationales coded from Phase 1 were quantitized and patterns explored. *Quantitizing* refers to the process of assigning numerical values to qualitative data for further analyses (Sandelowski et al., 2009). The goal of Phase 2 was to separately explore the quantitative findings (for teaching moves and rationales) using descriptive statistics. In *Phase 3*, I connected qualitative and quantitative findings for teaching moves and rationales. The goal of Phase 3 was not only to explore the relationship between teaching moves and rationales but also to connect qualitative and quantitative findings to draw inferences across the three research questions.

Phase 1: Qualitative Analysis of the Problem-Solving and Stimulated-Recall Interviews

In the first phase, I began to qualitatively analyze both the problem-solving and stimulated-recall interviews to gain a general sense of the PSTs' teaching moves and rationales. It was important to explore the landscape of teaching moves and rationales to identify initial categories for preliminary coding and to determine units of analysis. To do so, I watched each problem-solving interview and stimulated-recall interview and wrote unstructured memos to capture broad ideas I noticed about teaching moves and rationales. Next, I transcribed the problem-solving and stimulated-recall interviews and matched the sections of the conversations from the stimulated-recall interviews to the corresponding teaching moves in the problem-solving interviews. I combined the transcripts from the two interviews into a single document in which the problem-solving interview transcript was in the left column and the corresponding sections of the stimulated-recall interview transcript were in the right column. Although I analyzed the teaching moves and rationales separately in Phase 1, this matching was necessary for analyzing the rationales because the teaching moves provided the context for understanding the rationales.

Using these combined transcripts and the videos, I developed coding schemes for the teaching moves and rationales. I coded not only from the transcripts but also from the videos because videos provided intonations and facial expressions that helped to convey meaning that the transcripts did not show. Overall, I built on my unstructured memos for teaching moves and rationales by using a constant comparative analysis involving multiple iterations until categories were clear and coding schemes solidified (Glaser & Strauss, 1967). For the problem-solving interviews, I used provisional coding, beginning with a list of categories generated from the literature, and then added and revised categories as new ideas arose in the data (Miles & Huberman, 1994). For the stimulated-recall interviews, I used *in vivo* coding because there was not an established framework of rationales in the literature from which to build (Saldaña, 2016). *In vivo* coding allowed me to start with the spoken language of PSTs and cluster their words and phrases based on similarity of meaning. Coding inductively in this way allowed their words to become initial categories for the coding scheme.

During this phase of analysis, I also determined the best unit of analysis for coding each interview. For the problem-solving interviews, the unit of analysis was individual talk turns. Talk turns were based on the PSTs' and children's verbal and non-verbal ways of communicating (Myers, 2000). Non-verbal exchanges were taken into account because often PSTs or children would move manipulatives, point, or nod as a way of communicating with each other, and these non-verbal actions sometimes indicated the beginning or ending of a PST's or child's turn. Note that even though I coded individual talk turns within the problem-solving interview, I used surrounding talk turns to provide context and help determine the code. For the stimulated-recall interviews, the unit of analysis for coding was an idea unit (Jacobs & Morita, 2002). Specifically, I looked at the section of transcript of the stimulated-recall interview that was linked with a single teaching move (from the problem-solving interview), and I identified a coherent idea (or idea unit)—each idea unit was considered a rationale. Most teaching moves linked with one idea unit —and thus one rationale—but some teaching moves linked to two or three idea units—and thus two or three rationales.

For reliability, a second individual coded approximately 20% of the data, which included 93% of the teaching moves and 70% of the rationales. Reliability was above 80% for both teaching moves and rationales, and discrepancies resolved through discussion. After reaching reliability, I conducted one additional iteration of coding to address any codes refined during the reliability discussions.

Finally, I reviewed the coded data from both the problem-solving and stimulated recall interviews to begin to identify major themes for the study. Specifically, I created a list of observations about what was prevalent, interesting, or unexpected.

Phase 2: Quantitative Analysis of the Problem-Solving and Stimulated-Recall Interviews

In the second phase, I quantitized the teaching moves and rationales coded from the qualitative analyses in Phase 1. My goal was to explore patterns that may not have been visible in the qualitative analysis. I transformed the data using counting (Sandelowski et al., 2009) and then analyzed the quantitized data using summary tables and descriptive statistics.

In the upcoming findings chapters, I will introduce categories of teaching moves and categories of rationales as well as multiple forms within each category. For simplicity, I describe my quantitizing process for categories of teaching moves only because the same process was used for all categories (and forms) of teaching moves and all categories (and forms) of rationales. Specifically, for each category of teaching moves, I tracked three things. First, I counted how many of the 11 PSTs enacted each category of teaching moves at least once during their problem-solving interview (Collinridge, 2013). Second, I calculated the mean number of instances of each category of teaching moves across the 11 PSTs. Third, I calculated the mean percentage of the total number of teaching moves for each category of teaching moves. However, because each PST posed a different number of story problems and used a different amount of teaching moves, I first calculated the percentage for each PST and then took the mean of those percentages. In that way, I could ensure the equal representation of each PST's data.

Note that the PSTs made a total of 532 teaching moves and offered 315 rationales across the 11 problem-solving interviews. However, I made a decision to exclude some of the teaching moves from further analysis. About a fourth of the teaching moves (143) were less substantive and were coded as *other*, such as transitional words (e.g., "Okay" or "hmmm"), phrases and conversational fillers between talk turns (e.g., "Alright, so keep going" or "Show me what you are going to do"), praise after correct answers (e.g., "Good job" or "Awesome"), and simple repetition of phrases or answers shared by children. I decided to exclude those teaching moves from all further analyses to better understand the more substantive teaching moves made by PSTs. Therefore, this dissertation focuses exclusively on the 389 teaching moves that were not coded as *other*.

To analyze the quantitized data from both the problem-solving and stimulated-recall interviews, I created summary tables to display the number of PSTs, the mean number of instances, and the mean percentages for each teaching-move category (and form) and each rationale category (and form). I reviewed the summary tables and made a list of observations from the quantitized data that I compared to my list of observations from the qualitative data that I made in Phase 1. The comparisons allowed me to confirm, adjust, or elaborate emerging themes for the study.

Phase 3: Exploration of the Relationship Between Teaching Moves and Rationales

In Phases 1 and 2, my focus was on separately analyzing the 389 teaching moves from the problem-solving interviews to address the first research question and the 315 rationales from the stimulated-recall interviews to address the second research question. In Phase 3, I conducted an exploratory analysis to build on those earlier analyses and examine initial relationships between categories of teaching moves and categories of rationales to address the third research question.

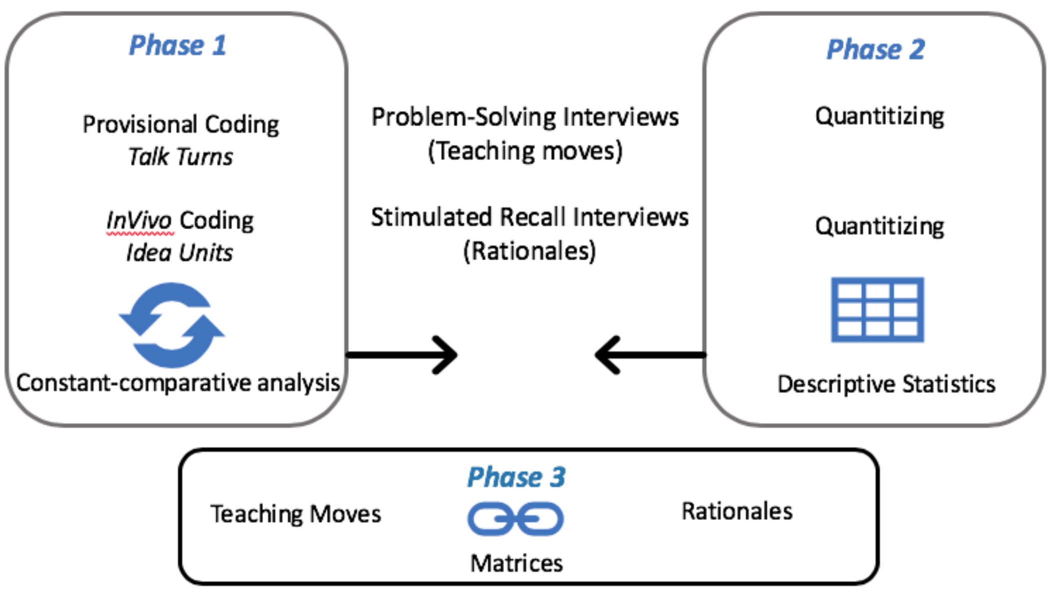
For this analysis, I used a reduced number of teaching moves because only 258 of the 389 teaching moves—about two-thirds—were discussed in the stimulated-recall interviews. Therefore, this analysis focused on 258 teaching moves and their corresponding 315 rationales. Note that the number of rationales is greater than the number of teaching moves because PSTs often shared more than one rationale for a single teaching move. For each category of teaching moves, I created matrices (Miles & Huberman, 2014) to examine the categories of rationales shared and their relative frequencies, including the number of PSTs who used each of those rationale categories. Additional details are provided in the third findings chapter, but an important distinction for this analysis is that I focused on the teaching moves and corresponding rationales as a single data set, independent of which PST generated these teaching moves and rationales.

This decision was necessary given the small number of instances of the multiple pairings of teaching-move categories and rationale categories for each PST. Further, this small sample size means that the findings presented in the third chapter are exploratory, with the goal of having them serve as a foundation on which future research can build.

Similar to previous phases, I reviewed both the qualitative data and the summary tables of quantitized data and made lists of observations that I compared to my lists of observations from Phases 1 and 2. In this way, my emerging themes for the study continued to evolve as I interpreted data across the three research questions to make inferences. See Figure 3.1 for a summary of the phases of data analysis.

Figure 3.1

Phases of Data Analysis



CHAPTER IV

FINDINGS ABOUT PSTS' TEACHING MOVES

This chapter is the first of three findings chapters, each addressing one of the research questions. The findings presented in this chapter address the first research question: *What teaching moves do PSTs make when engaging in problem-solving interviews with children around story problems?* I answered this research question by examining the teaching moves made by PSTs across 11 problem-solving interviews. Although I initially drew the categories from the literature, the teaching moves reported here have captured the nuances in the ways PSTs enacted teaching moves with children in an interview setting. Note that the focus of this study was to identify the PSTs' teaching moves, not the quality of enactment.

Overall, PSTs utilized a range of teaching moves, and in the following sections, I will describe and illustrate major categories of teaching moves as well as provide their frequencies. Following this overview of the PSTs' collection of teaching moves, I highlight three unexpected, but promising skills PSTs demonstrated, and I explore each in more depth. My goal in capturing the PSTs' current practices is to have them serve as a foundation that teacher educators can use to be responsive to the thinking of PSTs.

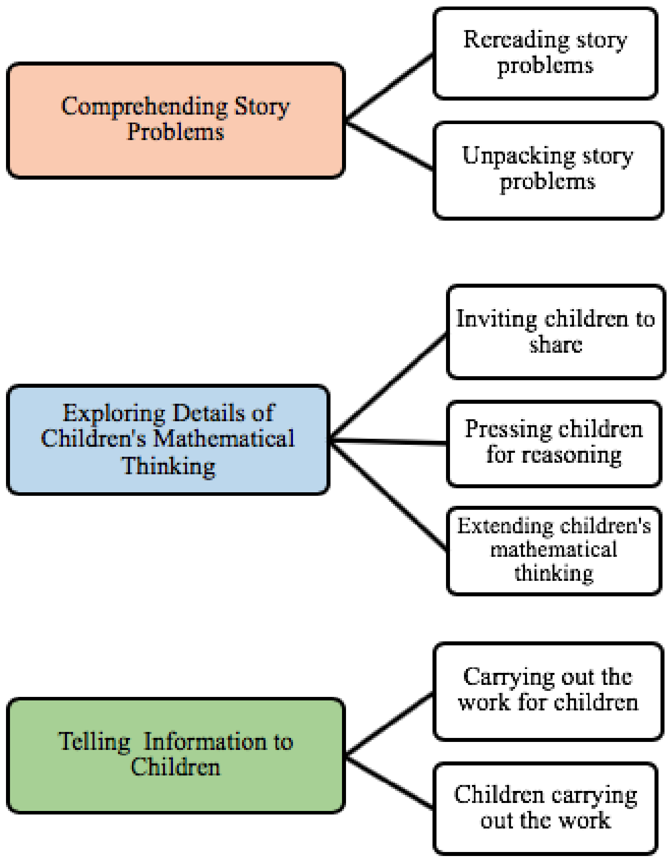
Range of Teaching Moves

Across the problem-solving interviews, the PSTs enacted a total of 389 teaching moves, and the number of teaching moves in each problem-solving interview ranged from 11–53, with a mean of 35.4 teaching moves. The teaching moves were coded into three categories: (a) comprehending story problems, (b) exploring details of children's mathematical thinking, and (c) telling information to children. *Comprehending story problems* describes teaching moves in which PSTs aimed to ensure children understood the context of the story problem and recalled pertinent information from the problem accurately. *Exploring details of children's mathematical thinking* describes teaching moves in which PSTs attended to the mathematical details of what children said or did along with teaching moves that created space for children to share those details. *Telling information to children* describes teaching moves in which PSTs took on the mathematical work for children. See Figure 4.1 for an overview of the categories of teaching moves (and their multiple forms).

Figure 4.1

Overview of Categories of Teaching Moves Enacted by PSTs

Category Form



The following sections describe and illustrate the three categories of teaching moves, drawing examples from across the 11 interviews (See Table 4.1 for a summary of their frequencies).

Table 4.1

Frequency of Categories of Teaching Moves Enacted by PSTs

Categories (and Forms) of Teaching Moves	Mean number of teaching moves	Mean percentage of the total number of teaching moves	Number of PSTs enacting teaching moves
Comprehending story problems	**9.5**	**29%**	**11**
Rereading story problems	6.4	20%	11
Unpacking story problems	3.1	9%	8
Exploring details of children's mathematical thinking	**10.4**	**33%**	**11**
Inviting children to share	2.8	9%	9
Pressing children for reasoning	7.4	23%	11
Extending children's mathematical thinking	0.2	1%	1
Telling information to children	**15.5[a]**	**39%**	**10**
Carrying out the work for children	4.1	10%	9
Children carrying out the work	11.5	29%	10

[a] The mean number of teaching moves for the forms of telling information to children do not sum to 15.5 because of rounding error.

PSTs Enactment of Teaching Moves for: Comprehending Story Problems

The first category of teaching moves—comprehending story problems—on average comprised more than one fourth of the total number of teaching moves for each PST (29%). Enacted by all 11 PSTs, this category of teaching moves focused on making sure children understood story contexts and recalled correct quantities of the story. The teaching moves in this category took two forms: (a) rereading story problems and (b) unpacking story problems. *Rereading story problems* describes teaching moves in which PSTs read aloud (repeated) part or all of the story problems. *Unpacking story problems* describes teaching moves in which PSTs corrected quantities children had mistakenly recalled or took time to explain story problem contexts. The PSTs reread story problems about twice as often as they unpacked story problems. The next sections will further describe and provide examples of the two forms of comprehending story problems.

Rereading Story Problems

In the first form, rereading story problems, PSTs used teaching moves to help children make sense of story problem contexts after initially posing the story problems to children. At times, PSTs repeated the problems because children requested specific numbers or other information about the contexts. Other times, PSTs initiated repeating the problems because children appeared to be confused. For instance, PST 2 posed the blueberry problem (eating 8 of 20 blueberries), and the child selected two ten rods, held them, and appeared to think for a few moments. The PST, perhaps assuming this child was confused, interjected and asked, “Do you need me to repeat it?” After the child agreed, the PST reread the problem in its entirety, as originally written.

Unpacking Story Problems

In the second form, unpacking story problems, PSTs used teaching moves that clarified the problem quantities, expanded details of the story context, or rephrased the question. For example, PST 11 expanded the story context by connecting it to the child’s life for the books problem (getting more books to go from 18 to 25 books). She posed the problem, reread it twice (at the request of the child), and then used wait time while the child started solving. The child added 25 unifix cubes to 18 unifix cubes and produced an incorrect answer of 43. The PST confirmed with the child how many books they needed at the end (25 books) and then unpacked the story:

> And then her dad, on her birthday, decided he was going to give her some more books. So, he gave her enough books that she didn't have 18 anymore, she had 25. So, if she started at 18, how many books would he have had to give her to get her to 25 books?

In this example, the PST went beyond the initial wording of the story problem to highlight that the initial 18 books were part of the 25 books rather than a separate set of books. Note that she elaborated on the story problem context *without* telling the child how to solve the problem.

PSTs Enactment of Teaching Moves for: Exploring Details of Children's Mathematical Thinking

The second category of teaching moves—exploring details of children's mathematical thinking—on average, comprised about one third of the total number of teaching moves for each PST. Enacted by all 11 PSTs, this category of teaching moves, focused on asking about specific details in children's strategies or providing space for mathematical details to surface to better understand children's ideas. The teaching moves in this category took three forms: (a) inviting children to share, (b) pressing children for reasoning, and (c) extending children's mathematical thinking. *Inviting children to share* describes teaching moves in which PSTs posed a general question to provide space for children to verbalize the details of their strategies. *Pressing children for reasoning* describes teaching moves wherein PSTs asked children to explain further about something they had already shared. *Extending children's mathematical thinking* describes teaching moves in which PSTs asked children to think about mathematical ideas that were related, but beyond their initial strategies. Pressing for reasoning was the most frequently used form, with these teaching moves accounting for about one fourth of the total number of teaching moves. The following sections will further describe and share examples of the three forms of exploring details of children's mathematical thinking.

Inviting Children to Share

In the first form, inviting children to share, PSTs used teaching moves that gave children general, open-ended opportunities to share their thinking. Sometimes PSTs asked children to think aloud prior to problem solving, by posing questions such as "How do you think we should do that?" Other times, PSTs asked children to share their thinking after problem solving. For example, PST 8 posed the buttons problem (putting 33 buttons in 3 bags), and the child arranged unifix cubes into 3 groups of ten and 3 ones. Without moving or visibly counting the manipulatives, the child correctly answered "11." The PST, sounding surprised, said, "Yeah, yeah! So how did you do that?" In this example, the PST asked the child about their thinking after they had finished solving and had provided an answer. The PST did not assume that they knew the child's strategy but instead afforded the child space to share the details of their mathematical thinking. In this way, the inviting-children-to-share form of exploring details of children's mathematical thinking is different than the subsequent forms in which the teaching moves explicitly connect to specific strategy details.

Pressing Children for Reasoning

In the second form, pressing for reasoning, PSTs used teaching moves that asked children to elaborate on or provide information about specific strategy details PSTs heard or observed. In the following example, PST 11 demonstrated two teaching moves that pressed for reasoning. While reading the candy problem aloud (giving away 20 pieces of candy in groups of 2), the child chose unifix cubes to show two groups of ten candies. The PST and the child collaboratively unpacked the problem and then the child decided to break the unifix cubes into groups of two, organized in two columns with 5 groups of two in each column. After the PST and child verified the total of 20 pieces of candy in the child's arrangement of twos, the PST probed further:[1]

Pressing children for reasoning	PST: Okay, so he got 20 pieces altogether. And then you said he broke them up into twos, right? So why did you do that?
	Child: *(while explaining, the child moved the groups of two out of the two columns so that they had more space between them and were spread across the table).* The reason I broke up the twos is because he gave two pieces of candy to his friends.
Pressing children for reasoning	PST: Right, so now you've got two pieces of candy and they're in separate blocks, so what do you think these blocks represent? *(PST points to one of the groups of 2).*

In the above example, the PST used language such as, "why did you" and "what do you think" in conversing with the child to understand why the child used groups of two, a

[1] Transcripts are structured to show the episode on the right and the category (or form) on the left.

mathematically important detail. Further, the PST aimed to reinforce the connection to the story problem as they focused on one group of two candies.

Extending Children's Mathematical Thinking

Only one PST used the final form, extending children's mathematical thinking, to inquire about the child's mathematical thinking beyond, but related to, the initial strategy. Although only 1 out of 11 PSTs utilized the move, I chose to include this form because research has shown it is mathematically important (see e.g., Jacobs & Empson, 2016), and this PST's data provide an existence proof that PSTs can use this teaching move. Specifically, PST 8 posed the toys problem (4 boxes with 12 toys in each box). The child initially pulled out 4 stacks of 10 unifix cubes that were already connected. The child then adjusted those stacks by adding on cubes to each stack so that there were 4 stacks with 12 unifix cubes each. Next, the child counted the total by saying "12" and then, starting with the second stack, counted out loud until reaching 48. The PST began the interaction by asking about the unifix cubes:

Pressing children for reasoning	PST: Okay. So, do you think that adding onto the [10-stack of unifix cubes] makes it easier? Child*: (Child shakes head yes)*
Pressing children for reasoning	PST: Why do you think that was easier? Child: Because usually if you add all of them together and you take them apart—and these help you more better than, to get the answer more faster.

Extending children's mathematical thinking	PST: More faster? Do you think there was another way to get the problem? Just give me a guess, anything like, what is a way to get the problem? Child: By paper?
Extending children's mathematical thinking	PST: Yeah but what kind of mathematics could you use? Could you use subtraction? Addition? Multiplication? Division?

To start, the PST explored the mathematical details of the child's strategy in the form of pressing for reasoning. Next, the PST asked the child to solve in another way, an example of extending the child's thinking beyond the initial strategy. The PST phrased the teaching move in a narrow way which prompted a questioning response from the child ("By paper?"), which was likely not what the PST had intended. The PST attempted to extend the thinking of the child again, aiming for the child to think beyond counting by ones and to consider alternate operations to arrive at the same solution. Although the PST could have improved on the clarity of her phrasing, she first explored the mathematical details of the child's strategy in the form of pressing children for reasoning, and then broadened the conversation beyond the child's initial strategy, which is an example of extending children's mathematical thinking.

PSTs Enactment of Teaching Moves for: Telling Information to Children

The third category of teaching moves—telling information to children—on average comprised more than one third of the total number of teaching moves for each PST (39%). In this category—enacted by all but 1 PST—PSTs foregrounded their own ideas and took on the mathematical work for children. In many cases, PSTs told children what to do not only through their words but also through their facial expressions, tone of voice, or movement of manipulatives. Similar language and actions applied to two forms of the teaching moves within this category with the primary difference being who completed the actions after PSTs told information to children. In other words, these teaching moves involved PSTs (a) carrying out the work for children or (b) children carrying out the work. *Carrying out the work for children* describes a form of teaching moves where the PST took on the mathematical work for the child but also completed the actions (e.g., the PST moved the manipulatives). In contrast, *children carrying out the work* describes a form of teaching moves where the PST took on the mathematical work for the child, but the child completed the actions (e.g., the child counted the blocks). When telling information to children, PSTs asked children to carry out the work almost three times as often as they carried out the work themselves. The following sections will further describe and share examples of the two forms of telling information to children.

Carrying Out the Work for Children

In the first form, carrying out the work for children, PSTs used teaching moves where they not only took on the mathematical work for children but also completed the work. For example, PST 10 posed the books problem (getting more books to go from 18 to 25 books). The child counted 18 individual unifix cubes and then added cubes until reaching 25 cubes, without keeping the added cubes separate from the initial 18 cubes. Without addressing the strategy the child had used, the PST asked, “So how many books did her dad give her if she had 18 to begin with?” The PST then became more specific by motioning first toward an imaginary set of 18 books and then to the child’s set of 25 books, stating "So, she had 18 and now she has 25." At this point, the child (incorrectly) answered 5 books because 5 of the 7 cubes that had been added happened to be black whereas all the rest of the cubes were white. The PST disregarded this answer and suggested, "Let's think about that a different way. Okay. So, let's count 18 one more time." The child ignored the 5 black cubes and then miscounted the pile of 20 white cubes as 18. The PST continued her telling: "There's a few you didn't count." She then carried out some of the work by assisting with the re-counting of the pile of white cubes to ensure the starting quantity of 18. Specifically, the PST moved each cube off to the side—one by one—as the child re-counted. This time 18 cubes were successfully counted and the problem solving continued. This example illustrates one of numerous instances where PSTs ignored strategies or answers provided by children and not only continued conversations in ways PSTs were thinking about the problem but also executing the actions themselves.

Children Carrying Out the Work

In the second form, children carrying out the work, PSTs used teaching moves where they took on the mathematical work for children but allowed children to complete the next steps. For example, PST 6 posed the buttons problem (putting 33 buttons in 3 bags). The child counted out 33 unifix cubes and asked the PST to confirm the number of bags, which she did by rephrasing the story:

Children carrying out the work	PST: She put the buttons into three bags with each bag having the same number of buttons. So how could you do that? Maybe would drawing something help? Maybe drawing the bags would help?

	Child: Maybe
Children carrying out the work	PST: Maybe. Draw the big bags.
	Child: (*Child draws three circles on a piece of paper)*
Children carrying out the work	PST: So now she has 3 big bags and you have the 33 buttons. How could you put each one of those into the bag (*pointing to the 33 unifix cubes the child had counted*) so that each bag has the same number of buttons? *(pointing to the inside of each of the three drawn circles)*

In this case, the PST instructed the child what to do step by step, but she offered the child the chance to do the work in between her directions. This distinction between children carrying out the work versus PSTs carrying out the work for children provided additional nuance to the nature of telling information to children.

Promising Skills in PSTs' Problem-Solving Interviews

In addition to having a range of teaching moves, PSTs also demonstrated some evidence of three unexpected, yet promising skills throughout their problem-solving interviews. In the following sections, I highlight these skills and then explore each in more depth: (a) prioritizing story contexts during problem solving, (b) noticing and asking about specific strategy details, and (c) using wait time with children. *Wait time* describes a holistic assessment of the extent of moments of silence during the interview that provided children space to think for themselves.

Prioritizing Story Contexts During Problem Solving

I expected PSTs to focus on children taking quantities out of context and performing operations, but instead all 11 PSTs showed some evidence of prioritizing story contexts during problem solving (i.e., on average, 29% of the total number of teaching moves focused on comprehending story problems). However, PSTs' efforts to help children comprehend story problems mainly consisted of rereading part or all of the problem, and upon closer look, this rereading may not always have been necessary. Although rereading story problems reminds children of story contexts, doing so unprompted interrupts children's thought processes. In short, PSTs seemed to have a sense that keeping contexts and quantities together was important, which research has shown to be effective. Teacher educators can build on this inclination by helping PSTs develop other ways to support children in using contexts when solving story problems.

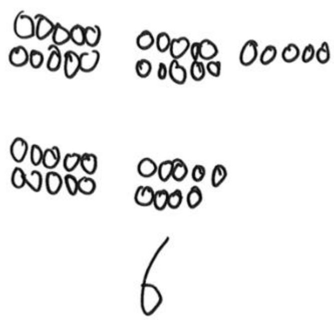
Noticing and Asking About Specific Strategy Details

I expected PSTs to ask few questions that pressed children for reasoning about specific details in their strategies. Instead, I found all 11 PSTs pressed children for reasoning, and they primarily did so in two specific ways: (a) pressing for explanations and (b) pressing for factual information. *Pressing for explanations* describes how PSTs asked children to share their reasoning in open-ended ways. *Pressing for factual information* describes how PSTs asked children to share their reasoning in narrowed ways, typically prompting simplistic responses such as a "yes", "no", or the value of a number.

To illustrate pressing for explanations, PST 1 posed the balloons problem (comparing 25 balloons to 19 balloons) and the child drew 25 circles in groups of fives (and tens). Next, the child drew 19 circles arranged in groups of fives (and tens) and offered 6 as the solution. (See Figure 4.2 for the child's written work.)

Figure 4.2

Child's Written Work for Balloons Problem (Comparing 25 Balloons to 19 Balloons)



The PST invited the child to share their strategy to which they responded, "Because there was 25 for Logan and so I counted to see how many more he had, and he had 6 more than [Sofia]." The PST then pressed for an explanation, "So, can you tell me why you're doing your circles in groups of five?" In this example, the PST used language such as "can you tell me why," as was typical of other PSTs who enacted pressing for explanations because this language left space for children to share their reasoning.

In a different example, PST 8 posed the books problem (getting more books to go from 18 to 25 books) and used teaching moves that demonstrated both pressing for explanations and pressing for factual information. To begin, the child counted 18 books by making two stacks of connected unifix cubes that were side by side—one stack of 10 cubes and one stack of 8 cubes. The PST proceeded to press for an explanation of a strategy detail, "So why did you put them together like that, side by side?" In this instance, the PST used language such as "why did you," which was an open-ended phrasing that allowed the child to provide their reasoning about a detail in their strategy.

After the child continued solving and produced the (incorrect) answer of 8 books, the PST invited the child to share their strategy but this time, she pressed for factual information:

	Child: Now, when I said I thought 18. I was like, she said, her, her dad gave her presents some more and she had 25 left. So then I do that [start at 18 and count to 25] and I got 8.
Pressing children for reasoning	PST: So you added 8 to the 18 up [to the 25]?
	Child: (*Shakes their head yes*).

The PST noticed the details in both what the child demonstrated and what the child verbalized, and she asked more about adding 8 cubes to the 18 cubes. However, the PST phrased her question in a way that prompted a simplistic response, as the child nodded in agreement. In short, pressing for factual information gathered isolated details about strategies and pressing for explanations gave additional space for children to share their mathematical thinking. It also should be noted that PSTs used pressing for factual information twice as often as pressing for explanations. Thus, PSTs demonstrated some ability to notice and ask about specific details in children's strategies, but they need additional support in framing their questions in ways that elicit explanations rather than factual information.

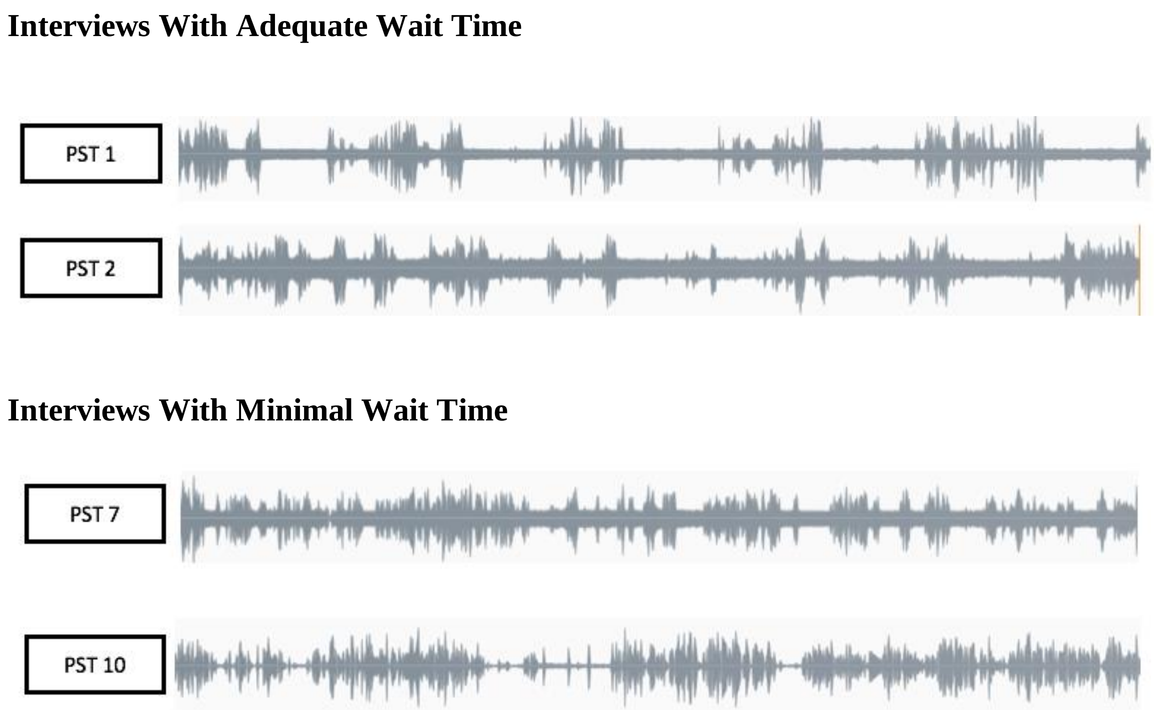
Using Wait Time with Children

I expected PSTs to use minimal wait time but instead they showed some evidence of using adequate wait time with children. I holistically assessed wait time throughout each interview rather than measuring exact wait-time amounts. Specifically, I holistically considered the ways PSTs created space for children to think for themselves without interruption, and two distinct groups emerged from the data: (a) adequate wait time and (b) minimal wait time. *Adequate wait time* describes problem-solving interviews in which children were able to solve problems and arrive at solutions, without interruption some or most of the time. *Minimal wait time* describes problem-solving interviews in which PSTs spoke for the majority of time and children were given little space to share their ideas. Seven PSTs demonstrated use of adequate wait time whereas 4 PSTs demonstrated use of minimal wait time.

I also used soundwaves of problem-solving interviews as a whole—as a visual to help me understand what wait time may have looked like within groups. I did not analyze wait time by the soundwaves because I had not planned on using this technology in my analysis, and many of the interview recordings presented technical challenges (e.g., background noise due to poor microphone placement). However, I found that the visual patterns depicted an overall sense of the differences between interviews with adequate versus minimal wait time. Therefore, I share visuals of the soundwaves from four of the interviews—two with adequate wait time and two with minimal wait time (see Figure 4.3). The heightened areas of the soundwaves generally showed PSTs speaking. The gaps between the heightened areas generally showed children either speaking, moving manipulatives, or thinking to themselves in silence. Note how the PSTs who used adequate wait time within their problem-solving interviews provided more space for children to think and verbalize their ideas whereas those PSTs who used minimal wait time provided less space.

Figure 4.3

Soundwaves Illustrating Wait-Time Groups in Interviews



I also expected to find a pattern in which wait time supported teaching moves in the category of exploring details of children's mathematical thinking and minimized teaching moves in the category of telling information to children. Therefore, I descriptively looked at the connections between wait-time groups and these two categories of teaching moves. As expected, I found that interviews categorized as having adequate wait time had a higher mean percentage of the total number of teaching moves focused on exploring details of children's mathematical thinking and a lower percentage of teaching moves focused on telling information to children (see Figure 4.4). In summary, the wait-time group distinctions appear to be connected with the teaching-move data, suggesting that wait time may be important to investigate more systematically in the future.

Figure 4.4

Mean Percentage of the Total Number of Teaching Moves for Teaching-Moves Categories by Wait-Time Group

Category of Teaching Moves	**Wait-Time Group**	
	Adequate Wait Time (*N*=7)	Minimal Wait Time (*N*=4)
Exploring details of children's mathematical thinking	38.8%	21.8%
Telling information to children	29.0%	54.8%

Summary of Key Findings About PSTs' Teaching Moves

For the first research question, I explored teaching moves PSTs used in conversations with children around mathematical story problems. My goal was to better understand the teaching moves PSTs used prior to participation in a teacher education program so that teacher educators can be responsive to the thinking of PSTs. In short, it is important not to underestimate the incoming skills of PSTs. Specifically, PSTs demonstrated a range of teaching moves, some appreciation for the significant role of the story context during problem solving, some capability of exploring details of children's mathematical thinking, and some use of adequate wait time. Teacher educators can build on these initial strengths.

CHAPTER V

FINDINGS ABOUT PSTS' RATIONALES FOR THEIR TEACHING MOVES

The findings presented in this chapter address the second research question: *What rationales do PSTs give as to why they make the teaching moves they do?* During the stimulated-recall interviews, I elicited the PSTs' rationales for making teaching moves during the problem-solving interviews. The purpose of this chapter is to introduce a framework that captures the range of these rationales and to explore their frequencies.

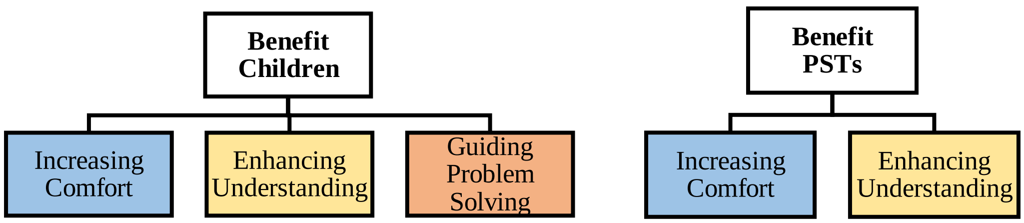
Framework of Rationales

Across the stimulated-recall interviews, PSTs shared a total of 315 rationales for teaching moves they used during the problem-solving interviews, and the number of rationales for each stimulated-recall interview ranged from 11–47, with a mean of 19.5 rationales. Sometimes PSTs' rationales focused on benefits for children and other times focused on benefits for themselves. All PSTs offered both types of rationales, with about two-thirds of the rationales focused on benefitting children and about a third on benefitting themselves.

Within these two broad types of rationales, there were five categories of rationales offered with some parallels. Specifically, PSTs offered rationales to increase children's comfort as well as their own comfort and to enhance children's understanding as well as their own understanding. There was one additional rationale category PSTs used that benefitted children—PSTs made teaching moves to guide problem solving by moving children toward an answer. See Figure 5.1 for an overview of the rationale categories.

Figure 5.1

Overview of PSTs' Rationales for Teaching Moves



The following sections describe and illustrate the rationale categories—first focusing on the parallel categories of increasing comfort and enhancing understanding of children and PSTs and then focusing on guiding problem solving of children (See Table 5.1 for a summary of their frequencies).

Table 5.1

Frequency of Categories of PSTs' Rationales for Teaching Moves

Rationale Categories	Mean number of rationales	Mean percentage of the total number of rationales	Number of PSTs providing rationale
Benefit Children	**19.4**	**67.3%**	**11**
Increasing comfort	0.9	3.4%	7
Enhancing understanding	9.8	34.8%	11
Guiding problem solving	8.7	29.1%	11
Benefit PSTs	**9.2**	**32.7%**	**11**
Increasing comfort	0.9	3.4%	6
Enhancing understanding	8.3	29.3%	11

Increasing Comfort of Children and PSTs

The parallel rationale categories—increasing comfort of children and increasing comfort of PSTs—captured the PSTs' desire to make children (or themselves) feel more at ease. For example, PSTs wanted to be supportive of children and tried to avoid their discomfort by offering reassurance when they thought children were "completely flustered" or they just "wanted [children] to be more comfortable" during problem-solving. Other times PSTs made teaching moves because they "wanted to compliment" children to make them feel confident. Likewise, PSTs made teaching moves to reassure themselves. For example, the PSTs sometimes made teaching moves when they needed help recalling information about the story problem or when they were "genuinely confused" about children's strategies. On average, rationales for these two categories comprised less than 10% of the total number of rationales for each PST (3.4% for rationales focused on increasing children's comfort and 3.4% focused on increasing PSTs' comfort). However, the parallelism in how the PSTs thought about comforting children and themselves was striking and thus worthy of investigation. Table 5.1 shows the frequencies of the two categories of rationales focused on comfort as well as the frequencies for the forms of these categories, and the following sections provide further description and examples.

Increasing Children's Comfort

PSTs made teaching moves because they wanted children to feel supported and less apprehensive. Seven of the 11 PSTs discussed making teaching moves to increase children's comfort. Some of the time PSTs used nonverbal cues from children that hinted they could be uncomfortable (e.g., a pause or a look) and other times, they simply wanted to make sure children remained at ease. For example, PST 10 made a teaching move to increase the child's comfort during their discussion about the blueberries problem (eating 8 of 20 blueberries). During the problem-solving interview, the child worked with blue base-ten blocks to create a group of 20 blueberries and another group of 8 blueberries. When the child paused to look up at the PST before adding the two groups together, the PST told the child,

> Alright, so he had 20 blueberries. You did awesome with that (*PST moved the group of 20 blueberries closer to the child*). That is the blueberries that he has. Well that's perfect because they're blue too. Little square blueberries. He ate 8 of them.

During the stimulated-recall interview, the PST shared her rationale for pointing out there were initially 20 blueberries: "I didn't want to kill her dreams that she counted 20 and then counted 8. So, I really wanted to compliment her—you did awesome with that." In this instance, the PST wanted to begin the exchange with praise to increase the child's comfort before helping the child make sense of the story problem context, which seemed to be misunderstood.

Increasing PSTs' Comfort

PSTs also made teaching moves because they needed to feel more at ease. Six of the 11 PSTs offered rationales in this category, and the rationales took three forms: (a) changing strategies, (b) recalling information, and (c) organizing strategies. First, PSTs made teaching moves to help children change their strategies because part or all of children's strategies confused them or they simply felt more comfortable using a strategy that was familiar to them. Second, PSTs made teaching moves to give themselves a chance to recall information, typically from the story problem. Third, PSTs made teaching moves because they wanted children to organize their strategies differently—in a way that the PST had learned or could better understand. I share two examples to illustrate the three forms of rationales focused on increasing PSTs' comfort.

To illustrate the changing-strategies and recalling-information forms of rationales to increase PSTs' comfort, I share PST 4's work with a child solving the buttons problem (putting 33 buttons in 3 bags). The child used base-ten blocks and counted out 3 ten rods and 3 unit cubes for the 33 buttons and then looked toward the PST. In response, the PST reread the story problem, and during the stimulated-recall interview, the PST shared her changing-strategies rationale with the author who was the interviewer (Int):

> PST: I [reread] the question or what he needed to do because I, I knew these [blocks] wouldn't work, or with my logic they wouldn't work. I never really liked these things to be honest.
>
> Int: So, you weren't sure how he could use those [blocks] to be able to solve the problem?
>
> PST: Yeah, because these, you can't separate the blocks.

In this case, the PST reread part of the story problem because the child's tool was uncomfortable for her—she "never really liked" blocks and she was not sure the child would be able to separate the base-ten blocks into 3 bags—so she hoped that the child would change strategies. Subsequently in the problem-solving interview, the child did change strategies (with the PST's help) and used paper and a marker to draw the 3 bags. At this point, the PST pointed to the drawn bags and said: "Alright, so now she has 33 buttons that she's going to put into these bags equally." During the stimulated-recall interview, the PST shared that her rationale for rephrasing the problem was to help her in recalling information about the problem: "I was trying to remind myself that it's 33 buttons total and then when I said 33 buttons equally into each bag, I meant [to say] divide…divide equally into each bag."

To illustrate the final form of rationales to increase PSTs' comfort—organizing strategies—I share PST 5's work with a child on the toys problem (counting toys in 4 boxes with 12 toys in each box). The child had counted 4 piles of 12 toys using blocks, but the piles were close together and close to the unused cubes, making the child's work hard to distinguish. The PST asked the child, "Do you want to separate them out a little more? It's getting crowded over there." In the stimulated-recall interview, the PST shared her rationale for making that comment:

> PST: It was more offering a suggestion. I didn't want to tell him to do it because it's his problem. He can answer it however he wanted. But, I wanted him to think about the fact that there was a cleaner way of doing it.
>
> Int: So you made that suggestion why?
>
> PST: For clarity in a sense. Just a visual.
>
> Int: For him to have more clarity or you?
>
> PST: Probably myself. It was probably selfish thinking since I didn't know how [he was counting]… so I just wanted to see what he was thinking as he was doing it.

In this example, the PST wanted to organize the child's strategy so that she was more comfortable following his counting.

Enhancing Understanding of Children and PSTs

The second parallel rationale categories—enhancing understanding of children and enhancing understanding of PSTs—captured the PSTs' desire to help children (or themselves). For example, PSTs wanted to make sure children had a "good understanding of what the problem was asking" or had an opportunity to reflect on their completed strategy and "understand how [they] got there." Similarly, PSTs made teaching moves to enhance their own understanding of "what [the child] was understanding" or just to explore their curiosity about the child's strategy. Table 5.2 shows that on average, rationales for these two categories comprised almost two-thirds of the total number of rationales for each PST (34.8% for rationales focused on enhancing children's understanding and 29.3% focused on enhancing PSTs' understanding).

Table 5.2

Frequency of PSTs' Rationales Focused on Enhancing Children's and PSTs' Understanding

Rationale Categories (and Forms)	**Mean number of rationales**	**Mean percentage of the total number of rationales**	**Number of PSTs that shared rationales**
Enhancing Children's Understanding	**9.8**	**34.8%**	**11**
Comprehending contexts	6.1	22.4%	11
Reflecting on strategies	3.3	10.6%	10
Linking to mathematical topics	0.4	1.8%	3
Enhancing PSTs' Understanding	**8.3**	**29.3%**	**11**
Expanding understanding	5.0	17.7%	10
Confirming understanding	1.9	7.3%	8
Developing understanding	1.4	4.3%	6

Enhancing Children's Understanding

PSTs shared rationales for enhancing children's understanding.[2] All 11 PSTs offered rationales in this category and the rationales took three forms: (a) comprehending contexts, (b) reflecting on strategies, and (c) linking to mathematical topics. First, PSTs made teaching moves because they wanted children to understand essential information from the story problem contexts or to connect mathematical details of their strategies to the story problem contexts. Second, PSTs wanted children to reflect on how they had solved problems. Third, PSTs wanted children to link mathematical topics to their strategies. On average, PSTs expressed rationales to comprehend contexts, at least twice as often as the other two forms. I share two examples to illustrate the three forms of rationales focused on enhancing children's understanding.

To demonstrate rationales centered on comprehending contexts and reflecting on strategies, I present PST 5's work with a child solving the books problem (getting more books to go from 18 to 25 books). After the PST posed the problem, the child counted 18 cubes and the PST asked, "And why'd you pick out a group of 18?" In the stimulated-recall interview the PST reasoned, "I wanted him to explain where, where he got his numbers from and what it represents." In this comprehending-context rationale, the PST wanted to enhance the child's understanding by asking the child to describe the relationship between the 18 cubes and the 18 books in the story problem context. After

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[2] Note that the PSTs' rationales sometimes focused on enhancing children's understandings in mathematically significant ways and other times in superficial ways. Those distinctions would be worthy of future research, but in this study, I considered all rationales together as my goal was to capture the underlying intentions for PSTs' teaching moves at a broad level.

this exchange in the problem-solving interview, the child added 6 cubes to the initial 18 cubes for a total of 24 cubes and incorrectly answered that 6 more books were needed. The PST moved the 24 books closer to the child and said, “Alright, so you added your 6, now let’s count up how many books there are.” When I asked the PST why she summarized how the child had added 6 more books, she shared, “I summarized it just because I want him to realize what he had already done. Keeping that in check.” In the rationale provided, the PST wanted the child to reflect on the part of the strategy in which 6 books had been added to the initial 18 books to make sure the child understood that action before moving on.

To illustrate rationales in which PSTs wanted children to link mathematical topics to their strategies, I share PST 11’s work with a child solving the blueberry problem (eating 8 of 20 blueberries). The child initially arranged 20 unifix cubes in two groups of 10 to represent the 20 blueberries. Then, the child separated 8 blueberries and correctly counted the remaining 12 cubes. The PST invited the child to share her mathematical thinking, and in the stimulated-recall interview, the PST explained her rationale for why she made this inquiry:

PST: Well she got the right answer, but I wanted to see if she could explain how she came to it … I wanted to see how well she could relate how she found her answer, because I mean that kind of helps them learn. I feel like —like telling how they got the answer.

Int: So, you thought if she explained her answer it would help her learn something?

PST: Well, maybe not so much that I guess as it would to maybe reinforce the concept. Maybe?

Int: Okay. What kind of concept?

PST: Addition—I don't—subtraction, not addition.

In this example, the PST described wanting the child to explain how she got her answer because verbalizing her process provided the child a chance to enhance her understanding of the underlying mathematical concept of the strategy—subtraction. Although the PST's phrasing could have better supported the child to do so, this example provides evidence that PSTs may be capable of keeping particular mathematical concepts in mind when working with children.

Enhancing PSTs' Understanding

PSTs shared rationales for enhancing their own understanding of children's mathematical thinking. All 11 PSTs shared rationales in this category and the rationales took three forms: (a) expanding understanding, (b) confirming understanding, and (c) developing understanding. First, PSTs made teaching moves to expand their understanding about children's strategies when they followed how children had solved problems but were perhaps unfamiliar with the strategies. In some cases, PSTs made inquiries about children's strategies prior to children solving problems so that the PSTs were prepared to help children later in the conversations if needed. Second, PSTs wanted to confirm their understanding of strategies to make certain their interpretations of children's ideas were correct or that they knew children's final answers. Third, PSTs made teaching moves to develop understanding because they *did not* understand part or all of children's strategies and had interest in learning more. Note that, on average, PSTs shared expanding-understanding rationales the most, at least twice as much as the other two forms. In the following sections, I provide two examples to illustrate the three forms of rationales focused on enhancing PSTs' understanding.

To illustrate an expanding-understanding rationale, I describe PST 6's work with the blueberries problem (eating 8 of 20 blueberries*)*. The child used a hundreds chart and correctly offered 12 as the answer after pointing to 20 and counting backwards 8. The PST made a series of teaching moves because although she understood the child's strategy, she was curious to learn more about the child's use of the hundreds chart.

PST: 12? How'd you figure that out?

Child: So I started on 20 and then I counted back by 8.

PST: Okay, and why did you like using the chart? Was it because you can see the numbers a little bit better?

(Child shakes her head yes)

PST: Okay, that's understandable. Would you have solved that a different way?

In the stimulated-recall interview, the PST shared that she first invited the child to explain how she solved because, "I wanted to see what she was thinking. See how did she count back. Maybe to see her process…to see how she was thinking." I then asked the PST about why she asked the child about "liking" the hundreds chart to which she responded, "To see why she liked using the chart...to see if she had any more background knowledge with the chart" Finally, I inquired about why the PST asked the child if she would have used an alternate strategy and the PST shared,

> To see if she knew how to do the problem multiple ways. Because…some problems might not be able to be solved with the hundred chart. So, just to see if she's able to work the problem out in a different way.

In this example, the PST understood the child's strategy and made a series of teaching moves with the intention to learn more about how (and why) the child had chosen that strategy and if the child was flexible in her use of multiple strategies. In other words, the PST wanted to know more about the child's thinking to expand the PST's own understanding.

To illustrate the confirming- and developing-understanding forms of rationales to enhance PSTs' understanding, I share PST 9's work with the books problem (getting more books to go from 18 to 25 books). The first part of the interaction illustrates a developing-understanding form of rationale and the second part of the interaction illustrates a confirming form of rationale. The child used base-ten blocks to count out a rod of 10 and 8 unit cubes for the 18 books. The child then added 7 more cubes for a total of 25 books. Ignoring the child's strategy, the PST reread part of the story problem, "How many more did her dad give her?" The child counted an additional 7 cubes to represent the (correct) answer of 7 more books, and the child placed these cubes above the strategy showing the 25 books. The PST incorrectly thought the answer was 6 books and therefore did not understand the child's strategy, which she mistakenly thought was incorrect. She asked the child to revisit his strategy by stating, "7, Okay. And you had 10, the 18, and then how many more did you add?" In the stimulated-recall interview, the PST shared a developing-understanding rationale for wanting the child to say the answer again; she shared, "I was also genuinely confused about why he put those [7 additional cubes] up top. I didn't know exactly why he put [the cubes] up there…the top ones…I don't know what [those cubes] were. What he was doing with those?"

In the problem-solving interview, the interaction continued, with the PST re-creating the child's strategy and essentially demonstrating how to solve the problem with the child's base-ten blocks. Toward the end of the interaction, the PST pointed to a group of 7 cubes and suggested, "let's count these," after which she and the child counted the group several times, eventually determining that there were 7 cubes and that the answer to the problem was that 7 more books were needed. During the stimulated-recall interview, the PST described a confirming-rationale for her demonstration and request for the child to re-count, "I had gotten the wrong answer and he had counted right. So, I guess for both of us, [it] was a double check to make sure [of the right answer]."

Guiding Children's Problem Solving

The final category of rationales—guiding children's problem solving—captured the PSTs' desire to help children move in the direction of the answer. All the PSTs gave rationales in this category, and on average, 29.1% of the total number of rationales for each PST were in this category. These rationales took three forms: (a) preventing errors, (b) correcting errors, and (c) providing direction. All three rationale forms focused on guiding children's problem solving but, in the first two, the PSTs emphasized children's errors—and their desire to provide direction to either prevent or correct those errors—whereas in the final form, PSTs focused on providing direction unrelated to children's errors. Table 5.3 provides frequency information for this rationale category, and the following sections further describe and illustrate the forms of this category.

Table 5.3

Frequency of PSTs' Rationales Focused on Guiding Children's Problem Solving

Rationale Category (and Forms)	**Mean number of rationales**	**Mean percentage of the total number of rationales**	**Number of PSTs that shared rationales**
Guiding Children's Problem Solving	**8.7**	**29%**	**11**
Preventing errors	1.5	5.3%	5
Correcting errors	2.6	8.2%	8
Providing direction	4.5	15.5%	11

Preventing Errors

In the first form of rationales to guide children's problem solving—*preventing errors*—PSTs made teaching moves because they wanted to make sure children were "on the right path" and if children got stuck, PSTs wanted to "try to help push [them] along" preventing any future missteps. About half of the PSTs (5) gave rationales focused on preventing errors, and on average, 5.3% of the total number of rationales for each PST had this form.

For example, PST 3 engaged a child with the toys problem (counting toys in 4 boxes with 12 toys in each box). After the PST read the story problem aloud, the child began to draw small boxes and the PST interjected by rereading part of the problem and suggesting, "You should draw [the boxes], big." The child proceeded to draw the 4 boxes a bit larger and put 12 small circles (toys) in each box. In the stimulated-recall interview, the PST explained why she decided to tell the child to redraw the boxes larger before counting the circles:

> What I have seen happen before is kids draw small boxes because they think it's enough space to count [the circles]. But then they don't [have enough space], so they just mess up…kind of misunderstand their drawing. So, I said, you can draw a big, big box.

The PST had predicted that the child would attempt to draw 12 toys (in this case, circles) within each box and miscount because of the lack of space. To prevent the counting error from occurring, the PST asked the child to redraw the boxes.

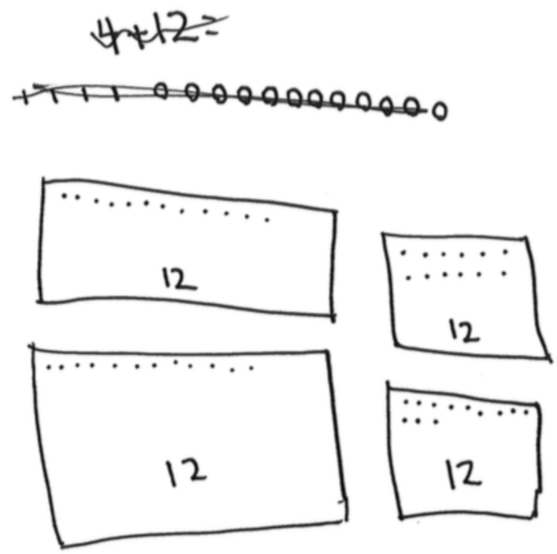
Correcting Errors

In the second form of rationales to guide children's problem solving—correcting children's errors—PSTs made teaching moves because they either wanted to directly point out errors or they wanted children to "rethink" or "re-look at [their] answer" to recognize their own mistakes in strategies and answers. Most of the PSTs (8) gave rationales focused on correcting children's errors, and on average, 8.2% of the total number of rationales for each PST had this form.

For example, PST 2 worked with a child to solve the toys problem (counting toys in 4 boxes with 12 toys in each box). Initially, the child wrote the number sentence "12 + 4 =" and drew a picture of 4 tally marks and 12 circles underneath. The child then counted all the tally marks and circles together to incorrectly answer 16. In response, the PST reread the problem to the child and in the stimulated-recall interview, the PST shared that she wanted the child to correct the error: "So for that one, I read it again because I wanted to give her another chance to see why it wasn't 16 and why it was going to be 48. So, she could see where to go from there." In this case, the PST wanted the child to see and make the correction on her own, which she did. (See Figure 5.2 for the child’s written work in which the initial representation and number sentence were crossed out, and the subsequent work showed that the child had drawn 4 boxes and placed 12 dots in each box, which allowed her to determine the correct answer of 48 toys.)

Figure 5.2

Child’s Revised Written Work for the Toys Problem (Counting Toys in 4 Boxes with 12 Toys in Each Box)



Providing Direction

In the third form of rationales to guide children’s problem solving—providing direction—PSTs described finding ways to move children forward in the problem-solving process with the end goal in mind, but without a focus on preventing or correcting errors. Instead, they wanted to generally “point [children] to the right answer,” or give children “a way to figure out [the problem] and make [the problem] easier to solve.” All the PSTs gave rationales focused on providing children direction, and on average, 15.5% of the total number of rationales for each PST had this form.

For example, PST 11 worked with a child on the candy problem (giving away 20 pieces of candy in groups of 2). The child counted two stacks of 10 unifix cubes for the 20 candies and then took away 2 cubes, seemingly trying to understand the story context. After engaging the child in a discussion of the story context, the child separated the 20 pieces of candy into 10 groups of 2 cubes and shared, “So he gave all his pieces of candy away.” The PST decided to explore the details of the child’s mathematical thinking by making a teaching move to link the child’s strategy to the story context:

PST: Right, so now you’ve got two pieces of candy and they're in separate [groups], so what do you think these [groups] represent? (*PST points to one of the groups of 2 cubes*).

Child: Twos

PST: Mmhmm. Twos? So how would we find out how many friends he gave them to? So if these are the two pieces of candy he gave away *(PST points to some of the groups of 2 cubes)*, how do we find out how many friends he had to give them to?

Child: By just…counting them… count by twos or something like that.

PST: Okay, when you say by counting them, how would you count them to show how many friends he has? Because remember, he gave two pieces to each friend. So one friend has two candies.

During the stimulated recall interview, the PST shared, “I was just trying to explain, you know, what was going on in the problem and trying to get her in the right direction of finding the correct answer by showing that two candies belonged to one person.” In this example, the PST described further explaining the context to help the child move forward in a direction that would allow her to find the correct answer. Although the PST shared her desire to help the child understand part of the story context, her underlying rationale for doing so was to guide the child toward the answer.

Summary of Key Findings About PSTs' Rationales for Their Teaching Moves

For the second research question, I explored rationales PSTs gave as to why they made the teaching moves they did during conversations with children. My goal was to introduce a framework that captures PSTs' reasoning to support teacher educators in being responsive to the thinking of PSTs when making instructional decisions. (See Appendix D for the framework of rationales.)

Three big ideas emerged from the data to support our understanding of the rationales PSTs make for using the teaching moves they do. First, PSTs made teaching moves for either the benefit of children or for the benefit of themselves. Second, PSTs made teaching moves for increasing children's comfort and enhancing children's understanding as well as for increasing their own comfort and enhancing their own understanding. Third, PSTs made teaching moves that guided children's problem solving. In short, it is important to elicit specific rationales from PSTs because we can learn about PSTs' thinking from these rationales.

CHAPTER VI

FINDINGS OF RELATIONSHIP BETWEEN PSTS' TEACHING MOVES AND RATIONALES

The findings presented in this chapter address the third research question: *What is the relationship between PSTs' teaching moves and their rationales for making them?* To explore the relationship between teaching moves and rationales, I matched teaching moves from 11 problem-solving interviews with the rationales elicited for these teaching moves from corresponding stimulated-recall interviews. Examination of the paired teaching moves and rationales revealed occasions rationales aligned with the teaching moves and occasions when they were misaligned. PSTs need to see rationales and teaching moves as connected (and aligned) for them to be responsive to children's mathematical thinking. Thus, considering PSTs' alignment (and misalignment) of rationales and teaching moves helps us understand when PSTs are acting in ways that will (or will not) be likely to achieve their goals.

The previous findings chapters highlighted that PSTs enacted a range of teaching moves and provided a range of rationales. In this chapter, I share what I learned from exploring the alignment at the category level for both teaching moves and rationales. Note that I purposefully chose to work only at the category level of both teaching moves and rationales for these analyses given the small number of instances of the multiple pairings of categories for each PST. Given the small sample size, I consider these analyses to be exploratory. My goal was to provide directions for future research that can help teacher educators support PSTs learn to enact—in the moment—teaching moves that align with their goals and thus are more likely to help them achieve these goals.

I examined a total of 258 teaching moves linked to 315 rationales. The number of rationales was greater than the number of teaching moves because sometimes PSTs had more than one rationale for enacting a teaching move. Analysis focused on the three categories of teaching moves: (a) exploring details of children's mathematical thinking, (b) comprehending story problems, and (c) telling information to children. Alignment and misalignment of teaching-move and rationale categories was determined from my perspective—informed by the literature—on the ideal goal of each teaching-move category. Note that each teaching-move category includes a variety of forms, but for this exploratory analysis, as long as one of the forms of that teaching-move category aligned with the rationale category, that pair was considered aligned. (See Figure 6.1 for a summary of alignment/misalignment of teaching-move and rationale categories.)

Figure 6.1

Alignment/Misalignment of Categories of Teaching Moves and Rationales

		Categories of Teaching Moves		
		Exploring details of children's mathematical thinking	*Comprehending story problems*	*Telling children information*
Categories of Rationales		*Aligns*	*Aligns*	*Aligns*
Benefit Children	*Increasing children's comfort*	Yes	Yes	No
	Enhancing children's understanding	Yes	Yes	No
	Guiding children's problem solving	No	No	Yes
Benefit PSTs	*Increasing PSTs' comfort*	Yes	No	No
	Enhancing PSTs' understanding	Yes	No	No

For each category of teaching moves, I noted the number of corresponding rationales that fell into each of the 5 rationale categories and the percentage of the total number of rationales for that teaching-move category. I also calculated the number of PSTs who shared at least one rationale in that rationale category for that teaching-move category (see Figure 6.2). The rest of the chapter explores these pairings between categories of teaching moves and rationales, showcasing alignment (or misalignment). To illustrate these ideas, I use one rich example—PST 7 working with a child on the buttons problem (putting 33 buttons in 3 bags)—throughout the chapter. I begin with an overview of that interaction and then separately address each of the three categories of teaching moves.

Figure 6.2

Frequencies of Paired Categories of Teaching Moves and Rationales

		Categories of Teaching Moves					
		Exploring details of children's mathematical thinking 94 teaching moves 116 rationales		*Comprehending story problems* 58 teaching moves 76 rationales		*Telling children information* 106 teaching moves 123 rationales	
Categories of Rationales		*N* (%) of rationales[a]	*N* of teachers	*N* (%) of rationales[a]	*N* of teachers	*N* (%) of rationales[a]	*N* of teachers
Benefit children	*Increasing children's comfort*	1 (1%)	1	4 (5%)	3	6 (5%)	5
	Enhancing children's understanding	26 (22%)	9	41 (54%)	11	42 (34%)	10
	Guiding children's problem solving	18 (16%)	6	17 (22%)	9	60 (49%)	10
Benefit PSTs	*Increasing PSTs' comfort*	1 (1%)	1	4 (5%)	2	4 (3%)	3
	Enhancing PSTs' understanding	70 (60%)	11	10 (13%)	6	11 (9%)	6

[a] Percentage refers to the percentage of the total number of rationales for that teaching-move category. The percentages for comprehending story problems do not add to 100% due to rounding errors.

Note. This chapter focuses on the rationale categories that provided greater than 10% of the total number of rationales for a particular teaching-move category.

Overview of PST 7's Work with the Buttons Problem

PST 7 posed the problem: *Deja had 33 buttons. She put the buttons into 3 bags with the same number of buttons in each bag. How many buttons did she put in each bag?* The child chose to use the hundreds chart and shared that she was going to continually take away 3 at a time until she had taken away 33 and then she would see what number she landed on. However, when she started counting, she *counted up* 3 at a time, starting at 33. The child counted and re-counted for some time and when she moved past 50, the PST interrupted and asked her to explain her strategy. Although the explanation was unclear, the child seemed to indicate that she was counting up by threes, 33 times. However, when pressed, the child described her work as counting "down." The PST reiterated the problem and then encouraged the child to keep going so that the PST could learn what the child was doing. The child continued her counting up by threes on the hundreds chart and when she got to the end of the chart (100) and was not finished counting, she pulled out the base-ten blocks and continued counting by threes on a ten-rod. The PST interrupted to revisit the problem situation and encourage a change of strategy. The child chose to make 33 with the unit cubes and as she was counting the cubes, she put them into 3 groups but not in any systematic way. She ended with 35 cubes in uneven groups of 10, 12, and 13. The PST asked her to count the three groups and the child announced that she did "Less, middle, biggest." The PST again revisited the problem situation to emphasize the need for equal-sized groups. For the rest of the interaction, the child counted (and recounted) both the individual piles and the whole set as well as moved cubes among piles. When the interaction ended (because of a time limitation), the child had 33 cubes in 3 piles of 12, 11, and 10.

Rationale Categories Linked to the Teaching-Move Category of Exploring Details of Children's Mathematical Thinking

Across the 11 interviews, PSTs enacted 94 teaching moves categorized as exploring details of children's mathematical thinking, and they shared 116 rationales for enacting these teaching moves. Teaching moves in this category provided space for children to share their strategies and mathematical ideas. Frequently used rationale categories that aligned with providing this space were focused on enhancing children's understanding or enhancing PSTs' understanding. Specifically, when PSTs shared rationales focused on enhancing children's understanding, they wanted children to better understand the story problem context, their own strategy, or the underlying mathematical topics. Similarly, when PSTs shared rationales focused on enhancing their own understanding, they wanted to expand, confirm, or develop understanding of children's mathematical thinking. As shown in Figure 6.2, all 11 PSTs provided rationales in one of these two rationale categories and most provided rationales in both. In addition, 82% of the total number of rationales provided for exploring details of children's mathematical thinking were in one of these two rationale categories. In short, PSTs showed convincing evidence that their rationales for exploring details of children's mathematical thinking aligned with teaching moves that provided space for children to share their strategies and mathematical ideas.

However, I also found evidence of categories of rationales that were misaligned with teaching moves that explore details of children's mathematical thinking. Specifically, the frequently used rationale category of guiding children's problem solving did not align with providing space for children to share their strategies and mathematical ideas. More than half of the PSTs (6) shared rationales in this category, and these rationales were 16% of the total number of rationales provided for exploring details of children's mathematical thinking. PSTs' rationales indicated that they thought they were helping children move toward an answer by providing specific directions (including preventing or correcting errors). However, they tried to accomplish these goals with teaching moves that provided space for the children's thinking, which misaligns. Although we want to encourage PSTs to explore the details of children's mathematical thinking, the rationale for doing so should lie in enhancing understanding and *not* in trying to get children to the answer.

To illustrate these ideas, I draw on the example of PST 7 who enacted teaching moves to explore the details of a child's mathematical thinking while providing rationales that were sometimes aligned and sometimes misaligned with this category of teaching moves. At the beginning of the interaction, PST 7 explored the child's mathematical thinking even before she started solving by inviting the child to share her problem-solving plans. The PST's rationale for enacting this teaching move aligned as she was trying to enhance her own understanding of the child's thinking as seen in Figure 6.3.

Figure 6.3

Example of Alignment: Exploring Details of Children's Mathematical Thinking and Enhancing PSTs' Understanding

Problem-Solving Interview	**Stimulated-Recall Interview**
So what are you going to do there? *(teaching-move category of exploring details of children's mathematical thinking)*	Because I really, I wanted to, again know the process she was going to do before she did it so if there was a chance that I need to stop her [and explain] this is what you're trying to do. Because if I went into it blank, I wouldn't know what she was doing to be able to help her. *(rationale category of enhancing PSTs' understanding)*

As the interaction continued, the child began her strategy by counting up from 33—by threes—on the hundreds chart. After the child had counted past 50, the PST interrupted and explored the child's mathematical thinking by pressing for more details about her strategy. The PST's rationale for enacting this move was twofold (see Figure 6.4). The first part aligned with providing space for the child to share reasoning about her strategy because the PST was confused about what the child was doing. Thus, the PST tried to enhance her own understanding of the child's mathematical thinking. However, the second part of the PST's rationale was misaligned with providing space for the child to reason about her strategy because rather than explore the child's thinking, the PST was trying to guide the child's problem solving or, in the PST's words, "re-route her."

Figure 6.4

Example of Alignment and Misalignment of Rationales: Exploring Details of Children's Mathematical Thinking

Problem-Solving Interview	**Stimulated-Recall Interview**
Explain to me what you are doing right now. *(teaching-move category of exploring details of children's mathematical thinking)*	This is one of those places where I stopped her because I did not understand how she was going all the way into the 50s. And I wanted to see where she was *(rationale category of enhancing PSTs' understanding)* and if there was any way I could re-route her and to, I guess one of the correct ways of doing it. *(rationale category of guiding children's problem solving)*

These examples from the first part of the interaction showcase how PST 7 used several teaching moves to explore the details of the child's mathematical thinking. At times, the PST's rationales aligned with providing space for the child to share the details of her mathematical thinking and other times it did not. Note that, in this example, the PST did not provide rationales to enhance the child's understanding that aligned with the teaching move category of exploring details of children's mathematical thinking, but as discussed above, this alignment did exist in the broader sample.

Rationale Categories Linked to the Teaching-Move Category of Comprehending Story Problems

Across the 11 interviews, PSTs enacted 58 teaching moves categorized as comprehending story problems, and they shared 76 rationales for enacting these teaching moves. Teaching moves in this category helped children understand story contexts and recall correct quantities within problems. The frequently used rationale category that aligned with understanding story contexts and recalling story quantities focused on enhancing children's understanding. Specifically, when PSTs shared rationales focused on enhancing children's understanding, they wanted children to better understand the story problem context, their own strategy, or the underlying mathematical topics. As shown in Figure 6.2, all 11 PSTs used this rationale category that aligned—in fact, more than half (54%) of the total number of rationales provided for comprehending story problems were in this rationale category.

However, I also found evidence of two frequently used rationale categories that misaligned with teaching moves that made sure children understood contexts and recalled correct quantities within story problems. Specifically, the rationale categories of guiding children's problem solving and enhancing PSTs' understanding did not align with helping children understand contexts and recall quantities. First, more than half of the PSTs (9) shared rationales focused on guiding children's problem solving, and these rationales were 22% of the total number of rationales for comprehending story problems. PSTs' rationales revealed they thought they were helping children move toward an answer by providing specific directions (including preventing or correcting errors). However, they tried to accomplish these goals by using teaching moves that aimed to help children understand contexts of story problems, which misaligns. Second, more than half of the PSTs (6) shared rationales focused on enhancing PSTs' understanding, and these rationales were 13% of the total number of rationales for comprehending story problems—which seemed peculiar. In closer examination of their rationales, PSTs' rationales showed they wanted to understand the story problem contexts for themselves such as recalling quantities or mentally solving the problem themselves. However, they tried to accomplish this goal by using teaching moves that aimed to help children understand contexts or recall quantities of story problems, which misaligns.

To illustrate these ideas, I return to the example of PST 7 who enacted teaching moves to ensure comprehension of story problems while providing rationales that sometimes aligned and sometimes misaligned with this teaching-move category. In the second part of the interaction, PST 7 made a series of teaching moves to help the child comprehend the story problem, emphasizing how the buttons had to go into the 3 bags. She first unpacked the story problem to enhance her own understanding, which misaligns with making sure the child comprehends the context or recalls quantities from the story problem. As seen in Figure 6.5, the PST made a teaching move to give herself time to better understand the story problem and how she could prepare herself to get the child, in the PST's words, "back on track."

Figure 6.5

Example of Misalignment: Comprehending Story Problems and Enhancing PSTs' Understanding

Problem-Solving Interview	**Stimulated-Recall Interview**
Okay. So when she is putting them into 3 bags, she is grouping them into different ones, right? *(teaching-move category of comprehending story problems)*	Because I was trying to explain it—no—I was trying to better understand it myself because I didn't know, how do I get her back on track? *(rationale category of enhancing PSTs' understanding)*

After the PST and the child discussed the use of the hundreds chart, she suggested the child change her strategy and enacted a teaching move for the child to comprehend story problems. Specifically, the PST wanted to clarify quantities in the context and ensure the child understood there was a total of 33 buttons and the buttons needed to go into the bags, which aligned (see Figure 6.6).

Figure 6.6

Example of Alignment: Comprehending Story Problems and Enhancing Children's Understanding

Problem-Solving Interview	**Stimulated-Recall Interview**
So it is saying that if… she has 33 buttons, so that's all she's going to have is 33. And then if she's breaking them up into three different bags. So how are you going to break them up?	I stopped it there because I think she was going, she was going over 33, and I needed her to understand that they're only 33 and she's not getting any more buttons. It's all she's got and she's putting them away. She's not getting more and I think she was adding more to it and so that's why I was telling her, there's only 33.
(teaching-move category of comprehending story problems)	*(rationale category of enhancing children's understanding)*

These examples from the second part of the interaction showcase how PST 7 used several teaching moves focused on comprehending story problems. At times, her rationales aligned with making sure the child understood contexts and recalled quantities and other times it did not.

Rationale Categories Linked to the Teaching-Move Category of Telling Information to Children

Across the 11 interviews, PSTs enacted 106 teaching moves categorized as telling information to children, and they shared 123 rationales for enacting these teaching moves. Teaching moves in this category emphasized providing information to children that PSTs had determined pertinent for problem solving. The rationale category that aligned with providing information to children was focused on guiding children's problem solving. Specifically, when PSTs shared rationales focused on guiding children's problem solving, they provided specific direction to children (including preventing or correcting errors). As shown in Figure 6.2, all but one PST provided a rationale that guided children's problem solving and these rationales were almost half (49%) of the total number of rationales provided for telling information to children. Note this alignment was not always productive for children. Telling information to children to help guide them to the answer is not responsive to children's mathematical thinking but it suggests PSTs may know they are telling and consider it beneficial for children.

I also found evidence of rationales that were misaligned with teaching moves that provided information to children. Specifically, the frequently used rationale category of enhancing children's understanding did not align with providing information to children. All but one PST shared rationales in this category, and these rationales were 34% of the total number of rationales provided for telling information to children. PSTs' rationales revealed they wanted children to better understand the story problem context, their own strategy, or the underlying mathematical topics. However, they tried to accomplish these goals by using teaching moves that provided specific directions (including preventing or correcting errors), which misaligns.

To illustrate these ideas, I focus on the end of PST 7's interaction. For the teaching-move category of telling information to children, the PST only provided rationales that focused on providing children direction and thus all of her rationales aligned. This example takes place after the child had divided 35 (instead of 33) cubes into 3 groups of (10, 12, and 13), arranged from least to greatest. These errors prompted the PST to guide the child's problem solving by rereading the part of the problem that emphasized needing equal groups and then providing specific directions to count how many were in each group. As seen in Figure 6.7, the PST explained that her directions to focus on equal grouping were meant to correct the child's error—a rationale that aligned with the teaching-move category of telling information to children.

Figure 6.7

Example of Alignment: Telling Information to Children and Guiding Children's Problem Solving

Problem-Solving Interview	**Stimulated-Recall Interview**
Each group should have the same amount. Alright, how many are in this one? *(teaching-move category of telling information to children)*	PST: Because she moved 12 over here and I knew that [the groups] weren't going to be equal. Int: Were you trying to make sure that she was paying attention to equal groups? PST: I was trying to reiterate that they [had to] be equal *(rationale category of guiding children's understanding)*

This example from the last part of the interaction showcases how PST 7 used a teaching move to tell information to the child, and her rationale of guiding the child to the correct answer aligned. Note that, in this example, the PST did not provide rationales to explore details of children's mathematical thinking that misaligns with the teaching-move category of telling information to children, but this misalignment did exist in the broader sample.

Summary of Key Findings About the Relationship Between PSTs' Teaching Moves and Rationales

For the third research question, I conducted an exploratory analysis of the relationship between rationales shared by PSTs and the teaching moves they enacted. My goal for examining the teaching moves and rationales together was to understand their alignment so that teacher educators can find ways to support PSTs in learning to enact teaching moves that will support their goals and ultimately a vision of responsive teaching. Overall, for each teaching-move category, PSTs shared rationales in categories that sometimes aligned and sometimes misaligned. About half or more of the rationales in each teaching-move category were in rationale categories that aligned. These findings provide a starting point for teacher educators as they try to help PSTs better understand teaching moves, rationales, and the relationship among them.

CHAPTER VII

CONCLUSION

In this study, I explored the teaching moves PSTs used in one-on-one problem-solving interviews with children and the rationales they shared for making those teaching moves. I also examined the relationship between teaching moves and rationales. My overarching goal was to understand the teaching moves and rationales of PSTs prior to participation in a teacher education program. Further, I wanted to identify PSTs' strengths and ways teacher educators could build on them to support PSTs in developing expertise in responsive teaching. In this chapter, I summarize key findings and share theoretical, practical, and research implications as well as study limitations.

Key Findings

Examination of teaching moves PSTs enacted during problem-solving interviews showed they used a range of teaching moves that fell into three main categories, (a) comprehending story problems, (b) exploring details of children's mathematical thinking and (c) telling information to children. Further, when PSTs enacted teaching moves in each category, these moves took a variety of forms. Findings also revealed unexpected strengths of PSTs as they showed some evidence of prioritizing story contexts during problem solving, noticing and asking about specific strategy details, and using wait time with children. However, in each of these areas, PSTs could continue to grow in their expertise.

Exploration of rationales PSTs shared during stimulated-recall interviews indicated that PSTs had specific rationales for enacting their teaching moves and these rationales sometimes focused on benefitting children and sometimes focused on benefitting themselves (see also, Rich & Hannafin, 2008). Findings revealed 5 categories of rationales. Four were parallel categories within the two broad types of rationales including rationales focused on increasing comfort (both for children and PSTs) and rationales focused on enhancing understanding (both of children and PSTs). The final rationale category focused on benefitting children by guiding their problem solving. Many of these categories also had a variety of forms.

An exploratory analysis of the relationship between categories of teaching moves and rationales showed that PSTs' rationales sometimes *aligned* and sometimes *misaligned* (see also, Webel et al., 2018). About half or more of the rationales in each teaching-move category were in rationale categories that aligned.

Theoretical Implications

This study contributes to the research base on responsive teaching with children, in particular as it relates to PSTs working with children, and I highlight three implications. First, the research on teaching moves has primarily included practicing teachers, many of whom already have some knowledge of children's mathematical thinking (Jacobs & Empson, 2016; Franke et al., 2015). For example, researchers have focused on practicing teachers with a wealth of experience in teaching mathematics (Ball, 1993; Boaler & Brodie, 2004; Hiebert & Wearne, 1993) or practicing teachers who participated in a professional development project focused on children's mathematical thinking (Cengiz et al., 2011; Franke et al., 2009; Jacobs et al., 2010). Studying the teaching moves of teachers who have developed expertise is important for teacher educators to understand what a well-developed toolbox of teaching moves looks like for responsive teaching. However, if we think about building a toolbox of teaching moves over time like a continuum, it would also be important to know what that toolbox looks like at the very beginning—on the other end of the continuum. My focus on PSTs contributes information about their specific categories (and forms) of teaching moves prior to engagement in a mathematics methods course—providing a baseline of their expertise (Shaughnessey & Boerst, 2018). Prospective teachers do not come with *empty* toolboxes that must be filled by teacher educators—my study shows what teaching moves may already be in their toolboxes and what they are capable of enacting before explicit instruction in teaching mathematics.

Second, I extend the research on PSTs' teaching moves. Some of the findings confirm and extend what has been found in earlier research. For example, PSTs' frequent use of the teaching-move category of telling information to children showed that they often took on the mathematical work for children, funneling their thinking (see also Moyer & Milewicz, 2002; Sun & van Es, 2015, Wood, 1998). Further, my findings suggest that who is carrying out the work—children or PSTs—may provide additional nuance for this conversation about the dilemma of whether to tell or not to tell (Ball, 1993; Baxter & Williams, 2010; Chazan & Ball, 1999). Also in alignment with other research was that although PSTs showed capabilities in attending to some of the details of children's' thinking, they did not always know how to build on those details effectively

(Shaughnessey & Boerst, 2018; Sleep & Boerst, 2012; Sun & van Es, 2015; Webel et al., 2018). For example, although PSTs showed evidence of pressing children for reasoning, their press was more likely to elicit factual information than explanations.

In contrast, other findings were inconsistent with prior work. For instance, Moyer & Milewicz (2002) found that PSTs used teaching moves in a "rapid-fire" manner that did not allow adequate time for children to respond, but more than half of the PSTs in this study showed capability of using adequate wait time. Prior research on wait time found that, on average, teachers wait one second before responding (Rowe, 1986). Although my study did not systematically capture the amount of time PSTs waited before responding throughout their problem-solving interviews, I did informally consider how often PSTs waited more than a few seconds before responding when I was determining wait time holistically.

Third, research on responsive teaching has focused on foregrounding listening closely to what children say and do as a tool for instructional decision making (Jacobs & Empson 2016; Robertson et al., 2016). I argue that teacher educators should use the same approach with PSTs. Thus, I developed a framework that captured the specific rationales PSTs shared about the teaching moves they enacted, and of particular interest was that PSTs made teaching moves not only to benefit children but also to benefit themselves (see Appendix D for the framework of PSTs' rationales). Overall, I elevated the PSTs' voices by categorizing their rationales into a framework, which can provide a starting point for future studies and assist teacher educators in learning from PSTs and their perspectives.

Practical Implications

I argue that teacher educators need to build on the skills and perspectives of PSTs at the start of teacher education programs, particularly in supporting them in aligning their rationales and teaching moves. I draw on the findings from this study to highlight 4 specific suggestions for working with PSTs on responsive teaching: (a) increasing PSTs' access to research based frameworks of children's mathematical thinking, (b) using artifacts of practice from PSTs' work with children, (c) expanding PSTs' repertoire of teaching moves for helping children comprehend story problems, and (d) asking PSTs to reflect on their practice in specific ways.

Increasing PSTs' Access to Research-Based Frameworks of Children's Mathematical Thinking

The PSTs in this study expressed curiosity and interest in learning more about children's strategies, even prior to a mathematics methods course. Teacher educators can build on this curiosity by increasing PSTs' access to research-based frameworks of children's mathematical thinking. Further, access to these frameworks might increase PSTs' comfort with children's strategies and talking about the mathematics in their strategies. PSTs shared rationales focused on enhancing children's understanding, but upon closer look, there was some evidence that PSTs may be less comfortable talking about mathematical strategy details or underlying mathematical topics, because they instead focused more often on story contexts. Increased access to frameworks of children's mathematical thinking can also help PSTs see children as capable problem solvers with diverse ways of thinking (Fennema & Franke, 1996). Perhaps, with this understanding, PSTs would be less inclined to tell information and take on the mathematical work for children.

Using Artifacts of Practice from PSTs' Work with Children

Teacher educators often use artifacts of practice of K-12 classrooms to help practicing teachers (and PSTs) learn to teach responsively (Ball & Cohen, 1999; van Es & Sherin, 2008). I argue that using artifacts of practice *depicting PSTs working with children* would provide additional relevant and contextualized ways for PSTs to learn from their own teaching. Artifacts of PSTs' work with children could include video (or transcripts) as well as soundwaves. The next two sections describe how these artifacts could give PSTs specific opportunities to attend to the phrasing in their teaching moves and their use of wait time. One caveat in using these artifacts is that teacher educators would need to be sure to be sensitive to their personal nature and use them only as private learning tools.

Attention to Phrasing in Teaching Moves

PSTs could examine their own phrasing within transcripts or video recordings of them working with children. For instance, PSTs often pressed children for reasoning and although this teaching move is a way to explore the details of children's mathematical thinking, PSTs more often pressed for factual information—using phrasing that did not prompt children to fully explain the details of their strategies but instead give short responses (see also, Boaler & Brodie, 2004). In contrast, PSTs could learn to press for explanations—open-ended phrasing that provides space for children to share the details of their thinking (Kazemi & Stipek, 2001 & Franke et al., 2009). By examining their phrasing and how children responded to their phrasing, PSTs may see not only occurrences when their rationales and teaching moves may be misaligned, but also occurrences of alignment thereby recognizing their own strengths as they navigate the complexity of responsive teaching.

Examination of Wait Time

PSTs' could examine their own use of wait time as captured by soundwaves along with video recordings (or transcripts) of their work with children. Soundwaves could provide PSTs the chance to see a graphic representation of the amount of time they speak in relation to the child and thus how often and how much time the child had to share their mathematical thinking. Further, these representations could be the foundation for discussions about which teaching moves afford children opportunities to talk as well as the benefits—for both PSTs and children—of using wait time and listening (Ingram & Elliott, 2016; Mahmud, 2018; Rowe, 1986; Staples & King, 2017). Note that although soundwaves are a promising instructional tool, attention needs to be paid to the technology. To effectively capture soundwaves representative of the conversations between PSTs and children, PSTs would need a quiet space that limited background noise and interruptions, and their voice would need to be isolated in relation to the child.

Expanding PSTs' Repertoire of Teaching Moves for Helping Children Comprehend Story Problems

An unexpected strength of PSTs was their interest in helping children comprehend story problems and their attempt to keep quantities connected to story problems. However, PSTs had a limited repertoire of teaching moves for doing so. PSTs mainly relied on rereading story problems, and although this teaching move does emphasize story problem contexts and keeps quantities contextualized, there are other forms of teaching moves that could better enhance children's understanding of the story context. For example, PSTs could use teaching moves that connect story problem contexts to prior knowledge (Ball, 1993) or ask children to summarize story problems in their own words (Jacobs & Empson, 2016). In short, teacher educators could introduce alternative teaching moves to expand PSTs' repertoires of teaching moves for helping children comprehend story problems.

Asking PSTs to Reflect on Their Practice in Specific Ways

Teacher educators can refine the ways they ask PSTs to reflect on their practice. Typically, after teaching experiences, teacher educators ask PSTs to reflect on how they felt, what they learned, or what they might have done differently (Crespo & Nicol, 2003; Dunphy, 2010, Vacc & Bright, 1999; Webel et al., 2018). Although reflecting in these ways is well documented as an effective learning tool (Schön, 1987), PSTs' *recalling* their in-the-moment decision making—as they did in the stimulated-recall interviews—would be an additional tool for learning. Asking for reflections on specific teaching moves might help PST move beyond describing what generally occurred and toward analyzing (Davis, 2006) the details of children's mathematical thinking, which can support development of their expertise in eliciting and building on children's mathematical thinking. Further, asking PSTs to reflect on why they used particular teaching moves and noting how children responded can help PSTs better align their teaching moves with their goals. Finally, asking PSTs to reflect on their practice in more specific ways gives teacher educators an opportunity to learn about the thinking of PSTs, gain a deeper understanding of their perspectives, and use those insights to make their instructional decisions.

Research Implications

This study was an initial exploration into PSTs' enactment of teaching moves and their rationales for making those teaching moves prior to explicit instruction. Future research in this area can build on my decision making and findings, and I share four of these implications. First, determining the unit of analysis for a teaching move is complex and critical. In this study, I chose to start with talk turns to capture the range of teaching moves PSTs made. However, sometimes it was challenging to assign one teaching move to every talk turn because a series of teaching moves can work together to accomplish particular goals. Although I considered the broader context of each talk turn to assign a code, future research should also consider a larger unit of analysis, such as clusters of teaching moves that are related. As research explores teaching moves beyond talk turns, it would also be interesting to examine enactment of teaching moves before and after correct answers as prior research has documented distinctions that I informally noticed in this study (Fraivillig et al., 1999; Jacobs & Ambrose, 2008).

Second, PSTs' prior experiences may play a role in their teaching moves and rationales and thus are worthy of investigation. In this study, I engaged PSTs in a conversation about their backgrounds to build rapport. We specifically discussed their experiences as learners of mathematics and their prior experiences working with children. I did not systematically analyze these conversations and their links to teaching moves and rationales, but future research should do so. Each PST came from a unique context and possessed a lens in which they viewed the world, and these lenses may provide insight into the variety of toolboxes of teaching moves and rationales that PSTs bring to teacher education.

Third, wait time needs to be considered a substantive teaching move and researchers need to find ways to capture and systematically analyze wait time. In this study, I used my professional judgment to holistically categorized PSTs' wait time before responding to children. The soundwaves that I used to visually contrast the conversations of teachers in the adequate vs. limited wait-time groups is promising. However, soundwaves were not initially considered in the design of the study and thus, in some interviews, interference from extensive background noise and poor placement of the microphone meant that the soundwaves did not always accurately represent the PSTs' and children's voices. These technological challenges would need to be remedied to capitalize on the potential of this data collection tool.

Fourth, honoring PSTs' perspectives during the research process takes time, and future research needs to recognize this constraint during study design. In this study, I wanted to give the PSTs as much ownership over the stimulated-recall conversations as possible. Specifically, my goal was that PSTs would initially be in charge of choosing when to stop the video of the problem-solving interviews and share their rationales. However, due to time constraints, I had to control the video stopping in 4 of the 11 interviews. Thus, when possible, I recommend multiple pilot studies and the inclusion of extra time to achieve the full benefits of using stimulated recall as a method for data collection that honors learning from PSTs.

Study Limitations

As with any study, it is important to address limitations as we move forward with future research, and I share two suggestions. First, this study included 11 PSTs and thus provides only initial insights about the range of PSTs' teaching moves, rationales, and the relationship between them. Sample size was particularly limited in the exploration of the relationship between teaching moves and rationales. Given the substantial misalignment in each of the teaching-move categories, the findings suggest that this area merits further investigation. In short, replicating the study with a larger sample size is needed to confirm and extend the findings.

Second, this study only included story problems involving whole numbers, which was chosen because PSTs tend to be most comfortable with whole numbers. Thus, similar studies with other mathematical content in which PSTs are less comfortable are needed. In particular, it would be interesting to see what teaching moves PSTs enact and what rationales they share when working with fraction story problems—a content area that has historically been challenging for PSTs. Comparing the teaching moves and rationales across content areas could illuminate additional areas of strengths and areas of needed support for PSTs.

Final Thoughts

This study points toward the ways we can build on the strengths of PSTs. The current vision for mathematics teaching and learning is complex and responsive teaching practices are not only challenging for practicing teachers but also difficult for PSTs. It is important to remember PSTs may be holding onto narratives of the past, and the current vision for mathematics teaching and learning can be overwhelming for beginning teachers. PSTs do have a lot to learn but in preparing responsive teachers, teacher educators can also find ways to be responsive to PSTs' thinking. PSTs are capable, diverse in their thinking, and have unique assets on which teacher educators can build. We can learn from PSTs and use their thinking to make informed instructional decisions.

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APPENDIX A

RECRUITMENT SCRIPT FOR PROSPECTIVE TEACHERS

Hello everyone, I hope you all are doing well this week and are enjoying the elementary education program so far. My name is Montana Smithey and I work with other future teachers like you. My research study will aim to better understand future teachers' experiences with mathematics teaching and learning as well as the questions they ask children when working with them. I am asking if you are interested in taking part in this research study.

I am asking you to participate in two tasks during your scheduled internship time this semester. In the first task, you would take a brief survey and then I would ask you some questions about your learning experiences in mathematics. In the second task, there are two parts; you would work one-on-one with a child, posing story problems and asking questions to learn more about their thinking and afterwards, participate in a follow-up interview with me that will ask about your decision-making when working with the child. These two tasks would take place at your internship site and require about 1 1/2 hours during your scheduled internship time. For completing the tasks, you would be compensated with a $25 gift card.

If you choose to participate, there are multiple benefits. First, you would have an opportunity to reflect on your own learning. Second, you would have the opportunity to work with children, which would provide you with additional teaching experience. Finally, you would be helping other educators learn about how future teachers think so you would be helping to improve math methods courses for future teachers at other universities. Because of all these benefits, the chair of your elementary education program is allowing this study to take place during your scheduled internship time. If you choose to participate, I will work closely with your supervisor to ensure we are not taking away from other responsibilities you may have.

Your participation in the study is completely voluntary. You may choose to join or not to join, and at any point, you may withdraw your consent to be in the study, for any reason, without penalty.

If you wish to participate, you will be asked to sign a consent form and given a copy to keep. If you are interested, I would love to talk more with you at the back of the room. Do you have any questions?

APPENDIX B

PROBLEM-SOLVING INTERVIEW PROTOCOL

Read to prospective teacher before child enters the room:

- Thank you for helping me by having a one-on-one conversation with a child today. I am trying to learn more about the questions future teachers ask children about their mathematical thinking because asking follow-up questions is challenging, even for teachers who have been teaching a long time. You will be helping future teachers through your participation because this study will help me make adjustments to my own math methods courses.
- (See the sample math story problems at the end of this protocol). Please pose the first 3 problems in any order you wish. If time permits, you may pose any of the additional problems listed. It doesn't matter how many problems you finish. Your task is to understand the child's mathematical thinking, and you will have about 15 minutes to work with the child.
- After you pose each problem, you may ask any follow-up questions you would like to better understand the child's thinking. There is not a set amount of follow-up questions that should be asked.
- The child is free to use or not use, any of the materials provided. The materials include blank paper, base-ten blocks, unifix cubes, and a hundreds chart. Are you familiar with each of these?
- Do you have any questions?
- I am going to give you a few minutes to read over this problem set while I go get the child from their classroom. I will be back shortly and begin by explaining the interview to the child and give you some time to share a little about yourself to them. Feel free to ask them a question or two about themselves before you get started.

Read to child before beginning:

- Thank you for helping us by solving some math problems today. We are trying to learn more about how kids think about math and we are glad you can help us!
- In front of you, you will see lots of different tools to choose from. There is blank paper, base ten blocks, unifix cubes and a hundreds chart. Have you used any of these tools before?
- This is ___________, and they are studying to be a teacher. They are going to ask you to solve a few problems and you can solve them in any way you want. This means you can choose any of the tools here when solving, or you can use nothing at all and solve problems in your head—whatever works best for you. After you solve, _______ is going to ask you some questions about how you solved the problem so we can see how you were thinking. We are not looking for a particular explanation. We really want to hear how you are thinking!

- This interview will be recorded so we can listen to it later to learn all about how kids solve math problems; that means you are our math teacher today.
- Do you have any questions?
- [At the end of the interview]: Thank you very much for teaching us about how you solve math problems, I think we learned a lot! I am going to walk you back to your classroom.

Story Problems

I. Please pose these 3 problems.

  - Jackson had 20 blueberries. He ate 8 of them. How many blueberries does Jackson have left?
  - Ebony had 18 books. Her dad gave her some more books for her birthday. Then she had 25 books. How many books did Ebony's dad give her for her birthday?
  - Marcos had 4 boxes of toys. There were 12 toys in each box. How many toys did Marcos have altogether?

II. Choose one of these additional problems to pose.

  - Luke had 15 lego pieces. Sarah gave Luke 5 more lego pieces. How many lego pieces does Luke have now?
  - Gabriel had 20 pieces of candy. He gave 2 pieces of candy to each friend. How many friends did he give his candy to?
  - Deja had 33 buttons. She put the buttons into 3 bags with the same number of buttons in each bag. How many buttons did she put in each bag?
  - Logan had 25 balloons. Sofia has 19 balloons. How many more balloons did Logan have than Sofia?

APPENDIX C

STIMULATED-RECALL INTERVIEW PROTOCOL

Introduction

- Thank you for participating in this study. I know your time is valuable. This interview will last no longer than 45 minutes and it will be recorded so that I can pay more attention to the conversation instead of focusing on my notes.
- I am interested in your decision-making as a teacher when working with children and their mathematical thinking. Asking children follow-up questions based on their work is challenging, even for teachers who have been teaching a long time. The purpose of this interview is to enhance my understanding of what questions you asked and why you decided to ask them after you posed the problems. There are no right answers, and I am really interested in your reasoning. You will be helping other teachers through your participation as the findings will guide me in making adjustments to the courses I teach with preservice teachers.

Interaction Experience

- Before we begin, tell me a little bit about what stood out in the interaction with your child today.
- I am most interested in why you decided to ask the follow-up questions you did. We are going to watch the problem-solving interview and discuss the problems you posed one at a time. You may pause the video at any time to share the reasons for your decisions. For example, any time you had to make a decision, ask a question, gesture, or make a comment, you can pause the video and share your thoughts.
- [At the end of each problem, if particularly interesting moments were passed without the preservice teacher pausing the video, I will return to that portion of the video. I will back up at least 30 seconds prior to the segment of interest to give the preservice teacher context before asking him or her to respond.]
    - I was curious about this part of the interaction. What were you thinking at this moment?
    - Why did you decide to ask the question you did?

Thank you for participating in this interview. Is there anything else you would like to share that you think would be helpful to me to better understand your decision-making during that problem-solving conversation?

APPENDIX D

FRAMEWORK OF RATIONALES