THE EFFECTIVENESS OF TEACHING SELECTED NUMBER CONCEPTS

TO KINDERGARTEN CHILDREN

by

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A Thesis Submitted to
the Faculty of the Graduate School at
The University of North Carolina at Greensboro
in Partial Fulfillment
of the Requirements for the Degree
Master of Science in Home Economics

Greensboro
June, 1966

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## ACKNOWLEDGMENTS

For their interest, assistance and suggestions concerning various phases of this study, the writer is grateful to Dr. I. V. Sperry, Professor of Home Economics and Director of the Institute of Child and Family Development; Dr. Barbara James, Assistant Professor of Home Economics; and Dr. Marian Franklin, Associate Professor of Education.

Sincere appreciation is expressed to Dr. Nancy White, Assistant Professor of Home Economics, who served as the writer's thesis director and teacher of the control group during the study. Dr. White was of great assistance in her suggestions concerning techniques of analysis, in her reading of the manuscript and in her wise counsel throughout the writer's course of study. Especially is the writer grateful to Dr. White for her infinite patience.

She is appreciative of the helpful advice and loan of many relevant materials by Mrs. Floyd Moser, Elementary School Supervisor of Greensboro City Schools.

She is grateful for the assistance provided by Miss Betty High, Teacher of the St. Francis Episcopal Kindergarten, in participating in the Pilot Study.

The writer is particularly thankful for the cooperation, assistance, and interest of the Curry School Staff, including Mr. Herbert Vaughn, Principal and Mrs. C. T. Leonard, Jr., Teacher of the Kindergarten.

She is appreciative of the assistance given by Robert Milam in the analysis of data.

The writer is indebted to her family for their continued confidence, assistance and encouragement throughout her course of study.

Above all, she is deeply indebted to the children of St. Francis Episcopal Kindergarten and Curry Kindergarten, for their enthusiastic approval of "playing games."

POINDEXTER, FLORENCE CAROLYN. The Effectiveness of Teaching Selected Number Concepts to Kindergarten Children. (1966) Directed by: Dr. Nancy White. pp. 84.

The objective in this study was to determine the effectiveness of purposive teaching of selected number concepts to kindergarten children. The subjects were the nineteen children enrolled in the Curry School Kindergarten of the University of North Carolina at Greensboro for the spring semester, 1966. The number concepts which were used for the six experiments were: (1) Comparison of Sets, (2) Rational Counting, (3) Cardinal Property, (4) Place Value, (5) Ordinal Property, and (6) Conservation of Number.

Little research using the test-teach-test method to ascertain the concepts which kindergarten children could be taught was found. Sindwani (1964) and White (1963) reported use of the test-teach-test method with nursery school children; whereas, Smedslund (1961c), Suppes and Ginsberg (1962), and Wohlhill and Lowe (1962) reported somewhat similar methods with kindergarten children.

Tests and lesson plans were devised, pre-tested and revised. Prior to the beginning of the study, normal randomization procedures were used to assign the children to experimental groups, to receive teaching and testing, and to control groups, to receive testing only. Testing materials were in all instances parallel, differing only in manipulative media. The purposive teaching period was thirty-five minutes in length and individual testing periods required five minutes per child.

The analysis of covariance was used for interpretation of the data. The

analysis revealed that the difference between adjusted means was not significant for the first five experiments; Experiment VI, Conservation of Number, was significant. However, four of the tests which were not significant did show a small difference in favor of the experimental group. Test III, Cardinal Property of Number, showed a small difference in favor of the control group.

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# CHAPTER I

## INTRODUCTION

Parents and educators have been concerned about their role in the intellectual development of the preschool child. Should young children be taught symbolic, abstract behavior as in reading or counting exercises? There have been instances in which normal two-year-old children have learned to read, count, write the alphabet, and type, at least in a certain sense of the word. Obviously, these activities were carried on in a partial vacuum. It was improbable that such young children had had sufficient experience with abstractions and early logic exercises to make symbolic behavior meaningful.

However, much of the research cited in this study indicated that some young children do exhibit readiness to learn. Such research included that of Brace and Nelson (1965); Sindwani (1964); Suppes and Ginsberg (1962); Sussman (1962); and White (1963). Indeed, the most reasonable, as well as the most prevalent, belief asserted that readiness does exist and that it relates to a state of preparedness for learning, which is dependent upon the child's level of maturation and total development, upon his motivation, upon interest and drive, and upon previous training and experience.

Assuming that intellectual abilities did not emerge in the absence of environmental stimulation, McCandless and Hodges (1965) suggested that the goals of early childhood educators should be: (1) to develop in the child the desire to learn, (2) to develop learning skills, and (3) to provide a broad base of experiences which have been shared and talked about through adult-child interaction.

The method for attaining these goals at the preschool level is play. When a child is given love and security, further development of the mind may be facilitated by stimulation, practice, and planned intellectually related activity. However, the use of such purposive play does not imply the exclusion of substantial blocks of time each day in which children can engage in unstructured play, within the limits of their own and others' safety.

McCandless and Hodges (1965, p. 10-11) gave eight principles to guide educators in the selection and execution of the tasks to be used in reaching the goals of purposive play.

1. Whatever can be done with many tasks might well be done in a humorous manner.
2. The educator should choose and create games in which he can expect the child to be successful.
3. The educator should modify and shape his own behavior to that of the young child and gradually increase the complexity of the tasks. Such a cue to movement to a more complex game is based on the child's success in the previous task, his enthusiasm, and the amount of mutual adult-child interest that can be maintained.
4. Tasks selected should be terminated before the child tires of the game.
5. Demonstration of a task may well precede the point at which the educator elicits the child's own descriptions and labels for his activities.
6. As the opportunity presents itself, the educator should attempt to help the child express what he sees and what he does.
7. The selective use of rewards on the part of the educator should be used.
8. The educator should realize that transfer of learning may be facilitated by varying the manipulative materials and solutions to problems concerning a particular concept.

If these principles of purposive play were credible, a second question was raised. What were some areas which would lend themselves to directed teaching? From reading various publications and from talking with early childhood educators and specialists, the investigator believed that the following areas represented some of those suitable for purposive teaching: rational counting, comparisons of sets, cardinal property of number, ordinal property of number, place value, and conservation of number.

## PURPOSE OF THE STUDY

The purpose of the study was to determine the effectiveness of purposive teaching of number concepts in the kindergarten. There appeared to be conflicting evidence on the status of the young child's concept of number when he enters school. Some researchers, notably Piaget (1952), claimed that the preschool child's knowledge of number was very superficial, and although he may have some ideas of number, these ideas were not firmly established. On the other hand, some studies (Mott, 1945) created the impression that young children acquired a fairly extensive knowledge of number prior to systematic instruction. Unlike many studies reviewed, this study attempted to determine the child's understanding of the selected number concepts by his manipulations of objects rather than by verbalizations of number names and combinations alone. Sigel (1964) pointed out that attempts to teach new concepts to young children were not generally successful. He hypothesized that such failure might be attributable to insufficiently long training periods; or failure to ascertain, with any precision, the necessary sequences as preludes to learning. Therefore, in view of the conflicting connotations of readiness and the insufficient knowledge of how concepts are acquired, studies of the various areas of conceptualization seemed worthwhile.

## IMPORTANCE OF THE STUDY

In many instances, educators resorted to guesses about a child's development and when he learns. While it was known that a child's learning ability did not automatically start at age six, the investigator deemed it important to explore this area and make recommendations for pacing this phase of the educational system.

Cronbach (1965, p. 113) emphasized that many principles of learning, while sensible and supported by some research, were not truisms. He stated:

> We shall have to reconsider the evidence on which the principles rest, the value judgments they conceal, and their theoretical underpinnings. Ultimately, we may hope to distill out their essential truth and define the conditions in which they apply, and also learn the conditions under which some other generalization holds.

Moreover, the scarcity of experimental studies concerning concept formation was emphasized. Most concept-formation experiments with children have not been concerned with the teaching of concepts and with testing to determine the concepts acquired at different ages (Spiker, 1960). The investigator found a limited number of studies which attempted to evaluate effectiveness of teaching certain skills and concepts (Sindwani, 1964; Smedslund, 1961c; Suppes and Ginsberg, 1962; and White, 1963).

## DEFINITION OF THE TERMS USED

Included in the list below are terms used in the study.

1. Control group--children who were tested, not taught.
2. Experimental group--children who were taught and tested.
3. Flipchart--spiral book (containing testing media) in which pages could be turned by the investigator or by the subjects.
4. Number concepts--the subject's mental image about numbers.
   a. Rational counting--the subject's concept of number names and his ability to make a one-to-one correspondence.
   b. Comparisons--the subject's ability to ascertain equivalent and nonequivalent groups.
   c. Cardinal property of number--the subject's recognition (without counting) of the number of objects in a set.
   d. Ordinal property of number--the subject's ability to recognize the natural order of numbers.
   e. Place value--the subject's concept of the principles of the Hindu-Arabic System.
   f. Conservation of number--the subject's concept of invariance of a given number of total quantity.
5. Perception cards--five by eight inch cards containing testing media.

6. Set--a group or collection of things listed or specified well enough so that one can say exactly whether a certain thing does or doesn't belong to it.

7. Teacher--a kindergarten teacher who taught the experimental group each week.

8. Teaching period--purposive teaching used with the experimental group to help the subjects accomplish the perceptual concepts of number.

## ORGANIZATION OF THE REMAINDER OF THE THESIS

There are four subsequent chapters. Chapter II, Review of the Literature, includes some of the sources of research and theory relevant to this study. Chapter III, Procedures, includes an evaluation of the pilot study, an explanation of securing cooperation from the kindergarten teacher, the subjects, and the method by which the experimental and control groups were determined for each experiment. Chapter IV, Analysis of the Data, reports the statistical analyses for each experiment. Chapter V, Summary, indicates some findings, limitations of the study, conclusions drawn from the experiments, and recommendations for further research.

## CHAPTER II

## REVIEW OF THE LITERATURE

The question of when to begin instruction in number concepts, as well as in other academic subjects, has persistently challenged educators. Constructing their programs on the cumulative findings in the area of child development and growth and on established principles of learning, educators have made frequent use of the concept of readiness for learning. The term, readiness, per se, is of relatively recent origin. Indeed, it first appeared as an entry in the 1935-38 volume of the Education Index. Since that time, readiness for learning in the elementary and preschool has assumed an increasingly prominent role in the production and design of curriculum proposals.

Prior to the introduction of the concept of readiness, the teaching of arithmetic was dominated by the drill method of instruction (Howell, 1914). Pupils in the elementary grades were expected to master the basic number facts and skills by means of many scheduled periods of drill and practice. Emphasis was on the manipulation of number symbols with a minimum amount of pupil motivation, purpose, meaning, and concrete experiences.

The social-utility theory of teaching arithmetic replaced the drill method as a result of testing programs which pointed to the tedious and mechanical learning and rapid forgetting of the drill system. The new approach to teaching arithmetic emphasized those skills and processes which could be employed in adult life. For the most part, arithmetic learning was socially oriented, frequently related to group projects and the social studies. Buckingham and McLatchy (1930) were among those who helped lead the way in demonstrating the values of this approach.

During the period, predominately characterized by the social-utility approach, the concept of readiness for learning arithmetic was introduced. Co-inciding with the development of reading readiness, it became a vital force in shaping arithmetic curriculum patterns. Researchers (Benezet, 1935) claimed that, as in reading, a minimum mental age was required before pupils could be expected successfully to undertake arithmetic instruction. It was recommended that systematic and organized arithmetic instruction should be postponed and that an unplanned and informal type of arithmetic program be emphasized. Therefore, the incidental learning theory was characterized by the belief that pupils would learn the necessary arithmetic skills and processes by participating in natural classroom and social situations.

Brownell (1938) was instrumental in challenging this proposal of delayed arithmetic instruction. His criticism took the form of questioning both the re-search techniques and the conclusions reached by Washburne and his associates. Thus, the criticisms of Brownell as well as those of Buckingham (1930), Bush-well (1938), and Brueckner (1938) led to a weakening and eventual undermining of the proponents of deferred arithmetic instruction. Subsequently, Brownell (1945; 1956) demonstrated that a meaningful approach to the teaching of arithmetic was an appropriate and valid method. The meaning theory emphasized basic and underlying mathematical principles, planned for pupil discovery of generalizations and relationships, and stressed the importance of pupil understanding. Real situations were utilized abundantly for the establishing of motivation and purpose. The meaning theory was widely accepted, particularly since it incorporated the most worthy features of both the drill and social-utility theory. Researchers supporting the meaning theory included Morton (1953), Dawson and Ruddell (1961), Brueckner and Grossnickle (1963) and Swenson (1964).

The various modern elementary mathematics programs differ to some degree in their specific content and amount of emphasis on various number concepts. However, Burns (1965, p. 33) suggested some common strands of new content:

1. sets (basic concepts and notation)
2. system of numberation (number vs. numeral, place value, expanded notation, exponents, non-positional numeration systems, other bases)
3. number sentences (equations, inequalities, frames, variables)
4. properties of numbers (basic concepts and operations with numbers, prime and composite numbers, factors and multiples)
5. measurement
6. geometry (points, lines, planes, angles, figures, measurement, construction)
7. symbols

Suppes (1965) believed that the experimental program at the Institute for Mathematical Studies in the Social Sciences at Stanford University provided considerable evidence that children could master a modern and enriched elementary-school mathematics curriculum.

Suppes (1965) reported that some test results suggested that introduction of many new concepts did not decrease proficiency in work with concepts ordinarily taught in more conventional programs. Both experimental and control groups of first grades were taught conventionally. In addition, experimental groups were taught set operations. In 1960-61, 25 first grade classes in the San Francisco Bay Area were included in the study. The control and experimental groups were matched on the basis of student achievement levels and ranges of ability, staff capabilities, and comparable socio-economic background of the children. In terms of the proportion of children scoring correctly on individual items, the results indicated that there were not significant differences between experimental and control groups on items involving simple recognition of groups and of Arabic numerals, sequence of numerals, telling time, ordinals, and fractional part. The experimental group was superior on items involving decomposition of tens and ones, place value, and writing numerals. This group was markedly superior on items involving arithmetical operations.

## LITERATURE ON GENERAL NUMBER ABILITIES

Numerous investigators developed their own evaluation instruments to survey the arithmetic knowledge, skills, and abilities of young children. McLaughlin (1935) administered number exercises to preschool children and indicated that growth in number ability was a gradual process dependent upon both mental maturity and previous experience. Carpenter (1957) surveyed the abilities of kindergarten and primary-grade pupils and recommended the initiation of a systematic program to extend the arithmetical competencies of children.

Bjonerud (1960) tested pupils early in their kindergarten year in order to assess their arithmetical understandings. Buckingham and MacLatchy (1930) were among those who discovered that pupils entering the first-grade possessed a considerable amount of number knowledge. Martin (1951) studied the spontaneous expressions of children and reported that their ability to understand numerical concepts increased with age.

Piaget's (1952) extensive analysis of mathematical concepts were of particular interest to those concerned with the teaching of arithmetic in the elementary school. Based on a combination observation and interview method, he presented age levels for the attainment of concepts of number and measurement. He found that the concept of number has become operational by the time the child is six to seven-and-a-half years old. Up to age four-and-a-half to five, Piaget found an inaccurate concept of one-to-one correspondence, no conservation of discontinuous or continuous quantities, inability to seriate, no formation of ordinal correspondence, no understanding of cardinal number, ordinal number or unit, and non-existence of part-whole relations. From five to six, he found a one-to-one correspondence constructed but not conserved, gradual awareness of conservation of continuous and discontinuous quantity, trial and error seriation, no ordinal correspondence, gradual awareness of cardinal number, ordinal number, and units, intuitive trial and error understanding of whole and part relations, gradual awareness of addition of sets, and gradual awareness of transitivity of one-to-one correspondence.

Some criticism of Piaget's work has been based on his method of experimentation of reporting results (Russell, 1956). Piaget examined individuals in an unstructured situation in which the experimenter followed the lead of the child's responses and made a verbatim record of the interview. These records of Swiss children were then analyzed and categorized into developmental stages according to content and observed behavior.

Sallee and Gray (1963) reported failure to confirm Piaget's formulation of number perception in children three to six years old. Other investigators such as Dodwell (1960), Wohlwill and Lowe (1962), Elkind (1961) have conducted research which supported, to a certain extent, the findings of Piaget.

The trend appeared to be moving steadily toward the introduction of mathematical concepts earlier than ever before. Sussman (1962) suggested that kindergarten children knew as much about arithmetic at the beginning of kindergarten as first graders knew a few decades ago. Lambert (1960) pointed out the necessity of arithmetic concepts for five-year-olds in order that they may carry on their small affairs. Dutton (1963) suggested that kindergarten children are ready to learn concepts dealing with shape, size, relationships and measurement.

Sussman (1962) reported that two-thirds of the kindergarten pupils in a group of 595 children possessed a status of readiness for the undertaking of first-grade arithmetic work. Montague (1964) reported that the disadvantaged child entered school in need of individualized help due to deprivation of the background and educational experience.

Williams (1965) reported in his study of over 500 kindergarten children that three-fourths of the group scored correctly on test items requiring (1) identification of three dots, (2) marking of three dots in a larger group of dots, (3) identification of the thermometer with the highest temperature, and (4) identification of the longest pencil. More than fifty per cent of this group was able to identify five and nine dots and a single pair of socks, read the numeral four, make three marks, and locate the first object in a series. Less than fifty per cent of this group could reproduce 7, 9, and 14 by marking the corresponding number of dots; read the numerals 6, 9, 0; make ten marks upon request; demonstrate an understanding of the cardinal meaning of 2 and 5; locate objects in a series that were "second" and "between"; recognize half of a whole and half of a group of four dots; distinguish the group with the fewest objects; or, use one-to-one correspondence to match elements of equivalent sets.

His findings further showed that as the chronological age increased, the mathematical achievement of the subjects increased. Also, those children whose parents' occupations placed them in the higher socio-economic classes made significantly higher scores on the mathematical test than did those whose parents' occupations were categorized as lower socio-economic class.

Suppes and Ginsberg (1962b) conducted an experiment concerned with incidental learning. Thirty-six kindergarten children, in three groups of twelve each, were run for sixty trials a day on two successive days of individual experimental sessions during which they were required to learn equipollence of sets. On the first day, the stimulus displays, presented to the subjects on each trial, differed in color among the three groups, but otherwise were the same.

In Group 1, all displays were black; and in Group 2, equipollent sets were red and nonequipollent sets, yellow. For the first twelve trials in Group 3, equipollent sets were red and nonequipollent sets, yellow; for the remaining forty-eight trials on that day the two colors were gradually fused until discrimination between them was not possible. On the second day, all sets were presented to all three groups in one color, black.

Group 1 experienced two days of practice, under the same conditions, with the concept of equipollence. In Group 2, the child did not actually need to learn the concept of equipollence, but could simply respond to the color difference on the first day. (It has been shown that such a color discrimination for young children is a simple task.) If the child in Group 2 learned anything about equipollence of sets the first day, it was assumed to be a function of incidental learning. If incidental learning was effective, his performance on the second day, when the color cue was dropped, was assumed to be at least better than the performance of children in Group 1 on the first day. In Group 3, it was assumed that the child should continue to search the stimulus displays very closely for a color stimulus and thus be obliged to pay close attention to the stimuli.

Of the three groups, only Group 2 approached perfect learning on the first day. In this group, only color discrimination was necessary. Both the other groups did not improve over the first sixty trials, although Group 3 showed some initial improvement when the color cues remained discriminable. On the second day, Group 1 showed no improvement, and the learning curves for this group and Group 2 were almost identical. The results of the experiment with

Group 3 were conspicuously better on the second day than were those in the other two groups. The investigators suggested that the results indicated that the tasks chosen were relatively difficult for the age of the children because essentially, no improvement was shown by Group 1 in the entire 120 trials. The conditions in Group 3, where the children were forced to pay very close attention to the stimuli, did not seem to have enhanced learning significantly.

## LITERATURE ON SELECTED ASPECTS OF NUMBER ABILITY

Numerous investigators have directed their attention to various aspects of the arithmetic understanding and number knowledge of preschool and kindergarten children. Many of the studies focused upon one or more areas of arithmetic vocabulary, counting, writing numerals, numeral recognition, and conservation of numbers.

In an inclusive study of a sample of kindergarten children, Brace and Nelson (1965) reported a positive relationship between the children's knowledge of cardinal number and their ability to conserve number, but this relationship decreased with age; relationship of counting to the child's knowledge of ordinal number and place value was found to increase with the age of the child; four-fifths of the children tested showed an almost complete lack of knowledge of the invariance of number. The majority of the children counted beyond twenty. No significant differences were found between number concepts of boys and girls. Highly significant differences were found in the number knowledge of children six years of age and over and those below the age of six.

The investigators concluded that a thorough understanding of cardinal knowledge is necessary before the child can have real facility with ordinal number and before he can appreciate the significance of the counting process. Also, it was indicated that a thorough understanding of cardinal number, ordinal number and rational counting must be established before children are able to understand place value. A final suggestion was to include activities to develop the concept of number before requiring the child to undertake any activities involving the manipulation of number symbols.

## Arithmetic Vocabulary

Horn (1951) tabulated the ten most widely-used words relating to arithmetic at kindergarten age. These words included: one, some, little, all, big, no, two, more, any, and three.

Clark (1950) recorded the remarks made by twelve three-year-old children during a four week period at nursery school. She reported that number terms "came in constantly" during their conversations with the teacher or with one another. The words most frequently used were: big, little, many, high, up, down, and more.

Ausubel (1958) disclosed that the concepts of "biggest" and "littlest" became evident at the early age of fourteen months.

Bjonerud (1960) found that 80 per cent of the kindergarten children tested in one study responded accurately to situations requiring an understanding of largest, smallest, tallest, longest, most, inside, beside, closest, and farthest. He also noted that 70 per cent of these children understood "middle" and "last" and that 50 per cent of them understood shortest, few, underneath, and some.

McLaughlin (1935) surveyed the counting ability of 125 preschool children ranging from three through five years of age. She reported that the typical five-year-old child counted accurately to 33. Ilg and Ames (1951) indicated that one-third of the five-year-old children in their survey could count to 30 or higher. Coward (1940) found a marked increase in the ability to count as children increased in age.

Carper (1942) observed forty-eight kindergarten pupils and indicated that 50 per cent of them recognized groups of 3, 4, and 5 when the objects were arranged in compact groups and placed in a regular pattern. Bjonerud (1960) tested 127 kindergarten children and reported that the mean in counting for the entire group was 19. This investigator also reported that 25 per cent of the kindergarten pupils studied exhibited the ability to count by tens.

In their analysis of the counting skills of preschool and kindergarten children, several researchers tabulated the abilities of the children to do both rote counting and rational counting. The preschool children of the McLaughlin study (1935) counted by rote to 33 and counted rationally to 28.

Riess, cited in Sussman (1962), suggested that ordinal counting appeared first with children and gradually led to arithmetical operations with cardinal numbers. Spitzer (1956, p. 16) agreed with this finding and stated:

> The ordinal concept is the first . . . that the child acquires and is essential to the use of counting to determine the "how many" or the quantitative aspect of number.

In a recent study, Bjonerud (1960) found that 95 per cent of the kindergarten children in his group understood the numbers "second" and "fourth." Piaget (1952) stated that there was a mutual relationship between ordinal and cardinal numbers and that these systems were "acquired simultaneously, the two concepts being interdependent."

Harding (1963, p. 85) stressed the importance of the counting experiences at kindergarten level. He asserted:

> The cornerstone of all mathematical concepts may be laid squarely in the kindergarten, as the child learns to count functionally.

Writing Numerals

The assessment of the ability of preschool and kindergarten children to write numerals has received only limited attention in research. Ilg and Ames (1951) indicated that the five-year-old child could write a few numerals from dictation, usually 1 to 4. The chief mistakes were reversals of the numbers 3, 7, and 9.

Mott (1945) interviewed forty-four nursery school and kindergarten children and discovered that 72 per cent of them could write the numeral 3; 64 per cent of the children could write the numeral 5; and 20 per cent, the numeral 10.

Swenson (1964) pointed out that the writing of numerals should not preceed in isolation from the understanding of number meanings. Spitzer (1956) stated that some children grasp the principles of notation and prior to school entrance some learn to write numbers.

Numeral Recognition

Several investigators exhibited an interest in the ability of young children to recognize numerals. McLaughlin (1935) noted that numeral recognition progressed steadily from year to year and that nearly all five-year-old children identified at a glance the numerals 2 and 3. Mott (1945), in testing forty-four nursery and kindergarten children found that 89 per cent of them recognized the numeral 3; 72 per cent recognized the numeral 5; and 66 per cent recognized the numeral 10. Bjonerud (1960) stated that the majority of beginning kindergarten children possessed the ability to recognize number symbols. Coward (1940) indicated, however, that numerals were scarcely recognized by children until they had reached the age of six.

In an experiment concerning binary numbers, Suppes and Ginsberg (1962a) required five- and six-year-old subjects to learn the concepts of the numbers four and five in the binary number system, each concept being represented by three different stimuli. The child was required to respond by placing one of two cards directly upon the stimulus. On one card was inscribed a large Arabic numeral 4 and on the other was a large Arabic numeral 5. All children were told on each trial whether they made the correct or incorrect response, but half of them were also required to correct their wrong responses. Thus, a concern of the experiment was the examination of the effect upon learning of requiring the subject to correct overtly a wrong response. There were twenty-four subjects in each of the two groups. A significant difference between the two groups in rate of learning was found. The proportion of correct responses for the corrected group was higher.

In the experiment, the investigators treated the concept itself as the single stimulus; in this case there were two concepts, one for the number 4 and one for the number 5. The criterion for the learning of the concept was correct responses to the first three presentations of each stimuli. On this basis, data were divided into two parts. The data from the group meeting the criterion were arranged for concept-learning analysis, in this case a two-item learning task. The remaining data were assumed to represent paired associate learning involving six independent stimulus items. For the paired associate group, over the first ten trials, there were 81 cases; and for the concept-formation group there were 21 cases with 48 trials in each. The Chi Square Test of Stationarity was not significant for either group.

In summary, research indicated that the ability of preschool and kindergarten children to recognize numerals developed at about the same rate as their ability to write numerals. Some children could identify many numerals while most could identify only a few or none of the numerals. Preschool and kindergarten children appeared then, to be considerably less skilled in these areas than they were in either arithmetic vocabulary or in counting.

Problem Solving

A number of studies has been undertaken to determine the ability of young children to solve simple arithmetic problems. Buckingham and McLatchy (1930) found that 37 per cent of 313 kindergarten children responded correctly to various addition combinations. Mott (1945) discovered that almost all nursery school and kindergarten children tested dealt successfully with simple number combinations when they were presented concrete objects which they could manipulate. Bjonerud (1960) noted that 19 per cent of 127 kindergarten pupils successfully solved addition combinations and 75 per cent of these pupils successfully solved subtraction combinations when the sums or differences were less than 5. Approximately 35 per cent of these pupils were able to solve combinations when the sums or differences were more than 5. On the basis of his findings, Bjonerud stated that kindergarten children possessed a high degree of skill in solving work problems that involve simple addition and subtraction facts. Sussman (1962) indicated that beginning kindergarten pupils possessed modest problem solving abilities. On the other hand, Coward (1940) reported that children under six years of age showed only slight ability to combine numbers.

## Conservation of Number

In Piaget's (1952) theory of intellectual development, a central role was assigned to the child's conceptualization of the principle of conservation, i.e., his realization of the principle that a particular dimension of an object may remain invariant under changes in irrelevant aspects of the situation. Experiments were carried out by Piaget in order to understand children's comprehension of continuous quantity of weight. For example, in some experiments, materials used were two clay balls of identical size. One ball was used as the standard and the other was manipulated by the experimenter who cut it, rolled it out, or similarly altered its shape. After the shape of one ball was changed, the child was asked whether the two pieces of clay weighed the same, more, or less. Such experiments have suggested that there is a natural ordinal scale of conceptualization of quantity, weight and volume. Quantity was said to be understood in conservational terms by the age of seven or eight, weight by the ages of nine and ten, and volume around the age of eleven or twelve.

In a systematic replication of Piaget's studies, Elkind (1961) confirmed the developmental sequence for quantity and weight even to the extent of age ranges (in a cross-sectional study). The studies of Dodwell (1960) and Wohlwill (1960) have also given strong support to the notion that the attainment of the level of conservation marks a clearly defined stage in the formation of the number concepts.

Wohlhill and Lowe (1962) attempted to determine more specifically the nature of the processes at work in the development of the conservation of number as studied by Piaget. Their investigation was in the form of a nonverbal, matching-from-sample type of learning experiment, preceded and followed by verbal questions to measure the child's understanding of the conservation priciple. The 72 kindergarten subjects were trained under these conditions: role of reinforced practice on conservation, of dissociation of biasing perceptual cues, and of inferential mechanisms based on the recognition of the effects of addition and subtraction of elements. The results indicated an overall increase in nonverbal conservation responses from a pre-test to a posttest, within the limited context of the learning task; but, they showed no significant differences attributable to the conditions of training. It was concluded that whatever learning may have taken place was of a rather restricted type, representing more likely the formation of an empirical rule than the understanding of a general principle.

Zimiles (1963, p. 692), in an attempt to account for perplexing results in the study of Wohlwill and Lowe (1960) suggested that these investigators helped to obscure the issue that "the role of lack of differentiation between numerical and perceptual estimates of quantity may have been insufficiently considered." This was accomplished by repeatedly describing the tendency to judge quantity on the basis of "greater length or density as a response to irrelevant perceptual cues." Zimiles emphasized the concept of quantity which exists for the child before the concept of conservation is developed. Such early ideas about quantity were based exclusively on perceptual cues of length, density, height, weight, etc., according to Zimiles. Such dimensions constitute the definition, insofar as there is a definition, of quantity for the preschool child.

In a study cited by Siegel (1964), Kooistra (1963) worked with a sample of intellectually superior children of four to seven years of age, with emphasis on the importance of the child's intellectual level in performance of a conservation task. The results, using mental age as a criterion, were similar to previous findings of Piaget (1952), using chronological age as a criterion. When the children's explanations of the various conservation procedures were classified according to chronological age, 50 per cent or more of their responses displayed that the concept of conservation had been achieved at age five for mass, at age six for weight, and at age seven for volume. Only seven of the ninety-six children in Kooistra's experiment showed any deviation from the mass-weight, volume sequence reported by Piaget.

Smedslund, cited in Sigel (1964), reported some change in the development of conservation of substance. Smedslund held that the essential condition for the shift from nonconservation to conservation was the introduction of cognitive conflict. Such a conflict induced a cognitive reorganization which resulted in the concept of conservation. This hypothesis was tested by Smedslund (1961b) with five and one-half to six and one-half-year-old children who indicated no prior evidence of conservation of substance. These children were subjected to a training period in which each child was presented with two pieces of plasticine, one piece intact and the other transformed in shape and either a piece of it removed or another piece added to it. The child was asked "Do you think there is more or the same amount or less in this one than in that one?" To create conflict, the change in shape was always in opposition to the addition or subtraction. Four of the five subjects, who had consistently answered only in terms of addition or subtraction, correctly ignoring shape change, gave a number of conservation responses in the post-training test, complete with the logical rationale.

In a later experiment, Smedslund (1961c) used a similar training procedure, but added a control group that received no training. Whereas in previous studies training had been ineffective, with the introduction of cognitive conflict and with no rewards for correct responses, the training groups demonstrated better performance than did the controls. Sigel (1964, p. 241) suggested that this finding had implications for education, suggesting that to teach a concept, "juxtaposition of two competing ones will force a child to reflect and think rather than to respond with what he already knows." In addition, when a comparable conflict was inherent in the task, rewarding the child was not necessary.

In a third study, Smedslund (1961a) reported that children who had acquired conservation in the course of their normal experience did not give up that concept in the face of challenging experimental conditions. Subjects were presented with two plasticine objects. One was changed in shape, but the experimenter surreptitiously stole a small bit from it. Upon the child's response that the quantity was the same despite change in shape, the experimenter proved this answer wrong by weighing the plasticine on a scale. Children who had acquired the concept of conservation naturally were resistant, insisting for example, that a bit had fallen on the floor. Children who had acquired conservation experimentally, in contrast, quickly reverted to nonconservation.

In summarizing the findings of the studies concerning number conceptualization, it seemed necessary to emphasize that a child could not be taught the concept in question unless he had already attained a particular cognitive level of maturity. It has long been known that experience and language influence ability to conceptualize. Therefore, exposure to a wide variety of relevant experiences and encouragement in the acquisition of verbal skills may increase both quality and quantity of a child's understanding. Hopefully, such experiences will facilitate his application of concepts by providing a more coherent and stable cognitive organization. However, Sigel (1964) invoked the concern of building concepts on a foundation of insufficient intellectual maturity.

In conclusion, the investigator found only a few studies which were concerned with testing the effect of treatment through use of manipulative media. Two studies, Sindwani (1964) and White (1963), reported use of a test-teach-test method with children aged three and four years. Therefore, the investigator believed that a similar study with children who were five years of age seemed feasible.

# CHAPTER III

# PROCEDURES

The investigator used the following procedures in endeavoring to identify some of the number concepts that kindergarten children could grasp.

## SELECTION OF TOPICS FOR THE STUDY

After having examined some of the research relative to number concepts, professional texts and workbooks concerning prenumber experiences, previous tests on number concepts for young children, and having discussed the proposed topic with professional persons engaged in the education of young children, the investigator determined that it was necessary to limit the areas of concentration for the experiments to: comparisons of sets, rational counting, cardinal property of number, ordinal property of number, place value, and conservation of number. These areas were selected because of their suggested relationship to prenumber experiences as a background for first grade mathematics experience. Through the survey of professional materials and from the advice of professional personnel, the investigator determined to limit the study to six separate experiments, one each week for a six-week period from February 22-March 31, 1966. All experiments involved a basic concept of set terminology. A lesson on sets was taught to the entire group of kindergarten children on February 14, 1966.

After careful consideration and study, the experimenter constructed the twelve tests for the study, six pre-tests and six post-tests. In addition, six lesson plans were prepared by the investigator. A copy of each test and each lesson plan is included in Appendix B and C, respectively.

## ARRANGEMENTS FOR THE STUDY

After having discussed the research with the Principal of Curry School, the experimenter arranged several conferences with the kindergarten teacher. Procedures for the testing and teaching were explained to her. The teacher expressed interest in the study and offered her support. No general number readiness activities were planned for teaching to the kindergarten group until the completion of the study.

The investigator secured permission to conduct a pilot study, during the first three weeks in February, at St. Francis Episcopal Church Kindergarten. Several conferences were held with the teacher of this kindergarten in order to explain various procedures to be used in the pilot study.

## THE PILOT STUDY

A pilot study of twenty-four children who were five-years-old was conducted at the St. Francis Church Kindergarten in Greensboro, North Carolina. The purposes of the pilot study were: (1) to determine the feasibility of the lesson plans; (2) to determine the level of difficulty of the test items; and (3) to forestall other problems which might be incurred in the study.

The pilot study consisted of two separate, full-length experiments conducted during the first two weeks. The two experiments were concerned with place value and rational counting. The range of scores of the first experiment was 0 to 10 and on the second 2 to 10. Thus, the difficulty of the tasks involved seemed adequate.

During the third week of the pilot study, six children were tested on each of the remaining tests, ordinal property of number, conservation of number, comparison of sets and cardinal property of number. Such capsule testing enabled the investigator to test more fully the level of difficulty of the tasks involved in the remaining experiments. Although some of the instructions and the order of items were changed as a result of the pilot testing, the basic elements of the tests were unchanged. An explanation of the testing items, materials used, and teaching procedures is included in Appendices B and C, respectively.

Through the pilot study the experimenter learned that the time element was important and that it would be necessary to work more efficiently. This was facilitated through practice with the testing materials, through increased awareness of the importance of gaining rapport with the children, and through awareness of the need for economy of motion in recording and handling manipulative media.

The investigator allotted five minutes per subject for a testing sequence which consisted of ten items for each test. Such a number had been suggested as adequate for each experiment, both in terms of allocation of time and of evaluation of the subject's responses (White, 1963).

The pilot study required an experimental and control group, randomly selected, for each experiment. To eliminate bias, a table of random numbers was used to assign subjects to groups. A coin was then flipped to determine the experimental group.

## SELECTION OF THE SUBJECTS

An alphabetized list of the kindergarten children was used and each child was given a code number, 00-18. Before the experiment started, a table of random numbers was used to establish different experimental and control groups for each of the experiments.

The purpose of the establishment of different experimental and control groups for each experiment was to prevent bias for whatever reasons subjects differed at a given point in time. For each experiment, the first ten children whose code names appeared in the table constituted Group A; the following nine, Group B. A coin was then flipped to determine the experimental group, to receive the treatment, teaching.

In order to be able to adjust the groups, in the event of absences, the examiner constructed a code sheet for each experiment and entered the code number by each subject's name, determined by its appearance in the table of random numbers. If more than one child in either the experimental or control groups was absent during any portion of the experiment, the next number was moved into the other group before the experiment started.

## CONFERENCES

The pre-tests and the post-tests were administered to all subjects by the experimenter who, in order to reduce bias, did not do the purposive teaching. Specifically, the investigator had knowledge of the materials used in testing. However, the kindergarten teacher was more aware of the personalities within her class. Thus, her assistance was secured for the teaching periods. The investigator's teaching was limited to the introductory lesson of set terminology. This lesson was taught to the entire group prior to the beginning of the experimental sequence.

A copy of the lesson plan was presented to the teacher on Friday before the teaching period on the following Wednesday. Each lesson plan, including the manipulative media, was discussed with the teacher. General procedures for each week were established. On Tuesday, all subjects received a pre-test. On Wednesday, the experimental group was taught during a thirty-five minute period, beginning at nine-fifteen. Testing was planned for a period beginning at nine-thirty a.m. and extending through eleven o'clock a.m.

The lesson plans, as well as the testing materials and manipulative materials for teaching and testing, were prepared by the investigator. During the conferences with the kindergarten teacher, valuable suggestions were received from the teacher for inclusion of additional materials in the lesson plans. In all cases, the manipulative materials used for testing were different from those used for teaching. The testing and teaching materials were constructed inexpensively by the investigator or were available in the kindergarten.

The testing periods were conducted in a small conference room adjacent to the playroom. The teaching periods were conducted in the playroom where the children were seated in a circle on the rug. The experiment concerning conservation of number was conducted at a table in the art room. Such a setting was chosen due to the nature of the manipulative materials.

During the teaching period, the investigator observed the experimental group. The control group was taken to the art room where they engaged in free play or in quiet activities, under the direction of a second kindergarten teacher. During the fifth experiment, due to the illness of the regular kindergarten teacher, the control group teacher taught the experimental group. Thus, the substitute kindergarten teacher supervised activities for the control group in the art room.

## DESCRIPTION OF THE SETTING

The Curry School Kindergarten, Laboratory School for the School of Education on the campus of the University of North Carolina at Greensboro, consisted of a spacious playroom, a cloak room, an art room, and an office for the kindergarten teacher, who was the sole member of the Kindergarten Staff.

The testing room contained a desk, three chairs, and a filing cabinet. There were no toys, or other play equipment to distract the subjects. The one window opened at a height which did not facilitate looking outside.

The subjects were very accessible. The investigator merely entered the adjoining playroom or walked through the playroom to the art room to secure the subjects. Occasionally, a child requested that he select the next subject for testing. He was allowed to do this. This setting seemed conducive to an effective use of materials in a testing situation.

## TESTING AND TEACHING MATERIALS

The testing and teaching materials were presented to the children in the form of games. On Tuesday of each week, the investigator tested both the experimental and control groups. On Wednesday, a thirty-five minute purposive teaching period was given to the experimental group. On Thursday, both the experimental and control groups were again tested by the investigator. Post-tests and pre-tests were parallel in all instances.

Prior to the beginning of the study, the investigator visited the kindergarten several times in order to become acquainted with the routines and with the children. The investigator taught the initial lesson on set terminology to the entire group. A copy of the lesson plans is included in Appendix C.

During the first testing period, the kindergarten teacher assisted the experimenter by asking the children to "play games" with the investigator. Thereafter, the subjects seemed eager to "play games," and securing their cooperation presented no problem. In fact, the children often requested longer games and, as members of the experimental group, participated enthusiastically during the teaching periods.

The six experiments were carried out in the following order: comparison of sets, rational counting, cardinal property of number, place value, ordinal property of number, and conservation of number.

## THE SUBJECTS

The subjects included the nineteen children enrolled in the Curry School Kindergarten for the Spring Semester, 1966. Their ages ranged from five years 4 months to six years 4 months. Ten boys and nine girls composed the class. The IQ for the group, as measured by the Peabody Picture Vocabulary Test, ranged from 92 to 135.

## SCORES

At the end of the six weeks, the experimenter had twelve scores for every child who was present each week during the study. Obviously, the subjects who were absent during any part of the study had fewer scores. A summary of the number of children in each experiment is included in Appendix A. The analysis of covariance and the pre-test helped to provide a comparable group and provided evidence for making implications concerning the effectiveness of purposive teaching of the selected number concepts at the kindergarten level. The analyses are considered in Chapter IV, Analysis of the Data.

# CHAPTER IV

# ANALYSIS OF THE DATA

The purpose of this study was to determine the effectiveness of the purposive teaching of selected number concepts to kindergarten children. The study was designed to compare two groups, one which received teaching and one which did not. Both groups were given pre- and post-tests.

Analysis of covariance was used for statistical interpretation of the data. This analysis is concerned with two or more measurable variables where no exact control has been exercised over measurable variables regarded as independent. It makes use of the concepts of both analysis of variance and of regression. According to Steel and Torrie (1960, p. 305), the most important uses of covariance analysis are:

(1) to assist in the interpretation of data;
(2) to partition a total covariance or sum of cross products into component parts;
(3) to control error and increase precision;
(4) to adjust treatment means of the dependent variable for differences in sets of values of corresponding independent variables.

The fourth use has particular implications for this study. That is, although normal randomization procedures were used to assign the children to the twelve groups, 6 experimental and 6 control (thus eliminating assignment bias), the children within each group could not be considered homogeneous. Differences existed, such as age, IQ, and previous stimulation, and were sources of uncontrolled variation in the children. However, when covariance is used as a method of error control, that is, control of variance, it is in recognition of the fact that observed variation in the dependent variable is partly attributable to variation in the independent variable. In turn, this implies that variation among dependent treatment means should be adjusted to make them the best estimates of what they would have been if all independent treatment means had been the same.

Thus, having made the adjustment, the significance between the means may be tested with added assurance that the difference between means is in fact due to treatment effects and not simply a result of difference which existed between children previous to treatment.

In the following paragraphs, the analysis of the six experiments for the kindergarten group is reported according to: (1) the adjusted means; (2) the range of scores on the pre-tests and post-tests for the experimental and control groups; (3) the significance or non-significance of the difference between the adjusted treatment means. All F-ratios are reported at the .05 level. Tables containing the original data are included in Appendix A.

Analysis of the data in Experiment I, Comparison of Sets, revealed an adjusted mean of 7.41 for the control group and 7.90 for the experimental group. Although a small difference (in favor of the experimental group) was noted, the difference in the adjusted means was not significant. The range of scores for the experimental group was 3 to 9 on the pre-test and 6 to 10 on the post-test; for the control group, 7 to 9 on the pre-test and 5 to 10 on the post-test.

In Experiment II, Rational Counting, the difference in the adjusted means was not significant. However, analysis revealed a difference (in favor of the experimental group) of 1.21 points. The adjusted means for the control group was 7.24 and for the experimental group, 8.45. For the experimental group the range was 5 to 10 on the pre-test and 8 to 10 on the post-test. The range for the control group was 2 to 9 on the pre-test and 1 to 9 on the post-test.

In Experiment III, Cardinal Property of Number, the adjusted mean for the control group was 7.69 and for the experimental group, 7.54. In contrast to the other five experiments, the difference in the control group mean exceeded that of the experimental group by .15. However, the difference between the means was not significant. The range of scores for the experimental group was 3 to 10 on the pre-test and 5 to 10 on the post-test. For the control group, the range was 2 to 10 on the pre-test and 0 to 10 on the post-test.

It seemed probable that the successful performance of the tasks in Experiment IV, Place Value, represented a level which superceded readiness for concept formation of many kindergarten children. Specifically, the adjusted mean for the experimental group was 4.65 and for the control group, 2.30. Although a difference of 2.35 points (in favor of the experimental group) existed, the difference was not significant at the .05 level. The range of scores for the control group was 0 to 9 on the pre-test and 0 to 10 on the post-test. The range for the experimental group was 0 to 9 on the pre-test and 0 to 9 on the post-test.

In Test V, Ordinal Property of Number, the adjusted mean for the experimental group was 8.62 and for the control group, 7.10. However, the difference of 1.52 (in favor of the experimental group) was not significant. The range for the control group was 3 to 10 on the pre-test and 3 to 10 on the post-test. For the experimental group, the range was 0 to 9 on the pre-test and 2 to 10 on the post-test.

In Test VI, Conservation of Number, the difference between the adjusted means was significant at the .05 level. For the control group, the adjusted mean was 6.51 and for the experimental group, 8.83. Thus a difference of 2.32 points (in favor of the experimental group) was found. The range of scores for the control group was 4 to 10 on the pre-test and 2 to 10 on the post-test. For the experimental group, the range was 2 to 9 on the pre-test and 3 to 10 on the post-test.

Out of the six experiments, only Test VI, Conservation of Number, was significant at the .05 level. However, four of the other tests showed a difference (in favor of the experimental group) ranging from .49 to 2.35 points. In contrast, Experiment III, Cardinal Property of Number, showed a difference of .15 in favor of the control group.

The relatively high means for both the experimental and control groups on both pre-tests and post-tests in Experiment I, Comparison of Sets; Experiment III, Cardinal Property of Number; and Experiment V, Ordinal Property of Number; may have indicated that both the experimental and control groups were approaching a ceiling. As previously stated, Test IV, Place Value, was probably too difficult for the children. Performance of the tasks in Test IV required the subject to use a base of ten in responding to the testing items. Since one child consistently counted rationally to 5 only, and a second child to 4 only, it was unlikely that these children could accomplish tasks requiring knowledge of numbers through ten or above. In fact, these children did not accomplish tasks involving counting skills past 4 or 5.

In conclusion, performance of the tasks, throughout the experiments, required basic counting skills, through ten or above. Within the group, two individuals did not exhibit such skills. Other children exhibited operational skills consistently, while other children were inconsistent. Such semi-operationalism of counting skills was suggested by Piaget (1952) for a group of five-year-old Swiss children. Finally, the difference (in favor of the experimental groups) in the adjusted means, though not significant at the .05 level, may indicate that purposive teaching might produce significant differences if given longer training periods on concepts more directly related to basic counting skills with larger samples.

Chapter V indicates some findings, limitations of the study, conclusions drawn from the experiments, and recommendations for further research.

# CHAPTER V

## SUMMARY

This study was conducted to determine the effectivenss of purposive teaching of selected number concepts to kindergarten children. The subjects were the nineteen children enrolled in the Curry School Kindergarten of the University of North Carolina at Greensboro for the spring semester, 1966.

## PROCEDURES

The number concepts which were used for the six experiments in this study (in order) were: (1) Comparison of Sets, (2) Rational Counting, (3) Cardinal Property, (4) Place Value, (5) Ordinal Property, and (6) Conservation of Number. In selecting and defining tasks for these concepts, the investigator made a survey of professional literature and held conferences with teachers and specialists in early childhood education.

In the literature reviewed, the investigator found only a little research using the test-teach-test method to ascertain the number concepts which could be taught to kindergarten children. Much of the research reviewed was concerned with reporting levels of conceptualization at age five years. Such research included that of Brace and Nelson (1965), Piaget (1952), and Sussman (1962). Sindwani (1964) and White (1963) reported use of the test-teach-test method with nursery school children; whereas, Smedslund (1961c), Suppes and Ginsberg (1962), and Wohlhill and Lowe (1962) reported somewhat similar methods with kindergarten children.

Beard (1965) was among those who offered suggestions for a kindergarten program with emphasis on present theory. She advocated a program which was systematic, meaningful, purposeful, and incidental, as well as planned.

After having discussed the proposed research with the Kindergarten teacher at St. Francis Episcopal Church, arrangements for a pilot study during the first three weeks in February, 1966, were made. The pilot study consisted of two separate, full-length experiments and a third week of capsule testing of the remaining tests. From the pilot study, the investigator learned to use her time more efficiently and increased her awareness of the importance of gaining rapport with the children. It was found that an average of five minutes for testing each child was feasible. After the pilot study, some instructions and order of items were changed, but the basic elements of the tests were not altered since the range of scores for the tests seemed to indicate an appropriate level of difficulty of the tasks.

During the fall semester, 1965, the proposed research was also discussed with the Principal of Curry School; after which, several conferences were held with the Kindergarten teacher to explain the procedures for the study.

The research consisted of six experiments of the test-teach-test sequence, one each week for a six-week period from February 22-March 31, 1966. Prior to the beginning of the study, normal randomization procedures were used to assign the children to experimental groups, to receive teaching and testing, and control groups, to receive testing only. Each Tuesday, the investigator tested all nineteen children. On Wednesday, the Kindergarten teacher taught the experimental group for thirty-five minutes, while the investigator observed. (Prior to each teaching period, the lesson plan, prepared by the investigator, was discussed with the Kindergarten teacher.) On Thursday, all nineteen children were given a post-test, which was in all instances parallel to the pre-test, differing only in manipulative media.

Analysis of covariance was used for statistical interpretation of the data, which consisted of ten measures on each of the twelve tests, six pre-tests and six post-tests. The analysis revealed that the difference between adjusted means was not significant at the .05 level for the first five experiments; Experiment VI, Conservation of Number was significant. However, four of the tests showed a difference in favor of the experimental group, and the remaining test showed a difference in favor of the control group.

## FINDINGS

Because of the conflicting connotations of readiness and the insufficient knowledge of how concepts are acquired, the following findings, based upon the data obtained in this study, have implications for those concerned with conceptualization in young children and with planning of pre-school curricula.

1. The difference between the adjusted means for Experiment I, Comparison of Sets, was not significant. The adjusted means were

7.41 for the control group and 7.90 for the experimental group. Overall, the performances of the subjects in this study were similar to those studied by Bjonerud (1960). That is, he found that 80 per cent of the kindergarten children in the group understood such terms as most, smallest, tallest, and largest. However, it is noted that Bjonerud was concerned only with testing the level of conceptualization of children aged five years, and not with the purposive teaching of kindergarten children.

2. The difference in the adjusted means in Test II, Rational Counting, was not significant. The relatively high means for both the control and experimental groups, 7.24 and 8.45, respectively, were consistent in direction with McLaughlin (1935) and Brace and Nelson (1965) who reported that the typical child of kindergarten age could count rationally past twenty, without consideration of effects of training.

3. In Test III, Cardinal Property of Number, the difference between the adjusted means was not significant. In contrast to the other five experiments, the control group adjusted mean exceeded that of the experimental, but the difference was so small that this may have been due to children who had been placed in the groups by the randomization process.

4. The difference between adjusted means was not significant in Test IV, Place Value. Since the adjusted mean for the control group was

2.30 and for the experimental, 4.65, it seemed probable that the test was too difficult for the children. Such a finding was consistent with a conclusion of Brace and Nelson (1965) who indicated that a thorough understanding of cardinal number, ordinal number and rational counting must be established before children are able to understand place value. The performance of the control group was consistent with that reported by Bjonerud (1960) who found that 25 per cent of the group of kindergarten children could count by tens, without consideration of the effects of teaching.

5. In Test V, Ordinal Property of Number, the difference between adjusted means was not significant. The high means for both the control and experimental groups were somewhat consistent with the finding of Bjonerud (1960) who reported that 95 per cent of the kindergarten children in a study understood ordinal numbers below ten; therefore, the test might have been too easy for the subjects.
6. Analysis of the data for Test VI, Conservation of Number, revealed a significant difference between the adjusted means. This finding is consistent with that of Smedslund (1961b) who trained five year old children in a conservation experiment. Also, the finding is interesting in light of Piaget's (1952) sequence of development of conservation, in which the general level of conceptualization of quantity was found to occur at ages seven or eight years, weight by ages nine and ten years, and volume around age eleven or twelve years.

Wohlhill and Lowe (1962) concluded, from an experiment in conservation of number with kindergarten children, that whatever learning had taken place was more likely the formation of an empirical rule rather than the understanding of a general principle. Such a conclusion may have implications for this study.

From the data, it appeared that purposive teaching of number concepts to kindergarten children was not effective in five instances and effective in one instance.

## CONCLUSIONS

In formulating conclusions from the study, the investigator recognized the following limitations involved in the research: (1) Test IV, Place Value, required a basic knowledge of counting skills which some children did not exhibit, or exhibited non-operationally; (2) the small sample size seemed to be a limiting factor; (3) duration and intensity of the teaching periods seemed to be a limiting factor; (4) incorporation of too many different concepts within a single testing and consequently teaching period probably was a shortcoming; and (5) possible failure to ascertain, with sufficient precision, the necessary sequences as preludes to learning was a final factor.

Considering the limitations of the study, the following conclusions seemed justifiable.

1. The subjects were not ready for further learning, through purposive teaching as defined in this study, of the concepts of comparison of

sets, rational counting cardinal and ordinal property of number, and place value.

2. The children in this study were ready to learn conservation of quantity, weight, and volume.
3. The subjects exhibited, at the time of the experiment, rather definite concepts of comparisons of sets, rational counting, and cardinal and ordinal property of number. Therefore, it was concluded that the tasks were not sufficiently difficult to measure learning through limited purposive teaching.

## RECOMMENDATIONS

Sigel (1964) suggested that the evidence on the reaching of concepts, like that on the invariant order of their emergence, was not yet conclusive. While some theorists held that external influence of teaching can not affect stage development, other writers considered the question to be open. Further experimental studies of the attainment of concepts in young children would provide potential application of the insights gained in curriculum development and in diagnosis of the intellectual status of the child. With consideration of the limitations of this study, the following recommendations are made.

1. A test-teach-test study of one to three concepts, developed for the kindergarten child, with several weeks of purposive teaching for each concept before the post-test might provide further evidence of the effectiveness of cognitive training.

2. An experiment in which the tests provide more measurement of fewer subconcepts within a single experiment might provide a more accurate evaluation of purposive teaching.
3. A larger sample of kindergarten children might provide more conclusive evidence for making inferences.
4. Similar studies with culturally deprived children, brain damaged children, and emotionally disturbed children might provide interesting implications for sequence of development of concepts and effectiveness of teaching.
5. Follow-up studies of the psychological, cognitive, and affective consequences of the use of purposive teaching might be advisable.
6. Longitudinal studies of the relative permanence of the attainment of concepts through purposive teaching might provide valuable information for those concerned with early childhood education.
7. Investigators should be cognizant of the fact that the child's observed facility in producing concepts does not necessarily mean that underlying intellectual processes are accurately reflected. That is, the underlying process can be ascertained only with appropriate questioning to determine the limits to which meaning are applied by the children.

Finally, the investigator recommends that preschool educators be familiar with evolving theory in the cognitive development of young children, and its implications for teaching and child rearing. Only when preschool educators provide for mental development, as well as for affective and social development, can optimum growth and development be expected or achieved.

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# APPENDIX A

TABLE I

RANGE OF SCORES AND NUMBER OF SUBJECTS IN

EACH EXPERIMENT

| Experiment | RANGE OF SCORES | | | | SUBJECTS | |
|---|---|---|---|---|---|---|
| | Experimental Group | | Control Group | | Experimental Group | Control Group |
| | Pre | Post | Pre | Post | | |
| I | 3- 9 | 6-10 | 7- 9 | 5-10 | 9 | 7 |
| II | 5-10 | 8-10 | 2- 9 | 1- 9 | 7 | 7 |
| III | 3-10 | 5-10 | 2-10 | 0-10 | 10 | 8 |
| IV | 0- 9 | 0- 9 | 0- 9 | 0-10 | 7 | 9 |
| V | 0- 9 | 2-10 | 3-10 | 3-10 | 7 | 6 |
| VI | 2- 9 | 3-10 | 4-10 | 2-10 | 9 | 9 |

TABLE II

ORIGINAL MEANS

| Experiment | Experimental Group | | Control Group | |
|---|---|---|---|---|
| | Pre | Post | Pre | Post |
| I | 6.50 | 7.66 | 8.0 | 7.71 |
| II | 7.57 | 9.28 | 5.85 | 6.42 |
| III | 7.1 | 7.9 | 6.12 | 7.25 |
| IV | 1.56 | 4.33 | 2.07 | 2.36 |
| V | 5.86 | 8.14 | 7.0 | 7.66 |
| VI | 7.22 | 8.88 | 7.11 | 6.44 |

TABLE III

RAW SCORES FOR EACH EXPERIMENT

| Experiment | Rate | Experimental | | Control | |
|---|---|---|---|---|---|
| | | X | Y | X | Y |
| I | A | 3 | 7 | 9 | 10 |
| | B | 4 | 8 | 8 | 8 |
| | C | 7 | 8 | 8 | 5 |
| | D | 7 | 6 | 8 | 7 |
| | E | 7 | 6 | 7 | 7 |
| | F | 9 | 10 | 8 | 9 |
| | G | 8 | 9 | 8 | 8 |
| | H | 5 | 6 | | |
| | I | 9 | 9 | | |
| | J | | | | |
| II | A | 10 | 10 | 5 | 8 |
| | B | 6 | 8 | 2 | 1 |
| | C | 8 | 9 | 4 | 3 |
| | D | 9 | 10 | 8 | 8 |
| | F | 7 | 10 | 6 | 9 |
| | G | 8 | 10 | 7 | 8 |
| | H | 5 | 8 | 9 | 8 |
| | I | | | | |
| | J | | | | |
| III | A | 10 | 10 | 10 | 10 |
| | B | 0 | 1 | 7 | 8 |
| | C | 9 | 10 | 10 | 10 |
| | D | 3 | 5 | 2 | 6 |
| | E | 5 | 6 | 10 | 10 |
| | F | 8 | 10 | 2 | 5 |
| | G | 10 | 10 | 0 | 0 |
| | H | 7 | 8 | 8 | 9 |
| | I | 9 | 9 | | |
| | J | 10 | 10 | | |

TABLE III (Continued)

| Experiment | Rate | Experimental X | Experimental Y | Control X | Control Y |
|---|---|---|---|---|---|
| | A | 1 | 8 | 0 | 0 |
| | B | 1 | 7 | 0 | 0 |
| | C | 1 | 0 | 0 | 0 |
| | D | 1 | 2 | 0 | 0 |
| IV | E | 9 | 9 | 9 | 10 |
| | F | 0 | 0 | 8 | 8 |
| | G | 1 | 4 | 0 | 1 |
| | H | 0 | 9 | | |
| | I | 0 | 0 | | |
| | J | | | | |
| | A | 7 | 10 | 10 | 10 |
| | B | 0 | 10 | 6 | 10 |
| | C | 7 | 2 | 3 | 3 |
| | D | 7 | 10 | 3 | 3 |
| V | E | 3 | 5 | 10 | 10 |
| | F | 9 | 10 | 10 | 10 |
| | G | 8 | 10 | | |
| | H | | | | |
| | I | | | | |
| | J | | | | |
| | A | 9 | 10 | 9 | 10 |
| | B | 9 | 10 | 10 | 10 |
| | C | 7 | 10 | 7 | 7 |
| | D | 8 | 10 | 10 | 8 |
| VI | E | 2 | 3 | 6 | 3 |
| | F | 8 | 7 | 4 | 2 |
| | G | 7 | 10 | 9 | 10 |
| | H | 8 | 10 | 4 | 6 |
| | I | 7 | 10 | 5 | 2 |
| | J | | | | |

DATA SHEET

Pre-test date: __________

Post-test date: __________ Experiment number___

| Code | Name | Items | | | | | | | | | | Comments |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
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Code: E (Experimental)
C (Control)
Ab (Absent)
+ (correct response)
- (incorrect response)

Pre-test in pencil
Post-test in ink

# APPENDIX B

# TESTS

Experiment I

Comparison of Sets

Purpose: to determine the subject's ability to compare equivalent and nonequivalent groups by asking him to establish a one-to-one correspondence between objects in one set and those in another.

Pre-test

Materials: Flipchart, perception cards

1-7 Flipchart

Directions: HERE ARE SEVERAL GROUPS OF PICTURES. AFTER YOU HAVE LOOKED AT EACH GROUP, I WILL ASK YOU A QUESTION ABOUT THE GROUP. YOU MAY ANSWER THE QUESTION BY POINTING TO THE GROUP YOU HAVE CHOSEN.

1. HERE ARE THREE SETS OF SQUARES. WHICH GROUP CONTAINS THE SMALLEST NUMBER OF SQUARES? (3)
2. HERE ARE GROUPS OF APPLES. WHICH GROUP CONTAINS THE FEWEST APPLES? (2)
3. HERE ARE SETS OF CIRCLES. WHICH GROUP CONTAINS THE MOST CIRCLES? (18)
4. HERE ARE SETS OF TRIANGLES. WHICH GROUP CONTAINS THE GREATEST NUMBER OF TRIANGLES? (9)
5. HERE ARE SETS OF BOOKS ON DIFFERENT SHELVES. WHICH SET OF BOOKS IS ON THE LOWEST SHELF?
6. HERE IS A SLIDE, A SANDBOX, AND A SEESAW. WHICH OF THE TOYS IS BETWEEN THE TREES?
7. HERE IS A SET OF EIGHT TOOTHBRUSHES. WHICH TOOTHBRUSH IS THE LARGEST?

8-10 Perception cards

Directions: HERE ARE THREE CARDS. TAKE ONE. AFTER YOU HAVE LOOKED AT IT, I HAVE A QUESTION TO ASK YOU. (Variable order on the following questions)

8. LOOK AT YOUR CARD. NOW LOOK AT MY CARD. WHICH SET CONTAINS FEWER PENCILS? NOW TAKE ANOTHER CARD.
9. LOOK AT YOUR CARD. NOW LOOK AT MY CARD. WHICH SET CONTAINS MORE CHILDREN. NOW TAKE ANOTHER CARD.
10. LOOK AT YOUR CARD. WHICH OF THE GIRLS IN YOUR SET IS THE SHORTEST?

Post-test

Materials: Flipchart, perception cards

1-7 Flipchart

Directions: HERE ARE SEVERAL GROUPS OF PICTURES. AFTER YOU HAVE LOOKED AT EACH GROUP, I WILL ASK YOU A QUESTION ABOUT THE GROUP. YOU MAY ANSWER THE QUESTION BY POINTING TO THE GROUP YOU HAVE CHOSEN.

1. HERE ARE THREE SETS OF RECTANGLES. WHICH GROUP CONTAINS THE SMALLEST NUMBER OF RECTANGLES? (3)
2. HERE ARE SETS OF DIAMONDS. WHICH GROUP CONTAINS THE FEWEST DIAMONDS? (2)
3. HERE ARE SETS OF UMBRELLAS. WHICH GROUP CONTAINS THE MOST UMBRELLAS? (18)
4. HERE ARE SETS OF HEMICIRCLES. WHICH GROUP CONTAINS THE GREATEST NUMBER OF HEMICIRCLES? (9)
5. HERE ARE SETS OF DOLLS ON DIFFERENT SHELVES. WHICH SET OF DOLLS IS ON THE LOWEST SHELF?
6. HERE IS AN APPLE, ORANGE AND BANANA. WHICH PIECE OF FRUIT IS BETWEEN THE PLATES? (Banana)
7. HERE IS A SET OF EIGHT BEETLES. WHICH BEETLE IS THE LARGEST?

8-10 Perception cards

Directions: HERE ARE THREE CARDS. TAKE ONE. AFTER YOU HAVE LOOKED AT IT, I HAVE A QUESTION TO ASK YOU. (Variable order for the following questions)

8. LOOK AT YOUR CARD. NOW LOOK AT MY CARD. WHICH SET CONTAINS FEWER PAINTBRUSHES? NOW TAKE ANOTHER CARD.
9. LOOK AT YOUR CARD. NOW LOOK AT MY CARD. WHICH SET CONTAINS MORE BUTTONS?
10. LOOK AT YOUR CARD. WHICH OF THE MOTHERS IS THE SHORTEST?

Experiment II

Rational Counting

Purpose: to determine the subject's knowledge of number names and his ability to make a one-to-one correspondence of number names with objects counted.

Pre-test

Materials: Perception cards; pennies and candies; two pages of paper containing drawings of balls

1-4 Perception cards

Directions: LOOK AT EACH OF THE TWO SETS. AFTER YOU HAVE LOOKED, I WILL ASK YOU A QUESTION. YOU MAY ANSWER BY POINTING TO THE SET YOU HAVE CHOSEN.

1. HERE IS A SET OF CIRCLES AND A SET OF TRIANGLES. ARE THERE MORE CIRCLES OR MORE TRIANGLES? COUNT THEM. (4 large circles, 1 small triangle)
2. HERE IS A SET OF STARS AND A SET OF SQUARES. ARE THERE MORE SQUARES OR MORE STARS? COUNT THEM. (7 small stars, eight large squares)
3. HERE IS A SET OF TREES AND A SET OF HEXAGONS. ARE THERE MORE TREES OR MORE HEXAGONS? COUNT THEM. (7 large trees, 9 small hexagons)
4. HERE IS A SET OF CIRCLES AND A SET OF OCTAGONS. ARE THERE MORE CIRCLES OR MORE OCTAGONS? COUNT THEM. (6 large circles, 6 small octagons)

5-7 Pennies and candies

Directions: Experimenter places nine pennies in a horizontal row.

5. HOW MANY PENNIES ARE THERE? COUNT THEM. (9)
6. Experimenter places ten gumdrops above the nine pennies. HOW MANY GUMDROPS ARE THERE? COUNT THEM. (10)
7. Experimenter removes seven gumdrops from the table. SHOW ME HOW MANY PENNIES YOU WOULD NEED TO BUY THE GUMDROPS ON THE TABLE. ONE PENNY WILL BUY ONE GUMDROP. COUNT THEM. (3 pennies)

8-10 Drawings of balls

Directions: HERE ARE PICTURES OF BALLS.

8. FIND A SET CONTAINING ONLY THREE BALLS.
9. FIND A SET CONTAINING ONLY SIX BALLS.
10. FIND A SET CONTAINING ONLY EIGHT BALLS.

Post-test

Materials: Perception cards; pennies and candy canes; two pages of paper containing drawings of squares

1-4 Perception cards

Directions: LOOK AT EACH OF THE TWO SETS. AFTER YOU HAVE LOOKED, I WILL ASK YOU A QUESTION. YOU MAY ANSWER BY POINTING TO THE SET YOU HAVE CHOSEN.

1. HERE IS A SET OF BALLS AND A SET OF BATS. ARE THERE MORE BALLS OR MORE BATS? COUNT THEM. (4 large balls, 1 small bat)
2. HERE IS A SET OF MOONS AND A SET OF HEMICIRCLES. ARE THERE MORE MOONS OR MORE HEMICIRCLES? COUNT THEM. (7 small moons, eight large hemicircles)
3. HERE IS A SET OF CIRCLES AND A SET OF CUPS. ARE THERE MORE CIRCLES OR MORE CUPS? COUNT THEM. (7 large circles, 9 small cups)
4. HERE IS A SET OF RECTANGLES AND A SET OF BEES. ARE THERE MORE RECTANGLES OR MORE BEES? COUNT THEM. (6 large rectangles, 6 small bees)

5-7 Pennies and candy canes

Directions: The experimenter places nine pennies in a horizonal row.

5. HOW MANY PENNIES ARE THERE? COUNT THEM. (9)
6. The experimenter places ten candy canes above the nine pennies. HOW MANY CANDY CANES ARE THERE. COUNT THEM. (10)
7. The experimenter removes seven candy canes from the table. SHOW ME HOW MANY PENNIES YOU WOULD NEED TO BUY THE CANDY CANES ON THE TABLE. ONE PENNY WILL BUY ONE CANDY CANE. (3 pennies)

8-10 Drawings of squares

Directions: HERE ARE PICTURES OF SQUARES

8. FIND A SET CONTAINING ONLY THREE SQUARES.
9. FIND A SET CONTAINING ONLY SIX SQUARES.
10. FIND A SET CONTAINING ONLY EIGHT SQUARES.

Experiment III

Cardinal Property of Number

Purpose: to determine the extent to which the subject can recognize, without counting, the number of objects in a group

Pre-test

Materials: Six perception cards containing varying numbers of trading stamps; 1" X 3" blocks

HERE ARE SOME CARDS. LOOK AT THIS CARD AND TELL ME HOW MANY TRADING STAMPS YOU SEE.

1. HOW MANY TRADING STAMPS DO YOU SEE? (4)
2. HOW MANY TRADING STAMPS DO YOU SEE? (6)
3. NOW HOW MANY TRADING STAMPS DO YOU SEE? (8)
4. NOW HOW MANY? (2)
5. HOW MANY TRADING STAMPS DO YOU SEE HERE? (10)
6. NOW HOW MANY? (12)

HERE ARE SOME BLOCKS.

7. HOW MANY BLOCKS DO YOU SEE? (5)
8. HOW MANY BLOCKS DO YOU SEE? (7)
9. NOW HOW MANY BLOCKS DO YOU SEE? (9)
10. NOW HOW MANY? (11)

Post-test

Materials: Six perception cards containing varying numbers of seeds which have been glued onto card; dominos

HERE ARE SOME CARDS. LOOK AT THIS CARD AND TELL ME HOW MANY SEEDS YOU SEE.

1. HOW MANY SEEDS DO YOU SEE? (4)
2. HOW MANY SEEDS DO YOU SEE? (6)
3. NOW HOW MANY SEEDS? (8)
4. NOW HOW MANY? (9)
5. HOW MANY SEEDS DO YOU SEE HERE? (10)
6. NOW HOW MANY? (12)

HERE ARE SOME DOMINOS.

7. HOW MANY DOMINOS DO YOU SEE? (5)
8. HOW MANY DOMINOS DO YOU SEE? (7)
9. NOW HOW MANY DOMINOS DO YOU SEE? (9)
10. NOW HOW MANY? (11)

HOW DID YOU KNOW THE NUMBER IN EACH SET? DID YOU GUESS? DID YOU COUNT? DID YOU LOOK AT THE NUMERAL?

Experiment IV

Place Value

Purpose: to determine the subject's concept of the Hindu-Arabic system of notation.

Pre-test

Materials: Soda straws (ten straws per group) tied with yarn; set of ten paper clips; extra paper clips and loose straws

1-5 Straws--HERE IS A GROUP OF TEN STRAWS. HERE ARE SOME EXTRA STRAWS.

1. MAKE A GROUP OF ELEVEN STRAWS.
2. HERE IS A GROUP OF TEN STRAWS. MAKE A GROUP OF FIFTEEN STRAWS.
3. HERE IS A GROUP OF TEN STRAWS. MAKE A GROUP OF NINETEEN STRAWS.
4. HERE IS A GROUP OF TEN STRAWS. MAKE A GROUP OF TWENTY STRAWS.
5. HERE ARE TWO GROUPS OF STRAWS. THERE ARE TEN STRAWS IN EACH GROUP. MAKE A GROUP OF TWENTY-FIVE STRAWS.

6-10 Paper clips--HERE IS A SET OF TEN PAPER CLIPS. HERE ARE SOME EXTRA PAPER CLIPS. YOU MAY ADD ADDITIONAL PAPER CLIPS OR REMOVE PAPER CLIPS.

6. MAKE A GROUP OF TWELVE PAPER CLIPS.
7. MAKE A GROUP OF THIRTEEN PAPER CLIPS.
8. MAKE A GROUP OF FOURTEEN PAPER CLIPS.
9. MAKE A GROUP OF FIFTEEN PAPER CLIPS.
10. MAKE A GROUP OF EIGHTEEN PAPER CLIPS.

Post-test

Materials: Groups of candy canes (ten per group) tied with yarn; a set of ten nickles; extra nickles and loose candy canes

1-5 Candy canes--HERE IS A GROUP OF CANDY CANES. HERE ARE SOME EXTRA CANES.

1. MAKE A GROUP OF ELEVEN CANES.
2. HERE IS A GROUP OF TEN CANDY CANES. MAKE A GROUP OF FIFTEEN CANDY CANES.
3. HERE IS A GROUP OF TEN CANDY CANES. MAKE A GROUP OF NINETEEN CANES.
4. HERE IS A GROUP OF TEN CANES. MAKE A GROUP OF TWENTY CANDY CANES.
5. HERE ARE TWO GROUPS OF CANES. THERE ARE TEN CANES IN EACH GROUP. MAKE A GROUP OF TWENTY-FIVE CANES.

6-10 Nickles--HERE IS A SET OF TEN NICKLES. HERE ARE SOME EXTRA NICKLES. YOU MAY ADD ADDITIONAL NICKLES OR REMOVE NICKLES.

6. MAKE A GROUP OF TWELVE NICKLES.
7. MAKE A GROUP OF THIRTEEN NICKLES.
8. MAKE A GROUP OF FOURTEEN NICKLES.
9. MAKE A GROUP OF FIFTEEN NICKLES.
10. MAKE A GROUP OF EIGHTEEN NICKLES.

Experiment V

Ordinal Property of Number

Purpose: to determine the subject's ability to recognize ordinal numbers

Pre-test

Materials: cardboard parking lot with fifty cars in five rows of ten each; paper racing cars; tagboard doll on cardboard steps.

1-4 Cardboard parking lot

Directions: HERE IS A PARKING LOT WITH MANY TOY CARS. HERE IS ROW ONE. HERE IS THE FIRST CAR IN ROW ONE. I HAVE SOME QUESTIONS TO ASK YOU. YOU MAY ANSWER BY POINTING TO THE RIGHT ONE.

1. FIND THE THIRD ROW.
2. FIND THE FIRST CARD IN THE SIXTH ROW.
3. FIND THE SECOND CAR IN THE FOURTH ROW.
4. FIND THE TENTH CAR IN THE FIFTH ROW.

5-7 Paper racing cars

Directions: HERE ARE SOME PAPER RACING CARS. HERE IS THE RACING TRACK. HERE IS THE CAR IN FIRST PLACE.

5. FIND THE CAR IN FOURTH PLACE.
6. FIND THE CAR IN SIXTH PLACE.
7. FIND THE CAR IN EIGHTH PLACE.

8-10 Tagboard doll

Directions: HERE IS A GIRL WALKING UP THE STEPS. HERE IS STEP ONE.

8. WHICH STEP IS THE GIRL STANDING ON NOW? (6)
9. WHICH STEP IS SHE STANDING ON NOW? (7)
10. WHICH STEP IS SHE STANDING ON NOW? (9)

Post-test

Materials: cardboard floor plan of seating arrangement for cardboard dolls (five rows with ten dolls in each row); ten cardboard Indians; tagboard doll on cardboard tree ladder.

1-4 Cardboard floor plan

Directions: HERE IS A MOVIE THEATER WITH MANY PEOPLE. HERE IS THE FIRST ROW. HERE IS THE FIRST PERSON IN ROW ONE. I HAVE SOME QUESTIONS TO ASK YOU. YOU MAY ANSWER BY POINTING TO THE RIGHT ONE.

1. FIND THE THIRD ROW.
2. FIND THE FIRST PERSON IN THE SIXTH ROW.
3. FIND THE SECOND PERSON IN THE FOURTH ROW.
4. FIND THE TENTH PERSON IN THE FIFTH ROW.

5-7 Cardboard Indians

Directions: HERE ARE SOME INDIANS. THEY ARE RUNNING A RACE. HERE IS THE INDIAN IN FIRST PLACE.

5. FIND THE INDIAN IN FOURTH PLACE.
6. FIND THE INDIAN IN SIXTH PLACE.
7. FIND THE INDIAN IN EIGHTH PLACE.

8-10 Tagboard doll

Directions: HERE IS A BOY CLIMBING THE LADDER TO HIS TREE HOUSE. HERE IS STEP ONE.

8. WHICH STEP IS HE STANDING ON NOW? (6)
9. WHICH STEP IS HE STANDING ON NOW? (7)
10. WHICH STEP IS HE STANDING ON NOW? (9)

Experiment VI

Conservation of Number

Purpose: to determine the subject's concept of the invariance of a given number or a total quantity.

Pre-test

Materials: Cardboard cutouts of hats and faces; red cards and blue cards; colored water and plastic cups of varying size; 4 equal balls of grey clay.

1-4 Cutouts of hats and faces arranged randomly on the table top.

Directions: DO YOU KNOW WHAT THESE ARE?

1. IS THERE A HAT FOR EACH FACE? (9 hats and 9 faces)
2. Investigator raises hats twelve inches above heads. ARE THERE MORE HATS OR MORE FACES? (There is the same number of hats as faces.)
3. Investigator pushes hats together, while faces remain spaced apart. ARE THERE MORE HATS OR MORE FACES?
4. Investigator pushes faces together and places hats farther apart. ARE THERE MORE HATS OR MORE FACES?

5-7 Red cards and blue cards (ten of each) arranged randomly on table top.

Directions: HERE ARE SOME RED CARDS AND SOME BLUE CARDS.

5. IS THERE A RED CARD FOR EACH BLUE CARD? (yes)
6. Investigator pushes blue cards twelve inches above red cards. ARE THERE MORE BLUE CARDS OR MORE RED CARDS?
7. Investigator pushes blue cards together, while red cards remain spaced apart. ARE THERE MORE RED CARDS OR MORE BLUE CARDS?

8. Pretend lemonade--investigator pours pretend lemonade into two identical cups. HERE IS SOME PRETEND LEMONADE. THERE IS AS MUCH LEMONADE IN THIS CUP AS IN THIS CUP. Investigator pours one cup of lemonade into three smaller cups. DO YOU THINK THERE IS MORE, THE SAME AMOUNT, OR LESS IN THIS SET THAN IN THAT SET? (the same) WHY?

9-10 Grey clay--two balls of equal size.

Directions: HERE ARE TWO BALLS OF CLAY. THEY ARE THE SAME SIZE. EACH CONTAINS THE SAME AMOUNT OF CLAY. Investigator flattens one of the balls with her hand.

9. DO YOU THINK THERE IS MORE, THE SAME AMOUNT, OR LESS IN THIS SET THAN IN THAT SET? (the same) WHY? Investigator removes the two balls.

10. Investigator shows child two new equally sized balls of clay. HERE ARE TWO MORE BALLS OF CLAY. EACH CONTAINS THE SAME AMOUNT OF CLAY. Investigator separates one of the balls into three smaller ones. DO YOU THINK THERE IS MORE, THE SAME AMOUNT, OR LESS IN THIS SET THAN IN THAT SET? (They are the same.) WHY?

<u>Post-test</u>

Materials: Plastic cups and saucers; red and black checkers; colored liquid and five plastic cups of varying size; four equal balls of play dough.

1-4 Cups and saucers arranged separately and in random order on the table top.

Directions: DO YOU KNOW WHAT THESE ARE?

1. IS THERE A CUP FOR EACH SAUCER? (9 cups and 9 saucers)
2. Investigator raises saucers twelve inches above cups. ARE THERE MORE CUPS OR MORE SAUCERS? (There are the same number of cups as saucers.)
3. Investigator pushes saucers together, while cups remain spaced apart. ARE THERE MORE CUPS OR MORE SAUCERS?
4. Investigator pushes cups together and places saucers farther apart. ARE THERE MORE CUPS OR MORE SAUCERS?

5-7 Red checkers and black checkers (ten of each) arranged randomly on the table top.

Directions: HERE ARE SOME RED CHECKERS AND SOME BLACK CHECKERS.

5. IS THERE A RED CHECKER FOR EACH BLACK CHECKER? (yes)
6. Investigator places black checkers twelve inches above red checkers. ARE THERE MORE BLACK CHECKERS OR MORE RED CHECKERS?
7. Investigator pushes black checkers together, while red checkers remain spaced apart. ARE THERE MORE RED CHECKERS OR MORE BLACK CHECKERS?

8. Pretend hot chocolate--investigator pours pretend hot chocolate into two identical cups. HERE IS SOME PRETEND HOT CHOCOLATE. THERE IS AS MUCH HOT CHOCOLATE IN THIS CUP AS IN THIS CUP. Investigator pours one cup of hot chocolate into three smaller cups. DO YOU THINK THERE IS MORE, THE SAME AMOUNT, OR LESS IN THIS SET THAN IN THAT SET? (the same) WHY?

9-10 Play dough--two balls of equal size.

Directions: HERE ARE TWO BALLS OF PLAY DOUGH. THEY ARE THE SAME SIZE. EACH CONTAINS THE SAME AMOUNT OF PLAY DOUGH. Investigator flattens one of the balls with her hand.

9. DO YOU THINK THERE IS MORE, THE SAME AMOUNT, OR LESS IN THIS SET THAN IN THAT SET? (the same) WHY? Investigator removes the two balls.

10. Investigator shows child two new equally sized balls of play dough. HERE ARE TWO MORE BALLS OF PLAY DOUGH. EACH CONTAINS THE SAME AMOUNT OF CLAY. Investigator separates one of the balls into three smaller ones. DO YOU THINK THERE IS MORE, THE SAME AMOUNT, OR LESS IN THIS SET THAN IN THAT SET? (They are the same.) WHY?

# APPENDIX C

# LESSON PLANS

Introduction of Sets

Objective: To introduce the set concept and the use of set terminology, including the idea that a set may have only one member or even no members at all.

Vocabulary: Set, collection, member, set with one member, set with no members

Materials: Variety of set materials, tagboard table top with accompanying paper cutouts of four table settings, puzzles, paint brushes, scissors, blocks, and books.

Set: Since the children are familiar with the set of dishes in the playhouse area and in the home, this may be used as a point of departure. Arrange the tagboard table top on the floor in the center of the circle of children. Have a set of paper dishes, silverware, and glasses. Such questions as the following may be used as a guide:

> How many children have helped to get the table ready for dinner? What do we put on the table? Allow the children to help in placement of members of sets on the table. Does each plate belong to our set of dishes? Does this (point to a particular knife, fork, etc.) belong to our set of silverware? Where is our set of people to eat the dinner?

Member of a set: Read Make Way for Ducklings to the children. After reading the story have them describe various sets in the story. Further attention can be focused upon the things that belong to each set. Make reference to the group of animals in the story as members of a particular set. Explain that the children of the class form a set of children. Explain that the girls are members of the set of girls in the classroom. Explain that the boys are members of a set of boys. Continue using as many references as needed to clarify the meaning of member of a set or group.

Set with one member: Ask all the girls wearing a particular color of dress to stand. (Continue with a variety of set descriptions. The teacher should be seated.) Ask the set of boys to stand. Ask for the set of girls to stand. Ask for the set of teachers to stand. Explain that the teacher is the only member of the set of teachers in the room. Ask if there are other sets in the room with one member only (set of pianos, teacher's desk, set of goldfish). Explain that sets can have many members or just one member. Discuss other sets the children have seen.

Sets with no members: (The teacher should wear a dress or apron with two pockets, one empty and one filled with various small articles which form a set in that particular pocket.) Before the game, select as secret helpers at least three children who have clothing with pockets. Place small objects in a pocket of each child's clothing. After the class has begun, call the helpers together, one at a time. Inform the children that the pockets contain surprise sets and ask them to describe the set as it is placed on the floor. The teacher should empty her own pocket. Then, ask, "What are the members of the set of things in the empty pocket (no members)?" See if the children can suggest other sets with no members. Explain that sets may have few, many, one, or no members.

Some concrete suggestions were incorporated from materials by the following:

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Eicholz, R. E., Martin, E., Brumfiel, C. F. and Shanks, M. E. Teacher's edition, elementary school mathematics primer. Reading, Mass.: Addison-Wesley, 1963.

Lucas, J. S. and Neureld, Evelyn. Developing pre-number ideas work-text. Chicago: Holt, Rinehart, and Winston, 1965.

Nelson, Eileen, and Gundlach, B. H. Sets, numbers, numerals primer, teacher's edition. River Forest, Ill.: Laidlaw, 1965.

School Mathematics Study Group. Mathematics for the elementary school. Book K, teachers' commentary. Stanford: Stanford Junior University, 1963.

Lesson Plan I

Comparison of Sets

Purpose: To compare equivalent and nonequivalent groups by establishing a one-to-one correspondence between objects in one set and those in another.

Manipulative Materials: Crayons, bottle caps, blocks, paper clips, construction paper circles, yarn, two small plastic people, soda straws.

Vocabulary: More than, fewer than, shortest, largest, greatest, most, smallest, fewest, lowest, between

More Than, Fewer Than

(10-15 crayon sets, 8-10 large blocks, 8-10 bottle caps)

Demonstrate to the children that we can see which set has more members by pairing one member of each set with a member of the other set. Do this with the children. When pairing is complete, tell which set has more members and which set has fewer members.

Place a set of crayons (with as many as 10 to 15 members) on the floor where all students can see. Place another set (with as few as 2 to 5 members) on the floor near the first set. Ask the children if one set has more members than the other. If the children choose the correct set, ask the children why they chose that group. If the child replies, "It is bigger," substitute some very large blocks for the set with fewer members and ask again which set has more members. Explain that we say the set of blocks has fewer members than the set of crayons because the set of crayons contains more members than the set of blocks.

Place on the floor a set of crayons and a set of bottle caps, each with nearly the same number of members (8 to 10). Ask the children which set has more members. There may be some disagreement. Clarify this by demonstration.

Look at the set of children in the room. Look around the room. Does the set of blocks have more members than the set of children? Pair the members of these sets, if necessary. Ask the children to find a set which has fewer members than the set of children. Ask for sets with more members than the set of children.

Between

(yarn, two small plastic people, construction paper circle to represent home)

Using yarn, make a simple closed curve on the rug, large enough for all of the children to see. At one point on the curve, place a paper circle to represent a home. Using two small plastic people, called Mary and Sharon, place these two "people" at points a distance from and on opposite sides of the "house." Explain that Mary wishes to go home. Ask the children to explain the position of home in relation to Mary and Sharon. Various arrangements may be used to illustrate "between."

The children may stand in partners while a single child goes between the two members forming the couple.

Lowest

Ask the children to look at the window panes in the room. Describe the lowest window pane. Ask them to look around the room for other examples of "lowest" ....... lowest shelf, lowest drawer of the desk, lowest section of the door.

Largest, Greatest, Most

(Paper clips, crayons)

Place three sets of paper clips on the floor, one set with 4 to 5 members, another set with 7 to 8 members, and the last set with 10 to 12 members. Describe the set with the greatest number of members. Explain that (by counting or one-to-one correspondence) one may ascertain the number in each set. Explain that the set with 10 to 12 members may also be described as the "largest" or the set with the "most" members.

Place a set (1 crayon) on the floor along with a second set with four members and a third set with eight members. Describe the sets in terms of "largest," "greatest," and "most."

Additional descriptions may be needed here, depending on the interest of the children and their grasp of the concepts of the given words. If so, various combinations of three sets (up to twenty members) may be used. Included should be those sets with nearly the same or the same number of members in order to emphasize "close" discrimination.

Shortest

(Soda straws)

Show the children five soda straws or sticks of varying length. Explain that one of the straws is the "shortest" of the set of five straws. Describe the shortest straw.

Ask the children to stand. Ask them to select the shortest child, the shortest boy, and the shortest girl.

Smallest, Fewest

(Bottle caps, paper circles)

Place three sets of bottle caps on the table or the floor, one set with 5 to 6 members, one with 8 to 10 members, and one with 12 to 15 members. Describe the set with the smallest number of bottle caps. Explain that this set also contains the fewest bottle caps.

Place construction paper circles on the floor or table. Use three sets with varying number of members. Ask the children to describe the set with the smallest number of members and the set containing the fewest circles.

Lesson Plan II

Rational Counting

Purpose: To further the subject's knowledge of number names and his ability to make a one-to-one correspondence of number names with objects counted.

Manipulative Materials: Ten dimes, nine boxes of crayons, cutouts of sets illustrating one-to-one correspondence; flipchart of pictures of sets, illustrating number in the set through ten; supplementary materials such as marbles and blocks of varying size

Counting and Simple Problem Solving

(Crayon boxes, and dimes)

Place nine crayon boxes on the floor where the children in the circle can see and identify them. Place ten dimes in a set beside the set of crayon boxes. Ask the children how many dimes there are in the given set, and how many crayon boxes there are in the given set. Use both counting and one-to-one correspondence. Explain that each box of crayons costs one dime. Remove four crayon boxes. Ask the children how many dimes are needed to buy the remaining boxes of crayons. Repeat with varying numbers of boxes of crayons and dimes, adding and removing the crayons and dimes.

One-to-One Correspondence Illustration

(Cutouts of sets to be pinned to an easel)

The various sets illustrate the necessity of one-to-one matching in order to determine equivalence or nonequivalence of sets. The sets also illustrate variations in size within individual sets and between sets of equivalent and nonequivalent members. In addition to one-to-one correspondence, the children may count. It is important to emphasize that equivalence or nonequivalence is determined by the number of members of each set and not by size variations within a set or between the two or more sets.

Number in a Set Through Ten

(Flipchart)

Explain that we discover through one-by-one matching of many equivalent sets that, although these sets may differ in physical content, they possess a common property called number. Thus, one will be the number name which is given to all sets containing one and only one object. The symbol which we use to represent this number idea of one is the numeral 1. (Introduction of the numerals is optional.) Continue to identify number (and numerals, if desired) in sets through ten by use of the flipchart of pictures of sets.

Additional Experiences

Materials such as marbles or blocks of various sizes may also be used to teach rational counting. Children may also count the number of students in the classroom and may divide into sets of varying size. The song "Ten Little Indians" (with use of fingers) may be taught.

Lesson Plan III

Cardinal Property of Number

Purpose: To provide practice in identifying the number of a set through ten

Manipulative Materials: Blocks, flipchart, perception cards, box, abacus, ten boxes of crayons

Learning: There are many ways to count. We may choose different ways to count.

Simple Counting to Ten

(Flipchart)

Introduce sets varying in number of members from one to ten. Introduce the corresponding numerals. Crayon sets may be used to emphasize the given number as being one more than the preceding one.

Counting by Two's

(Abacus, perception cards with pictures of bicycles)

Another way of counting is to group the objects or members of the set by two's. For example, we often walk with partners. We can see sets of two's within the sets of bicycles. Let's count by groups of two.

We can use the abacus to count by two's.

Counting by Three's

(Abacus, perception cards with pictures of tricycles)

We can use the abacus to group the counting beads by three's.

We can group our set of children by three's.

We can see groups of three's within our sets of tricycles.

Counting by Five's

(Abacus, perception cards with sets of crowns with five points)

We can use the abacus to count groups of five.

Have the pupils looks at their hands. Help them to see their hands as a number line. Stress five to help in recognizing sets which show patterning. Lead pupils to realize that the complete hand picture always contains five, so that they can think at once five and so many more. Try various combinations of five and so many more through ten (composed of two fives).

The perception cards may further reinforce the principles of counting by five.

Additional Experience

Place a number of blocks in a box. Have various children remove a portion of the blocks. Ask another child to identify the number of remaining blocks. Blocks may be added in a similar way. Ways of counting may be varied from simple counting through counting by fives.

Lesson Plan IV

Place Value

Purpose: To provide practice in using the Hindu-Arabic system of notation, using ten as a base

Manipulative Materials: Macaroni shells, abacus, flipchart, numeral cards through twenty-five, 25 sticks, three-pocket chart

This lesson stresses the meaning of ten. An attempt is made to help the children realize that ten may be used as a base to which an additional number of objects may be added. Numbers through twenty-five are used.

Meaning of Ten
(Fingerplay and flipchart)

I have ten little fingers
And they all belong to me.
I can make them do things.
Just you watch and see.
I can hold them in front.
Or out at the side.
I can let them all show
Or I can make them all hide.
I can hold them up high
Or hold them down low.
I can make them go "clap,"
Then fold them just so.

--selected--

Chart, Numeral Cards, and Twenty-five Sticks

Place a varying number of sticks in the right hand pocket of the chart. Continue counting and placing the sticks, through nine sticks. Use the corresponding numeral cards. Illustrate the placeholder for ten. Continue this operation varying numbers and numerals through twenty-five.

Adding Members to the Set of Ten

The following story may be used in teaching this principle. Each child is given a set of ten macaroni shells; an additional large set of fifty or more shells (depending on the number of children) is placed in the middle of the circle. As the story is told, additional members may be added to the set of ten from the large set in the center of the circle.

> Sally and Jack are spending the week at the beach with their parents. On the first day, Sally and Jack each find ten shells on the sand. Each of you has a set of ten shells. Let's count our ten shells. Early the next morning, Sally and her Dad walk along the beach and she finds another shell to add to her set. Now she has eleven. (Each child may add an additional shell.) After lunch that day, Sally and Jack each find five more shells. Now Sally has sixteen shells. (Continue the story until the children have collected twenty-five shells, twenty-five explained as a set of two tens plus five ones.)

Additional Games

Have the children practice taking large steps around the room, a set of ten, plus varying additional steps.

An abacus may be used in introducing place value.

Lesson Plan V

Ordinal Property of Number

Purpose: To stengthen the subject's understanding of the natural order of numbers

Manipulative Materials: Number cards, one through ten; ten cards with balls of varying designs and colors

Learnings: In counting, the numbers are used in their natural order, i.e. each number is one more than the number preceding it.

Number cards

Pass out ten number cards bearing numerals one through ten. Have the children arrange them on the chalk tray so that they show the natural order of numbers, reading from left to right. After they are in place, have the children hide their eyes while you remove one of the cards. Ask, "What numeral is missing? How can you tell?" Try removing two or three cards at one time and see if the children can determine which ones are missing. Ask, "Which numeral is sixth in the row, starting at the left?" "Point to the ninth card." Continue.

Cards with Balls

Discuss the position of each ball (arranged on the chalk tray or floor in a horizonal line of ten). Ask such questions as, "What design does the seventh ball from the left have on it?" Which balls have stars on them?" "Which balls have no designs?" Etc.

Game: I'm Thinking of a Number

The child who is IT says "I'm thinking of the number that comes after six." He then calls on a volunteer for the answer. If the answer is correct, the volunteer beomes IT. The clues may be varied to include such phrases as "the number which comes before nine," "the number which comes between seven and nine," and the "number which is one more than six."

Lesson Plan VI

Conservation of Number

Purpose: To strengthen the subject's understanding of the invariance of a given number or a total quantity

Manipulative Materials: Orange Koolade, two large paper cups, three small paper cups, three identical candy bars, ten gumdrops, ten pennies, scales

Invariance of Number

(Gumdrops and pennies)

Place ten gumdrops and ten pennies in two separate horizonal rows (ten inches separating the rows). The pennies and gumdrops are positioned to correspond with each other. Ask the children how many members are in each set (use counting and one-to-one correspondence). Push the pennies together and ask the children how many members are now in each set. Check by counting. The gumdrops may be regrouped, as may be the pennies, in a number of ways, in order to emphasize the invariance of a given number.

Invariance of Weight

(Candy bars and scales)

Place two pieces of equally sized candy bars in the circle. Explain that each piece of candy contains the same amount of food. Demonstrate that both bars weigh the same--using the scales. (Candy bars are unwrapped.) Use a knife to flatten one bar. Ask the children which bar now contains more candy, or if they are both equal in the amount of candy contained. Again, check weights on the scales. Explain that each bar still contains an equal amount of candy but that one bar has been changed in shape.

Remove the flattened bar and substitute a second bar equal in size and weighing the same as the first bar. Use a knife to separate one bar into three pieces. Ask the children if the set composed of the large candy bar contains more candy than the set composed of the three pieces of candy. Check the weights of the scale. Explain that each set contains an equal amount of candy, but one set has been divided into three smaller pieces.

Invariance of Liquids

(Koolade and paper cups)

Fill two large cups with orange Koolade. Help children understand that each cup contains the same amount of liquid. A wax mark may be placed on each cup to illustrate the equal levels of liquid. Into three small paper cups, empty the contents of one of the large cups. Ask the students which set contains more Koolade. Explain that each set contains an equal amount of Koolade. Demonstrate by pouring the Koolade (in the set of three small cups) into the large cup which was emptied.

Typed by Marie Teague