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We develop a theory for the study of the absorption and disper-
sion of a sound wave propagating in a binary mixture of gases by using
both the 13-moment approximation and the Navier - Stokes theory. This
general theory is valid for a mixture of any two monatomic gases, how-
ever, we limit our study to the mixture of helium and argon.

After lengthy mathematical formulation we obtain a set of linear
equations in the general eigenvalue - eigenvector form. Using numerical
methods to solve the eigenvalue equation, we compute the absorption and
dispersion.

We compare the absorption calculated by both the 13-moment approx-
imation and the Navier - Stokes theory with experimental data for mix-
tures containing 5%, 10%, 25%, 50% and 75% argon. Both theories give
values which compare favorably with experiment for low and mid-range
values of frequency/pressure. The Navier - Stokes absorption values are
consistently closer to experiment for higher values of f/p.

For the 50% argon mixture, we find that the 13-moment absorption
curve peaks sharply for $f/p \approx 280$. The corresponding dispersion curve
dips at the same value. The Navier - Stokes theory yields neither of
these features.

For each of the remaining cases, the dispersion curves for the
13-moment approximation start at ≈ 1 and decrease until leveling off.
The Navier - Stokes dispersion curves also start at 1 but decrease
steadily, with the final dispersion value being greater than the corre-
sponding absorption by .01.

THE KINETIC THEORY OF SOUND PROPAGATION

IN BINARY GASEOUS MIXTURES

by

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ii

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TABLE OF CONTENTS

413101

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Our research is a study of the absorption and dispersion of a sound wave propagating in a binary mixture of monatomic gases. Kohler[1] in 1949 studied sound propagation in a gaseous mixture by using the single temperature Navier - Stokes theory. Goldman[2] applied the two fluid theory to arrive at the absorption and dispersion of the sound wave. Bramlette and Huang,[3] using the discrete ordinate method, studied forced and free wave propagation in binary mixtures.

We approach the problem by using Walker's[4] 13-moment approximation equations for a mixture of gases consisting of Maxwell molecules. We assume that the deviations of flow velocities and temperatures are arbitrary, while heat flux and stress deviations are small. We also apply the dimensionless collision integrals, as derived by Williams,[5] to these equations.

As a second approach we develop the two temperature Navier - Stokes theory, which differs from Kohler's[1] approach in that his theory dealt only with deviations in the temperature of the mixture, while our theory is based on the deviations in the temperatures of each gas.

The absorption calculated by both theories is compared with experimental values for the helium - argon mixture as determined by Prangsma, Jonkman and Beenakker.[6] The dispersion for both theories is also plotted.

In Chapter II we develop the dimensionless forms of the 13-moment

and Navier - Stokes equations, and express them in the eigenvalue - eigenvector form. We explain the numerical solutions to these equations in Chapter III. Chapter IV contains the plotted solutions and a discussion of the results. In Chapter V we give a summary of the research.

CHAPTER II
MATHEMATICAL FORMULATIONS

A. 13-Moment Approximation

We begin with Walker's[4] 13-moment approximation equations for a binary gaseous mixture of Maxwell molecules. These equations are a closed set of differential equations for the density, flow velocity, temperature, stress, and heat flux of the mixture. For a one dimensional problem such as sound propagation in the x-direction, these equations can be written in the form:

continuity,

$$\frac{\partial n_1}{\partial t} + n_1 \frac{\partial u_1}{\partial x} = 0 , \tag{1}$$

$$\frac{\partial n_2}{\partial t} + n_2 \frac{\partial u_2}{\partial x} = 0 , \tag{2}$$

momentum,

$$m_1 n_1 \frac{\partial u_1}{\partial t} + \frac{\partial P_1}{\partial x} + k T_1 \frac{\partial n_1}{\partial x} + n_1 k \frac{\partial T_1}{\partial x} = n_1 \nu_{12} m^* (u_2 - u_1) , \tag{3}$$

$$m_2 n_2 \frac{\partial u_2}{\partial t} + \frac{\partial P_2}{\partial x} + k T_2 \frac{\partial n_2}{\partial x} + n_2 k \frac{\partial T_2}{\partial x} = - n_2 \nu_{21} m^* (u_2 - u_1) , \tag{4}$$

energy,

$$\frac{3}{2} n_1 k \frac{\partial T_1}{\partial t} + \frac{\partial q_1}{\partial x} + n_1 k T_1 \frac{\partial u_1}{\partial x} = 3 n_1 \nu_{12} \frac{m_1 m_2}{(m_1 + m_2)^2} k (T_2 - T_1) , \tag{5}$$

$$\frac{3}{2} n_2 k \frac{\partial T_2}{\partial t} + \frac{\partial q_2}{\partial x} + n_2 k T_2 \frac{\partial u_2}{\partial x} = - 3 n_2 \nu_{12} \frac{m_1 m_2}{(m_1 + m_2)^2} k (T_2 - T_1) , \tag{6}$$

stress,

$$\frac{\partial P_1}{\partial t} + \frac{8}{15}\frac{\partial q_1}{\partial x} + \frac{4}{3}n_1 kT_1 \frac{\partial u_1}{\partial x} = \left[\nu_{11}(\beta-1) + 2\beta\nu_{12}\left(\frac{m_2}{m_1+m_2}\right)^2\left(1 - \frac{m_1+m_2}{\beta m_2}\right)\right]P_1$$
$$+ 2\beta\nu_{12}\left(\frac{m_2}{m_1+m_2}\right)^2 \frac{n_1}{n_2}\frac{m_1}{m_2} P_2 , \tag{7}$$

$$\frac{\partial P_2}{\partial t} + \frac{8}{15}\frac{\partial q_2}{\partial x} + \frac{4}{3}n_2 kT_2 \frac{\partial u_2}{\partial x} = \left[\nu_{22}(\beta-1) + 2\beta\nu_{21}\left(\frac{m_1}{m_1+m_2}\right)^2\left(1 - \frac{m_1+m_2}{\beta m_1}\right)\right]P_2$$
$$+ 2\beta\nu_{21}\left(\frac{m_1}{m_1+m_2}\right)^2 \frac{n_2}{n_1}\frac{m_2}{m_1} P_1 , \tag{8}$$

heat flux,

$$\frac{\partial q_1}{\partial t} + \frac{kT_1}{m_1}\frac{\partial P_1}{\partial x} + \frac{5n_1 k^2 T_1}{2m_1}\frac{\partial T_1}{\partial x} =$$
$$\left\{\frac{1}{2}(\gamma-2) - 2\nu_{12}\left(\frac{m_2}{m_1+m_2}\right)^3\left[\gamma - (1+\gamma)\left(\frac{m_1+m_2}{m_2}\right) + \frac{3}{2}\left(\frac{m_1+m_2}{m_2}\right)^2\right]\right\} q_1$$
$$+ 2\nu_{12}\left(\frac{m_2}{m_1+m_2}\right)^3 \frac{m_1}{m_2}\frac{n_1}{n_2} \gamma q_2 , \tag{9}$$

$$\frac{\partial q_2}{\partial t} + \frac{kT_2}{m_2}\frac{\partial P_2}{\partial x} + \frac{5n_2 k^2 T_2}{2m_2}\frac{\partial T_2}{\partial x} =$$
$$\left\{\frac{1}{2}(\gamma-2) - 2\nu_{21}\left(\frac{m_1}{m_1+m_2}\right)^3\left[\gamma - (1+\gamma)\left(\frac{m_1+m_2}{m_1}\right) + \frac{3}{2}\left(\frac{m_1+m_2}{m_1}\right)^2\right]\right\} q_2$$
$$+ 2\nu_{21}\left(\frac{m_1}{m_1+m_2}\right)^3 \frac{m_2}{m_1}\frac{n_2}{n_1} \gamma q_1 , \tag{10}$$

where, for gas i (i=1 and 2), n_i is the particle density; u_i is the flow velocity in the x-direction; m_i is the mass; P_i is the viscous stress; T_i is the temperature; and q_i is the heat flux. Also in these equations $m^* = m_1 m_2/(m_1+m_2)$ is the reduced mass of the system; k is Boltzmann's constant; $\beta = 1-3A_2(5)/4A_1(5) = .9661$; $\gamma = 2-A_2(5)/A_1(5) = .2246$ (see reference 7); $A_1(5)$ and $A_2(5)$ are dimensionless collision cross sections defined by Chapman and Cowling;[8] and $\nu_{ij} = 2 A_1(5)(K_{ij}/m^*)^{1/2}n$, is the frequency of collision between type i and type j molecules, with K_{ij} being the force constant in the Maxwell intermolecular force law, $F_{ij} =$

$K_{ij}/r^5.$

The right hand side of Eqs. (3) - (10) are the collision integrals for gases with type 1 and type 2 molecules as derived by Williams.[5] The left hand side of Eqs. (1) - (10) are similar to Creech's,[9] except that we are not interested in the internal energy terms. Also we have ten equations whereas he had six, since we are working with a mixture of gases, while he studied a polyatomic gas.

To facilitate the solving of these equations by numerical methods, we find it helpful to write them as

$$\frac{\partial n_1}{\partial t} + n_1 \frac{\partial u_1}{\partial x} = 0 \quad , \tag{11}$$

$$\frac{\partial n_2}{\partial t} + n_2 \frac{\partial u_2}{\partial x} = 0 \quad , \tag{12}$$

$$m_1 n_1 \frac{\partial u_1}{\partial t} + \frac{\partial P_1}{\partial x} + kT_1 \frac{\partial n_1}{\partial x} + n_1 k \frac{\partial T_1}{\partial x} = n_1 m_1 a_1 \nu (u_2 - u_1) , \tag{13}$$

$$m_2 n_2 \frac{\partial u_2}{\partial t} + \frac{\partial P_2}{\partial x} + kT_2 \frac{\partial n_2}{\partial x} + n_1 k \frac{\partial T_2}{\partial x} = - n_2 m_2 a_2 \nu (u_2 - u_1) , \tag{14}$$

$$\frac{3}{2} n_1 k \frac{\partial T_1}{\partial t} + \frac{\partial q_1}{\partial x} + n_1 kT_1 \frac{\partial u_1}{\partial x} = 3 n_1 b_1 \nu k (T_2 - T_1) , \tag{15}$$

$$\frac{3}{2} n_2 k \frac{\partial T_2}{\partial t} + \frac{\partial q_2}{\partial x} + n_2 kT_2 \frac{\partial u_2}{\partial x} = - 3 n_2 b_2 \nu k (T_2 - T_1) , \tag{16}$$

$$\frac{\partial P_1}{\partial t} + \frac{8}{15} \frac{\partial q_1}{\partial x} + \frac{4}{3} n_1 kT_1 \frac{\partial u_1}{\partial x} = (a_{11} P_1 + a_{12} P_2) \nu , \tag{17}$$

$$\frac{\partial P_2}{\partial t} + \frac{8}{15} \frac{\partial q_2}{\partial x} + \frac{4}{3} n_2 kT_2 \frac{\partial u_2}{\partial x} = (a_{21} P_1 + a_{22} P_2) \nu , \tag{18}$$

$$\frac{\partial q_1}{\partial t} + \frac{kT_1}{m_1} \frac{\partial P_1}{\partial x} + \frac{5 n_1 k^2 T_1}{2 m_1} \frac{\partial T_1}{\partial x} = (b_{11} q_1 + b_{12} q_2) \nu , \tag{19}$$

$$\frac{\partial q_2}{\partial t} + \frac{kT_2}{m_2} \frac{\partial P_2}{\partial x} + \frac{5 n_2 k^2 T_2}{2 m_2} \frac{\partial T_2}{\partial x} = (b_{21} q_1 + b_{22} q_2) \nu . \tag{20}$$

In these equations we have set $\nu = \nu_{11}$; and the coefficients of the

right hand side of Eqs. (11) - (20) are given by:

$$a_1 = \frac{1}{m+1} \left(\frac{m+1}{2}\right)^{1/2} \frac{1}{n} \left(\frac{K_{12}}{K_{11}}\right)^{1/2} ,$$

$$a_2 = m n a_1 ,$$

$$b_1 = \left(\frac{m+1}{2}\right)^{1/2} \frac{1}{n} \left(\frac{K_{12}}{K_{11}}\right)^{1/2} \frac{m}{(m+1)^2} ,$$

$$b_2 = n b_1 ,$$

$$a_{11} = -\left[(1-\beta) + \frac{2}{(m+1)^2} (m+1-\beta) \left(\frac{m+1}{2}\right)^{1/2} \frac{1}{n} \left(\frac{K_{12}}{K_{11}}\right)^{1/2} \right] ,$$

$$a_{12} = \frac{2m\beta}{(m+1)^2} \left(\frac{m+1}{2}\right)^{1/2} \left(\frac{K_{12}}{K_{11}}\right)^{1/2} ,$$

$$a_{21} = -\left\{ (1-\beta) m^{1/2} \left(\frac{K_{22}}{K_{11}}\right)^{1/2} \frac{1}{n} + 2 \left(\frac{m+1}{2}\right)^{1/2} \left(\frac{K_{12}}{K_{11}}\right)^{1/2} \frac{m}{(m+1)^2} \left[1 + m(1-\beta) \right] \right\} ,$$

$$a_{22} = 2\beta \left(\frac{m+1}{2}\right)^{1/2} \left(\frac{K_{12}}{K_{11}}\right)^{1/2} \frac{m}{(m+1)^2} \frac{1}{n} ,$$

$$b_{11} = -\left\{ \frac{1}{2}(2-\gamma) + \frac{2}{(m+1)^3} \left(\frac{m+1}{2}\right)^{1/2} \frac{1}{n} \left(\frac{K_{12}}{K_{11}}\right)^{1/2} \left[\gamma - (1+\gamma)(m+1) + \frac{3}{2}(m+1)^2 \right] \right\} ,$$

$$b_{12} = \frac{2m\gamma}{(m+1)^3} \left(\frac{m+1}{2}\right)^{1/3} \left(\frac{K_{12}}{K_{11}}\right)^{1/2} ,$$

$$b_{21} = 2 \left(\frac{m+1}{2}\right)^{1/2} \left(\frac{K_{12}}{K_{11}}\right)^{1/2} \left(\frac{m}{m+1}\right)^3 \frac{1}{mn} ,$$

$$b_{22} = -\left\{ \frac{1}{2}(2-\gamma) m^{1/2} \left(\frac{K_{22}}{K_{11}}\right)^{1/2} \frac{1}{n} + 2 \left(\frac{m+1}{2}\right)^{1/2} \left(\frac{K_{12}}{K_{11}}\right)^{1/2} \left(\frac{m}{m+1}\right)^3 \right.$$
$$\left. \cdot \left[\gamma - (1+\gamma)\left(\frac{m+1}{m}\right) + \frac{3}{2}\left(\frac{m+1}{m}\right)^2 \right] \right\} ,$$

where $m = m_1/m_2$, and $n = n_1/n_2$.

In order to study the problem of sound propagation in the gaseous mixture, we assume plane wave solutions to Eqs. (11) - (20) of the form:

$$n_j = n_{oj} + n_j' e^{i(\omega t - \gamma x)} \tag{21}$$

$$u_j = u_j' e^{i(\omega t - \gamma x)} \quad , \tag{22}$$

$$T_j = T_o + T_j' e^{i(\omega t - \gamma x)} \quad , \tag{23}$$

$$P_j = P_j' e^{i(\omega t - \gamma x)} \quad , \tag{24}$$

$$q_j = q_j' e^{i(\omega t - \gamma x)} \quad , \tag{25}$$

where $j = 1$ and 2 for gases one and two respectively; T_0 is the ambient temperature; n_{01} and n_{02} are the ambient densities; the primed quantities are the deviations from equilibrium; and $\gamma = \omega/c - i\alpha$ is the complex wave number, where ω is the angular frequency, α is the linear absorption coefficient, and c is the speed of the wave.

Upon substitution of Eqs. (21) - (25) into Eqs. (11) - (20) and dropping prime notation, we obtain:

$$i\omega n_1 - i\gamma n_{10} u_1 = 0 \quad , \tag{26}$$

$$i\omega n_2 - i\gamma n_{20} u_2 = 0 \quad , \tag{27}$$

$$i\omega m_1 n_{10} u_1 - i\gamma P_1 - i\gamma k T_o n_1 - i\gamma n_{10} k T_1 = n_{10} m_1 a_1 \nu (u_2 - u_1) , \tag{28}$$

$$i\omega m_2 n_{20} u_2 - i\gamma P_2 - i\gamma k T_o n_2 - i\gamma n_{20} k T_2 = -n_{20} m_2 a_2 \nu (u_2 - u_1) , \tag{29}$$

$$i\omega \frac{3}{2} n_{10} k T_1 - i\gamma q_1 - i\gamma n_{10} k T_o u_1 = 3 n_{10} b_1 \nu k (T_2 - T_1) , \tag{30}$$

$$i\omega \frac{3}{2} n_{20} k T_2 - i\gamma q_2 - i\gamma n_{20} k T_o u_2 = -3 n_{20} b_2 k (T_2 - T_1) , \tag{31}$$

$$i\omega P_1 - i\gamma \frac{8}{15} q_1 - i\gamma \frac{4}{3} n_{10} k T_o u_1 = (a_{11} P_1 + a_{12} P_2) \nu , \tag{32}$$

$$i\omega P_2 - i\gamma \frac{8}{15} q_2 - i\gamma \frac{4}{3} n_{20} k T_o u_2 = (a_{21} P_1 + a_{22} P_2) \nu , \tag{33}$$

$$i\omega q_1 - i\gamma \frac{k T_o}{m_1} P_1 - i\gamma \frac{5 n_{10} k^2 T_o}{2 m_1} T_1 = (b_{11} q_1 + b_{12} q_2) \nu , \tag{34}$$

$$i\omega q_2 - i\gamma \frac{k T_o}{m_2} P_2 - i\gamma \frac{5 n_{20} k^2 T_o}{2 m_2} T_2 = (b_{21} q_1 + b_{22} q_2) \nu . \tag{35}$$

We can write Eqs.(26) - (35) in dimensionless form by introducing the following variables: $\mu = \nu/\omega$, which is the ratio of self-collision frequency of particles of gas one to angular sound frequency; $c_0 = (5kT_0/3m_{eff})^{1/2}$, which is the adiabatic or low frequency sound speed with $m_{eff} = \left(\dfrac{n_1}{n_1+n_2}\right) m_1 + \left(\dfrac{n_2}{n_1+n_2}\right) m_2$; and

$$\lambda^{-1} = \frac{c_0 \gamma}{\omega} = \frac{c_0}{c} - i\frac{\alpha c_0}{\omega} \quad . \tag{36}$$

The resulting dimensionless equations are:

$$X_2 = \lambda X_1 \quad , \tag{37}$$

$$\frac{3}{5} m_1^* X_1 + \frac{3}{5} m_1^* X_3 + m_1^* X_4 = \lambda \phi_1 X_2 + \lambda \alpha_1 X_7 \quad , \tag{38}$$

$$\frac{3}{5} X_2 + X_5 = \lambda \frac{9}{10} \epsilon_1 X_3 + \lambda \frac{9}{5} \beta_1 X_8 \quad , \tag{39}$$

$$\frac{4}{5} X_2 + \frac{8}{15} X_5 = \lambda \theta_1 X_4 + \lambda \frac{\mu_{12}}{n} X_9 \quad , \tag{40}$$

$$\frac{9}{10} m_1^* X_3 + \frac{3}{5} m_1^* X_4 = \lambda \delta_1 X_5 + \lambda \frac{\rho_{12}}{n} X_{10} \quad , \tag{41}$$

$$X_7 = \lambda X_6 \quad , \tag{42}$$

$$\frac{3}{5} m_2^* X_6 + \frac{3}{5} m_2^* X_8 + m_2^* X_9 = \lambda \alpha_2 X_2 + \lambda \phi_2 X_7 , \tag{43}$$

$$\frac{3}{5} X_7 + X_{10} = \lambda \frac{9}{5} \beta_2 X_3 + \lambda \frac{9}{10} \epsilon_2 X_8 \quad , \tag{44}$$

$$\frac{4}{5} X_7 + \frac{8}{15} X_{10} = \lambda n \mu_{21} X_4 + \lambda \theta_2 X_9 \quad , \tag{45}$$

$$\frac{9}{10} m_2^* X_8 + \frac{3}{5} m_2^* X_9 = \lambda n \rho_{21} X_5 + \lambda \delta_2 X_{10} , \tag{46}$$

where $X_1 = \dfrac{n_1}{n_{10}}$, $X_2 = \dfrac{u_1}{c_0}$, $X_3 = \dfrac{T_1}{T_0}$, $X_4 = \dfrac{P_1}{m_{eff} n_{10} c_0^2}$, $X_5 = \dfrac{q_1}{m_{eff} n_{10} c_0^3}$,

$X_6 = \dfrac{n_2}{n_{20}}$, $X_7 = \dfrac{u_2}{c_0}$, $X_8 = \dfrac{T_2}{T_0}$, $X_9 = \dfrac{P_2}{m_{eff} n_{20} c_0^2}$, $X_{10} = \dfrac{q_2}{m_{eff} n_{20} c_0^3}$, $m_1^* = \dfrac{m_{eff}}{m_1}$,

$m_2^* = m \, m_1^*$,

and for j, 1 = 1 and 2

$$\alpha_j = i a_j \kappa \quad ,$$
$$\beta_j = i b_j \kappa \quad ,$$
$$\mu_{jl} = i a_{jl} \kappa \quad ,$$
$$\rho_{jl} = i b_{jl} \kappa \quad ,$$
$$\phi_j = (1 - \alpha_j) \quad ,$$
$$6_1 = (1 + 2\beta_j) \quad ,$$
$$\theta_j = (1 + 2\mu_{jj}) ,$$
$$\delta_j = (1 + \rho_{jj}) \quad .$$

Eqs. (37) - (46) are a set of ten equations which can be written in the general eigenvalue - eigenvector form:

$$A\vec{X} = \lambda B \vec{X} , \tag{47}$$

where

$$\vec{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{10} \end{bmatrix}$$

is the eigenvector, and A and B are the matrices whose elements are the coefficients of the equations. We wish to solve Eq. (47) for the eigenvalue λ. By Eq. (36) the absorption , $\alpha c_0 / \omega$, and the dispersion, c_0/c, of the sound wave can be found once we have obtained λ.

In order to solve this set of equations we use numerical methods similar to those developed by Creech.[9] The IBM subroutines which he used to calculate the eigenvalues cannot handle our set of equations, since we have zeros on the diagonal of matrix A. In order to obtain equations which can be handled by the subroutines we follow a proce-

dure similar to that used by Creech.[9] We eliminate variables X_2, X_7, X_4, X_9, X_5, and X_{10} analytically by expressing them in terms of the remaining variables. The eliminated variables are given by the following:

$$X_2 = \lambda X_1 \quad , \tag{48}$$

$$X_7 = \lambda X_6 \quad , \tag{49}$$

$$X_4 = \frac{1}{m_1^*}\left[\left(\lambda^2\phi_1 - \frac{3}{5}m_1^*\right)X_1 - \frac{3}{5}m_1^* X_3 + \lambda^2\alpha_1 X_6\right] \quad , \tag{50}$$

$$X_9 = \frac{1}{m_2^*}\left[\lambda^2\alpha_2 X_1 + \left(\lambda^2\phi_2 - \frac{3}{5}m_2^*\right)X_6 - \frac{3}{5}m_2^* X_8\right] \quad , \tag{51}$$

$$X_5 = \lambda\left[\frac{9}{10}\delta_1 X_3 + \frac{9}{5}\beta_1 X_8 - \frac{3}{5}X_1\right] \quad , \tag{52}$$

$$X_{10} = \lambda\left[\frac{9}{10}\delta_2 X_8 + \frac{9}{5}\beta_1 X_3 - \frac{3}{5}X_6\right] \quad . \tag{53}$$

By substituting Eqs.(48) - (53) into Eqs.(40), (41), (45) and (46), we arrive at the following set of equations:

$$\left(\frac{3}{5}\theta_1 + \frac{12}{25}\right)X_1 + \left(\frac{3}{5}\theta_1 + \frac{12}{25}\delta_1\right)X_3 + \frac{3}{5}\frac{\mu_{12}}{n}X_6 + \left(\frac{3}{5}\frac{\mu_{12}}{n} + \frac{24}{25}\beta_1\right)X_8 =$$
$$\lambda^2\left\{\left(\frac{9\,\phi_1}{m_1^*} + \frac{\alpha_2\,\mu_{12}}{n\,m_2^*}\right)X_1 + \left(\frac{\alpha_1\,\theta_1}{m_1^*} + \frac{\mu_{12}\,\phi_2}{n\,m_2^*}\right)X_6\right\}, \tag{54}$$

$$-\frac{9}{25}m_1^* X_1 + \frac{27}{50}m_1^* X_3 = \lambda^2\left\{-\frac{3}{5}\left(\delta_1 + \phi_1\right)X_1 + \left(\frac{9}{10}\delta_1\delta_1 + \frac{9}{5}\frac{\beta_2\rho_{12}}{n}\right)X_3\right.$$
$$\left. -\frac{3}{5}\left(\alpha_1 + \frac{\rho_{12}}{n}\right)X_6 + \left(\frac{9}{5}\delta_1\beta_1 + \frac{9}{10}\frac{\rho_{12}}{n}\delta_2\right)X_8\right\}, \tag{55}$$

$$\frac{3}{5}n\mu_{21}X_1 + \left(\frac{3}{5}n\mu_{21} + \frac{24}{25}\beta_2\right)X_3 + \left(\frac{3}{5}\theta_2 + \frac{12}{25}\right)X_6 + \left(\frac{3}{5}\theta_2 + \frac{12}{25}\delta_2\right)X_8 =$$
$$\lambda^2\left\{\left(\frac{\alpha_2\,\theta_2}{m_2^*} + \frac{\phi_1\,n\mu_{21}}{m_1^*}\right)X_1 + \left(\frac{\phi_2\,\theta_2}{m_2^*} + \frac{\alpha_1\,n\mu_{21}}{m_1^*}\right)X_6\right\}, \tag{56}$$

$$-\frac{9}{25}m_2^* X_6 + \frac{27}{50}m_2^* X_8 = \lambda^2\left\{-\frac{3}{5}\left(\alpha_2 + n\rho_{21}\right)X_1 + \left(\frac{9}{5}\beta_2\delta_2 + \frac{9}{10}n\rho_{21}\delta_1\right)X_3\right.$$
$$\left. -\frac{3}{5}\left(\delta_2 + \phi_2\right)X_6 + \left(\frac{9}{10}\delta_2\delta_2 + \frac{9}{5}n\rho_{21}\beta_1\right)X_8\right\} . \tag{57}$$

These equations are also of the form of a general eigenvalue equation

$$A \vec{X} = \lambda^2 B \vec{X},$$ (58)

where

$$\vec{X} = \begin{bmatrix} X_1 \\ X_3 \\ X_6 \\ X_8 \end{bmatrix},$$

λ^2 is the eigenvalue, and where A and B are complex matrices given by

$$A = \begin{bmatrix} \frac{3}{5}\theta_1 + \frac{12}{25} & -\frac{3}{5}\theta_1 + \frac{12}{25}\zeta_1 & \frac{3}{5}\frac{\mu_{12}}{n} & \frac{3}{5}\frac{\mu_{12}}{n} + \frac{24}{25}\beta_1 \\[2mm] -\frac{9}{25}m_1^{*} & \frac{27}{50}m_1^{*} & 0 & 0 \\[2mm] \frac{3}{5}n\mu_{21} & \frac{3}{5}n\mu_{21} + \frac{24}{25}\beta_2 & \frac{3}{5}\theta_2 + \frac{12}{25} & \frac{3}{5}\theta_2 + \frac{12}{25}\zeta_2 \\[2mm] 0 & 0 & -\frac{9}{25}m_2^{*} & \frac{27}{50}m_2^{*} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\theta_1\phi_1}{m_1^{*}} + \frac{\alpha_2\mu_{12}}{n m_2^{*}} & 0 & \frac{\alpha_1\theta_1}{m_1^{*}} + \frac{\mu_{12}\phi_2}{n m_2^{*}} & 0 \\[2mm] -\frac{3}{5}(\delta_1+\phi_1) & \frac{9}{5}\frac{\beta_2\rho_{12}}{n} + \frac{9}{10}\zeta_1\delta_1 & -\frac{3}{5}\left(\alpha_1 + \frac{\beta_{12}}{n}\right) & \frac{9}{10}\frac{\rho_2\zeta_2}{n} + \frac{9}{5}\delta\beta_1 \\[2mm] \frac{\alpha_2\theta_2}{m_2^{*}} + \frac{\phi\,\mu_{21}n}{m_1^{*}} & 0 & \frac{\theta_2\phi_2}{m_2^{*}} + \frac{\alpha_1\eta\mu_{21}}{m_1^{*}} & 0 \\[2mm] -\frac{3}{5}(\alpha_2+\eta\beta_{21}) & \frac{9}{5}\beta_2\delta_2 + \frac{9}{10}n\rho_{21}\zeta_1 & -\frac{3}{5}(\delta_2+\phi_2) & \frac{9}{10}\zeta_2\delta_2 + \frac{9}{5}n\beta_{21}\beta_1 \end{bmatrix}$$

This eigenvalue problem can be solved by use of the IBM subroutines because the matrices A and B do not have zeros on the diagonal. Another benefit of reducing the number of equations analytically is that computer execution time is reduced.

B. Navier - Stokes Theory

In our second approach to the problem we make use of the two tem-

perature Navier - Stokes equations. This closed set of differential equations is similar to the 13-moment approximation equations except for the stress and heat flux equations. In these we assume that the change in the flow velocities and temperatures are the largest deviations. Thus we neglect the changes in viscous stress and heat flux in the equations. Upon making the same basic assumptions and substitutions as with the 13-moment equations, we obtain the following equations corresponding to Eqs.(17) - (20) of the 13-moment approximation. These are:

stress,

$$\frac{4}{3} n_1 k T_1 \frac{\partial u_1}{\partial x} = \left(a_{11} P_1 + a_{12} P_2 \right) \nu , \tag{59}$$

$$\frac{4}{3} n_2 k T_2 \frac{\partial u_2}{\partial x} = \left(a_{21} P_1 + a_{22} P_2 \right) \nu , \tag{60}$$

heat flux,

$$\frac{5}{2} \frac{n_1 k^2 T_1}{m_1} \frac{\partial T_1}{\partial x} = \left(b_{11} q_1 + b_{12} q_2 \right) \nu , \tag{61}$$

$$\frac{5}{2} \frac{n_2 k^2 T_2}{m_2} \frac{\partial T_2}{\partial x} = \left(b_{21} q_1 + b_{22} q_2 \right) \nu . \tag{62}$$

These equations, combined with Eqs.(11) - (16) from the 13-moment theory, constitute the basic set of equations in the Navier - Stokes theory. There are two fewer terms on the left hand side of each of Eqs.(59) - (62) than in the corresponding 13-moment equations. Since, for the case of a single component gas, there is no appreciable difference in the absorption and dispersion calculated for high values of κ, we expect the mixture of gases to behave the same way. We believe that the absorption and dispersion calculated for the two theories will differ at low values of κ, since they differ for the single component gas.

In order to study the problem of sound propagation with this theory, we follow the same approach as with the 13-moment equations, i.e., we assume plane wave solutions and define our variables the same as before. We thus obtain the following set of equations:

$$X_2 = \lambda X_1 \, , \tag{63}$$

$$\frac{3}{5} m_1{}^* X_1 + \frac{3}{5} m_1{}^* X_3 + m_1{}^* X_4 = \lambda \, \phi_1 X_2 + \lambda \alpha_1 \, X_7 \, , \tag{64}$$

$$\frac{3}{5} X_2 + X_5 = \lambda \frac{q}{10} 6_1 X_3 + \lambda \frac{q}{5} \beta_1 X_8 \, , \tag{65}$$

$$\frac{4}{5} X_2 = \lambda \, \mu_{11} X_4 + \lambda \, \frac{\mu_{12}}{n} \, X_9 \, , \tag{66}$$

$$\frac{q}{10} m_1{}^* X_3 = \lambda \, \rho_{11} X_5 + \lambda \, \frac{\rho_{12}}{n} \, X_{10} \, , \tag{67}$$

$$X_7 = \lambda X_6 \, , \tag{68}$$

$$\frac{3}{5} m_2{}^* X_6 + \frac{3}{5} m_2{}^* X_8 + m_2{}^* X_9 = \lambda \, \phi_2 X_7 + \lambda \alpha_2 X_2 \, , \tag{69}$$

$$\frac{3}{5} X_7 + X_{10} = \lambda \frac{q}{5} \beta_2 X_3 + \lambda \frac{q}{10} 6_2 X_8 \, , \tag{70}$$

$$\frac{4}{5} X_7 = \lambda n \, \mu_{21} X_4 + \lambda \, \mu_{22} X_9 \, , \tag{71}$$

$$\frac{q}{10} m_2{}^* X_3 = \lambda \, n \rho_{21} X_5 + \lambda \, \rho_{22} X_{10} \, . \tag{72}$$

Again we wish to reduce the number of variables by eliminating analytically variables X_2, X_7, X_4, X_9, X_5 and X_{10}, where

$$X_2 = \lambda X_1 \, , \tag{73}$$

$$X_7 = \lambda X_6 \, , \tag{74}$$

$$X_4 = \frac{1}{m_1{}^*} \left[\left(\lambda^2 \phi_1 - \frac{3}{5} m_1{}^* \right) X_1 - \frac{3}{5} m_1{}^* X_3 + \lambda^2 \alpha_1 X_6 \right] \, , \tag{75}$$

$$X_9 = \frac{1}{m_2{}^*} \left[\lambda^2 \alpha_2 X_1 + \left(\lambda^2 \phi_2 - \frac{3}{5} m_2{}^* \right) X_6 - \frac{3}{5} m_2{}^* X_8 \right] \, , \tag{76}$$

$$X_5 = \lambda \left[\frac{9}{10} \sigma_1 X_3 + \frac{9}{5} \beta_1 X_8 - \frac{3}{5} X_1 \right] , \tag{77}$$

$$X_{10} = \lambda \left[\frac{9}{10} \sigma_2 X_8 + \frac{9}{5} \beta_2 X_3 - \frac{3}{5} X_6 \right] . \tag{78}$$

Substituting Eqs.(73) - (78) into Eqs.(63) - (72) we arrive at these four equations:

$$\left(\frac{3}{5} \mu_{11} + \frac{4}{5} \right) X_1 + \frac{3}{5} \mu_{11} X_3 + \frac{3}{5} \frac{\mu_{12}}{n} X_6 + \frac{3}{5} \frac{\mu_{12}}{n} X_8 =$$
$$\lambda^2 \left\{ \left(\frac{\mu_{11} \phi_1}{m_1^*} + \frac{\mu_{12} \alpha_2}{m_2^* n} \right) X_1 + \left(\frac{\mu_{11} \alpha_1}{m_1^*} + \frac{\mu_{12}}{m_2^*} \frac{\phi_2}{n} \right) X_6 \right\}, \tag{79}$$

$$\frac{9}{10} m_1^* X_3 = \lambda^2 \left\{ -\frac{3}{5} \rho_{11} X_1 + \left(\frac{9}{10} \rho_{11} \sigma_1 + \frac{9}{5} \frac{\beta_2 \rho_{12}}{n} \right) X_3 - \frac{3}{5} \frac{\rho_{12}}{n} X_6 \right.$$
$$\left. + \left(\frac{9}{5} \rho_{11} \beta_1 + \frac{9}{10} \frac{\rho_{12} \sigma_2}{n} \right) X_8 \right\}, \tag{80}$$

$$\frac{3}{5} n \mu_{21} X_1 + \frac{3}{5} n \mu_{21} X_3 + \left(\frac{3}{5} \mu_{22} + \frac{4}{5} \right) X_6 + \frac{3}{5} \mu_{22} X_8 =$$
$$\lambda^2 \left\{ \left(\frac{\mu_{22} \alpha_2}{m_2^*} + \frac{n \mu_{21} \phi_1}{m_1^*} \right) X_1 + \left(\frac{\mu_{22} \phi_2}{m_2^*} + \frac{n \mu_{21} \alpha_1}{m_1^*} \right) X_6 \right\}, \tag{81}$$

$$\frac{9}{10} m_2^* X_8 = \lambda^2 \left\{ -\frac{3}{5} n \rho_{21} X_1 + \left(\frac{9}{15} \rho_{22} \beta_2 + \frac{9}{10} n \rho_{21} \sigma_1 \right) X_3 - \frac{3}{5} \rho_{22} X_6 \right.$$
$$\left. + \left(\frac{9}{10} \sigma_2 \rho_{22} + \frac{9}{5} n \beta_1 \rho_{21} \right) X_8 \right\} . \tag{82}$$

These equations are also of the form:

$$A \vec{X} = \lambda^2 B \vec{X} , \tag{83}$$

where again

$$\vec{X} = \begin{bmatrix} X_1 \\ X_3 \\ X_6 \\ X_8 \end{bmatrix} ,$$

is the eigenvector, λ^2 is the eigenvalue, and A and B are complex matrices given by

$$
A = \begin{bmatrix}
\frac{3}{5}\mu_{11} + \frac{4}{5} & \frac{3}{5}\mu_{11} & \frac{3}{5}\frac{\mu_{12}}{n} & \frac{3}{5}\frac{\mu_{12}}{n} \\[8pt]
0 & \frac{9}{10}m_1^{*} & 0 & 0 \\[8pt]
\frac{3}{5}n\mu_{21} & \frac{3}{5}n\mu_{21} & \frac{3}{5}n\mu_{21} + \frac{4}{5} & \frac{3}{5}\mu_{22} \\[8pt]
0 & 0 & 0 & \frac{9}{10}m_2^{*}
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
\frac{\mu_{11}\phi_1}{m_1^{*}} + \frac{\mu_{12}\alpha_2}{m_2^{*}n} & 0 & \frac{\mu_{11}\alpha_1}{m_1^{*}} + \frac{\mu_{12}\phi_2}{m_2^{*}n} & 0 \\[8pt]
-\frac{3}{5}\rho_{11} & \frac{9}{10}6_1\rho_{11} + \frac{9}{5}\frac{\beta_2\rho_{12}}{n} & -\frac{3}{5}\frac{\rho_{12}}{n} & \frac{9}{5}\rho_{11}\beta_1 + \frac{9}{10}\frac{\beta_{12}6_2}{n} \\[8pt]
\frac{\mu_{22}\alpha_2}{m_2^{*}} + \frac{n\mu_{21}\phi_1}{m_1^{*}} & 0 & \frac{\mu_{22}\phi_2}{m_2^{*}} + \frac{n\mu_{21}\alpha_1}{m_1^{*}} & 0 \\[8pt]
-\frac{3}{5}n\rho_{21} & \frac{9}{5}\rho_{22}\beta_2 + \frac{9}{10}n\rho_{21}6_1 & -\frac{3}{5}\rho_{22} & \frac{9}{10}\rho_{22}6_2 + \frac{9}{5}\beta_1n\rho_{21}
\end{bmatrix}
$$

In a manner identical to the 13-moment case, we use numerical methods to solve Eq.(83) and find the absorption and dispersion from the λ^2. In the following chapter the numerical method used to calculate the eigenvalues is explained.

CHAPTER III

NUMERICAL METHODS

We wish to calculate the absorption and dispersion of the sound wave using both the 13-moment approximation and the Navier - Stokes theory. This involves solving the eigenvalue - eigenvector equation, $A\vec{X} = \lambda^2 B\vec{X}$, for the eigenvalue λ^2. In order to do this, we first multiply Eq.(58) by the inverse of B to obtain the reduced eigenvalue equation

$$A_1\vec{X} = \lambda^2\vec{X} \quad ,$$

where $A_1 = B^{-1}A$.

We developed a PL/1 computer program to solve this equation which, like the program used by Creech,[9] employs IBM subroutines. The program has as its input parameters the dimensions of the expanded matrix, N; the mass ratio, M; the fraction of particles of gas two, X2; and the viscosity of the mixture, VIS. After the data is read in, the program computes the elements of the 4 x 4 matrices A and B in terms of the variable f/p, where $f = \omega/2\pi$ is the frequency; and p is the pressure. Matrix B is then passed to subroutine MATINV to form B^{-1}, which is multiplied by matrix A to form matrix A1. Matrix A1 then is expanded into the 8 x 8 matrix which is passed to subroutine MATE.[10] This subroutine reduces a real, expanded matrix to Hessenberg form, after which subroutine MEAT[10] calculates the eigenvalues of the Hessenberg matrix.

We make use of the new independent variable, f/p, since the exper-

imental values for the absorption are given in terms of this variable. We thus need a relationship between the rarefaction parameter, $\mathcal{M}$, and f/p.

It can be shown that the self - collision frequency ν is given by $\nu = c_1 p/\mu$, where p is the total pressure, μ is the viscosity of the mix-ture and

$$C_1 = \frac{\mu(a_{21} - a_{22}) + a_{12} - a_{11}}{(n+1)(a_{11}a_{22} - a_{12}a_{21})} \cdot {}^{11}$$

Thus the rarefaction parameter, $\mathcal{M} = \nu/\omega$, can be written in terms of f/p as:

$$\mathcal{M} = \frac{C_1}{2\pi\mu} \frac{1}{f/p} \quad .$$

Eight eigenvalues are calculated from the expanded matrix of which only four are the eigenvalues of our original matrix A1. To find these four eigenvalues we form the matrix A2 which has as its diagonal elements the diagonal elements of matrix A1 minus the calculated eigenvalues. When the eigenvalues of the expanded matrix are equal to the eigenvalues of the matrix A1, then matrix A2 is singular. To test for singularity matrix A2 is passed into subroutine MATINV which is designed for doing this. This step is repeated for all eight eigenvalues. When the singu-larities occur, the program calculates the reciprocal square root of the eigenvalues. By Eq. (36) the absorption, $\alpha c_0/\omega$, is the negative imagi-nary part of the reciprocal square root; and the dispersion, c_0/c, is the real part.

Of the four values of dispersion and absorption calculated for each value of f/p, only one is physically meaningful. The others, as in Creech's[9] case, represent waves that are either absorbed before they

can be measured, or they are amplified by the gas.

Both the 13-moment approximation equations and the Navier - Stokes equations are solved by using this program. The only changes in the program for these two cases are the different elements of matrices A and B for each theory.

The program is designed to calculate the absorption and dispersion of a sound wave traveling in a mixture of any two monatomic gases, such as helium, neon, argon, and xenon. We restricted our research to the mixture of helium and argon since the most recent experimental data obtained has been for this mixture. We are thus more readily able to check the agreement of our theory with experiment. The results are given in the following chapter.

CHAPTER IV

RESULTS

The absorption, $\alpha c_0/\omega$, and dispersion, c_0/c, of a sound wave propagating in a mixture of helium and argon have been calculated for argon concentrations of 5%, 10%, 25%, 50%, and 75%. The viscosities of the mixtures for these concentrations were obtained from Thornton and Baker.[12] The results of the calculations for both the 13-moment approximation and the Navier - Stokes theory for the different concentrations are presented in Figs. 1 - 5. In each case the absorption and dispersion are plotted as functions of f/p. The f/p values vary from 10 to 10000 in units of MHz/atm, while the absorption and dispersion are dimensionless fractions. Also experimental absorption values found by Prangsma, Jonkman and Beenakker[6] are plotted on each graph.

Our first case, Fig. 1, is for a mixture containing 5% argon. The dispersion curves start at approximately 1 for both the 13-moment and Navier - Stokes theories, but soon begin to separate. The 13-moment dispersion decreases gradually until leveling off at approximately .5. The Navier - Stokes curve decreases steadily to a value of .155. For the absorption, both curves begin at .024 and increase together up to an f/p value of 70. The two curves separate with the 13-moment absorption curve attaining a maximum value of .2 for an f/p value of 400. It then decreases to a value of .017. The Navier - Stokes curve extends to higher values, peaking at .31 for f/p equal to 1000. The Navier - Stokes absorption curve ends at a value of .145, showing a difference between
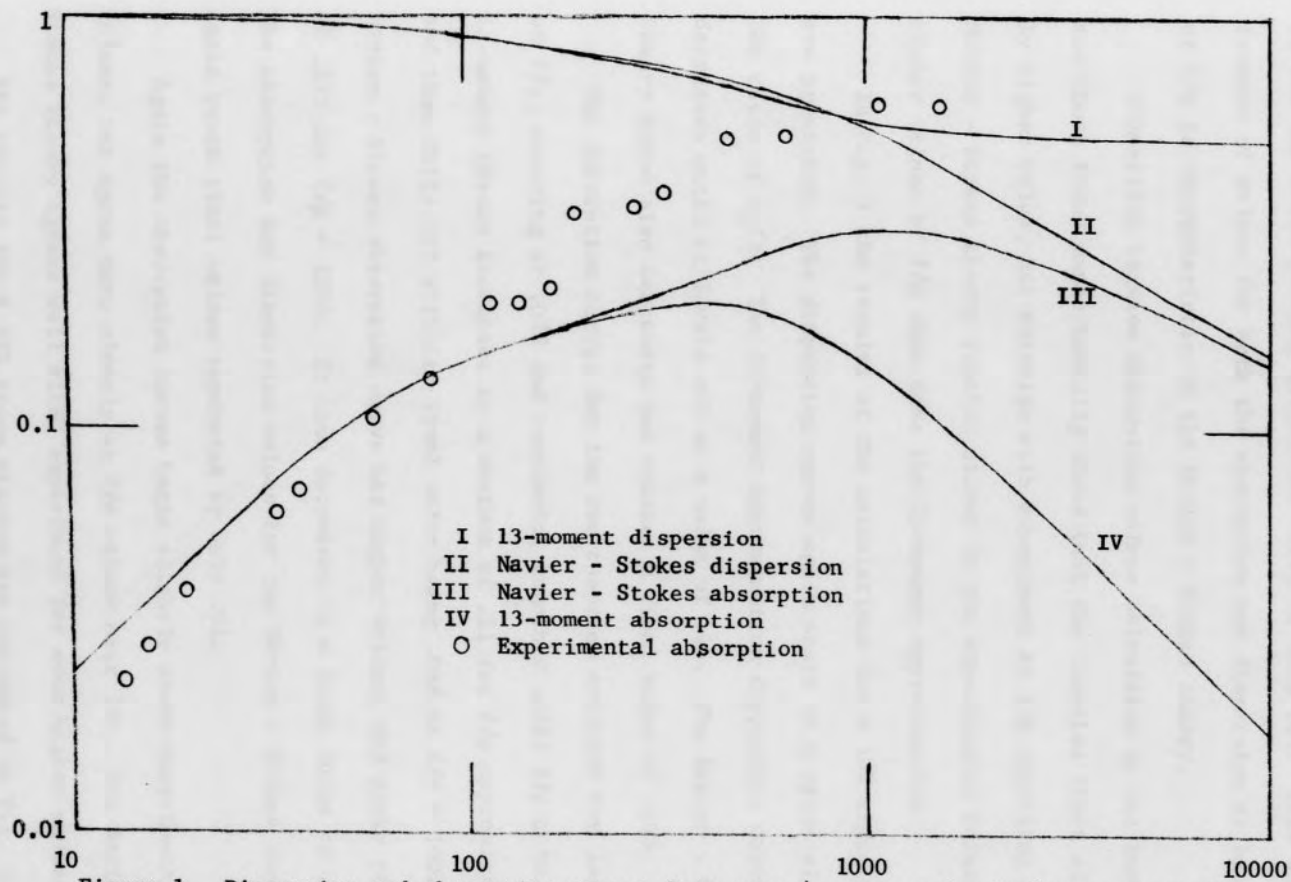
Figure 1. Dispersion and absorption versus frequency / pressure for helium - argon mixture
% argon = 5,   Viscosity = .0002 atm/Mhz

it and the corresponding dispersion curve of only .01. This small difference of values for both the absorption and dispersion at high values of f/p is characteristic of the Navier - Stokes theory.

Comparison between absorption values calculated by the two theories and those found experimentally shows that the theories start at a slightly higher value, but coincide with experiment at f/p equalling 90. The Navier - Stokes theory remains closer to the experimental values for higher values of f/p than does the 13-moment approximation.

In Fig. 2 the results of the calculations for a 10% argon mixture are presented. The dispersion curves again start at a value of 1 for the ratio of c_0/c. The 13-moment approximation dispersion curve slowly decreases until it levels off at a value of .44. The Navier - Stokes theory curve also decreases and reaches a final value of .145.

The absorption curves for the two theories coincide for low values of f/p, starting at .029 and increasing together until $f/p \approx 50$. The 13-moment theory increases to a maximum of .21 for f/p approximately 180, and then falls off with its final value being .018 at $f/p = 10000$. The Navier - Stokes absorption curve has higher values, and peaks at a value of .275 for $f/p = 1200$. It then decreases to a final value of .135. The absorption and dispersion values for the Navier - Stokes theory again reach final values separated by only .01.

Again the absorption curves begin slightly above experimental values, but agree more closely at f/p values near 130. The Navier - Stokes theory agrees well with experiment for even higher values of f/p.

The results for a 25% argon mixture are presented in Fig. 3. Both dispersion curves begin at 1 and each decreases, the 13-moment curve
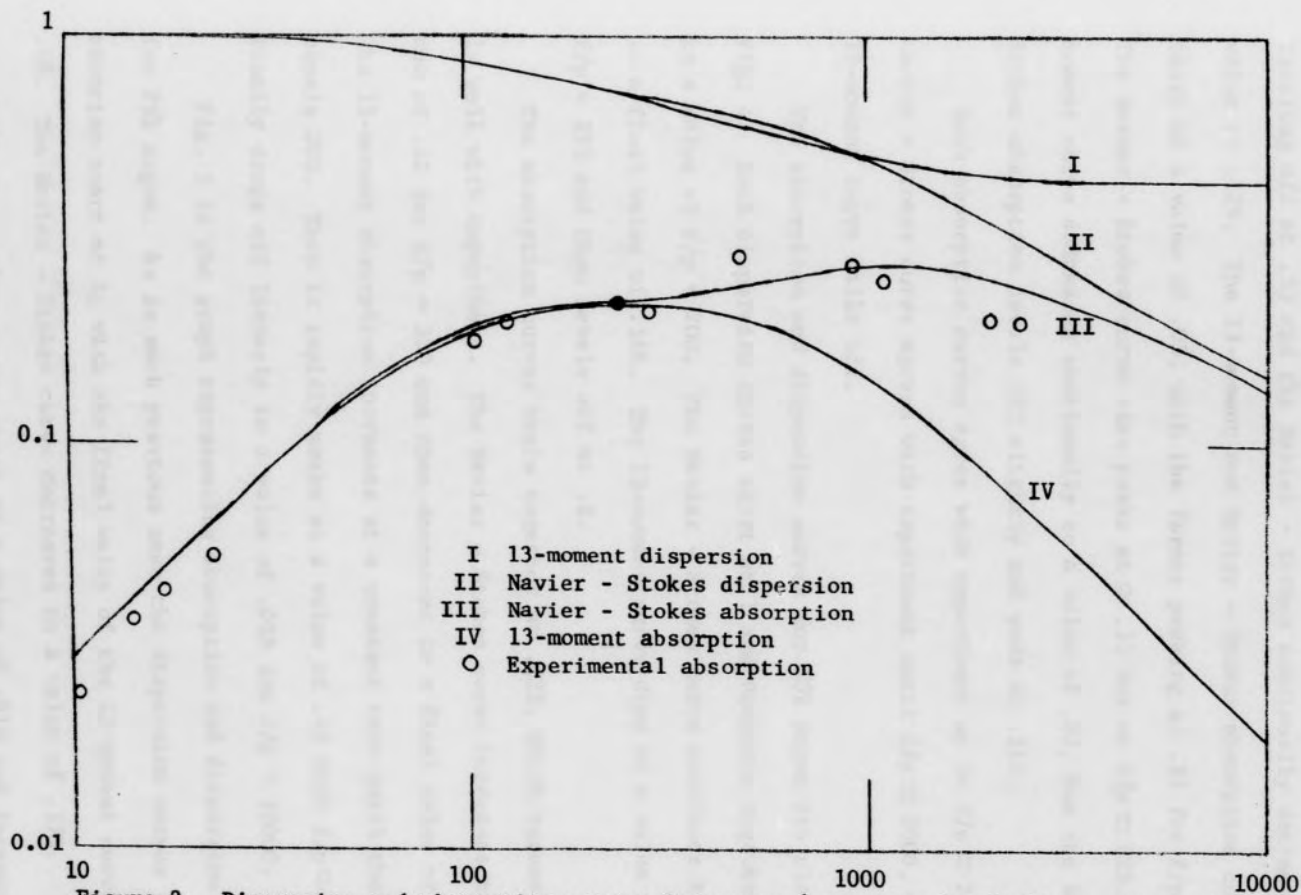
Figure 2. Dispersion and absorption versus frequency / pressure for helium - argon mixture
% argon = 10. Viscosity = .000207 atm/Mhz

I   13-moment dispersion
II   Navier - Stokes dispersion
III Navier - Stokes absorption
IV   13-moment absorption
O   Experimental absorption

leveling off at .33 and the Navier - Stokes continually decreasing to a value of .129. The 13-moment and Navier - Stokes absorption curves both start at a value of .03, with the former peaking at .31 for $f/p \approx 250$. The Navier - Stokes curve also peaks at $\approx$ .31 but at $f/p \approx 225$. The 13-moment curve decreases continually to a value of .02, but the Navier - Stokes absorption levels off slightly and ends at .119.

Both absorption curves agree with experiment up to $f/p \approx 250$. The Navier - Stokes curve agrees with experiment until $f/p \approx 2000$, while the 13-moment curve falls off.

The absorption and dispersion curves for 50% argon are plotted on Fig. 4. Both dispersion curves start at 1 and decrease together slightly to a value of $f/p = 200$. The Navier - Stokes curve continues to decrease to a final value of .166. The 13-moment curve dips to a value of .76 for $f/p = 275$ and then levels off at .8.

The absorption curves begin together at .023, which agrees extremely well with experiment. The Navier - Stokes curve increases to a maximum of .42 for $f/p = 300$ and then decreases to a final value of .156. The 13-moment absorption increases at a constant rate until the f/p value equals 200. Then it rapidly peaks at a value of .45 with $f/p \approx 290$. It finally drops off linearly to a value of .018 for $f/p = 10000$.

Fig. 5 is the graph representing absorption and dispersion curves for 75% argon. As in each previous case the dispersion curves for both theories start at 1, with the final value of the 13-moment curve being .68. The Navier - Stokes curve decreases to a value of .173.

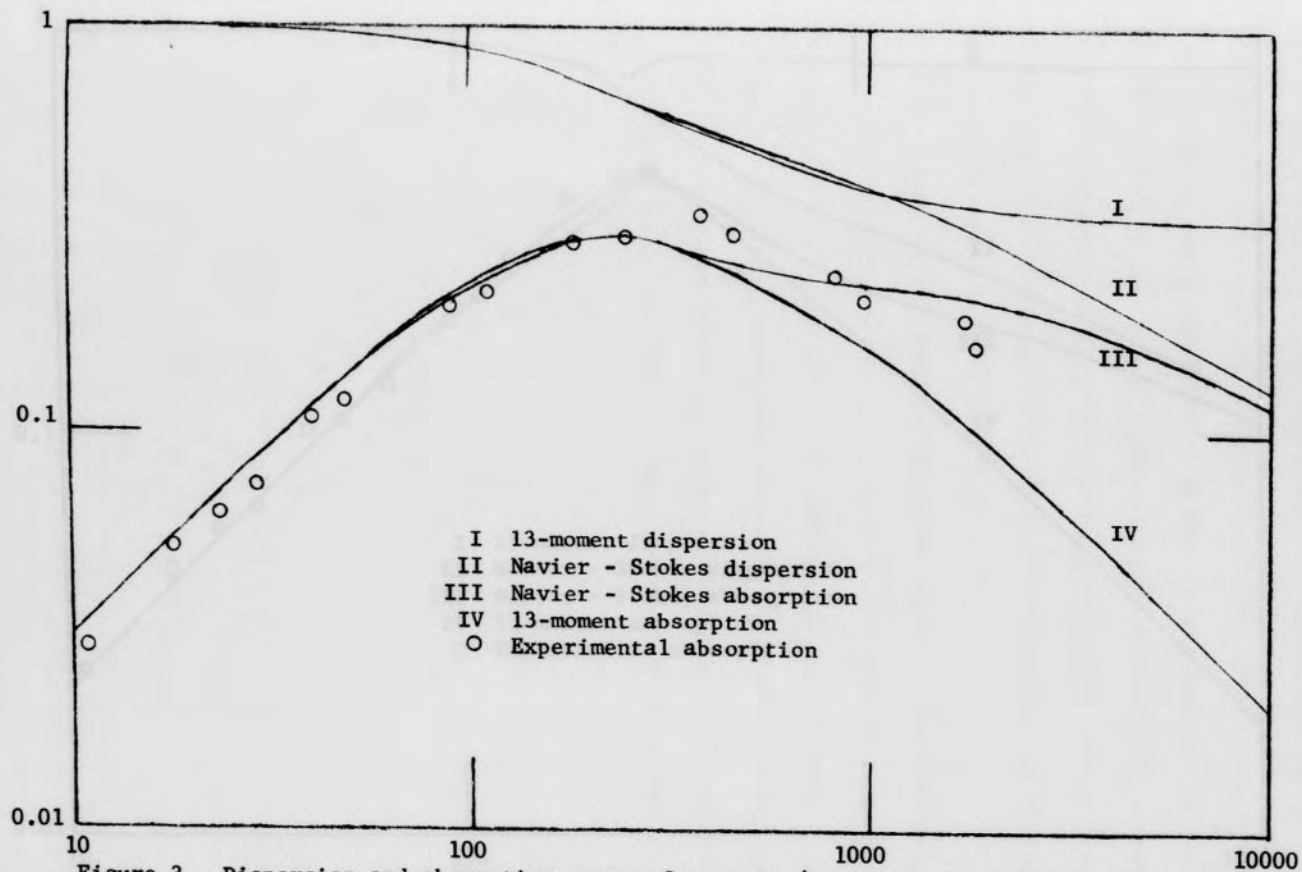Both absorption curves start at a value of .016 and increase togeth-

Figure 3. Dispersion and absorption versus frequency / pressure for helium - argon mixture
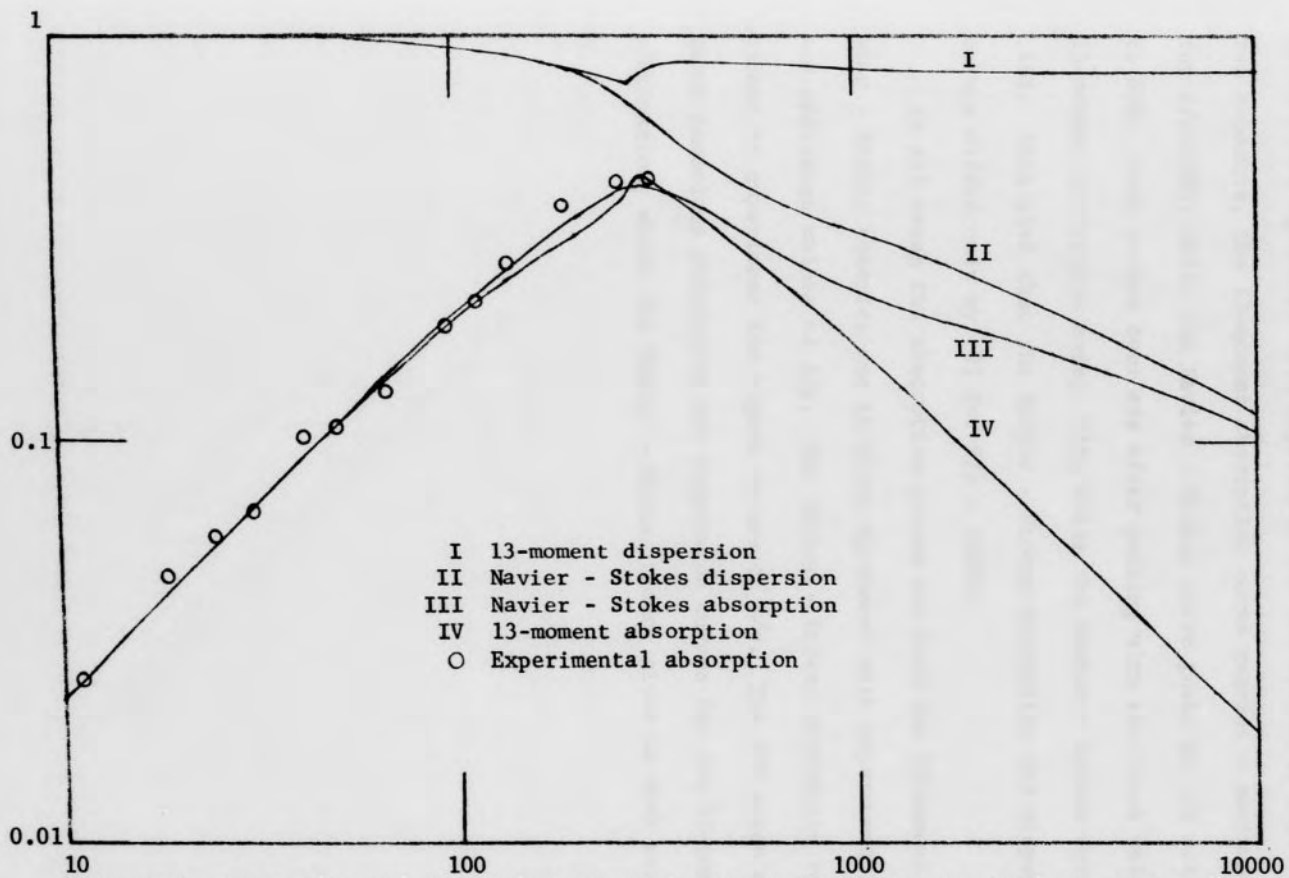% argon = 25, Viscosity = .000222 atm/Mhz

I   13-moment dispersion
II   Navier - Stokes dispersion
III   Navier - Stokes absorption
IV   13-moment absorption
O   Experimental absorption

Figure 4. Dispersion and absorption versus frequency / pressure for helium - argon mixture
% argon = 50 , Viscosity = .000226 atm/Mhz

er in close agreement with experiment for low values of f/p. After the two separate, the 13-moment absorption curve reaches a maximum of .28 for $f/p \gtrsim 300$, while the Navier - Stokes curve peaks at .43 near f/p equal to 600. Both curves decrease after peaking with the final value of the 13-moment absorption being .016, while the Navier - Stokes curve ends at .163. Note also that the Navier - Stokes absorption and dispersion curves differ only by .01 for $f/p = 10000$.

In all cases the absorption curves for both the 13-moment and Navier - Stokes theories are in close agreement with experiment for low and mid-range values of f/p. The Navier - Stokes absorption remains closer to experiment for higher values of f/p. The 50% argon mixture shows anomolous absorption and dispersion curves for the 13-moment approximation, while the Navier - Stokes theory gives no such results.
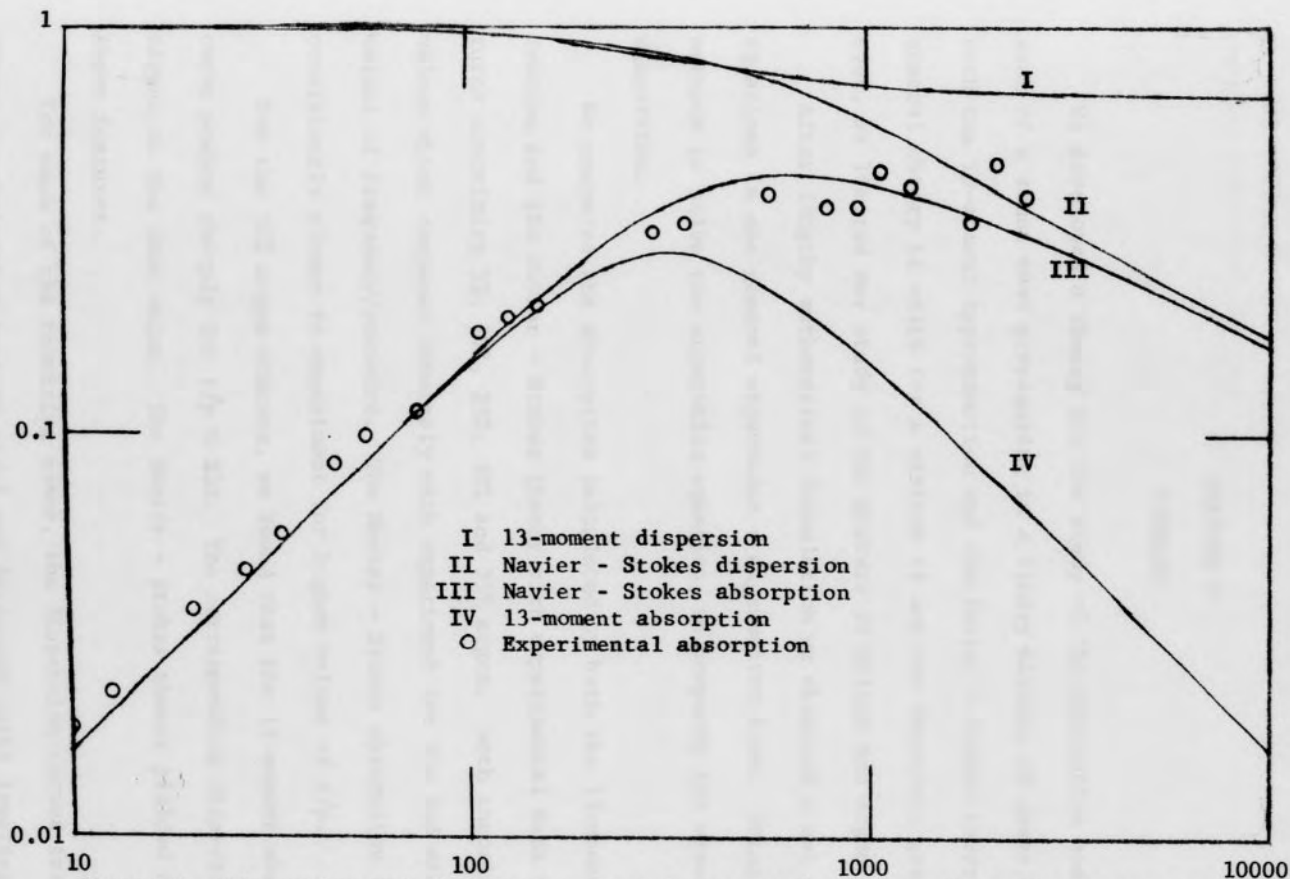
Figure 5.   Dispersion and absorption versus frequency / pressure for helium - argon mixture
% argon =   75, Viscosity = .000223 atm/Mhz

CHAPTER V

SUMMARY

We developed a theory for the study of the absorption and dispersion of a sound wave propagating in a binary mixture of gases by using both the 13-moment approximation and the Navier - Stokes theory. This general theory is valid for a mixture of any two monatomic gases, however, we limited our study to the mixture of helium and argon.

After lengthy mathematical formulation we obtained a set of linear equations in the general eigenvalue - eigenvector form. Using numerical methods to solve the eigenvalue equation, we computed the absorption and dispersion.

We compared the absorption calculated by both the 13-moment approximation and the Navier - Stokes theory with experimental data for mixtures containing 5%, 10%, 25%, 50% and 75% argon. Both theories gave values which compared favorably with experiment for low and mid-range values of frequency/pressure. The Navier - Stokes absorption values were consistently closer to experiment for higher values of f/p.

For the 50% argon mixture, we found that the 13-moment absorption curve peaked sharply for $f/p \approx 280$. The corresponding dispersion curve dipped at the same value. The Navier - Stokes theory yielded neither of these features.

For each of the remaining cases, the dispersion curves for the 13-moment approximation started at ≈ 1 and decreased until leveling off. The Navier - Stokes dispersion curves also started at 1 but decreased

steadily, with the final dispersion value being greater than the corresponding absorption by .01.

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APPENDIX I

Program Listing

```
//THESIS JOB ECS.UNCG.PY500187,CRAVEN,T=(1,30),P=50
// EXEC PLC,REGION=250K
//SYSIN DD *
*PLC   (NOATR,NOXREF,TIME=(1,30),PAGES=50)
  EIGEN:PROC OPTIONS(MAIN);
  DCL(N,IN,IER) BIN FIXED;
  GET LIST(N);
IN=N/2;
  BEGIN;
  DCL(A1,A21(N/2,N/2) BIN FLOAT CPLX;
  DCL(A01,A02,A11,A12,A21,A22,B1,B2,B11,B12,B21,B22)  BINARY;
  DCL(ALPHA1,ALPHA2,BETA1,BETA2) CPLX BIN FLOAT;
  DCL(MU11,MU12,MU21,MU22) CPLX BIN FLOAT;
  DCL(RHO11,RHO12,RHO21,RHO22,THETA1,THETA2) CPLX BIN FLOAT;
  DCL(DELTA1,DELTA2,SIGMA1,SIGMA2) CPLX BIN FLOAT;
  DCL(PHI1,PHI2) CPLX BIN FLOAT;
  DCL(SUM,X) CPLX BINARY;
  DCL(R,D,M,M1S,M2S,M1,M2,M3,M4,K12,K22,BETA,GAM) BIN FLOAT;
  DCL((AA,B,AINV)(N/2,N/2),EIG)CPLX BINARY;
  DCL C1 BIN FLOAT;
  DCL(A(N,N),RR(N),RI(N),H(N,N)) BINARY,
  (IP(N),I,J,K,P) BINARY FIXED,
  ANA(N) BIT(1);
  DCL(FOP,VIS) BIN FLOAT;
  PI=3.14159;
  BETA=.2246; GAM=.9661;
  K12=1; K22=1;
  GET LIST(M);
LL1:GET LIST(X2,VIS);
  PUT PAGE DATA(M,X2,VIS);
  D=1/X2-1;
  M1S=(M*D+1)/(M*(D+1)); M2S=(M*D+1)/(D+1); M1=((M+1)/2)**.5;
  M2=(M+1)**2; M3=(M+1)**3; M4=M/(M+1);
  A01=(M1*K12**.5)/((M+1)*D); A02=M*D*A01;
  B1=(M1*K12**.5*M)/(D*M2);B2=D*B1;
  A11=-((1-BETA)+(2*(M+1-BETA)*M1*K12**.5)/(M2*D));
  A12=(2*M*BETA*M1*K12**.5)/M2;
  A21=(2*BETA*M1*K12**.5*M)/(M2*D);
  A22=-(((1-BETA)*M**.5*K22**.5)/D+(2*M1*K12**.5*M
  *(1+M*(1-BETA)))/M2);
  B11=-((2-GAM)/2+((2*M1*K12**.5)/(M3*D))*(GAM-(1+GAM)*(M+1)+
  (3*M2)/2));
  B12=(2*M*M1*K12**.5*GAM)/M3;
  B21=(2*M1*K12**.5*M4**3*GAM)/(M*D);
```

```
B22=-(((2-GAM)*M**.5*K22**.5)/(2*D)+(2*M1*K12**.5*M4**3)
*(GAM-(1+GAM)/M4+3/(2*M4**2)));
C1=(D*(A21-A22)+A12-A11)/((D+1)*(A11*A22-A12*A21));
DO FOP=10 TO 90 BY 10, 100 TO 180 BY 20, 200 TO 300 BY 50,
400 TO 900 BY 100, 1000 TO 1800 BY 200, 2000 TO 3000 BY 500,
4000 TO 10000 BY 1000;
R=C1/(2*PI*VIS*FOP);
ALPHA1=1I*A01*R; ALPHA2=1I*A02*R;
BETA1=1I*B1*R; BETA2=1I*B2*R;
MU11=1I*A11*R; MU22=1I*A22*R; MU12=1I*A12*R; MU21=1I*A21*R;
RHO11=1I*B11*R; RHO22=1I*B22*R;
RHO12=1I*B12*R; RHO21=1I*B21*R;
THETA1=1+MU11; THETA2=1+MU22;
DELTA1=1+RHO11; DELTA2=1+RHO22;
SIGMA1=1-2*BETA1; SIGMA2=1-2*BETA2;
PHI1=1-ALPHA1; PHI2=1-ALPHA2;
AA(1,1)=3/5*(THETA1+4/5);
AA(1,2)=3/5*(THETA1+4/5*SIGMA1);
AA(1,3)=(3*MU12)/(5*D);
AA(1,4)=3/5*(MU12/D+8/5*BETA1);
AA(2,1)=(-9*M1S)/25;
AA(2,2)=(27*M1S)/50;
AA(2,3),AA(2,4)=0;
AA(3,1)=(3*D*MU21)/5;
AA(3,2)=3/5*(D*MU21+8/5*BETA2);
AA(3,3)=3/5*(THETA2+4/5);
AA(3,4)=3/5*(THETA2+4/5*SIGMA2);
AA(4,1),AA(4,2)=0;
AA(4,3)=(-9*M2S)/25;
AA(4,4)=(27*M2S)/50;
B(1,1)=(THETA1*PHI1)/M1S+(ALPHA2*MU12)/(D*M2S);
B(1,2),B(1,4)=0;
B(1,3)=(ALPHA1*THETA1)/M1S+(MU12*PHI2)/(D*M2S);
B(2,1)=-3/5*(DELTA1+PHI1);
B(2,2)=(9*SIGMA1*DELTA1)/10+(9*BETA2*RHO12)/(5*D);
B(2,3)=-3/5*(ALPHA1+RHO12/D);
B(2,4)=(9*DELTA1*BETA1)/5+(9*RHO12*SIGMA2)/(10*D);
B(3,1)=(ALPHA2*THETA2)/M2S+(PHI1*D*MU21)/M1S;
B(3,2),B(3,4)=0;
B(3,3)=(PHI2*THETA2)/M2S+(ALPHA1*D*MU21)/M1S;
B(4,1)=-3/5*(ALPHA2+D*RHO21);
B(4,2)=(9*BETA2*DELTA2)/5+(9*D*RHO21*SIGMA1)/10;
B(4,3)=-3/5*(DELTA2+PHI2);
B(4,4)=(9*SIGMA2*DELTA2)/10+(9*D*RHO21*BETA1)/5;
CALL MATINV(IN,B,AINV,IER);
DO I=1 TO N/2;
DO J=1 TO N/2;
SUM=0;
DO K=1 TO N/2;
SUM=SUM+AINV(I,K)*AA(K,J);
END;
```

```
 A1(I,J)=SUM;
 END;   END;
 DO I=1 TO N; DO J=1 TO N;
 IF I<=N/2&J<=N/2 THEN A(I,J)=REAL(A1(I,J));
 IF I>N/2&J>N/2 THEN A(I,J)=REAL(A1(I-N/2,J-N/2));
 IF I<=N/2&J>N/2 THEN A(I,J)=-IMAG(A1(I,J-N/2));
 IF I>N/2&J<=N/2 THEN A(I,J)=IMAG(A1(I-N/2,J));
 END; END;
 CALL MATE(A,N,IP);
 H=A;
 CALL HEAT(A,N,RR,RI,ANA);
 PUT SKIP(2) EDIT(FOP)(F(5));
DO P=1 TO N;
A2=A1;
EIG=COMPLEX(RR(P),RI(P));
DO I=1 TO N/2;
A2(I,I)=A1(I,I)-EIG;
END;
CALL MATINV(IN,A2,AINV,IER);
IF IER=0 THEN DO;
X=1/SQRT(EIG);
PUT SKIP EDIT(REAL(X),-IMAG(X))(COL(5),2(F(10,5)));
END;
END;
END;
 GO TO LL1;
END;
 END EIGEN;
*DATA
8
.100195
.05,.0002,.1,.000207,.25,.000222,.75,.000223,.5,.000226
/*
//
```

The matrix elements for the Navier - Stokes theory are given by:

```
AA(1,1)=(3*MU11/5+4/5);
AA(1,2)=3*MU11/5;
AA(1,3)=(3*MU12)/(5*D);
AA(1,4)=(3*MU12)/(5*D);
AA(2,1),AA(2,3),AA(2,4)=0;
AA(2,2)=9*M1S/10;
AA(3,1),AA(3,2)=(3*D*MU21)/5;
AA(3,3)=(3*MU22/5+4/5);
AA(3,4)=3*MU22/5;
AA(4,1),AA(4,2),AA(4,3)=0;
AA(4,4)=9*M2S/10;
B(1,1)=(MU11*PHI1/M1S+MU12*ALPHA2/(M2S*D));
B(1,2),B(1,4)=0;
B(1,3)=(MU11*ALPHA1/M1S+MU12*PHI2/(M2S*D));
B(2,1)=-3*RHO11/5;
B(2,2)=((9*SIGMA1*RHO11)/10+(9*BETA2*RHO12)/(5*D));
B(2,3)=-3*RHO12/(5*D);
B(2,4)=((9*RHO11*BETA1)/5+(9*RHO12*SIGMA2)/(10*D));
B(3,1)=(MU22*ALPHA2/M2S+(D*MU21*PHI1)/M1S);
B(3,2),B(3,4)=0;
B(3,3)=(MU22*PHI2/M2S+(D*MU21*ALPHA1)/M1S);
B(4,1)=-(3*D*RHO21)/5;
B(4,2)=((9*RHO22*BETA2)/5+(9*D*RHO21*SIGMA1)/10);
B(4,3)=-3*RHO22/5;
B(4,4)=((9*RHO22*SIGMA2)/10+(9*BETA1*D*RHO21)/5);
```