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The purpose of this study is to explore the importance of linear programming techniques in economic policy-making, particularly in underdeveloped countries.

In the first chapter we review the general framework of linear programming and its techniques, particularly the simplex algorithm, duality and sensitivity analysis.

The second chapter discusses development planning models that utilize linear programming techniques extensively. We study the following models:

1. Eckaus Model

This is a dynamic linear programming model with 10 periods and 30 years, that was developed for India. The model mainly focuses on the problems of determining the optimal levels of savings and investment over time, and the related problems of intersectoral and intertemporal distribution of investment and output and use of foreign exchange resources.

2. Adelman Model

This is a 4 period, 20 year dynamic programming model, with investment in the education sector is optimized simultenously with investment in real capital. The model was intented for Argentine. It mainly focuses on the determination of the optimal extent and composition of resource allocation to education.

3. Blitzer Model

This is a 5 period, 15 year dynamic programming model that was

developed for Turkey. It includes a current account interindustry matrix, capital coefficients, trade balance improvement activities, and macroeconomic variables. In its formulation it maximizes the level of gross domestic product at the terminal year. It attempts to merge economic and human resources planning through a general equilibrium approach which incorporates both into the same model.

At the end of the second chapter we conclude that the planning models using programming techniques are superior tools for policy-makers, and continuous efforts are necessary to improve the techniques and the models for the achievement of higher economic performance and development.

LINEAR PROGRAMMING AND
DEVELOPMENT PLANNING MODELS

by

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CHAPTER I

INTRODUCTION

Historically optimal allocation has been the major concern in the economic analysis. Such problems were dealt with classical optimization techniques such as differential calculus or the calculus of variations. A new class of optimization models has since become of considerable interest, related to problems of optimum allocation of limited resources in a given state of the economy. These new models are different in that they employ new solution techniques to arrive in their solutions. The most flourishing of these methods are linear programming, input-output analysis and game theory.

The first to be developed was the game theory by John Von Neumann.[1] The theory of games attempts to study economic behaviour by concentrating on individuals or groups with conflicting interests. Neumann showed that under certain assumptions each participant can act so as to be guaranteed at least a certain minimum gain or maximum loss. When each participant acts so as to secure his minimum guaranteed return, then he prevents his opponents from attaining any more than their minimum guaranteeable gains. Thus the minimum gains become the actual gains, and the actions and returns for all the participants are determinate.[2]

The second method that was developed by W. Leontief is input-output analysis.[3] Leontief's input-output analysis is based on the idea that a very considerable proportion of the efforts of an economy is devoted to the production of intermediate goods, and the output of intermediate goods is closely linked to the output of final products. A change in the output of any final product implies changes in the outputs of the intermediate goods used in producing that final product and indeed in producing goods used in producing those intermediate goods, and so on.[4]

In its original version, input-output analysis dealt with an entirely closed economic system--one in which all goods were intermediate goods, consumables being regarded as the intermediate goods needed in the production of personal services. Equilibrium in such a system exists when the outputs of the various products are in balance in the sense that just enough of each is produced to meet the input requirements of all the others.

The focal point of input-output analysis is an array of coefficients variously called the "input-output matrix." A column of this matrix represents the input requirements of various commodities for the production of one unit of a particular commodity. There is exactly one column for each commodity produced in the economy. Thus the production of a commodity corresponds to the concept of an activity in a linear programming model. The input-output analysis makes it possible to determine each industry's rate of output to meet specified direct demand by the people and also to trace the indirect effect on each industry. After the second World War interests shifted to a different view of Leontief's model. In this view final demand is regarded as being exogenously determined, and input-output analysis is used to find levels of activity in the various sectors of the economy consistent with the specified final demand.

The linear programming method is among the most important scientific advances of the mid-twentieth century. In 1939 the Russian mathematician Kantorovich[5] formulated production problems as linear programming problems and suggested a possible way of solving such models. He examined a wide range of problems of organization and planning of production where the problem consisted in selecting the optimal one among a large number of different alternatives. In his works he showed that it is theoretically possible to apply mathematical methods in such types of economic problems as that of assignment of machine-time to different jobs or of land to different types of cultivation, the planning of transportation, the processing of complex raw materials, etc.

Between the years 1947-1949 intensive research on linear programming began in the United States. At first it was organized to respond to military problems, but soon it acquired a wider scope. The results of these works found diverse applications in fields of industrial planning. The primary contributor in solving linear programming problems was George Dantzig. His general algorithm is known as the simplex algorithm, developed in 1947.[6] Later on, interest in this area grew, and in 1949 T. C. Koopmans organized in Chicago the Cowles Commission Conference on Linear Programming. The papers presented in this conference were collected by Koopmans in 1951 in the book entitled _Activity Analysis of Production and Allocation_.[7] A. Charnes, W. W.

Cooper, and A. Henderson wrote the first book on linear programming.[8] They have also worked on transportation problems, and Charnes and Cooper applied linear programming to oil refining industry (the problem of the optimal mixture of different kinds of petrol is examined). Charnes and Cooper also studied different variants of the warehousing and of the stock-management problems.[9] In 1956, Dantzig, Ford and Fulkerson worked out a method for the simultaneous solution of the primal and the dual problem of linear programming. The first interpretation of the transportation problem is by Hitchcock.[10] This problem was also investigated by Koopmans.[11] The solution however to the transportation problem by Simplex Method is given by Dantzig.[12] Linear programming is widely applied in the so-called theory of the firm in the works of Dorfman, Samuelson and Solow.[13] Since the early works on linear programming, there has been a wide range of research with applications in fields of economics, engineering, statistics, mathematics and business.

Briefly, linear programming deals with the problem of allocating limited resources among competing activities in the best optimal way. The variety of situations to which the description applies is diverse, ranging from allocation problems to inventory problems and so on. However, the one common point in each of these situations is the necessity for allocating resources to activities. The linear programming models applied in business and industry can be classified into groups according to the processes involved in their use. G. Roccaferrera, in his <u>Introduction to Linear Programming Processes</u>, classifies these models into eight major groups.[14] These groups are:

1. Allocation, Transportation and Distribution Processes

In this type of processes the problem becomes one of combining activities and resources in order to maximize the overall effectiveness, or to minimize the total cost of allocating, distributing or shipping demanded quantities of products produced in several plants.

2. Inventory Processes

Inventory type problems deal with the question of how many items should be produced, ordered or stored in a given period of time, how and under what conditions these items should be produced, ordered or stored, for balancing the inventory carrying costs against the run setup costs, storage costs, and all costs associated with the changing of level of production or purchasing.

3. Sequencing and Scheduling Processes

Scheduling type problems simply try to find the optimal scheduling of the sequence of the operations in production or assembling items.

4. Routing Processes

Routing type problems deal with the routing of a person through a sequence of locations with the purpose of minimizing the distance that he travels or minimize the total cost of movement.

5. Quening Processes

In this type processes the problem occurs as a result of the arrivals of units to one or more service facilities in order to receive a service. The objective is to minimize the time wasted by units waiting for service, and to minimize the time wasted by servicemen in waiting to render services.

6. Search Processes

Search processes deal with solving assignments or location problems by maximizing the efficiency of the assigned activity, or minimizing relative distances, in case of location search.

7. Replacement Processes

Here the problem is simply to find out the appropriate time to replace working equipment or machine parts.

8. Competitive Processes

The problems involved in this case are similar to the situation existing in a game. The decision of the player influences the decision of the other players. In business the decision of a firm may influence the behaviour of the competitors.

A game is specified by the number of players, the rule of the game, and the rewards or losses involved. Each player has his own strategy and tactics to observe, and a payoff is always associated with each possible flow of decisions. The minimization of the total loss, or the maximization of the return, are the objectives of the decision makers.

LINEAR PROGRAMMING: DEFINITION, EVOLUTION AND TECHNIQUE

The term linear programming evolved from a series of earlier names for a technique which selects the best program from a series of feasible alternatives. This program has to do with the allocation of limited resources in a manner that maximizes or minimizes some objective of the planner. For instance, the planner for a firm may be concerned with the best production mix of items for a given time period knowing both production requirements as well as equipment availability. Using linear programming this problem and many similar others can be solved vey easily.

The essence of linear programming method is to serve as a tool in helping the planner (decision maker) when he is facing a problem that cannot be solved solely by past experience. Linear programming models are by-products of the scientific method of solving problems in which an optimum solution is sought from among many possible ones which are subject to a set of constraints. For example, problems of determining the optimal mix of products under given selling prices and known purchasing investment, problems concerned with productivity in relation to labour and machine capacities, problems involved with the determination of the optimum storage or distribution of commodities, problems seeking to minimize the time usage of existing machines, problems of maximizing the firm's profits, or of optimizing labour allocation, and many other production or economic problems.

A businessman may define linear programming techniques as useful tools for seeking from among many solutions one which matches his clealy stated objectives. On the other hand, an economist may define linear programming as a method for allocating a group of limited resources in a manner which satisfies a certain group of competing demands under known and fixed limitations. No matter how it is defined, certain basic requirements must be present before this technique can be used in the solution of a problem.

It is necessary for the organization to have a well stated objective which it is attempting to achieve, in order to use linear programming. It may be necessary to find a way to produce a certain order at the least cost using a given limited amount of productive factors; or to find the way for obtaining the highest profit by utilizing only the resources available under certain conditions; or to determine the best distribution of the productive factors within a fixed period of time. These are well stated objectives. The resources of the system which are to be allocated for the objective of the organization must be limited in supply. There must be a series of feasible alternatives available to the organization. From among the elements considered--for instance, men, machinery, money, methods, and markets--it must be possible to make a selection for reaching a solution which satisfies the objective of the organization. It may be possible to choose between the use of manpower and machinery or to choose among workers from skilled personnel, the use of special machines, and the application of a process taken from a different set of possible processes. All relationships representing the objective as well as the resource limitations considerations must be expressed with inequalities or equations, which must be linear in nature.

We mentioned that the organization must be able to establish a goal or an objective, in terms of an objective function. Suppose a firm is attempting to maximize profits obtained through production of two separate goods, A and B. Total profit then is a function of the number of goods sold. Suppose that for each unit of good A that is sold there is a profit of \$2, and for each unit of good B sold a profit of \$1 is realized. (Here the price level is assumed to be constant.) Therefore, total profit resulting from the sale of these goods, P, is given by the expression

$$P = 2A + 1B \quad ,$$

where A represents the number of units of good A sold, and B represents the number of units of good B sold. This relationship formulates the objective of the organization, namely to earn profit through producing and selling these two goods. The objective function is a linear relationship between the profit and the sales level of each of these goods.

We have also mentioned above that the resources to be allocated for achievement of the objective of the organization must be limited in supply. The mathematical relationship which explains this limitation is called an equation or an inequality. The limitation itself is referred to as a constraint. Let us continue with the example. If the total cost to produce the two goods are not to exceed \$100 and it costs \$5 to produce the first good and \$6 for the second good, then the relationship that states the cost constraint will be

$$5A + 6B \leq 100$$

The linear programming problem is, then, to choose the optimal values of A and B.

GENERAL FRAMEWORK OF LINEAR PROGRAMMING PROBLEMS

Linear programming can be used when a problem under consideration can be described by a linear objective function, to be maximized or minimized, subject to linear constraints which may be expressed as equalities or inequalities. When completely written out, a maximization problem in n variables and subject to m constraints will appear as follows:

$$\text{Maximize} \quad z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

subject to

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &\leq b_2 \\ \ldots \quad \ldots \quad\quad \ldots \quad &\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &\leq b_m \end{aligned}$$

$$\text{and} \qquad x_j \geq 0 \qquad \text{for all } j = 1,2,\ldots,n$$

A substantial saving in space can be achieved by expressing the linear programming problem in summation notation. The problem appears as follows:

$$\text{Maximize} \quad z = \sum_{j=1}^{n} c_jx_j$$

$$\text{subject to} \quad \sum_{j=1}^{n} a_{ij}x_j \leq b_i \qquad (i = 1,2,\ldots,m)$$

$$\text{and} \qquad x_j \geq 0 \quad (j = 1,2,\ldots,n)$$

A linear programming problem can also be written in matrix form. If we define the following matrices:

$$c = \begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ c_n \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \quad A = \begin{bmatrix} a_{11} & a_{12} \cdots a_{1n} \\ a_{21} & a_{22} \cdots a_{2n} \\ \cdot & \cdot \quad\quad \cdot \\ \cdot & \cdot \quad\quad \cdot \\ a_{m1} & a_{m2} \cdots a_{mn} \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_m \end{bmatrix}$$

The objective function can be expressed by the equation

$$z = c'x$$

And the set of constraints can be summarized in a single inequality as follows:

$$Ax \leq b$$

Similarly, we can express the nonnegativity restrictions by the single inequality

$$x \geq 0.$$

So the linear programming problem in matrix notation can be expressed as follows:

$$\begin{aligned} &\text{Maximize} \quad z = c'x \\ &\text{subject to} \quad Ax \leq b \\ &\quad\text{and} \quad\quad x \geq 0. \end{aligned}$$

The numbers a_{ij}, b_i and c_j are known constants that describe the problem. The variables x_j are to be chosen in such a way that the constraints are satisfied and the objective function is maximized (or minimized).

The first line of the statement of the problem indicates the objective which may be, for example, the profits of a firm where c_1 is the profit to be realized in producing a unit of product 1. The number of units of product 1 to be produced is a variable x_1, whose value is determined in the course of solution. The m inequalities are the explicit constraints. They state that a maximum amount of each resource, such as resource i, is available, and the amount is b_i. Each product, say product j, requires a_{ij} units of i to produce a unit of product j. Thus the entire constraint states that the amount of resource used in the production cannot exceed the amount that is available. The final constraints are called nonnegativity constraints which specify that certain variables cannot be negative.

GEOMETRIC (GRAPHICAL) SOLUTION

If the linear programming problem has two choice variables x_1 and x_2, then the problem can be solved graphically. To illustrate this, let us assume that a firm produces two lines of products, A and B, with a plant that consists of three production departments, 1, 2, and 3. The equipment in each department can be used for eight hours a day (a daily capacity in each department). The process of production can be summarized as follows: (1) Product A is first processed in department 1, then processed in department 3. Each ton of this product uses up one half hour of the first department's capacity and one third hour of the third department's capacity. (2) Product B is first processed in department 2 and then processed in department 3. Each ton of this product uses up one hour of the second department's capacity and two-thirds hour of the third department's capacity. Finally, products A and B can be sold at prices of \$80 and \$60 per ton, repectively, but after deducing the variable costs incurred, they yield on a net basis \$40 and \$30 per ton respectively. What output combination should the firm choose in order to maximize the total profit? The problem can be summarized in a table as follows:

Department	Hours of Processing needed per ton		Daily Capacity
	product A	product B	
1	½	0	8
2	0	1	8
3	1/3	2/3	8
Profit per ton	\$40	\$30	

If we let x_1 and x_2 be the amounts of products A and B in tons to be produced respectively then the problem becomes:

Maximize $z = 40x_1 + 30x_2$

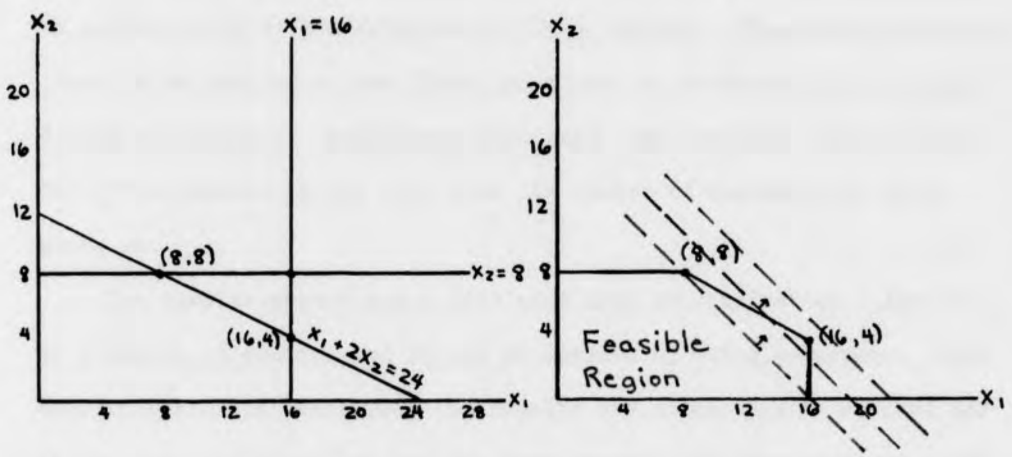
subject to

$$\begin{aligned} x_1 \quad\quad &\leq 16 \quad \text{Department 1} \\ x_2 &\leq 8 \quad \text{Department 2} \\ x_1 + 2x_2 &\leq 24 \quad \text{Department 3} \\ x_1, x_2 &\geq 0 \end{aligned}$$

By virtue of the nonnegativity constraints, the problem is confined to the nonnegative quadrant, in which we can draw the three departments' capacity constraints. The first department's capacity border (x = 16) and the second department's capacity border (x = 8) plot, respectively, as a vertical line and a horizontal line whereas the third department's capacity border appears as a slanting line that intersects the other two borders at points (16,4) and (8,8), in Figure 1.

Figure 1.

(a) (b)



We select the extreme point (16,4) as representing the best output combination. That is, the optimal solution is $x_1^* = 16$ tons per day, and $x_2^* = 4$ tons per day. Substituting these values into the objective function, we can then find the maximized profit to be $z^* = \$760$ per day.

SIMPLEX METHOD (ALGORITHM)

In the previous section it was shown that the graphical approach could be used to obtain solutions to linear programming problems involving two variables. However, for larger problems the graphical approach would fail us completely. In cases where the linear programming problem is large we can employ a technique, namely, the "simplex method" or "simplex algorithm"[15] in the solutions of these complex problems.

The simplex method is an iterative process which approaches, step by step, an optimum solution in such a way that an objective function of maximization or minimization is fully reached. The number of iterations to be applied is not fixed and cannot be predicted with a high degree of accuracy. Experience indicates that the most frequent number of iterations is not less than the number of inequalities in a given set.

The simplex method has a very wide span of utilization. Its use is a matter of routine and it can be applied by using computers. When each iteration is completed, the results are automatically checked and if the required objective has not been reached, another complete cycle is repeated by computing the data obtained from the previous iteration. If, for example, a decision maker (manager) does not use the simplex method for finding the best production schedule in order to maximize (or minimize) his objective, he must try all possible combinations of the quantities that he can produce, and determine, under certain restrictions, the most convenient production program. It is clear that this is a tremendously tedious and painful task, especially when the number of products is large.

The simplex method is able to select in a planned and scientific way only those arrangements suitable for consideration, with the aim of determining the optimum solution. The process involved is by itself not difficult, but in linear programming problems of substantial dimensions, the computation task will inevitably be lengthy and tedious. Fortunately, the modern computer is well adapted to precisely the type of repetitive calculations that a linear programming problem entails. By properly giving the computer a set of detailed instructions, we can rely on the computer to carry out the successive steps of the simplex algorithm faithfully and at a real maximum speed. High dimensionality of the problems then poses little problem.

DUALITY

One of the most important discoveries in the early development of linear programming was the concept of duality. It was shown that corresponding to every minimization problem (maximization) there always exists a counterpart maximization problem (minimization) with the property that

$$z^* = z^{*\prime} \quad \text{(optimal value of the objective function).}$$

The original linear programming problem is usually referred to as the "primal" and its counterpart is known as the "dual". If the primal problem is given as:

Maximize $z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

subject to

$$\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \le b_2 \\ \dots \quad \dots \quad \dots \quad \dots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \le b_m \end{array}$$

and $x_j \geq 0$ for all $j = 1,2,\ldots,n$

or in matrix notation

maximize $z = c'x$

subject to

$$Ax \leq b$$

and $x \geq 0$

Then the dual becomes:

Minimize $z' = b_1y_1 + b_2y_2 + \ldots + b_my_m$

subject to

$$a_{11}y_1 + a_{21}y_2 + \ldots + a_{m1}y_m \geq c_1$$

$$a_{12}y_1 + a_{22}y_2 + \ldots + a_{m2}y_m \geq c_2$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$a_{1n}y_1 + a_{2n}y_2 + \ldots + a_{mn}y_m \geq c_n$$

and $y_i \geq 0$ $i = 1,2,\ldots,m.$

where y_i are the dual variables. And in matrix notation the dual becomes:

Minimize $z' = b'y$

subject to

$$A'y \geq c$$

and $y \geq 0$

Note that if the primal has m constraints and n choice variables, so that the matrix A is mxn, then the dual will have n constraints and m choice variables because the matrix A', being the transpose of A is nxm.

The economic interpretation of the dual problem is based upon the interpretation of the primal problem. Let the primal problem be a standard production problem, that is, the problem of determining the profit maximizing output levels for the firm's various products, subject to a number of scarce input (capacity) constraint limitations. To make the analysis simple, let us assume a case where there are two products and two constraints. The primal problem then appears as follows:

$$\text{Maximize} \quad z = c_1x_1 + c_2x_2$$

subject to

$$a_{11}x_1 + a_{12}x_2 \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 \leq b_2$$

$$\text{and} \quad x_1, x_2 \geq 0$$

Accordingly, the dual can be written as:

$$\text{Minimize} \quad z' = b_1y_1 + b_2y_2$$

subject to

$$a_{11}y_1 + a_{21}y_2 \geq c_1$$

$$a_{12}y_1 + a_{22}y_2 \geq c_2$$

$$\text{and} \quad y_1, y_2 \geq 0$$

In the primal problem z denotes total profits in dollars. In view of the fact that $z^* = z^{*\prime}$, the symbol $z^{*\prime}$ in the dual should also be in dollars, as must be the expression $b_1y_1 + b_2y_2$ as well. Since the symbol b_i refers to the total quantity of the ith resource in the firm's plant the symbol y_i must obviously be expressed in units of dollars per unit of the ith resource, for only then the term b_iy_i will come out in dollars. That is to say, y_i must signify some kind of valuation of the resource in question. However, this value is not a market price, rather it is a value to be imputed to the resource. For this reason the value of y_i is referred to as an "accounting price" or "shadow price" for the ith resource. It is also useful for our analysis to regard y_i alternatively as representing the "opportunity cost" of using the ith resource.

Now let us examine the dual problem. First, what the nonnegativity restrictions $y_i \geq 0$ means is that we are not allowed to impute to any resource a value of less than zero. This is definitely an economically sensible requirement. In fact, we should always impute a positive value to a resource, unless that particular resource happens not to be fully utilized so that a zero opportunity cost is incurred in putting it to productive use. This means that a positive opportunity cost for a resource is always to be associated with the full utilization of the resource in the optimal solution.

Turning next to the constraints in the dual problem, let us examine the first constraint,

$$a_{11}y_1 + a_{21}y_2 \geq c_1$$

Since the coefficient a_{ij} denotes the amount of the ith resource used in the production of a unit of the jth product, the left side of the constraint represents the total opportunity cost of production of a unit of the first product (j=1). The right hand term c_1 denotes the per unit gross profit of the first product. Thus, what this constraint requires is that the opportunity cost of production be imputed at a level at least as the gross profit from the product. If the opportunity cost of production is actually to exceed the profit, then the resource allocation must certainly be nonoptimal, because by simply dropping the first product, resources will be released from the use of the first product can be utilized to better advantage somewhere else. Lastly, let us look at the dual objective function. Recalling that b_1 and b_2 are the total quantities of available resources in the firm's plant, the expression

$$z' = b_1y_1 + b_2y_2$$

evidently denotes the total value to be imputed to those resources. It is the idea of the dual problem to minimize this total while fulfilling the constraints as interpreted above. Thus the correspondence between the primal problem and the dual problem suggests that to maximize profit by finding the optimal output levels is the same as to minimize the total imputed value or the opportunity cost of the resources in the plant, with the condition that the opportunity cost of production of each product must be no less than the gross profit from that product. And the fact that $z^* = z^{*\prime}$ means that, in the optimal solution, the total gross profit must be imputed or allocated in its entirety to the resources in the plant via the shadow prices.

SENSITIVITY ANALYSIS

The prime motivation for constructing a linear programming model is to determine an optimal strategy for the process under consideration. Such a strategy is represented by the optimum solution to the linear programming problem. Finding such a best strategy is important, but frequently one would like to know more about the solution of the problem, especially one would like to know why the suggested optimal solution is the best solution. In addition, planning managers and other decision makers who use linear programming techniques would like to know which parameter variations the solution is most sensitive. For example, future variable operating costs are bound to be somewhat inaccurate. If the linear programming model suggests entirely different strategies for small variations in the future variable cost of a product, then the solution is highly sensitive to this cost and it may be wise to develop a more accurate estimate of such a cost element. If the model indicates that a solution remains constant for wide variations in most cost factors, then we consider that the operation is relatively insensitive to those cost factors.

Similar conditions apply to the availability of limited resources. In some cases it may be found that what was expected to be a limited resource actually turns out to be not limiting. In such a case one may be interested to know how much of this particular resource is not needed. Also, having more than the initially specified quantity available will not change either the suggested optimal solution or the cost or profit of the total operation because increasing the availability of a resource that is already surplus will only increase the surplus, and not the use of the resource.

In many cases the unit cost or profit of an activity or available resource cannot be determined exactly; the planner may not have perfect information on the coefficients in the model. Under these conditions one can analyze the model with a technique commonly referred to as "sensitivity analysis."

Sensitivity analysis is concerned with an investigation to determine whether or not a solution should be changed when one or more coefficients vary. In practical problems the manager is concerned with what happens to the optimal solution when changes in the values of the coefficients of the cost matrix occur. These changes are determined by real changes in the original data by considering the uncertainty of the determination of some or all of these data. The confidence in the optimal solution obtained by the model depends on the accuracy of the values entered in the first matrix (a_{ij}, b_i, c_j). The changes in the model have effect when one of the following events are present[16]:

1. Variations in the coefficient matrix (the a_{ij} values).

2. Variations in the right hand side of the model, namely, in the b_i constants.

3. Addition of new variables (x_{n+r}).

4. Addition of new constraints when one or more restrictions have been overlooked during the first formulation of the model or when the original conditions of the problem are no longer present.

5. Variations in the costs (or profit) of the objective function (the c_j values).

With reference to the variations in the coefficient matrix (a_{ij}), it can be said that there are no coefficient matrixes which perfectly reproduce the situation of a problem under study. This is caused inevitably by approximating the original data by numerical terms and describing the behaviour of a phenomenon by linear constraints. Sometimes there are factors not considered because they are estimated to have no influence on the model. Later, during the application of the optimal solution, it may be discovered that it is necessary to change some coefficients. This is the case when one or more coefficients must be changed because the original conditions of the problem have changed. This may occur due to either internal or external factors. Internal causes may be encountered, for example, when available working time, tools, materials or manpower, are expressed by values different from those entered in the original problem. External factors may influence changes in the optimal solution because new raw materials are usable, new technological processes can be applied or sometimes government regulations may cause a revision in the model.

In the second case, the changes in the original problem is encountered when one or more values on the right hand side of the first set of inequalities or the set of equations are changed. These changes of b_i values may occur when, for example, the availibility of raw materials is varied, the quantity (or quality) of goods produced is changed, or the usable time in performing a certain job is reduced or extended.

When a new variable not originally considered is added to the model, as in the third case, a new problem is created. When the variable is inserted into the model, the dimensional space required by the description of the problem is increased by one. This means that when x_n variables are increased by k, x_i, $_{n+k}$, it may possibly affect the optimal solution by adding a new set of coefficients $a_{i,n+k}$ and a new cost coefficient c_{n+k} of the objective function.

The fourth case considers an improvement of the original matrix by the insertion of one or more constraints. A new equation, or equations, is added to the existing ones. Usually these added constraints are ignored when the problem is first stated. It is possible that the necessity of being more precise in the description of the problem is detected during the application of the problem. Other events may occur, for example, when external or internal factors, such as new laws and new governmental regulations in production or control, impose a change on the original problem. If the optimal solution satisfies the new constraints, the model can be reworked without recomputing the whole problem.

The last case is concerned with variations that may occur in the cost coefficients of the objective function. Such a case may occur when the planner has already applied the program suggested by the optimal solution when he can take advantage of some reduction of costs or increase in profit, or when he may encounter higher cost or less return per item. The planner is eager to know whether the optimal solution adopted is still valid after a certain amount of change in the objective function.

Sensitivity analysis is a very useful technique which can be employed to answer all the above questions in those five cases without reworking the entire problem any time a change is necessary in the model.

CHAPTER II

ECONOMIC DEVELOPMENT PLANNING MODELS

The allocation of resources among sectors over given time periods is among the major determinant of the costs and benefits of economic growth and therefore central issues in economic development planning. In our survey we present a set of linear programming models for planning and analyzing these resource allocations. These models are applied to development planning in various underdeveloped countries to illustrate their uses and scopes. The study on which this survey reports is primarily a study of the capability of linear programming techniques in overall development planning. However, we believe that the analyses also provide some insights into economics of underdeveloped countries, mainly into their economic policies and the operation of their economy.

Development planning always has many different economic and social goals. In general, these goals conflict with each other to varying degrees. A greater achievement with respect to one goal often means a lesser achievement with respect to another, so that compromises are necessary. For example, if a higher rate of growth of the gross national product is desirable, it may be necessary to accept a lower rate of improvement in the average standard of living. Similarly, reduction in income distribution inequalities may result in a lower growth rate for total income. Making compromises among conflicting objectives requires political value judgements, which in turn, implies the existence of at least an ordinally social welfare function embodying these objectives. Yet in practice, comprehensive and precisely stated goals for development are rarely created by social decision processes. Thus, if economists confine themselves to the conventionally defined role for economists in policy-making of analyzing the consequences of well defined objectives, they have only narrowly and inexactly defined terms of reference. In such circumstances, the economist can contribute to economic policy-making not in the conventional manner only, but also by describing alternative economic policies corresponding to different feasible combinations of objectives. For example, economic analyses may show that 8 percent growth rate of national income can be achieved if the rate of increase in the standard of living is limited to 2 percent, but that only a 5 percent growth rate can be achieved if the standard of living is to rise by 3 percent. Unless such alternatives and their implications are made explicit, informed choices cannot be made. The models presented here generate policy alternatives and explore the implications of any given set of objectives. Consequently, these models can also be used to test the feasibility, consistency, and political acceptibility of plans made by other, less formal methods.

The complexity of development processes and the limitations of data necessitate the formulations of models, or conceptual simplifications, of real relationships for both theoretical analysis and formulation of policy. Models are intended to bring the most relevant issues into focus without doing any damage to our understanding of them through the means by which other problems are put into the background. The more implications of a model which can be deduced, the more useful the model will be. Modern, highspeed computers can carry out detailed investigations of relatively large and complex quantitative models, which otherwise could either not be analyzed at all or only in a qualitative manner. An improvement in the speed with which the process of logical deductions can be performed is, therefore, a potential increase in our powers of analysis and adds insight and flexibility to policy making.

Although there exists some drawbacks in the use of linear programming models, we believe that no other known method does as well in providing a consistent analysis of intertemporal and intersectoral relationships and economic goals. Whatever the means of implementation, decisions will have to be made on amounts of government savings, on whether to start another steel plant next year, or built more power facilities, or allocate foreign exchange to importing capital equipment, etc. These decisions should be coordinated with all the sectoral development plans and the national goals. We believe the models that will be analyzed in our survey indicate how this can be done in a manner superior to that of existing techniques and work on similar models.

ECKAUS MODEL

The planning model that was developed by Parish and Eckaus is mainly focused on the problems of determining the optimal levels of savings and investment over time, and the related problems of intersectoral and intertemporal distribution of investment and output and use of foreign exchange resources. The model defines an economy with the following characteristics:[17]

1. Production processes in all sectors require fixed capital and intermediate inputs in fixed proportions to output and is characterized by constant returns to scale.
2. Most sectors require imports in exogenously fixed proportions to output. Additional imports are permitted in certain sectors to supplement domestic production in amounts determined endogenously within specified ceilings.
3. A balance of payments constraint must be met in each period, which limits total imports to the total amount of exports and net foreign capital inflow in that period.
4. Private consumption is a composite commodity the sectoral proportions of which are fixed exogenously. Furthermore, consumption is required to increase monotonically in successive periods at least at specified minimum rates.
5. In order to create new capacity, investment must be made in the periods immediately preceeding the period when the new capacity becomes available, as determined by a fixed gestation process, which varies among sectors but can be as long as three periods.

6. In each period, investment in inventories in each sector is linearly, related to the change in output to be realized in the next period.

7. Fixed capital stocks and foreign exchange are the only scarce factors. Labor and new materials are assumed to be adequate and exogenous to the model. Labor supply is assumed to be unlimited and, thus, not a constraint on output.

Ideally in planning models the objective function should be a social welfare function. Since this is unknown in the basic model[18], the objective function, which is maximized, is the sum of aggregate consumption in each of the plan periods, discounted by a social discount rate.[19]

$$W=\sum_{t=1}^{T} C(t)/(1+w)^{t-1}$$

The specified objective function maximized is the present value discounted value of aggregate private consumption W over the planning period t=1,...,T, where C(t) is aggregate consumption in period t and w is the social discount rate applied to future consumption. The solution of the basic model achieves the highest value of this function that is consistent with all of the constraints. This particular objective was chosen because it reflects directly one of the major objectives of development: improvement in the average standard of living.[20] We must note however that in a programming model, goals of economic policy can be stipulated not only by what is chosen to be maximized but also by the content of the constraints.

Consumption growth constraints in the model require that aggregate consumption grow by at least a stipulated minimum rate. This rate, when compared to the population growth rate, indicates a required minimum rate of growth in the average standard of living. The consumption growth constraint is given by

$$C(t+1) \geq C(t)\{1+\delta(t)\}, \quad \text{for } t=0,\ldots,T-1$$
$$C(0) = \overline{C(0)},$$

where $\delta(t)$ is the prescribed minimum growth rate for aggregate private consumption in period t and $\overline{C(0)}$ is the aggregate private consumption in the preplan period.[21]

Savings constraint relates the maximum permissible level of net savings to the net national product. It is yet another way of introducing social goals and a behavioral constraint into the model, for it describes, though indirectly, the limits on the willingness of society to sacrifice present for future consumption.[22]

Production accounting relationships stipulate that the total requirements for each commodity in each period not exceed its availability in that period. The total demand consists of the requirements for the good as an intermediate input, which are determined by use of an input-output matrix, and of a number of final demands. These include the demands for inventories, new fixed investment, replacement investment, public and private consumption, and exports. The availability is the sum of domestic production and imports. The production accounting relationships are given by:[23]

$$J(t)+H(t)+N(t)+Q(t)+F(t)+G(t)+E(t) \leq M(t)+X(t),$$

for $t=1,\ldots,T$.

The first seven terms represent uses of the output of each sector: J(t) is intermediate inputs, H(t) is deliveries for inventory accumulation, N(t) is deliveries of investment goods for new fixed capital, Q(t) is deliveries of investment goods for restoring depreciated fixed capital, F(t) is private consumption, G(t) is government consumption, and E(t) is exports. The last two terms, M(t) imports and X(t) domestic production, are the sources of availability of the products.

The intermediate requirements for output in each period are determined by an nXn matrix of input-output coefficients a(t) where $a_{ij}(t)$ is the amount of good i required as an intermediate input in period t to produce one unit of good j.[24]

$$J(t) = a(t)\ X(t), \qquad \text{for } t=1,\ldots,T.$$

Input-output coefficients reflect technology, relative factor prices, the degree of plant integration, and the internal composition of the sectors. The time subscript t of the a(t) matrix indicates that it is possible to change the intermediate requirements ratios over time. Inventory accumulation is determined in a set of accelerator type relationships using an nXn matrix of inventory coefficients s(t).[25] An element $s_{ij}(t)$ is the amount of good i required as inven-ory in period t-1 to produce one unit of good j in period t.

$$H(t) = s(t)\ [X(t+1) - X(t)], \qquad \text{for } t=2,\ldots,T.$$

$$H(1) = s(1)\ [X(2) - (I+\alpha_o)\ \overline{X(0)}],$$

where I is the identity matrix. Thus, deliveries in period t for inventory are a function of the forward difference of output X(t+1)-X(t).

Production for inventory in the first period H(1) is based on the difference between output levels in the first period as anticipated in the preplan period.[26] With the diagonal matrix of anticipated sectoral growth rates and $\overline{X(0)}$ the preplan year of output levels, $s(1)(I+\alpha_o)\overline{X(0)}$ gives total stocks of inventories at the beginning of the plan. Inventory coefficients are only partially determined by technical requirements.

The vector F(t) of deliveries to private consumption is related to aggregate consumption C(t) by a coefficient vector c(t), which defines the proportions of sectoral consumption in the aggregate.[27]

$$F(t) = c(t)\ C(t), \qquad \text{for } t=1,\ldots,T.$$

$c_i(t)$ is the amount of good i in one unit of aggregate consumption C(t) and so $\sum_{i=1}^{T} c_i(t)=1$.

The specification of c(t) thus fixes the composition of aggregate private consumption in period t. Since substantial variability in consumption composition is unlikely in the short run, c(t) is kept constant for all the periods of the plan in the solution of the model. The amounts required from each sector for government consumption in each period are specified externally. If G(t) is the vector of government consumption in period t, then[28]

$$G(t) = \overline{G(t)}, \qquad \text{for } t = 1,\ldots,T.$$

E(t), the vector of exports in period t, is determined outside the model structure, and[29]

$$E(t) = \overline{E(t)}, \qquad \text{for } t = 1,\ldots,T.$$

This is not a fully satisfactory procedure, since, exports depend on domestic prices, which in turn depend on the amount and composition of productive resources and, finally, on comparative advantage. Capacity restraints[30] insure by means of the diagonal matrix b of capital-output ratios that the output of each sector in each period does not exceed that producible with the fixed capacity available in the sector at the beginning of that period. If K(t) is the vector of fixed capital available at the beginning period t,

$$b(t)X(t) \leq K(t), \qquad \text{for } t = 1,\ldots,T.$$

The total capital in each sector, represented by an element of the vector K(t), is a composite commodity with a fixed composition. This composition is defined by a proportion matrix p, in which element p_{ij} represents the good i held as fixed capital by sector j per unit of composite fixed capital K_j.

Sectoral capacities may be increased in any period t by the delivery of additions to capacity Z(t). These increments of capacity, in turn, are formed by deliveries of investment goods from the sectors that produce them. The deliveries are in fixed proportions and with fixed time leads of one, two, and three periods prior to the completion of the addition to capacity. The amount of Z(t) that must be furnished by each sector in each period is determined by the three investment lag proportions matrices p', p", p"'. The coefficients p'_{ij}, p''_{ij}, p'''_{ij} in these matrices indicate the proportions of the total increment to capacity in sector j in period t that must be supplied by sector i in periods t-1, t-2, and t-3.[31] Thus the total amount of deliveries of investment goods in each period is[32]

$$N(t) = p'Z(t+1) + p''Z(t+2) + p'''Z(t+3), \quad \text{for } t=1,\ldots,T.$$

Real depreciation in this model depends on the passage of time rather than of rate of use. In this way capital can produce services at a constant rate over a lifetime independent of the rate of use of the capital. At the end of this lifetime, all the productive capability of the capital dissappears. Since the lifetimes of plant and equipment are chosen to be, respectively thirty-three and twenty years in the model, the dissappearence of productive capacity through depreciation is predetermined in full for twenty years and in part for thirteen more years.

The capital originating in sector i that wears out in sector j is $D_{ij}(t)$. The total depreciated capital in each sector in each period is then[33] $D_j = D_{ij}(t)$, for the sector i and

$$D(t) = \overline{D(t)}, \quad \text{for } t=1,\ldots,T+3.$$

Different lifetimes for different components of capital imply that the plant and equipment depreciate in an unbalanced manner. This, in turn, provides the opportunity for restoring capacity by an unbalanced production of capital of the plant or equipment type. Since the components of capital stock in each sector wear out at different rates, the capacity immobilized by the depreciation of the components must be computed in the model. The depreciation composition matrix r is defined with element $r_{ij} = D_{ij}/D_j$. Then if $D_j(t)$ is multiplied by r_{ij}/p_{ij}, the amount of capacity that would be lost due to depreciation on each component $D_{ij}(t)$ in sector j can be computed.[34] The actual amount of capacity lost through component and is determined by the maximum of (r_{1j}/p_{1j}, r_{2j}/p_{2j},...,r_{nj}/p_{nj}) D_j. Therefore, the diagonal matrix d can be formed whose element d_{ij} = Max (r_{1j}/p_{1j}, r_{2j}/p_{2j},... ...,r_{nj}/p_{nj}).[35] The capacity lost through depreciation in each sector is, therefore,

$$V(t) = [d]\ D(t), \qquad \text{for } t=1,\ldots,T+3.$$

It is up to the optimizing mechanism to determine R(t), the amount of the capacity lost through depreciation that will be restored. The model solution provide for restoration of only part of the depreciated capacity.

The deliveries Q(t) from each sector for capacity restoration R(t) are assumed, like new capital formation, to require up to three periods.[36] So the deliveries for this purpose in any one period look three periods ahead. Where r', r'', and r''' are restoration lag proportions matrices similar to p',p'', and p''',

$$Q(t) = r'[d]^{-1}\ R(t+1) + r''[d]^{-1}\ R(t+2) + r'''[d]^{-1}\ R(t+3),$$

$$\text{for } t=1,\ldots,T.$$

The coefficients r'_{ij}, r''_{ij}, and r'''_{ij} indicate the proportions of the total capital replacement $R_j(t)/d_{ij}$ to restore capacity[37] $R_j(t)$ that must be supplied by sector i in periods t-1, t-2, and t-3.

Capital accounting relationships[38] determine capacity at the beginning of each period as the capacity previously available, less depreciation, plus the newly completed additions to capacity, plus that part of the depreciated capacity which is restored. The accounting[39] relationships for capacity in each sector is given by

$$K(t+1) \leq K(t) + Z(t+1) + R(t+1) - V(t+1), \qquad \text{for } t=1,\ldots,T+2.$$

This merely states that K(t+1), the capital available at the beginning of period t+1, cannot be greater than the capital available in the preceeding period plus the new, completed additions to capacity, plus that part of depreciated capacity that is restored, less the capacity depreciating in period t. Since both the restored capacity R(t+1) and new capacity Z(t+1) can be zero, decumulation of capital to the extent of V(t+1) is possible.

Since a unit of capacity can be created more cheaply by restoring a worn-out component than by supplying the entire set of components of the composite capital, the model has to be restrained from restoring more capacity than is depreciated in any period.[40]

$$R(t) \leq V(t), \quad \text{for } t=1,\ldots,T+3.$$

Balance of payments constraints[41] require that total imports in each period not exceed the foreign exchange availability as determined by exports and the stipulated net foreign capital inflow in that period. The total amount of imports in each period is limited by the availability of foreign exchange. This in turn depends on the total amounts of exports, foreign aid from government sources, private foreign investment, and whatever changes in reserves will be tolerated. The latter three components, lumped together, are designated net foreign capital inflow and are specified exogenously in the model as $\overline{A(t)}$. The balance of payments is given by[42]

$$uM(t) \leq \overline{A(t)} + uE(t), \quad \text{for } t=1,\ldots,T.$$

where u is a unit row vector 1,1,1,...,1. By changing A(t) over time, a schedule of progress toward a condition of self-sufficiency can be enforced in this constraint.[43]

Imports are divided into two categories in the model. Noncompetitive imports for each sector are determined by stipulated import-output ratios, but the stipulations change over time. Competitive imports are allocated by the model with limits set on the extent to which this type of import can be absorbed in any one sector. The vector of noncompetitive imports $M'(t)$ is related to output levels by fixed coefficients. The vector of competitive imports $M''(t)$ merely supplements the output of the corresponding domestic sector. Total imports in each sector are the sum of the two types:[44]

$$M(t) = M'(t) + M''(t), \qquad \text{for } t=1,\ldots,T.$$

Initial conditions are estimates of production capacities, stocks of inventories, and the unfinished capital-in-process actually available at the beginning of the plan period.

Terminal conditions must be provided in some manner, in order to relate the events of the plan period to the postplan period, so the model will not behave as if time stopped at the end of the plan. These terminal conditions are the final capital stocks on hand in process of completion. They are either completely specified from some source outside the model, or they are partially derived in the solution of the model.[45] In general the terminal requirements state the desired minimum levels of the final capital stocks:

$$K(T+1) \geq \overline{K(T+1)}$$

$$K(T+2) \geq \overline{K(T+2)}$$

$$K(T+3) \geq \overline{K(T+3)}$$

and

$$s(t)\ X(T+1) \geq \overline{X_s(T+1)}$$

where $\overline{X_s(T+1)}$ is the vector of stocks of inventories at the beginning of the first postplan period T+1, i.e. at the end of the plan.[46]

For any set of values of the parameters, a solution of the model, if it exists, will be a point in consumption space defined by the intersection of the binding constraints. Variation of the relative weights on consumption in each period in the objective function will move the solution to a different point on the production feasibility surface. Variation of the postterminal conditions will change the solution by shifting the feasibility surface. Likewise, changes in the production parameters will change the production feasibility surface itself and consequently the value of the maximand for any given objective function. For each value of the maximand there is a specific allocation of resources and outputs in each period. A solution of the model determines the unknown variables remaining after all the possible substitutions have been made. These are the gross domestic outputs in the model given by X(t), the level of aggregate consumption given by C(t), competitive imports given by M"(t), capital stocks given by K(t), new capital given by Z(t), and restored capacity given by R(t). With the solution values of these variables, it is possible to generate for each period a detailed list of gross-output levels, imports and final demands, interindustry transactions, investment allocations, and capital stock uses that will achieve the maximand.[47] This sectoral and temporal detail, along with the associated set of national income accounts, facilitates overall appraisal of the implications of the solution of the model.

In addition to allocations of physical quantities, a solution includes a set of shadow prices, each of which is related to one of the constraints. These shadow prices are the variables of the minimizing valuation problem, which is the dual of the maximizing problem. In the minimizing problem, prices are found for the scarce resources, in this case the sectoral capacities and foreign exchange, which exhaust the value of the total product and minimize the cost of production within the behavioral as well as technological constraints.

From the solution of the model and its dual, the following shadow prices are determined:[48] $V_{x(t)}$, vector of shadow prices associated with the production accounting relationships and interpretable as shadow prices of output X(t); $V_{k(t)}$, vector of shadow rentals of capital K(t) obtained as values associated with the capacity constraints; $V_{z(t)}$, vector of shadow prices of net capital stock Z(t) obtained as values associated with the capital accounting relationships; $V_{CR(t)}$, shadow prices associated with the consumption growth constraints; $V_{FX(t)}$, shadow prices associated with the foreign exchange balance requirements; $V_{M''C(t)}$, vector of shadow prices associated with the ceilings imposed on the competitive imports; $V_{R(t)}$, vector of shadow prices of restorable capacity obtained as values associated with the restoration ceilings; $V_{IK(t)}$, vector of shadow prices of initial capital-in-process obtained as values associated with initial capital-in-process constraints. Since the optimal solutions of the primal and the dual describe the same state of the economy, the quantity allocations and the valuations must be considered equally valid.

As we noted earlier, the shadow price associated with a constraint is the value of change in the objective function when there is a marginal change in the right hand side of the particular constraint and all other constraints are left unchanged. Whenever a restraint represents an inequality which stipulates that requirements must be less than or equal to availabilities, the shadow prices associated with the constraint can be interpreted as the shadow price of the quantity, because in this case a marginal modification in the right hand side of the constraint amounts to a marginal change in the availability of the quantity.

Most of the constraints are descriptions of real technical or physical relationships that must be met if any economic system is to function viably. These real descriptions are not completely accurate because of limitations of data or computational capacity, or because of analytical restrictions, such as the assumption of linearity. The deficiencies create undesirable results in the solutions. For example, due to the linear form of the objective function in the planning models, there occurs a flip-flop tendency in the solutions. In order to avoid such undesirable features other constraints are added to the model. These constraints are artifacts, effective in compensating for limitations in other parts of the models' structure only if the quantity results obtained in the solution are the results that would have been obtained if the limitations were not present.[49] The artifact constraints can be given an economic interpretation, although their primary purpose is not the addition of the economic content they may be interpreted to embody. The shadow prices associated with these constraints can also be given an economic interpretation, such interpretation must be made, however, with the source and precise meaning of the constraint constantly in mind.

Finding solutions to the model is practicable only with high-speed computers with large memory, and even so requires relatively large amounts of computer time. Many of the simplifications embodied in the model's descriptions of the economy are required because of limitations of computer capacity. Others are due to unavailable data. Every simplification has a cost in terms of the realism and usefulness of the models, and one of the most important aspects of model building and use is the appreciation of the consequences of simplifications.

Although consumption is the only criterion in the objective function, some additional social goals, such as steady growth in average consumption levels and in national self-sufficiency, are introduced as linear constraints, and still others could have been introduced in the model. For example, government consumption, which is specified externally, provides for expenditures on education, medical, and other welfare goals and can be specified to account changes in military defense expenditures. Similarly, the savings constraint reflects the limits of the willingness of the present generation to sacrifice consumption for the future. If sectoral employment coefficients were available to the model, a minimum employment constraint could have been easily introduced. Income distribution goals can also be introduced as a constraint by means of data relating the distribution of income generated in each sector to the output of the sector.

The form of the objective function has a major influence on the intertemporal distribution of consumption and investment. A linear objective function, as in the model described above, will unless modified by other relationships, result in a solution in which consumption tends to be concentrated either at the beginning or at the end of the plan. However, this flip-flop tendency can be modified by means of the constraints specifying a minimum consumption growth rate and those relating savings and net national product. Such constraints have an effect on the solution similar to nonlinearities in the objective function.[50]

With the necessary specification of these constraints, any desired time path of consumption which is feasible can be obtained by the model. It is clear that the present formulation has the advantage that the significance of the constraints is more readily apparent and meaningful and, in the absence of any empirical data on the social utility of consumption, involves no more arbitrariness than would be required by the specification of a social utility function.[51]

The sectoral composition of consumption is stipulated in the model in externally fixed proportions or is determined endogenously by fixed elasticity relations with total consumption.[52] No opportunity is provided for variations in the composition of consumption depending on price elasticities, mainly due to lack of empirical data. The only means of adjusting sectoral demands for consumption is through changes in total consumption, in the model. As a result, bottleneck situations in a particular sector can have the effect of constraining overall consumption.

The high degree of aggregation in the model makes substitution only marginally important, and in a practical application on a more disaggregated basis some amount of flexibility could have been permitted with additional computational costs.

In the model the availability of natural resources is assumed to be reflected in the productivity of capital in the primary goods sector.[53] Labor is assumed to be free except that education expenditures are subsumed in government consumption. The gestation period specified for capital provides another device for taking into account the problems of developing managerial skills. It would be fairly a simple matter to expand the model to require various types of labor inputs for production and to specify the methods of their supply in models of this type. These additional production relationships have not been introduced in the present analysis, i.e. the model described above, mainly because data on the types of labor skills required in the different sectors are limited, and there is almost no information on how these skills are acquired in the Indian economy.[54] Furthermore, scarcity of labor with limited skills is not critical in India, and the effects of neglecting such labor as an input to production would be marginal. The types of labor skills which are most likely to be bottlenecks in expanding production, i.e. those of foreman, technicians, engineers, and managers, are the most difficult to quantify and take into account in any type of planning framework.

The production function in the model is homogeneous of the first degree, i.e. they show constant returns to scale and have fixed input coefficients. In addition, all production relationships are accounted for directly in inputs and outputs, so there are no external economies or diseconomies. Thus there is no provision for increasing or decreasing returns to scale, substitution of relatively scarce factors, or increasing productivity by learning-by-doing. Decreasing returns to scale can easily be incorporated into the model. As far as they can be predicted in advance, marginal rather than average capital-output ratios can be used to treat the increasing returns to scale. Similarly, in order to account for the effects of learning-by-doing, a higher capital-output ratio can be used for capacity in the first few years after its creation than in subsequent years. At the high level of aggregation of eleven sectors with which the model's calculations have been made, substitution among the outputs of different sectors may be unimportant. The explicit accounting of interindustry interdependencies in the model is a satisfactory treatment of the so called pecuniary externalities.

Though inventory coefficients are fixed, inventory accounting is an explicit part of the model's framework rather than being omitted or estimated in a completely arbitrary manner as has often been done in planning exercises.

The estimation of depreciation externally to the model follows from the assumption that the wearing out of capital is a function of time rather than of use, and the capital lifetimes are long in relation to planning periods. The alternative assumption that depreciation is a linear function of the use of capital could have been easily introduced in the model.

The structure of foreign trade sector in the model is less sophisticated, because it is linear with fixed coefficients, than much of current trade theory. Yet the model does have the virtue of relating foreign trade to the rest of the economy quantitatively and in considerable detail.

To plan a program of import substitution by means of the model requires a level of sectoral aggregation which is far more detailed than the eleven sector aggregation that the Eckaus model has. However, by externally specifying noncompetitive import coefficients that diminish with time, planned programs of import substitution can be introduced in the model. Exports were specified externally using projections based on studies for particular commodities.[55]

In view of the importance in economic development of changing production techniques, externalities, import substitution, variations in inventory-holding relations and in capital maintenance, a special comment is required on the simple treatments of these issues in the model. As already pointed out in the model, that treatment is usually determined not so much by theoretical preference as by limited data and computational necessity. Therefore, an evaluation of the potentiality of the model should not be based on the present restricted treatment of these issues. However it is necessary to keep these limitations in mind when assessing the empirical results presented in the model for their insights into past and future economic policy.

ADELMAN MODEL

In the model that is worked out by Adelman,[56] a dynamic linear programming model is developed for educational planning; the model was intended for Argentina's economy, and was applied to the available Argentine data to explore the potential usefulness of this approach in determining the optimal extent and composition of resource allocation to education. The model is a four period, twenty year dynamic programming model, with investment in the education sector optimized simultaneously with investment in real capital. The optimal time patterns of production, imports, and exports for each of the several sectors of the economy are also determined concurrently.

The Adelman model treats investment in the educational sector in a manner entirely analogous to investment in real capital. The pattern through time of resource use in education is optimized simultaneously with the structure of production, investment, imports, and exports for the entire economy. The optimization is carried out under a set of linear constraints which represent the economic, technological and socio-cultural limitations upon the country's economic policy. The end result of the computations is a description of a dynamic pattern of investment in human resources which best meets the overall planning objectives, given the constraints under which the economy must operate.

The general structure of the Adelman model is similar to the general structure of any given maximization (or minimization) problem; an objective, such as the maximization of the economy's rate of growth, the maximization of GNP, or the minimization of unemployment, is specified. The objective function is optimized subject to a set of linear constraints of several types.

For the educational system, the major exogenous constraints are the initial supply of teachers for each type of school, the supply of school buildings, and the school-age population. In addition, because the initial experiments were characterized by radical shifts in the utilization of several types of secondary schools between periods, it was necessary for the model to subject the optimization to a set of socio-economic constraints which stipulate that enrollment in each type of school be nondecreasing through time.[57]

For the productive sectors of the economy, the constraints specify the technological conditions of production and investment. They also limit the economy's use of primary resources to available stocks for productive capacity (by sector), manpower (by skill), and foreign exchange and savings. The optimal program is required to obey certain behavioral constraints:[58] it must provide certain minimal amounts of each industry's product for domestic consumption; it must not exceed specified absorptive capacity[59] constraints upon investment in each sector; and it cannot export more than a certain amount of each industry's output. Finally, certain terminal conditions must be met by the optimal program:[60] sectoral investment in the last period of the program must cover at least that period's depreciation.

The maximization process results in a specification of the optimal levels at which all the endogenous variables[61] in the system must be operated in each period of the program. The decision variables[62] for the educational system allocate graduates and dropouts in each time period to a particular type of employment: labor of a given skill within the school system (either as a student or as a teacher).[63] In setting the levels of these decision variables, the driving force for the educational part of the model is basically the over-all labor scarcities by skill generated in the rest of the model. The demand for labor of each class is translated into educational requirements through the assignment of different productivities to different levels of schooling within labor of a given skill. Anticipated student, teacher, and school building requirements and availabilities also have a strong impact upon the optimal levels of production and the use of graduates and dropouts. The decision variables relating to the noneducational portion of the model specify the optimal sectoral levels of domestic production, imports, fixed capital formation, inventory accumulation, labor use, and over-all foreign capital inflow as a function of time.[64] The labor availability-labor use constraints in the model provide the major link between the educational and the noneducational portions of the model.

Even though only six educational processes are considered in an economy disaggregated into nine productive sectors, the resultant model specifications still involve about 70 equations per period.[65] The complete four-period, twenty-year model contains 284 equations.[66] The computer time cost imposes a severe limitation upon the number of parametric studies which could be carried out with the model. Since all decision periods in the program must be of the same length, and since the average school course is five years, a five-year period is used for the calculations in the model.

In the model, the Argentine educational system is disaggregated into the following types of schools:[67] (1) primary; (2) general secondary; (3) commercial and industrial (technical); (4) normal (primary teacher training); (5) vocational and trade; (6) university and superior.

All the constraints in the model are linear. They express the accounting, technological, socio-cultural, and the resource balance restrictions which the optimization process must obey. The constraints of the model can be analyzed in two different groups: constraints for the educational sector and constraints for the noneducational sector. The model has the following constraints for the educational sector:[68]

Equalities for graduates

These equations specify that for each type of school, the total number of new graduates available to the system from the previous period must be precisely equal to the sum of those allocated to the work force, to teaching, and to continuation in school in the current period.

Equalities for dropouts

These constraints state that the total number of dropouts available to the system from each type of school must be precisely equal to the sum of those allocated to various categories of the work force, or, for university dropouts, the work force and primary teaching. It is assumed in the model that on the average dropouts leave school during the middle of each period.

Teacher constraints

These constraints require that the number of students who can enroll in a school of type r is limited by the number of teachers available for that kind of school.

School building constraints

These constraints limit the number of students enrolled in primary and secondary schools to the number which can be accomodated by the stock of primary and secondary school buildings. Similarly, the number of university students must obey the analogous constraint.

Population constraint

This constraint states that the number of entrants into primary schools plus those entering the work force with no education cannot exceed the population in the five to fourteen age group.

Socio-cultural constraints

These constraints state that enrollment in each type of school must be nondecreasing through time. They express the socio-cultural requirement for a certain amount of continuity in school enrollments and provide terminal constraints for the educational system.[69]

The labor force change equations provide the main link between the educational system and the productive sectors of the economy in the model. They define the contribution of graduates and dropouts from schools of a given type to the supply of labor of a particular skill.

The noneducational section of the model consists of the following sets of constraints:[70]

Total use-total availability constraints

These constraints specify that, for each sector, the total amount of product available from production, imports, and opening inventories must equal its total use in intermediate production, investment, exports, final consumption and stockpiling activities.

Capacity constraints

These constraints state that domestic production in the jth sector in the tth time period is limited by the capacity available in that sector during that period. The available capacity is set equal to the initial capacity at the start of the plan plus the net investment that has taken place during all previous periods.

Labor supply constraints

These constraints state that the sum of labor of a given skill demanded for production cannot exceed the availability of labor of that grade during that period.

Balance of payments constraint

This constraint states that for each time period, the total peso value of imports must be equal to the sum of the peso value of exports, the net foreign capital commitments available for that period,[71] and the accumulated balance of payments surplus.

Savings-investment constraint

This constraint expresses the requirement that the number of pesos used for investment shall not exceed the finance available for this purpose from demestic savings and from foreign sources.

Export constraint

This constraint states that exports of the jth commodity are not allowed to exceed a preassigned upper limit.

Upper limit on investment

Investment in every industry is constrained not to exceed prescribed limits.

Lower limits on investment in the terminal period

These constraints require that the final period's investment cover at least the final period's depreciation in each sector.

The Objective Function

a) Maximize the discounted sum of GNP.

b) Maximize the change of GNP.

c) Minimize the discounted sum of net foreign capital inflows.

The shadow price of each resource constraint in a linear program indicates the change in the objective function which would result from an additional unit of the resource indicated by the constraint. The shadow prices of the conservation equalities for graduates (or dropouts) from the several educational processes in the model therefore indicate the marginal social products of graduates (or dropouts) from the respective schools.[72] When the objective function maximized is the discounted value of GNP, the social benefit is measured in terms of the discounted peso value of additional product which could be generated by an additional graduate (or dropout) over the twenty years of the program. When the objective function is the minimization of foreign capital inflows, the social benefit from education is expressed in terms of the number of foreign exchange (in pesos) which could be saved by an additional graduate or dropout from each school process. When the objective is the maximization of the economy's rate of growth, the social benefit from education is measured in pesos of the final period's output which could be produced by an extra graduate or dropout.

In general, the model suggests that the shadow prices of commercial and vocational school graduates are considerably lower than those of secondary and normal school graduates.[73] This explains why these schools are not utilized in the optimal school network in Argentina. The shadow price of university dropouts is quite large relative to that of university graduates. Since the characteristics of dropouts in the calculations of the model are that they are available for work for half the period during which they enter their respective course, and their productivity is halfway between that of secondary and university graduates, the model suggests the desirability of instituting extensive two to three year junior college programs.

The difference between the shadow prices of graduates and dropouts from each type of school sets an upper limit to the subsidy which it would be economically desirable to pay in order to reduce the dropout rate.[74] To the extent that students are compelled to leave schools for economic reasons, the difference between these two shadow prices indicates the maximal grants which could be offered to keep students in school. The difference between the shadow prices of graduates of various types and their respective average earnings may offer an indication of the extent to which the market incentives reflect the true marginal social benefit of each type of education.

The data in the model's calculations is also used to evaluate the marginal social cost of a student of each type. The major elements are the opportunity cost of student and teacher time and the opportunity cost of school buildings.[75] In terms of cost per productive graduate, normal school and general secondary are found to be the cheapest of all secondary education. In estimating the amount of investment in the educational sector, student and teacher costs are measured at their social opportunity cost, and investment in buildings is taken at its actual value in the optimal program.[76]

The allocation of graduates and dropouts from different schools to labor of various skill classes is determined endogenously in the model. It is therefore interesting to inquire into educational level of labor force additions during the life time of the program. A typical assignment pattern results in additions to the entrepreneurial group having, on the average, nine years of formal education, with 28 per cent university dropouts and 62 per cent high school dropouts.[77] The managerial and professional category is staffed entirely with university graduates and university dropouts, leading to an average of fifteen years of formal education. Additions to skilled and unskilled labor, on the other hand, are almost wholly uneducated; their average number of years of schooling is less than one, with 12 per cent secondary school dropouts and the rest with no formal education whatsoever.[78]

The calculations of the Adelman model display a significant lack of sensitivity of the educational optima to the details of the industrial structure of the economy. For the economy described in the model's calculations, one can therefore formulate a reasonable plan without reference to developments in the noneducational portion of the model. This does not mean that such a procedure is indicated in the more general case in which one feature of the economy (the scarcity of high-level manpower) does not completely dominate all other considerations. What the results of the model suggest, then is a multistage approach of the following nature.[79] The first step is to ascertain, by any means whatever, whether or not some particular aspect of the economy exerts an overwhelming influence upon the optimal development plan. If it does, the conclusions are obvious. If not, it appears desirable to explore the situation with a fairly coarse model of the educational subsector coupled with rather aggregative description of the rest of the economy, as in the present model. The final stage of analysis would utilize the shadow prices obtained in the fairly aggregative calculations to set up an objective function which maximizes the net benefit from education subject to a considerably more detailed model of the educational subsector.

The complementary question to the one discussed above is the extent to which the development plan for the productive sectors of the economy can be programmed without including the educational sectors. To investigate this point the model was run without the educational sector, with the supply of each labor skill growing exogenously at the rate of 2 per cent per year.[80] A comparison of the optimum obtained for this case with that obtained with the same initial conditions when the educational sector is included, indicates that the omission of the educational sector from the model tends seriously to distort the allocation of resources in the productive sectors at both late and early periods of the plan.[81] In the model's calculations, the economy constrained by the need to educate its labor force devoted a significantly smaller percentage of its resources to invest, achieved a considerably lower rate of growth, was more agricultural, and concentrated a larger share of its industrial product in light consumer goods industries.[82]

The Adelman model indentifies a new point of departure for the programming models involving the educational sector in that unlike the previous attempts, the model explicitly involved optimization simultaneously in the education and noneducation sectors. This innovation allowed the demand for education to be generated endogenously by the development of the optimal pattern of noneducational as well as the educational growth. This approach, as Adelman indicated, was suggested by the highly sensitive nature of the optimal solution to a programming model of Argentina, when the availability of technical and managerial manpower was raised. The optimal profile of the economic structure was changed in the direction of a higher degree of industrialization, as well as a greater concentration of manufacturing in heavy rather than light industry.

The model is in the form of a dynamic linear program, covering several time periods. It represents a compromise between the manpower planning approach and the rate of return approach[83] in that fixed labor-output coefficients are used and the desirability of labor is a function of the earnings which one related to the level of schooling. The significant departure, however, which the Adelman approach takes is in determining the rate of return along with the production profile and the pattern of education rather than in the use of historical data.[84] The Adelman approach also shifts the emphasis from the unilateral determination of labor requirements typical of the more conventional manpower planning approach, and allows instead for the optimal determination of supply and demand.

In the convention of the linear programming format, the model specifies an objective function to be optimized subject to a set of constraints. The objective function that is maximized can assume many forms, and three such functions were considered in the model. The constraints are of several types and refer to the educational system and the productive system. For the educational system, the constraints involve the usual initial conditions (supply of students and teachers and school buildings), production function for the educational system in which it is specified how students move through the educational system and a set of exogenously specified lower limits to enrollment in each type of school, to prevent radical shifts in the pattern of school enrollment during the program. For the productive sectors, the technological conditions of production and investment, and the usual programming requirement limiting the use of resources, both labor in the form of skill, and sectoral capacity, as well as foreign exchange and savings are specified. Behavioral constraints and terminal conditions complete the list of constraints.

Maximization of the objective function subject to the constraints results in identification of optimal levels at which the various processes should be operated in each period of the program. In addition to that, the dual of the linear programming problem generates shadow prices with which to evaluate constraints in the optimal program. In short, there is significant relation between the number of limited resources and the number of processes in the solution of a linear programming problem. Resources which are not binding in the sense that they are not used to capacity will have a zero shadow price in the dual. If the number of resources is greater than the number of processes, some of the excess resources will have zero shadow prices in the dual; if the number of limited resources is exactly equal to the number of processes, all resources will have positive dual values; if the number of processes is greater than the number of resources in limited supply, some processes will not be used in the optimal program.

For the Adelman model, the dual of the program gives shadow prices for the graduates and dropouts of the various schools (or school levels) used in the system for each optimization problem. These values are used to determine social costs and benefits of education, and also to identify the subsidies that are justifiable to encourage dropouts to remain in school. Other results of the experiments include determination of investment in education, and the educational level of the labor force. Perhaps the most crucial part of the model comprises of the labor force change equations which provide the link between the educational and noneducational sectors. Labor demand per class of labor (a) workers, (b) managers, white collar workers, and professionals, and (c) proprietors is translated into demand for education via productivity differentials for different schooling levels within each skill class. Adelman assumed that labor within each skill class was highly substitutable, but even with equivalent education, substitution of labor between skill classes was not possible. Productivity parameters which were used are merely estimates of true parameters.

In the Adelman, model, as Professor Adelman indicated, the linearity assumptions comprise an essential limitation on the usefulness of the model, though they do not entirely invalidate its insight. Other perhaps more crucial issues involve the economic reality implied by the assumptions governing the productivity coefficients, and the issue of substitutability of skills in the production function. While on the one hand, it may not be entirely costless to convert, for example, proprietor skills into managerial skills; on the other hand, it is not easily defended that productivity differentials are constant, which is implied by a constant marginal rate of substitution for different levels of education within a given skill class. Labor market conditions would eventually be the deciding factor and perhaps earnings differentials would have to be used as an indicator of productivity differentials, despite their obvious limitations. Given these issues, Adelman found that with respect to the optimal educational allocation, the model was quite insensitive to changes in industrial structure and to the goals of the planners in that the alleviation of the high level manpower bottleneck emerged as the policy of highest priority.

In addition to testing the properties of the model via the use of different objective functions, Adelman could have combined the true objectives in any number of ways, perhaps by giving values to each objective in a composite preference function, or by regarding one of the three forms as the maximand, and listing the others as among the constraints of the model. While the first method would involve some real difficulties in the specification of the parameters, the second method would lend itself exceedingly well to the quantification of the costs of alternate levels of the instrument variables on the attainment of the specified objective of the program. Other parametric studies which could have been made, include variations of several groups of parameters, for example, teacher/student ratios, passing rates, duration of schooling, and the establishment of the universal primary education. Perhaps also one could study the effect of out-migration on the demands to be made on the educational system. A major departure would be to introduce nonconvexities into the model so as to study the problem of scale of economies.

BLITZER MODEL

The Blitzer model[85] was developed as a natural extension of the earlier five year plans for Turkey. However, the model differs from the earlier works in several important ways.

In developing the First and Second Five Year Plans, the Turkish State Planning Office (SPO) utilized macroeconomic planning models to test the overall consistency of the major macroeconomic target variables such as investment, savings, trade gap, consumption, and GNP. A one sector model of the Harrod-Domar type was built for the first plan. Using alternative estimates for the incremental capital-output ratio, various rates of capital formation were estimated consistent with the given percentage increases in GNP. In preparing the Second Plan, the economy was broken into five sectors (agriculture, mining, manufacturing, construction, and services). Neither plan was of the optimizing type. In order to calculate the requirements for foreign aid the rate of import substitution was specified exogenously for the target year.

The Blitzer model is more disaggregated in both its foreign trade sectors and in the number of producing sectors. Also its technical structure is based on newer and more reliable data and its base year and horizon date have been moved forward to 1969 and 1984 respectively. However the main difference between the Blitzer model and the earlier models is its inclusion and treatment of human resources. The earlier models all took little account of labor, either as a limiting factor in the economic growth rate or as a factor influencing the optimal sectoral composition of output and investment. It was assumed that Turkey had a labor surplus economy.[86] The First and Second plans calculated total employment and the requirements for skilled labor on the basis of sectoral growth rates which were derived independently of manpower supply considerations. Since the earlier models of the economy have not included labor inputs and manpower constraints, the shadow prices generated by these models imputed zero marginal productivity to labor and therefore equated the marginal product of capital with the marginal capital-output ratio for the economy as a whole.[87]

The Blitzer model attempts to merge economic and human resource planning using the general equilibrium approach which incorporates both into the same model. Human resources are introduced directly into the planning model through constraints on the supply and demand for various labor skills and through endogenous activities for the creation of human capital. Labor is no longer treated as surplus.

Any economic model can only be an approximation to the real world, and as such can only concentrate careful attention on several aspects of the economy. In the Blitzer model the major emphasis is on the importance of the utilization of human resources.

In the model, the behavior of the economy is examined at three year intervals between 1969 and 1984 to ensure that some supply and demand relationships are met. These supply and demand relationships include balances for labor skills, goods and services, foreign exchange, and domestic savings. In most respects, the model resembles dynamic input-output models which have been built during the last decade.[88] The model includes a current account interindustry matrix, capital coefficients, trade balance improvement activities, and macroeconomic variables. Labor is treated in an analogous way with physical capital. That is, it is disaggregated into skill levels, and activities are included in the model which create additions to the stocks of human capital for use in the production of goods and services.

The model breaks the economy down to eight sectors. These sectors include:[89]

1. Agriculture, livestock, forestry, fishing
2. Mining and quarrying
3. Manufacturing
4. Utilities
5. Construction
6. Commerce and trade
7. Transportation and communication
8. Services

For the base year 1969, it is assumed in the model that all quantities are known except for the distribution of investment outlays by sectors of destination.

In its basic formulation the model maximizes the level of Gross Domestic Product (GDP) at the terminal year 1984, subject to the various constraints on the technology, initial capital stocks (both physical and human), foreign exchange availability, and a constant upper limit on the marginal propensity to generate domestic savings.

Blitzer's inclusion of human resources in the model is not the first attempt at using programming techniques to analyze the problem of human resource planning. During the 1960s some programming models were developed and reported on. In general these models can be classified as belonging to one of two broad types; manpower requirement approach or choice of techniques in education and training approach.

In the models which use the manpower requirements approach the supplies of various labor skills are projected exogenously, and these projections are either used directly in the general model of the economy to insure a feasible allocation of resources[90], or are used indirectly with the general model to project skill gaps during the planning period.[91]

The other approach is choice of techniques in education and training. In this approach an education and training sector is modeled to provide needed manpower at some minimum cost.[92] The required quantities of skilled manpower are projected exogenously in some models, while in others the benefits of skill upgrading are included as coefficients in the objective function, where these coefficients are based on some sort of rate of return analysis.

The Blitzer model attempts a synthesis of the approaches, combining the skill requirements approach with its inclusion of labor constraints within the planning model, and the choice of techniques approach, in which these activities for the creation of various kinds of human capital. This approach allows the model to gain insight into the role of labor in project and sectoral evaluation.

The model computes balances for a number of labor skill classes in the economy. These balances ensure that there is sufficient manpower in each skill class to supply the needs of producing the economy's output of goods and services.

It is not possible to examine any of the human resource problems without disaggregating the labor supply in some meaningful way. In the model a simple disaggregation scheme in which the labor supply is divided into six skill categories is adopted. These categories are defined as follows:[93]

1. Scientists, engineers and professors include all university professors regardless of field, natural scientists, physicians, dentists, veterinarians, engineers and architects. In general the education level of this group corresponds to more than three years of higher education.

2. Other technical and professional workers are defined as all other workers, except those included in skill level 1, included in the population census as technical and professional workers.

3. Managerial and clerical workers includes all those whose education level corresponds to roughly three years of high school.

4. Skilled and semi-skilled workers are defined as all those urban workers whose jobs require the equivalent of a middle school education.

5. Unskilled urban workers form the remainder of the non-agricultural work force.

6. Unskilled agricultural workers form the bulk of the labor force. The group is primarily farmers.

This six way skill classification is an oversimplification of reality. Within each skill level the workers are taken to be homogeneous when they are not. However without this breakdown it wouldn't be possible for the model to construct the labor input requirement coefficients.

The model computes labor balances for skill levels 1 through 5.

These ensure that there is sufficient manpower to supply the demand of the producing sectors. Skill level 6 is assumed to be in surplus throughout the planning period in the model.

The model is formulated as a dynamic linear programming problem with 298 constraints and 300 linear programming variables.[94] The constraints of the model are as follows:[95]

For labor skill classes 1 through 5, the labor balance constraints ensure that there will be sufficient supply of manpower to meet the various demands for workers of that skill level. Urban labor is utilized either in the production of goods and services or in the production of additional skilled labor.

The supply of manpower in the model can be determined in three sources. There are exogenous supplies of manpower for each skill level during each time period. The human capital formation activities both add to these available supplies and deplete them (through training for higher skills). Additions to the supply of manpower of class 5 come from rural-urban migration. Finally, labor downgrading activities provide some scope for skill substitution.

The material balance constraints ensure that for each sector i producing goods and services, net domestic output (that is gross output less interindustry demands) plus imports at least meet the demands from deliveries to consumption, investment, exports, and urban transformation costs.

The foreign exchange balance constraints ensure that in each time period export earnings and net foreign loans and other invisibles at least meet the cost of all imports. There are rigid import requirements implicit in the noncompetitive import coefficients for intermediate goods, consumption goods, and investment goods. The model allows some scope of substitution in the choice of competitive imports and export activities.

Although the Blitzer model has five distinct export earning activities (agricultural goods, mined goods, manufactures, freight and shipping, and tourism) this is clearly insufficient disaggregation for a detailed examination of the comparative advantage of the country. For each of the eight producing sectors the output capacity constraints insure that gross output levels do not exceed additions to created capacity. This assumes that whatever the excess capacity in the base period, the absolute amount will remain unchanged during the planning period. These constraints are written as though time were continuous; the requirement for investment resources remains constant for the 3-year period centered around instant t itself; and there is an average lag of 1.5 years between resource input and the availability of capacity from that input. Since there are 3 years between periods and since the requirement for investment resources refers to the annual increment in capacity, a time factor of 3 years appears in these constraints.

Education activities are limited by their plant and equipment in a similar way to the productive sectors' output. Since the education activities are measured in man-years while the physical capacity creation activities are measured in monetary units, a student-investment ratios, q_i, are used to properly convert the units of measurement. The

q_i are defined as the increase in capacity (measured in man-years) of education sector i per unit increase in corresponding physical capacity measured in money terms.

Each export activity Z_i is constrained to grow at no more than e_i percent per year during any three year period. These upper bounds on the rates of growth of exports earnings are based on commodity projections of the SPO.

The lower bounds are placed on exports as a result of the fact that production for export markets cannot always be rapidly converted into production for domestic markets. These stipulate that exports of any particular item during any time period must be at least as great as they were in the previous period.

Sectoral investment levels are related to the capacity creation activities by sectoral capital output ratios.

Aggregate investment in each time period is the sum of the investment levels in each of the eight sectors producing goods and services plus the sum of the capacity creation activities of the three education sectors.

Sectoral gross output levels during period t are defined as base year gross output levels plus the increase in gross output between the base year and period t.

During each time period, the marginal propensity to consume must be at least as great as .74; this puts an upper bound on the marginal propensity to save at .26.[96]

Aggregate consumption is related to increases in per capita consumption through exogenous growth in population and the base year per capita consumption level, $\overline{C_o}$. The aggregate consumption is the sum of the population times the base year per capita consumption and increases over base year per capita consumption.

Gross domestic product is defined as the sum of the aggregate consumption and domestic savings. Domestic savings is aggregate investment less net foreign loans and other invisibles.

The terminal investment level constraints guarantee that investment in all sectors will be in proportions during the terminal period and are included in order to minimize horizon effects. They are created by assuming that after the terminal period investment within each sector will grow at 8 percent per year and the economy must be in a position to sustain this growth.[97]

The terminal education level constraints are intended to avoid horizon effects in the human capital formation sectors.

The objective function in the Blitzer model is simply the maximization of the gross domestic product in the terminal year.

The key issues to which this model has been focused are the importance and feasibility of including human resource questions within this multi-sector dynamic model. Comparing the projections from running the model with and without effective labor constraints, Blitzer concludes that the presence of human resources does have an important impact on the results.[98] The model also gives some clues regarding Turkey's future employment problems. Studies conducted by the SPO indicate that approximately 8 percent of Turkey's urban work force is unemployed. The studies also project a 7 percent annual rate of growth in the urban work force during the 1970s. If these figures are combined with the model's projections that the demand for labor can increase at only 6 percent per annum, we can conclude that the unemployment situation will get worse.[99]

Finally the model points out misallocation of human resources. The model's results indicate that the skill composition should be more pyramid shaped, in the sense of lower ratios of higher skills to lower skills. Special emphasis should be given to development of schools for managerial personnel to avoid the problems of overtraining.[100]

While the model projects different growth rates for the producing sectors and foreign trade activities, it is not sufficiently disaggregated to give many meaningful insights as to the profitability of subsectors. The model should concentrate on increasing the disaggregation, especially in the manufacturing sector.

CONCLUSIONS

The conclusion that the planning models using programming techniques are important advances in the techniques of economic policy-making when compared with their uses by the planners in practise creates the paradox.

One argument made against the planning models is that a relatively long time is required to formulate these models, to collect the necessary information and develop them to the stage where they can be used by the planners efficiently. If it is necessary to produce a plan document in two months and no analytical structure and only limited information is available then it will not be possible to use a model to help prepare the plan under consideration. But if the models are really superior tools, which we believe they are, the implication of this argument is that development of the models should not be delayed.

Another argument made against the multi-sector planning models is that the demands which these models make for data are greater than the demands made by simpler models which do not use the programming techniques in their formulations. Yet it is possible to build a multi-sectoral model with no more data input requirements than an aggregate model by assuming that all sectors have the same input and output structure. On the other hand, there is always some information available which would improve on that simple assumption. Unless the model structure is complex enough to permit that information to be used, it will have no effect on planning. Thus the argument reveals a failure to appreciate the significance of the assumptions involved in simpler models and the flexibility of the more sophisticated models.

Another argument made against the multi-sectoral planning models is that they require more resources than the simpler planning models, such as, economists, statisticians and computer programmers and computers. In many underdeveloped countries the limited availability of resources of this type may constitute a real problem. No amount of talk or explanation of the advantages of the models will overcome such a constraint. Nor is it realistic to argue that a more effective use of the manpower available in the country would overcome the constraint. While it is nearly always possible to use manpower and other resources more effectively there are often organizational and institutional constraints which make that impossible. On the other hand, professional manpower and other resource constraints are not unchangeable facts of life. They can be substantially modified within a relatively short time by appropriate resource allocations. If domestic conditions permit, foreign manpower can be imported to assist in the preparation of the models and in training of manpower to use them. This of course brings other problems in the political scene of the given country. So exercise of a reasonable amount of foresight on the side of the planners and a modest allocation of resources to the planning models would make it possible to break most bottlenecks relatively quickly. This kind of effort should come not only from the political structure of the given country but also from its individuals.

The returns to good economic policy-making as a result of planning models are quite large: the costs of mistakes, especially in the less developed countries where these type of models are developed, are great. There can be no doubt that many mistakes of economic policy have been made. While it takes careful study to verify mistakes the existence of widespread idle industrial capacity, repeated foreign exchange crisis and repeated requirements for emergency food imports at high prices is certainly suggestive. As compared to the costs of mistakes of this type the costs of improved methods of policy-making are trivial. The manpower and other resource costs of undertaking the improvement of planning methods might represent substantial diversions of resources currently allocated to planning, but they are modest in size. In a period no longer than is necessary to construct an integrated steel mill and at a tiny fraction of the cost, methods can be developed which can improve policy-making substantially.

Economists have been modest in the requests which they have made for resources to undertake the tasks of effective policy-making. There are situations in which simple calculations are sufficient to identify the major problems and solutions. However, most of the development problems cannot be resolved by simple calculations or even by more sophisticated but partial equilibrium calculations. A comprehensive approach, which is embodied in multi-sectoral programming models is necessary. We should not claim too much for these models but neither should we claim too little. There are many new developments which should be pursued. Yet policy-makers should not wait for the ultimate technique. The ones which now exist can substantially improve economic performance and help economic development.

FOOTNOTES

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[9]A. Charnes and W.W. Cooper, " Generalizations of the Warehousing Model ", Operations Research Quarterly 6 (1955): 131-172.

[10]F.L. Hitchcock, " Distribution of a Product From Several Sources to Numerous Localities ", Journal of Mathematical Physics 20 (1941): 224-230.

[11]Koopmans, " Optimum Utilization of the Transportation Systems ", Econometrica 17 [supplement] (1940): 136-146.

[12]Dantzig, " Application of the Simplex Method to a Transportation Problem, " T.C. Koopmans (ed.), pp. 359-373.

[13]Dorfman, Samuelson and Solow, p. 2.

[14]G. Di Roccaferrera, Introduction to Linear Programming Processes (Cincinnati, Ohio: 1967), pp. 182-184.

[15]For more details on the Simplex Algorithm see Hamdy Taha, Operations Research (New York: MacMillan Publishing Company, 1967), and George Dantzig, Linear Programming and Extensions (Princeton, New Jersey: Princeton University Press, 1963).

[16]Hamdy Taha, Operations Research, p. 94.

[17]R.S. Eckaus and K.S. Parish, Planning for Growth (Cambridge, Massachusetts: 1968), p. 21.

[18]The social welfare function can be calculated theoretically, but in practice, due to the lack of sufficient data, it is unknown to the planning model.

[19]Eckaus and Parish, p. 22.

[20]Improvement in the standard of living is the major objective of development planners in most cases. By optimizing consumption on the aggregate level this goal is automatically satisfied.

[21]Eckaus and Parish, p. 23.

[22]This social behavioral constraint can be imposed on the society by the central planning officials. But in mixed economies, this constraint cannot be imposed on the society.

[23]Eckaus and Parish, p. 24

[24]Ibid., p. 24

[25]Ibid., p. 24

[26]Ibid., p. 25

[27]Ibid., p. 26

[28]Ibid., p. 26

[29] Ibid., p. 27.

[30] Ibid., p. 28.

[31] The matrices p', p'', p''', have a simple relationship to p, p_{ij} being equal to $p'_{ij}+p''_{ij}+p'''_{ij}$.

[32] Eckaus, p. 28.

[33] Ibid., p. 28.

[34] Ibid., p. 28.

[35] Ibid., p. 29.

[36] Ibid., p. 29.

[37] Ibid., p. 29.

[38] Ibid., p. 29.

[39] Ibid., p. 29.

[40] Ibid., p. 30.

[41] Ibid., p. 30.

[42] Ibid., p. 30.

[43] Ibid., p. 30.

[44] Ibid., p. 31.

[45] Ibid., p. 31.

[46] Ibid., p. 31.

[47] Ibid., p. 46.

[48] Ibid., p. 48.

[49] These constraints are useful to the model in finding solutions when the available data is insufficient to generate the solutions.

[50] Eckaus, p. 16.

[51] Ibid., p. 16.

[52]*Ibid.*, p. 17.

[53]*Ibid.*, p. 17.

[54]*Ibid.*, p. 18.

[55]*Ibid.*, p. 18.

[56]I. Adelman, "A Linear Programming Model of Educational Planning : A Case Study of Argentina," in I. Adelman and E. Thornbecke (eds.), *The Theory and Design of Development* (Baltimore: Johns Hopkins Press, 1966), pp. 385-412.

[57]*Ibid.*, p. 386.

[58]*Ibid.*, p. 388.

[59]For more details on absorptive capacity see J. H. Adler, *Absorptive Capacity--The Concept and Its Determinants* (Washington D. C.: Brookings Institution, 1964).

[60]Terminal conditions relate the events of the plan period to the postplan period, so there will not be any discontinuity in the model at the end of the planning period.

[61]In the model, the endogenous variables are the numbers of new students, new teachers, and new graduate and dropout workers by skill category in the educational sector.

[62]The decision variables in a planning model are the determinant variables of the optimal decisions. These variables determine the optimal solutions.

[63]Adelman, p. 388.

[64]*Ibid.*, p. 389.

[65]*Ibid.*, p. 389.

[66]*Ibid.*, p. 389.

[67]*Ibid.*, p. 389.

[68]For the mathematical model formulation see Adelman, pp. 400-403.

[69]This constraint can be emphasized by government regulations on the school-age children by making the participation in education compulsory. Indeed, this is what is done in underdeveloped countries to assure this particular socio-cultural constraint.

[70]Adelman, p. 396.

[71]The foreign capital commitments can be analyzed in two ways, foreign aid commitments and foreign investment in that particular country.

[72]Adelman, p. 404.

[73]Ibid., p. 406.

[74]Ibid., p. 407.

[75]Ibid., p. 407.

[76]Ibid., p. 408.

[77]Ibid., p. 408.

[78]Ibid., p. 408.

[79]Ibid., p. 411.

[80]Ibid., p. 412.

[81]Ibid., p. 412.

[82]Ibid., p. 412.

[83]See S. Bowles, Planning Educational System for Economic Growth (Cambridge, Massachusetts: Harvard University Press, 1969).

[84]See S. Bowles, "The Efficient Allocation of Resources in Education," Quarterly Journal of Economics 81 (May 1967), pp. 188-219. Bowles uses the historical data in determining the rate of return.

[85]C. Blitzer, "A Perspective Planning Model for Turkey: 1969-1984," dissertation submitted to the Department of Economics, Stanford University, California, August 1971.

[86]It was assumed that there were no limitations on the supply

side of the labor.

[87]The earlier models did not take the limitations on the supply of labor and calculations of these models ignored the productivity of labor. They imputed zero marginal productivity to labor.

[88]Look at S. Chakravary and L. Lefeber, "An Optimizing Planning Model," Economic Weekly (1965) and M. Bruno, "A Programming Model for Israel," in L. Adelman and E. Thornbecke (eds.), 1966.

[89]Blitzer, p. 12.

[90]See Bruno, 1966.

[91]See F. Harbison and C. A. Myers, Education, Manpower and Economic Growth. Strategies of Human Resource Development (New York: McGraw-Hill Publishing Company, 1964).

[92]See Bowles, 1967, 1969.

[93]Blitzer, pp. 17-18.

[94]*Ibid.*, p. 30.

[95]*Ibid.*, pp. 31-41.

[96]*Ibid.*, p. 42.

[97]*Ibid.*, p. 44.

[98]The model was run without the labor constraints to see the changes in the macroeconomic variables within the model.

[99]Blitzer concluded that by 1984 the unemployment rate would increase up to 20 per cent.

[100]Blitzer, p. 47.

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