BUSINESS INTERRUPTION, INCOME LOSS & VALUE-AT-RISK TO CATASTROPHES

by

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Abstract

Not all risks are insurable. In accordance with the natural and fundamental operation of the practice of insurance, insurers envision certain characteristics that they attribute to "ideally insurable" risks. One of these key elements of an insurable risk is the *degree of loss* caused by the risk, if loss were to occur. For an insurer, an insurable risk would ideally not result in devastatingly destructive loss; in other words, the risk must not be *catastrophic*. However, the difficulty of insuring against catastrophes does not lessen the importance for companies to be able to estimate how their own performance will be impacted by the occurrence of a catastrophic loss. This paper aims to estimate the extent of a firm's business interruption, income loss, and value-at-risk to a catastrophic loss event. The study involves a Poisson-Pareto calamity simulation to estimate business interruption and income loss, and a modified VaR simulation that offers a customized estimation of value-at-risk to catastrophe. The data utilized to run these simulations is gathered from the financial statements of a thoroughly and realistically imagined hand-tool manufacturing company—Kingston Tools, Inc.—in order to provide an estimation of the firm's risk in a catastrophic event.

List of Exhibits

Contents

Company Background and Introduction

Kingston Tools, Inc. is conceived as an American-based but international manufacturer of hand tools, power tools, diagnostic tools, tool storage and shop equipment for a variety of industrial and commercial businesses, including: auto, marine, and aviation dealers; farmers; and repair shops. The company operates in three core segments: (1) Tools; (2) Diagnostics and Repair Information; and (3) Equipment. Its Tools segment includes the manufacturing of hand tools, power tools, and tool storage products. The Diagnostics and Repair Information segment is comprised of handheld and computer-based diagnostics products, diagnostics software, service and repair information products, and business management services. The Equipment segment spans (1) automotive uses—like wheel alignment equipment, tire changers, and vehicle lifts—and (2) industrial diagnostic and service equipment—such as troubleshooting equipment and air conditioning service equipment.

I was originally introduced to Kingston Tools, Inc. through the capstone course, Enterprise Risk Management, of my Risk Management & Insurance degree program here at Appalachian State. Dr. Karen Epermanis and David D. Wood devised the company as the focus of a cumulative case study to span the entirety of the course. The aim of the case for students is to analyze various information about Kingston's operations, performance (from financial statements), and loss history (such as an extensive log of the company's recent workers' compensation claim history), and use this analysis to serve the role of risk consultant and provide Kingston management with a comprehensive risk audit and improved risk management & insurance program.

This thesis is an extension of the case work that I completed for the Enterprise Risk Management course. Dr. Lori Medders—my thesis director—and I wanted to expand upon the risk management analysis and recommendations that I made in the original case; we wanted to see if we could use the information in the case to investigate the potential impact of a catastrophic loss event on the company's operations and performance.

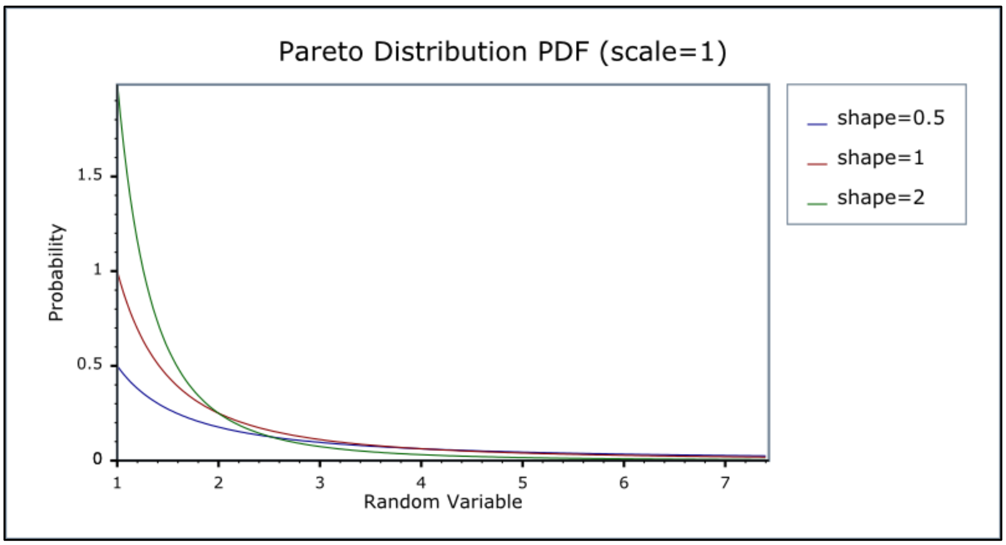
The importance of our work here seems particularly timely considering the catastrophic nature of the COVID-19 pandemic, of which we all as a society are navigating together. We hope that these difficult times, in addition to our work on this thesis, can serve as a reminder of the reality and tangibility of catastrophes. Such events are not something to dismiss because of their statistical rarity; rather, they are occurrences which affect all industries, all companies, and all people, and it is crucial that we do all we can to anticipate them and prepare for them.

Poisson-Pareto Model: A Calamity Simulation

The fallout of business interruption is a direct risk that companies like Kingston face in the event of a catastrophe. Such huge loss events can often limit a business's capability to operate at normal levels. Business interruption refers to this reduction in or total stoppage of operations and "the actual loss of income the insured sustains during the necessary suspension of its operations during the period of restoration" (Levin, 2008).

The Pareto distribution is a skewed, heavy-tailed power-law probability distribution. For the sake of conceptualizing an application for the Pareto model, a general visual of which is provided below in Exhibit 1.1, consider that the Pareto is often used to model the distribution of incomes in a population. The model reflects that the vast majority of the wealth in a population is typically owned by only a small minority of the population's people, and that the remaining minute minority of the population's wealth is dispersed across the majority of its people. This idea can be translated to the concept of catastrophic loss. The majority of a firm's possible total amount of loss resides in catastrophic risk—risk that is relatively much less likely to occur than the less severe risks that constitute a smaller portion of the firm's total loss amount.

***Exhibit 1.1*: Pareto Distribution**



I*mage credit*: https://valelab4.ucsf.edu/svn/3rdpartypublic/boost/libs/math/doc/sf_and_dist/graphs/pareto_pdf2.png

This study aims to utilize a Pareto simulation to do just this—to estimate the extent of business interruption and income loss that Kingston would suffer in the event of catastrophic loss. The first component of this model involves a simulation of the *frequency* of catastrophic loss that Kingston might face. Catastrophic loss events exhibit a discrete loss frequency distribution; that is, in any given time period—say, in one year—Kingston might experience no catastrophic loss, one catastrophic loss, or more than one catastrophic loss. To achieve a simulation of catastrophe frequency, we built a Poisson-distributed frequency model. The Poisson distribution is "a discrete probability distribution for the counts of events that occur randomly in a given interval of time"

(Filippi, 2015). Using the Poisson, we can determine the probability of observing *x* number of events in the given time interval with the equation:

$$P(X = x) = e^{-\lambda}\frac{\lambda^x}{x!}$$

Where:

x = 0, 1, 2, 3, etc. (the number of losses per year)
λ = the mean number of events per interval (the frequency of losses per year)

We first calculate the probability of each *x* value occurring in a given year, ranging from 0 catastrophic events to 16 catastrophic events, as shown below in Exhibit 1.2 in the column "P(X = x)." However, the infrequency of catastrophic loss quickly becomes clear, as shown in the column "P(X ≤ x);" there is nearly a 96% chance that the number of catastrophic events (*x*'s) in a given year will *not* exceed two events, and each *x* value above two approaches even closer to statistical impossibility.

***Exhibit 1.2*: Simulating Loss Frequency with Poisson Model**

Poisson Distributed Frequency Process

X ≡ number of losses per year
λ ≡ frequency of losses per year

λ 0.75
$p(x) = e^{-\lambda}\lambda^x/x!$

x	P(X = x)	P(X ≤ x)	Low	High	Value
0	0.47237	0.47237	0	0.47237	0
1	0.35427	0.82664	0.47237	0.82664	1
2	0.13285	0.95949	0.82664	0.95949	2
3	0.03321	0.99271	0.95949	0.99271	3
4	0.00623	0.99894	0.99271	0.99894	4
5	0.00093	0.99987	0.99894	0.99987	5
6	0.00012	0.99999	0.99987	0.99999	6
7	1.3E-05	1	0.99999	1	7
8	1.2E-06	1	1	1	8
9	9.8E-08	1	1	1	9
10	7.3E-09	1	1	1	10
11	5E-10	1	1	1	11
12	3.1E-11	1	1	1	12
13	1.8E-12	1	1	1	13
14	9.7E-14	1	1	1	14
15	4.8E-15	1	1	1	15
16	2.3E-16	1	1	1	16

Simulation of X	
Mean	0.70000
Variance	0.81000
Standard Deviation	0.90000
10 Simulated Time Periods	x
1	1
2	0
3	1
4	0
5	1
6	0
7	0
8	3
9	0
10	1

Once we have these probability values, we can conduct a simulation of X that includes ten simulated time periods. Our simulation results over the ten time periods are shown above in Exhibit 1.2. In five of the ten simulated years, *no* catastrophic loss events occurred. In four of the remaining five simulated years, *one* single catastrophic loss event occurred. In the final remaining simulated year, *three* catastrophic loss events occurred, representing what would be an extremely unlikely but very severe period for Kingston's operations.

Now that we have conducted a *frequency* simulation of Kingston's potential catastrophic loss events, we simulate loss *severity* to estimate how these simulated catastrophes would impact Kingston through business interruption and income loss. First, we use information from Kingston's income statement, as provided in the original case material, to estimate the company's net income before taxes and continuing expenses—the sum of which would represent Kingston's dollar loss amount in the event that a catastrophe completely halts normal operations. As shown below in Exhibit 1.3, from Kingston's income statement information we can determine each key business segment's net income before taxes, as well as each division's contribution to the company's overall net sales. All values are in thousands:

***Exhibit 1.3*: Contribution to Net Sales, By Division**

Net Sales Actual)	2019	2018	2017
Tools	$223,873.00	$266,211.00	$247,553.00
Diagnostics & repair information	$ 95,010.00	$ 92,638.00	$ 98,220.00
Equipment	$ 84,458.00	$ 89,308.00	$ 85,147.00
Totals	$403,341.00	$448,157.00	$430,920.00
Net Income before taxes (Actual)	$ 9,792.00	$ 38,420.00	$ 51,593.00
Tools (Estimated)*	$ 5,435.02	$ 22,821.97	$ 29,638.92
Diagnostics & repair information (Esimated)*	$ 2,306.58	$ 7,941.75	$ 11,759.64
Equipment (Estimated)*	$ 2,050.41	$ 7,656.28	$ 10,194.44

Contribution to Net Sales (Actual)	2019	2018	2017
Tools	55.50%	59.40%	57.45%
Diagnostics & repair information	23.56%	20.67%	22.79%
Equipment	20.94%	19.93%	19.76%

After calculating each division's net income before taxes and contribution to net sales, we need to estimate each division's continuing expenses. First, we estimate the company's total continuing expenses by summing: (1) 60% of the company's estimated utilities expense; (2) 42% of the company's actual salaried payroll; and (3) 100% of the company's interest expense. Once we calculate the company's total estimated continuing expenses, we make the assumption that each division's percentage contribution to net sales would be the same as its percentage contribution to the company's continuing expenses, as shown in Exhibit 1.4. Again, values in thousands:

***Exhibit 1.4*: Estimation of Continuing Expenses**

Continuing Expenses		
Continuing Selling, G&A Expenses:		
60% of Utilities (Estimated)	$ 8,894.40	$ 7,467.60
42% of Payroll (Actual salaried payroll)	$ 27,951.42	$ 29,631.42
100% of Interest Expense	$ 6,066.00	$ 5,329.00
Total	$ 42,911.82	$ 42,428.02
Continuing Expenses (Estimated)		
Tools (Estimated)*	$ 23,818.05	$ 25,202.79
Diagnostics & repair information (Esimated)*	$ 10,108.20	$ 8,770.25
Equipment (Estimated)*	$ 8,985.56	$ 8,454.99

Now, we are able to find the sum of estimated net income before taxes and continuing expenses for the previous two periods (years 2018 and 2019) and use these values to reach the expected value of net income before taxes and continuing expenses in 2020, which we placed at $60,000,000. We can move forward with this value to use as an average claim size within our Pareto-based loss severity simulation.

Pareto distributions are commonly used to represent heavy-tailed loss possibilities, such as catastrophes. For example, a 2013 study reviewing data of "earthquake disaster loss from 1969 to 2011" yielded a distribution featuring "the characteristics of right skew peak, excess kurtosis and heavy-tail;" the study identified the Pareto distribution as fitting "the earthquake loss perfectly," and significantly improving estimation precision (Pu & Pan, 2013). The use of a truncated Pareto allows us to incorporate decision strata into our Pareto work. A truncation point of 100,000, for instance, allows us to denote a loss in excess of $100,000,000 as having moved into a different decision stratum (e.g., to insure a risk rather than to retain it, or to insure it in an excess layer of insurance rather than a primary layer). Defining the average size of a loss below the truncation point—the $60,000,000 sum of net income before taxes and continued expenses referenced above—helps us to simulate realistic loss amounts within that stratum. Beyond the truncation point, our choice of Pareto location and shape parameters help us to simulate realistic loss amounts in the Pareto (tail of the distribution) stratum or multiple Pareto strata.

t = truncation point = 100,000
s = average loss size of losses below truncation point of 100,000 = 60,000
p = probability loss is smaller than truncation point = 0.40
β = pareto location (scale) parameter = 600,000
α = pareto shape parameter = 4.00

In our model, F*(y) is a random number (between 0 and 1) generated by Excel that can be used to simulate a loss amount. If the random number (F*(y)) is less than or equal to the probability *p* that the loss is smaller than the truncation point (i.e., falls within the lowest loss stratum and is thus not in the tail), we use one probability density function (pdf) to convert the random number into a corresponding simulated loss amount (such as the pdf associated with the normal or exponential distribution). If the random number (F*(y)) is greater than the probability *p* of the loss being smaller than the truncation point (i.e., falls within a higher loss stratum and is thus in the tail), we use a different probability density function (pdf)—the Pareto—to convert the random number into a corresponding simulated loss amount.

F**(y) and F***(y) are transformations of F*(y)—the initial random number generated—that we use to convert to higher simulated loss amounts (i.e., our Pareto loss amounts). Some of these simulated Pareto losses may fall just above the truncation point but others can fall quite far out into the tail of the Pareto distribution, indicating a catastrophic loss amount. The location and shape parameters chosen for the Pareto determine the likelihood of smaller and larger loss amounts being simulated beyond the truncation point. Once we possess simulated values for F*(y) and F**(y), we can calculate Kingston's simulated dollar loss amount F***(y) in Excel with:

$$= Exp\{ln(t + \beta) - [ln(1 - F^{**}(y)) - ln(1 - p)]/\alpha\} - \beta$$

Exhibit 1.5 below shows a simulation of F***(y), which provides Kingston with a tangible dollar loss amount that it would face with each catastrophic event. Naturally, the 8th period trial—with three simulated catastrophes in the same year—would be especially damaging to Kingston's operations, with a total business interruption and income loss of approximately $2.8 billion. As illustrated in this Kingston case, we envision this Poisson-Pareto model as a valuable tool for any group that wishes to simulate its catastrophe risk and loss exposure so that it can develop an adequate plan for those difficult times.

***Exhibit 1.5*: Pareto Loss Severity Simulation**

Simulation of a Pareto Density Function

Y ≡ Severity of losses	
$F(y) = 1 - (1-p)\,[(t + \beta)/(y + \beta)]^{\alpha}$, y>t	
F*(y) = Rand()	
F**(y) = F(600,000) + F*(y)·[1-F(600,000)]	
F***(y)	
= Exp{ln(t + β) – [ln(1 – F**(y)) – ln(1 – p)]/α} – β	
0 ≤ F*(y) ≤ 1	
t = truncation point = 100,000	100000
s = average claim size of losses below truncation point of $100,000	60000
p = probability claims are smaller than truncation point = 0.40	0.4
β = pareto location parameter = 600,000	600000
α = pareto shape parameter = 4	4
F(600,000)≡Prob(Loss < Strata Limit)	0.956628111

		Trial number of loss amount: F***(y)							
10-Year Period Trials	Losses	1	2	3	4	5	6	7	Total
1	1	808,895	-	-	-	-	-	-	**808,895**
2	0	-	-	-	-	-	-	-	**-**
3	1	1,237,426	-	-	-	-	-	-	**1,237,426**
4	0	-	-	-	-	-	-	-	**-**
5	1	910,737	-	-	-	-	-	-	**910,737**
6	0	-	-	-	-	-	-	-	**-**
7	0	-	-	-	-	-	-	-	**-**
8	3	1,254,876	760,371	807,723	-	-	-	-	**2,822,970**
9	0	-	-	-	-	-	-	-	**-**
10	1	821,925	-	-	-	-	-	-	**821,925**
Total	**7**								

Value-at-Risk Simulation

In its typical form, value-at-risk statistically measures the riskiness of an entity or asset portfolio. VaR modeling estimates the potential loss in value within the firm or portfolio over a specific period of time and at a predetermined confidence interval. In this study, we designed a customized VaR-based simulation that estimates Kingston's value-at-risk to catastrophic loss based on the asset value and returns of the company's three key operating segments—Tools, Diagnostics and Repair Information, and Equipment.

To begin this analysis, we gather the excess returns for each of Kingston's three key divisions over the last 48 months, benchmarked against an industry index. For modeling purposes, we take the natural logarithm of these excess return values to achieve an approximation of a normal distribution. The average of each division's "Ln of excess returns" is represented by "Ln of Asset Return" in Exhibit 2.1. Likewise, the variance of each division's log of excess returns is represented by "Variance" in Exhibit 2.1. Using these two values, we generate each division's expected return. Asset values in the exhibit below are in thousands:

***Exhibit 2.1*: Division Asset Values and Returns**

	Tools	Info	Equip
Asset Value	29253	12415	11036
Ln of Asset Return	0.01291	0.00828	0.02986
Variance	0.00524095	0.00289	0.00266
Expected Return	0.01565424	0.00978	0.03169
Time Months	3		
Confidence Level	95.00%		
z-value	1.64485363		

Next, we construct a correlation matrix (Exhibit 2.2) among Ln Excess Returns for each division over the 48-month period; the matrix correlates each division's set of Ln Excess Returns against the same respective values over the same period for the two other divisions, as well as against itself—hence the presence of the value "1" at the three conjunctions in the matrix where a division is pitted against itself.

***Exhibit 2.2*: Correlation Matrix**

	Tools	Info	Equip
Tools	1	0.50253	0.23379
Info	0.50253359	1	0.20048
Equip	0.23378737	0.20048	1

At this point, we have the values necessary to calculate not only each division's relative and absolute value-at-risk, but also the entire portfolio's—the firm's—relative and absolute VaR.

The relative VaR calculation for each asset, or division, is as follows:

$$Relative\ VaR = Asset\ Value \times z - value \times \sqrt{Variance \times Time}$$

Where:

- *"Asset Value" refers to the division's asset value*
- *"z-value" refers to the z-value corresponding with our chosen confidence interval*
  **in this case, an internal of 95%*
- *"Variance" refers to the division's variance of Ln Excess Returns*
- *"Time" refers to our chosen time period*
  **in this case, one quarter (3 months)*

The absolute VaR calculation for each division is as follows:

$$Absolute\ VaR = Relative\ VaR - (Asset\ Value \times Expected\ Return \times Time)$$

To calculate the portfolio's relative VaR, we utilize Excel's "MMULT" function to construct a multiplication matrix between the previously mentioned correlation matrix and the values of each division's relative VaR, and then take the square root of the product to reach the portfolio relative VaR calculation. We follow the same process to calculate the portfolio's absolute VaR, except the multiplication matrix includes the correlation matrix and the values of each division's *absolute* VaR, rather than relative. Our portfolio VaR calculations are shown below in Exhibit 2.3, in thousands.

***Exhibit 2.3*: VaR Calculations**

VaR Asset Rel		6033.41757	1901.5	1621.81
VaR Asset Abs		4659.61715	1537.39	572.731
VaR Port Rel		7745.3548		
VaR Port Abs		5762.3322		

Absolute VaR is simple VaR calculated, but with respect to a mean of zero, as the maximum loss that can occur at a certain confidence level over a specific period of time. Relative VaR is typically given at a 95% confidence level as 1.645*volatility*Value of Portfolio, whereas Absolute VaR will take into consideration the overall loss, including the gain from the positions that can be expected for a given confidence level. Absolute VaR is the loss relative to zero (0) and relative VaR is the loss compared to the mean, μ. One way in which the Absolute VaR is particularly useful lies in its ability to consider potential loss against what would otherwise have been the expected future gain, rather than just considering potential loss against the current financial position (which is what Relative VaR does). As such, Kingston can consider the "Var Port Abs" calculation to mean that at 95% confidence, and with consideration of what would have otherwise been expected future gain under normal circumstances, it faces a possible loss of approximately $5.8 million to firm value over a three-month time period due to catastrophe.

Discussion

The simulation methods that we have provided—the Poisson-Pareto simulation of catastrophic loss frequency and severity to estimate the extent of Kingston's business interruption and income loss risk due to catastrophe, and the VaR simulation to estimate how much of Kingston's firm value is at risk over a certain time period due to catastrophe—combine to provide an in-depth illustration of Kingston's catastrophic risk. We hope that they can serve together as a useful device for any group aiming to simulate their catastrophic loss exposures and develop a plan to manage these large risks.

We encourage, for Kingston and other firms alike, the adoption of key risk management strategies that can help a group navigate catastrophe, especially as we all endure this current COVID-19 situation. In a 2018 study of catastrophic risk management strategies across both domestic and international publicly-traded firms, Howard Kunreuther and Michael Useem find that firms "who have already put in place a risk management strategy that enables them to take deliberative actions in response to an adverse event are better prepared to recover from that disruption and stay true to their firm's core values." For example, a company that has developed a thorough business-continuity plan will be better prepared to conduct productive, albeit limited, post-catastrophe operations than a company that did not exhibit the foresight to develop such a plan. Their study identifies several steps that can be taken towards mastering catastrophic risk.

As we discussed earlier in this paper, decision makers should resist the temptation to "perceive the likelihood of a disastrous event to be so small that they view it below their threshold level of concern" (Kunreuther & Useem, 2018)—do not assume that "this will not happen to us" just because of how unlikely a catastrophic event might seem. Companies should adopt long-term mindsets and “stretch time horizons,” as getting stuck in a short-term perspective can make it hard to look beyond the high upfront costs that might be required to establish the protective measures to adequately defend against catastrophic events in the future. Scenario planning and sensitivity analyses can help a firm gauge how its operations will be impacted by loss scenarios of different severities. Lastly, we hope that companies follow Kunreuther and Useem’s advice to view risk management as a long-term, value-creating investment for the firm by “creating sustainable value and protecting the firm and its reputation, rather than a short-run burden on management’s time and the company’s budget” (Kunreuther & Useem, 2018). A risk management department’s ability to develop a quality strategy of preparation for and defense against catastrophe can be the difference between demise and survival.

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APPENDIX A - Net Income + Continuing Expenses Estimation

(all values in thousands)

Mitchell & Medders

Net Sales Actual)	2019	2018	2017
Tools	$ 223,873.00	$ 266,211.00	$ 247,553.00
Diagnostics & repair information	$ 95,010.00	$ 92,638.00	$ 98,220.00
Equipment	$ 84,458.00	$ 89,308.00	$ 85,147.00
Totals	$ 403,341.00	$ 448,157.00	$ 430,920.00

Contribution to Net Sales (Actual)	2019	2018	2017
Tools	55.50%	59.40%	57.45%
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Net Income before taxes (Actual)	$ 9,792.00	$ 38,420.00	$ 51,593.00
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Continuing Expenses		
Continuing Selling, G&A Expenses:		
60% of Utilities (Estimated)	$ 8,894.40	$ 7,467.60
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Continuing Expenses (Estimated)		
Tools (Estimated)*	$ 23,818.05	$ 25,202.79
Diagnostics & repair information (Esimated)*	$ 10,108.20	$ 8,770.25
Equipment (Estimated)*	$ 8,985.56	$ 8,454.99

*Assuming % contribution of each division equals % contribution to net sales

Net Income BT + Continuing Expenses (Estima	2019	2018
Tools	$ 29,253.07	$ 48,024.76
Diagnostics & repair information	$ 12,414.78	$ 16,712.00
Equipment	$ 11,035.97	$ 16,111.26
	$ 52,703.82	$ 80,848.02

2020 Expected Value	
Net Income BT + Continuing Expenses	**$60,000**

(Exponentially distributed)

APPENDIX B - Poisson-Pareto Simulation

Poisson Distributed Frequency Process

Mitchell & Medders

**note that the Pareto Appendix included here (and subsequent uses of the live Excel workbook) will show different values than are shown in the Exhibits included in thesis write-up, as the simulation produces new values each time it runs*

X ≡ number of losses per year
λ ≡ frequency of losses per year

λ 0.75
$p(x) = e^{-\lambda}\lambda^{x}/x!$

x	P(X = x)	P(X ≤ x)	Low	High	Value
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9	9.8E-08	1	1	1	9
10	7.3E-09	1	1	1	10
11	5E-10	1	1	1	11
12	3.1E-11	1	1	1	12
13	1.8E-12	1	1	1	13
14	9.7E-14	1	1	1	14
15	4.8E-15	1	1	1	15
16	2.3E-16	1	1	1	16

Simulation of X

Mean	0.60000
Variance	0.44000
Standard Deviation	0.66332

10 Simulated Time Periods	x
1	0
2	0
3	1
4	1
5	0
6	0
7	0
8	1
9	1
10	2

Simulation of a Pareto Density Function

Mitchell & Medders

Y ≡ Severity of losses
F(y) = 1 – (1-p) [(t + β)/(y + β)]$^{\alpha}$, y>t
F*(y) = Rand()
F**(y) = F(600000) + F*(y)·[1-F(600000)]
F***(y) = 60000, F*(y) ≤ 0.40
= Exp{ln(t + β) – [ln(1 – F**(y)) – ln(1 – p)]/α} – β
0.40 ≤ F*(y) ≤ 1

Sample Simulation of Y

Trial	1	2	3	4	5	6	7
F*(y)	0.440466971	0.483602811	0.767204421	0.692921572	0.223888843	0.241778	0.176673
F**(y)	0.975731996	0.977602878	0.989903216	0.986681429	0.966338593	0.967114	0.964291
F***(y)	960908.1079	992530.6555	1343524.799	1213516.208	60000	60000	60000

t = truncation point = 100,000	100000	
s = average loss size of losses below truncation point of $100,000	60000	0.4
p = probability loss is smaller than truncation point = 0.40	0.4	
β = pareto location (scale) parameter	600000	
α = pareto shape parameter = 4.00	4	
F(600,000)≡Prob(Loss < Strata LImit)	***0.9566***	

Using frequency simulation results

OR

10 Period Trials	Losses	Trial number of loss amount: F*(y) 1	2	3	4	5	6	7
1	0							
2	0							
3	1	0.500226133						
4	1	0.991532949						
5	0							
6	0							
7	0							
8	1	0.987581937						
9	1	0.856323411						
10	2	0.282257573	0.19372053					

Simulation of a Pareto Density Function

Mitchell & Medders

Y ≡ Severity of losses

$F(y) = 1 - (1-p)\,[(t + \beta)/(y + \beta)]^{\alpha}$, y>t

$F^*(y) = Rand()$

$F^{**}(y) = F(600000) + F^*(y)\cdot[1-F(600000)]$

$F^{***}(y) = 60000,\ F^*(y) \le 0.40$

$= Exp\{\ln(t + \beta) - [\ln(1 - F^{**}(y)) - \ln(1 - p)]/\alpha\} - \beta$

$0 \le F^*(y) \le 1$

t = truncation point = 100,000	100000
s = average claim size of losses below truncation point of $100,000	60000
p = probability claims are smaller than truncation point = 0.40	0.4
β = pareto location parameter = 2,000,000	600000
α = pareto shape parameter = 4.00	4
F(750,000)≡Prob(Loss < Strata Limit)	0.956628

Sample Simulation of Y

Trial	1	2	3	4	5	6	7
F*(y)	0.314207097	0.802007354	0.806700542	0.574702131	0.24772543	0.027102	0.035464
F**(y)	0.975731996	0.991412685	0.991616237	0.981554028	0.967372431	0.957804	0.958166
F***(y)	60000	1423818.23	1435992.197	1071708.178	60000	60000	60000

10-Year Period Trials	Losses	Trial number of loss amount: F**(y)						
		1	2	3	4	5	6	7
1	0							
2	0							
3	1	0.978323863						
4	1	0.999632768						
5	0							
6	0							
7	0							
8	1	0.999461405						
9	1	0.993768475						
10	2	0.968870155	0.965030136					

Simulation of a Pareto Density Function

Mitchell & Medders

Y ≡ Severity of losses

$F(y) = 1 – (1\text{-}p) [(t + \beta)/(y + \beta)]^{\alpha}$, y>t

F*(y) = Rand()

F**(y) = F(600,000) + F*(y)·[1-F(600,000)]

F***(y)

= Exp{ln(t + β) – [ln(1 – F**(y)) – ln(1 – p)]/α} – β

0 ≤ F*(y) ≤ 1

t = truncation point = 100,000	100000
s = average claim size of losses below truncation point of $100,000	60000
p = probability claims are smaller than truncation point = 0.40	0.4
β = pareto location parameter = 600,000	600000
α = pareto shape parameter = 4	4
F(600,000)≡Prob(Loss < Strata Limit)	0.956628111

		Trial number of loss amount: F***(y)							
10-Year Period Trials	Losses	1	2	3	4	5	6	7	**Total**
1	0	-	-	-	-	-	-	-	**-**
2	0	-	-	-	-	-	-	-	**-**
3	1	1,005,611	-	-	-	-	-	-	**1,005,611**
4	1	3,850,417	-	-	-	-	-	-	**3,850,417**
5	0	-	-	-	-	-	-	-	**-**
6	0	-	-	-	-	-	-	-	**-**
7	0	-	-	-	-	-	-	-	**-**
8	1	3,444,085	-	-	-	-	-	-	**3,444,085**
9	1	1,592,741	-	-	-	-	-	-	**1,592,741**
10	2	866,701	824,664	-	-	-	-	-	**1,691,365**
Total	**6**								

APPENDIX C - VaR Calculations

VaR Solutions **Mitchell & Medders**

Portfolio Detail

	Tools	Info	Equip
Asset Value	29253	12415	11036
Ln of Asset Return	0.01291	0.00828	0.02986
Variance	0.00524	0.00289	0.00266
Expected Return	0.01565	0.00978	0.03169

Portfolio Analysis Criteria

Time Months	3
Confidence Level	95.00%
z-value	1.64485

	Tools	Info	Equip
Tools	1	0.50253	0.23379
Info	0.50253	1	0.20048
Equip	0.23379	0.20048	1

Portfolio VaR Analysis Results

VaR Asset Rel	6033.42	1901.5	1621.81
VaR Asset Abs	4659.62	1537.39	572.731

VaR Port Rel	7745.35
VaR Port Abs	5762.33

Exhibit **Mitchell & Medders**

Market Data
Excess Returns for Industry Index + 3 Divisions Over 48 Months

Month	Industry	Tools	Info	Equip
1	1.038731	0.980199	1.077884	1.077884
2	0.974725	0.962809	1.052323	1
3	1.003506	0.910374	1.033654	1.039459
4	1.02593	1.087194	1.074118	0.943461
5	1.10506	1.127835	0.997703	1.122659
6	1.040395	1.011162	1.040082	1.087411
7	1.004309	1.155346	1.017959	1
8	1.042477	0.979905	1.063005	1.022448
9	0.96464	0.926075	0.940165	0.964737
10	1.029322	1.09834	1.042477	1.004008
11	1.0008	1.009848	0.936786	1.058656
12	0.960886	0.995112	0.960213	0.969185
13	1.002303	0.936318	0.960886	1.076591
14	0.963773	0.910283	1.012275	1.011566
15	1.013896	1.002904	1.04446	0.971805
16	1.054535	1.099439	1.016332	1.027368
17	1.037901	1.084696	1.011465	0.988171
18	0.989852	0.941011	0.941011	1.052007
19	1.023471	1.026136	1.039147	0.993124
20	1.019793	0.994913	0.968119	1.069509
21	0.972777	0.913018	0.907738	1.112155
22	0.972194	1	1.055801	1
23	0.965509	0.931276	0.95466	1
24	1.044251	1.22067	1.032931	1.056541
25	1.004912	1.018061	1.053692	1.003707
26	0.996108	0.958103	0.932674	1.007427
27	1.07907	1.125357	1.08937	1.06844
28	1.020303	1	1.09812	1.044042
29	1.017451	1.004711	1.009041	1.036967
30	1.045087	1.087846	1.041019	1.054008
31	0.982161	1.057598	0.962135	1.036967
32	1.057598	1.071865	0.99551	1.16416
33	1.049381	1.038627	1.103735	1.085999
34	1.017959	1.06812	0.971805	0.968797
35	1.034585	1.043416	1.028087	1.043312
36	0.995709	0.996606	1.012376	1.033034
37	1.092862	1.17539	1.096474	1.107827
38	1.018978	1.054113	1.057809	1.102963
39	1.078423	1.025315	1.062793	1.071222
40	1.082962	1.023369	1.050746	1.124569
41	0.924595	0.857443	0.915669	0.950659
42	0.975505	0.979807	0.909919	1.043938
43	0.984915	0.966958	0.978142	0.994117
44	0.996307	1.026752	1.04697	1.014301
45	0.962809	0.923116	0.979317	0.991834
46	0.989852	1.023574	0.996207	0.897628
47	1.024393	0.967345	0.984816	1.058973
48	0.963291	0.953897	0.911194	0.967248

Coversion of Return into Approximation of Normal Distribution
Ln Excess Returns for Industry + 3 Divisions Above

Month	Industry	Tools	Info	Equip
1	0.03800	-0.02000	0.07500	0.07500
2	-0.02560	-0.03790	0.05100	0.00000
3	0.00350	-0.09390	0.03310	0.03870
4	0.02560	0.08360	0.07150	-0.05820
5	0.09990	0.12030	-0.00230	0.11570
6	0.03960	0.01110	0.03930	0.08380
7	0.00430	0.14440	0.01780	0.00000
8	0.04160	-0.02030	0.06110	0.02220
9	-0.03600	-0.07680	-0.06170	-0.03590
10	0.02890	0.09380	0.04160	0.00400
11	0.00080	0.00980	-0.06530	0.05700
12	-0.03990	-0.00490	-0.04060	-0.03130
13	0.00230	-0.06580	-0.03990	0.07380
14	-0.03690	-0.09400	0.01220	0.01150
15	0.01380	0.00290	0.04350	-0.02860
16	0.05310	0.09480	0.01620	0.02700
17	0.03720	0.08130	0.01140	-0.01190
18	-0.01020	-0.06080	-0.06080	0.05070
19	0.02320	0.02580	0.03840	-0.00690
20	0.01960	-0.00510	-0.03240	0.06720
21	-0.02760	-0.09100	-0.09680	0.10630
22	-0.02820	0.00000	0.05430	0.00000
23	-0.03510	-0.07120	-0.04640	0.00000
24	0.04330	0.19940	0.03240	0.05500
25	0.00490	0.01790	0.05230	0.00370
26	-0.00390	-0.04280	-0.06970	0.00740
27	0.07610	0.11810	0.08560	0.06620
28	0.02010	0.00000	0.09360	0.04310
29	0.01730	0.00470	0.00900	0.03630
30	0.04410	0.08420	0.04020	0.05260
31	-0.01800	0.05600	-0.03860	0.03630
32	0.05600	0.06940	-0.00450	0.15200
33	0.04820	0.03790	0.09870	0.08250
34	0.01780	0.06590	-0.02860	-0.03170
35	0.03400	0.04250	0.02770	0.04240
36	-0.00430	-0.00340	0.01230	0.03250
37	0.08880	0.16160	0.09210	0.10240
38	0.01880	0.05270	0.05620	0.09800
39	0.07550	0.02500	0.06090	0.06880
40	0.07970	0.02310	0.04950	0.11740
41	-0.07840	-0.15380	-0.08810	-0.05060
42	-0.02480	-0.02040	-0.09440	0.04300
43	-0.01520	-0.03360	-0.02210	-0.00590
44	-0.00370	0.02640	0.04590	0.01420
45	-0.03790	-0.08000	-0.02090	-0.00820
46	-0.01020	0.02330	-0.00380	-0.10800
47	0.02410	-0.03320	-0.01530	0.05730
48	-0.03740	-0.04720	-0.09300	-0.03330