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Conceptual instruction in developmental algebra and its effect on student achievement and affect

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CONCEPTUAL INSTRUCTION IN DEVELOPMENTAL

ALGEBRA AND ITS EFFECT ON STUDENT

ACHIEVEMENT AND AFFECT

by

Sara Brame Skinner

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APPROVAL PAGE

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TABLE OF CONTENTS

LIST OF TABLES

SKINNER, SARA BRAME, Ed.D. Conceptual Instruction in Developmental Algebra and Its Effect on Student Achievement and Affect. (1993) Directed by Dr. George W. Bright. 110 pp.

The primary purpose of this research was to investigate differences in achievement of students of developmental algebra who received two different methods of instruction. The method of instruction for the experimental group was conceptual instruction; the method of instruction for the control group was procedural instruction. A secondary purpose of the research was to investigate the link between method of instruction and two affective issues: self confidence in learning mathematics and effectance motivation. The sample for the study consisted of 65 community college students (36 students in the experimental classes and 29 in the control classes). The experimental period of instruction was four weeks in length, 5 hours per week.

Post-instruction achievement was measured at the end of approximately two weeks of instruction and again at the end of four weeks of instruction. Each content test consisted of two subtests: (a) a skill-based test and (b) a transfer of knowledge test. The Assessment and Placement Test (APT) was given as the pretest. ANCOVAs using the APT score as the covariate showed that there was no statistically significant difference in the post-test scores of the students from the two groups. In addition, a t-test did not indicate statistically significant differences between groups for either of the two affective variables.

For these two particular groups of community college developmental algebra students who were taught by alternative means for a four-week period, instructional style did not affect posttest scores. Additionally, method of instruction for these two groups did not have a bearing on the affective measures.

CHAPTER I

INTRODUCTION

Recommendations for reform in mathematics education in the last decade (National Council of Teachers of Mathematics [NCTM], 1989, 1991) have called for drastic changes in the focus of mathematics for students in grades K-12. Across all grade levels and all subjects, these include an increased emphasis on meaningful mathematical experiences and a decreased emphasis on repeated practice of algorithmic procedures. The goal of meaningful instruction is to produce students who are mathematically literate individuals and who possess mathematical power. Such individuals value mathematics, are confident in their own mathematical abilities, are problem solvers, and can communicate and reason mathematically (NCTM, 1989). These students become their own authorities rather than relying on the teacher as sole authority.

Meaningful experiences are in sharp contrast to those provided by traditional rule-based instruction which has been the cornerstone of the mathematics classroom for some time. Despite the fact that many of the recommendations presently offered are not new (NCTM, 1980), research indicates that instruction in typical classrooms has changed very little over the last 20 years (National Research Council, 1989; Weiss, 1989; Welch, 1978). Instruction in mathematics classrooms continues to include practices which most teachers experienced as students, specifically, rote learning and mechanical answer-finding. In a typical classroom, answers to homework problems are given, a major portion of the class time is spent answering questions on the homework, a brief explanation of new material is given, and homework on the new material is assigned (Welch, 1978). If the goals of the current recommendations are to be met, then mathematical instruction must provide experiences in which students are encouraged to explore, conjecture, and problem solve.

Because the understanding of algebra can be the gateway to success in higher mathematics, the study of algebra by all students, not just a select few, is being urged by reformers (NCTM, 1989). The algebra that is being recommended, however, is not the current school approach to algebra, which has been described by Steen (1992) as an "unmitigated disaster." Currently, only 75% of high school students take any algebra; the remaining 25% are relegated to general or consumer mathematics in which they learn very little if any new mathematics (Steen, 1992). In addition, Steen observes that of the students who do take algebra in high school, half leave school with a "lifelong distaste for mathematics" and little or no appreciation of the value of algebra.

"Algebra for everyone" involves authentic problems which can be approached in a variety of ways. As a part of the recommendations for sweeping changes throughout the mathematics education of students K-12, there are specific recommendations for algebra. The Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) enumerated topics for each subject which should be emphasized or deemphasized in order for the recommended goals to be met. For elementary algebra the list included increased attention to developing an understanding of variables, expressions, and equations, along with a use of a variety of methods to solve linear equations. The topics to receive decreased attention were manipulating symbols, memorizing procedures, and drilling on equation solving (NCTM, 1989).

While the recommendations of the Standards were directed to the teaching and learning of mathematics in grades K -12, they are also applicable to developmental mathematics courses. The term "developmental mathematics" is used interchangeably by researchers to refer to more than one level of mathematics. Some researchers (Berenson, Carter, & Norwood, 1992; Garafalo, 1988; Higbee & Dwinell, 1988) use the term in reference to those noncredit courses offered at a four-year college or university to freshmen who are placed in those courses as a result of their scores on placement tests. However, at the community college, developmental mathematics is the term used to refer to those courses which are usually offered at the precollege level (Haney & Testone, 1990; Robinson, 1990) . In this vein, developmental mathematics courses include arithmetic, general mathematics, elementary algebra, intermediate algebra, and sometimes geometry. For this study, the term indicated the second meaning referred to above, and in particular, developmental algebra referred to a first course in algebra.

Although these courses are taught to postsecondary students, the content falls within the curriculum usually taught in grades K-12. Therefore, the argument has been made (Garofalo, 1988) that the standards which are recommended for comparable courses offered in middle or high school should also be applied to developmental courses offered at the college level. Furthermore, Garofalo projected that the answer to "What is the main purpose of developmental mathematics programs at colleges?" comes in forms and levels of generality from "to give students another chance" to "in order to prepare students for taking calculus." Certainly, these reasons are not mutually exclusive. It is evident, however, that one major goal of developmental mathematics courses is to assist individuals in meeting entrance requirements for a desired curriculum.

Requirements for different curricula vary, and therefore the reasons students take courses vary. For some curricula (e.g., business administration), an algebra credit is a prerequisite of entrance into the curriculum, and therefore a passing grade in developmental algebra will fulfill the prerequisite. For other curricula (e.g., nursing) the course prepares the student for an entrance exam (i.e., Assessment and Placement).

Statement of the Problem

An outgrowth of the reform movement was a search for instructional approaches which would produce conceptual understanding leading to the desired mathematical literacy for each student. Specifically for introductory algebra, the challenge was to find instructional methods which would help students understand the underlying concepts of algebra rather than execute algorithms merely by manipulation of symbols. It was to meet this challenge that this study was dedicated.

In this study two types of instruction were used to teach selected topics to developmental algebra students. The two types of instruction used were procedurally-based instruction, provided to the control group, and conceptually-based instruction, provided to the experimental group. Procedurally-based instruction (henceforth called procedural instruction) has often been referred to as rule-based instruction.

Students were taught in the traditional manner; that is, emphasis was placed on the learning of procedures. The teaching style was that of teacher example. Instruction for the experimental group was conceptually-based instruction (henceforth called conceptual instruction). The emphasis in this type of instruction was the building of understanding of the underlying concepts behind procedures. The teaching style here was that of providing situations from which students could construct their own knowledge.

Student achievement was assessed by testing the students after instruction. Both groups were given the same tests. Test items included both skill-oriented items and transfer of knowledge items. Transfer of knowledge items were those which tested content that was covered in instruction, but in situations different from those in instruction. The purpose of the experiment was to investigate the differences in achievement of students who had received the two different methods of instruction.

Although the main thrust of this study was the link between teacher instruction and student performance, two affective issues which have been hypothesized to be related to the study and/or learning of mathematics were included, namely, confidence in learning mathematics and effectance motivation. Acquisition of mathematical power for each student is a recommendation of the Standards. The development of personal self-confidence in one's ability to learn mathematics is an important aspect of having mathematical power. Effectance motivation is similar to a problem solving attitude (Kagan, 1964) and is an attribute which might be more present in students who are able to transfer current knowledge to new situations.

Research Questions

Specifically, the research questions addressed by the study were:

1. Do developmental algebra students who are exposed to instructional strategies which encourage conceptual understanding achieve differently on a skill-based content test from students who are exposed to procedural instruction?
2. Do developmental algebra students who are exposed to instructional strategies which encourage conceptual understanding achieve differently on a transfer of knowledge test from students who are exposed to procedural instructional?
3. Do developmental algebra students who are exposed to instructional strategies which encourage conceptual understanding achieve differently on an effectance motivation scale from students who are exposed to procedural instruction?
4. Do developmental algebra students who are exposed to instructional strategies which encourage conceptual understanding achieve differently on a confidence in learning mathematics scale from students who are exposed to procedural instruction?

Significance of Study

While the study was conducted on subjects who were enrolled in developmental algebra at the community college level, there are implications for instruction in introductory algebra classrooms across all grade levels. Therefore, the answers to these research questions are significant for several reasons.

First is the current focus on Algebra I instruction throughout North Carolina. In answer to the call for "algebra for everyone," the North Carolina Board of Education established a requirement of an algebra credit for graduation from high school begining with students who entered the ninth grade in the fall of 1992. This requirement means that the typical approach used to teach algebra, which yields effective mathematics education for the few but not for the many, must be altered. It is imperative, therefore, that instructional methods be found that will motivate every student to achieve in elementary algebra. This study adds to the research base for one alternative method of instruction for any introductory algebra classroom, whether at the middle school, high school, or the community college level.

Second is the immediate applicability of the research results. Booth (1989) points out the importance of performing research whose results can be viewed as having an immediate impact on the classroom. Research on instructional methods, such as those used in this study, could be viewed as being immediately applicable to the introductory algebra classroom across all grade levels.

Third is the addition to the theoretical base for instruction of elementary algebra at the postsecondary level. Many of the students who are registered in any developmental algebra class are those who have not previously had successful experiences in algebra. Traditionally, the dropoutand repeat rates are high. It is important to look for instructional methods which will help these students have successful experiences. Although the research base is rather broad for alternative instructional methods in elementary algebra at the secondary level, the research is limited at the post-secondary level. The results of this study, therefore, would be important for understanding instructional methods which lead to success in post-secondary developmental mathematics.

A fourth important impact that the study might have is in the evaluation portion of the instruction. The role of evaluation is a critical component of reform. Schoenfeld (1988) stated that next to texts, the major force which drives the curriculum is testing. He further stated that if mathematics educators "really intend to have an impact on practice, we will need to become deeply involved in the development and testing of instructional materials" (p. 165). As goals of instruction change, so must the instruments of evaluation.

Because instruction in most mathematics classrooms has procedural knowledge as a goal, most tests are skill-based tests of procedural skills only. It seems reasonable to assume that students who receive procedural instruction might perform better on skill-based items than students who receive conceptual instruction. Likewise, in light of the work of Hiebert and Wearne (1992), it would be expected that students who receive conceptual instruction might perform better on the transfer items on a test than those who receive procedural instruction. In this study, the experimental and the control groups were each given tests which included both skill-based and transfer items. There was, therefore, a potential misfit between the method of instruction and the method of evaluation in this study.

For this reason, the answers to the research questions then became important in that they add to the theoretical base concerning student achievement on skill-based items after being exposed to conceptual-based instruction. In addition, the study provided information concerning the achievement of students who receive procedural instruction on transfer-of-knowledge items. The lack of such research in each of these areas adds to the importance of the study.

CHAPTER II

REVIEW OF LITERATURE

This study was designed around the conjecture that, although both the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) and the Professional Standards for Teaching Mathematics (NCTM, 1991) were presented as recommendations for the reform of mathematics education in grades K-12, those recommendations could and should be extended to include developmental mathematics. Therefore, this literature review will begin with a review of these two documents as they relate to the study. The remaining topics reviewed are classified into six specific areas: (a) conceptual versus procedural Instruction, (b) constructivism, (c) algebra instruction, (d) problem solving in algebra, (e) developmental algebra, and (f) the relationship between computational skill and success in algebra.

Mathematics Education Reform

The Standards (NCTM,1989, 1991) were the response of the National Council of Teachers of Mathematics to a national call for reform in the teaching and learning of mathematics. This call for reform suggested that new goals were needed to meet the changing needs of society. The educational system which adequately served the needs of an industrial age have not met the needs of the technological age of the approaching 21st century (Office of Technology Assessment, 1988a). Traditional notions of a basic arithmetic competence for the majority of the population with the reservation of the more advanced topics for the selected few is no longer acceptable. Requirements in the workplace have changed, demanding different skills, work habits, and attitudes. Furthermore, employment counselors have projected that, on average, workers will change jobs at least four to five times during the next 25 years, with each job requiring retraining (OTA, 1988a). As society changes, its schools must change if the needs of the people are to be met. Thus, the Standards were offered in the hope that all students will have an opportunity to become mathematically literate, will be capable of becoming lifelong learners, and will become informed citizens capable of understanding the complex issues that face members of a technological society (NCTM, 1989).

A view that "knowing" mathematics is, at least in part, "doing" mathematics suggests the active involvement of students in constructing and applying mathematical ideas (Brophy, 1982; Cobb, 1988; Cobb & Steffe, 1983). The traditional emphasis on practice in manipulating expressions and in using algorithms as a precursor to solving problems ignores the fact that learning often emerges from the act of solving problems (NCTM, 1980). Therefore, problem solving becomes a means as well as a goal of instruction (Schoenfeld, 1988).

A call for a shift in emphasis from a curriculum dominated by memorization of isolated facts and manipulation of symbols to one that emphasizes conceptual understandings (Brownell, 1947; NCTM, 1989, 1991; Whitney, 1985) suggests a need for a change in instructional patterns and in the role of teachers. Teachers will move from dispensers of knowledge to facilitators of learning. A variety of instructional methods should be used in classrooms in order to cultivate students' abilities to investigate, to make sense of and construct meanings from new situations, to conjecture and generalize, and to use a variety of strategies to solve problems. Traditional teacher-led demonstrations and teacher-led discussion should be supplemented by (a) opportunities to work in small groups, (b) individual explorations, (c) peer instruction, and (d) whole class discussion among students with the teacher as moderator (National Research Council, 1989).

In addition to these overall recommendations for the teaching and learning of mathematics, there are specific recommendations for the teaching of algebra. It should be noted that the recommendations for algebra in the ninth grade were made with the assumptions that students had received the recommended instruction in grades five through eight. All students would, therefore, come to the ninth grade understanding how to bridge from arithmetic to algebra. This, of course, is not true for students currently enrolled in developmental algebra. Therefore, many of the standards directed to students in grades five through eight might reasonably be applicable to developmental algebra students.

In grades 5 - 8, the mathematics curriculum should include explorations of algebraic concepts and processes so that students can -

- understand the concepts of variable, expression, and equation;
- represent situations and number patterns with tables, graphs, verbalrules, and equations and explore the interrelationships of these representations;
- analyze tables and graphs to identify properties and relationships;

- develop confidence in solving linear equations using concrete informal, and formal method;
- investigate inequalities and nonlinear equations informally;
- apply algebraic methods to solve a variety of real-world and mathematical problems. (NCTM, 1989, p. 102)

While an acceptable level of computational proficiency is suggested for all students in grades K - 8, an important recommendation for students in grades 9 - 12 is that no student be denied entrance into the 9 - 12 curriculum because of a lack of computational facility. It is recommended, therefore, that no student will be denied the opportunity to study algebra because of a lack of proficiency in arithmetic skills (NCTM, 1989).

Two of the six standards of teaching mathematics (NCTM, 1991, p. 19) address discourse in the classroom. Discourse refers to the interactions among students and the teacher as they engage in tasks. As the facilitator, the teacher should orchestrate both oral and written discourse in ways that contribute to the students' understanding of mathematics. Several methods of doing this are recommended (NCTM, 1991, p. 35).

The teacher poses questions and tasks which will cause each student to become involved, and will challenge each students' thinking. As students respond to the questions, the teacher listens carefully to students ideas, sometimes asking students to explain and justify their ideas. The teacher decides which of the students' ideas should be pursued in length. Using the students' ideas as a springboard, the teacher decides when more information should be provided, when to clarify an issue, when to model, when to lead, and when to let a student struggle to come to a conclusion (p. 35). Throughout this discourse, the teacher is assessing student understanding by monitoring students' participation in discussion and a decision is made as to when and how to encourage each student to participate.

The students also has a role in the discourse. The teacher should promote discourse which allows students to listen and question both the teacher and one another. In orchestrating the discourse in the classroom, the teacher should allow the students to become active participates in the construction of their own mathematics. Students are encouraged to explore, make conjectures, and generalize. In this way, the teacher relinquishes the role of authority in the classroom and encourages the students to become their own authorities.

Conceptual Versus Procedural Instruction

An understanding of how procedural and conceptual knowledge relate to mathematical expertise is a subject of great concern to the mathematics education community. Unfortunately, there is some disagreement on the labels used to describe the two kinds of knowledge. Hiebert and Lafevre (1986) suggest that perhaps the most widely used distinction has been that between skill (procedural) and understanding (conceptual), but they also note that the debate is not new. In the early twentieth century, McLellan and Dewey argued for understanding while Thorndike endorsed the case for skill learning. Brownell (1947) took the position that the teaching of mathematics (arithmetic in particular) should be "meaningful," in contrast to "meaningless" (p. 257). More recently, Gagne (1983) seemed to place emphasis on procedural knowledge when he called for "automatization of computational skills" (p. 18), and suggested that practice (speed drills, conducted under competitive rules) will lead to automatization.

With the recent emergence of the constructivist theory of learning, procedural and conceptual knowledge in mathematics learning have again become a focus of attention. Comparisons have been made between syntactics and semantics (Hiebert & Wearne, 1988), textbook (procedural) and conceptual instruction (Hiebert & Wearne, 1992), instrumental and relational understanding (Skemp, 1982), and form and content (Byers & Erlwanger, 1984). Other labels sometimes used for procedural instruction are traditional instruction (Ball, 1990) and teaching for rote learning (Peterson, 1988).

In spite of the use of different terminology, however, underlying concepts seem common. Conceptual knowledge is sometimes defined as knowledge which is understood (Hiebert & Carpenter, 1992). It is rich in relationships and has as its basis a network of information through which existing knowledge and new knowledge are connected. It is through this connection that new knowledge becomes a part of the network (Hiebert & Lafevre, 1993). In contrast, procedural knowledge is referred to as a "sequence of actions" (Hiebert & Carpenter, 1992). It includes formal language, rules, algorithms for completing mathematical tasks, and symbol manipulation (Hiebert & Wearne, 1992).

As stated earlier, the goal of conceptual instruction is to impart understanding of content, so conceptual knowledge must, therefore, be learned with meaning (Hiebert & Lafevre, 1986). This understanding leads to a development of meaning for symbols and for good retrieval of procedures and information as well as a transfer of knowledge to novel situations (Hiebert & Wearne, 1992). The goal of procedural instruction is the learning of a sequence of actions which will lead to a desirable result.

Procedural instruction may or may not be taught with meaning (Hiebert & Lafevre, 1986), but if procedures are learned with meaning, they become a part of a network of knowledge, permitting access to other information in the network (Hiebert & Carpenter, 1992).

While it might seem that the goal of instruction is either procedural or conceptual knowledge, the mathematical literate person must possess both kinds of knowledge. One goal of instruction, therefore, should be the acquisition of both kinds of knowledge (Byers & Erlwanger, 1984; Hiebert & Lafevre,1986). The primary question is not "Which kind of knowledge is more important?" but "How do conceptual and procedural knowledge interact?" (Hiebert & Carpenter, 1992). A related question is "Should students be taught first to understand concepts, or to manipulate symbols?" A number of studies has addressed this question and investigated the effects of conceptually based versus procedurally based instruction.

Schoenfeld (1988) described case studies which were conducted as a part of a year-long study of the teaching and learning of geometry in a tenth-grade geometry class. Instruction in the class was termed by the researcher to be "well-taught" (p. 145) in that, for the most part, everything that took place in the classroom went as intended. The teacher followed the curriculum and ran the class well and the students scored well on standardized examinations. Schoenfeld concluded that although the teaching was "good" the results were "bad" (p. 145). As an example of why the results were deemed as "bad," Schoenfeld gives two examples. First, when students did constructions, the goal was to construct accurately; they made no connection between their constructions and the reasoning which made their constructions accurate. That is, the students learned how to construct by rote, without understanding the theory behind the construction. Second, students perceived that the form of the answer was what really counted, and even geometry proofs were committed to memory. As a result of his observations throughout the school year, Schoenfeld reported that the instruction focused on mastery of algorithmic procedures as isolated skills, and that there was little evidence of deep understanding of the mathematics being taught.

In a study conducted with 150 college freshman engineering students, Clement (1982) reported that when students were asked to solve for x in simple equations ($5x = 50$, $6/4 = 30/x$, and $9a = 10x$), 99%, 95% and 91% answered correctly. However, when the students were presented three problem situations and asked to solve (or write an equation which exemplified the conditions of the problem) the success rate fell substantially (93%, 63%, and 27%). Because analysis of the incorrect responses indicated that there was a strong pattern in the errors, the researcher was not inclined to contribute the mistakes to carelessness. His analysis caused him to conclude that the students had been successfully taught to manipulate equations, but that they failed to understand how equations are used to symbolize meanings.

In an investigation of links between instruction and cognitive change in mathematics (Hiebert & Wearne,1988), 29 students in Grades 4, 5, and 6 were given nine lessons designed to promote conceptual knowledge in adding and subtracting decimal numbers. Although the students in Grade 4 had not received previous instruction in the adding and subtracting of decimals, five fifth graders and all ten sixth graders had been previously exposed to syntactic instruction for working with decimals; that is, the decimal symbol had been introduced and time had been spent practicing rules for solving decimal problems. Prior to instruction each of the students was interviewed individually on direct measures (tasks which had been specifically taught) and transfer measures (novel situations). Six weeks after instruction, students were interviewed again and presented with alternate forms of the tasks given in the preinstruction interview. The delay of the postinterview was intended to allow for the effects of instruction that represented short-termed change to be eliminated.

Based on the interviews, student processes on each task were classified into syntactic execution, semantic analysis, or indeterminate. This classification was based on students' responses and explanations. Syntactic execution meant that the response was generated by manipulating the symbols in a learned procedure. Semantic analysis meant that students used meanings of the symbols to generate the responses. Indeterminate included cases of no response, guesses, and responses that were unclear and could not be explained by the student. In reporting results, the authors labeled syntactic and indeterminate as nonsemantic. On the postinterview, 100% of the semantic-produced responses on direct measures were correct and 85% of the nonsemantic-produced responses were correct. Of more interest, however, is the fact that across all transfer measures, 80% of the responses generated from semantic analyses were correct, while only 17% of the nonsemantic-produced responses were correct. The results on the transfer measures seem to be important since they indicate not only that semantic instruction leads to better transfer but also that the concepts learned through semantic instruction are retained for a longer period of time than those learned syntactically.

In a project conducted with 153 students in six first grade classrooms, Hiebert and Wearne (1992) investigated the effect of alternative instruction on the meaning of place value to 100 and two-digit addition and subtraction without regrouping. The primary focus of the project was the change from conventional textbook-based instruction to instruction that placed a greater emphasis on conceptual understandings before moving to procedural skills.

All students were pretested using a written group test in December, prior to instruction. The pretest (designated Test 1) contained 34 broad-ranged items designed to measure students' entry knowledge on content relating to the study. In order to assess the change after instruction, all students received two written tests. The first post-instruction test (designated Test 2) was administered in January after the first set of lessons and the second post-instruction test (designated Test 3) was administered in May after the second set of lessons. The two post-instruction tests contained three kinds of items: place value, two-digit addition and subtraction without regrouping, and two-digit addition and subtraction with regrouping. Because no students had received instruction on adding and subtracting before Test 2, the results on this portion of the test were reported together. Test 2 contained 14 items (8 on place value and 6 on addition and subtracting, both nonregrouping and regrouping) while Test 3 contained 20 items (6 on place value, 11 on addition and subtracting nonregrouping, and 3 on addition and subtraction, regrouping). None of the students had received instruction concerning the third type of test item during the school year.

The researchers collapsed the data into an alternative-instruction group and a textbook-based instruction group. The differences between the two groups were statistically significant for both tests on place value tasks with the alternative group having a higher mean. The researchers interpreted this as an indication of better understanding of the meaning of place value by the alternative instruction group. The mean on the two-digit addition and subtraction problems (Test 2, nonregrouping and regrouping and Test 3, nonregrouping) was higher for the alternative group, although not significantly so. The mean score on addition and subtraction with regrouping, which was not taught to either group, was significantly higher for the alternative group, indicating a transfer of knowledge to novel situations. The results on the tests indicated that both student understanding and transfer of knowledge is enhanced by conceptual instruction.

Schoenfeld's study with geometry students indicated that while students who are taught from a procedural perspective may score well on assessments which measure procedures only, they do not appear to be able to connect this knowledge to underlying concepts. Students who are taught to manipulate symbols in solving equations are able to solve equations, but are not able to translate problem situations into equations, and thereby use equations to help them solve the problem (Clement, 1982). Hiebert and Wearne (1986) presented a strong argument in favor of the view that mathematical competence is a result of connections between conceptual and procedural knowledge. In addition, students who were given instruction which placed a greater emphasis on conceptual understanding before moving to procedural skills in adding and subtracting decimals were able to score as well on direct measures and significantly better on transfer measures (Hiebert & Wearne, 1992) .

Throughout the studies, it appears that instructing for understanding of underlying concepts prior to procedural instruction allowed students to transfer the knowledge they gain to novel situations. All of these results support a major criticism of current mathematical instruction in which students are taught procedures without understanding (Romberg & Carpenter,1986; Resnick & Ford, 1981; Whitney, 1985).

The presence of conceptual knowledge has several benefits for procedural knowledge (Hiebert & Lefevre, 1986). Among them are: (a) developing meaning for symbols (Schoenfeld, 1986), (b) recalling procedures (Skemp, 1982), (c) using procedures (Silver, 1979), and (d) transferring (Brownell, 1947; Carpenter, 1986; Hiebert & Wearne, 1986). On the other hand, the presence of procedural knowledge also holds benefits for conceptual knowledge. Among these are: (a) providing tools for dealing with complex ideas (Skemp, 1982) and (b) applying concepts to solve problems (Hiebert & Lefevre, 1986). Mathematical literate students are those who know concepts, know symbols and procedures, and know how to relate them (Hiebert, 1984; Hiebert & Lefevre,1986)

Constructivism

Although the prevailing model of instruction in today's classrooms continues to be that of "absorption" (Schoenfeld, 1988, p. 148), in recent years there has been a move in mathematics education toward what is called a constructivist perspective in the learning and teaching of mathematics (Nickson, 1992). In the constructivist approach, mathematics is not a fixed body of knowledge which is taught under the assumption that learners absorb the content to which they have been exposed (Romberg & Carpenter, 1986). Rather, constructivists recognize that learners do not come to the classroom with a lack of knowledge, but rather bring with them much knowledge, some of it correct, and some not (Carpenter & Peterson, 1988). The assumption that meanings lie in words, actions, and objects independent of an interpreter is challenged by the constructivist (Cobb, 1988).

Furthermore, all teachers are guided by their understanding of students' mathematical realities and by their own mathematical knowledge (Cobb & Steffe, 1983). In the constructivist view, however, teachers must make a conscious attempt to see not only the student's actions but also their own actions from the student's point of view (Cobb & Steffe, 1983). To the constructivist, teaching is primarily a matter of communicating with the student. Teaching is no longer transmission of knowledge, but presentation of activities in which the students engage. In this manner, teachers influence the problems their students attempt to solve, and thereby the knowledge their students construct (Cobb, 1988).

From the constructivist perspective, teaching becomes an interaction between the teacher and the learner and between the learner and the content. The teacher acts with an intended meaning, and the students interpret the teacher's actions within their own mathematical realities, thereby constructing new knowledge. Each student brings to a situation different knowledge, and therefore, the new knowledge which is constructed will be different for each learner. The teacher initiates activities; the learner (a) reflects on these activities, (b) pulls patterns from them, (c) generalizes, and (d) conjectures (Cobb & Steffe, 1983). Student contributions to the discussion are acknowledged in view of the potential value which they may possess for further mathematical construction. Based on the student responses, the teacher recognizes those which have value, and initiates new activities to pursue in depth those ideas which have mathematical merit (Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, & Perlwitz, 1991). Thus, Cobb (1988) noted that teaching moves along a line from imposition to negotiation. If teaching is viewed as transmission of knowledge, then it is imposition, but if teaching is viewed as facilitation, then it becomes negotiation.

There are those who contend that, in addition to the construction of knowledge which students gain from their involvement with mathematical content, knowledge is also constructed from the social interaction between the teacher and the student and among the students themselves (Yackel, Cobb, Wood, Wheatley, & Merkel, 1990). When students are given the opportunity to interact with others, to verbalize their thinking, and to support and defend their positions, then new knowledge is constructed (Yackel et al., 1990).

Research examining teaching from a constructivist prospective has used two particular techniques: teaching episodes (e.g., Cobb, 1988; Lawler 1981) and the teaching experiment (e.g., Cobb et al., 1991). Most of the research on mathematical construction of knowledge has been conducted in the subject of arithmetic within the early grades using few subjects (Romberg & Carpenter, 1986). This fact has generated a criticism from Brophy (1986) who contends that many mathematics educators have a "limited and distorted view of the existing research on classroom teaching" (p. 323). Nevertheless, several such studies will be referenced here in the belief that what happens in these studies could in fact have implications for mathematics for other subjects at other grade levels.

Cobb (1988) reported an episode in which a single first grade student (Melissa) is observed in her interactions with her teacher in instruction concerning two addends up to ten. When Melissa displayed a reliance on direct modeling (counting on her fingers) in order to get the solution, the teacher intervened. Knowing that many children at the same conceptual level as Melissa count-on rather than count all items in both sets, the teacher made several highly directive interventions to motivate Melissa to subtract by counting backwards. Nevertheless, Melissa insisted on subtracting by direct modeling when asked to complete the sentence 15-3. When she was asked to solve it by counting backwards, Melissa hesitated, and then said she could not do it. After a move to an alternate activity for a short time the teacher again asked Melissa to complete sentences comparable to 13 - 4, which she again did by finger counting. Then the teacher presented the sentence 21 - 4, which Melissa could not do on her fingers. After muttering quietly to herself for a few seconds Melissa began to whisper "21, 20, 19, 18, 17." Presented with 32 - 5, Melissa again found the solution by counting backwards, although she struggled with the problem for almost 2 1/2 minutes before giving the correct solution.

Cobb made three observations in concluding that Melissa did not merely do as she was told, but did, indeed, construct a backward counting method that expressed her own concepts. First, a new method was not constructed until the situation was such that the old method would not work. Second, the amount of time it took the student to solve the two sentences infers that more was involved that recall. Third, the method she used differed from the one which she had been taught. She did not use any method of keeping track in finding the solution for 21 - 4, and she closed fingers as a way of keeping track when she solved 32 - 5. In fact, it appeared to the researcher that the struggle with 32 - 5 was in finding a way to keep track of her backward counting. Cobb's conclusion was that this episode characterizes Melissa as an "active constructor of knowledge who strove to overcome problems that arose as she interacted with the teacher" (p. 95).

In a major project which examined construction of mathematical knowledge in classroom settings, students from ten second-grade classes participated in a year-long study in which instruction was compatible with the constructivist theory (Cobb et al., 1991). The students in the study attended three schools which contained both project and nonproject classes. The project classes were taught by ten second-grade teachers who volunteered to participate in the project and used the instructional activities developed by the researchers. The eight nonproject teachers used a regular second-grade textbook as the basis for their instruction. At the end of the school year, the ten project classes were compared with eight nonproject classes on the mathematics portion of a standardized achievement test and on a computational proficiency test.

The instructional activities chosen by the researchers lent themselves to two types of classroom organizations. In the first type of organization, students worked in pairs for approximately 20 minutes while the teacher observed and interacted with the students. The students had at their disposal several types of manipulatives, but the decision to use the manipulatives lay with the students. At the end of the 20 minute period, the teacher pulled the class together for whole-class discussion of students' solutions and interpretations. In the second type of organization, instructional activities were used in a whole class setting. The teacher would begin these activities by posing questions, usually with the aid of the overhead projector and manipulatives. In both types of organizations, the teacher was the facilitator and encouraged dialogue in which solutions and interpretations could be defended and justified. In the project classrooms written work was not graded and there was no individual paper-and-pencil seatwork.

Two arithmetic tests were administered to both the project and the nonproject classes. The first, which was administered in early March, was the state-mandated multiple choice achievement test. This test had two mathematical subtests: a computation subtest and a concept and applications subtest. The second arithmetic test was the Project Arithmetic Test, which was developed by the project staff. It was comprised of two scales, Instrumental and Relational scales. It was possible to score well on the Instrumental part of the test by using computational algorithms without conceptual understanding. In contrast, the Relational scale contained items designed to assess students' conceptual understanding of place-value numeration and computation in nontextbook formats. An example of a place-value numeration item in nontextbook format is "What number do 12 ones and 3 tens make?" (p. 15) In contrast to "How many tens in 28?" (a textbook format problem). An example of a computation item on the Relational scale was a missing two-digit missing addend items presented in a horizontal sentence.

Analysis on the two arithmetic tests revealed that while there were no significant differences in the scores on the computation subtest of the achievement test, there were significant differences in the scores on the concepts and applications subtest, with the students from the project classes scoring higher. In addition there were no significant differences in scores on the Instrumental section of the test, but the differences in the Relational section were significant, with the higher scores belonging to the project classes. The researchers concluded that project students developed a higher level of reasoning in arithmetic than did nonproject students.

At the core of constructivists' argument is that students must construct their own mathematical knowledge even if they have been instructed in a traditional manner (Cobb, Yackel, & Wood, 1992). Cobb, et al. (1992, p. 28) maintained that "the central issue is not whether students are constructing, but the nature or quality of those constructions."

Byers and Erlwanger (1984) reported the results of an interview which illustrate this point. The interview involved a twelve-year-old, grade-6 pupil, Benny. He was deemed a "good student" (p. 272) who had manufactured his own rules, which he applied consistently, some of which were valid and some of which were invalid. The point of interest is that Benny did not tie things together; he did not connect content and rule. The rules which were investigated in this particular interview were those pertaining to adding fractions, converting fractions into decimals, and adding decimals.

While Benny had discovered that the same answer could be expressed in different forms, (1/2 and 2/4; 4/4 and 1) he had generalized what the authors termed as his own "theory of content;" that is, the correct answer to a problem depends on how the problem is worked. For example: 2 + 3 is 5, but 2 + .3 is .5 unless it is done with models, then it is 2.3. If it is done with fractions (2 + 3/10), then would give 2 3/10. It is apparent that Benny did not connect mathematical form with content, and that he had fundamental misconceptions regarding the nature of mathematics.

In summary, the teaching episode reported by Cobb (1988), the interview reported by Byers and Erlwanger (1984), and the teaching experience reported by Cobb et al. (1991) indicate that students do construct their own mathematical knowledge based on experiences, both past and present. The constructions they make may or may not be correct. The implications for mathematics instruction are important. The constructivist perspective calls for teachers to "become mature, autonomous professionals who take responsibility for the development of their own practices" (Cobb, 1988, p. 101). The project of Cobb et al. (1991) is significant in that it illustrates how conceptual instruction from the constructivist perspective can be successfully implemented in the classroom.

In conclusion, it can be said that the constructivist perspective of teaching is learner-focused in that it centers around the "learners' personal construction of mathematical knowledge" (Kuhs & Ball, cited in Thompson, 1989). The view of the constructivist is that learning is not absorbed, but is constructed; it is not direct, but indirect. The teacher is a facilitator and stimulator of student learning, providing interesting situations for students to investigate which will provoke them to think. For the constructivist, learning is a joint venture between student and teacher.

Algebra Instruction

When students move from arithmetic to algebra, many of their difficulties center on two areas: (a) the shift of a set of conventions different from those used in arithmetic and (b) the meaning of letters. Since elementary algebra can be viewed as generalized arithmetic, students' prior experiences with the structure of numerical expressions have an important effect on their success, or lack of success, in making sense of algebra (Kieran, 1989).

One problem arises because arithmetic students view operational symbols differently from the way they are used in algebra (Ginsburg, 1977). They do not view $3 + 4 = 7$ as a true statement, but as $3 + 4$ makes 7, and the "+" and "=" are calls for action. While the "+" seems to call for the operation, the "=" says "put down the answer." When Kieran (1979), in case studies of six, seventh grade students, asked them to write an example in which they would use an equals sign, each example contained an operation with two numbers on the left of an equals sign and the answer on the right.

A second problem which elementary algebra students have is with the meaning of variable and its use in equations. Rosnick (1981), in an extension of a body of research by the Cognitive Development Project at the University of Massachusetts which focused on students' ability to translate English sentences into algebraic expressions, studied the misconceptions which students have concerning the use of letters in equations. One of the problems on which much of the original research (Clement, Lochhead, & Monk, 1981) was based is the Students and Professors problem. The problem reads as follows:

> Write an equation, using the variables, S and P to represent the following statement: "At this university there are six times as many students as professors." Use S for the number of students and P for the number of professors.

Of a group of 150 entering engineering students at the University of Massachusetts, 37% were unable to write the correct equation, S = 6P, in any form. Taped interviews supported the belief that students considered the S as a label rather than the number of students, and they would read the equation 6S = P as "there are six students for every one professor." Rosnick gave a version of the Student and Professor problem to 33 sophomore and junior business majors in his statistics course and 119 students in a second-semester calculus course designed for the social sciences. In this version the students were given the equation S = 6P and asked to respond to the questions "In this equation, what does the letter P (S) stand for?" by choosing from multiple choice selections. Over 40% (43%) of the students were not able to pick "number of professors (students)" as the only appropriate answer. Again, this indicated that students viewed the letters as labels that refer to concrete entities. Furthermore, over 22% chose the answer "S stands for professor." Rosnick hypothesized that students who chose "S stands for professor" believed 6S = P was really the correct equation. In fact, he hypothesized, they believed it so strongly that when presented with S = 6P, they assumed the variables have been interchanged, and therefore, S does stand for professors.

Wagner (1981) investigated students' understanding of relations using the conservation methodology devised by Piaget. A conservation-of- equation task was given to 29 students (15 middle school and 14 high school) with a wide range of achievement levels. The students were asked to view two equations which both read 7 x W + 22 = 109 and respond to the question "Are these two statements the same?" When the student seemed satisfied that the statements were the same, the W in one of the statements was changed to N, and the student was asked, "If the number was found which would make each statement true, which would be larger, W or N?" Whatever answer the student gave, the interviewer asked, "Why?" A response that W and N would be the same represented conservation of equation. Saying that one or the other was larger was classified as nonconserving, and a conflicting answer (e.g., "I'd have to figure it out to see." or "They might be the same.") was considered a transitional response. Of the 29 interviewed, 13 were classified as nonconserving, 5 as transitional, and 11 as conserving. For the students with no formal algebra background, 9 were nonconserving, 3 traditional, and 2 conserving; for students with formal algebra background, 4 were nonconserving, 2 transitional, and 9 conserving. While there was a strong association between mathematical background and responses, 6 out of 15 students who had formal algebra were either nonconserving or transitional. This is an indication that even students with formal algebra experience may not have a clear grasp of the fact that a relation does not depend on the letter used.

After being introduced to the notion of using letters to represent numbers, the next topic for an elementary algebra student is usually operating on these letters in the context of simplifying expressions. Getting the answer becomes a problem for students when they are faced with simplifying an expression like 5a + 2b (Booth, 1988). In this case, 5a + 2b indicates the procedure as well as the answer, and the student may have difficulty accepting the lack of closure that is evidenced.

Chalouh and Herscovics (1988) conducted a teaching experiment investigating the cognitive obstacles children have concerning algebraic expression. The subjects for the study were six children (12 to 13 years of age) who had not had formal algebra experience. The teaching outline employed three types of problems based on geometric representation. In the first type, the students were asked to represent the total number of dots in an array for which one dimension was shown; in the second type, students were asked to represent the length of a line segment part of which was hidden; and in the third type, students were asked to find the area of a figure with only one dimension known. By using these models, the students were led to generate algebraic expressions moving first from placeholder (box) to letter representation when one quantity was hidden, to letter representation which involved a hidden quantity, to letter representation involving an unknown quantity.

Finally, the students were introduced to algebraic expressions with multiple terms. It was found that students were able to develop meaning for expressions like 2a + 5a, but that most of the children were unable to interpret this as 7a. The results seem to indicate that developing meaning for an algebraic expression does not automatically extend to simplifying the expression. However, the study seemed to indicate that students quickly moved to an awareness that the conventions of arithmetic are not the same as those of algebra, and sometimes asked the researcher if an answer was wanted "in algebra."

Booth (1984) found similar results when seventh and eighth grade students were asked to write algebra expressions representing lengths of lines which were diagramed with a part of the diagram not shown. A typical problem was: "A spaceship travels in 'stages' that are all the same distance long. If each stage is 11 light-years long, what could you write for how far the spaceship goes in y stages?" (p. 21). One of the students was able to give the correct expression 11y, but she was concerned that she had not given a "proper answer."

Whitman's study (cited in Kieran, 1989) of 156 seventh-grade students researched the relationships between formal and informal methods of solving equations. Students were taught in one of three ways: (a) intuitive techniques only, (b) formal techniques only, and (c) intuitive techniques followed by formal techniques. Intuitive techniques might include cover-up, and/or reasoning through a solution. For example, if the equation $69 - 96 / (7 - b) = 37$, the learner might cover-up $96 /(7 - b)$ to reason $69 - 32$ gives 37, then reason that 96/what gives 32. Since 96/3 gives 32, 7 - b must equal 3, and therefore b must equal 4.

Formal techniques would include multiplying both sides by 7 - b, distributing 69 and 37 times 7 - b, combining terms, adding 37b to both sides, subtracting 387 from both sides, and dividing both sides by -32 to get b is 4. Students who learned to solve equations only intuitively performed better than those who learned both ways, one soon after the other, but students who learned to solve only by formal methods performed worse than the other groups. Whitman concluded that students who are taught formal techniques seem to lose their initiative to solve by informal methods.

Kuchemann (1983) hypothesized that because students cope more readily with positive whole numbers than with negative numbers, fractions, and decimals, what are sometimes seen as problems with formal methods in solving equations might well be problems in coping with unfamiliar numbers. In order to investigate his hypothesis, Kuchemann used ten equations which he divided into three groups. The equations in Group 1 ($a + 5 = 8$, $3e + 7 = 28$, and $24 - 5k = 9$) were designated as easily solved by informal methods (e.g., trial and error, inspection, cover-up, and intuition); their solutions were all positive whole numbers. Group 2 contained equations which the researcher considered more difficult ($14 / (e + 2) = 2$, $27 + 4e = 15$, and $8t = 9$) but which could perhaps still be solved by informal methods, although one of the solutions was negative and one was a mixed number. Group 3 were designated the most difficult equations ($3e + 5 = e + 13$, $26 = 11 - r$, $4 / n = 3$, and $29 = 14 - 5n$). The equations in group three, he argued, were more easily solved by formal methods.

The three groups of equations were given to two samples of students. The first sample contained 70 14-year-olds of above average ability. The second sample contained 200 mostly 15-year-olds who were "generally more able than the first sample" (p. 18). As the researcher had projected, the groups of equations did prove to be differentially difficult, with the percentage of students who solved the equations correctly ranging from a high of 100% for item 1 to 11% for item 10. Kuchemann concluded that the evidence supported the hypothesis that students favor informal methods to formal methods. He suggested that if students are to be taught formal methods, it should be only after their own methods are shown to be inadequate. This was an interesting study, but some of the procedures were not clear. For example, it was not clear how the researcher decided which technique was used by the students. There were indications that interviews were used, but this was not made clear.

"Change Side-Change Sign" (transposing) is an equation solving model which has been investigated by O'Brien (cited in Kieran,1989) and Kieran (1988). O'Brien studied 23 third-year high school students who had previously learned some algebra and, at the beginning of the study, were already solving equations by using the "Change Side-Change Sign" method. The subjects were divided into two groups. The researcher attempted to teach one group the meaning for equations and for the manipulations performed by using concrete materials, that is, by removing objects from both sides or adding objects to both sides of the concretely modeled equation. The other group was taught the meaning for manipulation of symbols by a generalization of the addition/ subtraction relation (e.g., $2 + 3 = 5$ implies that $2 = 5 - 3$). Near the end of the study, the second group was also taught to solve equations by performing the same operation to both sides of the equation. O'Brien found that the second group became better equation solvers than did the group who were exposed to the concrete materials. The fact that the students who were exposed to solving equations by concrete materials were already solving equations by transposing seems to be of significance in this study. It may indicate that students who are taught to solve equations initially by transposing do not understand the symmetry of an equation, and have a difficult time imposing meaning to the procedures. This conclusion seems to be supported by the following study by Kieran.

Kieran's (1988) work with six average ability seventh graders who had no previous algebra instruction showed that on simple equations, the students preferred two distinct methods of solving equations: (a) arithmetic methods, like substitution and known number facts and (b) transposing. In the teaching experiment the students were taught the formal method of doing the same thing to both sides of the equation, first with arithmetic equalities, and then with algebraic equations. It was found that those students who initially favored transposing were not able to make sense of the procedure of performing the same operation to both sides of an equation. This suggest that, although transposing is often referred to as a shortcut to performing the same operation to both sides of an equation, students who transpose do not understand that they are operating on both sides of the equation. The procedure of transposing does not emphasize the symmetry of an equation as operating on both sides does (Kieran, 1988). That is, transposing is a manipulation without understanding that an equation is a statement of equality.

To summarize, much of the research on algebra instruction has centered around four specified topics: (a) difficulties students have moving from conventions in arithmetic to those in algebra, (b) cognitive obstacles which students have when learning to attach meaning to letters and expressions, (c) various techniques students use in solving equations, and (d) difficulties which students have in generating equations which will show relationships stated in word problems. Given that one of the primary recommendations in the Standards for algebra is instruction which provides explorations of algebraic concepts so that students can understand the concepts of variables, expressions, and equation, the focus of this research seems appropriate. Additionally, none of the reported studies in algebra instruction involved subjects who were postsecondary, and therefore, the need for research with these students is needed.

Problem Solving in Algebra

It is well known that one of the areas where students have the most difficulty in algebra is in writing equations which represent the relationships stated in typical word problem situations. Traditionally, word problems in algebra have been solved either by the direct translation method or by a principle-driven approach. The direct-translation approach involves a phrase-by-phrase translation of the word problem into numbers, variables, and operations. To be done correctly some semantic knowledge is needed; however, students generally use nothing more than syntactic rules. In the principle-driven approach, the student is encouraged to classify each problem into a specified type of problem (Hinsley, Hayes, & Simon,1977). A popular strategy for solving problems is to use the solution of an analogous problem. However, one of the limitations of this strategy is the fact that students often are not able to recognize analogous problems (Reed, Dempster, & Ettinger, 1985). Reed et al. found that the categorization of problems facilitates the use of analogy.

Furthermore, research has indicated that many students appear to have much difficulty solving certain types of fairly simple algebraic word problems. The difficulty seems to lie in their inability to translate written language to mathematical language. Often when students are asked to read a problem stating a relationship and then to write an equation which expresses this relationship they write the reverse of what is expected

(Clement, Lochlead, & Monk, 1881). The misconceptions do not seem to come from misreading but rather from misconceptions concerning the structure and interpretation of algebraic statements.

In an effort to implement the Standards, an alternative first-year- algebra project was developed in California (Kysh, 1991). One of the learning goals of the project was to provide experiences in which students would become more aware of their own thinking about problems. As a vehicle for this goal, students were given what was described as memorable problems. These problems were investigations, sometimes real situations, that required the students to pull together several of the mathematical concepts which they were studying.

A second emphasis was the use of variables to represent problems algebraically. Often, typical word problems from the textbook were used. Students were shown how to use several problem solving strategies (e.g., make a table, guess and check) to move from specific examples to an algebraic equation. Although there had been initial apprehensions among some of the project teachers concerning students' acquisition of algebra skills, it was found that there was no noticeable difference between the algebra skills of project students and those from traditional classes. Further, the students from the algebra pilot classes were far better at reading and tackling problems.

It is important to note that an overall standard for all of mathematics, including algebra, is the projection of mathematics as problem solving. In addition, one of the topics recommended to receive decreased instructional attention (NCTM, 1989) is categorized word problems; for example, coin, digit, and work. This recommendation was accompanied by one which urges increased attention to real-world problems to motivate and apply theory.

Developmental Algebra

A review of the literature that focused on developmental algebra revealed few studies and no studies addressing specific content in developmental algebra. The few studies which were found are discussed here.

In a discussion concerning what mathematics should be taught in developmental programs, Garafalo (1988) asserted that the role of developmental algebra as a pre-course for students heading to calculus warrants investigation, and that consideration should be given to including topics which might prepare students for other mathematics courses in addition to or instead of calculus. He pointed out that many of the topics which the

Standards recommended be de-emphasized in secondary school should also be de-emphasized at the developmental mathematics level. Although the Standards were written for the secondary curriculum, Garafalo asserted that a careful and thorough analysis of the curricula served by developmental algebra courses would reveal similar topics, and even perhaps more, which should receive less emphasis. As with the recommendations for secondary school algebra, problem solving is projected as the major focus for the developmental algebra curriculum.

Developmental programs differ from college to college, and two varying programs will be discussed here. Haney and Testone (1990) described an after-semester workshop for developmental students enrolled in either Elementary or Intermediate Algebra who had not met the proficiency level at the end of the semester and therefore were in danger of receiving a grade of "U" (unsatisfactory). Before the after-semester program began, these students had the option of repeating the course for another full semester or dropping out of the course. At the time of the writing, Haney and Testone reported that the workshop for Elementary Algebra students had been taught four times with a success rate of 100%, and the workshop for Intermediate Algebra students had been held three times with a success rate of 80% or better each time. A follow-up study showed that the students who participated in the workshop had the same success rate in the next level course as the students who did not need to attend the workshop. Furthermore, workshop participants had a greater sense of accomplishment and a more positive attitude toward mathematics than they did before participating in the workshop.

Developmental courses are often taught in mathematics laboratory (math lab) situations. Robinson (1990), in a two-year study, compared the traditional lecture method to the math lab approach. In the first year of the study, all students were taught by the traditional lecture method; in the second year, students who needed both developmental arithmetic and developmental algebra were placed in a math lab format, and students who needed only algebra were placed in the lecture format. Those students taught by the lab format in the second year scored significantly higher ($p < .05$) on post course assessments than students taught by the traditional lecture method.

Berenson, Carter, and Norwood (1992) studied students who had been placed in developmental algebra as college freshman. In the two-part study, factors which were thought to contribute to student success in developmental algebra were investigated. Data from 263 students who were placed in remedial algebra as college freshman were collected on three academic variables (Scholastic Aptitude Test, high school grade point average, and the Group Assessment of Logical Thinking), two affective variables (attitude toward mathematics and anxiety), and mandatory class attendance policy. Moderate correlation between the six variables was not found, indicating that no predictive model existed for this group. A second part of the study compared the final grade for the 1988 class who attended class under a mandatory attendance policy and those in 1987 who did not have the attendance policy. No significant different was found and it was concluded that the mandatory attendance did not affect student scores.

Higbee and Dwinell (1988) studied the relationship between affective variables and academic success among high-risk college freshmen. The affective variables investigated were the student goals, learning styles, mathematics and test anxiety, other sources of stress, and level of development on achievement among developmental studies students. Among the findings were (a) developmental students were likely to prefer a hands-on learning style and learning through interaction and visual stimuli rather than through traditional lecture and text and (b) stress and other affective variables may account for a greater proportion of variance in first quarter grades than does high school grade point average or Scholastic Aptitude Test scores. The authors concluded that affective variables are significantly related to performance among freshman who take developmental studies.

In summary, the literature concerning developmental algebra is limited. However, there seems to be evidence that teaching methods which are oriented toward conceptual ideas (i.e., labs, stress-free settings, interaction, and visual stimuli) produce better results in developmental studies. This evidence seems to support the legitimacy of the treatment which the subjects in this study received. The fact that no studies were found which directly investigated instructional styles in developmental algebra makes the study significant.

Relationship Between Computational Skills and Success in Algebra

Several studies investigated the relationship between computational skills and success in order to establish criteria for a student's admittance into elementary algebra. However, these studies involved using various scores of eighth-grade students in order to make decisions about their placement for the ninth grade.

Barnes and Asher (1962) conducted a study in which the subjects were 192 ninth-grade students with six different algebra teachers from two junior high schools. Data included 11 variables available from students' school records. These 11 variables included (a) seventh-grade mathematics grade, (b) eighth grade mathematics grade, (c) grade equivalent on the arithmetic part of the Iowa Every-Pupil Tests of Basic Skills for the seventh grade, and (d) grade equivalent on the arithmetic part of the Iowa Every-Pupil Tests of Basic Skills for the eighth grade. It is reasonable to assume that each of these four measures would have been heavily based on the students computational skill. Multiple regression was used to establish the correlations between each of these variables and the ninth-grade algebra grade, which was taken as an indication of success in algebra. The best single predictor of success in algebra for this school system was the eighth-grade mathematics grade. Actually, an equation which used eighth-grade mathematics grade and the grade equivalent on the arithmetic achievement test given during seventh grade yielded a multiple correlation of .6245 which was only a slight reduction from the maximum correlation .6600 using all the variables.

The findings reported above were supported by a study by Rothenberger (1967) who found that if a single variable is to be used to predict student success in algebra, performance in eight-grade mathematics would be the best single predictor. Several other studies have reported positive corrections between arithmetic achievement and success in elementary algebra (Mogull & Rosengarten, 1972; Sabers & Felt, 1969; Taylor, Brown, & Michael, 1976).

Summary

In addition to a discussion aimed at showing how this study was designed around recommendations endorsed by the Standards, the literature review was divided into six specific areas: (a) conceptual versus procedural Instruction, (b) constructivism, (c) algebra instruction, (d) problem solving in algebra, (e) developmental algebra, and (f) the relationship between computational skill and success in algebra. The current focus in research in mathematics is on conceptual instruction from the constructivist viewpoint. This perspective asserts that the teacher of mathematics is not a transmitter of knowledge, but a facilitator of learning. Much of the research in algebra instruction has centered on difficulties students have in understanding the meaning of variable and equations. Because of a lack of understanding of the meaning of equation, generating equations which exemplify specific problem situations has emerged as one of the main difficulties of algebra students.

This review of literature indicates that another study on the outcomes of procedural versus conceptual instruction is justified, specifically at the level chosen for this study.

Many of the studies which have investigated student performance following alternative instructional methods have focused on arithmetic with young children (e.g., Hiebert & Wearne, 1988). While there has been much research concerning the teaching and learning of algebra, the subjects have typically been drawn from grades K - 12; therefore, research is needed which focuses on subjects at the postsecondary level, especially students of developmental algebra.

CHAPTER III

METHODS AND PROCEDURES

The study involved instruction to introductory developmental algebra students at a community college in a midsize city in North Carolina. The purpose of the experiment was to investigate the differences in achievement of students who received two different methods of instruction. As explained earlier, the research questions were:

1. Do developmental algebra students who are exposed to instructional strategies which encourage conceptual understanding achieve differently on a skill-based content test from students who are exposed to procedural instruction?
2. Do developmental algebra students who are exposed to instructional strategies which encourage conceptual understanding achieve differently on a transfer of knowledge test from students who are exposed to procedural instructional?
3. Do developmental algebra students who are exposed to instructional strategies which encourage conceptual understanding achieve differently on an effectance motivation scale from students who are exposed to procedural instruction?
4. Do developmental algebra students who are exposed to instructional strategies which encourage conceptual understanding achieve differently on a confidence in learning mathematics scale from students who are exposed to procedural instruction?

Subjects

The subjects for the study were students registered in the four developmental elementary algebra (MAT 003) classes taught in the summer session. Two of the classes were day classes, and two were night classes. Each of the two treatment groups consisted of a day class and a night class (see Table 1).

Table 1
Number of Students in Each Section

Section	Began study	Completed study
Experimental		
Night	20	16
Day	24	20
Control		
Night	21	12
Day	21	17

The reasons there is a discrepancy between the number of students at the beginning of the study and the number whose data was used are varied. From the night experimental class, two of the students did not have the Assessment and Placement Test (APT) score; a third student missed $Test_1$ because he was absent, and one student dropped the class. From the day experimental class, one student did not have the APT score, two students did not take $Test_2$, and one student dropped the class.

The night control class had eight students who had not taken the APT test. As has been previously stated, the APT is a placement test given to students who make application for admittance into a specific program of study. This class had an unusual number of students who were taking the class as special credit students, and the APT is not a requirement for this classification of student. Since these students did not have scores for the test used as the covariate, their scores were not used in the data sets. One student from this class did not take $Test_2$ and therefore was not used in the study. The day control class had 2 students who had not taken the APT, and two students dropped the class.

The 65 students whose scores were used in the data analysis each had an APT score and scores on $Test_1$ and $Test_2$. Five students who did have complete sets of tests scores did not complete the attitudes scales because they were absent on the day the scales were completed. Three of these were from the day experimental class, one from the night control class, and one from the day control class. Therefore, there were 33 students from the experimental and 27 from the control group whose responses were used in the analysis of the attitudes scales.

Design

Since random assignment of subjects to classes was not possible, the study employed a nonequivalent control group design. Although students could not be randomly assigned to the classes, they were not assigned to MAT 003 classes in any systematic way. Rather, students chose classes which would best fit their schedules. The assignment of classes to control and experimental groups was random (i.e., toss of a coin) within the constraints presented by the day/night situation. Two classes served as the experimental control group and two as the control group.

Procedure

Before the study began, the researcher met with each of the four instructors to discuss the plans for completing the project. Instructors agreed to allow the researcher to provide the instruction in their classes for a four-week period. It was agreed that instruction would run from the beginning of Chapter Two of the textbook (Wright & New, 1990) through the first two sections of Chapter Four.

Because classes were not synchronized, instruction for the study did not begin in each class on the same day; however, the same amount of time was devoted to the study in each class. Each of the night classes met two nights a week. One night class (experimental) met on Monday and Wednesday nights from 8:00 p.m. until 10:20 p.m. with a 15-minute break at 9:00 p.m. The experimental day class met from 12:20 p.m. until 1:10 p.m., Monday through Friday. One class (control) met on Tuesday and Thursday nights from 5:25 p.m. until 7:45 p.m. with a 15-minute break at 6:30 p.m. The control night class met from 8:00 a.m. until 8:50 a.m., Monday through Friday. Each class contained a total of 250 minutes per week.

At the researcher's initial meeting with each class, students completed a consent of human subjects form as required by the university (Appendix A). In addition, the university required that an oral presentation describing the research be delivered to the students prior to the beginning of the study. A copy of the oral presentation was signed and dated by the researcher (Appendix B). Each student was also asked to complete the demographic survey (Appendix C).

The researcher was the instructor of each of the four classes for a four-week interval. In order to assess the accuracy of implementation of the treatment, each lesson was audiotaped. The researcher later listened to the tapes and recorded any discrepancies between written lesson plans and the implementation of the lessons.

After six lessons, students were given $Test_1$, the first content test, termed the midtest. At the end of the 4-week period, the second content test $Test_2$, termed the posttest, was given (see Appendices G & H).

The length of the study was four weeks within the 10-week summer quarter. There were 18 50-minute lessons, with two 50-minute test periods. This resulted in 20 sessions for the day classes (18 instruction sessions with two test sessions). The schedule for the night classes was altered on the two nights that the tests were given. The test was given for the first 50 minutes, with a 15-minute break following the test. After the break, the class session resumed with instruction.

On the final day the researcher was in the class, students were asked to complete a 24-item attitude scale. The scale was designed to measure self-confidence as a learner of mathematics (12 items) and effectance motivation (12 items).

Because many of the items on the content tests were skill-based in nature, there was little opportunity to observe how the students arrived at their answers. In order to assess whether there was a difference in the ways students approached problems, two students from each class were interviewed during the week following the posttest. The purpose of the interviews was to elicit explanations from the students concerning the strategies they used to solve the problems on the test and similar problems.

Treatment

Two different instructional styles were used. The control group received procedural instruction, which stressed skills and rules for manipulating symbols. The experimental group received instruction which was designed to build conceptual understanding. Lesson plans for both groups are included in Appendices D and E.

Instruction was planned around exercises in Introductory Algebra (Wright & New,1990), which was the adopted text for the course. A course outline developed by members of the college mathematics department delineated the topics to be covered in the course. From this outline, the units chosen for study were signed numbers, solving first-degree equations, and exponents. The outline of these units included operations with signed numbers, absolute value, simplifying and evaluating expressions, solving first-degree equations, writing algebraic expressions, solving word problems, solving for any term in a formula, using formulas to solve application problems, and exponents.

Because the study was focused on the effects of a single change in instruction (i.e., the change from procedural instruction using the textbook as guide to conceptual instruction), other variables (e.g., content covered, homework exercises assigned) were kept as constant as possible. The control group received lecture, with little opportunity for dialogue, so itwas easy to monitor the kinds of examples given. In the experimental group, however, there was more interaction and dialogue between teacher and student, and among students, so it was not as easy to control the particular examples used. As much as possible, the same examples were used in each of the classes.

For the control group, the instructor was the giver of knowledge, and the student was the receiver. Definitions of new concepts were given to the student, new rules were given, problems using the new rules were demonstrated, and students were asked to do the same kind of problem which had been demonstrated. Practice exercises were assigned as homework. At the beginning of the next lesson, an opportunity was given to the students to ask questions on the homework before new material was presented. The instructional style was not completely lecture, as the students were given every opportunity to ask questions on examples, and immediate feedback was given to the students on guided practice problems. However, even with these opportunities, there was a limited amount of interaction between student and instructor.

Instruction for the experimental group differed in that new content was presented in situations for the student to investigate. Through discussion of ideas and justification of generalizations, it was intended that students would build new knowledge, and gain conceptual understanding before learning procedures. This instructional style included several different teaching strategies; for example, demonstrations with algebra tiles, problem solving, generalization, and guess and check. Dialogue among students and between student and instructor was encouraged and there was much interaction between the instructor and the students, as well as among the students themselves. The treatment was similar to that used by Cobb et al., (1991) and Heibert and Wearne (1992).

In order to demonstrate the differences in instructional styles, two specific topics have been chosen. The first illustration deals with adding signed numbers; the second deals with developing meaning for term and expression.

Signed numbers.

For the control group, adding signed numbers was the third lesson which dealt with the concept of signed numbers. The previous two lessons had (a) developed the concept of positive and negative numbers and their location on the number line and (b) introduced the concept of absolute value of a number and solved simple absolute value equations.

The objectives of this lesson were that the students would learn to add signed numbers and to determine if given numbers were solutions for specified equations. The format of the lesson closely followed the textbook presentation. The lesson began by going over the previous homework assignment which had covered problems on absolute value. Questions concerning these exercises were answered.

The lesson covering new content began with a demonstration by the instructor of adding signed numbers by jumps on the number line which had been drawn on the chalkboard. As each of the different possible combinations of signed numbers were demonstrated, the rule used was given. After each combination was demonstrated, all the rules were displayed on the overhead. While the rules were displayed, several examples of each were given by the instructor, each time pointing out which rule was used. In order to give students an opportunity to use the rules, exercises from the text were used as oral exercises; individual students were asked to answer the exercise, and then give the rule they used to get the answer. The instructor answered questions as needed.

In order to address the second objective of the lesson the instructor demonstrated substituting a given signed number in a specified equation to determine if the given number was a solution to the equation. Several examples were demonstrated, giving the students opportunity to ask questions in order to clarify their understanding. The lesson was closed by going over the rules for adding signed numbers, and again the rules were displayed on the overhead. Practice problems from the textbook were assigned for homework.

The experimental group had been exposed in a previous lesson to combining signed numbers by playing a spinner game which required the player to add points if the spinner stopped on an even number and to deduct points if the spinner stopped on an odd number. Although the rules of the game referred to "having points" or "being in the hole" the students quickly moved to referring to their scores as "positive" or "negative." This group had also had a lesson in which they developed the meaning for absolute value and were able to give solutions to simple absolute value equations. The objectives for this lesson were for students to develop rules for adding signed numbers, and to be able to verify if a specified number was the solution to a given equation.

After reviewing absolute value and going over homework problems, this lesson began by displaying on the overhead green disks and red disks, one color representing positive numbers and one color representing negative numbers. By consensus, it was decided that green would be positive and red would be negative. After representing several examples of opposites, it was established that a number combined with its opposite equals zero. After this was established, students were asked to provide situations for discussion (e.g., four reds and six greens). By matching opposites, and taking out the zeros, students were able to get the sum. Students were asked to give all possible situations (i.e., positive plus positive, positive plus negative with more negatives, positive plus negative with more positives, and negative plus negative).

Finally, after examining all possibilities, students were asked to write in words how they would tell someone to get the sum of signed numbers. They were encouraged to talk with a neighbor as they developed the rules. Coming back together as a whole class, the "rules" were discussed, and after reaching consensus, they were written on the chalkboard. With the rules they had established on display, individual students were then asked to give answers to exercises in the text. Discussion followed concerning which rule was used, and why it was used.

To address the second objective, an equation, $x + (-7) = 10$, was written on the chalkboard and the students were asked if -3 was a solution to the equation. Students gave suggestions about how to determine if the given number was a solution. Several examples were discussed. The lesson was closed by asking students to give different situations which could arise in adding signed numbers and to explain how they would get the answers. Practice problems from the text were assigned from the text.

<u>Terms and expressions</u>.

This lesson on terms and expressions was the lesson following the first content test. It began the chapter in the text which presents the skills required to solve linear equations. The lesson format for the control group followed closely the presentation of the text. The objectives of the lesson were (a) to learn what terms and expressions are, (b) to learn to simplify algebraic expressions by combining like terms and, (c) to evaluate expressions for given values of the variable.

For the control group, the class began by a display on the overhead of examples of terms, like terms, coefficients, and expressions. Students were told what is meant by "like terms" and were shown how to combine like terms by adding or subtracting coefficients. The instructor did several examples of combining like terms on the chalkboard and students were given the opportunity to ask questions if necessary. Through guided practice, students were asked to work a problem, and then it was explained to them in order to clarify any questions.

Objectives (b) and (c) were addressed by instructor demonstration. Several examples of each of the types of problems they would encounter on their homework exercises. Practice problems from the textbook were given for homework.

The objectives for the experimental group were the same as those for the control group. The experimental classes were introduced to a geometric interpretation of terms and expressions. The first activity was a review of area of rectangle by use of a coordinate grid transparency on the overhead. Several rectangles were drawn on the grid, and students were asked to give the area and to explain what is meant by the area. Several arithmetic examples are given so that students were confident of what was meant by area and how to find area of rectangle.

The second activity reviewed additive inverses by using two different colored tiles of the same dimension. This was to remind students that $a + (-a) = o$ for all real a.

Next, overhead algebra tiles were used but without the grid so that sides could represent variables. By discussion, meaning was established for tiles of different dimensions and different colors. By modeling different expressions, students were able to connect the model and the symbols. For example, a term was modeled, and students were asked, "How would you write this expression is symbols?", or a term was given (i.e., $3x^2$) and students were asked to draw a picture of the model.

Finally, several terms were modeled, and students were asked to represent them in symbols, and then to decide if there was a way to represent the same value in fewer pieces. Through discussion, modeling, and drawing diagrams, the students were led to combining terms by combining those that had the same dimension. Students were asked to verbalize when terms could be combined, and they were referred to as "like terms."

Students were asked to examine exercises in text, and mentally picture how each could be modeled. Could they be simplified? Could they be modeled with the tiles which we are using? Did we need other tiles to model some of the problems? What might those tiles be? Several sample homework problems were worked through and discussed among class members. Practice problems from the text were assigned for homework.

Instruments

The instruments used in the study were a pretest, a demographic survey, two content tests, a Mathematical Attitude Test, and an interview. Each of these is discussed below.

Existing scores on the Assessment and Placement Test (APT) were used as the pretest. The APT is a standardized test distributed by the Educational Testing Service (ETS) for use in two-year colleges. It is required by the admissions office of this community college before a student is allowed to enter a program of study. The score is used as a predictor of performance. It consists of six subtests, four of which are mathematics. The mathematics subtests are on computation, applied arithmetic, elementary algebra, and intermediate algebra. When applying for admission to the college, students are given the sections of the APT specified by their curriculum choice. Because there is research (Mogull & Rosengarten, 1972; Sabers & Felt, 1969; Taylor, Brown, & Michael, 1976) which indicates that performance on computation tests are predictors of performance in algebra, the score from the computation subtest was used in this study as the pretest score. This computation subtest is a timed (20 minutes) test containing 35 arithmetic questions. It is multiple choice with 4 choices provided on each item. Scores are scaled so that the standard scores range from 23 (0 correct) to 65 (35 correct), with a mean score of 45.27 and a standard deviation of 8.28. The reliability coefficient (KR-21) for the test is .88 (Educational Testing Service, 1985).

Predictive validity was reported for the full mathematics test, but not for the computation subtest only. Performance on the tests are positively related to performance in mathematics. The correlations given were obtained from scores reported by colleges that were regular users of the APT tests. The correlations are between test scores students earned during the year prior to beginning college work and grades earned at the end of the first or second semester at college. The reported correlations include various types of courses within the respective subjects. The correlations reported in the summary are based on 29 validity studies, each of which had at least 100 students. Three correlations were reported: the median (.43), the highest (.55) and the lowest (.06). The community college in this study had not computed validity correlations for this particular institution.

The demographic survey (Appendix C) was developed by the researcher. The eight questions on the survey were used to establish the diversity of the students. The first four questions determined demographic information, and the last four pertained to previous mathematical experiences .

Two content tests ($Test_1$ and $Test_2$) were written by the researcher. Each test consisted of both transfer and skill-based items (Appendix H). The transfer items covered material for which neither group received explicit instruction but which the students might be able to do by extension. An example of a transfer item is, "Write an equation in the form ax + b = c so that the solution to the equation is 2." Skill-based items were those which required the students to use a learned algorithm. An example of a skill item on the same concept is, "Solve for x. 3x + 7 = 2."

For analysis purposes, each of the content tests was divided into two subtests, one containing skill-based items, and one containing transfer items. $Test_1$ (Appendix F) included 14 items (designated T_{1S}) which were skill-based in nature and 7 items (designated T_{1T}) which required the students to use transfer of knowledge. $Test_2$ (Appendix G) included 13 skill-based items (designated T_{2S}) and 6 items which required transfer of knowledge (T_{2T}). A KR-21 reliability test established relibility coefficients of .74 for $Test_1$ and .82 for $Test_2$. Because the subtests were used in the analyses, relibility coefficients were established for the subtests as well. The results were: T_{1T}, r=.42; T_{1S}, r=.69; T_{2T}, r=.45; and T_{2S}, r=.74.

Both tests (all 4 parts) were piloted in the spring quarter preceding the study in three introductory developmental algebra at the community college where the study was held. The pilot was administered to students in their last week of classes after they had covered all the topics in the course. The purposes of the pilot test were to field test the items so that revisions could be made and to establish preliminary relibility coefficients for each of the subtests. $Test_1$ was piloted in one class of 20 students and $Test_2$ was piloted in two classes with a total of 40 students. Test time for the pilot was 50 minutes as that was to be the time allowed in the study for the test.

Based on student responses, some of the pilot test items were reworded for clarity. For example, item 1 on $Test_1$ was changed from "If $|a + 1| = 5$, then what is the value of a? " to "If $|a + 1| = 5$, then what are the possible values for a"? The other items which were changed were items 5 and 14 on $Test_1$ and item 19 on $Test_2$.

The Fennema-Sherman Mathematics Attitudes Scales (Fennema & Sherman, 1976) consist of nine scales which measure nine different attitudes which have been hypothesized to be related to the study and/or learning of mathematics. The attitude test used in this study (Appendix I) was a 24-item test which was a compilation (using random ordering) of two of the Fennema-Sherman scales: Confidence in Learning Mathematics and Effectance Motivation. Each of these scales consists of 12 items which require a Likert-type response. The confidence scale has a split-half reliability coefficient of .93 and the effectance motivation scale has a reliability coefficient of .87 determined by testing 1600 high-school students (Fennema & Sherman, 1976).

The interview consisted of 5 questions (Appendix J). Question 1 had appeared on $Test_2$; Question 2 was very similar to Question 1, but it contained an equation with a variable in the denominator of a fraction. Such an equation had not been a part of the classroom instruction for either group and therefore was considered a transfer item. Both Question 1 and Question 2 were included to investigate strategies which the students used to solve equations. Question 3 was included to assess student understanding of variable and meaning of equation. Question 4 consisted of two equations which were the same except a different letter designated the variable. The question was included to shed light on the meaning of variable for the interviewees and to see if they had what Wagner (1981) referred to as conservation of equation. Question 5, which had been on $Test_2$ was the familiar "student and professor" problem from the studies of Clement (1982) and was used in order to analyze student thought processes as they tried to write an equation which stated conditions given to them in a written problem.

The interviews, which lasted between 15 and 30 minutes, were audio taped. Eight students were interviewed, two from each class. The researcher chose each interviewee using the following criteria. From each treatment group (which consisted of two classes) four students were chosen, two who had prior algebra experience and two who did not have prior algebra experience. Of the two who had prior algebra, one scored above the class mean on both the midtest and the posttest and one scored below the class mean on each of these tests. Of the two who did not have prior experience, the same criteria was followed; one scored above the class mean and one scored below.

Statistical Analysis

The design of the study incorporated several dependent variables and one independent variable. The dependent variables were scores on T_{1S}, T_{1T}, T_{2S}, T_{2T}, the self-confidence scale, and the effectance motivation scale. The independent variable was method of instruction. Analysis of covariance (ANCOVA) was used to analyze the data obtained from the content tests. The existing score on the computational subtest of the APT was used as covariate in order to adjust the midtest and posttest scores for initial pretest differences. A t-test was used to determine whether the differences in the scores on each attitudes scale were significant.

CHAPTER IV

RESULTS

The focus of this research study was whether students of developmental algebra who are exposed to instructional strategies which encourage conceptual understanding achieve differently on content tests and have different affective responses from students who are exposed to procedural instruction. Chapter 4 is presented in five sections. Demographic data are presented first, followed by the results of the content tests, data from the affective scales, and results of the interviews. The final section is a summary of the results.

Demographic Data

An analysis of the data received from the demographic survey indicated great diversity among the students. The statistics given here are those for the students who had complete data sets, and therefore their scores were used in the statistical analysis.

In the experimental group, 78% were female and 69% were white, while in the control group 76% were female and 66% were white. The youngest student for the experimental group was 18 (for the control group, 19) and the oldest was 44 (for the control group, 42). However, the average age for each group was 27.3 years. For the experimental group, 89% of the students had completed high school while for the control group only 72% had (Table 2).

In reponse to the question "Have you taken an elementary algebra course prior to this one?" 64% of the students in the experimental group answered yes. Of this group, 59% said their prior experience was in high school. However, 47% responded that the highest mathematics class they had completed in high school was general mathematics, indicating that although an algebra course had been taken in high school, it was not completed. Twenty-eight percent in this group had completed Algebra I in high school and 22% had completed Algebra II in high school. For the control group 66% answered yes to prior algebra experience, with 48% having experiences in high school. However, 59% in this group responded that general mathematics was the highest mathematics they had completed in high school, with 28% completing Algebra I and 14% completing Algebra II (Table 3).

Traditionally, MAT 003 has a high attrition rate, with many students dropping the course before the quarter is over, and repeating the course at a later date. For the experimental group, only 19% of the students had taken MAT 003 previously, and of those who had previously taken the course, 29% had dropped the course before it was completed. For the control group, 41% of the group had previously taken MAT 003, and of those who had previously taken the course, 58% had dropped before the course was completed.

Content Tests

Evaluation of student performance was assessed with two content tests, each of which contained two specific types of items, skill and transfer. For analysis purposes, the content test were separated into subtests by type of item. Those subtests with transfer items were designated T_{1T} and T_{2T}; those with skill items were designated T_{1S} and T_{2S}.

$Test_1$ (midtest) was administered after six instructional periods, and $Test_2$ (posttest) was administered at the conclusion of instruction. Descriptive data for the content tests are presented in Table 2. With the exception of T_{1T} (on which both means were 4.0) the mean scores on each of the remaining content tests (T_{1S}, T_{2T} and T_{2S}) were higher, although not significantly so, for the experimental group than for the control group.

The four content subtests were analyzed separately with the APT score used as the covariate in each analysis. APT scores were used as the covariate on the premise that the APT was a predictor of content tests scores. The analyses indicated no statistically significant differences between groups on either of the content tests (See Table 3). Taken together, APT and method of instruction predicted scores on posttests, but the only important factor was APT. Method of instruction did not account for differences in the means on either the midtest or posttest.

Table 2

Demographic Data

Group	Av. Age	HS Grad[a]	GED[b]	Gender M	Gender F	Race B	Race W
		All students registered in class					
Experimental							
Night	29.6	19	1	4	16	4	16
Day	21.6	21	3	6	18	9	15
Control							
Night	29.6	13	8	8	13	4	17
Day	25.0	16[c]	3	5	15	8[d]	11
		Students used as subjects					
Experimental							
Night	28.9	15	1	3	13	4	12
Day	26.0	17	3	5	15	7	13
Control							
Night	29.2	7	5	4	8	2	10
Day	26.1	14[c]	2	3	14	7[d]	9

[a] Number of students who were high school graduates. [b] Number of students who had received General Equivalency Diplomacy. [c] Discrepancy in totals due to no response by one student. [d] Discrepancy in totals due to response of one student who was of Hispanic ethnicity.

Table 3

Mathematics Background Data

	Prior Elem. Alg.		Highest H.S. Math		
Group	H.S.	College	Gen. Math	Alg. I	Alg. II
	All students registered in class				
Experimental					
Night	14	1	8	8	4
Day	11	4	11	5	6
Control					
Night	12	2	10	9	2
Day	8	3	2	5	2
	Students used as subjects				
Experimental					
Night	10	0	8	6	2
Day	11	2	9	4	6[a]
Control					
Night	5	2	7	3	2
Day	9	3	10	5	2

[a]These numbers do not total 20 because one student wrote the response "Algebra III", which was not one of the choices.

Table 4

Means (Standard Deviations) for Content Tests

	Group			
Test (number of items)	Experimental (n=29)		Control (n=36)	
APT (35)	45.83	(6.22)	47.03	(6.48)
T_{1T} (7)	4.00	(1.55)	4.00	(1.77)
T_{1S} (14)	10.53	(2.30)	10.52	(2.64)
T_{2T} (6)	2.36	(1.76)	2.03	(1.32)
T_{2S} (13)	5.44	(3.53)	4.93	(2.56)

Table 5

Summary of Analyses of Covariance

Test	Source	df	Type III SS	F Value	p
T_{1T}	Group	1	0.21	0.09	0.7673
	APT	1	23.33	9.73	0.0027
T_{1S}	Group	1	0.67	0.13	0.7167
	APT	1	66.65	13.18	0.0006
T_{2T}	Group	1	3.43	1.72	0.1950
	APT	1	33.27	16.64	0.0001
T_{2S}	Group	1	11.10	1.57	0.2153
	APT	1	181.55	25.63	0.0001

Affective Scales

Data on the affective variables, effectance motivation and self- confidence as a learner of mathematics, are presented in Table 4. Scores were higher for the experimental group than for the control group, but the differences were not statistically significant.

Table 6

Data from Attitudes Scales

	Group		
Affective Variable	Experimental (n = 33)	Control (n = 27)	t
Effectance Motivation	42.87	40.89	0.183
Self Confidence	41.46	38.70	0.251

Interview Data

Eight students, two from each class, were interviewed. The researcher chose the interviewees using the following criteria. For identification purposes, the experimental students who were interviewed are referred to as E1, E2, E3, E4, and control students as C1, C2, C3, C4. From each treatment group (which consisted of two classes) four students were chosen, two who had prior algebra experience (E1, E2, C1, and C2), and two who did not have prior algebra experience (E3, E4, C3, C4). Of the two who had prior algebra experience, one scored above the class mean on the content tests (E1 and C1) and one scored below the class mean (E2 and C2).

Similarly, of the two who had no prior algebra experience, one who scored above the class mean on the content tests was chosen (E3 and C3) and, again, one who scored below the class mean was chosen (E4 and C4). For each question, the students were asked to respond orally, telling the interviewer their thoughts while they were addressing the problem. In each case, the student was given a paper with the question written on it, and told that it was acceptable to solve the problem mentally or on paper.

Question 1: If $2(x + 1) = 12$, what is the value of $x + 1$?

Students had previously solved problems in this format, and this particular problem was on $Test_2$. It was considered one of the transfer items because the question asked for x + 1, and not x. The point of interest was whether the students used formal rules for solving equations (i.e., distribute, subtract 2 from both sides, and divide by 2) or if they used reasoning skills (i.e., 2 times some number is 12, so that number is 6).

Since this problem had appeared in the same format on Test 2, the interview responses were compared to the written responses on the test. The students had been asked to record all necessary work on the test paper.

E1 solved the equation by inspection during the interview. On the written test there was no written work shown to support the solution. E2 solved the equation by the formal rules on the test, but in the interview performed part by rote and part by observation. The distributive property was used first, and then by inspection E2 decided that 2 times 5 + 2 is 12, so x is 5, and x + 1 is 6. E3 gave the same response on the test that was given in the interview. The equation was solved by the rules, and each step was written out. The value of x was found, and then the value of x + 1 was computed. E4 gave the answer 5 on the test without showing any work to support the answer and did not notice that the value of x + 1 was requested. In the interview, it became clear that the distributive property was not understood; the solution was sought by trial and error, and the 2 was not distributed. Instead, E4 multiplied the 2 by x and added the product to 1 (e.g., 2*5 + 1). None of the guesses used in the trial and error process were fractions, therefore no solution could be found.

On the test C1 had not written any work for the solution. The answer was simply given. However, in the interview, the solution was computed on paper. The 2 was distributed, but equivalent equations were not given. Instead, C1 wrote $12 - 2 = 10 \div 2 = 5$, and then wrote $x + 1 = 6$. C2 solved for x by manipulation of symbols on the written test, but in the interview while the distributive property was used properly, 2x + 2 was then combined to get 4x and it was concluded that x was 3 and x + 1 was 4. C3 solved the equation by inspection; no written work was shown on the test and an immediate response was given in the interview. C4 had not written any solution steps on the test. The solution which was given was 10. In the interview it became clear that 2x + 1 was being viewed as 2 + x + 1, and that x + 1 was therefore 10.

In summary, two of the students, one control and one experimental, used the rules for solving equations to find the correct solution. Two, one control and one experimental, used inspection only to correctly solve the equation. One experimental student used rules and observation together to solve the equation correctly. Two students, one control and one experimental, seemed to misunderstand the meaning of the symbols and were therefore unable to find a correct solution. One control student solved by manipulation of symbols, but combined terms incorrectly and therefore did not find the correct solution.

Question 2: If $\frac{14}{x+2} = 2$, solve for x.

The students had not been asked previously to solve equations which had a variable in the denominator of a fraction, and therefore a similar question did not appear on either test. The question was included in order to assess whether the students could solve a problem which they had not previously seen. Three of the experimental students (E1, E2, E3) were able to solve the problem by reasoning that 14 divided by something gave 2. The denominator had to be 7, so x was 5. E4 thought that 14 over x + 2 meant x + 2 divided by 14, and therefore responded that x was 26.

Two of the control students (C1, C3) were able to solve the problem by reasoning that 14 divided by something gave 2, the denominator had to be 7, and therefore x was 5. C2 seemed to have no idea how to attack the problem. When asked why it was different, the student responded that the x in the bottom made it different. No attempt was made to solve the problem. C4 had a misunderstanding of fraction notation and thought the denominator was subtracted from the numerator. Therefore the correct solution was not found.

Question 3: Make up an equation in the form $ax + b = c$ so that the value of x is 2.

Neither group had been taught how to make up equations that met specified conditions. However, this question had appeared on Test 2 in a different form. (If $ax + b = c$, give values for a, b, and c so that the solution of the equation is 2.)

E1 and E3 had been able to write an acceptable equation on the test. In the interview they wrote a correct arithmetic equation (e.g., 3*2 + 1 = 7) but had to be prompted by the interviewer to write the equation $3x + 1 = 7$. E2 was confused about the word "solution" on the written test and thought that the equation should equal 2. However, when the problem was worded differently, as in the interview, a correct arithmetic equation was given immediately, but again the interviewer had to remind the students to leave the variable in the equation. E4 had not been able to give an equation on the test but was able to give one almost immediately in the interview. C1 set the equation equal to 2 on the written test, but was able to give a correct equation in the interview when the question was reworded. Both C2 and C3 set up equations on the written test in $ax + b = c$ form, and solved them formally to get $x = 2$. C2 was not able to give an equation in the interview, but C3 could. C2 wrote $2x + 2 = 2$, then $4x = 2$, and $x = ...$ and could not complete the process. C4 followed through with the misconceptions that have been exhibited before and, after reading the questions silently, wrote $2 + a + b = c$ and gave the equation $2 + 8 + 6 = 16$.

In summary, two students from the control group were not able to render an equation which satisfied the given conditions. It should be pointed out that even the students who did give an equation which met the conditions had to be prompted to give the equation with the x left in the equation. They left 2 in the equation in place of x.

Question 4: The interviewer uncovered the equation $7W + 22 = 109$, and asked the student to look at the equation, thinking about how to solve the equation. Then the equation $7N + 22 = 109$ was uncovered. The question was asked: "If the first equation were solved to get a solution for W, and then the second equation were solved for N, which would be larger, W or N?"

The students had not been previously asked to answer this problem, nor a similar problem. Actually, this did not prove to be a problem for any student from either group. Each of them immediately knew that the solutions would be the same. When asked by the interviewer, "Why?", all responded that they knew because the equations were the same, with only a different variable.

Question 5: At this college there are 6 times as many students as instructors. If I represents the number of instructors and S represents the number of students, write an equation which will show the relationship between the number of instructors and the number of students.

This question was on Test_2. E1 remembered that the problem was on the test and that an incorrect response had been given at that time, so a correct response was given immediately in the interview. E2 and E4 both gave the equation $6(S) = I$ on the test and were only able to write a correct equation after much prompting from the interviewer. E3 gave the equation 6*I = S on the test, but in the interview immediately wrote 6*S = I, thought about it, realized that it was backwards, and changed it to 6*I = S.

C1 gave the equation I*6 = S on the written test. In the interview, the equation 6*I = S was given, but when the student put numbers into the equation for verification, it was decided that this equation was incorrect, and that equation should be S/6 = I. The equivalency of the equations was not recognized. C2 gave several equations on the written test, without indicating which was correct. One of the equations given was 6(I) = S. In the interview, several equations were again written, but none of the equations represented the correct relationship. C3 gave the equation 6*S = I on the written test; however, in the interview situation, the equation 6*I = S was immediately given. C4 gave an expression, not an equation on the written test. In the interview the student paired I with 1 and S with 6 by drawing an arrow between them. When the interviewer asked for an equation, the correct equation I/S = 1/6 was given.

In summary, three of the students, one experimental and two control, were able to write an equation which stated the correct relationship between number of students and number of instructors without any prompting. With little prompting, two others, one from each group, were able to write an acceptable equation. After being led to substitute an arithmetic example, two others, both experimental, were able to write an acceptable equation. Even after the suggestion from the interviewer, one control student was unable to write an acceptable equation.

In analyzing responses to all five interview questions, it appeared that some students from both groups were able to transfer knowledge, and some from both groups were not able to transfer knowledge. For Question 3, each of the experimental students was able to write a correct equation. However, two of the control students who gave a correct equation formally solved the equation to verify that the solution was 2. This perhaps was an indication that students from the control group felt more "rule bound" than did the experimental students.

Summary

Data from the demographic survey indicated great diversity among the students. In both groups, most of the students were white female.

The diversity of the students was more pronounced in their educational background. Both groups contained students who had not completed high school. While the majority reported that general mathematics was the highest mathematics course completed in high school, both groups contained students who had completed Algebra II and one student had compoleted Algebra III.

An ANCOVA was used to test for significant differences with the APT computation score used as covariate. No statistically significant differences were found between groups on either of the content tests. However, mean scores on each of the content tests, with the exception of the transfer subtest of the midtest, were higher for the experimental group than for the control. Scores on the affective scales were higher for the experimental group than for the control group, but again the differences were not statistically significant.

Analysis of the interviews did not indicate differences in strategies used. Some students from each group were able to transfer knowledge. However, there were some indications that the control group felt more "rule bound" that did the experimental students.

CHAPTER V

SUMMARY AND DISCUSSION

This study involved instruction to introductory algebra students in four classes at a community college. The length of the study was four weeks within the 10-week summer quarter. Each of the classes received instruction on the same content using different instructional styles. Two of the classes, designated the control group, received procedural instruction, and the remaining two, designated the experimental group, received conceptual instruction.

After six lessons, the students were given the first content test. A second content test was given at the end of instruction. Each test had items which were considered skill items and items which were considered transfer items. Student scores on the content tests were analyzed using an analysis of covariance with a computational pretest score (APT score) used as covariate.

The study also examined the link between instructional style and two affective issues which have been hypothesized to be related to the study and/or learning of mathematics: confidence in learning mathematics and effectance motivation. These qualities were measured by two of the Fennema-Sherman Attitudes Scales. The students were asked to respond to the attitude scales after the second content test.

A third part of the study was the interviewing of eight students, two from each class. The purpose of the interviews was to elicit from the students explanations of the strategies used to solve problems like those on the tests.

The study was designed around four research questions. These questions were:

1. Do developmental algebra students who are exposed to instructional strategies which encourage conceptual understanding achieve differently on a skill-based content test from students who are exposed to procedural instruction?
2. Do developmental algebra students who are exposed to instructional strategies which encourage conceptual understanding achieve differently on a transfer of knowledge test from students who are exposed to procedural instruction?

3. Do developmental algebra students who are exposed to instructional strategies which encourage conceptual understanding achieve differently on an effectance motivation scale from students who are exposed to procedural instruction?
4. Do developmental algebra students who are exposed to instructional strategies which encourage conceptual understanding achieve differently on a confidence in learning mathematics scale from students who are exposed to procedural instruction?

For purposes of analysis, the content tests were divided into four subtests. These four tests were analyzed separately with the APT computational score used as covariate in each analysis. The analysis indicated no significant differences between groups.

A t-test was used to test significance of differences on the attitude scales. Although scores were higher for the experimental group than for the control group on both effectance motivation and self confidence as a learner of mathematics, neither of the differences proved to be significant.

While the interviews uncovered interesting strategies and misconceptions, they did not indicate that method of instruction had a bearing on the way students' chose to solve problems. It appeared that some students from both groups were able to transfer knowledge, and some from both groups were not able to transfer. However, the responses from control students were more structured for some questions, and this was perhaps an indication that the control students felt more rule-bound.

Conclusions

Even though there was a change in some of the test items on the pilot content test in order to eliminate ambiguity, the reliabilities established by the administration of the content tests in the study were comparable to those in the pilot. The fact that the reliabilities of the content tests were low must be considered in the drawing conclusions about the study. However, based on these content tests scores, it appears that for these two particular groups of students who were taught by alternate means for a four-week period, instructional style did not affect posttest scores or affect scale scores.

It should also be noted that the performance of both groups was much poorer on the second test than on the first test. The scores of the two groups were similar on both tests, so the poor performance on the second test would tend to amplify small differences between the two groups. There are several things which might have contributed to this. The time allowed for instruction of the concepts was very structured, and there was not much flexibility. Much content had to be covered in a short period of time. Little time was allowed for reflection and understanding. The first content test covered material which was basically arithmetic, and therefore the students were more familiar with the material covered and reflection time was not essential. However, the second test covered material which is usually the first real introduction to algebra, namely, terms and expressions and solving linear equations. This material was completely foreign for many of the students, and more time was needed for understanding. This perhaps accounts for the poor performance on the second test.

Although the analyses did not reveal statistical significance, there are results which might indicate a move in the direction of significance. One indication is that on every test except the transfer subtest of the midterm (on which the means were equal) the means for the experimental group were higher than the means for the control group.

Another indication that there was a move toward significance was the fact that for the experimental group, the variance increased on the posttest for both the skill-based subtest and the transfer subtest, while the variance decreased for the control group for both tests. In addition, the F value also increased for both subtests of the second content test. The p value moved from approximately .7 for the first content test to .2 on the second content test. Again, this seems to be a clear indication that there was a trend toward significance.

It is also noteworthy that even though the APT scores were lower for the experimental group than for the control group, the scores for the experimental group were not lower on the skill-based tests than for the control group. Despite the fact that they received less drill and practice, the students were still able to answer the skill-based questions as well as those who had received instruction which concentrated on "how" rather than "why." This would seem to contradict the belief that conceptual instruction, leading to less drill and practice, will cause a decrease in algebraic skills.

Although a t-test of significance did not indicate statistically significant differences between groups for either of the two affective variables, the mean scores on both measures were higher for the experimental group. Again, this may indicate that if the treatment period had been longer, significance might have been reached.

It must be concluded from the interviews that mode of instruction probably had little bearing on the strategies which students used to solve problems. However, there are some slight indications which again suggest that a longer period of intervention might cause more differences. These will be discussed here.

Question 1 asked the students to solve for x + 1, not for x. Five of the students solved for x + 1 without solving first for x. They seemed to be able to view x + 1 as an entity itself, an important concept in understanding the structure of algebra. Of these five people, three were from the experimental group. The fact that all four of the experimental students and only two of the control students interviewed were able to find the solutions for Questions 2 and 3, which were both transfer items, also indicates that perhaps the experimental students were learning with more understanding than were the control students.

Question 4 proved not to be a problem for any of the students interviewed as each student from both groups answered the question correctly immediately, without reservations. This result does not support the finding of Wagner (1977) who investigated conservation of equation. In her study, which involved 69 middle and high school students, only 38% of the students answered correctly. This leads one to question whether older students, as in this study, are able to conserve equation better than younger students. Further investigation concerning this question is warranted.

Question 5 was a variation of the "Student-Professor" problem which Clement (1982) used in a study which found that 37% of engineering students tested answered the question incorrectly. Only two of the students in this study (25%) gave the correct solution on the written test. However, three (37%) of the students, two of whom were from the control group, were able to give a correct solution in the interviews without any prompting and two others, one from each group, with little prompting. (i.e., Write an equation. Think about it.) Two remaining students, both from the experimental group, were able to give a correct equation when it was suggested that they use numbers to help them finalize the relationship. Success using this strategy supports the findings of Chalouh and Herscovics (1988) who found that sixth and seventh graders who never had algebraic experience were able to attach meaning to algebraic expressions when they were taught to move from arithmetic examples to algebraic expressions. Further studies are warranted to investigate an instructional style which would teach students to instantiate with familiar arithmetic examples in order to move to an algebraic equation which models the relationship.

While the interviews did not contribute much to the answer to the research questions, they were informative, and seemed to have merit. Student misconceptions often became apparent during the interviews, although they had not been apparent from the written tests. Therefore, it seems that the interviews pointed out the value of the interview as a tool for evaluating student understanding, and a means to address student misconceptions.

Limitations and Discussion

Although the data suggested that instructional style might have some effect on post-instruction-tests scores, none of the group differences were significant. Perhaps one explanation of this is the low reliability coefficients of the content tests.

Another explanation of the lack of significant differences might be the design of the study. This explanation takes two parts: length of the study, and change of instructor and instructional style. Both of these will be discussed here.

The short duration of the the instructional period was a limitation of the study. However, four weeks was the maximum time which could be negotiated. As mentioned earlier, the fact that the means on both of the content subtests at the end of four weeks of instruction were greater for the experimental group than for the control group indicates a move in the right direction. This trend in the data is possibly an indication that the intervention period was too short.

A second possible explanation arose from the apparent resistance of some of the students to a change of instructor. This resistance had been expected, but not to the degree which was exhibited by some students. None of the resistance was felt by the researcher from the students in the control classes, but it was definitely felt from several of the students in the experimental classes. It seems that it was not just a resistance to change in instructor, but also a change in instructional style. The resistance was evidenced in several different ways.

One student voiced a concern in the change of instructor at the first meeting. The reason for the concern was not expressed at that time, but it seemed that the concern was relieved, as the consent form was signed. However, this student was one of two who often vocally expressed an apprehension about the mode of instruction. On three occasions, this student left at break, and did not return for the second part of the class. One or two students from both experimental classes went to their instructors to mention their apprehension about the change in instructors in the middle of the course. Both instructors tried to reassure them, and informed the researcher of the students' concerns.

Another possible limitation of the study might have been the variation in the time of day that the classes were offered. Does time of day have a bearing on the number of absences? on attrition? on student alertness? on student participation? Does a student learn as well in attending a class from 8:00-10:20 p.m. after a full work day as the same student might learn from attending an 8:00 a.m. class? These questions were not a part of this study, but they warrant investigation.

A further limitation of the study was the higher number of elementary algebra repeaters in the control group. Certainly, it would seem that this might have had an impact on the affective issues addressed, and perhaps on the performance of the students on the content tests. The affect of repeating the course is not known.

A final limitation is the constraint caused by attempting to address too many topics in a limited amount of time, leaving no time available for true development of conceptual understanding and reflection. This limitation was placed on the study by the course outline which delineates the topics which are to be covered in a 10-week period, and therefore determines the number of topics which must be covered in a 4-week period. This limitation is not different from the limitation which is felt by any instructor who has been given the mission to address too many topics in too little time. To address this problem was not a purpose of this study, but the problem deserves attention.

Implication for Further Research

Based on the data of this study, mode of instruction does not seem to have an effect on posttest scores when the intervention period is not longer than 50 minutes, five times a week, for four weeks under circumstances similar to those of the project. Many questions concerning the relationship between mode of instruction and student achievement remain unanswered.

While the study was designed around the concept that the Standards which were written for grades K - 12 were also applicable to developmental algebra, whether developmental algebra students learn the same way as algebra students at other grade levels is an important question. While the goals of the Standards, that is, the mathematical power and literacy for all students, it is possible that instruction needs to be specified for the age and experience of the student. This is an important question which needs to be researched.

Does conceptual instruction affect achievement over a longer term? Should the method of measuring achievement following conceptual instruction be altered? It remains clear that if the goals of the Standards are to be met for students of mathematics at any grade level, alternative instructional methods must be investigated. Yet, to successfully implement the Standards, there must be evidence of positive effects associated with those alternative methods.

Certainly one implication of the study is the need for development of reliable content tests which will measure not only procedural knowledge but conceptual knowledge as well. It seems that an appropriate and important follow-up study would be to develop such tests to be used in the developmental algebra classroom.

This study has several implications for mathematics instruction at the community college level. Based on the fact that there is little research addressing alternative instructional styles in developmental mathematics, it can be assumed that the reform movement which is being implemented in K-12 schools has had no or little influence on instruction at the community college level. Therefore, studies are needed which investigate instructional styles which can be used to produce conceptual understanding for older students. Students in developmental algebra are very diverse. They bring to the class varied experiences. Do these experiences have an impact on how these students learn algebra? If so, in what way is their learning affected by their prior experiences? What classroom experiences would help them become better students of mathematics?

Specifically, there are two important implications for instruction in developmental algebra. One calls for new study concerning instruction in developmental algebra which would extend the length of the study, perhaps over a full quarter. The second is to continue to investigate alternative instructional styles for students in developmental algebra. The research base is not broad here, and more research at this level is warranted.

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APPENDIX A

THE UNIVERSITY OF NORTH CAROLINA AT GREENSBORO

Consent to Act as a Human Subject
(Short Form)

Subject's Name Jane Doe

Date of Consent 11 June 1992

I hereby consent to participate in the research project entitled Problem Solving Instruction in Developmental Algebra.

An explanation of the procedures and/or investigations to be followed and their purpose, including any experimental procedures, was provided to me by Sara Brame Skinner ____________. I was also informed about any benefits, risks, or discomforts that I might expect. I was given the opportunity to ask questions regarding the research and was assured that I am free to withdraw my consent to participate in the project at any time without penalty or prejudice. I understand that I will not be identified by name as a participant in this project.

I have been assured that the explanation I have received regarding this project and this consent form have been approved by the University Institutional Review Board which ensures that research projects involving human subjects follow federal regulations. If I have any questions about this, I have been told to call the Office of Research Services at (919)334-5878.

I understand that any new information that develops during the project will be provided to me if that information might affect my willingness to continue participation in the project. In addition, I have been informed of the compensation/treatment or the absence of compensation/treatment should I be injured in this project.

____________________ ____________________

Subject's Signature | Witness to Oral Presentation and Signature of Subject

If subject is a minor or for some other reason unable to sign, complete the following:

Subject is ____ years old or unable to sign because ____________________

____

____

Parent(s)/Guardian Signature

APPENDIX B

Oral Presentation to Students

The purpose of the research in which you are asked to participate is to investigate differences in student achievement after receiving different types of instruction. Your contribution by participating will allow instructors to use teaching methods which will better foster student understanding. There are no risks for you.

You are simply to attend class each day for the next 4 weeks just as you usually do. You will have a different instructor. You will spend no more time in class than you usually spend. You will be given two tests on the material covered. Your instructor will use these tests scores in evaluating your achievement.

The data collected will be kept completely confidential. While it may be later used for research reports, neither your name nor the name of the college will be revealed. You have the option to withdraw from participation in this research without penalty. You now have the opportunity to ask any questions you may have concerning this project.

Signature of Person Obtaining Consent on Behalf of UNCG	Date Presentation Delivered

APPENDIX C

Survey for MAT 003 Project

1) Social Security Number: _____-_____-_____

2) Sex: ____Female ____ Male

3) Race/Ethnicity: (check one)
   _____American Indian
   _____Asian
   _____Black
   _____Hispanic
   _____White

4) Age _____

5) Are you a high school graduate? ____Yes ____No
   If no, have you passed the GED? ____Yes ____No

6) Have you taken an elementary algebra course prior to this one?
   _____Yes _____No
   If yes, check the appropriate place. _____High School _____College

7) What is the highest level mathematics class you have completed beforetaking this class?
   _____General Mathematics _____Algebra I _____Algebra II

8) Have you taken MAT 003 before? _____Yes _____No
   Did you drop the course before the quarter was over? ____ Yes ___ No

APPENDIX D

LESSON PLANS FOR EXPERIMENTAL GROUP

The approved text for the MAT 003 classes is Introductory Algebra (3rd ed.) by Wright and New. When exercises and pages are referenced in the lesson plans, they are from the aforementioned text.

Experimental Lesson 1

Objectives: Provide opportunities for the students to construct meaning to signed numbers, combining of signed numbers, and the location of signed numbers on the number line.

Activities

I. Using a transparency on the overhead which contains a spinner with a pointer which could point to the numbers 1-6, a game is played in which the students are asked to combine numbers according to the following rules:
(a) If the spinner points to an even number, the student scores that many points,
(b) if the spinner points to an odd number, the student deducts that many points,
(c) scores will be designated "having points" or being "in the hole".

Procedure: After several spins ask the students to:
(a) Calculate their scores,
(b) Explain how they calculated their scores,
(c) After 2 (3, 4, etc.) spins, how could you get 0? 5? 2 "in the hole", etc.
(d) After 2 (3, 4, etc.) spins, what is the highest score you could get? Lowest score?

II. Two different colored dice are used. One is designated as the "in the hole" die; the number of dots on this die are deducted even if the player goes "in the hole."

Procedure: The same kinds of questions are asked as with the first game. In addition, students are asked to diagram consecutive tosses which would result in specific scores.

III. Play game II again, but this time put the scores on a number line (transparency). Lead students to the fact that an orientation point is needed, (0), that some scores go above that point, and some below.

Words to use: positive number, negative number, opposites, coordinate of a point, integer.

IV. Invite students responses concerning some situations in their lives which bring to mind negative numbers.

V. Investigate order of numbers by relating to the relationships between their scores on the games previously played. Talk about a need for symbols to show relationship. Agree on the meanings of the symbols $<$, $\leq$, $>$, $\geq$, $=$, and $\neq$. By questioning and with examples establish that if two numbers are put on the number line, the one on the right is larger than the one on the left. Using several examples compare all possible combinaionas (i.e., two positives, two negatives, a negative and a positive, zero and both positive and negative).

VI. Solicit examples of numbers which are not integers. Locate them on the number line, using key words to talk about the numbers.

Additional words to use: Rational numbers, signed numbers, natural numbers, whole numbers.

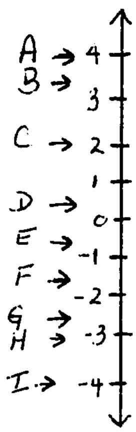
Homework: pages 49 and 50; exercises 1 - 55, odds

Experimental Lesson 2

Objectives: To provide opportunities for the student to develop the meaning for absolute value and to be able to give solutions to simple absolute value equations.

Activities

I. Review. Transparency with the following number line is displayed.



Instigate discussion by asking questions like:
Which arrow points to -2? -2 1/2, 3 1/2, 1/2, etc.
Which arrow points to the greatest number?
Which arrow points to the smallest number?
What is relationship between -2 and -3?
Use symbols to show this relationship.
Classify the numbers referred to.

II. Answer homework questions, encouraging students to answer each othersques-tions and explain their answers.

III. Using the same number line (above) develop the meaning of absolute value by talking about numbers which are the same distance from zero. Talk about the differences in a number and the "absolute value" of a number. Develop a formal definition for absolute value of using examples. Tell them the symbol for absolute value (parallel bars).

IV. In preparation for the homework assignment, go over the following problems, giving students opportunities to discuss and answer.

pages 52 and 53; 4, 12, 18, 22, 34, 42, 46.

Homework: pages 52 and 53; 1-19 odds, 21, 25, 27, 29, 31, 35, 37.

Experimental Lesson 3

Objectives: Provide opportunities for students to establish that "adding" signed numbers is combining, and to construct the "rules" for adding signed numbers. In addition, students should be able to verify if a specified number is the solution to an equation by substitution.

Activities

I. Review the meaning of absolute value (use the same transparency as for lesson 2) by asking question like:

> Which arrow points to the number whose absolute value is 1?
> Which arrow points to the number with the greatest absolute value? Least?
> Which number points to a number whose absolute value is 2? 0? etc.

II. To foster further understanding of absolute value, ask students to discuss with a neighbor exercises 49 - 54 (never, sometimes, always exercises) on page 53 of text. For each exercise the students should give examples to support their response. (Transparency)

Go over these exercises, soliciting responses from students, especially on those exercises where there is disagreement on the correct answer.

III. Discuss homework questions.

IV. On overhead screen, use disks to remind students of the arithmetic meaning of addition (combining sets). By letting one color disk represent positive and another negative, develop the property that a number combined with its opposite equals 0. Further examples will model adding integers by combining sets of disks. Ask students for all possible situations (positive plus positive, etc.) and then ask them to write in words how they would tell someone to get the sum. Come to a consensus on what the "rules" should be, and write them on the board. With the students, go through the even-numbered exercises on page 56. (2 - 20).

V. On the chalkboard write the equation x + (-7) = 10. Ask if the solution to the equation is -3. Solicit suggestions about how to do the problem.

Homework: pages 56 - 57; 1, 3, 7, 13, 15, 25-39, odds; 45, 49, and 53.

Experimental Lesson 4

Objectives: Help students realize that subtracting signed numbers is adding the opposite of the number and give them opportunities to use this property.

Activities

I. A. Review additive inverse (opposite) of a number.

Suppose a can represent any number. Ask students to give values to a to fill in the following chart. Take several examples. (Transparency)

value of a	additive inverse of a	sum of a and its additive inverse

Ask students to make observations about a and the inverse of a. Try to solicit that $a + (-a) = 0$.

B. Reinforce the rules for adding signed numbers by going through the "never, sometimes, always" exercises on page 57. (Transparency)

II. Go over homework questions.

III. Talk about the meaning of arithmetic subtraction and the relationship between addition and subtraction in arithmetic, i.e., $7 - 3 = 4$ because $4 + 3 = 7$. Note that the same thing is true for signed numbers, and let the students find the answers for the following subtraction problems.

$5 - 2 = ?$ because $? + ? = ?$; $-5 - (-2) = ?$ because $? + ? = ?$

$-5 - 2 = ?$ because $? + ? = ?$; $5 - (-2) = ?$ because $? + ? = ?$

After establishing the answers, ask the students to do the following problems:

$5 + (-2) =$	and compare with	$5 - 2 =$
$-5 + (-2) =$		$-5 - 2 =$
$-5 + (+2) =$		$-5 - (-2) =$
$5 + (-2) =$		$5 - (-2) =$

Try to get them to notice that in subtraction, the opposite of the number being subtracted is added. That is, for any signed numbers a and b, $a - b = a + (-b)$. This is not easy, but it is important!

IV. Talk about pluses and positives, minuses and negatives. Admit up front that using the same sign for both can be confusing, and therefore it is important to see the difference. Go through equations and specify if a sign is a minus sign or a negative sign. For example, in $-4 - 6$, the first sign is a negative, while the second is a minus, and the 6 is a positive number. This is an important concept, and needs to be addressed early.

V. Do, with the students, sample problems on page 61 (12-22 and 36, 40, 42, and 46.). Each time, write down the problem, visually changing the subtraction to addition of the inverse.

Homework: page 61; exercises 11-21, odds; 35-45 odds

Experimental Lesson 5

Objectives: Help students to develop meaning for multiplication and division of signed numbers and provide opportunities for them to use these meanings.

Activities

I. Review Opener

Number line and following questions on transparency. In each case, answer yes or no; for a - f, give examples to support your answer.

(a) If x is to the right of zero, then -x is to the left of zero.
(b) If x is to the left of y, then x is greater than y.
(c) The opposite (additive inverse) of any number to the right of zero is always to the left of zero.
(d) The farther a number is from zero, the greater it is.
(e) If x is to the left of zero, then -x is to the right of zero.
(f) -x is sometimes positive.
(g) Zero is neither positive or negative. Explain.

II. Go over homework, expecting many questions. Generate a lot of discussion.

III. With disks on overhead screen, model the equivalent sets of sets arith metic meaning of multiplication. Model several examples, each time asking the student to tell what is modeled. (Example: 4 sets of 5 is 4 times 5.) Talk about the fact that arithmetic multiplication is the same as positive number times positive number.

Ask how to model 4 times -5? Discuss. Take several examples. Lead to "positive times negative is negative."

How can we handle -4 times 5? We can't have -4 sets. Commutative property. Give examples from arithmetic.

Summarize so the students will realize what situations we have covered. The one situation not covered is "negative times negative."

Note that modeling "negative times negative" is not as apparent as the other situations, and an intuitive method will be used to help them construct the result of multiplying a negative times a negative. We will begin with something we know, and look for a pattern.

Give the correct product based on the "rules" we just developed, and look for a pattern:

-4 times 3 = ? (-12)	-4 times 0 = ? (0)
-4 times 2 = ? (-8)	Soooooo
-4 times 1 = ? (-4)	-4 times -1 = ?

This is tricky, but they can see it if its done right.

Ask the students to summarize the "rules" for multiplication. Practice on page 66, exercises 1-12 by soliciting student answers, and discussing as necessary.

IV. Note the relationship between multiplication and division in arithmetic.

In arithmetic, 12 ÷ 4 = 3 because 3 * 4 = 12; we do the same thing with signed numbers. Solicit an example for every situation, and let the students decide what the correct signs are. Practice on page 66, exercises 21-31 by soliciting student answers and discussing as necessary.

Ask the students to do exercise 54 on page 67. It's a true-false. Ask for answer support.

Ask the students to do exercise 64 on page 67. Ask for an explanation to support answer.

Homework: page 67; exercises 47-55, odds; 57-65 odds.

Experimental Lesson 6

Objectives: To pull it all together. Review for test to be given next session.

Activities

I. Review multiplication and division of signed numbers by going through exercises 36-45 on pages 66 and 67 together, asking students to give an example in every case to support their answers. (Transparency)

II. Students will work in pairs on Chapter 2 Review exercises 1-44 odds or evens, but not both, on page 71, and 52-61 on page 72. Instructor will move about, answering questions and generating discussion between students.

IV. Bring class back together. As a group go through exercises 45-51 (never, sometimes, always), discussing as necessary.

Homework: Study for test for next session.

Experimental Lesson 7

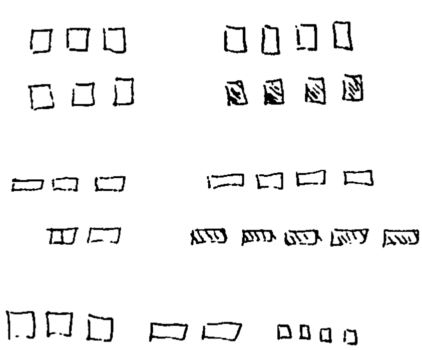
Objectives: To provide experiences so that the student can establish meaning to terms and expressions. In addition, students should be able to combine like terms and evaluate expressions for specific values of the variables.

I. Review: The concept of area is reviewed by using a coordinate transparency on the overhead. Diagrams are given to remind students that if a square has a side of length of 3, then the area is 3^2, or 9. Several arithmetic examples are given and discussed. Then an diagram of a rectangle (e.g., 3 units by 5 units) is given and students are reminded that the area of a rectangle is length times width (3 x 5, or 15). Again, several arithmetic examples are given and discussed.

II. By using different colors tiles which are the same dimensions, the meaning of opposites, or additive inverses is reviewed so that students will remember that $a + (-a) = 0$ for all real a.

III. Overhead algebra tiles are used (without grid so that the sides can represent variables) to give geometric meaning to terms and expressions in one variable. The tiles are used to give students an understanding for term, like terms and unlike terms. By modeling expressions and putting them together under the meaning of inverses, students are led to the idea of combining expressions and encouraged to verbally give a name to the term. Then the model and verbal representation are attached to written symbols. Expressions are modeled and students are asked to write the symbol for them. Then students are given expressions, and asked how they would model them.

Examples: Models — Written expressions



$3x^2 + 4x^2$

$3x^2 - 4x^2$

$3x + 4x$

$2x - 5x$

$3x^2 + 2x + 4$

IV. Ask students to examine exercise #1 on page 78. Mentally, picture how it could be modeled. Then picture how you could combine it, and think how the combination could be written. Do the same with # 2, encouraging discussion as each new exercise is addressed. Ask for verbalization of how the expressions can be simplified without the model. Continue with several of the first exercises. Ask student tocontinue through the exercises, stopping when they reach a problem which we could not model with our tiles. (The first such exercise is #11.) Discuss this problem, asking why it could not be modeled with our tiles, and what we could do to model it. Talk about how the combination could be made.

V. Choose sample homework problems, and work through them with the students, always encouraging discussion among the students.

Sample problems: page 78; exercises 16, 24, 28, 34, 44, 50, and 58.

Homework: pages 78 and 79; 15, 19, 23, 27, 37, 41, 45, 51, 55, 59.

Resource for this lesson:

Howden, H., (1985). Algebra tiles for the overhead projector. New Rochelle, NY: Cuisenaire Company of America, Inc.

Experimental Lesson 8

Objectives: To establish the difference between an expression and an equation. To investigate a variety of ways to solve linear equations, encouraging the students to use informal methods before moving to formal methods.

I. Review like and unlike terms by displaying the following terms on the overhead. Ask students to access whether the terms are like or unlike, and be prepared to explain **why** they answer as they do.

$-4y$ and $3y$	$3x^2y$ and $4xy^2$
$3x$ and $4y$	$3x$ and $-2x^2$
-8 and $2x$	$-4xy^2$ and $-5xy^2$
6 and -4	$\frac{2x^2}{6}$ and $\frac{2x}{6}$
$3x^2$ and $7x^2$	

II. Accept homework questions, soliciting discussion from students as questions are answered.

III. Give the expression $3x + 5x$. Ask students the value of the expression. Through discussion, establish that the expression has no value unless a value is assigned to x, and x can take on any value. Ask individual students to assign values to x, and ask the class to then find the value of the expression.

Give the equation $3x + 5x = 16$. Can x take on any value here? Go with the class response encouraging discussion. Ask for the value of x which will make the equation true. Investigate methods of finding the solution, asking students to explain. Develop a consensus on the difference between expression and equation.

IV. Use informal methods (e.g. guess and check, cover-up and basic arithmetic facts) for solving equations, letting the students explain how they would solve the equation.

Use equations from page 86; exercises 2, 4, 24, 26, 28, 30, 32, 34.

Homework: page 86; exercises 3, 5, 23, 25, 27, 29, 31, 33.

Experimental Lesson 9

Objectives: To develop more fully informal methods. To provide equations which will show students that informal methods are not always adequate, (e.g., with equations which have fractional solutions) and move toward some formal methods.

I. Go through homework problems thoroughly, asking for discussion on the methods used to solve each equation, focusing on a variety of strategies. Encourage much verbalization from students about processes used.

II. What is the solution to the equation $x = 10$? $x = -2\ 1/2$? $x = a$?
One strategy might be to get an equation in the form $x = a$, where the coefficient of x is 1. Then the solution is a.

Now investigate equations whose solution is evident, and establish what happens if both sides of the equation are multiplied by the same number.

Examples: What is the solution to the equation $4x = 8$? (2)
Suppose both sides of the equation were multiplied by 3.
Would the solution still be 2?
Suppose both sides of the equation were divided by 2?
Would the solution still be 2?
What happens if we divide both sides by 4?
Still have a solution of 2?

Investigate equations in the form $ax = b$ which have solutions. How could we get the coefficient of x to be 1?

Examples: $-12x = 32$ $\quad$ $5x = -2$

$\frac{5}{4}x = \frac{8}{10}$ $\quad$ $\frac{1}{5}x = \frac{2}{3}$

Homework: page 86; exercises 35-44 all.

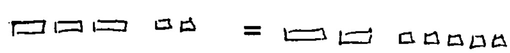
Experimental Lesson 10

Objective: To provide opportunities for students to solve equations in the form $ax + b = cx + d$ by adding the same quantity to both sides of the equation.

I. Review the quantities represented by the algebra tile pieces, giving examples of the variable term, and constant terms. Review the zero principal by modeling with algebra tiles on the overhead.

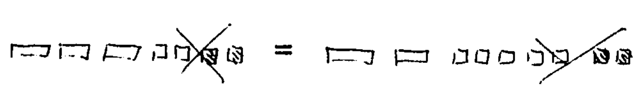
II. Model equations on the overhead with algebra tiles. Ask for ideas of what the equals sign means, leading to the idea of balance. The value of the variable can be found by adding equal amounts to both sides until we have one variable alone on one side. For example, to solve $3x + 2 = 2x + 5$ physically we would:

Represent $3x + 2 = 2x + 5$ using tiles

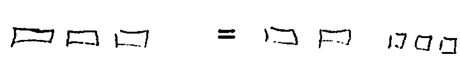


Add -2 to both sides

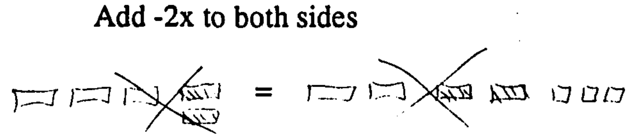
Use the zero principal



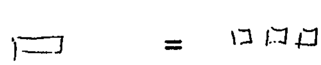
representation of new equation



Add -2x to both sides



representation of new equation



Verbalize the solution (x equals 3). Substitute in original equation to see if it is in fact the solution.

Physically solve several equations, soliciting the process from students. When they seem to be comfortable with the physical manipulation, then write what is done in algebraic notation as the physical event takes place. Then move to algebraic notation only.

Equations used:

$2x + 4 = x + 5$
$4x = 3x + 7$
$5x + 3 = 4x + 8$
$4x - 3 = x + 3$
$3x - 2 = 2x + 1$

Examine what happens if we have more than one of the variable. Relate back to the previous lesson of equations in the form $ax = b$.

Examine $4x + 1 = 2x + 7$

Summarize by pulling in equations in many forms from pages 86 and 91. Sample equations: page 86; exercises14, 16,18, 20; page 91; exercises 14, 18, 22, 32. If informal methods can be used, then encourage students to use them.

Resource used for this lesson:

Kinach, B. (1985). Solving linear equations physically. Mathematics Teacher, 78, 437-447.

Experimental Lesson 11

Objective: To provide opportunities to practice equation solving using both informal and formal strategies.

I. Review equation solving methods by going over homework problems, referring to physical model for solving as needed. Encourage students to use both informal and formal methods, depending on the format of the and/or solution of the equation itself.

II. Present equations that involve several operations to solve (e.g., distribution and combining terms before performing operatons on both sides.) Solicit student discussion about methods used to solve the equations.

Sample equations: page 92; exercises 36, 40, 42, 52, 56, and 60.

III. Present situations in which a formula is given, and values are known for some of the variables (given in word problem format). Give students opportunities to express their ideas about how to do the problems, and then how solutions for the remaining equation can be found.

Sample problems: page 92; exercises 62, 64, and 66.

Homework: page 92; exercises 35, 39, 43, 45 - 59, odds; 61 - 63, odds.

Experimental Lesson 12

Objective: To provide opportunities for the student to change English phrases to algebraic expressions.

I. Review meanings for sum, difference, product, and quotient.

II. In order to help students translate from work phrases to symbolic notation, begin with arithmetic examples, in table form, and move to the algebraic form. For example:
If phrase is:

4 less than a number	let number be	value of the phrase
	6	6 - 4 or 2
	11	11 - 4 or 7
	25	25 - 4 or 21
	n	n - 4

III. With the students do several sample problems, discussing along the way.
Sample exercises: page 95; exercises 20 22, 24, 26, 34, 36, 38, 40, 42, 44.

Homework: pages 95 and 96; exercises 29 - 43 odds.

Experimental Lesson 13

Objective: To provide opportunities for students to write equations which exemplify situations stated in word problems.

I. Use homework problems to review the method used in the previous lesson to write algebraic expressions.

II. Use a guess and check approach to solving word problems. First students will guess arithmetic numbers and check with problem situation to see if it satisfies the conditions. If not, then they will make another guess, etc. After a solution is found, then the guess will be x, and an equation is generated which exemplifies the relationship stated in the problem.

Example: One number is five more than a second number. The product of the numbers is 3300. Find the numbers.

Begin with a table .

Guess	Guess plus five	Product	3300
10	15	150	too low
100	105	10500	too high
50	55	2750	too low
55	60	3300	YEAH!
x	x + 5	x (x + 5)	equal to

Using this method, do exercises 5, 8, and 16 on page 100 along with students.

Homework: Use this method to generate an equation for the following exercises on page 100; exercises 9, 11, 13, 15, 17.

Resource for this lesson:

Kysh, J. (1991). Implementing the curriculum and evaluation standards: First-year algebra. Mathematics Teacher, 84, 715-722.

Experimental Lesson 14

Objective: To provide opportunities for students to write equations which exemplify situations stated in word problems.

I. Use homework problems to review the method used in the previous lesson to generate equations for word problem situations.

II. Move to problems which are application problems, and stress the equation, not the solution. Talk about different strategies (e.g., guess and check, make a table, draw and label a diagram, and work backwards). Encourage students to take arithmetic examples and work through the problem if that will help generate an equation.

For example: The length of a field is 40 yards greater than twice the width. If the field is 400 yards long, find the width. (page 100; exercise 32)

Strategies: Draw a figure.
What do we know? Label the figure accordingly.
Make a guess for the width. Use the given relationship to check it out. Use a variable in place of the guess, an write an expression for the length.

Search for the statement of equality in the problem. Use it to write an equation.

III. With the students do exercises 34, 36, 38, 42, on page 100, talking through the strategies that they use to get the equation.

Homework: (Important task is to write an equation which shows the relationship stated in the problem.) pages 100 and 101; exercises 35, 43, 45, 47, and 49.

Experimental Lesson 15

Objective: To establish the meaning of solving a formula for a specified variable and to provide opportunities for students to solve such formulas.

I. Begin with the formula $A = \frac{m+n}{2}$. What relationship does the formula state? Do you recognize the formula? What words could the letters be representing? What variable is the formula solved for? Suppose m is 100 and n is 20, what is the value of A? (Do several more examples.) Suppose A is 25 and m is 8, what is the value of n? (Do several more examples.) Would this calculation be easier if the formula was solved for n? Solve the equation for n, using an equation in the same format.

   Ask the students what they would do to solve the equation on the left. Do the same thing to the problem on the right. Remind them that while we can multiply 2 x 5, we can only **imply** multiplication of 2 x A, and ask how we write it.

$5 = \frac{6 + x}{2}$	$A = \frac{m + n}{2}$
$10 = 6 + x$	$2A = m + n$

   What would you do next to solve the equation on the left? Do the same thing to the equation on the right. Again, talk about subtracting 6 from 10, but implying subtraction of m from 2A.

II. Do several examples, always asking the students what would you do if....? If necessary, present an equation in the same format so that they can see the analogous steps.

   Sample exercises: pages 105 and 106; exercises 2, 8, 12, 16, 28, and 38.

Homework: pages 105 and 106; exercises 1, 5, 9, 15, 17, 21, 27, 31, 35, and 39.

Experimental Lesson 16

Objective: To review the meaning of integral exponents and to provide opportunities so that students can generalize about properties of integral exponents, and how to saimplify expressions by using these properties.

I. Review how symbols are used in algebra to communicate mathematical ideas. Begin with what is meant by 2x, that it is a shorthand way of writing x + x and that repeated addition can be thought of as multiplication, so 2x means 2 times x. There are many shorthand ways of writing quantities in algebra, and it is imperative that we know what these symbols mean so that we can communicate with each other using these symbols.

Ask what x^2 means. x^3? 4^3? Keeping that meaning in mind, what could be a short hand way of writing x^2*x^3? After the property is established, go through the following examples:

x^4*x^5	$x*x^8$
$y*y$	9^3*9^8

Solicit discussion.

Similarly, help the students develop the property of division when exponents are involved.

II. After the division property is established, help the students develop an understanding for zero as an exponent.

$x^5/x^5 = x^0$ by the property just established.

However, x^5/x^5 is apparently 1, therefore x^0 must be 1. Discuss why the base cannot be zero.

III. Similarly, establish the meaning of negative exponents. By using many examples, help students generalize that

$a^{-n} = \frac{1}{a^n}$ and that $\frac{1}{a^{-n}} = a^n$

IV. Do sample problems with students stopping often to ask why and letting students discuss.

Practice problems: pages 119 and 120; even-numbered problems.

Homework: pages 119 and 120; exercises 1-49, odds.

Experimental Lesson 17

Objectives: To review the properties of integral exponents, and provide sample problems so that students can establish properties for power of a power.

I. Review yesterday's lesson by going over homework problems, soliciting discussion from students concerning their methods for finding the correct answers.

II. Help students establish how to simplify a product like $(5x^2)(2x^4)$. Talk about the difference between $5x^2$ and $(5x)^2$. Note that it is just a matter of convention, and that we accept it so that we can communicate. Do several examples, like -3^2 and $(-3)^2$.

Talk about the fact that $(5x^2)(2x^4)$ really means $5*x^2*2*x^4$ and that multiplicaton is commutative, so we can move it be be $5*2*x^2*x^4$, and can therefore be simplified to be $10x^6$.

III. Give the problem $(x^2)^3$ and ask students if there is another way they could write it, using the meaning of the exponent 3. Give several such problems, and try to get students to realize that they could multiply exponents as a short cut.

Give problem $(3x^3)^2$. Do the problem as $3x^3*3x^3$ first. then ask how we could use the short cut, leading them to square the 3 and multiply the exponents. Talk about the fact that 3 is not the base of the cube. Do even-numbered exercises 2 - 12 on page 127.

Homework: page 120; exercises 51 -75 odds
page 127; exercises1 -13 odds.

Experimental Lesson 18

Objective: To review for Test #2

I: Review exponent properties by going homework, fielding questions, and encouraging students to answer each others questions.

II. Review simplifying expressions and solving equations by taking sample problems from Chapter 3 Test on page 111-112. Do the review as a guided practice, letting students have the opportunity to do the problem, then as a whole class activity, give students the opportunity to tell others the methods they used to answer the question.

Sample exercises from pages 111 and 112:

exercises 1 - 11, all; 14 - 16, all; 17 and 18; 22-25, all.

Homework: Review for test. Suggested exercises for review: Chapter 3 Review on page 110.

APPENDIX E

LESSON PLANS FOR CONTROL GROUP

The approved text for the Mat 003 classes is Introductory Algebra (3rd ed.) by Wright and New. When exercises and pages are referenced in the lesson plans, they are from the aforementioned text.

Control Lesson 1

Objectives: Students will learn to determine the order of given numbers, determine if number statements are true or false, and to graph numbers on the number line.

Lecture:

I. Natural numbers (Counting Numbers) Give set notation.

II. Whole numbers (Include 0) Give set notation and on number line.

III. Integers (Whole numbers and their opposites) Give set notation and number line. Three sets: negatives, positives, and zero.

Discuss difference between "minus" and "negative", "plus, and "positive".

Demonstrate graphing sets on integers on number line. Graph both finite and infinite sets.

IV. Rational numbers. Use number line to show numbers between integers. Usually called fractions. Decimals are fractions. Give formal meaning of rational number. Give several examples, and show that they are rational by the definition.

V. Irrational numbers. Numbers which are not rational Examples given. Will discuss in detail later, probably in another course.

VI. Signed numbers. All rational and irrational numbers, positive and negative, can be referred to as signed numbers.

VII. Order of numbers. On the number line, of two numbers, the one on the left is smaller. Give symbols for order relationships. Give examples of each.

Homework: pages 49 and 50; execises 1 - 55, odds.

Control Lesson 2

Objectives: Students will learn to find absolute of numbers, to give solutions to absolute value equations, and to give solutions to absolute value inequalities.

Review by going over homework questions. Work problems which seem to give students most difficulty on the board.

Lecture:

I. Define absolute value as distance from 0. Give examples of the absolute value of specific numbers, covering all situations. Give the symbol for absolute value.

Give a formal definition of absolute: $|a| = \begin{cases} a & \text{if a is positive or zero} \\ -a & \text{if a is negative.} \end{cases}$

Give several examples covering of possible situations of absolute value, using symbols.

II. Demonstrate how to solve absolute value equations.

III. Demonstrate how to solve absolute value inequalities. Use the following exercises as sample problems.

pages 52 and 53: 4, 12, 18, 22, 34, 42, 46.

Homework: pages 52 and 53: 1-19, odds, and 21, 25, 27, 29, 31, 35, 37.

Control Lesson 3

Objectives: Students will learn to add signed numbers, and to determine if given signed numbers are solution for specified equations.

Go over homework problems, working those exercises with which students had difficulty.

Lecture:

I. Demonstrate addition of signed numbers on the number line. Display rules for adding signed numbers on overhead.

   Give several examples of each rule.

II. Use even exercises 2-20 on page 56 as oral exercises, asking the students to give the answers, answering questions as needed and pointing out which rule to use.

III. Demonstrate how to check to see if a given value is the solution to a problem. Do problems 48-54 as sample problems.

IV. Display rules on overhead again, talking through each situation.

Homework: pgs. 56-57, 1, 3, 13, 15, 25 - 39 odds, and 45, 49, and 53.

Control Lesson 4

Objectives: Students will be able to find the additive inverse of a number, subtract signed numbers, and evaluate signed number expressions.

Review previous lesson by going over homework questions.

Lecture:

I. Define: opposite of a number as the additive inverse

II. Give examples to demonstrate $a + (-a) = 0$

III. Give definition of subtraction as a - b = a + (-b). Demonstrate with the following exercises:

5 - 2	-5 - 2
-5 - (-2)	5 - (-2)

In each of the above problems, point out the minus signs, and the negative signs.

IV. As guided practice, do problems 12 -22 and 36,40, 42, and 46 on page 61. Each time, ask them to write the problem, visually changing the subtraction to addition of the inverse.

V. On overhead, show a - b = a + (-b). State the every subtraction should be changed to adding the inverse. Review rules for adding signed numbers.

Homework: page 61: exercises 11-21, odds, and 35-45, odds.

Control Lesson 5

Objectives: Students will learn rules for multiplication and division of signed numbers, and will use these rules in various situations.

Review rules for adding and subtracting signed numbers by giving examples of all possible situations. Go over homework exercises, checking for difficulities among students.

Lecture:

I. Give rules for multiplication of signed numbers (overhead transparency). Give several examples of each rule.

II. Do exercises 1-12 on page 66 as oral exercises, calling on different students to give answer. Point out the rule to use if there are questions.

III. Give rules for division of signed numbers, pointing out that they are the same as rules for multiplication. Give several examples of each rule.

IV. Do exercises 21-31 as oral exercises, calling on different student ot give answer. Point out the rule to use if there are questions.

V. Demonstrate how to determine whether or not a given number is a solution to a given equation by substitution. Work several examples.

VI. Display rules for multiplying and dividing on the overhead. Talk through them again, giving an example for each.

Homework: page 67: exercises 47-55, odds, and 57-65 odds.

Control Lesson 6

Objectives: Review for test next session.

I. Go through homework exercises, answering any questions.

II. Use the odd-numbered exercises in the Chapter 2 Review on page 71 as seat-work problems. Instructor moves about, working with individual students as necessary.

III. Bring class back together. Work examples from the even exercises on page 72 as samples.

Homework: Study for test for next session.

Control Lesson 7

Objectives: Students will learn what terms and expressions are. They will learn to simplify algebraic expressions by combining like terms and to evaluate expressions for given values of the variables.

Lecture:

I. (Transparency) Give examples of terms, like terms, coefficient, and expressions.

II. Give pairs of terms. Tell if they are like terms, and why.

III. Show students how to combine like terms by adding the coefficients. Point out that terms cannot be combined if they are not like terms. Do several examples as demonstration problems.

IV. Review the distributive property and its use in combining terms. Do sample problems.

V. Do exercises 1-12 as guided practice, explaining as needed.

VI. Demonstrate how to do homework problems by doing exercises 16, 24, 28, 34, 44, 50 and 58 on page 78.

Control Lesson 8

Objectives: Students will learn how to solve equations of the form x + b = c or equations which can be simplified to that form.

Review combining terms by going over homework exercises, answering questions as necessary.

Lecture:

I. Define and give examples of first-degree equations.

II. Give and explain the addition property of quality:

A = B and A + C = B + C have the same solutions, and are called equivalent equations.

III. Demonstrate how to use the addition property of equality to solve equation of the form x + b = c. Work several examples.

IV. Demonstrate how combining terms will sometimes change an equation to the desired form. Work several examples.

V. Demonstrate the steps to solving equations by getting the variable term on one side by itself. Do this by adding constant terms and/or variables terms to both sides as needed. Show how to check correctness of solution by substitution.

Homework: page 86: exercises 1-21 odds.

Control Lesson 9

Objectives: Students will learn how to solve equations of the form ax = c or equations which can be combined to that form.

Review: Go over sample homework exercises, reviewing how to solve equations of the form a + c = b.

Lecture:

I. Give and explain the multiplication property of equality:

A = B and AC = BC have the same solutions, and are called equivalent equations.

II. Demonstrate how to use the multiplication property of equality to solve equations. Work several examples being sure that sometimes the coefficient of the variable is an integer, and sometimes the coefficient of the variable is a fraction.

$$3x = 15 \qquad\qquad (3/4)x = 15$$

III. Demonstrate how combining terms will sometimes change an equation to the desired form. Work several examples.

$$10x - 2x = 36 - 100$$

V. Remind always that the goal is to get the variable on one side of the equation, alon, with a coefficient of 1. Show how to check to see if solution is correct by substitution.

Homework: page 86: exercises 23 - 43, odds.

Control Lesson 10

Objective: Students will learn to solve equations of the form $ax + b = c$, or that can be simplified to that form.

Review methods for solving equations of the form $ax = b$ and $a + b = c$. Go over any homework questions.

Lecture:

I. Demonstrate solving any first-degree equation by using the following rules:

(a) Simplify each side of the equation by distributing and combining terms.
(b) Use the addition property of equality to add the opposites of the constants and/or variables to each side so that constants are on one side and variables on the other.
(c) Use the multiplication property of equality to multiply both sides by the reciprocal of the coefficient (or divide both sides by the coefficient).

II. Give students the following equations to solve as guided practice.

$$2x + 4 = x + 5$$
$$4x = 3x + 7$$
$$5x + 3 = 4x + 8$$
$$4x - 3 = x + 3$$
$$3x - 2 = 2x + 1$$

Homework: page 91; exercises 5-29, odds.

Control Lesson 11

Objective: Students will practice solving more difficult first-degree equations (involving distribution and combining terms.)

Review by answering homework questions.

I. Demonstrate solving more difficult equations.

Sample equations: page 92; exercises 36, 40, 42, 52, 56, and 60.

II. Demonstrate how to substitute given values (given in word problem format) in a formula, and solve for the remaining variable.

Sample problems: page 92; exercises 62, 64, 66.

Homework: page 92; exercises 35, 39, 43, 45-59, odds, and 61-63, odds.

Control Lesson 12

Objective: Students will learn to change English phrases into algebraic expressions.

Review by going over homework questions.

Lecture:

I. Review meanings of sum, difference, product, and quotient, and other words which are the key to operations (e.g., more than, less than, twice).

II. Demonstrate how to change from words to algebraic expressions by doing the following same exercises:

Sample exercises: page 95; exercises 20, 22, 24, 26, 34, 36, 38, 40, 42, and 44.

Homework: pages 95 and 96; exercises 29-43, odds.

Control Lesson 13

Objective: Students will learn to solve word problems by writing and solving equations which exemplify the situations in the word problem.

Review changing from words to algebraic expressions by going over homework exercises.

Lecture:

Demonstrate solving word problems by setting up equations which exemplify the conditions stated in the word problem.

Sample problems: page 100; exercises 5, 8, and 16.

Homework: page 100; exercises 9, 11, 13, 15, 17.

Control Lesson 14

Objective: Students will learn to solve application word problems by writing and solving equations.

> Review by going over word problems assigned for homework.
>
> Demonstrate solving application word problems by setting up equations which exemplify the conditions stated in the word problem. Demonstrate drawing diagrams if appropriate.

Sample problems: page 100; problems 34, 36, 38, and 42.

Homework: pages 100 and 101; problems 35, 43, 45, 47, and 49.

Control Lesson 15

Objective: Students will learn to solve formulas for specified variables in terms of the other variable.

Do homework problems on chalkboard, stressing writing the equation that states the conditions.

Lecture:

> Give examples of formulas, and tell students which variable is solved for in each formula. Work several examples to demonstrate how to solve for a different variable.

Sample exercises: pages 105 and 106; exercises 2, 8, 12, 16, 28, and 38.

Homework: pages 105 and 106; exercises 1, 5, 9, 15, 17, 21, 27, 31, 35, and 39.

Control Lesson 16

Objective: Students will learn to simplify expressions by using the properties of integer exponents.

Go over homework questions.

Lecture:

I. Define exponent. Give both arithmetic and algebraic examples of exponents and what they mean. Note placement of parenthesis (e.g., $-3^2 \neq (-3)^2$. Do several examples which illustrate this notation.

II. Property 1 of exponents: If a is a nonzero number and m and are integers, then $a^m * a^n = a^{m+n}$.

Give examples to illustrate the rule.

III. Property 2 of exponents: If a is a nonzero number, then $a^0 = 1$.

Give examples to illustrate the rule.

IV. If a is a nonzero number and n is an integer, then $a^{-n} = (1/a^n)$.

Give examples to illustrate the rule.

V. Property 4 of exponents: If a is a nonzero number and m and n are integers, then $(a^m/a^n) = a^{m-n}$.

Give examples to illustrate the rule.

VI. Do even-numbered exercises on pages 119 and 120 as guided practice.

Homework: pages 119 and 120; 1-49, odds.

Control Lesson 17

Objective: Students will learn to simplify powers of expressions by using the properties of integer exponents.

Go over homework questions.

Lecture:

I. Property 5 of exponents: If a and b are nonzero numbers and n is an integer, then $(ab)^n = a^n b^n$. Give examples which illustrate the property.

II. Summarize all the properties, illustrating how they can all be used in one exercise.

Sample problems: page 127; exercises 2-12.

Homework: page 120; exercises 51-75, odds
page 127; exercises 1-13, odds

Control Lesson 18

Objective: To review for Test #2

I. Review exponent properties by going through homework and answering students' questions.

II. Review simplifying expressions and solving equations by taking sample problems from Chapter 3 Test on page 111-112. Do the review as a guided practice.

Sample exercises from pages 111 and 112; 1-11, all; 14-16, all; 17 and 18; 22-25, all.

Homework: Review for test. Suggested exercises for review: Chapter 3 Review on page 110.

APPENDIX F

Test # 1

Please show all necessary work on this paper. If the space provided is not enough, turn to the back of the page to continue your work.

1. If $|a + 1| = 5$, what are the possible values for a?

2. Suppose a rectangle has area of 24 square units. Write a value for the length and a value for the width of the rectangle. ($A = lw$)

3. Describe this pattern: 3, -6, 12, -24, 48, -96,

4. Graph the following numbers on the number line and label them using the appropriate letter.

   A. -0.4 B. $\frac{7}{3}$ C. $|-3|$ D. -2

5. The sum of -2, -8, 7, and -1 is -4. List three integers whose sum is -5.

6. What is 20% of 835?

7. Find the volume of a cylinder with a diameter of 20 feet and height of 2 feet. Use $\pi = 3.14$. ($V = \pi r^2 h$)

8. Perform the indicated operation. (2)(-6)(-3)

9. Perform the indicated operation. 16 - (-4)

10. Perform the indicated operation. $\frac{-20}{10}$

11. Find the value of the following expression.

    $3\,5 + 21 - 9 + 3$

12. If a b = 0 and a = 10, then what is the value of b?

13. Insert parentheses in the following expression to make the value of the expression be 39.

$$35 + 21 - 9 + 3$$

14. If | a | = 5, what are the possible values for a?

15. If a b = 1 and b = $\frac{3}{4}$, then what is the value of a?

16. Perform the indicated operation. -3 -13 + 20

17. Find the missing numerator.

$$\frac{5}{9} = \frac{\_\_}{108}$$

18. Divide the sum of $\frac{5}{8}$ and $\frac{3}{4}$ by $\frac{3}{2}$.

APPENDIX G

Test #2

Please show all necessary work on this paper. If the space provided is not enough, turn to the back of the page to continue your work.

1. The product of two expressions is a^5. What are the expressions?

2. Solve for y in terms of x. $3x - 4y - 8 = 0$

3. If $2(x + 1) = 12$, what is the value of $x + 1$?

4. Set up an equation and solve.

   A new car costs \$8,125. This is $\frac{1}{3}$ of the yearly salary of a certain student. Find the student's yearly salary.

5. Simplify $\dfrac{8x^5 x^2}{4x^3}$

6. Solve for x. $9x + 4.7 = -3.4$

7. At this college there are 6 times as many students as instructors. If S represents the number of students and I the number of instructors, write an equation using S and I to show the relationship between students and instructors.

8. Evaluate the following expression if $a = -3$, $b = 2$, and $c = -1$.

   $$2a + 3c - ab$$

9. If $3x + k = 5x + 7$, and if $x = -1$, what is the value of k?

10. Set up an equation and solve.

    Fourteen more than three times a number is equal to 6 decreased by the number. Find the number.

11. Solve for x. $-4x = 16$

12. Solve the formula $I = prt$ for r.

13. Solve for x. $8(3 + x) = -4(2x - 6)$

14. Solve for x. $\frac{1}{6}x + \frac{2}{3} = x - 1$

15. Combine like terms. $4(2x^2 - 3x) - 2(x^2 + 2x)$

16. Evaluate.

$$\frac{7^4}{7^5}$$

17. Simplify $(4x^2y)^3$

18. $2^x = 16$ and $4^y = 16$. What is the value of $x + y$?

19. If $ax + b = c$, give values for a, b, and c so that the solution of the equation is 2.

APPENDIX H

TRANSFER AND SKILL-BASED ITEMS

Each of the test consisted of both transfer and skill-based items. The items in $Test_1$ which were considered transfer items were items 1, 2, 3, 5, 12, 13, and 15. In $Test_2$ the items which were considered transfer items were 1, 3, 7, 9, 18, and 19. The remainder of the test items were considered skill-based items.

APPENDIX I

Directions for Attitude Scales

On the following two pages are twenty four statements. There are no correct answers for these statements. They have been set up in a way which permits you to indicate the extent to which you agree or disagree with the ideas expressed. Suppose the statement is:

	A	B	C	D	E
Example 1. I like mathematics.					

As you read the statement, you will know whether you agree or disagree. If you strongly agree, check under A on the scale beside the statement. If you agree but with reservations, that is, you do not fully agree, check under B. If you disagree with the idea, indicate the extent to which you disagree by checking under D for disagree or check E if you strongly disagree. But if you neither agree nor disagree, that is, you are not certain, check under C for undecided. Also, if you cannot answer a question, check under C. Now mark your answer. Do the same for example No. 2.

	A	B	C	D	E
Example 2. Math is very interesting to me.					

Do not spend much time with any statement, but be sure to answer every statement. Work fast but carefully.

There are no "right" or "wrong" answers. The only correct responses are those that are true for you. Whenever possible, let the things that have happened to you help you make a choice.

THIS INVENTORY IS BEING USED FOR RESEARCH PURPOSES ONLY AND NO ONE WILL KNOW WHAT YOUR RESPONSES ARE.

ATTITUDE SCALES

		A B C D E
1.	Once I start trying to work on a math puzzle, I find it hard to stop.	| | | | | |
2.	When a math problem arises that I can't immediately solve, I stick with it until I have the solution.	| | | | | |
3.	I am challenged by math problems I can't understand immediately.	| | | | | |
4.	I would rather have someone give me the solution to a difficult math problem than to have to work it out for myself.	| | | | | |
5.	The challenge of math problems does not appeal to me.	| | | | | |
6.	I do as little work in math as possible.	| | | | | |
7.	I have a lot of self-confidence when it comes to math	| | | | | |
8.	When a question is left unanswered in math class, I continue to think about it afterward.	| | | | | |
9.	I think I could handle more difficult mathematics.	| | | | | |
10.	I am sure I could do advanced work in mathematics.	| | | | | |
11.	I can get good grades in mathematics.	| | | | | |
12.	I am sure that I can learn mathematics.	| | | | | |

13.	I'm not the type to do well in math.	A B C D E
14.	I don't think I could do advanced mathematics.	A B C D E
15.	For some reason even though I study, math seems unusually hard for me.	A B C D E
16.	Generally I have felt secure about attempting mathematics.	A B C D E
17.	I like math puzzles.	A B C D E
18.	Figuring our mathematical problems does not appeal to me.	A B C D E
19.	Math has been my worst subject.	A B C D E
20.	Mathematics is enjoyable and stimulating to me.	A B C D E
21.	Most subjects I can handle O. K., but I have a knack for flubbing up math.	A B C D E
22.	Math puzzles are boring.	A B C D E
23.	I'm no good in math.	A B C D E
24.	I don't understand how some people can spend so much time on math and seem to enjoy it.	A B C D E

APPENDIX J

Interview Questions

Question 1

If $2(x + 1) = 12$, what is the value of $x + 1$?

Question 2

If $\frac{14}{x + 2} = 2$, solve for x.

Question 3

Make up an equation in the form $ax + b = c$ so that the value of x is 2.

Question 4

$7W + 22 = 109$

$7N + 22 = 109$

Question 5

At this college there are 6 times as many students as intructors. If I represents the number of instructors and S represents the number of students, write an equation which will show the relationship between the number of instructors and the number of students.