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The relationship between load and elasticity in the power squat

Bird, Michael, Ph.D.

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U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

The Relationship Between Load and Elasticity in the Power Squat

by

Michael Bird

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The purpose of this study was to examine the relationship between load and musculoskeletal elasticity in the power squat. Eight male subjects experienced in the power squat participated in this study (mean height: 1.756±0.072 m; mean mass: 77.5±10.4 kg). Subjects were videotaped performing a countermovement squat (CMS) and a purely concentric squat (PCS). Both the CMS and PCS were performed at four load percentages (40%, 55%, 70%, and 85%) of the subject's tested one repetition maximum (mean maxima: 166.9±51.9 kg). Segmental data were digitized, reduced to selected mechanical variables, and analyzed with repeated measures ANOVAs (α=0.05). Results for concentric time indicated significant main effects for condition (CMS or PCS) and load percentage and a significant interaction between condition and load. Lifters required greater amounts of concentric time in the PCS and at higher loads. The interaction indicated that the subjects required exponentially greater amounts of time at heavier PCS loads than heavier CMS loads. Average concentric work and average concentric power had significant main effects for both condition and load percentage; average work and power were greater in the CMS condition and less at the heaviest load. A significant main effect for load percentage was found for maximum concentric velocity, net concentric work on the system, and energy; velocities decreased with increased relative loads; net work increased as load percentage increased; and energy increased with increasing load. Elastic energy did not change with load. The variability of the elastic energy measure suggested that it was influenced by the subject's performance, the task characteristics, or both: The subjects' training regimes (i.e. heavy or light weights) may have influenced their performance favorably or detrimentally at different loads. The task may have influenced the elastic energy measure since the concentric phase could be completed anywhere within

the upward thrust, not necessarily at the top, as other tasks (e.g. jumping) require. In conclusion, although the measure of elastic energy was confounded in the present study, its mechanical benefits were still apparent in a loaded activity.

APPROVAL PAGE

This dissertation has been approved by the following committee of the Faculty of The Graduate School at The University of North Carolina at Greensboro.

Dissertation Advisor Jackie Hudson

Committee Members Kathleen ________

Diane L. Gill

Stephen C. Rafford

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Chapter I

INTRODUCTION

It is not difficult to recognize that athletes invest much time, effort, and expense to gain a competitive edge in their performance. New and often costly products are continually offered to athletes as performance enhancements. Ironically, some athletes may be overlooking simpler means to performance improvement. Rather than investing their resources elsewhere, athletes may only need to analyze and modify the mechanics of their performance to achieve considerable improvement. One advantage that athletes often fail to utilize is the inherent elastic properties of their musculoskeletal systems.

What is elasticity? It can be defined as the spring-like response to a stimulus. Specifically, it is the ability to deform and then return to an initial position. One example of this is a rubber band. Rubber bands have the ability to be stretched and then vigorously rebound to their initial size (if they are not stretched too far). The physical characteristics of the rubber band allow it to store the energy of the stretch and then return it later as it regains its initial position. The amount of energy stored is directly proportional to the amount of stretch and the stiffness/compliance of the rubber band.

While the human musculoskeletal system does not contain rubber bands, it does have elements that act, under certain circumstances, with elastic characteristics. For the musculature to act elastically a movement must have a period of eccentric braking of one action followed *immediately* by a concentric thrust of the opposite action (see Appendix A for additional descriptions of eccentric and concentric motion). In other words, the initial movement is counter to the direction of the subsequent primary movement, and the muscles which stop the countermovement are prestretched prior to initiating the primary movement. Even though a countermovement is considered necessary for eliciting elastic behavior, it is not sufficient unless the reversal from countermovement to the primary movement is immediate.

Several sport skills, such as weightlifting, jumping, striking, throwing, and running all have countermovements. When these countermovements are reversed quickly, the use of elastic energy is likely. Consider the following scenario from weightlifting: A person performs five successive repetitions of a bench press at 80% of maximum (for one repetition). The first repetition, which is started from chest level, involves no countermovement and therefore has no elasticity. The second repetition has a countermovement (from the descent of the weight after the first repetition) but minimal elasticity if the exercise is performed with the recommended slow, deliberate technique. The fifth (and last) repetition, if it is performed in a state of fatigue, may include a bounce at the bottom as the downward motion is quickly reversed to an upward motion. In this final lift, the fading energy of the muscle may be augmented by elastic energy. Of course, a performer does not have to be in a fatigued state to induce an elastic response.

To date, research has not provided conclusive evidence indicating which physiological components within musculature have elastic capabilities. Hypothesized elastic energy storage sites include ligaments, tendons, muscle sheaths, and the myosin heads within the muscle fibers. The present study provides no additional evidence to this cause, but instead seeks further understanding of the characteristics and benefits of elasticity within human motion.

The benefits of elastic energy to countermovement activities were noted by Asmussen and Bonde-Peterson (1974b) when discussing the results of their study on vertical jumping:

> When a countermovement was performed...a certain amount of [negative] energy...was implanted into the body in excess of the energy liberated by the muscle contractions, which are assumed to be maximal. Part of this must have degenerated into heat, but another part most probably was absorbed by the elastic components of the muscles, so that less of the energy subsequently liberated by the muscles was wasted as internal work. As a consequence more energy was available for external work, resulting in a greater [performance outcome]. (p.388-389)

The benefits of elasticity to performance outcome have since been studied by other researchers (Cavagna, 1977; Chapman, 1980; Chapman, Caldwell, & Selbie, 1985; deHaan, Van Ingen Schenau, Ettema, Huijing, & Lodder, 1989; Hudson and Owen, 1985; Shorten, 1987; Van Ingen Schenau, 1984; Wilson, Elliott, & Wood, 1991). The traditional paradigm involves the comparison of a primary movement with a countermovement to the same primary movement without a countermovement. When the countermovement was quickly reversed, favorable metabolic and mechanical performance outcomes resulted in the countermovement condition. The metabolic benefits to the countermovement included reduced muscular energy requirements, greater efficiency, greater work, and greater power. The mechanical benefits included greater concentric velocity, reduced time to peak concentric velocity, greater force, reduced time to peak force, greater work, greater power, and greater energy. In short, these performance enhancements, both metabolic and mechanical, are thought to reflect the use of elastic energy within the movement.

Because benefits to performance appear to exist due to elasticity, what characteristics of movement are associated with the use of elastic energy? According to

Cavagna (1977), "the amount of mechanical energy stored and re-utilized depends on the mechanics of the exercise" (p. 89). This statement can be expanded to include both the general characteristics of the task and the individual variations in technique employed by each performer (Hudson, 1986). In other words, certain characteristics of performance are correlated with the use of elastic energy. The time between eccentric and concentric phases is one such variable. Due to the degenerative nature of elastic energy in humans, it seems that greater amounts of time between eccentric and concentric phases results in lower elastic energy use and benefit (Aruin, Prilutski, Raitsin, & Savel 'ev, 1978; Wilson et al., 1991).

Also, the eccentric displacement (or range of motion) in the countermovement has been shown to influence elastic energy use (Bosco & Komi, 1979; Bosco, Tihanyi, Komi, Fekete, & Apor, 1982; Cavagna, 1977; Chapman, Caldwell, & Selbie, 1985; deHaan et al., 1989; Joyce, Rack, & Ross, 1974; Thys, Cavagna, & Margaria, 1975). Depending on the task, eccentric displacement may reduce or increase elastic energy benefits. While greater eccentric displacement has been negatively correlated with elastic energy benefits, no research indicates how less than "optimal" eccentric displacements would affect elastic energy and its benefits.

Peak eccentric velocity has also been shown to influence elastic energy use (Bosco, Komi, & Ito, 1981; Edman, Elzinga, & Noble, 1978; Thys et al., 1975). Specifically, higher peak eccentric velocities (up to a point) seem to be positively correlated with greater elastic energy use. Additionally, it seems that the intersegmental coordination of the performer may influence the amount of elastic energy used (Hudson, 1986). For example, when the optimal intersegmental coordination of a skill is theorized to be simultaneous (all of the limbs begin the concentric thrust at the same time), but the subject is early or late with some of the segments involved, the effectiveness of elastic energy use may be minimized or neutralized.

Much of the contemporary research on elastic energy has focused on the issues of time, range of motion, velocity, and intersegmental coordination in movement. No evidence, however, has been provided regarding how well elastic energy is utilized with different eccentric forces. Modifying load is one way of changing the eccentric forces. This change in load may change the behavior of the elastic elements that are stretched in the eccentric phase of the movement. Further, any changes in the behavior of elastic elements may lead directly to changes in elastic energy as well as indirectly to changes in variables which are correlated with the use of elastic energy.

In general, the influence of load on elastic behavior may be evident in two ways. First, the addition of load may affect the relationship between the countermovement and non-countermovement variations of the task. As with unloaded activities, significant differences between countermovement and non-countermovement conditions would be taken as evidence of elasticity. Second, the amount of load, from moderate to heavy, may influence the extent of elastic contribution. That is, as load increases, the evidence of elasticity may increase or decrease. Of course, load may also affect performance independent of elasticity.

The main effects of condition (countermovement or non-countermovement) and load (moderate to heavy) as well as the interaction of condition and load on elasticity can be studied with several variables. Time is one variable that may be modified with a countermovement. It is expected that with additional load the amount of time in the concentric phase of the activity will decrease due to the additional work performed by elastic structures in the body. Also, it is expected that subjects will require greater amounts of time as the load increases due to the additional effort needed.

Greater velocity is expected in the conditions involving a countermovement because of its benefit from the recoil of elastic structures (Thys et al., 1972). Lesser velocity is expected, however, with greater loads due to the limitations of the force-velocity relationship. That is, greater loads will require greater forces and thus result in lesser velocities (Hill, 1938).

The mechanical work performed is expected to increase with the use of a countermovement (Cavagna, 1968; Chapman, 1980; Chapman & Caldwell, 1985; Thys et al., 1972; Wilson et al., 1991). Also, the amount of work performed is expected to increase as load increases and velocity decreases (Hill, 1970). Power is expected to increase with the use of a countermovement (Bosco & Komi, 1979; Cavagna et al., 1971; Cavagna, 1977 Thys et al., 1972). No evidence indicates how power will change as a function of load.

Peak force values are expected to increase with the use of a countermovement due to the recoil of elastic elements (Bosco & Komi, 1979; Cavagna & Citterio, 1974; Thys et al., 1972). Also, peak force is expected to increase with load due to the inherent requirements of performing lifts of heavier weights. The force-velocity relationship is expected to shift along both force and velocity axes due to elastic energy use. This is expected because elastic recoil will provide both additional force and additional velocity to the lifter's performance outcome. Elastic energy, the primary characteristic in this study, is expected to change as a function of load, but no evidence exists indicating how.

No prediction can be made at this time regarding the interaction of condition and load with any variable. The evidence of elasticity may be greater at either moderate or heavy loads. Alternatively, the evidence of elasticity may be consistent across moderate to heavy loads.

In order to test the relationship between load and elasticity a weightlifting task will be used. In weightlifting, the load (amount of weight) that is used in a particular lift is easily modified. Specifically, the power squat is a dynamic weightlifting event that allows changes in load relative to the performance capability of the lifter. Moreover, skilled lifters tend to perform the power squat with a relatively quick reversal at the bottom of the lift.

Therefore, the purpose of this study is to examine the relationship between load and elastic behavior in the power squat. Several variables which represent the benefits correlated with elastic energy use will be analyzed as a function of condition and load. Also, an actual characteristic of countermovements, elastic energy, will be examined as a function of load. Specifically, the following hypotheses will be tested.

Hypotheses

I. Time.

a. Time will decrease with the use of a countermovement.

b. Time will increase with greater loads.

II. Velocity.

a. Velocity will increase with the use of a countermovement.

b. Velocity will decrease with greater loads.

III. Work (average and net).

a. Work will increase with the use of a countermovement.

b. Work will increase with greater loads.

IV. Power (average and peak).

a. Power will increase with the use of a countermovement.

b. Power will not change with greater loads.

V. Force.

a. Force will increase with the use of a countermovement.

b. Force will increase with greater loads.

VI. Force-Velocity Relationship.

a. The force-velocity relationship will shift horizontally and vertically with the use of a countermovement.

VII. Elastic Energy.

a. Elastic Energy will not change with load.

Chapter II

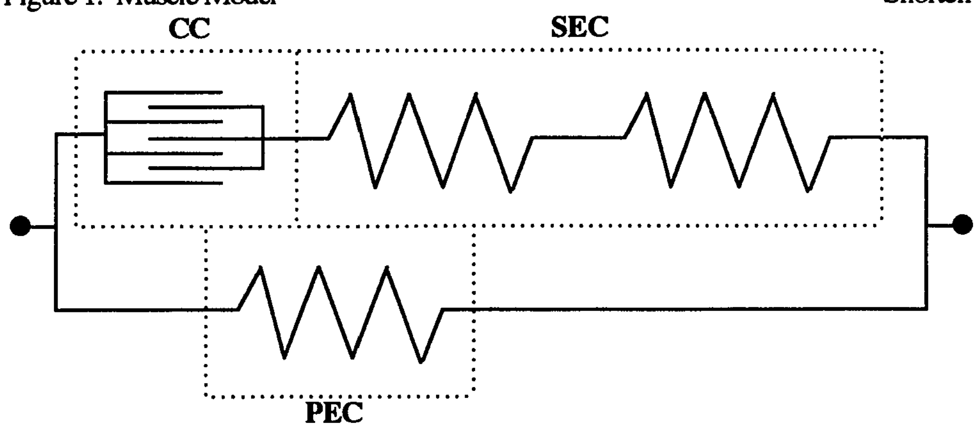
REVIEW

In explicating the issues surrounding elastic energy it is useful to begin with a model of the muscle. A structural model is provided to aid the discussion of where elastic energy originates within the muscle. A functional muscle model (i.e., a mass-spring model) is included to explain how elastic energy is stored and returned. Also, an alternative, non-elastic explanation is presented. Next, the characteristics of movement (e.g., range of motion, velocity) that contribute to elastic energy are evaluated. Then, the issues of what performance outcomes can benefit from elastic energy are assessed. The mechanical variables of interest include velocity, work, power, efficiency, and load. Finally, the primary issue of this study, load and its effect on elastic energy utilization, is reviewed.

Structural Muscle Model

A brief overview of the structural muscle model originally developed by Hill (1938) is provided in the following section. An adaptation of this model as used by Shorten (1987) is provided in Figure 1.

Figure 1. Muscle Model Shorten (1987)



This muscle model consists of three components. The contractile component (CC) provides the force generating processes that enable the muscle to contract and thus the body to move. Structurally, the thick and thin filaments of the CC are located within the sarcomeres of each myofibril of each muscle fiber. The thick filaments temporarily attach and reattach to the thin filaments via the myosin heads which protrude from the thick filaments. These connections have a mechanical function and are referred to as crossbridges. As the myosin heads uncouple from their current attachment sites, there is a chemical reaction (i.e., adenosinetriphosphate [ATP] is broken down to adenosinediphosphate [ADP]) and heat is released. Note that the uncoupling of a myosin head from an attachment site does not have to occur with an actual shortening of the muscle. It may occur as part of the crossbridge cycling that happens when muscles are tensed in isometric positions.

The other two components, the series elastic component (SEC) and the parallel elastic component (PEC), represent the nominal elastic structures within the model. These components are presented according to their geometric relationship with the CC in Figure 1. The SEC is described as joining end-to-end with the CC. The largest elements of the

SEC are tendon and other connective tissue. These elastic structures are tensed passively when the CC is tensed. The PEC is described as joining side-to-side with the CC. The largest elements of the PEC are muscle fascia and other connective tissue. To a limited extent, the PEC is stretched when the CC is stretched.

The actual pliant elements to elastic behavior are currently only theorized. Research to date has not provided conclusive evidence for any particular component or even any particular part of any component. The use of varying methodologies and specimens has produced conflicting results, subsequently leading to a confusing situation, at best. Shorten's (1987) review reflects many current beliefs. At this time, most believe that elastic behavior exhibited in muscles comes from a number of locations including tendons, crossbridges, myofibrils, and connective tissue. Each of these possibilities may have specific advantages or disadvantages, but none has been found false.

Currently, all of the elements that make up the SEC are considered as possible contributors to elastic behavior. Because the CC is tensed during eccentric braking, the pliant elements of the SEC would be passively tensed at the same time. Despite the stretching of the CC during eccentric braking, the passive elastic elements of the PEC are not believed to be a large contributor to elastic behavior. The problem associated with all of the passive elements, however, is that no explanation has been provided for the relatively short life of the tension generated.

The idea of the crossbridges within the sarcomere contributing to elastic energy has drawn recent attention (Bosco et al., 1982; Edman et al., 1978; Shorten, 1987). This type of elasticity is active rather than passive (Shorten) because it is within the contractile machinery and relies on the maintenance of the crossbridges for utilization of the elastic component. Bosco et al. believed that not only is the elastic energy stored in the crossbridges, but is specifically stored in the myosin heads as they are rotated backwards. This concept is appealing for two reasons. One is that it may help explain the transient nature of the elastic advantage. Elastic behavior seems to have a half-life between one and four seconds (the transient characteristic of elastic behavior will be discussed shortly). Another appeal of this concept is the release of heat that occurs when the energy is lost (Hill, 1961). That is, the loss of heat with the breakdown of ATP in an isometric contraction may be seen as a loss in elastic energy that could have been used in a subsequent movement.

Functional (Mass-Spring) Muscle Model

Rather than using a structural model, many researchers prefer to use a functional, mass-spring model to describe the elastic characteristics of musculature (Aruin et al., 1978; Bosco & Komi, 1979; Cavagna, 1970; Cavagna, 1977; Cavagna & Citterio, 1974; Cavagna, Citterio, & Jacini, 1981; deHaan et al., 1989; Edman et al., 1978; Joyce et al., 1974; Lensel-Corbeil & Goubel, 1990; Shorten, 1987; Van Ingen Schenau, 1984; Wells, 1967). Of course, "neither muscle nor tendon behaves like a perfect spring, but both possess mechanical properties that can be described by relatively simple elastic models" (Shorten, p. 1). In other words, just as the mass-spring system has specific laws which govern its reactions, so does the muscle.

In its simple form the mass-spring model consists of a spring and an oscillating mass. The spring has a constant stiffness and can be deformed a finite linear distance. The deformation of the spring is accomplished by the application of a force. When applying the mass-spring model to muscle, the elastic elements are considered to be like a spring. Specifically, the elastic elements have a stiffness and an ability to store energy that can be later recovered. Forces that are imparted to the body, either externally by inertial or gravitational forces or internally by muscular forces, have the ability to deform the elastic structures within the muscles and store energy. After certain conditions, such as eccentric prestretching of muscle followed immediately by a muscle contraction, this stored energy can be applied to a subsequent concentric contraction of that muscle.

As in any spring, the amount of energy that can be stored within the elastic elements is proportional to the stiffness of the elements and the square of the deformation of the elements (Aruin et al., 1978). Because the characteristics of the spring must be maintained by the elastic elements within the muscles, it should be possible to find out the stiffness of these elements. This value is crucial because a muscle system's ability to store beneficial amounts of elastic energy depends on its stiffness, as well as its inverse, compliance. Understanding the relationship between stiffness and compliance is important when considering elasticity. Elastic structures which have high stiffness are not easily deformable and therefore are not able to store large amounts of elastic energy without the application of large forces. Elastic structures which are compliant, however, are easily deformable and therefore able to store more energy, especially under the application of lesser forces.

The elastic structures within muscles must be able to handle the wide variety of forces (both internal and external) that are applied to it. As stated by Cavagna et al. (1981), "the contracted muscles behave mainly as elastic bodies and require a compliant structure capable of storing a large amount of energy during stretching without attaining excessively high and dangerous force values" (p. 131). Clearly, the muscles would benefit from a dynamic solution to this problem. While the stiffness of a spring is considered constant, the stiffness of the elastic elements within muscles may not be.

Some researchers (Aruin et al., 1978) argued that the stiffness constant of muscles remained the same across different loads (forces). This claim has not been supported by other research. Cavagna (1977), Cavagna and Citterio (1974), Cavagna et al. (1981), Lensel-Corbeil and Goubel (1990), and Van Ingen Schenau (1984) all argue for changes in compliance or elasticity as the mechanics involved in the movement change. Specifically, Cavagna and Citterio found evidence for an increase in the compliance of the muscle during rapid stretching. Cavagna (1977) and Van Ingen Schenau advocated the decrease in compliance (or the increase in stiffness) with increasing force.

The work of Lensel-Corbeil and Goubel (1990) specifically addressed the issue of stiffness and compliance in frog muscles. They found changes in the muscle stiffness with respect to velocity of stretch, amplitude of stretch, and initial length of the muscle. Under some conditions the stiffness of the muscle's elastic structures increased, and under other conditions the compliance of the muscle's elastic structures increased. More specifically, as the velocity of stretch increased the stiffness increased. Also, as the velocity of the stretch decreased, the compliance decreased, but only if the amplitude of the movement was smaller. If the amplitude of the movement was greater, then as the velocity of the stretch decreased the compliance increased.

These conclusions are important because they imply that the muscle utilizes a dynamic solution to the problems and dangers of constant stiffness of the elastic structures. In the words of Cavagna et al. (1981), the elastic properties of muscle "change according to need" (p. 140). That is, the elastic structures can alter stiffness during stretch-shorten cycles depending on the mechanical parameters present.

Alternative, Non-Elastic Explanation

While the elastic-like behavior of muscles is generally believed to come from one or more energy storing elements within the musculoskeletal system, other explanations have been offered for the advantages gained by a prestretch or countermovement. The most common alternative explanation is neuromuscular in nature. It is often referred to as reflex potentiation (Cavagna, 1977; deHaan et al., 1989), stretch reflex (Chapman et al., 1985; Shorten, 1987), or activation of the contractile system (Bosco & Komi, 1979; Cavagna, 1977; Cavagna & Citterio, 1974; Edman et al., 1978). Regardless of the terminology used, the concept is the same. The idea is that in the countermovement the muscles to be used in the concentric contraction are activated prior to the primary movement. This activation serves to bring the muscle to a more prepared state (i.e. more motor units are recruited) earlier in the concentric movement, thus resulting in performance benefits in the concentric phase.

Several researchers (Asmussen & Bonde-Peterson, 1974b; Auro & Komi, 1986; Bosco & Komi, 1979; Cavagna & Citterio, 1974; Chapman et al., 1985; deHaan et al., 1989; Edman et al., 1978; Shorten, 1987; Thys et al., 1972) have examined the role of contractile system activation in countermovement activities. In particular, the effect of a countermovement on electromyographic (EMG) activity has been studied. If the EMG activity of the muscle is increased with the use of a countermovement, then the increased activity would contradict claims that the advantages gained by a countermovement are due to an increased use of stored elastic energy.

Auro and Komi (1986) found that integrated EMG values were lower in eccentric exercises than in concentric exercises. This means that exercises that work the muscles eccentrically are not activating the muscles to the same degree as exercises that work the same muscles concentrically. Thus, muscles are activated to a greater degree in concentric exercises whether or not there is a countermovement. (Interestingly, if the eccentric activation is too low, there may not be sufficient force to load the elastic elements.)

Thys et al. (1972) also found high electrical activity during concentric movements. In fact, they found that electrical activity within the muscles was maximal in the first half of the concentric phase for both the static and countermovement conditions. What was less expected, however, was that the greater forces exerted in the countermovement condition were generated with less electrical activity over a shorter period of time. In short, the use of a countermovement does not seem to increase neuromuscular activation, and may, in fact, decrease it.

In a different approach to the neuromuscular question, Cavagna and Citterio (1974) found that curare did not affect the results of their experiments on the prestretch of frog muscles. The lack of effect is seen as an indication that there is an elastic behavior exhibited by the muscles that is not dependent on neuromuscular transmission. Also, Edman et al. found that their "biochemical data did not support the view that force enhancement during stretch is based on an increase in activation of the contractile system" (p.152). In sum, no researcher has provided conclusive evidence that the advantages gained by prestretch are due to a change in the electrical stimulation of the system rather than elasticity.

There is little doubt that the neuromuscular activation of the muscles plays at least some role in the elastic behavior of the muscles (deHaan et al., 1989; Shorten, 1987). The exact role, however, of neuromuscular activation is difficult to identify. As stated by Chapman et al. (1985) "the part played by reflex enhancement of contractile properties is difficult to predict during stretch due to the fact that force begins rising at the onset and that variable amplitudes of muscle stretch are possible" (p. 79). Given the difficulties of measurement and lack of experimental evidence for increased neuromuscular activation, it is commonly believed that the elastic elements within the musculature are the largest contributors to the elastic-like behavior exhibited after prestretch. Regardless of the source of enhancement (elastic or neuromuscular) the appropriate use of a prestretch is a benefit to performance.

Range of Motion

What is an appropriate prestretch? The amplitude of stretch or range of motion (ROM) is one factor that influences elastic behavior. Generally, to take full advantage of the elastic characteristics of muscles the amplitude of the movement should optimize the stiffness/compliance values. This will allow the greatest amount of deformation of the elastic elements to take place, and thereby, maximize the amount of elastic energy that can be stored and then returned in a subsequent movement.

In the literature ROM has been, not surprisingly, one variable that seems to influence the amount of elastic energy utilized (Bosco & Komi, 1979; Bosco et al., 1982; Cavagna, 1977; Chapman et al. 1985; deHaan et al., 1989; Joyce et al., 1974; Thys et al., 1975). For example, Thys et al. found that in hopping, as opposed to successive deep knee flexions, there was a much more limited shortening of the muscles, yet there was a higher utilization of elastic energy. Generally, research has shown that smaller amplitudes of eccentric work have a large effect on elastic energy use, but large amplitudes of eccentric work have lesser effects on elastic energy use (Cavagna; Chapman et al.; deHaan, et. al.).

Prestretch Velocity

Another aspect of the prestretch which can affect elastic behavior is prestretch velocity. While none of the previously mentioned studies included the effect of reduced amplitude on prestretch velocity, Chapman et al. (1985) found that the proportional difference between stretch velocity and resting velocity decreased as the amplitude of the stretch increased. A few researchers have found that higher prestretch velocities are associated with greater utilization of elastic energy (Bosco et al., 1981; Edman et al., 1978; Thys et al., 1975). Also, Bosco et al. found the velocity of the prestretch to be correlated with greater jumping performance.

Transient Behavior

While the velocity of the prestretch is important in the loading of the elastic elements, the delay time between the eccentric and concentric contractions is probably *the* most critical determinant of the amount of elastic energy provided in the subsequent concentric contractions. The elastic elements within muscles seem to have a transient characteristic (Aruin et al., 1978; Asmussen & Bonde-Peterson, 1974a; Bosco et al., 1981; Cavagna, 1977; Cavagna & Citterio, 1974; Cavagna et al., 1968; Cavagna et al., 1975; Chapman et al., 1985; Edman et al., 1978; Lensel-Corbeil & Goubel, 1990; Shorten, 1987; Thys et al., 1972; Wilson et al., 1991). That is, if a "too long" length of time elapses between the eccentric prestretch and the concentric contraction of the muscle, the elastic energy stored is lost as heat (Hill, 1961). How long is too long? The values for the half-life of elastic energy storage range from one or two seconds to seven seconds. The most consistently found values seem to indicate that four seconds is a good representation of the half-life of elastic energy storage (Aruin et al.; Shorten; Thys et al.; Wilson et al.). The decay of elastic energy as a function of time is represented in Equations (1) and (2) from Aruin et al. and Wilson et al., respectively.

$$E' = E * e^{-t/x} \qquad (1)$$

In this equation E' is the elastic energy stored at a given time, E is the energy at time zero (before any decay has taken place), t is the time of the muscle being stretched (measured from the onset of eccentric movement), and x is the relaxation constant. Based on this equation, after 5.88 seconds the amount of elastic energy stored is negligible (Aruin et al., 1978).

$$y = 100 + a * e^{-bt} \qquad (2)$$

In this equation y is the elastic energy stored at a given time, a and b are constants, and t is the pause duration between eccentric and concentric movements. Based on this equation, 4 seconds of delay between the eccentric and concentric movement would be sufficient to ensure that movements were performed without prestretch benefits (Wilson et al., 1991).

Part of the difference in the ability of the two equations to predict elastic energy decay is due to the differences between how the time value, t, is measured. Equation 1 is less effective because time must be measured from the initial eccentric movement and because the initial amount of elastic energy stored must be known. Equation 2 is more effective because time must be measured from the end of eccentric motion, and determining the end of eccentric motion is easily done. Also, Equation 2 is a better representation of the decay of elastic energy based on its lower residual sums of squares values. Equation 2 is perhaps more easily interpreted as well.

In equation 2 it is clear that as the time between the end of eccentric movement and the beginning of concentric movement approaches zero, the amount of elastic energy that is provided for the contracting muscle approaches the amount that is stored. It would be advantageous for the performer to move as quickly as possible, thus maximizing the amount of elastic return. The advantage of being able to move quickly from eccentric to concentric movement is evident in Wilson et al's. (1991) study of the bench press. The athletes performing the press had average delays of either 0.6 seconds or 1.27 seconds between eccentric and concentric phases. When there were delays in beginning the concentric portion of the movement, the force impulse was significantly reduced in the beginning of the concentric portion of the movement. The impulse found in the longer delay condition was similar to the impulse found in the static version (concentric phase only) of the lift. Even after the 1.27 second delay, however, they found elastic energy advantages gained from the prior loading of the elastic elements.

Force

The use of less muscle activity (as represented by lower EMG levels) to produce greater forces is an important part of the argument for the presence of elastic elements within our muscle systems. As the use of elastic energy increases so do the forces generated in the subsequent concentric phase of the movement. The examination of the forces (and torques) generated from muscle contractions has been measured for non-humans and humans alike. Edman et al. (1978) found that in frog muscles the force level during the prestretch of the elastic elements was dependent on the velocity of the prestretch and was not proportional to the overlap of the thick and thin filaments within the muscle fiber (i.e., length of the muscle). Furthermore, Edman et al. found that at higher velocities of prestretch the forces leveled off or rose slowly after their initial peak. This change in the response of the muscle could be due to the contribution of elastic energy being more apparent at the beginning of the contraction and the contractile elements of the muscle not being able to react fast enough to the recoil of elastic elements. In another animal study, Cavagna and Citterio (1974) found that frog muscles could attain $1^1/_2$-2 times greater force at a given length with a prestretch than at the same length under isometric conditions.

The advantages of prestretch have been demonstrated for humans as well. Bosco and Komi (1979) and Thys et al. (1972) found that ground reaction forces were enhanced with a preliminary countermovement. Chapman et al. (1985) found similar results in peak torque values: The countermovement condition resulted in greater peak torque values compared to the isometric condition. Wilson et al. (1991) also found greater elastic energy benefits and higher initial forces with shorter delays between the eccentric and concentric movements. Thus, the preceding researchers have verified a key point made by Cavagna et al. (1968): Not only does prestretch lead to greater force development but also to greater initial force development. Greater earlier forces are important when their effects are seen later in the movement.

Concentric Velocity

According to Cavagna et al. (1971), one advantage of greater forces at the beginning of the concentric movement is a greater acceleration and thus a shorter time to reach a given velocity. Included among the given velocities, of course, would be the maximum velocity of the movement. The data of Chapman and Caldwell (1985) confirm this point: maximum angular velocity was reached about 20% sooner when a countermovement was involved. The most striking difference in velocity between the countermovement and non-countermovement conditions occurred in the first 0.1 seconds of concentric movement. During this span the angular velocity for the countermovement condition was roughly double that of the non-countermovement condition. Indeed, as time goes on, the velocity advantage due to the countermovement either disappears or dissipates. Chapman and Caldwell found no difference in maximum angular velocity between countermovement and non-countermovement conditions in forearm supination. The subjects of Cavagna et al. (1971) showed a 2-12% improvement (mean = 6%) in jumping velocity with a countermovement. Thys et al. (1972) found a velocity increase of 2-41% (mean = 20%) in lifting the body from a deep flexion when there was a countermovement. This higher velocity, according to Thys et al., represented the sum of the speed of shortening of the contractile components and the speed of shortening of the series elastic elements stretched in the eccentric phase of the movement.

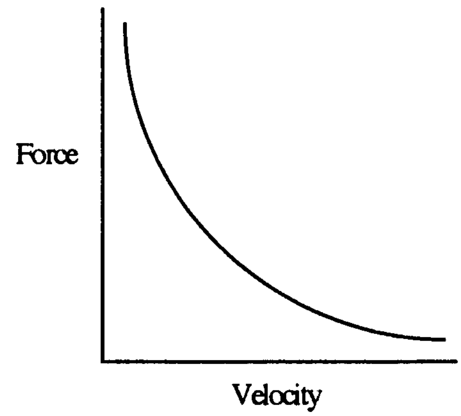
As mentioned previously, there may be difficulty utilizing all of the benefits of elastic recoil if the elastic elements contract at a faster rate than the contractile components are capable of contracting. Would this contraction rate vary from person to person? Perhaps it does. The Thys et al. (1972) study in particular showed a large variability among subjects.

Partial evidence for individual differences in contraction rate was provided by Bosco et al. (1982). They examined the relationship of elastic energy use and fast twitch (FT) and slow twitch (ST) fiber type. The subjects with predominantly FT fibers used a greater amount of elastic energy in the shorter time, shallow crouch jumps, and the subjects with predominantly ST fibers used a greater amount of elastic energy in the longer time, deep crouch jumps. The result of FT fibers utilizing greater amounts of elastic energy in the shallow, fast condition is not surprising. Fast twitch fibers are distinguished by faster recruitment of motor units and an increased number of motor units means more crossbridges and more elastic energy storage sites if the hypothesis that elastic energy can be stored in the cross bridges is valid. Further, greater results in the concentric phase of the movement could have been due to the FT fibers ability to 'keep up' or maintain the velocity started by elastic recoil. The ST fiber subjects were perhaps able to use more elastic energy in the deep, slow condition because ST fibers are able to retain crossbridge attachment for a longer period of time.

Force-Velocity Relationship

The relationship between concentric force and velocity is one that has been reliably found since it was discussed by Hill (1938). This relationship is shown in Figure 2.

Figure 2. Force-Velocity Curve. (Hill, 1938)



Does this relationship hold with respect to elastic energy return? This question has been examined by several researchers (Bosco & Komi, 1979; Cavagna & Citterio, 1974; Chapman et al., 1985; Edman et al., 1978). The resulting curves for the non-countermovement (non-elastic) condition were similar to the force-velocity curve shown above. (Chapman et al. also found a similar curve using torques and angular velocities.) In the countermovement (elastic) condition, however, the force-velocity curve was shifted along the velocity axis. Thus, with the same amount of force developed, the resulting velocity was greater.

Work

The greater forces and velocities produced with the use of elastic energy are beneficial to other mechanical aspects of movement. One mechanical aspect that is enhanced through the use of elastic recoil is work. In the mechanical sense, work (W) can be defined as the displacement (d) of a mass times the resultant force acting on it. (See equation 3).

$$W = F * d \tag{3}$$

This equation is deceptively simple. Work itself is a complicated variable that must be understood by its relationship with other mechanical variables. For example, the work performed can also be represented as the change in kinetic energy. (See equation 4).

$$W = \frac{1}{2}mv^2_2 - \frac{1}{2}mv^2_1 \tag{4}$$

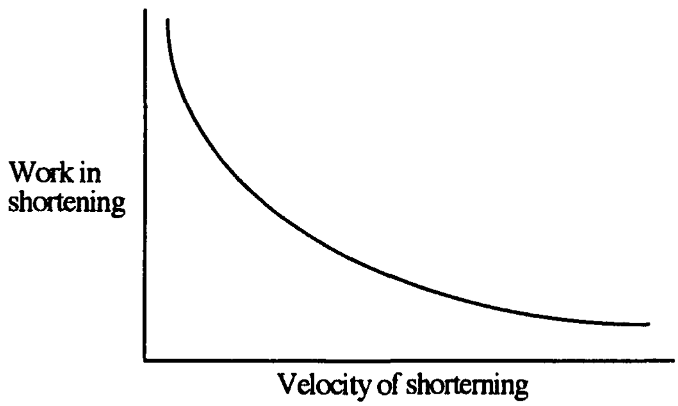
In the above equation, m represents the mass of the object and v represents the velocity of the object.

Given that force and velocity are central to work and given that the use of elastic energy leads to a greater force and velocity in the concentric phase of the movement, one would expect that the use of elastic energy would lead to greater work. Indeed, Cavagna (1968) found that active muscles that shorten following forcible stretching are able to do more work than possible when shortening at the same speed from a state of isometric contraction. Also, Chapman (1980) found that the wind-up in a rotational motion enhanced the work performed in inertial loading by a factor of 1.56 times.

The larger amount of work performed is seen immediately in movements involving a prestretch. During the first 0.1 seconds of the concentric phase in the countermovement condition there is a greater rate of performing work (Chapman & Caldwell, 1985; Wilson et al., 1991). After the first second the rate of work performed is the same as that in the non-countermovement condition.

Hill (1970) discussed the relationship between work in shortening and speed of shortening. As a result of the shift in the force-velocity curve previously described, the greater velocities associated with elastic recoil correspond with greater muscular forces instead of lesser muscular forces. With greater muscular forces there is a greater acceleration of the mass of the system, and therefore a greater change in the velocity of the system. Thus, more work can be performed in a given amount of time with the use of elastic energy. This shift along the velocity axis is depicted in Figure 3.

Figure 3. Work-Velocity Relationship. (Hill, 1970, p. 77)



Power

Cavagna et al. (1971) found that not only was the positive work done with a prestretch 10% greater than the work done without a prestretch, but the time of positive work was 55% greater without a prestretch of the muscle. This leads to the next mechanical aspect of movement benefiting from a prestretch, power. Mechanical power is the time rate of doing work. (See equation 5).

$$P = W/t = F * v \qquad (5)$$

Work and power can be considered as either instantaneous or average. Instantaneous work and power are calculated for relatively short intervals of time (much less than one second). They represent power and work performed at that moment.

Average work and power are different from instantaneous work and power because they represent the work and power over an entire movement.

Average power increased by 70% with a prestretch over movements without a prestretch (Cavagna et al., 1971). Other researchers (Bosco & Komi, 1979; Thys et al., 1972) also found an increase in the mechanical power as a result of a prestretch. According to Cavagna (1977), the effect of previous stretching is higher power output (average power) and not a greater amount of positive work (average work) performed. This increase in power is due to the increase in the speed of the whole muscle shortening and thus the speed of the concentric movement (Thys et al.).

Efficiency

Because the average work performed in the concentric phase of a given task would be the same (you move the same distance both with and without a prestretch), other variables may help explain the benefits gained with the use of a prestretch. For instance, efficiency is often mentioned (Aruin et al., 1978; Auro & Komi, 1986; Asmussen & Bonde-Peterson, 1974a; Cavagna, 1977; Cavagna, 1981; Lensel-Corbeil & Goubel, 1990) in this regard. Cavagna (1977) defined mechanical efficiency as the ratio of the positive work produced to the energy expended in producing that work. (See equation 6).

$$\text{mechanical efficiency} = \frac{\text{positive work produced}}{\text{energy expended}} \qquad (6)$$

Although the variable is easily defined, it is difficult to calculate because the energy expended must be found through the subtraction of a baseline measure (Auro & Komi, 1986). The use of various definitions and measurements for the baseline values produces variations in the values calculated for mechanical efficiency. Because of this it is difficult to compare the few studies involving efficiency and elastic energy. Some general findings are, however, pertinent. Aruin et al. (1978) observed no differences in the efficiency of knee bending with and without rebound. They are the only researchers who did not claim the converse. Thys et al. (1972) found that stretch-shorten cycle activity resulted in a more efficient performance based on oxygen consumption during the activity. Asmussen and Bonde-Peterson (1974a) found much higher efficiencies in the rebound condition (39-41%) than in the no-rebound condition (22-26%). Lensel-Corbeil and Goubel (1990) found an improved efficiency with active prestretch in frog muscles. They attributed the difference in efficiency to the recoil of elastic elements, which remain free of energy cost, adding to the energy of the contraction. In sum, the increases in mechanical efficiency above maximal values when prestretch occurs is evidence that a portion of the positive work measured does not derive from the transformation of chemical energy and is free due to the recoil of tense elastic elements (Cavagna, 1977).

Load

With few exceptions the preceding results were derived from experimental tasks with no external load. The effect of a change in the load, however, may alter many of the mechanical aspects of the movement on both kinematic and kinetic levels. For example, the overall efficiency of the movement would decrease with increases in the load or the intensity of the exercise. Despite such a decrease in efficiency, there may not be a decrease in the utilization of elastic energy.

The fact that Aruin et al. (1978) found contrary results in efficiency could be due in part to the effect of load. That is, higher intensity tasks recruit more FT fibers which are inherently less efficient than ST fibers (Auro & Komi, 1986). In addition to varying load, Aruin et al. varied the pause duration in the reversal of the lift as well as the depth of the lift. Both pause duration and depth had an effect on the energy returned in the concentric phase of the movement. Clearly, these variables confound any investigation on the effect of load on energy return and should be controlled or added to the analysis.

The resulting forces and velocities of movement have also been investigated as a function of load. Bober et al. (1980) found that improvements in resultant velocities of the performance were inversely proportional to the load increases. The velocities in Bober et al. did improve, up to a limiting value, with the increasing load. Improvements in velocity as the load increases are not expected according to the force velocity relationship discussed earlier. What is expected is that as the load increases so must the force necessary to produce the *same* velocity, let alone an the increase in force necessary for an *increase* in the velocity. The increase found by Bober et al. could have been due to an increase in the elastic energy brought on by a change in compliance with the greater load or by the increase in the number of elastic elements used. It is also possible that the hypothesized shift in the force-velocity curve only occurs as the eccentric load increases. Further, the shift that occurs in the force-velocity curve could be two dimensional, not one dimensional (along the velocity axis) as previously discussed. It is possible that the shift occurs along both the velocity and force axes, thus shifting the curve outward and upward from its original position.

The issue of stiffness/compliance with load has drawn limited attention. Aruin et al. (1978) varied external load but found no differences in the stiffness values that they calculated. Cavagna (1970), on the other hand, found that compliance decreased (or stiffness increased) with load. If a muscle's "capacity to store elastic energy is a function of the applied force and the compliance of the muscle-tendon complex" (p. 328), then the load could have both a positive and negative influence on the storage of elastic energy. That is, greater loads should increase the applied force yet decrease the compliance. If the load becomes too great, there is an additional negative influence on the recovery of elastic energy: Cavagna and Citterio (1974) pointed out that the crossbridges existing at the end of the prestretch are subjected to forces greater than those corresponding to the maximum isometric contraction and have a propensity to break under higher strain.

Only one researcher has provided relatively direct evidence regarding the effect of load on elasticity. Using an animal model, Wells (1967) applied various loads and measured the velocity of retraction in a countermovement. Given that the velocities remained constant while the load increased, the kinetic energy had to increase with load. Thus, more mechanical or elastic energy was associated with heavier load.

Elastic Energy Measurement

Several methods for assessing the contribution of elastic structures can be found in the literature. While the most direct way to evaluate elastic energy use is with energy values, indirect assessments of elastic energy are the most common. For example, Thys et al. (1972) used average power to reflect elastic energy use. They found that jumps with a rebound produced 14%-49% greater average power than jumps without a rebound.

Peak force has also been used to evaluate elastic energy use. Bosco et al. (1982) used the difference between the peak concentric force of a countermovement jump and static jump (potentiation effect) divided by the force at the end of eccentric motion to evaluate elastic energy benefit. They found that jumps with a countermovement had 17%-30% greater peak forces than jumps without a countermovement. Cavagna and Citterio (1974) found that striated frog muscles were able to generate 1.5-2.0 times greater force from a prestretch than from an isometric contraction. Wilson et al. (1991) used the ratio of a countermovement bench press impulse to a purely concentric bench press impulse to evaluate the use of elastic energy and its decay. They found that elastic energy benefit, as reflected by the impulse force in the bench press, improved by 6.6%-18.7% on the average, depending on the amount of time between the eccentric and concentric phases.

In the preceding studies with indirect assessment of elastic energy, the critical feature was that a task with elastic contribution (e.g., rebound, countermovement, prestretch) was compared to a task without elastic contribution (e.g., no rebound, static, isometric). Given such a task structure, average power, peak force, impulse force, and other indirect variables (e.g., time, velocity) are assumed to reflect the use of elastic energy. Therefore, the use of variables other than energy to evaluate elastic energy use is not unjustified, so long as the variables chosen are correlated with elastic energy use or accurately reflect elastic energy utilization.

A few researchers of elasticity have used more direct measurements of energy. That is, potential and/or kinetic energy have been combined in various ways to represent elastic energy. Asmussen and Bonde-Peterson (1974a) used the ratio of the change in kinetic energy between the baseline static condition and different countermovement conditions to the peak negative energy of the countermovement condition. They found that the mean percentage of improvement due to elastic energy ranged from 3%-22% for countermovement jumps of various heights. A similar method was used by Komi and Bosco (1978). Their values, however, had a much larger range (49%-91%). No explanations were offered for the greater magnitudes of elastic energy contribution.

Hudson and Owen (1985) extended the method of calculating the energy benefits of elastic structures used by Asmussen and Bonde-Peterson (1974a) and Komi and Bosco (1978). In their experiment on vertical jumping they used the total energy of the body as a sum of the potential energy, translational kinetic energy, and rotational kinetic energy of all the segments in the body. Eccentric and concentric energy values were obtained by summing the changes in total body energy for the entire eccentric or concentric motion, calculated from frame to frame. The equation for evaluating the use of stored elastic energy was a ratio identical to that used by Asmussen and Bonde-Peterson (1974a) and Komi and Bosco (1978). The subjects in the study by Hudson and Owen (1985) had average elastic energy values of 37% and 51% for the groups involved.

The preferred method of assessing elastic energy use is some evaluation of energy values. Clearly, other methods and variables can be used to assess the benefits of elastic energy, but those values are correlational and subject to errors regarding the differences in the actual amount of benefit accrued from elastic structures. If possible, a method similar to that incorporated by Hudson and Owen (1985) should best indicate the contribution of elasticity. Otherwise, a method similar to that of Bosco et al. (1982) or Wilson et al. (1991) should be appropriate.

Chapter III

METHODS

"One way of investigating the possible function of the elastic component in muscle is to compare the release of external mechanical energy without and with a previous stretching of the involved muscles" (Asmussen & Bonde-Peterson, 1974b, p. 385). With this in mind, an appropriate task to measure the effects of load on elastic energy use is the power squat.

Task

The power squat is a multijoint, lower extremity exercise. It is performed with a weight bar that rests across spines of the scapulae on the upper back (Lombardi, 1989). Usually the hands are also on the bar for the purpose of maintaining balance of the weight. The movement is performed with the back kept relatively upright at all times. For safety reasons involving support of the lower trunk, most lifters perform the squat with some device (usually a belt) tightly worn around the abdominal and lumbar regions of the trunk.

In keeping with standard practice, each subject used a weight belt in performing all lifts. No other devices used to enhance the lifter's capabilities, such as knee wraps or lifting suits, were allowed. To further insure the safety of the subject, all lifts were performed with the use of a power rack designed specifically for power squats. The rack was adjusted for each lifter, with safety bars at the lowest levels of the lift and an adjustable mainstay for the bar near the upright position of the lifter. All lifts were performed with an experienced spotter for the safety of the subject.

Two types of the power squats were used in this study. The traditional power squat was performed with a downward movement (crouch) and an upward movement (thrust). It was started in an upright position with the bar on the shoulders. The crouch was the portion of the lift from the beginning of downward movement until the lowest point that the subject reached. The thrust started at the end of the crouch and finished when the lifter was once again upright. Because the thrust immediately followed the crouch, this type of squat was termed the countermovement squat (CMS). By contrast, in the purely concentric squat (PCS) the subject only performed the concentric or thrust phase of the squat, starting from the bottom of the squat with the bar sitting on the rack. See Figure 4 for a representation of the CMS and Figure 5 for a representation of the PCS.

Figure 4. Countermovement Squat (CMS)

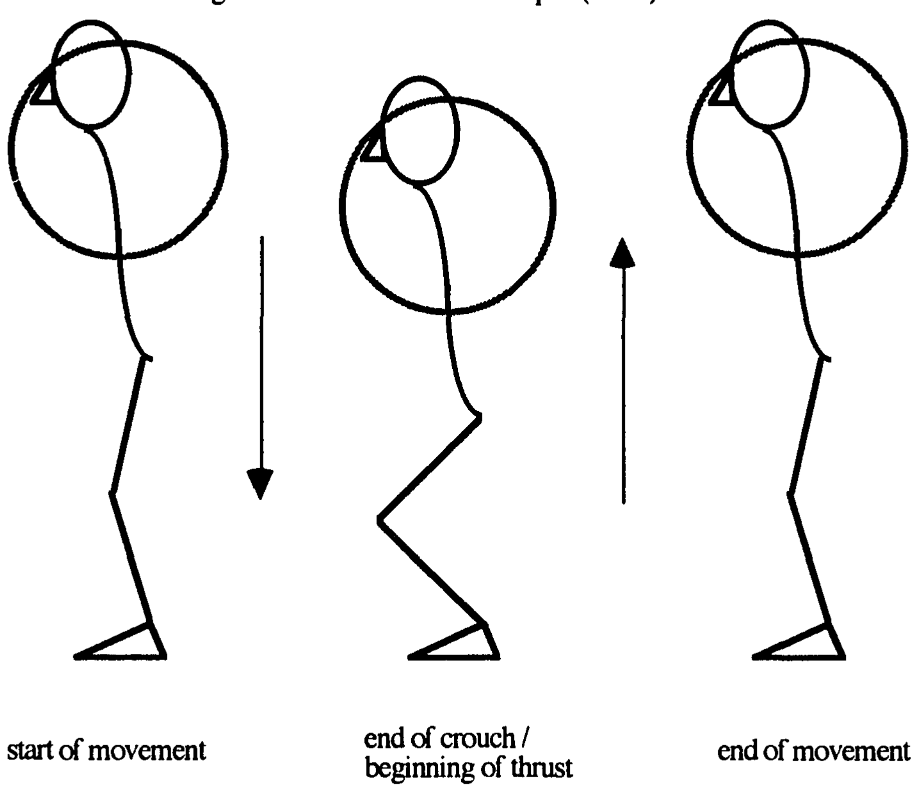
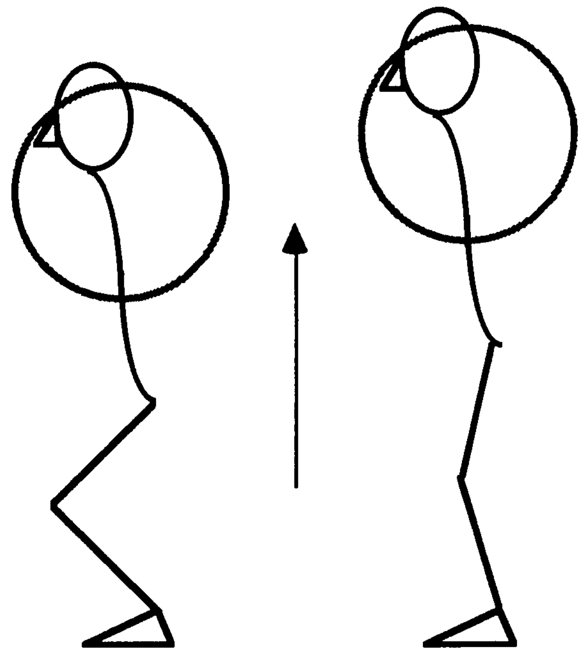


Figure 5. Purely Concentric Squat (PCS)



start of movement / beginning of thrust

end of movement

Each subject performed a countermovement squat as well as a purely concentric squat for each of four load percentages. All subjects crouched the same relative depth, ending at a knee angle of about 90 degrees. Thus, range of motion was standardized for each subject regardless of the condition or relative load.

The time between the eccentric and concentric motions is a critical determinant of the amount of elastic benefit gained. It is critical because elastic benefit is lost exponentially (see equation 1 and equation 2 in Chapter II) with the delay between eccentric and concentric phases (Aruin et al., 1978; Wilson et al., 1991). Thus, controlling for the time between phases is important for standardizing differences between conditions. Wilson et al. (1991) suggested that some stretch benefits may last as long as four seconds before dissipation as heat. Consequently, the present study required five seconds or more between the end of the downward movement and the beginning of the upward movement in the PCS to minimize the elastic energy benefit. During the five second delay the lifter was allowed to rest the bar on the rack, but not to change body position in relation to the bar.

Subjects

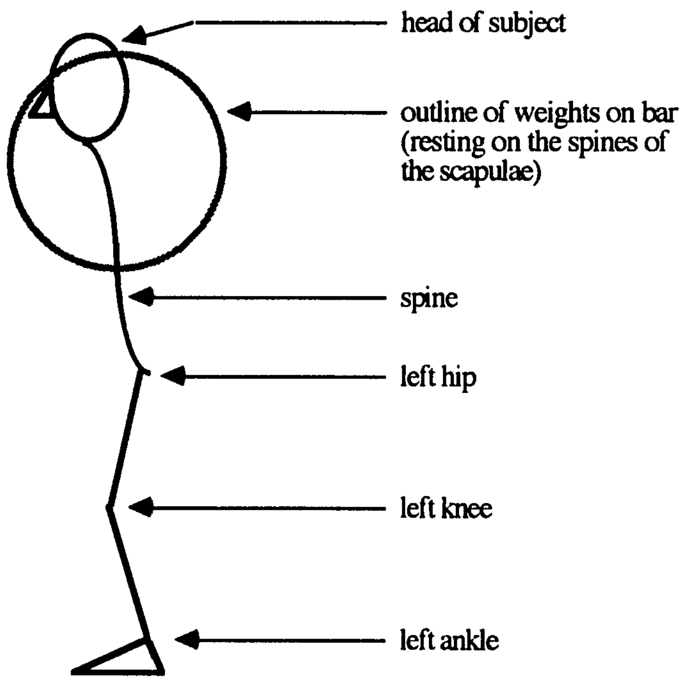
Subjects were primarily recruited from Greensboro area fitness centers and activity classes at the University of North Carolina at Greensboro. All subjects were in good apparent health, with no reported history of chronic or recent acute back, knee, or leg injuries. Every lifter trained with weights to some degree, but in widely differing types of resistance training programs. All lifters were experienced and well practiced in power squat techniques.

Of the fourteen subjects (twelve males and 2 females) from which data were collected, several were removed from the analysis at different points for various reasons. One subject was eliminated due to his inability (or lack of desire) to perform the required 1 repetition maximum (1 RM). Another subject was dropped during data reduction due to a framing error during videotape data collection. Four subjects were eliminated during statistical procedures. These subjects were removed because the statistical procedure required a complete data set for the subject to be included. Three of these subjects had incomplete data due to errors in data collection and one subject failed to perform the lifts correctly and subsequently had several trials removed. Eight subjects (all male) had complete data and were used for analysis.

Testing Protocol

Prior to data collection, each subject signed informed consent in accordance with the approved standards set forth by the University of North Carolina at Greensboro (see Appendix B). Anthropometric measures (height, weight (mass), thigh length, and shank length) were taken for each subject. The joint centers of the hip (greater trochanter), knee (lateral epicondyle), and ankle (lateral malleolus) were marked on the left side of the body with reflective markers. The bar center also was similarly marked with reflective tape. See Figure 6 for a representation of the stick figure.

Figure 6. Sample Lifter Stick Figure



Subjects then performed their preferred warm up in preparation for testing. They were provided with any bar weights necessary to prepare. When ready, each subject was tested to establish a 1 RM countermovement lift. Each subject then performed two lifts (one CMS and one PCS) at each of the following percentages of their 1 RM: 85%, 70%, 55%, and 40%. The order of the eight lifts was randomized for each subject by load percentage and condition (CMS or PCS) to reduce the effect of fatigue. As suggested by Wilson et al. (1991) each subject was asked to perform the movement at as high a force as possible. Following each lift the subject was allowed as much recovery time as desired (5 to 10 minutes was typical).

A pilot study validated the use of 85% as the maximum percentage that lifters could reliably perform in the PCS condition (suggested by Wilson et al., 1991). The subject's responses in performing the lifts also verified the appropriateness of the 85% load. Some lifters required more than one attempt to perform the 85% PCS lift and some seemed to require more recovery time following this lift (although recovery time was neither regulated nor measured).

Force Plate Data Collection

Ground reaction force data (in three dimensions) were collected from a Kistler force plate (type 9281B) operating at a sampling rate of 500 Hz. Prior to each lift the force plate was reset to zero. The recording of the force data was manually activated via specialized computer programs just before each subject's initiation of downward movement. The raw data were converted from analog to digital via a Kistler 9861A electronic unit, scaled, and stored on a Macintosh II computer. Force data were further analyzed and manipulated via specialized programs in BASIC and FORTRAN.

Videography and Digitization

All subjects were videotaped with a Panasonic camcorder (model PV-330D) at a rate of 30 Hz. Blur was eliminated through the use of a $^{1}/_{2000}$ second high-speed shutter.

The camera was positioned perpendicular to the sagittal plane of the subject. A 2 m scale with 0.5 m checkered sections was placed in the sagittal plane of movement and videotaped prior to data collection to provide a scale factor.

Data were reduced from video to digital horizontal and vertical coordinates representing relevant joint landmarks and the bar via a Peak Performance Motion Analysis System. This system grabbed and split each frame into two fields to provide an effective 60 Hz rate. Data were digitized with both an automatic digitizing program and manual manipulation of a cursor. The automated digitizing process searched a predetermined area of each field for the reflective markings placed on the subject. In the event of a marker not being located the point was manually digitized for that field. Several points, such as the top of the head, elbow, and shoulder were digitized manually because marker placement was impractical or impossible.

Digitized data were adjusted to a fixed origin within the field (to eliminate frame shifting and vibration errors inherent to the camera and data collection procedure) and scaled within the FORTRAN data smoothing program (see Appendix C). A four segment model was used to represent the rigid link system of the human body (Plagenhoef, Evans & Abdelnour, 1983). Segments in this model were: (1) the trunk, head, and neck, (2) the thigh, (3) the shank and foot, and (4) the bar. This model was used for calculation of the radii of gyration and moment of inertia for each segment as well as the center of mass (COM) of the subject. Segmental angles (angles with respect to the horizontal) were also calculated for each segment of the model.

Data Smoothing

A quintic spline smoothing program was incorporated for eliminating random errors in the digitized joint center data and the segmental angle data. Spline smoothing is an effective way to eliminate most of the noise inherent to the digitizing process and to retain most of the meaningful signal for further analysis (Wood, 1982). Spline smoothing works on the principle of connecting the least number of small curves in series to represent the actual signal and connect these small curves with knots. Cubic splines have been most commonly used, but have the disadvantage of forcing the second derivative to zero at the beginning and end of the data. The result of the second derivative (acceleration) data being forced to zero influences not only that derivative, but the two previous derivatives also. A quintic spline will force the data for the fourth derivative to zero at the ends. The result is that data at the null, first and second derivatives, which were of greatest interest here, were insignificantly influenced by the forced-to-zero action of the fourth derivative.

Effectiveness of the smoothing routine was assessed through visual examination of the second derivative and the residual pattern of the difference between the raw and smoothed data points (Zernicke, Caldwell, & Roberts, 1976). Sufficient smoothing of the data resulted in a residual plot without pattern and a second derivative (acceleration) with a definite pattern. The desired contour of the second derivative pattern was neither "too smooth" nor "too coarse". Input parameters of the spline smoothing subroutine were adjusted manually or iteratively until consistent results were found from trial to trial and subject to subject. The smoothing routine provided several variables as output, such as the number of knots used and the mean square error. These variables were monitored for all trials and subjects to assess smoothing consistency. Residual graphs and second derivative graphs of a random selection of trials also verified the consistency of the smoothing routine. Spline smoothing output was used to calculate smoothed position data (the null derivative), velocity data (the first derivative), and acceleration data (the second derivative) of all horizontal coordinates, vertical coordinates, and angles.

Biomechanical Variables

All biomechanical variables were computed from either force plate data or videotape data via specialized FORTRAN programs (see Appendix D). All kinematic variables represent the characteristics of the COM of the subject and load. All kinetic and energetic variables represent the characteristics (linear and angular) of the four segment model described above. The following methods were used to calculate the needed variables:

1. Vertical force data were taken from a specialized BASIC computer program interfaced with the Kistler equipment.

2. Displacements, velocities, and accelerations can be calculated from the force data or from the video data. In using the vertical force method the manner in which the equations were developed is shown below. (F is vertical force, m is the known mass of the subject and load, a is the vertical acceleration, v is the vertical velocity, and s is the vertical displacement).

$F = ma$	Newton's second law
$F/m = a$	dividing by the mass
$F/m = dv/dt$	acceleration is the derivative of velocity
$F/m\ dt = dv$	multiplying by dt
$\int F/m\ dt = \int dv$	integrating
$(F/m)\ t = v$	equation for vertical velocity
$\iint F/m\ dt = \iint ds$	double integrating for displacement

$(F/m)\ t^2 = s$ equation for vertical displacement

The above equations for vertical velocity and vertical displacement were used to compute the respective variables for each consecutive pair of values in the array of vertical force data. The time is known by the sampling frequency used in the data collection (500 Hz).

The procedure for using the video method was the converse of the above. Instead of integrating to reduce the data the derivatives were used. Displacements, velocities, and accelerations were calculated from the output of the spline smoothing routine. The manner in which the equations for vertical, linear data were developed is shown below. (*t* represents the time within the equation or function, *p* represents the position of the coordinate, *v* represents velocity, and *a* represents acceleration). Angular equations were developed in an analogous manner.

$$dy = \Delta p_y(t)$$

$$v_y = \frac{dy}{dt}$$

$$a_y = \frac{dv_y}{dt}$$

The computation of all variables above were performed for each array of smoothed horizontal, vertical, and angular data. Time was known by the sampling frequency used in the data collection (60 Hz).

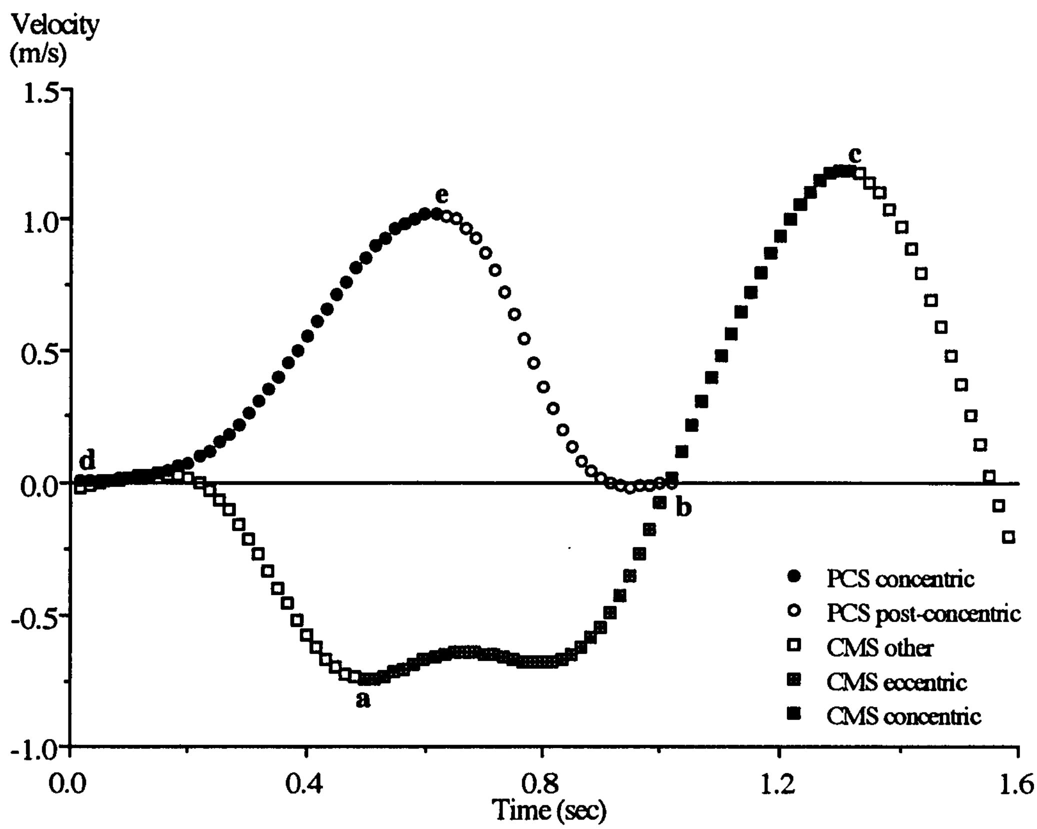
3. The vertical velocity array for the COM of the system was used to identify the eccentric and concentric phases of the countermovement squat as well as the

concentric phase of the purely concentric squat. Specifically, the start of the eccentric phase was identified by searching the COM vertical velocity array from the beginning to the minimum (negative) value. The end of the eccentric phase was subsequently detected by searching from the beginning of the eccentric phase until where and when the velocity reached zero. In the CMS condition the end of the eccentric phase also marked the beginning of the concentric phase. From the beginning of the concentric phase the array search continued until the maximum (positive) velocity was located, indicating the end of the concentric phase.

By definition, the purely concentric squat has no eccentric phase. The start of the concentric phase was identified by the minimum non-negative vertical velocity of the COM in the array of PCS vertical velocities (usually a value at or near zero). The array was subsequently searched from that point until the maximum (positive) velocity was located, indicating the end of the concentric phase.

Figure 7 has sample velocity-time curves for the CMS and PCS. These curves were used to locate the beginning and ending of the relevant eccentric and concentric phases. Note the lack of negative values in the PCS curve, an indication that no countermovement occurred.

Figure 7. Sample Velocity-Time Curve Indicating Phases of CMS and PCS



a -minimum CMS velocity, beginning of eccentric phase
b -zero CMS velocity, end of eccentric phase & beginning of concentric phase
c -maximum CMS velocity, end of concentric phase
d -beginning of PCS concentric phase
e -maximum PCS velocity, end of concentric phase

4. The relative upward displacement that the COM of the system moved during the concentric phase was measured as a percentage of the overall upward, vertical motion of the COM. This measure controlled for differences in the vertical movement ranges of the lifters. Because the concentric phase of the task does not necessarily end at the end of the upward thrust, this variable was used to monitor differences between conditions and lifters.

5. Time to minimum vertical force was measured from the beginning of movement (when the vertical forces began to decline) to the occurrence of minimum vertical force. Time to maximum vertical force was measured from the beginning of the concentric phase until peak force occurred.

Eccentric time was calculated for the CMS condition by using the time from the start of eccentric braking (minimum vertical velocity) to the end of eccentric braking (zero velocity). Concentric time was calculated in a similar manner for both CMS and PCS conditions. In both conditions concentric time was calculated as the difference in time between the start of concentric motion (zero velocity) and the end of concentric motion (maximum vertical velocity). (See Figure 7 for sample velocity-time curves.)

6. Minimum (negative) vertical velocity of the COM of the system for the CMS condition was identified as the start of the eccentric phase and located by the method described in #3 above. Similarly, the maximum (positive) vertical velocities of the COM of the system for the CMS and PCS conditions marked the end of the concentric phases of each respective lift and were also located by the method described in #3 above. (See Figure 7 for sample velocity-time curves.)

7. Work was computed for net, average, and non-dimensional values. Net work of the system was calculated as the change in mechanical energy of the four segment model from the beginning of the phase (eccentric or concentric in the CMS, concentric in the PCS) to the end of the phase. The equation below was used to calculate the net work performed by each of the four segments in each respective phase. In the equation m represents mass, v represents velocity

in the x (horizontal) or y (vertical) direction, g represents the gravitational constant, and h represents the height of the COM of the particular segment.

$$W_{net} = (\tfrac{1}{2} mv^2_{x\text{-}end} - \tfrac{1}{2} mv^2_{x\text{-}beg}) + (\tfrac{1}{2} mv^2_{y\text{-}end} - \tfrac{1}{2} mv^2_{y\text{-}beg}) + (mgh_{end} - mgh_{beg})$$

The overall net work performed by the system was then computed as the sum of work performed by each of the four segments. Net work can also be calculated as the overall change in the instantaneous work values for every frame in both lifting conditions.

Average work represented the mean change in mechanical energy between each frame in the respective phase. Average work was computed by calculating the instantaneous work values (from equation 4 in Chapter II, $W_{inst} = [\tfrac{1}{2}mv^2_2 - \tfrac{1}{2}mv^2_1] - [mgh_2 - mgh_1]$, see above for variable identification) for each frame. These values were then summed for the entire phase and divided by the number of samples in the phase, as shown below.

$$W_{avg} = (\sum W_{inst}) / n$$

Non-dimensional work was analyzed because of the possibility that the different heights and masses of the various lifters could confound the calculated work values. Lifters of greater height and mass (including the bar) were expected to perform greater amounts of work. Non-dimensional work was originally described by Garhammer (1979) as a representation of the work done on the bar during an Olympic lift. This variable controlled for different amounts of work being performed by lifters of different heights by dividing the net work performed on the bar by the height and mass of the lifter (see equation below).

$$W_{\text{non-dimensional(bar)}} = W_{\text{(bar)}} \,/\, \text{lifter height and mass}$$

This equation was used in an analogous manner to the above equation for net work, but only considers the bar segment, and not the other three segments of the system.

8. Instantaneous power was calculated over the entire array of instantaneous work values using equation 5 from Chapter II, $P_i = W_{inst} / t$. Minimum eccentric power was found by examination of the CMS instantaneous power array for the lowest (negative) value. Maximum concentric power was found by examination of the CMS and PCS instantaneous power arrays for the highest (positive) values for each of the respective concentric phases. Average power values were computed by summing the instantaneous values and dividing by the number of samples.

9. Maximum vertical eccentric force was found by searching the eccentric phase of the CMS vertical force array for the highest value. Similarly, the maximum vertical concentric forces for both CMS and PCS conditions were found by searching the concentric portions of the respective vertical force arrays for the highest values.

10. Energy was computed by summing the translational kinetic energy, the rotational kinetic energy, and the potential energy for each of the four segments in the model. The equations below represents the energy of one segment. In the equations I represents the moment of inertia of the segment, ω represents the angular velocity of the segment, PE represents the potential energy of the segment, TKE represents the translational kinetic energy, RKE represents the

rotational kinetic energy, and E represents the total energy of the segment (see #7 above for further variable identification).

$$PE = mgh$$

$$TKE = \frac{1}{2} mv^2$$

$$RKE = \frac{1}{2} I\omega^2$$

$$E = PE + TKE + RKE$$

Total energy of the system was calculated for each frame by the summation of the energies of each of the four segments. Minimum eccentric energy values were found by search of the CMS energy array. Maximum concentric energy values were found by search of the CMS and PCS energy arrays.

11. After summing the energy changes for the eccentric and concentric phases of the movements, the amount of stored elastic energy (SEE) was found (Hudson & Owen, 1985). Stored elastic energy was defined as the difference between the concentric energy summations of the CMS and PCS, divided by the eccentric energy summation of the CMS, and multiplied by 100% (Asmussen & Bonde-Peterson, 1974a).

$$SEE = [\,(E_{CMS_{conc}} - E_{PCS_{conc}}) / E_{CMS_{ecc}}\,] * 100\%$$

Statistical Methods

Statistics were computed via a SAS program (see Appendix E) on a VAX mainframe computer. A two-way (4x2) ANOVA with repeated measures on load percentage was used to test main effects for load percentages, main effects for conditions

(CMS or PCS), and the interaction effect for load and condition on all concentric variables. A one way (4x1) ANOVA with repeated measures on load percentage was used to test main effects for load percentage on all eccentric variables. All tests for significance were performed at the α=0.05 level.

The statistical power of the F test in analysis of variance was calculated for all main effects and interaction effects. Statistical power (1-ß) represents the probability of rejecting the null hypotheses when it is false (Cohen, 1988; Kirk, 1968). The closer the power value is to one the lower the probability is for the rejection of a false null hypothesis.

Significant main effects on load were further examined with a Scheffé post-hoc test (Ferguson, 1976). Scheffé was chosen because it is relatively conservative, thus less likely to produce Type I errors. This test was done with a specialized MathCAD application using the equation below.

$$F_{df1,df2} = \frac{(\bar{x}_1 - \bar{x}_2)^2}{s_w^2\left(\frac{2}{n}\right)}$$

Chapter IV

RESULTS

Lifter Performance

The following data represent the performance characteristics of the eight subjects (mean height: 1.756±0.072 m; mean mass: 77.5±10.4 kg) remaining after data reduction and calculation. In both absolute and relative terms the 1 RM results indicate a skilled sample of subjects (mean 1 RM: 166.9±51.9 kg; mean 1 RM/body weight: 2.13±0.50). The means and standard deviations for the bar weights at each of the four load percentages are represented in Table 1. A significant difference was found between the means by load percentage ($F_{3,21}=99.57$, $p<.05$). The statistical power of the main effect was found to be greater than 0.99. Scheffé post-hoc tests revealed that all of the means were significantly different from one another.

Table 1. Bar Weight (kg)

Load Percentage	Mean (SD)
40%	74.1 (21.3) * § ø
55%	101.9 (28.7) * §
70%	130.3 (36.8) *
85%	157.3 (44.8)

* significantly different from 85%
§ significantly different from 70%
ø significantly different from 55%

Table 2 contains the means and standard deviations for the same respective weights as Table 1, divided by the lifter's weight. Once again a significant difference was found between the means by load percentage ($F_{3,21}$=120.21, p<.05). Also, the statistical power of the main effect was greater than 0.99. Scheffé post-hoc tests further revealed that all of the means were significantly different from one another. The average 40% lift was almost 90% of the lifter's body weight. The average 85% lift was 190% of the lifter's body weight.

Table 2. Bar Weight / Lifter Weight

Load Percentage	Mean (SD)
40%	0.90 (0.23) * § ø
55%	1.24 (0.31) * §
70%	1.58 (0.40) *
85%	1.91 (0.49)

* significantly different from 85%
§ significantly different from 70%
ø significantly different from 55%

Force Data

An unforeseen error in force data collection invalidated the use of that information. This error involved the assumption that the contact of the bar with the rack would have little effect on both the force data and the corresponding calculations made from it. While the error initially seemed negligible, and was for many of the trials, repeated correction efforts with different mathematical approaches eventually proved it insurmountable. Thus, peak force, time to peak force, and the force-velocity relationship could not be analyzed. Fortunately, the error had no apparent effect on the videotape data. All subsequent analyses were based on variables reduced from that data.

Relative Displacement

The means and standard deviations for relative concentric vertical displacement of the COM by load percentage are represented in Table 3. (See Appendix F for the means and standard deviations for load by condition.) The relative upward distance that the COM of each subject moved did not differ between CMS and PCS conditions ($F_{1,7}$=0.24, p>.05). The statistical power of this nonsignificant test was approximately 0.06. Relative distance, however, did differ with relative load ($F_{3,21}$=1.58, p<.05). The statistical power of this significant main effect was approximately 0.94. For both conditions the 40% load had significantly less relative vertical displacement of the system prior to the end of the concentric phase. No interaction effect was found between load percentage and condition ($F_{3,21}$=0.41, p>.05). The statistical power of this test was about 0.08. In general, the lifters performed each lift with the same net range for the entire thrust regardless of condition, yet had significantly less concentric motion (according to post-hoc tests) relative to the entire thrust when the load was at 40% of their 1 RM. No pattern was apparent for the other means. The range of values calculated across conditions, loads, and subjects was large. The minimum relative distance that a subject moved within the concentric phase was 42.16% and the maximum relative distance that a subject moved was 83.41%. This large range contributed to the differences in the standard deviations.

Table 3. Relative Vertical Displacement of the COM by Load (%)

Load Percentage	Mean (SD)
40%	60.5 (8.6) * § ø
55%	68.9 (5.9)
70%	71.3 (4.6)
85%	71.3 (9.5)

* significantly different from 85%
§ significantly different from 70%
ø significantly different from 55%

Time

Eccentric time was examined for differences by load percentage in the CMS (there is no eccentric phase in the PCS). The means and standard deviations are indicated in Table 4. No significant differences were found between the means ($F_{3,21}$=2.48, $p > .05$). The statistical power of this test was about 0.32. Subjects spent about the same amount of time braking the motion of the system regardless of the relative amount of weight.

Table 4. Eccentric Time by Load (sec)

Load Percentage	Mean (SD)
40%	0.59 (0.27)
55%	0.58 (0.25)
70%	0.70 (0.34)
85%	0.77 (0.27)

While eccentric time did not change, concentric times did. Concentric time means and standard deviations for both CMS and PCS conditions are shown in Table 5, Table 6, Table 7, and Figure 8. As evident in Figure 8 and Table 5, the concentric PCS times were significantly longer than the CMS times, regardless of load ($F_{1,7}=15.29$, $p<.05$). The statistical power of this main effect was approximately 0.92. A main effect for load percentage was also found ($F_{3,21}=26.83$, $p<.05$). This main effect had a power that was greater than 0.99. Post-hoc results indicated that the CMS 40% and 55% loads were significantly lower in concentric time than the 70% and 85% loads. For the PCS condition the results were the same, except that the 70% load was also significantly shorter in time than the 85% load. Generally the time to perform the concentric phase of the lift increased as the relative load increased.

A significant interaction also existed between condition and load percentage ($F_{3,21}=7.15$, $p<.05$). The power of the interaction effect was approximately 0.89. The interaction effect was a result of the higher load percentages of the PCS condition taking proportionately longer times to perform than the CMS condition, especially for the 85% load.

Table 5. Concentric Time by Condition (sec)

CMS Mean (SD)	PCS Mean (SD)
0.56 (0.22)	0.77 (0.33)

Table 6. Concentric Time by Load (sec)

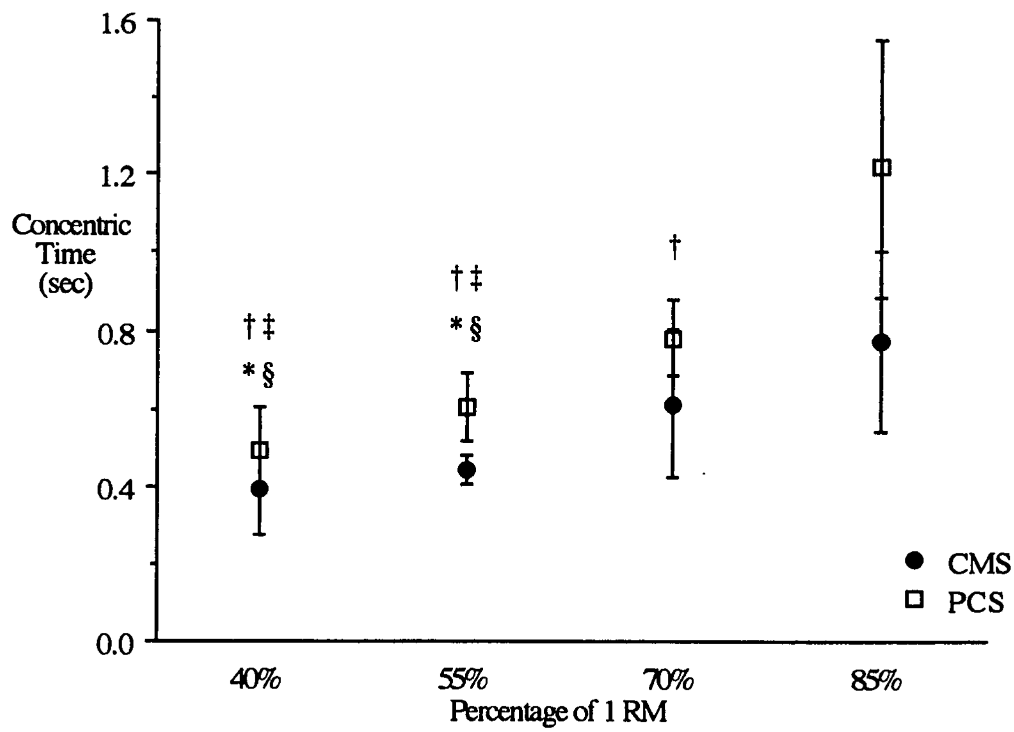
Load Percentage	Mean (SD)
40%	0.44 (0.12)
55%	0.52 (0.11)
70%	0.70 (0.17)
85%	1.00 (0.36)

Table 7. Concentric Time by Load and Condition (sec)

Load Percentage	CMS Mean (SD)	PCS Mean (SD)
40%	0.39 (0.12) * §	0.49 (0.11) † ‡
55%	0.44 (0.04) * §	0.60 (0.09) † ‡
70%	0.61 (0.19)	0.78 (0.10) †
85%	0.77 (0.23)	1.22 (0.34)

* significantly different from 85%, CMS
§ significantly different from 70%, CMS
† significantly different from 85%, PCS
‡ significantly different from 70%, PCS

Figure 8. Concentric Time by Load and Condition



* significantly different from 85%, cms
§ significantly different from 70%, cms
† significantly different from 85%, pcs
‡ significantly different from 70%, pcs

Note: Significant difference between CMS and PCS and significant interaction between condition and load.

Velocity

Maximum eccentric velocity values for the COM of the system in the CMS are shown in Table 8. These values represent how fast the lifters were prone to drop. A significant main effect was found by load ($F_{3,21}$=5.25, p<0.01). Statistical power calculations indicated a power of about 0.78. Post-hoc calculations indicated that only in the 85% condition were the lifters reducing the rate of decent.

Table 8. Maximum Eccentric Velocity by Load (m/s)

Load Percentage	Mean (SD)
40%	-0.83 (0.24) *
55%	-0.83 (0.14) *
70%	-0.74 (0.22) *
85%	-0.62 (0.18)

* significantly different from 85%

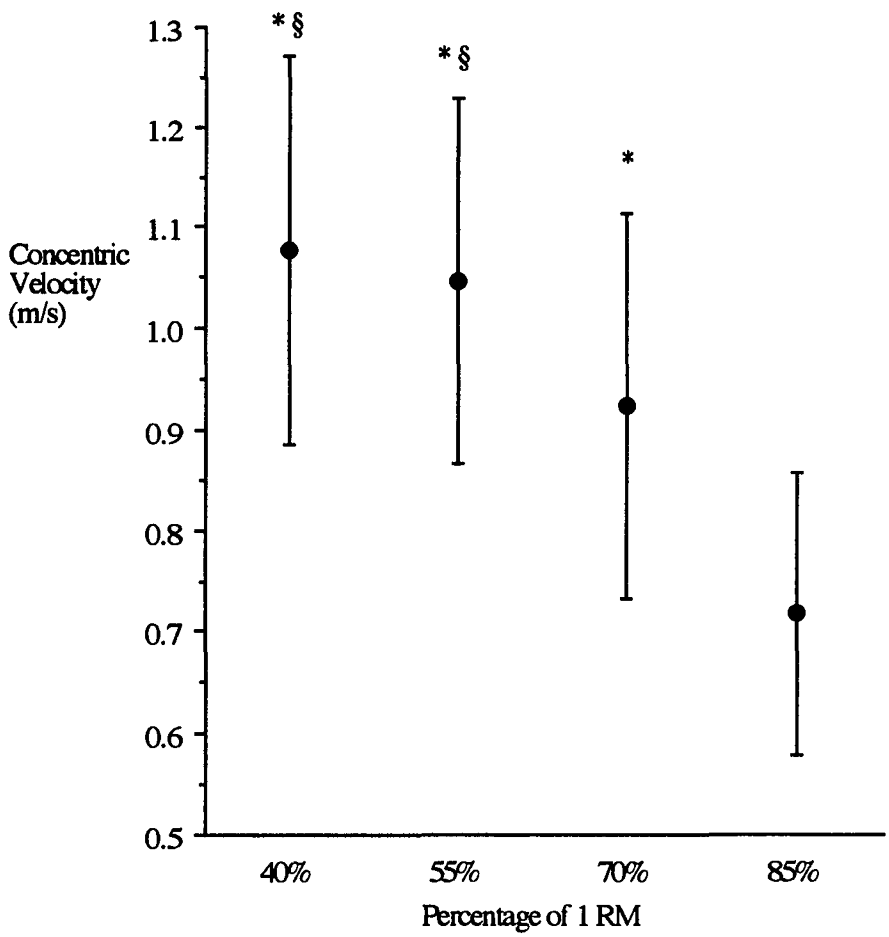
Mean maximum concentric velocities by load are represented in Table 9 and Figure 9. (See Appendix F for the means and standard deviations for load by condition.) No significant differences were found between CMS and PCS conditions ($F_{1,7}$=1.88, p>.05) and no interaction effect was evident ($F_{3,21}$=0.26, p>.05). The statistical powers of these nonsignificant results were about 0.13 and 0.07 respectively. A significant main effect, however, was found for load percentage ($F_{3,21}$=11.51, p<.05). The power of this main effect was greater than 0.99. Post-hoc calculations indicated that all of the means were significantly different except at the 40% and 55% loads. The downward trend in the means was evident as load percentage increased. Thus, the maximum concentric velocities generally decreased as load increased in the same manner as maximum eccentric velocities.

Table 9. Maximum Concentric Velocity by Load (m/sec)

Load Percentage	Mean (SD)
40%	1.08 (0.19) * §
55%	1.05 (0.18) * §
70%	0.92 (0.19) *
85%	0.72 (0.14)

* significantly different from 85%
§ significantly different from 70%
ø significantly different from 55%

Figure 9. Maximum Concentric Velocity by Load



* significantly different from 85%
§ significantly different from 70%

Work

Differing results were found for each of the three types of work calculated. Results for net eccentric work of the system are represented in Table 10 and Figure 10. A significant main effect was found for load percentage ($F_{3,21}$=19.11, p<.05). The statistical power of this main effect was greater than 0.99. Post-hoc tests revealed that all of the means were significantly different from one another. As load increased the net amount of eccentric work performed by the lifter increased.

Table 10. Net Eccentric Work of the System by Load (Nm)

Load Percentage	Mean (SD)
40%	-338 (173) * § ø
55%	-421 (148) * §
70%	-524 (183) *
85%	-620 (236)

* significantly different from 85%
§ significantly different from 70%
ø significantly different from 55%

Although net eccentric work had a definite pattern according to load, net concentric work of the system was not as clear. (See Table 11 and Figure 11 for the means and standard deviations of net concentric work by load percentage and Appendix F for the means and standard deviations for load by condition.) Statistical analyses indicated no main effect for net concentric work by condition ($F_{1,7}$=0.87, p>.05) and no interaction effect existed between condition and load ($F_{3,21}$=0.79, p>.05). The statistical powers of these nonsignificant results were about 0.06 and 0.08, respectively. A load main effect, however, was found ($F_{3,21}$=25.84, p<.05). The power of this main effect was greater than 0.99. Post-hoc test results revealed significant differences between net work averages at every load percentage except 70% and 85%. The means tended to increase with the increased loads.

Table 11. Net Concentric Work of the System by Load (Nm)

Load Percentage	Mean (SD)
40%	478 (153) * § ø
55%	637 (185) * §
70%	711 (157)
85%	757 (247)

* significantly different from 85%
§ significantly different from 70%
ø significantly different from 55%

The theoretical relationship between work of the shortening muscle and velocity of the shortening muscle proposed by Hill (1970) was discussed in Chapter II. Figure 12 depicts the relationship between work and velocity for the lifters in this study. The boxes represent mean values for net concentric work of the system and maximum concentric velocity of the COM. The horizontal bars represent the standard deviations for velocity and the vertical bars represent the standard deviations for net work.

Figure 10. Net Eccentric Work of the System by Load

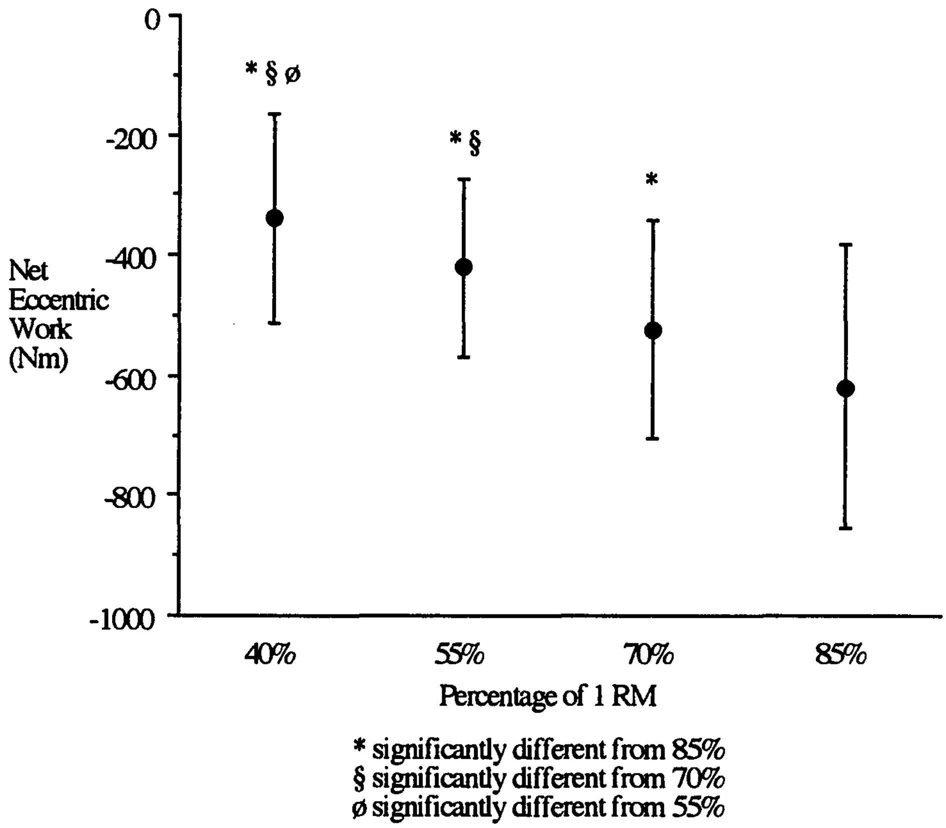
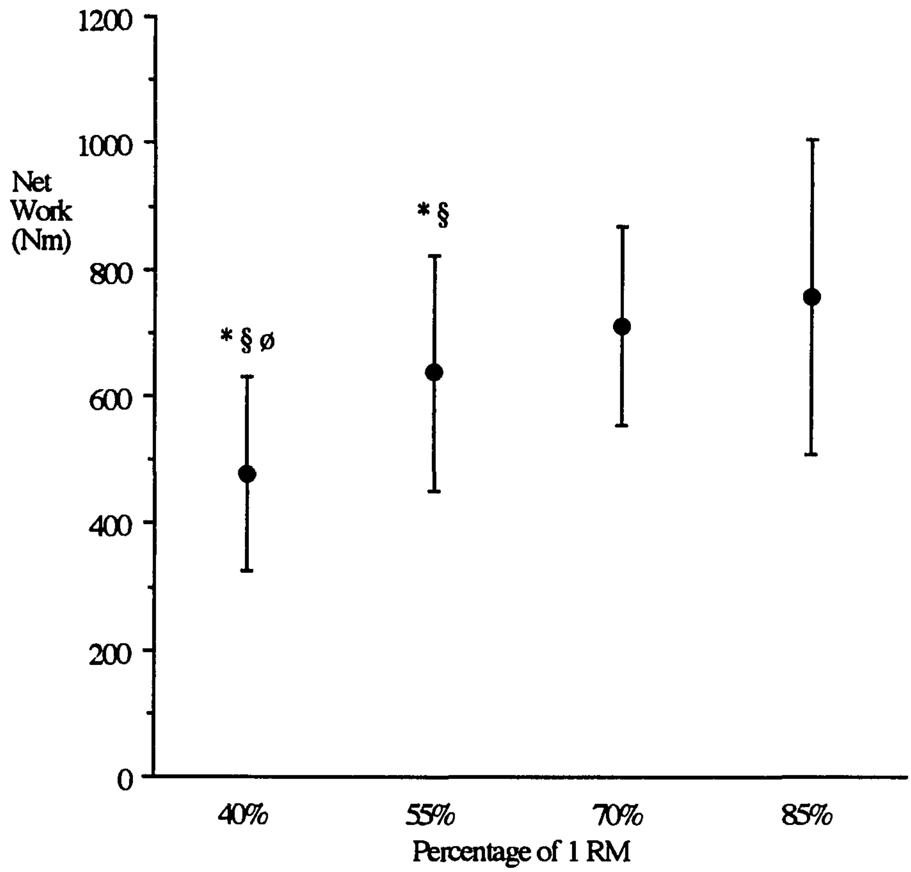
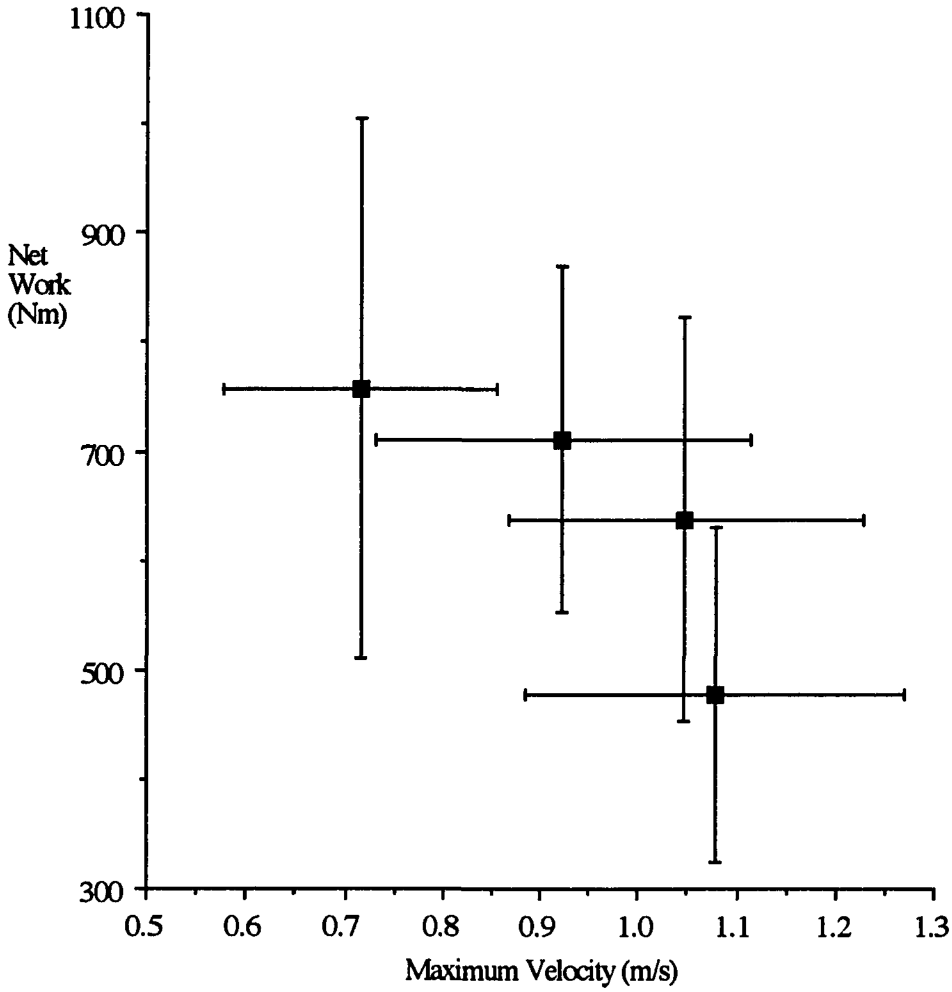


Figure 11. Net Concentric Work of the System by Load



* significantly different from 85%
§ significantly different from 70%
ø significantly different from 55%

Figure 12. Work-Velocity Graph



Non-dimensional eccentric work performed on the bar produced results similar to net eccentric work of the system. (See Table 12 and Figure 13 for the means and standard deviations of non-dimensional eccentric work.) A significant difference was found between load percentages ($F_{3,21}$=31.42, p<.05). As with net eccentric work, the statistical power of this main effect was greater than 0.99. Post-hoc tests revealed that all means were different from one another. As load increased the amount of non-dimensional eccentric work increased significantly. This was an indication that the amount of eccentric work performed on the bar increased, regardless of the height and mass of the lifter.

Table 12. Non-dimensional Eccentric Work Performed on the Bar by Load

Load Percentage	Mean (SD)
40%	-1.27 (0.71) * § ø
55%	-1.81 (0.65) * §
70%	-2.44 (0.86) *
85%	-3.10 (1.24)

* significantly different from 85%
§ significantly different from 70%
ø significantly different from 55%

Results for non-dimensional concentric work performed on the bar were also similar to those reported for net concentric work of the system. The means and standard deviations for load are represented in Table 13 and Figure 14. (See Appendix F for the means and standard deviations for load by condition.) While no condition main effect ($F_{1,7}$=1.34, p>.05) or interaction effect ($F_{3,21}$=0.90, p>.05) was found, a significant main effect for load did exist ($F_{3,21}$=39.78, p<.05). The nonsignificant test power values were 0.07 and 0.08 respectively and the significant main effect power value was greater than 0.99. Post-hoc tests indicated that non-dimensional concentric work averages were significantly different from one another. The means increased significantly as the load increased, regardless of condition. These results indicated that lifters performed greater amounts of work as the relative loads increased, regardless of their height and mass.

Table 13. Non-dimensional Concentric Work Performed on the Bar by Load

Load Percentage	Mean (SD)
40%	1.87 (0.66) * § ø
55%	2.87 (0.98) * §
70%	3.39 (0.77) *
85%	3.77 (1.25)

* significantly different from 85%
§ significantly different from 70%
ø significantly different from 55%

Average eccentric work values are presented in Table 14. No significant main effect was found for load percentage ($F_{3,21}$=0.96, p>.05). Unlike net eccentric work of the system and non-dimensional eccentric work on the bar, values previously reported as increasing as a function of relative load, average eccentric work results did not change with relative load.

Table 14. Average Eccentric Work by Load (Nm)

Load Percentage	Mean (SD)
40%	-14.19 (5.07)
55%	-16.64 (4.39)
70%	-16.39 (4.55)
85%	-15.77 (4.08)

Figure 13. Non-dimensional Eccentric Work Performed on the Bar by Load

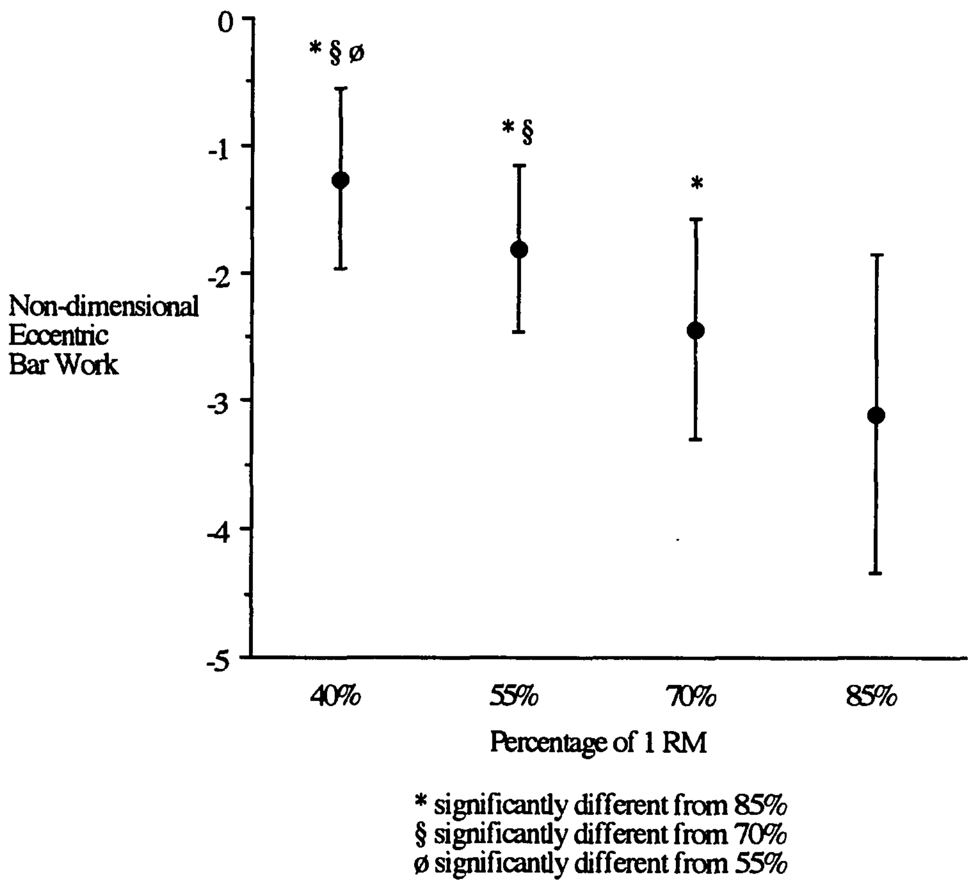
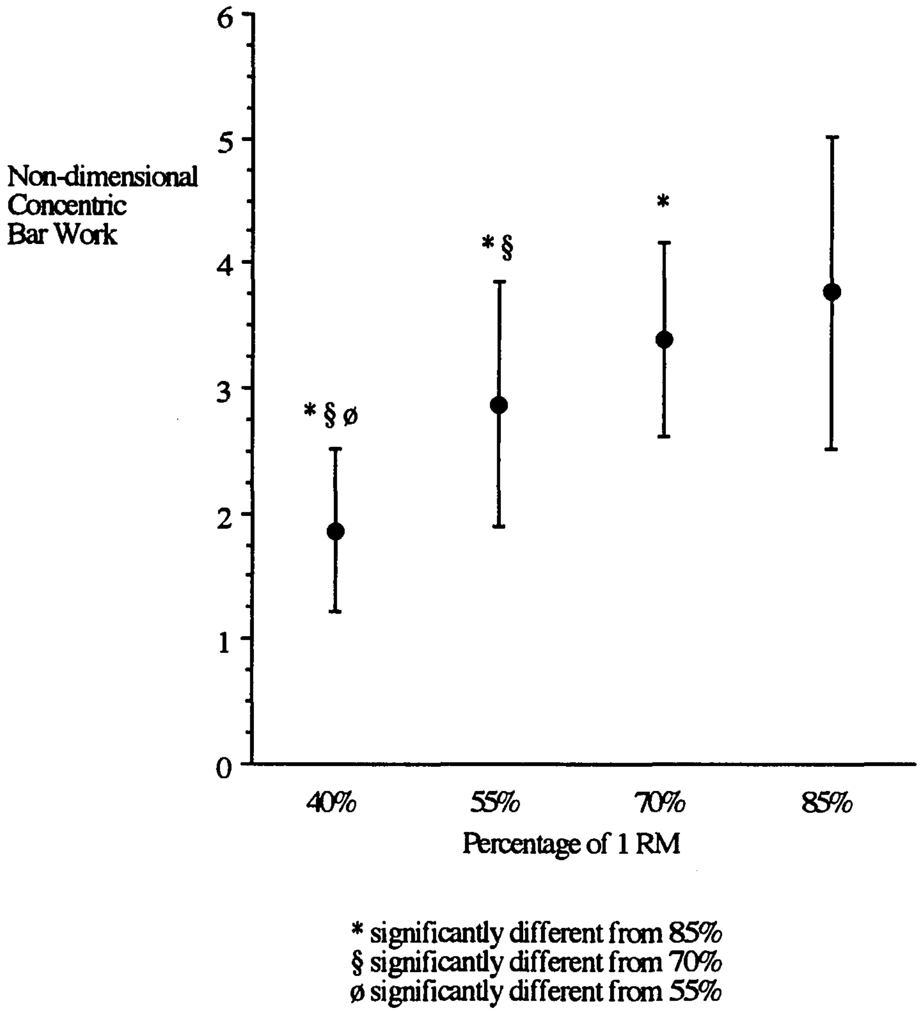


Figure 14. Non-dimensional Concentric Work Performed on the Bar by Load



While average eccentric work values remained similar across load percentages, average concentric work values differed. The means and standard deviations for condition and load are represented in Table 15, Table 16, and Figure 15. (See Appendix F for the means and standard deviations for load by condition.) A significant main effect was found for condition ($F_{1,7}$=48.61, p<.05). The statistical power of this main effect was greater than 0.99. In the CMS condition the lifters performed a significantly greater amount of average work than in the PCS condition. While no interaction effect was found ($F_{3,21}$=0.04, p>.05), a significant main effect was found for load percentage ($F_{3,21}$=6.50, p<.05). The power of the nonsignificant interaction effect was about 0.04 and the power of the significant load main effect was approximately 0.89. As evident in Table 16 and Figure 15, the lifters performed significantly less average work at the 85% load than at any other load for both CMS and PCS conditions. Also, the lifters performed significantly less work at the 40% load than at the 55% load.

Table 15. Average Concentric Work by Condition (Nm)

CMS Mean (SD)	PCS Mean (SD)
20.49 (5.99)	13.39 (4.57)

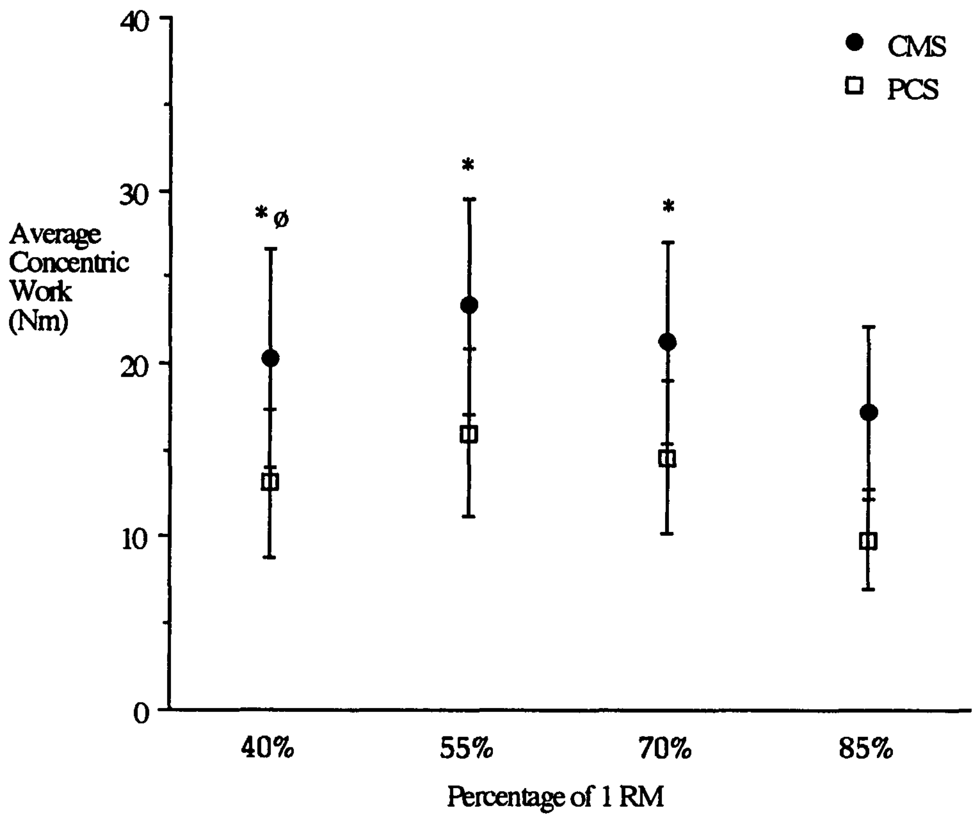
Table 16. Average Concentric Work by Load (Nm)

Load Percentage	Mean (SD)
40%	16.67 (6.36) * ø
55%	19.65 (6.56) *
70%	17.93 (6.03) *
85%	13.50 (5.44)

* significantly different from 85%
ø significantly different from 55%

As previously described, the CMS and PCS conditions for net concentric work of the system and non-dimensional concentric work on the bar were not significantly different. These finding were different than the findings for average concentric work, which had a significant main effect for condition. This is an indication that while the lifter in the CMS condition performed greater amounts of work than in the PCS condition on the average for each 60 Hz cycle, no more net work was performed for the concentric phase as a whole.

Figure 15. Average Concentric Work by Load and Condition



* significantly different from 85%
ø significantly different from 55%
(for CMS & PCS conditions collapsed)

Note: CMS and PCS are significantly different

Power

Power was analyzed for peak and average values. Minimum eccentric power results are shown in Table 17. No significant differences were found between the means ($F_{3,21}=0.37$, $p>.05$). The statistical power of this test was about 0.11. Regardless of load percentage the minimum power was about the same.

Table 17. Minimum Eccentric Power by Load (Nm/s)

Load Percentage	Mean (SD)
40%	-1544 (637)
55%	-1635 (210)
70%	-1651 (397)
85%	-1472 (260)

Maximum concentric power values are given in Table 18. No significant main effects were found for condition ($F_{1,7}=0.04$, $p>.05$) or load ($F_{3,21}=1.56$, $p>.05$), and no interaction effect was found ($F_{3,21}=0.87$, $p>.05$). The statistical power of these tests were 0.01, 0.15, and 0.11 respectively. Lifters exhibited about the same amount of maximum power regardless of condition or load. This indicated a relatively consistent maximum power output from the lifters across conditions and load percentages.

Table 18. Maximum Concentric Power by Load and Condition (Nm/s)

Load Percentage	CMS Mean (SD)	PCS Mean (SD)
40%	1981 (686)	2252 (760)
55%	2262 (585)	2501 (1026)
70%	2315 (626)	2122 (473)
85%	2014 (868)	1836 (585)

Minimum and maximum power values had results different from average power values. Average eccentric power means and standard deviations are shown in Table 19. No significant differences existed between any of the means ($F_{3,21}=0.96$, $p>.05$). The statistical power of this test was about 0.24. On the average, subjects expended the same amount of stopping power for each time interval, regardless of load. This result was similar to that of average eccentric work.

Table 19. Average Eccentric Power by Load (Nm/s)

Load Percentage	Mean (SD)
40%	-851.5 (304.0)
55%	-998.4 (263.3)
70%	-983.1 (272.7)
85%	-946.1 (244.9)

Unlike average eccentric power, average concentric power had some significant differences. Table 20 contains the means and standard deviations for the two conditions. Table 21 and Figure 16 contain representations of the means and standard deviations for concentric average power of the different load percentages. (See Appendix F for the means and standard deviations for load by condition.) A significant main effect was found for condition ($F_{1,7}=22.59$, $p<.05$). The statistical power of this main effect was approximately 0.98. Lifters in the CMS condition generated a significantly greater amount of average power than in the PCS condition.

Table 21 and Figure 16 contain the means and standard deviations for average concentric power for the different load percentages. A main effect was found for load percentage ($F_{3,21}$=7.09, p<.05), although no interaction effect was apparent ($F_{3,21}$=0.47, p>.05). The statistical power of the significant main effect for load was approximately 0.91 and the statistical power of the nonsignificant interaction effect was approximately 0.07. Post-hoc tests revealed that the lifters generated less average power at the 85% load than at any other load. While not significant, average power generated was usually lower at 40% than at 55% and 70%. This pattern was similar to that of average work.

Table 20. Average Concentric Power by Condition (Nm/s)

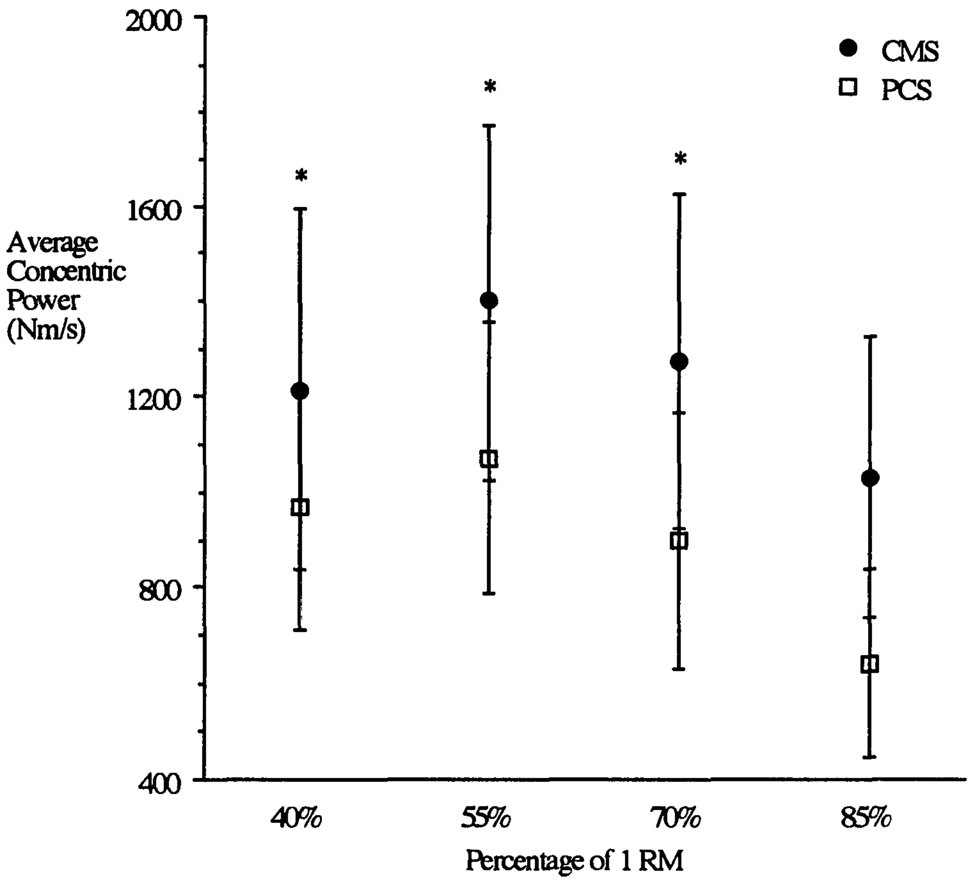
CMS Mean (SD)	PCS Mean (SD)
1229.4 (359.1)	894.1 (289.9)

Table 21. Average Concentric Power by Load (Nm/s)

Load Percentage	Mean (SD)
40%	1097 (337) *
55%	1234 (362) *
70%	1086 (358) *
85%	835 (314)

* significantly different from 85%

Figure 16. Average Concentric Power by Load and Condition



* significantly different from 85%
(for CMS & PCS conditions collapsed)

Note: CMS and PCS are significantly different

Energy

Energy (the sum of translational and rotational energy and potential energy) was evaluated for minimum (eccentric) values, maximum (concentric) values, and elastic energy values. Minimum eccentric energy values are represented in Table 22. No significant main effect was detected according to load ($F_{3,21}$=0.47, p>.05). The statistical power of this test was about 0.04.

Table 22. Minimum Eccentric Energy by Load (Nm)

Load Percentage	Mean (SD)
40%	486 (50.8)
55%	489 (49.5)
70%	485 (48.4)
85%	489 (47.3)

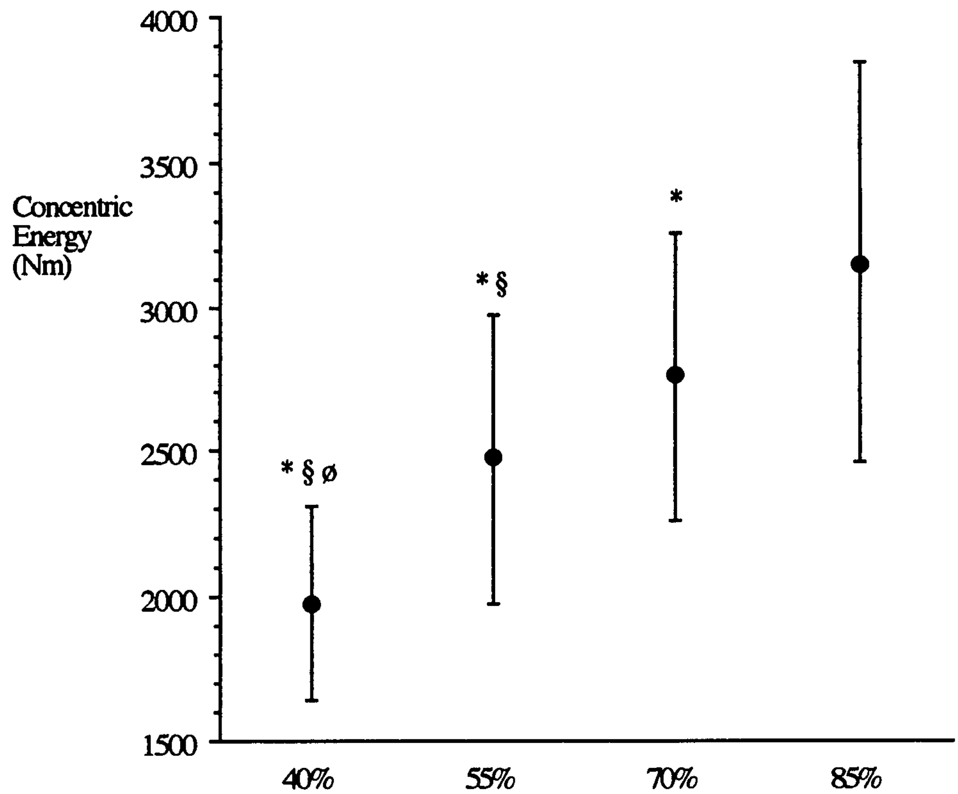
Maximum concentric energy had different results than minimum eccentric energy. Table 23 and Figure 17 present the means and standard deviations by load. (See Appendix F for the means and standard deviations for load by condition.) While no main effect was found for condition ($F_{1,7}$=0.17, p>.05) and no interaction effect was evident ($F_{3,21}$=0.62, p>.05), a main effect was found for load percentage ($F_{3,21}$=61.79, p<.05). The statistical powers of these tests were 0.04, 0.07, and greater than 0.99 respectively. Post-hoc tests indicated that all load percentages were significantly different from one another. Maximum concentric energy increased as load increased.

Table 23. Peak Concentric Energy by Load (Nm)

Load Percentage	Mean (SD)
40%	1976 (331) * § ø
55%	2476 (497) * §
70%	2759 (500) *
85%	3149 (688)

* significantly different from 85%
§ significantly different from 70%
ø significantly different from 55%

Figure 17. Maximum Concentric Energy by Load



* significantly different from 85%
§ significantly different from 70%
ø significantly different from 55%

Elastic Energy

Means and standard deviations for elastic energy are shown in Table 24. The overall mean and standard deviation for all loads was 3.43 ± 26.93%. The values for individual trials ranged from -69.95% to 64.72%. No significant differences were found between elastic energy values for each load percentage ($F_{3,21}=1.33$, $p>.05$). The statistical power of this test was about 0.11. No significant or non-significant pattern was apparent as a function of load.

Table 24. Elastic Energy by Load (%)

Load Percentage	Mean (SD)
40%	1.758 (13.597)
55%	-8.410 (35.897)
70%	19.975 (21.595)
85%	0.411 (28.028)

Chapter V

DISCUSSION

Elastic energy has been a topic researched by many biomechanists and physiologists because of their desire to understand how something as apparently simple as a muscle contraction can act in a complex number of ways depending on the circumstances of the contraction. After the development of a muscle model and its basic contraction mechanism, researchers began investigating how the muscle responded in different situations. They discovered that movements of apparent simplicity were often influenced by the very nature of how they were executed. For example, countermovement actions have been shown to change the kinetic and kinematic characteristics of a movement. While some of the changes were anticipated for mechanical or physiological reasons, others were not. Such was the case in this endeavor. Although some of the results of this study were anticipated, others were not. The task of lifting a weight a foot or so under two conditions resulted in different mechanical outcomes depending on the condition and relative load placed on the muscles.

Relative Displacement

The relative vertical displacement of the COM of the system during the concentric phase of the upward thrust was free to vary by subject, condition, load, or any combination of the three. This variability was impossible to control in this task because of the risks involving the lack of integrity between the lifter and the weighted bar. In other words, the bar was free to move independently of the lifter under certain circumstances and this possible bar motion provided unnecessary risks to the lifter. Because of this and the nature of the motion, which ended at the top of the upward thrust, the lifters could reach peak velocity at any point during the upward motion, thus ending the concentric phase. The end of the concentric phase also signaled the end of concentric displacement of the system. The relative concentric displacements ranged from 42.16% to 83.41% of the overall upward thrust distance. Indeed, these values were not outlying values. Smaller and larger relative displacements were distributed across the subjects and trials without much pattern. (See Table 3 for the means and standard deviations). No significant differences in relative concentric displacement were found between the countermovement squat (CMS) and purely concentric squat (PCS) conditions. A significant main effect for load was, however, found. Post-hoc tests revealed that the lifters exhibited significantly less concentric displacement at the 40% load than at all other loads. The abbreviated concentric displacement at 40% load and the rather variable concentric displacement across all trials confounded many of the other results in this study because of the interdependence of most variables with concentric displacement.

Time

Time is an important variable in evaluating any movement because most variables are either dependent on it or influenced by it. As previously discussed (see Chapter II and Chapter III), the transient behavior of elastic energy has implications for the movement that must be controlled, as they were in the present study.

Other than the transient variety, time has not typically been measured or evaluated in the studies of elasticity. Wilson et al. (1991) measured transient time and the time to peak force, but not eccentric time or concentric time. It is surprising that eccentric and concentric time has not been studied, because it seems that for tasks that can conclude anywhere within the upward thrust, time becomes a more relevant variable. In this study, both eccentric and concentric times were measured.

Eccentric time, the time from maximum eccentric velocity to the start of upward movement (see Figure 7), was not found to change as the relative load changed. While time increased slightly with load in this manner, no significant effects were found. It was expected that eccentric time would increase with greater loads because of the lifter's desire to control the weight. Using greater amounts of time to retard the motion of the system is one method of absorbing forces because braking through a longer time base (less intensity) requires less muscular force than braking through a shorter time base (more intensity).

The lack of significance for eccentric time could be related to the lifter's previous experience. That is, some lifters may have been conditioned to begin the eccentric phase at a particular time or place within the movement. Such differences in the lifter's strategy could have produced large variations in eccentric time between subjects but small variations in eccentric time between lifts for a given subject. In addition, if a lifter's training program involved primarily negative lifting routines, which involve a slow, eccentric decent of the weight but no concentric lifting, then the lifter's performance in the more traditional countermovement squat would probably be influenced.

Although eccentric times did not change as the load increased, the concentric times did. Not only did the PCS take significantly more concentric time than the CMS (0.774 seconds and 0.555 seconds respectively), the lifts also increased in time as the load increased regardless of condition. A significant interaction was also found between condition and load percentage.

In keeping with previous research, the time difference between the PCS and the CMS was expected. Cavagna et al. (1971) found the time of positive work to be 55%

greater without prestretch of the muscle. The results in the present work reflect that trend because the PCS condition took 39% more time than the CMS condition. At the 85% load the present results were most similar to the Cavagna et al. work given that the lifters took 58% longer in the PCS than in the CMS.

The amount of time spent in the concentric phase also increased as relative load increased. The two lighter loads required significantly less time to lift than the two heavier loads, regardless of condition. Also, the 70% PCS load required significantly less time than the 85% PCS load. The general trend for concentric time to increase as load increased was expected because loads of greater mass have greater amounts of inertia and require longer amounts of time to reach peak velocity (which determined the end of the concentric phase).

The fact that the 70%-85% load comparison was significantly different for the PCS and not for the CMS is probably most of the reason for the significant interaction effect between condition and load (the only significant interaction found for any variable). The interaction effect was a reflection of the difficulty of PCS lifts, especially at high load percentages. PCS lifts required more effort in overcoming the inertial properties of the mass than the CMS lifts, especially when the load approached the 1 RM value.

The significant main effect for condition and significant interaction indicates how elastic energy may help to reduce the time required to reach maximum concentric velocity even under moderate to heavy loads. The lower concentric times of the CMS lifts could be due to the recoil of elastic elements at the beginning of those lifts. The elastic recoil would reduce the concentric time by allowing the muscle to more easily overcome inertia at the beginning of the lift. The interaction effect was likely due to the greater time difference between conditions at the 85% load. Perhaps the elastic recoil was most helpful at the heavier load because inertia was greatest and the lifters struggled without the benefit. The main effect for load indicates that, as expected, greater loads required increased amounts of time for the lifter to reach maximum velocity.

Velocity

A significant main effect was found for maximum eccentric velocity by load percentage. Specifically, the results revealed that the lifters crouched at slower speeds with higher amounts of weight. The only load percentage, however, that was significantly different from the others was 85%. As noted in Table 7, this indicates that the lifters maintained a peak negative velocity of 83 cm/s until the load required greater amounts of effort. Typically, weights lifted at or near the 1 RM are not lifted as fast as lighter weights, so this result is not unusual or unexpected.

This decrease in velocity at heavier loads suggests that most lifters use caution with heavier weights. Even under the safest of circumstances all lifters want to feel as if the weight is always under control. Without control of the weight, lifters may believe that the risk of an acute injury would be unnecessarily high.

The rate of descent also may be influenced by the lifter's experience in lifting maximum weights. For example, a lifter who trains with high intensity (i.e. great velocity) and near maximal resistance may be favored in the current methodology. Few lifters, however, train in such a manner. Most lifters either train with greater weights and slower motions or lesser weights and faster motions. In other words, specificity of training may influence the performance (and hence the velocity and elastic energy) of lifters at given loads depending on the characteristics of their workout program.

Although there was no literature evaluating the relationship between eccentric velocity and load, prestretch velocity has been considered. Bosco et al. (1981) found that an enhancement in jump performance was correlated with prestretch velocity and short delay times. No indication was evident, however, as to how this relationship would hold when the prestretch velocity was a function of load. Cavagna et al. (1968) believed that the capacity to store (and possibly utilize) elastic energy would be greater with higher speeds of prestretching. It was expected that as prestretch velocity increased elastic energy benefits would increase (Thys et al., 1975), forces would increase (Edman et al., 1978), and concentric velocities at given forces would increase (Edman et al., 1978). If these relationships hold when prestretch velocity is altered by load, then the 85% lift (with the slowest prestretch velocity) should have reduced elastic energy benefit, reduced force, and reduced concentric velocity. While the effect of velocity on force was lost and the effect on elastic energy was undetermined, the effect on concentric velocity can be examined.

Unlike concentric time, no differences were found between CMS and PCS conditions for peak concentric velocity. Although the peak velocities occurred significantly earlier in the concentric phase of the CMS (see previous discussion of concentric time), they were no higher or lower as condition changed. While a significant interaction was not found between condition and load, the means for the heavier CMS loads were about 14% higher than the PCS loads and the means for the lighter CMS loads were nearly the same as the PCS loads. It was expected that peak concentric velocity would increase with the use of a countermovement. The increase in velocity was expected to come from the sum of elastic recoil and the shortening of the contractile component. One possible limitation to this scenario would be if the elastic elements recoiled at a faster rate than the contractile components (Cavagna, 1977; Edman et al, 1978). If so, a measurement of velocity which occurs relatively late in the thrust phase, as maximum concentric velocity does, may not be sensitive to elastic contributions created near the beginning of the thrust phase. Perhaps a different representation of velocity, such as average velocity or the velocity at a predetermined time or position within the concentric phase, would more likely support elastic energy as a benefit.

Although no comparable concentric velocity data were found in the literature, Bober et al. (1980) believed that with elastic benefits velocity would increase with load up to some point. Because the present velocity data lack any interaction effect, they do not support any change in elastic benefit with increased load. Because the loads Bober et al. were referring to were much lower percentages of the 1 RM maximum, they allowed much higher relative velocities. It is possible that the velocity increases with elastic benefit occurred up to a load magnitude less than presently tested (but higher than the relative loads of Bober et al.), although no evidence currently supports or refutes the idea.

For load a similar downward trend in concentric velocity was evident for both CMS and PCS conditions. Post-hoc tests of the collapsed load values by condition revealed that the peak velocity at the 85% load percentage was significantly slower than peak velocities at lower load percentages. Also, the peak velocity at the 70% load was significantly slower than the peak velocities at the 40% and 55% loads. As measured, the velocity data did not directly support the benefits of elasticity.

The downward trend in velocity as load increased may have occurred because of the force-velocity relationship. Although greater forces were not directly measured, it stands to reason that greater amounts of weight would require greater forces and thus result in lesser velocities. While these data cannot completely support this relationship, the trend was evident. The significant main effect for load indicated that the concentric velocity did significantly decrease at nearly all load percentages.

One assumption the force-velocity relationship relies on is the effort being maximal or exactly the same percentage of maximal for every load and condition. While all subjects were encouraged to perform maximally, conditions, background, and previous experience in maximal lifting may have confounded these results. If the neurological feedback and control mechanisms are not trained for lifting various percentages maximally, these mechanisms may reflexively control the motion characteristics of the lifter. Some lifters, due to training specificity, may perform maximally more easily at lower loads or at higher loads. Some lifters may not be experienced or trained adequately for maximal performance at all percentages of their 1 RM.

Work

Work was measured as a net value, as a non-dimensional value (work performed on the bar), and as an average value. Because the results for net work of the system and non-dimensional work on the bar were nearly identical, they will be discussed together. The results for net eccentric work of the system and non-dimensional eccentric work on the bar did not differ. In both cases the amount of eccentric work increased significantly as the load increased. Because net eccentric work is a reflection of the total change in mechanical energy of the system, the result was expected. That is, greater amounts of work were needed, whether net or non-dimensional, to stop the motion of greater relative amounts of weight. Because taller and heavier lifters were expected to perform greater amounts of work on the bar, controlling for this difference (with non-dimensional work) allowed only load percentage to influence this value. Inasmuch as net work and non-dimensional work were nearly identical, these results were a strong indication of the bar weight itself influencing the amount of work performed.

Like the eccentric work variables previously discussed, net concentric work of the system had results quite similar to that of non-dimensional concentric work on the bar. While no main effects for condition and no interaction effects between condition and load were found for either variable, some trends were evident as a function of condition and load. For both net and non-dimensional work the values were 6-7% greater at heavier loads in the CMS than in the PCS and no different at the lighter loads.

A main effect for load was found for both net and non-dimensional concentric work. Post-hoc tests revealed that all of the means were significantly different for net work except those for the 70% and 85% loads and all of the means were significantly different for non-dimensional work. Overall, the values tended to increase in magnitude as load increased (see Figure 11 and Figure 14). It was expected that positive work would increase as load increased for the same reasons that negative work increased. That is, as load increases the total change in the mechanical energy of the system must also increase. The increased work with greater load reflected the research findings of Wells (1967). He found 1.6 times more positive mechanical work during elastic shortening at a heavy load than at a light load. In the present study the amount of work increased with load by 1.55 times when comparing the lowest CMS load percentage with the highest CMS load percentage.

Perhaps the lifters performed less net concentric work at lower loads because the relative amount of weight lifted was low enough for them to perform the concentric phase more quickly and "efficiently." As with eccentric work, these results strongly supported the expected increased amount of mechanical work performed on the bar as the load increased. Because the concentric phase ends at peak velocity, it was not surprising that net work results were similar to the results from concentric time. Overall, net work performed was probably influenced by the mass of the system (which increased as load increased), time (which increased as load increased), and velocity (which decreased as load increased). The lifters were performing the concentric phase more quickly at lower percentages, achieving higher velocities, yet performing less net work. Based on the results of non-dimensional concentric work performed on the bar, the reasons for the differences must be assigned to the task, the load, or both.

While most of the tasks used to evaluate elastic energy conclude with maximum velocity at the end of the overall thrust motion, this task does not. Moreover, too much risk is involved in performing this task to maximum velocity at the end of the thrust phase. The result of such action would be a projection of the bar, lifter, or both. Naturally with this type of projection the take-off is not the actual problem, the landing is. Because greater bar mass required greater force and delayed the time that maximum velocity was reached the amount of net work performed increased, as seen here. The evaluation of power will provide further understanding because of the influence of time and velocity on the computation of power.

The lower amounts of net concentric (or non-dimensional concentric) work with lighter loads combined with the concentric velocity results are in general agreement with the work-velocity relationship proposed by Hill (1970, see Figure 3). From this relationship, it was expected that the work in shortening would increase as the velocity decreased. In the present study velocity decreased as load increased and work increased as load increased (see Figure 12). Even though the graph of work and velocity can be viewed as more convex than the concave curve expected, the present results neither support, nor refute the relationship theorized by Hill (1970). One explanation for the lack of direct support presently is that the relationship proposed by Hill was based on physiological arguments regarding the muscle fiber, while the present curve was based on mechanical characteristics of the entire system. This may have lead to the results differing slightly from the expected pattern. Another explanation is that the values represented in Figure 12 have large standard deviations for both the velocity and work means. These large variations in the data allow for many different types of curves to adequately fit the data. For example, a parabolic fit of the data would produce a curve more convex, as the data seem to indicate, but an exponential fit of the data would produce a concave curve similar to the theorized relationship. Finally, more data points distributed throughout the entire spectrum of loads relative to the one repetition maximum would be required for an adequate evaluation of this relationship.

While net and non-dimensional work had similar, definitive, straight forward patterns for eccentric and concentric phases as load changed, average work had altogether different patterns for eccentric and concentric phases and load percentages. While net eccentric and non-dimensional eccentric work changed significantly as a function of load, average eccentric work did not change as load changed. All of the lifters performed the same amount of eccentric work on the average for each finite time interval ($1/60^{th}$ of a second). The probable reason that the lifters required greater net eccentric work with load and not greater average eccentric work was because the time required for eccentric braking increased somewhat and the peak eccentric velocities decreased as load increased. Thus greater amounts of work were spread over greater time intervals, resulting in similar average amounts of work performed across loads.

Analysis of average concentric work yielded results unlike average eccentric work, net concentric work of the system, and non-dimensional work performed on the bar. The amount of average concentric work was significantly greater in the CMS condition than in the PCS condition. The greater average concentric work for the CMS condition could be due to similar amounts of net work (for the entire concentric phase) divided by fewer time intervals because the CMS required significantly less concentric time than the PCS.

Several research findings confirmed the present results. In general, most researchers believe that more work can be performed in a given amount of time with elastic energy utilization. The exact amount of benefit varies with the study. Cavagna et al. (1971) expected net work to be 10% greater with prestretch, but they expected average work to be the same. In the present study net and non-dimensional work was about 6-7% greater with prestretch at the two heavier loads but not at the two lighter loads. As for average work, the present results exceeded their claim (see Table 15), yet are similar to those reported by Chapman (1980). Chapman reported that the use of a wind-up enhanced work by 1.56 times. The present study found average CMS work was 1.53 times greater than average PCS work. The value of the countermovement actions having about 1.5 times greater work than non-countermovement actions was surprisingly similar for both average work in the present study and work for previous studies (Chapman, 1980; Wells, 1967).

The pattern for the average amount of concentric work by load was similar for the two conditions, yet different from the pattern of any other variable examined thus far. The 40% load required significantly less work than the 55% load and the 85% load required significantly less work than all of the other three load percentages. This parabolic pattern is depicted in Figure 15. It is not clear, however, why the 40% loads did not have higher average work values. If lighter loads were lifted with higher average velocities (as would be expected in the force-velocity relationship), then the 40% loads should have much higher average work values. Perhaps the fact that the 40% loads had significantly shorter relative vertical displacements accounted partly for these lower work values. In addition, the 40% load may have been too light for the lifters to comfortably perform at maximal levels because of the dangers previously described.

Based on the present results for work, the time of the concentric phase was more influential than velocity in net work performed, and condition was most influential in determining the average amount of work performed. Considering the main effects for condition, average work seemed to be a better indicator of elastic energy contribution in this task than net or non-dimensional work. Based on the lack of an interaction effect for average work, no change in the amount of elastic energy benefit as a function of load was apparent. However, some trend was evident across load and condition. Average work was about 50% greater in the CMS at the three lower loads, but about 74% greater at the 85% load. This may indicate some additional elastic benefit as a function of load in a manner similar to that of maximum concentric velocity at heavier loads.

The effect of load on work differed depending on the variable. For net work of the system and non-dimensional work on the bar, increased loads required increased amounts of work. For average work the lower load percentage and higher load percentage required less average work, while the middle load percentages required more average work. While the results of work performed by the lifters were quite varied, not all research indicated that anything substantial would be found. Cavagna (1977) stated that the effect of previous stretching would be higher power output, not greater positive work.

Power

Power was analyzed for both peak and average values. The results of this study produced somewhat unexpected, but not unprecedented, power values. Maximum eccentric power values, unlike net eccentric work values, did not change as load changed. This is possibly a reflection of a lifter's inherent ability to recognize the largest amount of power which can be reliably generated to counteract the downward motion of the system. This knowledge (conscious or not) allows a lifter to safely perform the activity.

Maximum concentric power results were representative of hypothesized maximal human power output values proposed by Wilkie (1960). He proposed that the external power output of the body would be limited to something less than 6 h.p. when the duration of the task was less than 1 second. The maximum value of 6 h.p. would be reached only in activities of much less than 1 second and would reduce exponentially from that point as time elapsed. Given the movement duration of the present task (ranging from 0.4 seconds to 1.2 seconds), the maximum power values of about 2 h.p. (for the better performers, most were between 1.0 and 1.5 h.p.) were quite reasonable.

The power values were also similar in magnitude to those reported by Garhammer (1980). In his study of Olympic lifts (i.e., the snatch and the clean & jerk) he found power output ranging from about 920 joules per second for a 52 kg lifter to about 2600 joules per second for a 142 kg lifter. These values were similar to the maximum power outputs exhibited by the lifters in the power squat, which ranged from 1250 joules per second for a 86 kg lifter to about 3900 joules per second for a 95 kg lifter. The larger range in the present study was largely attributable to the fact that the present power output was based on a system that included both the subject and the bar and Garhammer's results were based on only the power output related to vertical bar movement.

In comparing the CMS and PCS conditions no statistically significant differences were found for maximum concentric power. Also, while no interaction effect was found between condition and load, the maximum concentric power was about 10% greater in the PCS than the CMS at the two lighter loads and about 10% greater in the CMS than the PCS at the two heavier loads. In addition, none of the loads were significantly different for maximum concentric power. Load influenced concentric time (which was also influenced by condition), maximum concentric velocity, and concentric net work, but not maximum power. This result was not expected because power values were hypothesized to increase as load increased (Bosco & Komi, 1979; Cavagna, 1977). It is possible that the work differences may have reflected the lifters' inability to reach higher peak velocities with heavier weights, taking larger amounts of time to reach peak velocity, yet achieving a similar amount of peak power with the increased load. Maximum power was a reflection of how relative displacement, time, velocity, and work have interacted. Perhaps this variable is an indication that the subjects may have been performing at nearly the same relative intensity across conditions and loads. Again, the task or the lifters' training may have confounded these results.

Average concentric power results were nearly identical in pattern to those of average concentric work. As before, the CMS average power was significantly greater than the PCS average power. As with concentric work, there was no interaction effect between condition and load. Some divergence, however, was evident in the means as relative load increased. The difference in average power between the CMS and PCS conditions went from about 26% at the 40% load to 31% at the 55% load to 42% at the 70% load to 60% at the 85% load. It was likely that no significant results occurred because the standard deviations were about 30% of the means. The only difference between the results of average concentric work and average concentric power by load was one less significant difference in the load comparisons (the 40% load was not significantly less than the 55% load for average power, but was for average work). Because time was the variable of difference (power is work divided by time) and the time intervals were all the same ($^1/_{60}$ sec), these result were expected.

The results of the power analyses were similar to the results of Cavagna et al. (1971). They found that average power was 70% greater with prestretch, but that instantaneous power was not different. The present results indicated 37% greater average power in the CMS condition than in the PCS condition. The increased average power output in the CMS may have been due to an increased speed of the whole muscle shortening and therefore the speed of the concentric movement (Thys et al., 1972). The results of this study reflect the above statement only as far as the duration of the movement because the CMS took significantly less time in the concentric phase than the PCS, but not as far as actual peak concentric velocity, which did not differ according to condition.

Average power and average work may be good indicators of the benefits of elasticity, while net work and peak power may not. While no statistical evidence indicated change in the amount of elastic energy benefit as a function of load for average power, the divergence in the means may have been "hidden" by the variability of the data within the means. This possible evidence, in conjunction with the results from average work and concentric time may be tentative indicators of the possibility that heavier lifts used more elasticity. Alternatively, elasticity could be just as much a factor at moderate loads but obscured by the task characteristics. That is, lifters performing a submaximal lift in a maximal manner probably produced multiple solutions and consequently increased variability within the conditions and loads.

Energy

Minimum eccentric energy values did not differ according to load percentage. Minimum eccentric energy was influenced by the peak eccentric velocity and the depth of the crouch at that point. Potential energy of each lift was the primary contributor to the total energy value, whether minimum or maximum, and probably best explains the lack of differences presently found. Where and when the minimum energy values actually occurred within the lift was not measured.

No differences in maximum concentric energy were detected according to condition. Total energy was comprised of kinetic and potential energy. Kinetic energy was a product of a mass constant and the square of linear velocity. Potential energy was a product of gravitational and mass constants and the height of the COM of the system.

Neither of the two non-constant influences of this variable differed according to condition. Further, relative displacement and maximum velocity were consistent across conditions. Therefore, it was not surprising that no prestretch benefit was apparent in the maximum concentric energy. These results may have been due in part to the influence of maximum vertical velocity on kinetic energy. As discussed previously, neither maximum concentric velocity nor maximum concentric power (which is dependent on velocity) were reflective of changes in elasticity that may have occurred in the initial part of the upward thrust. Moreover, whatever effect elasticity had that could be detected by kinetic energy may have been diluted by the influence of potential energy (which accounted for 90% or more of the total energy in this movement). Because relative displacement did not differ between the two conditions, maximum potential energy measured at the end of relative displacement would not be affected by differences in elasticity between the conditions.

Maximum concentric energy, unlike minimum eccentric energy, differed according to load. All of the load percentages were significantly different from one another, with a clear linear pattern of increasing values with increasing relative load. Because peak concentric velocities decreased with load, the kinetic energies may also have decreased. Consequently, the increased maximum concentric energy may be due to increased potential energy, which increased as a function of load itself. The typically shorter times spent in the concentric phase of lower load percentages and the lower amount of relative vertical displacement possibly confounded this result. In other words, the shorter concentric phase meant the phase ended sooner, when the bar was further from the top (most upright) position and potential energy was therefore lower for those movements. This was most evident at the 40% load, which had lower relative displacement and maximum concentric energy values.

Elastic Energy

Although previous research provided no clear evidence as to how elastic energy would change with load, one possibility was that it would increase with larger relative loads because of changes in stiffness/compliance within the elastic structures (Cavagna, 1970; Wells, 1967). While elastic energy values were not found to differ according to load, that was not the most meaningful outcome of the analysis of that variable. More importantly, some elastic energy values were negative, a theoretically infeasible result. Negative values occurred as a result of the changes in the energies of the PCS being larger than the changes in the energies of the CMS.

Elastic energy calculations rely on several factors: maximum effort in both CMS and PCS conditions, a task which allows maximum effort at different loads, a consistent eccentric effort, and a consistent concentric effort. Of these factors, only consistent eccentric efforts were somewhat supported by these data. The variations in performance that may explain the lack of meaningful elastic energy values can be attributed to any combination of variables within the squat, including relative displacement, time, and peak velocity. Each of these three variables differed with respect to condition and load and were also critically important in the calculation of all subsequent variables. For example, if the relative displacement of the PCS was slightly greater than the relative displacement of the CMS, the greater potential energy of the PCS would more than offset any difference in kinetic energy in favor of the CMS.

Similarly, if the effort of the PCS was maximal and the effort in the CMS was marginally less than maximal, then greater amounts of energy would be found in the PCS. Although the subjects were encouraged to perform maximally (and appeared to comply) and the group means for maximum power output were relatively consistent across condition and load, some lifters on some trials may have given less than maximal effort. Also, if the nature of the task prevented maximum performance at some conditions or loads, the results would also have been affected. This situation was most likely at lower loads (and with lifters who train often with heavy weights) because it was easier for the lifters to create dangerous bar and body projectiles. Lifters may have produced less than maximal effort in this situation in an attempt to avoid injury. Indeed, peak power output for the subjects as a group was slightly higher (but not significantly) in the PCS than in the CMS at the two lighter loads.

The existence of negative scores may have contributed to the large variance in elastic energy at each load. In addition, the variance of many of the composite variables (e.g. relative displacement, maximum velocity, and maximum power) may have influenced the variance of elastic energy. Therefore, it is possible that the power squat was not necessarily the most appropriate task for the measurement of elastic energy. Other tasks such as jumping, however, have also been associated with high variance in elastic energy measures (Hudson & Owen, 1985).

Chapter VI

SUMMARY

The results of the present study seem to support some hypotheses, but not others. Time was hypothesized to decrease with the use of a countermovement and increase with greater loads. Both hypotheses were supported in this study, and an interaction between condition and load was also present. Lifters required greater amounts of time when they were not able to incorporate an eccentric braking phase in the movement. Also, they required greater amounts of time when the loads were closer to their 1 RM, especially when the relative load was highest and no countermovement was used.

Velocity was hypothesized to increase with the use of a countermovement and decrease with greater loads. In this study, the peak concentric velocity did not change significantly with the use of a countermovement. Peak concentric velocities did, however, decrease with greater loads as expected.

It was hypothesized that work would increase with the use of a countermovement and with loads closer to the 1 RM. While net concentric work did not change with the use of a countermovement, the amount of net work performed did increase with load. Unlike net work, the average amount of concentric work performed frame to frame was greater with the use of a countermovement. Also, the changes in average work by load were different. The results indicated that less average work was performed at the highest percentage of the 1 RM than any other.

Power was hypothesized to increase with the use of a countermovement and to remain unchanged with load. In the present study, peak concentric power did not change with the use of a countermovement. Also, as expected, the amount of peak concentric power did not change with load. Subjects produced a similar amount of power at each percentage of their 1 RM. Average power had results nearly identical to average work. The average amount of concentric power produced frame to frame was greater with the use of a countermovement. Also, the changes in average power by load were different than peak power and indicated that less average concentric power was produced at the highest percentage of the 1 RM compared to any other.

In the present study the force-velocity relationship was hypothesized to shift both horizontally and vertically with the use of a countermovement. Due to the deletion of the force data analyses, this relationship and any related changes to the countermovement cannot be directly evaluated. The relationship was inferred, however, from load percentages based on the premise that greater loads require greater forces for motion. Based on this postulate it was found that peak velocities decreased with greater loads in accordance with the force-velocity relationship. Further, the comparison of the PCS and CMS velocities by load seems to indicate no shift along the velocity axis. Shifts along the force axis cannot be evaluated.

Elastic energy was hypothesized to remain unchanged with modifications in load. Although the characteristic of elastic energy did not emerge as a function of load, several of its benefits were apparent in the loaded activity of this study. Specifically, the results of concentric time, average concentric work, and average concentric power all served as evidence for the elastic benefits of countermovements. The interaction effect of the concentric time results seem to further imply that the benefits of elasticity may be greater at the 85% load percentage. While important, this was the only variable that suggested this possibility. Neither of the other two indicators of elasticity had significant interaction effects, indicating that the elastic benefits may have remained unchanged as a function of load. The lack of interaction effects suggests that the elastic elements within musculature may have dynamically adjusted their stiffness/compliance value according to the need of the movement.

All of the variables changed according to the load requirements as expected. Relative concentric displacement, concentric time, net concentric work of the system, non-dimensional concentric work on the bar, and maximum concentric energy all increased as a function of load. Concentric peak velocities decreased as a function of load, as the force velocity relationship indicated. Concentric peak power values remained unchanged as a function of load, suggesting that the lifters were consistently near peak power output at all of the load percentages.

While the hypotheses regarding expected variable changes as a function of load were generally supported, the hypotheses regarding elastic energy and its expected benefits were only partially supported. Therefore, the effect of varying eccentric forces on the characteristic of elastic energy and its benefits remains largely unknown. It is possible that the measure of elastic energy has been confounded in this study, yet many of the benefits it provides may still be present.

Several explanations may account for the present results. First, the subjects' past experiences may have enabled a relatively better performance (in terms of both maximal effort and where within the thrust phase the concentric phase ended) at load percentages closer to the typical training regime. For example, subjects who trained with heavy, near maximal weights (e.g., lifters who train for powerlifting events) may have performed more skillfully in the higher load percentages. Conversely, subjects who trained with moderate weights (e.g., 8-12 RM, lifters who train for other sports or fitness) or with no weight (e.g., lifters who train with jumping tasks) may have been more skillful in the lower load percentages.

Second, even if each lift were performed with maximal effort, the concentric phase ended at varying positions during the upward thrust, never at the end of the thrust. This allowed for many different solutions in performing each condition and load percentage and possibly confounded elastic energy values. In addition, some variables had large variances with respect to the overall range of means for condition or load. This was important because statistical power was reduced by these higher variances.

Third, some may argue that because the isometric contraction prior to the purely concentric thrust only involved support of the subject's mass, and not the mass of the entire system (body mass and bar mass), the performance of the subsequent thrust would be affected. The present method was the best choice for three reasons. One reason was provided by Cavagna (1977) and states:

> In some exercises, kinetic and/or gravitational potential energy of the body are absorbed by contracted muscles while they are forcibly stretched. This mechanical energy is wasted more or less completely as heat if the muscle is *kept active at the stretched length or if it is allowed to relax after stretching*.[italics added] On the contrary, if the mechanics of the exercise is such that shortening of the muscle immediately follows stretching, an appreciable recovery of the work done on the muscle can take place. (p. 125)

Thus, the conditions necessary for the dissipation of muscle energy were met with the support of only body mass because the muscles were kept active when the lifter remained at the bottom of the lift during the five second delay.

The second reason that the isometric support of the bar mass was impractical was the relative load. Few, if any, lifters can hold 85% of their 1 RM in an isometric position for five seconds and then immediately lift the mass. Even if it were possible, fatigue would have greatly influenced the results of the motion.

The third reason for the present methodology was the need to control the range of motion in both conditions. The control of the range of motion required a limitation of the downward movement of the bar, thus preventing minute, unwanted countermovements at the beginning of the purely concentric condition and providing an identical range for the countermovement condition. This requirement affected the use of the force data for the CMS condition, but not the PCS condition.

Several modifications to the present methodology are suggested for future work. One is that the force plate data collection method be modified to avoid loss of that information in the CMS condition. Specifically, some signaling device (probably audio) should be incorporated to indicate to the subject when the proper depth of the crouch has been reached. This signal should not involve contact of the bar with the rack, but may still be based on the depth of the bar, as presently done.

Another possible modification could be to the task itself. Perhaps it should be modified with respect to its purpose. It may be better to use a movement in which the subject reaches peak velocity at the same relative distance within the concentric phase, preferably (and most easily controlled) at the end of the thrust. If the subjects have no safety concerns for additional mass, the same relative loads could be incorporated. It is not necessary that the motion involve propulsion of the subjects' body mass.

Regardless of the task modification, training of the subjects may be desirable. It may be best to train the subjects to perform with maximal efforts at various percentages of their 1 RM. This would help maximize subject performance and minimize the effects of fatigue and training specificity. A four-to-six-week period should be sufficient for familiarization and neurological adaptation to the movement.

It may also be helpful to modify the manner in which elastic energy and concentric energy are calculated. Perhaps using kinetic energy as well as total energy would be useful in understanding the benefits of elastic energy. This would be especially true for movements in which the concentric phase ends at different relative positions within the thrust, as it does here. This adjustment would effectively control for differences in potential energy, which is a large contributor to the total energy of the system.

Although the present study did not reach a conclusion regarding elasticity and relative load, the mechanical advantages of elastic behavior were still apparent in the loaded activity of this study. If athletes are to gain from the use of elastic energy it would be helpful to know the limits of those benefits. This would maximize the performers' investments in time, effort, and expense in their activity and lead to greater performance outcomes.

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APPENDICES

Appendix A: Definition of Terms

1. The concentric phase of the thrust is the time from zero vertical velocity until the time of maximum positive (upward) velocity.

2. A countermovement is any relatively quick reversal in a segment's movement about a given joint which contains a braking of motion counter to the direction of the primary movement. This braking is performed by the muscles that will be used in the primary movement phase.

3. The eccentric phase of the crouch is the time from the maximum negative (downward) velocity until the time of zero velocity. Or, in forces, it is the time from minimum unweighting vertical force until the time that the vertical force is equal to the weight of the subject and load. Only the end of downward movement is eccentric, as the muscles are used to brake the motion of the mass. In countermovement activities the muscles used to eccentrically retard motion are the same ones that perform the concentric motion that follows.

4. Mechanical energy, as used here, is the sum of potential energy, translational kinetic energy, and rotational kinetic energy.

5. Power is the time rate of performing work.

6. Prestretch is the stretch of elastic elements that occurs in the eccentric phase of the movement. The prestretch and the eccentric phase occur over the same time period.

7. Stored elastic energy is the amount of mechanical energy which is generated and saved during an eccentric contraction and then utilized in a subsequent concentric contraction.

8. Work is the displacement of a mass times the force(s) acting on it. Alternatively, work is the change in kinetic energy.

Appendix B: Consent Form

BRIEF STATEMENT OF PROJECT GOALS:

The aim of this study is to determine the relationship between load and elastic behavior in the power squat.

PROTOCOL:

Experienced weightlifters (n=10: 20-30 yrs. old) will serve as subjects for this study. Joint centers will be marked with reflective tape. Following a subject-selected warm-up, each subject will perform a one repetition maximum (1RM) for the power squat. All lifts will be performed within a power squat rack designed for the safe performance of this task. Further, all subjects will be spotted by an experienced weightlifter. For each lifter the squat rack will be adjusted so the bar can move no further down than the desired crouch depth. After a subject selected recovery time each lifter will perform 4 pairs of power squats at 85, 70, 55, and 40% of their 1 RM. Each pair will consist of a countermovement squat and a purely concentric squat. The countermovement squat is the typical power squat lift, while the purely concentric squat has the lifter perform only the thrust phase of the lift. In the purely concentric lift the bar will begin Lifters will be allowed as much recovery time as necessary between each lift. All lifts, including the 1RM, will be videotaped (sagittal plane view) with a camcorder operating at 30 Hz with a 1/2000th shutter. Further, all lifts will be performed with both feet on a 40 x 60 cm force plate mounted flush with the floor. Before leaving the laboratory anthropometric measures will be taken on each subject to determine body segment parameters. The procedures will require about 90 minutes per subject.

RISKS:

As with all vigorous human movement, there is a potential to fall, strain a muscle, or sprain a joint. This study does not pose more than the usual risk associated with vigorous movement. Because you will be performing a movement (i.e., the power squat) that is well-learned, we anticipate having no injuries. In case an injury does occur, first aid in the form of cold packs and wrappings will be applied.

Confidentiality of the data will be maintained by having coding numbers for each subject. Consent forms and coding numbers will be kept in one file drawer. Data with coding numbers and videotapes will be stored in another file drawer. The tapes will be viewed only by the investigators and subjects. There will be no public disclosure of the names or the tapes.

How would you describe the level of risk for subjects participating in this project?

_____ No risks

__X__ Minimal risks

_____ More than minimal risks

ORAL PRESENTATION

Please follow the following guidelines when preparing to participate in this study:

1) Wear comfortable t-shirt
2) Wear loose fitting athletic shorts (these will be taped on one side to expose the hip joint).
3) Wear shoes that you typically lift in.

Upon entering the biomechanics laboratory, we will mark your joint centers with small, reflective dots. Most of the joint markers are easily placed. However, it is often difficult to find the center of the hip joint. As a result, pressure will be applied at the hip so that we can find the greater trochanter (hip bone). This pressure may become uncomfortable, but the pressure is temporarily needed until we find the joint center. We will tape up your shorts in such a way that our view of the hip joint marker is unobstructed.

Following preparation for filming, you will be given time to warm up before lifting for the investigation. After warm-up, each subject will perform lifts for their 1 RM, as well as 4 pairs of squats at 85, 70, 55, and 40% of their 1RM. Each pair will consist of a countermovement squat and a purely concentric squat. We will film and collect force data for your 1 RM lifts as well as your pairs of submaximal lifts.

Based upon the observable and immediately obtainable results of the initial 15 maximal jumps, you will be assigned to an intervention treatment. Those in the balance intervention will be encouraged to reduce the amount of horizontal travel in jumping. Also, they will be told that adjusting the stagger of their feet in the starting position may be helpful. Subjects in the range of motion intervention will be informed that, for some people, a shallower crouch may produce a better jump. Subjects in the speed of motion intervention will be told to focus on being quick or springy. Subjects in the arm coordination intervention will be taught to perform a static jump with arm swing such that the thrust of the jump occurs as the arms pass vertical. All instruction will be verbally conveyed with the instructor also modeling the desired intervention procedure. Following the description of intervention, you will have a 15 minute practice session to incorporate the suggestions. Warnings about fatigue will be issued. Retesting on the style of jump which was practiced will occur immediately after the intervention period and will include three trials.

Please feel free to ask questions at any time before, during and after the testing session. We will answer all questions to the best of our abilities. However, we reserve the right to answer a question at a later time if we feel it will interfere with the testing protocol.

Our aims are to a) identify the state of your technique in the criterion (e.g., height of jump) and control (e.g., range of motion) parameters of jumping, b) intervene on one selected control parameter per subject to assess the efficacy of treatment on the process and product of jumping, and c) explore methods of observation and communication about biomechanical technique with your coach.

Through the results of this study we expect to gain insight about biomechanical variables that matter in skillful performance. Also, we expect to learn more about biomechanical intervention for the improvement of movement.

As with all vigorous human movement, there is a potential to fall, strain a muscle, or sprain the ligaments at a joint. This study does not pose more than the usual risk associated with vigorous movement. Because you will be performing a movement (i.e., jumping) that is well-learned, we anticipate having no injuries. In case an injury does occur, first aid in the form of cold packs and wrappings will be applied.

Confidentiality of the data will be maintained by using code numbers for each subject. Consent forms and code numbers will be kept in one file drawer. Data with code numbers and videotapes will be stored in another file drawer. The tapes will be viewed only by the investigators, subjects (and/or their parents), and coaches. There will be no public disclosure of your names or the tapes.

Although you may consent to participate in this project, initially, you always retain the right to withdraw your consent to participate at any time.

____
Signature of Person Obtaining Consent
on Behalf of UNCG

____
Signature of Auditor/Witness

Appendix C: Spline Smoothing Program

```
      program smooth
c
c  this program is designed to calculate segment centers, body COM,
c   segmental angles, and smooth raw, digitized data.  data smoothing
c   is accomplished through the spline subroutines provided.
c
c  output from this program is directed to several files.
c
c  generally, a 'c' in the variable name represents countermovement data
c  and a 'p' represents static [purely concentric] data.
c
c

      IMPLICIT REAL*8 (A-H,O-Z), LOGICAL (L)
      PARAMETER ( K=1, NN=200, MM=10, MM2=MM*2,
NWK=NN+6*(NN*MM+1))
      parameter ( z = 0.0, iz = 0, g = 9.7976, pi = 3.1415927 )
      DIMENSION WX(NN), WY(K), C(NN), WK(NWK), V(MM2)

      dimension x(MM,NN), y(MM,NN)
      dimension rsy(MM,NN), rsx(MM,NN)
      dimension ssy(MM,NN), ssx(MM,NN)
      dimension svy(MM,NN), say(MM,NN)
      dimension svx(MM,NN), sax(MM,NN)
      dimension resx(MM,NN),resy(MM,NN), segwt(9), dcom(8)
      dimension ytmp(NN), t(NN), resa(4,NN), sm(4), smi(4)
      dimension hang(4,NN), shang(4,NN), omega(4,NN), alpha(4,NN)
c
c  read in data
c     4= (i) paramters about ht, wt, etc.
c     6= (i) raw, digitized data
c     7= (o) output file of data: raw, smoothed, vel, acc, resid
c     9= (o) information about spline smoothing
c          - the mode for spline smoothing is quintic and the method
c            for determining completeness is based on the variance of
c            the first 10 points of the data (see smoothing subroutines).
c
      open(unit=4, file='params', status='old')
      open(unit=6, file='data', status='old')
      open(unit=7, file='smdat', status='new')
      open(unit=9, file='info', status='new')
c
c  for x and y arrays the first dimension is the joint/point
c       1 = head
c       2 = shoulder
c       3 = elbow
c       4 = wrist/bar
c       5 = hip
c       6 = knee
c       7 = ankle
```

```
c      8 = heel
c      9 = toe
c     10 = COM
c
c  for the segment model the array assignment is as follows:
c      1 = trunk
c      2 = thigh
c      3 = shank
c      4 = bar center
c      5 = head
c      6 = arm
c      7 = forearm/hand
c      8 = foot
c      9 = COM (system)
c     10 = HAT COM
c
c  angles are set up in a 4 segment system as follows:
c      1 = head, arms, & trunk (HAT)
c      2 = thigh
c      3 = shank
c      4 = bar
c   NOTE:  angles are segmental to vertical, with bx and by representing
c          the vertex in the equations
c
c  first read parameters from file
c
     read(4,*) ht, bodm, barm, iperc
c
c  read raw data
c
     nf = iz
     do 10 n = 1,NN
      nf = nf + 1
  10   read(unit=6,FMT=*,end=20) ((x(i,n), y(i,n), dumr), i=1,9)
c
  20  nf = nf - 1
c
c   assign segment weights according to model
c
     segwt(1) = 0.4684 * bodm * g
     segwt(2) = 0.2100 * bodm * g
     segwt(3) = 0.0950 * bodm * g
     segwt(4) = barm * g
     segwt(5) = 0.0826 * bodm * g
     segwt(6) = 0.0650 * bodm * g
     segwt(7) = 0.0504 * bodm * g
     segwt(8) = 0.0286 * bodm * g
     segwt(9) = (bodm * g * 0.551) + (bodm * g * 0.1154)
     totm = bodm + barm
c
c   assign dist to com from prox end
```

```
c
      dcom(1) = 0.630
      dcom(2) = 0.433
      dcom(3) = 0.434
      dcom(4) = z
      dcom(5) = 0.550
      dcom(6) = 0.436
      dcom(7) = 0.430
      dcom(8) = 0.500
c
c   subtract all y values from floor (heel) to have heights
c

      do 30 i = 1,8
        do 25 j = 1,nf
          y(i,j) = y(i,j) - y(8,j)
          x(i,j) = (-1.0 * x(i,j)) + 4.000
 25     continue
 30   continue
c
c calculate segmental moment of inertias based on plagenhoef, et al. (1983)
c  radius of gyration data
c
      sl1 = z
      sl2 = z
      sl3 = z
      sl4 = z

      do 40 i = 1,10
        sl1 = sl1 + sqrt((x(1,i)-x(5,i))**2 + (y(1,i)-y(5,i))**2)
        sl2 = sl2 + sqrt((x(6,i)-x(5,i))**2 + (y(6,i)-y(5,i))**2)
        sl3 = sl3 + sqrt((x(7,i)-x(6,i))**2 + (y(7,i)-y(6,i))**2)
        sl4 = sl4 + sqrt((x(4,i)-x(5,i))**2 + (y(4,i)-y(5,i))**2)
 40   continue

      gyr1 = (sl1 / 10.0) * 0.830
      gyr2 = (sl2 / 10.0) * 0.540
      gyr3 = (sl3 / 10.0) * 0.5290
      sl4 = sl4 / 10.0

      sm(1) = segwt(9) / g
      sm(2) = segwt(2) / g
      sm(3) = segwt(3) / g
      sm(4) = barm

      smi(1) = sm(1) * (gyr1**2)
      smi(2) = sm(2) * (gyr2**2)
      smi(3) = sm(3) * (gyr3**2)
      smi(4) = sm(4) * (sl4**2)
c
c   calculate position for COM of each segment
```

```
c
      do 50 j = 1,nf
        rsy(1,j) = ((y(3,j) - y(2,j)) * dcom(2)) + y(2,j)
        rsy(2,j) = ((y(6,j) - y(5,j)) * dcom(5)) + y(5,j)
        rsy(3,j) = ((y(7,j) - y(6,j)) * dcom(6)) + y(6,j)
        rsy(4,j) = y(4,j)
        rsy(5,j) = ((y(1,j) - y(2,j)) * dcom(1)) + y(2,j)
        rsy(6,j) = ((y(4,j) - y(3,j)) * dcom(3)) + y(3,j)
        rsy(7,j) = ((y(2,j) - y(5,j)) * dcom(4)) + y(5,j)
        rsy(8,j) = ((y(8,j) - y(7,j)) * dcom(7)) + y(7,j)
       rsy(10,j) = (rsy(1,j)*segwt(1) + rsy(5,j)*segwt(5) +
     #         rsy(6,j)*segwt(6) + rsy(7,j)*segwt(7))/segwt(9)

        rsx(1,j) = ((x(3,j) - x(2,j)) * dcom(2)) + x(2,j)
        rsx(2,j) = ((x(6,j) - x(5,j)) * dcom(5)) + x(5,j)
        rsx(3,j) = ((x(7,j) - x(6,j)) * dcom(6)) + x(6,j)
        rsx(4,j) = x(4,j)
        rsx(5,j) = ((x(1,j) - x(2,j)) * dcom(1)) + x(2,j)
        rsx(6,j) = ((x(4,j) - x(3,j)) * dcom(3)) + x(3,j)
        rsx(7,j) = ((x(2,j) - x(5,j)) * dcom(4)) + x(5,j)
        rsx(8,j) = ((x(8,j) - x(7,j)) * dcom(7)) + x(7,j)
       rsx(10,j) = (rsx(1,j)*segwt(1) + rsx(5,j)*segwt(5) +
     #         rsx(6,j)*segwt(6) + rsx(7,j)*segwt(7))/segwt(9)

   50 continue
c
c  calculate com of body
c
      do 80 j = 1,nf
       sumty = z
       sumtx = z
       do 70 i = 1,8
        sumty = sumty + rsy(i,j) * segwt(i)
        sumtx = sumtx + rsx(i,j) * segwt(i)
   70  continue

       rsy(9,j) = sumty / (totm * g)
       rsx(9,j) = sumtx / (totm * g)
   80 continue
c
c  set a few smoothing parameters for spline routines
c
      mode = 3
      m = 3
      AT = 1.0/60.0
      WY(K) = 1.0
c
c  smooth data
c
      do 200 i = 1,10
c
```

```
c   y data
c
       do 110 j = 1,nn
 110     c(j) = z

       do 120 j = 1,nwk
 120     wk(j) = z

       sy = z
       sysq = z

       do 130 j = 1,10
         sysq = sysq + rsy(i,j)**2
         sy = sy + rsy(i,j)
 130    continue

       vary = (((10.0*sysq)-(sy**2))/100.0)
       vary = vary * 2.0
       if (vary .lt. 0.00005) vary = 0.00005

       write(9,900) i,vary
 900    format( ' seg-y-pt#',i2,' -variance= ',f12.8)

       val = vary
c
c   assign weights, single dimension y array and time array
c
       do 140 j = 1,nf
        num = j
        wx(j) = 1.0
        ytmp(j) = rsy(i,j)
        t(j) = num * AT
 140    continue
C
C***    Assess spline coefficients and type resulting statistics
C
      nc = nf
      call gcvspl(t,ytmp,nc,wx,WY,m,nc,K,mode,val,c,nc,wk,ier)

       IF (IER.NE.0) THEN
         write(9,905) IER
         GO TO 999
       ELSE
         VAR = WK(6)
         IF (WK(4).EQ.0D0) THEN
           FRE = 5D-1 / AT
         ELSE
           FRE = SCALE * (WK(4)*AT)**(-0.5/M)
         ENDIF
         write(9,910) VAR, (WK(Ik),Ik=1,4), FRE
       ENDIF
```

```
905   format( ' error ',i3)
910   format( ' var =',1PD15.6,', GCV =',D15.6,',  msr =',D15.6/
  1    ' df =',0PF8.3,',          p =',1PD15.6,
  2   ', fre =',1PD15.6)

C
C***  Reconstruct data, type i, x(i), y(i), s(i), s'(i), s"(i) [D]
C***  Assess and type acceleration mean and standard deviation
C
      do 150 j = 1,nf
        kx = j
        q = splder(iz, m, nf, t(j), t, c, kx, v )
       ssy(i,j) = q
        r = splder(1, m, nf, t(j), t, c, kx, v )
       svy(i,j) = r
        s = splder(2, m, nf, t(j), t, c, kx, v )
       say(i,j) = s
      resy(i,j) = ssy(i,j) - rsy(i,j)

150   continue
c
c   x data
c
      do 160 j = 1,nn
160     c(j) = z

      do 165 j = 1,nwk
165     wk(j) = z

      sy = z
      sysq = z

      do 170 j = 1,10
        sysq = sysq + rsx(i,j)**2
        sy = sy + rsx(i,j)
170   continue

      vary = (((10.0*sysq)-(sy**2))/100.0)
      vary = vary * 2.0
      if (vary .lt. 0.00005) vary = 0.00005

      write(9,902) i,vary
902   format( ' seg-x-pt#',i2,' -variance= ',f12.8)

      val = vary
c
c  assign weights, single dimension y array and time array
c
      do 175 j = 1,nf
       num = j
       wx(j) = 1.0
```

```
        ytmp(j) = rsx(i,j)
        t(j) = num * AT
 175    continue
C
C***    Assess spline coefficients and type resulting statistics
C
       call gcvspl(t,ytmp,nc,wx,WY,m,nc,K,mode,val,c,nc,wk,ier)

       IF (IER.NE.0) THEN
         write(9,905) IER
         GO TO 999
       ELSE
         VAR = WK(6)
         IF (WK(4).EQ.0D0) THEN
           FRE = 5D-1 / AT
         ELSE
           FRE = SCALE * (WK(4)*AT)**(-0.5/M)
         ENDIF
         write(9,910) VAR, (WK(Ik),Ik=1,4), FRE
       ENDIF
C
C***    Reconstruct data, type i, x(i), y(i), s(i), s'(i), s"(i) [D]
C***    Assess and type acceleration mean and standard deviation
C
       do 180 j = 1,nf
          kx = j
          q = splder(iz, m, nf, t(j), t, c, kx, v )
         ssx(i,j) = q
          r = splder(1, m, nf, t(j), t, c, kx, v )
         svx(i,j) = r
          s = splder(2, m, nf, t(j), t, c, kx, v )
         sax(i,j) = s
        resx(i,j) = ssx(i,j) - rsx(i,j)

 180    continue

 200    continue
c
c  calculate segmental angles to vertical for use in elastic energy equation
c
      do 300 j = 1,nf
        ax = x(2,j)
        ay = y(2,j)
        bx = x(5,j)
        by = y(5,j)
       call calcan( ax, ay, bx, by, ang)
       hang(1,j) = ang
       hang(4,j) = ang

        ax = x(6,j)
        ay = y(6,j)
```

```
        bx = x(5,j)
        by = y(5,j)
       call calcan( ax, ay, bx, by, ang)
        hang(2,j) = ang

        ax = x(7,j)
        ay = y(7,j)
        bx = x(6,j)
        by = y(6,j)
       call calcan( ax, ay, bx, by, ang)
        hang(3,j) = ang
 300  continue

      do 320 i = 1,4
       iflag = z
       do 310 j = 1,nf
         if(((hang(i,j) .gt. 4.71) .and. (hang(i,j+1) .lt. 1.0))
     &          .or. (iflag .ne. z)) then

          hang(i,j+1) = hang(i,j+1) + (2.0 * pi)
          iflag = 13
        endif
 310    continue
 320  continue

c
c smooth angles
c
      do 380 i = 1,3

       do 340 j = 1,nn
 340     c(j) = z

       do 345 j = 1,nwk
 345     wk(j) = z

       sy = z
       sysq = z

       do 350 j = 1,10
        sysq = sysq + hang(i,j)**2
        sy = sy + hang(i,j)
 350    continue

       vary = (((10.0*sysq)-(sy**2))/100.0)
       vary = vary * 2.0
       if (vary .lt. 0.0001) vary = 0.0001

       write(9,921) i,vary
 921    format( ' ang #',i2,' -variance= ',f13.9)
```

```
      val = vary
c
c   assign weights, single dimension angle and time array
c
      do 360 j = 1,nf
        wx(j) = 1.0
        ytmp(j) = hang(i,j)
 360    continue
C
C***    Assess spline coefficients and type resulting statistics
C
      call gcvspl(t,ytmp,nc,wx,WY,m,nc,K,mode,val,c,nc,wk,ier)

      IF (IER.NE.0) THEN
        write(9,905) IER
        GO TO 999
      ELSE
        VAR = WK(6)
        IF (WK(4).EQ.0D0) THEN
          FRE = 5D-1 / AT
        ELSE
          FRE = SCALE * (WK(4)*AT)**(-0.5/M)
        ENDIF
        write(9,910) VAR, (WK(Ik),Ik=1,4), FRE
      ENDIF
C
C***    Reconstruct data, type i, x(i), y(i), s(i), s'(i), s"(i) [D]
C***    Assess and type acceleration mean and standard deviation
C
      do 370 j = 1,nf
         kx = j
         q = splder(iz, m, nf, t(j), t, c, kx, v )
        shang(i,j) = q
         r = splder(1, m, nf, t(j), t, c, kx, v )
        omega(i,j) = r
         s = splder(2, m, nf, t(j), t, c, kx, v )
        alpha(i,j) = s
        resa(i,j) = shang(i,j) - hang(i,j)

        if (i .eq. 1) then
          shang(4,j) = shang(i,j)
          omega(4,j) = omega(i,j)
          alpha(4,j) = alpha(i,j)
          resa(4,j) = resa(i,j)
        endif
 370    continue

 380  continue
c
c write to output files!
c
```

```
    write(7,950) nf
    write(7,955) ht, bodm, barm, float(iperc)
    write(7,955) sm(1), sm(2), sm(3), sm(4)
    write(7,955) smi(1), smi(2), smi(3), smi(4)
950 format(1h ,i6)
955 format(1h ,4(1x,f15.8))

    do 410 j = 1,nf
      do 400 i = 1,10
        write(7,960) rsx(i,j),rsy(i,j),ssx(i,j),ssy(i,j),
  !      svx(i,j),svy(i,j),sax(i,j),say(i,j),resx(i,j),resy(i,j)
960      format(1h , 4(1x,f9.5),2(1x,f10.4),2(1x,f11.3),2f11.6)
400    continue
410  continue

    do 430 j = 1,nf
      do 420 i = 1,4
        write(7,970) hang(i,j),shang(i,j),omega(i,j),
  #         alpha(i,j),resa(i,j)
970      format(1h ,2(1x,f9.5),1x,f10.4,1x,f11.3,1x,f11.6)
420    continue
430  continue

    close(unit=4)
    close(unit=6)
    close(unit=7)
    close(unit=9)

c234567890123456789012345678901234567890123456789012345678901234567890

 999 stop

    END

C GCVSPL.FOR, 1986-02-19
C
C Author: H.J. Woltring
C
C Organizations: University of Nijmegen, and
C            Philips Medical Systems, Eindhoven
C            (The Netherlands)
C
C************************************************************************
*
C
C SUBROUTINE GCVSPL (REAL*8)
C
C Purpose:
C *******
C
C     Natural B-spline data smoothing subroutine, using the Generali-
```

```
C    zed Cross-Validation and Mean-Squared Prediction Error Criteria
C    of Craven & Wahba (1979). Alternatively, the amount of smoothing
C    can be given explicitly, or it can be based on the effective
C    number of degrees of freedom in the smoothing process as defined
C    by Wahba (1980). The model assumes uncorrelated, additive noise
C    and essentially smooth, underlying functions. The noise may be
C    non-stationary, and the independent co-ordinates may be spaced
C    non-equidistantly. Multiple datasets, with common independent
C    variables and weight factors are accomodated.
C
C
C Calling convention:
C ******************
C
C    CALL GCVSPL ( X, Y, NY, WX, WY, M, N, K, MD, VAL, C, NC, WK, IER )
C
C Meaning of parameters:
C *********************
C
C    X(N)    ( I )  Independent variables: strictly increasing knot
C                   sequence, with X(I-1).lt.X(I), I=2,...,N.
C    Y(NY,K) ( I )  Input data to be smoothed (or interpolated).
C    NY      ( I )  First dimension of array Y(NY,K), with NY.ge.N.
C    WX(N)   ( I )  Weight factor array; WX(I) corresponds with
C                   the relative inverse variance of point Y(I,*).
C                   If no relative weighting information is
C                   available, the WX(I) should be set to ONE.
C                    All WX(I).gt.ZERO, I=1,...,N.
C    WY(K)   ( I )  Weight factor array; WY(J) corresponds with
C                   the relative inverse variance of point Y(*,J).
C                   If no relative weighting information is
C                   available, the WY(J) should be set to ONE.
C                    All WY(J).gt.ZERO, J=1,...,K.
C                   NB: The effective weight for point Y(I,J) is
C                   equal to WX(I)*WY(J).
C    M       ( I )  Half order of the required B-splines (spline
C                   degree 2*M-1), with M.gt.0. The values M =
C                    1,2,3,4 correspond to linear, cubic, quintic,
C                   and heptic splines, respectively.
C    N       ( I )  Number of observations per dataset, with N.ge.2*M.
C    K       ( I )  Number of datasets, with K.ge.1.
C    MD      ( I )  Optimization mode switch:
C                   |MD| = 1: Prior given value for p in VAL
C                          (VAL.ge.ZERO). This is the fastest
C                       use of GCVSPL, because no iteration
C                          is performed in p.
C                   |MD| = 2: Generalized cross validation.
C                   |MD| = 3: True predicted mean-squared error,
C                          with prior given variance in VAL.
C                   |MD| = 4: Prior given number of degrees of
C                          freedom in VAL (ZERO.le.VAL.le.N-M).
```

```
C              MD  < 0: It is assumed that the contents of
C                     X, W, M, N, and WK have not been
C                     modified because the previous invoca-
C                     tion of GCVSPL. If MD < -1, WK(4)
C                     is used as an initial estimate for
C                     the smoothing parameter p.
C             Other values for |MD|, and inappropriate values
C             for VAL will result in an error condition, or
C             cause a default value for VAL to be selected.
C             After return from MD.ne.1, the same number of
C             degrees of freedom can be obtained, for identical
C             weight factors and knot positions, by selecting
C             |MD|=1, and by copying the value of p from WK(4)
C             into VAL. In this way, no iterative optimization
C             is required when processing other data in Y.
C   VAL    ( I )  Mode value, as described above under MD.
C   C(NC,K) ( O )  Spline coefficients, to be used in conjunction
C             with function SPLDER. NB: the dimensions of C
C             in GCVSPL and in SPLDER are different! In SPLDER,
C             only a single column of C(N,K) is needed, and the
C              proper column C(1,J), with J=1...K should be used
C             when calling SPLDER.
C   NC     ( I )  First dimension of array C(NC,K), NC.ge.N.
C   WK(IWK) (I/W/O) Work vector, with length IWK.ge.6*(N*M+1)+N.
C             On normal exit, the first 6 values of WK are
C             assigned as follows:
C
C             WK(1) = Generalized Cross Validation value
C             WK(2) = Mean Squared Residual.
C             WK(3) = Estimate of the number of degrees of
C                   freedom of the residual sum of squares
C                    per dataset, with 0.lt.WK(3).lt.N-M.
C             WK(4) = Smoothing parameter p, multiplicative
C                    with the splines' derivative constraint.
C             WK(5) = Estimate of the true mean squared error
C                   (different formula for |MD| = 3).
C             WK(6) = Gauss-Markov error variance.
C
C             If WK(4) --> 0 , WK(3) --> 0 , and an inter-
C             polating spline is fitted to the data (p --> 0).
C             A very small value > 0 is used for p, in order
C             to avoid division by zero in the GCV function.
C
C             If WK(4) --> inf, WK(3) --> N-M, and a least-
C             squares polynomial of order M (degree M-1) is
C             fitted to the data (p --> inf). For numerical
C             reasons, a very high value is used for p.
C
C             Upon return, the contents of WK can be used for
C             covariance propagation in terms of the matrices
C             B and WE: see the source listings. The variance
```

```
C              estimate for dataset J follows as WK(6)/WY(J).
C
C     IER   ( O )  Error parameter:
C
C              IER = 0:     Normal exit
C              IER = 1:      M.le.0 .or. N.lt.2*M
C              IER = 2:     Knot sequence is not strictly
C                          increasing, or some weight
C                          factor is not positive.
C              IER = 3:     Wrong mode  parameter or value.
C
C Remarks:
C *******
C
C     (1) GCVSPL calculates a natural spline of order 2*M (degree
C     2*M-1) which smoothes or interpolates a given set of data
C     points, using statistical considerations to determine the
C     amount of smoothing required (Craven & Wahba, 1979). If the
C     error variance is a priori known, it should be supplied to
C     the routine in VAL, for |MD|=3. The degree of smoothing is
C     then determined to minimize an unbiased estimate of the true
C     mean squared error. On the other hand, if the error variance
C     is not known, one may select |MD|=2. The routine then deter-
C     mines the degree of smoothing to minimize the generalized
C     cross validation function. This is asymptotically the same
C     as minimizing the true predicted mean squared error (Craven &
C     Wahba, 1979). If the estimates from |MD|=2 or 3 do not appear
C     suitable to the user (as apparent from the smoothness of the
C     M-th derivative or from the effective number of degrees of
C     freedom returned in WK(3) ), the user may select another
C     value for the noise variance if |MD|=3, or a reasonably large
C     number of degrees of freedom if |MD|=4. If |MD|=1, the proce-
C     dure is non-iterative, and returns a spline for the given
C     value of the smoothing parameter p as entered in VAL.
C
C     (2) The number of arithmetic operations and the amount of
C     storage required are both proportional to N, so very large
C     datasets may be accomodated. The data points do not have
C     to be equidistant in the independant variable X or uniformly
C     weighted in the dependant variable Y. However, the data
C     points in X must be strictly increasing. Multiple dataset
C     processing (K.gt.1) is numerically more efficient dan
C     separate processing of the individual datasets (K.eq.1).
C
C     (3) If |MD|=3 (a priori known noise variance), any value of
C     N.ge.2*M is acceptable. However, it is advisable for N-2*M
C     to be rather large (at least 20) if |MD|=2 (GCV).
C
C     (4) For |MD| > 1, GCVSPL tries to iteratively minimize the
C     selected criterion function. This minimum is unique for |MD|
C     = 4, but not necessarily for |MD| = 2 or 3. Consequently,
```

```
C     local optima rather that the global optimum might be found,
C     and some actual findings suggest that local optima might
C     yield more meaningful results than the global optimum if N
C     is small. Therefore, the user has some control over the
C     search procedure. If MD > 1, the iterative search starts
C     from a value which yields a number of degrees of freedom
C     which is approximately equal to N/2, until the first (local)
C     minimum is found via a golden section search procedure
C     (Utreras, 1980). If MD < -1, the value for p contained in
C     WK(4) is used instead. Thus, if MD = 2 or 3 yield too noisy
C     an estimate, the user might try |MD| = 1 or 4, for suitably
C     selected values for p or for the number of degrees of
C     freedom, and then run GCVSPL with MD = -2 or -3. The con-
C     tents of N, M, K, X, WX, WY, and WK are assumed unchanged
C     if MD < 0.
C
C     (5) GCVSPL calculates the spline coefficient array C(N,K);
C     this array can be used to calculate the spline function
C     value and any of its derivatives up to the degree 2*M-1
C     at any argument T within the knot range, using subrou-
C     tines SPLDER and SEARCH, and the knot array X(N). Because
C     the splines are constrained at their Mth derivative, only
C     the lower spline derivatives will tend to be reliable
C     estimates of the underlying, true signal derivatives.
C
C     (6) GCVSPL combines elements of subroutine CRVO5 by Utre-
C     ras (1980), subroutine SMOOTH by Lyche et al. (1983), and
C     subroutine CUBGCV by Hutchinson (1985). The trace of the
C     influence matrix is assessed in a similar way as described
C     by Hutchinson & de Hoog (1985). The major difference is
C     that the present approach utilizes non-symmetrical B-spline
C     design matrices as described by Lyche et al. (1983); there-
C     fore, the original algorithm by Erisman & Tinney (1975) has
C     been used, rather than the symmetrical version adopted by
C     Hutchinson & de Hoog.
C
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C **********
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C     tamento de Matematicas, Faculdad de Ciencias Fisicas y Matema-
C     ticas, Universidad de Chile, Santiago.
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C     Wahba, G. (1980). Numerical and statistical methods for mildly,
C     moderately and severely ill-posed problems with noisy data.
C     Technical report nr. 595 (February 1980). Department of Statis-
C     tics, University of Madison (WI), U.S.A.
C
C Subprograms required:
C ********************
C
C     BASIS, PREP, SPLC, BANDET, BANSOL, TRINV
C
C***********************************************************************
*
C
      SUBROUTINE GCVSPL ( X, Y, NY, WX, WY, M, N, K, MD, VAL, C, NC,

     1                  WK, IER )
C
      IMPLICIT REAL*8 (A-H,O-Z)
      PARAMETER ( RATIO=2D0, TAU=1.618033983D0, IBWE=7,
     1         ZERO=0D0, HALF=5D-1 , ONE=1D0, TOL=1D-6,
     2         EPS=1D-15, EPSINV=ONE/EPS )
      DIMENSION X(N), Y(NY,K), WX(N), WY(K), C(NC,K), WK(N+6*(N*M+1))
      SAVE M2, NM1, EL
      DATA M2, NM1, EL / 2*0, 0D0 /
C
C***  Parameter check and work array initialization
C
      IER = 0
C***  Check on mode  parameter
      IF ((IABS(MD).GT.4) .OR.(  MD.EQ. 0  ) .OR.
     1 ((IABS(MD).EQ.1).AND.( VAL.LT.ZERO)).OR.
     2 ((IABS(MD).EQ.3).AND.( VAL.LT.ZERO)).OR.
     3 ((IABS(MD).EQ.4).AND.((VAL.LT.ZERO) .OR.(VAL.GT.N-M)))) THEN
        IER = 3      !Wrong mode value
        RETURN
      ENDIF
C***  Check on M and N
      IF (MD.GT.0) THEN
        M2  = 2 * M
        NM1 = N - 1
```

```
      ELSE
        IF ((M2.NE.2*M).OR.(NM1.NE.N-1)) THEN
          IER = 3   !M or N modified because previous call
          RETURN
        ENDIF
      ENDIF
      IF ((M.LE.0).OR.(N.LT.M2)) THEN
        IER = 1     !M or N invalid
        RETURN
      ENDIF
C***  Check on knot sequence and weights

      IF (WX(1).LE.ZERO) IER = 2
      DO 10 i=2,N
        IF ((WX(i).LE.ZERO).OR.(X(i-1).GE.X(i))) IER = 2
        IF (IER.NE.0) RETURN
   10 CONTINUE
      DO 15 J=1,K
        IF (WY(J).LE.ZERO) IER = 2
        IF (IER.NE.0) RETURN
   15 CONTINUE
C
C***  Work array parameters (address information for covariance
C***  propagation by means of the matrices STAT, B, and WE). NB:
C***  BWE cannot be used because it is modified by function TRINV.
C
      NM2P1 = N*(M2+1)
      NM2M1 = N*(M2-1)
C     ISTAT = 1            !Statistics array STAT(6)
C     IBWE  = ISTAT + 6    !Smoothing matrix BWE( -M:M  ,N)
      IB    = IBWE  + NM2P1  !Design matrix  B  (1-M:M-1,N)
      IWE   = IB    + NM2M1  !Design matrix  WE ( -M:M  ,N)
C     IWK   = IWE   + NM2P1  !Total work array length N + 6*(N*M+1)
C
C***  Compute the design matrices B and WE, the ratio
C***  of their L1-norms, and check for iterative mode.
C

      IF (MD.GT.0) THEN
        CALL BASIS ( M, N, X, WK(IB), R1, WK(IBWE) )
        CALL PREP  ( M, N, X, WX, WK(IWE), EL )
        EL = EL / R1 !L1-norms ratio (SAVEd upon RETURN)
      ENDIF
      IF (IABS(MD).NE.1) GO TO 20
C***    Prior given value for p
        R1 = VAL
        GO TO 100
C
C***  Iterate to minimize the GCV function (|MD|=2),
C***  the MSE function (|MD|=3), or to obtain the prior
C***  given number of degrees of freedom (|MD|=4).
```

```
C
 20  IF (MD.LT.-1) THEN
       R1 = WK(4)   !User-determined starting value
     ELSE
       R1 = ONE / EL       !Default (DOF ~ 0.5)
     ENDIF
     R2 = R1 * RATIO
     GF2 = SPLC(M,N,K,Y,NY,WX,WY,MD,VAL,R2,EPS,C,NC,
  1        WK,WK(IB),WK(IWE),EL,WK(IBWE))
 40  GF1 = SPLC(M,N,K,Y,NY,WX,WY,MD,VAL,R1,EPS,C,NC,
  1        WK,WK(IB),WK(IWE),EL,WK(IBWE))
     IF (GF1.GT.GF2) GO TO 50
       IF (WK(4).LE.ZERO) GO TO 100 !Interpolation
       R2  = R1
       GF2 = GF1
       R1  = R1 / RATIO
       GO TO 40
 50  R3 = R2 * RATIO
 60  GF3 = SPLC(M,N,K,Y,NY,WX,WY,MD,VAL,R3,EPS,C,NC,
  1        WK,WK(IB),WK(IWE),EL,WK(IBWE))
     IF (GF3.GT.GF2) GO TO 70
       IF (WK(4).GE.EPSINV) GO TO 100 !Least-squares polynomial
       R2  = R3
       GF2 = GF3
       R3  = R3 * RATIO
       GO TO 60
 70  R2  = R3
     GF2 = GF3
     ALPHA = (R2-R1) / TAU
     R4 = R1 + ALPHA
     R3 = R2 - ALPHA
     GF3 = SPLC(M,N,K,Y,NY,WX,WY,MD,VAL,R3,EPS,C,NC,
  1        WK,WK(IB),WK(IWE),EL,WK(IBWE))
     GF4 = SPLC(M,N,K,Y,NY,WX,WY,MD,VAL,R4,EPS,C,NC,
  1        WK,WK(IB),WK(IWE),EL,WK(IBWE))
 80  IF (GF3.LE.GF4) THEN
       R2  = R4
       GF2 = GF4
       ERR = (R2-R1) / (R1+R2)
       IF ((ERR*ERR+ONE.EQ.ONE).OR.(ERR.LE.TOL)) GO TO 90
       R4  = R3
       GF4 = GF3
       ALPHA = ALPHA / TAU
       R3  = R2 - ALPHA
       GF3 = SPLC(M,N,K,Y,NY,WX,WY,MD,VAL,R3,EPS,C,NC,
  1          WK,WK(IB),WK(IWE),EL,WK(IBWE))
     ELSE
       R1  = R3
       GF1 = GF3
       ERR = (R2-R1) / (R1+R2)
       IF ((ERR*ERR+ONE.EQ.ONE).OR.(ERR.LE.TOL)) GO TO 90
```

```
      R3  = R4
      GF3 = GF4
     ALPHA = ALPHA / TAU
      R4 = R1 + ALPHA
      GF4 = SPLC(M,N,K,Y,NY,WX,WY,MD,VAL,R4,EPS,C,NC,
   1       WK,WK(IB),WK(IWE),EL,WK(IBWE))
     ENDIF
     GOTO 80
  90  R1 = HALF * (R1+R2)
C
C***  Calculate final spline coefficients
C
 100  GF1 = SPLC(M,N,K,Y,NY,WX,WY,MD,VAL,R1,EPS,C,NC,
   1      WK,WK(IB),WK(IWE),EL,WK(IBWE))
C
C***  Ready
C
     RETURN
     END


C BASIS.FOR, 1985-06-03
C
C***********************************************************************
*
C
C SUBROUTINE BASIS (REAL*8)
C
C Purpose:
C *******
C
C     Subroutine to assess a B-spline tableau, stored in vectorized
C     form.
C
C Calling convention:
C ******************
C
C     CALL BASIS ( M, N, X, B, BL, Q )
C
C Meaning of parameters:
C *********************
C
C     M            ( I )  Half order of the spline (degree 2*M-1),
C                         M > 0.
C     N            ( I )  Number of knots, N >= 2*M.
C     X(N)          ( I )  Knot sequence, X(I-1) < X(I), I=2,N.
C     B(1-M:M-1,N)   ( O )  Output tableau. Element B(J,I) of array
C                         B corresponds with element b(i,i+j) of
C                         the tableau matrix B.
C     BL            ( O )  L1-norm of B.
C     Q(1-M:M)       ( W )  Internal work array.
```

```
C
C Remark:
C ******
C
C     This subroutine is an adaptation of subroutine BASIS from the
C     paper by Lyche et al. (1983). No checking is performed on the
C     validity of M and N. If the knot sequence is not strictly in-
C     creasing, division by zero may occur.
C
C Reference:
C *********
C
C     T. Lyche, L.L. Schumaker, & K. Sepehrnoori, Fortran subroutines
C     for computing smoothing and interpolating natural splines.
C     Advances in Engineering Software 5(1983)1, pp. 2-5.
C
C**********************************************************************
*
C
      SUBROUTINE BASIS ( M, N, X, B, BL, Q )
C
      IMPLICIT REAL*8 (A-H,O-Z)
      PARAMETER ( ZERO=0D0, ONE=1D0 )
      DIMENSION X(N), B(1-M:M-1,N), Q(1-M:M)
C
      IF (M.EQ.1) THEN
C***      Linear spline
         DO 3 I=1,N
           B(0,I) = ONE
    3    CONTINUE
         BL = ONE
         RETURN
      ENDIF
C
C***   General splines
C
      MM1 = M - 1
      MP1 = M + 1
      M2  = 2 * M
      DO 15 L=1,N
**       1st row
         DO 5 J=-MM1,M
           Q(J) = ZERO
    5    CONTINUE
         Q(MM1) = ONE
         IF ((L.NE.1).AND.(L.NE.N))
     1   Q(MM1) = ONE / ( X(L+1) - X(L-1) )
C***      Successive rows
         ARG = X(L)
         DO 13 I=3,M2
           IR = MP1 - I
```

```
        V = Q(IR)
        IF (L.LT.I) THEN
C***        Left-hand B-splines
          DO 6 J=L+1,I
            U  = V
            V  = Q(IR+1)
            Q(IR) = U + (X(J)-ARG)*V
            IR  = IR + 1
  6       CONTINUE
        ENDIF
        J1 = MAX0(L-I+1,1)
        J2 = MIN0(L-1,N-I)
        IF (J1.LE.J2) THEN
C***        Ordinary B-splines
          IF (I.LT.M2) THEN
            DO 8 J=J1,J2
              Y  = X(I+J)
              U  = V
              V  = Q(IR+1)
              Q(IR) = U + (V-U)*(Y-ARG)/(Y-X(J))
              IR = IR + 1
  8         CONTINUE
          ELSE
            DO 10 J=J1,J2
              U  = V
              V  = Q(IR+1)
              Q(IR) = (ARG-X(J))*U + (X(I+J)-ARG)*V
              IR  = IR + 1
  10        CONTINUE
          ENDIF
        ENDIF
        NMIP1 = N - I + 1
        IF (NMIP1.LT.L) THEN
C***        Right-hand B-splines
          DO 12 J=NMIP1,L-1
            U  = V
            V  = Q(IR+1)
            Q(IR) = (ARG-X(J))*U + V
            IR  = IR + 1
  12      CONTINUE
        ENDIF
  13  CONTINUE
      DO 14 J=-MM1,MM1
        B(J,L) = Q(J)
  14  CONTINUE
  15 CONTINUE
C
C***  Zero unused parts of B
C
    DO 17 I=1,MM1
      DO 16 K=I,MM1
```

```
        B(-K,   I) = ZERO
        B( K,N+1-I) = ZERO
  16    CONTINUE
  17  CONTINUE
C
C***   Assess L1-norm of B
C
      BL = 0D0
      DO 19 I=1,N
        DO 18 K=-MM1,MM1
          BL = BL + ABS(B(K,I))
  18    CONTINUE
  19  CONTINUE
      BL = BL / N
C
C***   Ready
C
      RETURN
      END


C PREP.FOR, 1985-07-04
C
C***********************************************************************
*
C
C SUBROUTINE PREP (REAL*8)
C
C Purpose:
C *******
C
C     To compute the matrix WE of weighted divided difference coeffi-
C     cients needed to set up a linear system of equations for sol-
C     ving B-spline smoothing problems, and its L1-norm EL. The matrix
C     WE is stored in vectorized form.
C
C Calling convention:
C *****************
C
C     CALL PREP ( M, N, X, W, WE, EL )
C
C Meaning of parameters:
C ********************
C
C     M          ( I )  Half order of the B-spline (degree
C                       2*M-1), with M > 0.
C     N          ( I )  Number of knots, with N >= 2*M.
C     X(N)       ( I )  Strictly increasing knot array, with
C                       X(I-1) < X(I), I=2,N.
C     W(N)       ( I )  Weight matrix (diagonal), with
```

```
C                  W(I).gt.0.0, I=1,N.
C    WE(-M:M,N)    ( O )  Array containing the weighted divided
C                  difference terms in vectorized format.
C                  Element WE(J,I) of array E corresponds
C                  with element e(i,i+j) of the matrix
C                  W**-1 * E.
C    EL          ( O )  L1-norm of WE.
C
C Remark:
C ******
C
C    This subroutine is an adaptation of subroutine PREP from the paper
C    by Lyche et al. (1983). No checking is performed on the validity
C    of M and N. Division by zero may occur if the knot sequence is
C    not strictly increasing.
C
C Reference:
C *********
C
C    T. Lyche, L.L. Schumaker, & K. Sepehrnoori, Fortran subroutines
C    for computing smoothing and interpolating natural splines.
C    Advances in Engineering Software 5(1983)1, pp. 2-5.
C
C***********************************************************************
*
C
      SUBROUTINE PREP ( M, N, X, W, WE, EL )
C
      IMPLICIT REAL*8 (A-H,O-Z)
      PARAMETER ( ZERO=0D0, ONE=1D0 )
      DIMENSION X(N), W(N), WE((2*M+1)*N) !WE(-M:M,N)
C
C***  Calculate the factor F1
C
      M2   = 2 * M
      MP1  = M + 1
      M2M1 = M2 - 1
      M2P1 = M2 + 1
      NM   = N - M
      F1   = -ONE
      IF (M.NE.1) THEN
        DO 5 I=2,M
          F1 = -F1 * I
   5    CONTINUE
        DO 6 I=MP1,M2M1
          F1 = F1 * I
   6    CONTINUE
      END IF
C
C***  Columnwise evaluation of the unweighted design matrix E
C
```

```
      I1 = 1
      I2 = M
      JM = MP1
      DO 17 J=1,N
        INC = M2P1
        IF (J.GT.NM) THEN
          F1 = -F1
          F = F1
        ELSE
          IF (J.LT.MP1) THEN
            INC = 1
            F  = F1
          ELSE
            F  = F1 * (X(J+M)-X(J-M))
          END IF
        END IF
        IF ( J.GT.MP1) I1 = I1 + 1
        IF (I2.LT. N) I2 = I2 + 1
        JJ = JM
C***      Loop for divided difference coefficients
        FF = F
        Y = X(I1)
        I1P1 = I1 + 1
        DO 11 I=I1P1,I2
          FF = FF / (Y-X(I))
 11     CONTINUE
        WE(JJ) = FF
        JJ = JJ + M2
        I2M1 = I2 - 1
        IF (I1P1.LE.I2M1) THEN
          DO 14 L=I1P1,I2M1
            FF = F
            Y  = X(L)
            DO 12 I=I1,L-1
              FF = FF / (Y-X(I))
 12         CONTINUE
            DO 13 I=L+1,I2
              FF = FF / (Y-X(I))
 13         CONTINUE
            WE(JJ) = FF
            JJ = JJ + M2
 14       CONTINUE
        END IF
        FF = F
        Y = X(I2)
        DO 16 I=I1,I2M1
          FF = FF / (Y-X(I))
 16     CONTINUE
        WE(JJ) = FF
        JJ = JJ + M2
        JM = JM + INC
```

```
 17  CONTINUE
C
C***   Zero the upper left and lower right corners of E
C
    KL = 1
    N2M = M2P1*N + 1
    DO 19 I=1,M
      KU = KL + M - I
      DO 18 K=KL,KU
        WE(   K) = ZERO
        WE(N2M-K) = ZERO
 18     CONTINUE
      KL = KL + M2P1
 19  CONTINUE
C
C***   Weighted matrix WE = W**-1 * E and its L1-norm
C
 20  JJ = 0
    EL = 0D0
    DO 22 I=1,N
      WI = W(I)
      DO 21 J=1,M2P1
        JJ    = JJ + 1
        WE(JJ) = WE(JJ) / WI
        EL    = EL + ABS(WE(JJ))
 21     CONTINUE
 22  CONTINUE
    EL = EL / N
C
C***   Ready
C
    RETURN
    END


C SPLC.FOR, 1985-12-12
C
C Author: H.J. Woltring
C
C Organizations: University of Nijmegen, and
C             Philips Medical Systems, Eindhoven
C             (The Netherlands)
C
C***********************************************************************
*
C
C FUNCTION SPLC (REAL*8)
C
C Purpose:
C *******
```

```
C
C     To assess the coefficients of a B-spline and various statistical
C     parameters, for a given value of the regularization parameter p.
C
C Calling convention:
C ******************
C
C     FV = SPLC ( M, N, K, Y, NY, WX, WY, MODE, VAL, P, EPS, C, NC,
C   1        STAT, B, WE, EL, BWE)
C
C Meaning of parameters:
C *********************
C
C     SPLC       ( O )  GCV function value if |MODE|.eq.2,
C                  MSE value if |MODE|.eq.3, and absolute
C                  difference with the prior given number of
C                  degrees of freedom if |MODE|.eq.4.
C     M          ( I )  Half order of the B-spline (degree 2*M-1),
C                  with M > 0.
C     N          ( I )  Number of observations, with N >= 2*M.
C     K          ( I )  Number of datasets, with K >= 1.
C     Y(NY,K)     ( I )  Observed measurements.
C     NY         ( I )  First dimension of Y(NY,K), with NY.ge.N.
C     WX(N)       ( I )  Weight factors, corresponding to the
C                  relative inverse variance of each measure-
C                  ment, with WX(I) > 0.0.
C     WY(K)       ( I )  Weight factors, corresponding to the
C                  relative inverse variance of each dataset,
C                  with WY(J) > 0.0.
C     MODE        ( I )  Mode switch, as described in GCVSPL.
C     VAL        ( I )  Prior variance if |MODE|.eq.3, and
C                  prior number of degrees of freedom if
C                  |MODE|.eq.4. For other values of MODE,
C                  VAL is not used.
C     P          ( I )  Smoothing parameter, with P >= 0.0. If
C                  P.eq.0.0, an interpolating spline is
C                  calculated.
C     EPS        ( I )  Relative rounding tolerance*10.0. EPS is
C                  the smallest positive number such that
C                  EPS/10.0 + 1.0 .ne. 1.0.
C     C(NC,K)     ( O )  Calculated spline coefficient arrays. NB:
C                  the dimensions of in GCVSPL and in SPLDER
C                  are different! In SPLDER, only a single
C                  column of C(N,K) is needed, and the proper
C                  column C(1,J), with J=1...K, should be used
C                  when calling SPLDER.
C     NC         ( I )  First dimension of C(NC,K), with NC.ge.N.
C     STAT(6)     ( O )  Statistics array. See the description in
C                  subroutine GCVSPL.
C     B (1-M:M-1,N)  ( I )  B-spline tableau as evaluated by subroutine
C                  BASIS.
```

```
C     WE( -M:M ,N)  ( I )  Weighted B-spline tableau (W**-1 * E) as
C                          evaluated by subroutine PREP.
C     EL            ( I )  L1-norm of the matrix WE as evaluated by
C                          subroutine PREP.
C     BWE(-M:M,N)   ( O )  Central 2*M+1 bands of the inverted
C                          matrix ( B  +  p * W**-1 * E )**-1
C
C Remarks:
C *******
C
C     This subroutine combines elements of subroutine SPLC0 from the
C     paper by Lyche et al. (1983), and of subroutine SPFIT1 by
C     Hutchinson (1985).
C
C References:
C **********
C
C     M.F. Hutchinson (1985), Subroutine CUBGCV. CSIRO division of
C     Mathematics and Statistics, P.O. Box 1965, Canberra, ACT 2601,
C     Australia.
C
C     T. Lyche, L.L. Schumaker, & K. Sepehrnoori, Fortran subroutines
C     for computing smoothing and interpolating natural splines.
C     Advances in Engineering Software 5(1983)1, pp. 2-5.
C
C***********************************************************************
*
C
      FUNCTION SPLC( M, N, K, Y, NY, WX, WY, MODE, VAL, P, EPS,
     1          C, NC, STAT, B, WE, EL, BWE)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      PARAMETER ( ZERO=0D0, ONE=1D0, TWO=2D0 )
      DIMENSION Y(NY,K), WX(N), WY(K), C(NC,K), STAT(6),
     1      B(1-M:M-1,N), WE(-M:M,N), BWE(-M:M,N)
C
C***    Check on p-value
C
      DP = P
      STAT(4) = P
      PEL = P * EL
C***    Pseudo-interpolation if p is too small
      IF (PEL.LT.EPS) THEN
        DP = EPS / EL
        STAT(4) = ZERO
      ENDIF
C***    Pseudo least-squares polynomial if p is too large
      IF (PEL*EPS.GT.ONE) THEN
        DP = ONE / (EL*EPS)
        STAT(4) = DP
      ENDIF
```

```
C
C***   Calculate BWE = B + p * W**-1 * E
C
      DO 40 I=1,N
        KM = -MIN0(M,I-1)
        KP =  MIN0(M,N-I)
        DO 30 L=KM,KP
          IF (IABS(L).EQ.M) THEN
            BWE(L,I) =          DP * WE(L,I)
          ELSE
            BWE(L,I) = B(L,I) + DP * WE(L,I)
          ENDIF
 30     CONTINUE
 40   CONTINUE
C
C***   Solve BWE * C = Y, and assess TRACE [ B * BWE**-1 ]
C
      CALL BANDET ( BWE, M, N )
      CALL BANSOL ( BWE, Y, NY, C, NC, M, N, K )
      STAT(3) = TRINV ( WE, BWE, M, N ) * DP !trace * p = res. d.o.f.
      TRN = STAT(3) / N
C
C***   Compute mean-squared weighted residual
C
      ESN = ZERO
      DO 70 J=1,K
        DO 60 I=1,N
          DT = -Y(I,J)
          KM = -MIN0(M-1,I-1)
          KP =  MIN0(M-1,N-I)
          DO 50 L=KM,KP
            DT = DT + B(L,I)*C(I+L,J)
 50       CONTINUE
          ESN = ESN + DT*DT*WX(I)*WY(J)
 60     CONTINUE
 70   CONTINUE
      ESN = ESN / (N*K)
C
C***   Calculate statistics and function value
C
      STAT(6) = ESN / TRN          !Estimated variance
      STAT(1) = STAT(6) / TRN      !GCV function value
      STAT(2) = ESN                !Mean Squared Residual
C     STAT(3) = trace [p*B * BWE**-1] !Estimated residuals' d.o.f.
C     STAT(4) = P                  !Normalized smoothing factor
      IF (IABS(MODE).NE.3) THEN
C***     Unknown variance: GCV
        STAT(5) = STAT(6) - ESN
        IF (IABS(MODE).EQ.1) SPLC = ZERO
        IF (IABS(MODE).EQ.2) SPLC = STAT(1)
        IF (IABS(MODE).EQ.4) SPLC = sqrt(( STAT(3) - VAL )**2)
```

```
      ELSE
C***      Known variance: estimated mean squared error
        STAT(5) = ESN - VAL*(TWO*TRN - ONE)
        SPLC = STAT(5)
      ENDIF
C
      RETURN
      END


C BANDET.FOR, 1985-06-03
C
C***********************************************************************
*
C
C SUBROUTINE BANDET (REAL*8)
C
C Purpose:
C *******
C
C     This subroutine computes the LU decomposition of an N*N matrix
C     E. It is assumed that E has M bands above and M bands below the
C     diagonal. The decomposition is returned in E. It is assumed that
C     E can be decomposed without pivoting. The matrix E is stored in
C     vectorized form in the array E(-M:M,N), where element E(J,I) of
C     the array E corresponds with element e(i,i+j) of the matrix E.
C
C Calling convention:
C ******************
C
C     CALL BANDET ( E, M, N )
C
C Meaning of parameters:
C *********************
C
C     E(-M:M,N)     (I/O)  Matrix to be decomposed.
C     M, N          ( I )  Matrix dimensioning parameters,
C                          M >= 0, N >= 2*M.
C
C Remark:
C ******
C
C     No checking on the validity of the input data is performed.
C     If (M.le.0), no action is taken.
C
C***********************************************************************
*
C
      SUBROUTINE BANDET ( E, M, N )
C
```

```
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION E(-M:M,N)
C
      IF (M.LE.0) RETURN
      DO 40 I=1,N
        DI = E(0,I)
        MI = MIN0(M,I-1)
        IF (MI.GE.1) THEN
          DO 10 K=1,MI
            DI = DI - E(-K,I)*E(K,I-K)
   10     CONTINUE
          E(0,I) = DI
        ENDIF
        LM = MIN0(M,N-I)
        IF (LM.GE.1) THEN
          DO 30 L=1,LM
            DL = E(-L,I+L)
            KM = MIN0(M-L,I-1)
            IF (KM.GE.1) THEN
              DU = E(L,I)
              DO 20 K=1,KM
                DU = DU - E(  -K,  I)*E(L+K,I-K)
                DL = DL - E(-L-K,L+I)*E(  K,I-K)
   20         CONTINUE
              E(L,I) = DU
            ENDIF
            E(-L,I+L) = DL / DI
   30     CONTINUE
        ENDIF
   40 CONTINUE
C
C***  Ready
C
      RETURN
      END


C BANSOL.FOR, 1985-12-12
C
C***********************************************************************
*
C
C SUBROUTINE BANSOL (REAL*8)
C
C Purpose:
C *******
C
C     This subroutine solves systems of linear equations given an LU
C     decomposition of the design matrix. Such a decomposition is pro-
C     vided by subroutine BANDET, in vectorized form. It is assumed
```

```
C     that the design matrix is not singular.
C
C Calling convention:
C ******************
C
C     CALL BANSOL ( E, Y, NY, C, NC, M, N, K )
C
C Meaning of parameters:
C *********************
C
C     E(-M:M,N)     ( I )  Input design matrix, in LU-decomposed,
C                          vectorized form. Element E(J,I) of the
C                          array E corresponds with element
C                          e(i,i+j) of the N*N design matrix E.
C     Y(NY,K)       ( I )  Right hand side vectors.
C     C(NC,K)       ( O )  Solution vectors.
C     NY, NC, M, N, K ( I )  Dimensioning parameters, with M >= 0,
C                          N > 2*M, and K >= 1.
C
C Remark:
C ******
C
C     This subroutine is an adaptation of subroutine BANSOL from the
C     paper by Lyche et al. (1983). No checking is performed on the
C     validity of the input parameters and data. Division by zero may
C     occur if the system is singular.
C
C Reference:
C *********
C
C     T. Lyche, L.L. Schumaker, & K. Sepehrnoori, Fortran subroutines
C     for computing smoothing and interpolating natural splines.
C     Advances in Engineering Software 5(1983)1, pp. 2-5.
C
C*********************************************************************
*
C
      SUBROUTINE BANSOL ( E, Y, NY, C, NC, M, N, K )
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION E(-M:M,N), Y(NY,K), C(NC,K)
C
C***  Check on special cases: M=0, M=1, M>1
C
      NM1 = N - 1
      IF (M-1) 10,40,80
C
C***  M = 0: Diagonal system
C
   10 DO 30 I=1,N
        DO 20 J=1,K
```

```
      C(I,J) = Y(I,J) / E(0,I)
 20   CONTINUE
 30  CONTINUE
    RETURN
C
C***   M = 1: Tridiagonal system
C
 40  DO 70 J=1,K
      C(1,J) = Y(1,J)
      DO 50 I=2,N         !Forward sweep
        C(I,J) =  Y(I,J) - E(-1,I)*C(I-1,J)
 50   CONTINUE
      C(N,J) = C(N,J) / E(0,N)
      DO 60 I=NM1,1,-1    !Backward sweep
        C(I,J) = (C(I,J) - E( 1,I)*C(I+1,J)) / E(0,I)
 60   CONTINUE
 70  CONTINUE
    RETURN
C
C***   M > 1: General system
C
 80  DO 130 J=1,K
      C(1,J) = Y(1,J)
      DO 100 I=2,N        !Forward sweep
        MI = MIN0(M,I-1)
        D  = Y(I,J)
        DO 90 L=1,MI
          D = D - E(-L,I)*C(I-L,J)
 90     CONTINUE
        C(I,J) = D
 100  CONTINUE
      C(N,J) = C(N,J) / E(0,N)
      DO 120 I=NM1,1,-1   !Backward sweep
        MI = MIN0(M,N-I)
        D  = C(I,J)
        DO 110 L=1,MI
          D = D - E( L,I)*C(I+L,J)
 110    CONTINUE
        C(I,J) = D / E(0,I)
 120  CONTINUE
 130 CONTINUE
    RETURN
C
    END


C TRINV.FOR, 1985-06-03
C
C Author: H.J. Woltring
```

```
C
C Organizations: University of Nijmegen, and
C          Philips Medical Systems, Eindhoven
C          (The Netherlands)
C
C***********************************************************************
*
C
C FUNCTION TRINV (REAL*8)
C
C Purpose:
C *******
C
C    To calculate TRACE [ B * E**-1 ], where B and E are N * N
C    matrices with bandwidth 2*M+1, and where E is a regular matrix
C    in LU-decomposed form. B and E are stored in vectorized form,
C    compatible with subroutines BANDET and BANSOL.
C
C Calling convention:
C ******************
C
C    TRACE = TRINV ( B, E, M, N )
C
C Meaning of parameters:
C *********************
C
C    B(-M:M,N)     ( I ) Input array for matrix B. Element B(J,I)
C                  corresponds with element b(i,i+j) of the
C                  matrix B.
C    E(-M:M,N)     (I/O) Input array for matrix E. Element E(J,I)
C                  corresponds with element e(i,i+j) of the
C                  matrix E. This matrix is stored in LU-
C                  decomposed form, with L unit lower tri-
C                  angular, and U upper triangular. The unit
C                  diagonal of L is not stored. Upon return,
C                  the array E holds the central 2*M+1 bands
C                  of the inverse E**-1, in similar ordering.
C    M, N          ( I ) Array and matrix dimensioning parameters
C                  (M.gt.0, N.ge.2*M+1).
C    TRINV         ( O ) Output function value TRACE [ B * E**-1 ]
C
C Reference:
C *********
C
C    A.M. Erisman & W.F. Tinney, On computing certain elements of the
C    inverse of a sparse matrix. Communications of the ACM 18(1975),
C    nr. 3, pp. 177-179.
C
C***********************************************************************
*
C
```

```
      REAL*8 FUNCTION TRINV ( B, E, M, N )
C
      IMPLICIT REAL*8 (A-H,O-Z)
      PARAMETER ( ZERO=0D0, ONE=1D0 )
      DIMENSION B(-M:M,N), E(-M:M,N)
C
C***  Assess central 2*M+1 bands of E**-1 and store in array E
C
      E(0,N) = ONE / E(0,N)  !Nth pivot

      DO 40 I=N-1,1,-1
        MI = MIN0(M,N-I)
        DD  = ONE / E(0,I)  !Ith pivot
C***     Save Ith column of L and Ith row of U, and normalize U row
        DO 10 K=1,MI
          E( K,N) = E( K, I) * DD  !Ith row of U (normalized)
          E(-K,1) = E(-K,K+I)      !Ith column of L
 10     CONTINUE
        DD = DD + DD
C***     Invert around Ith pivot
        DO 30 J=MI,1,-1
          DU = ZERO
          DL = ZERO
          DO 20 K=1,MI
            DU = DU - E( K,N)*E(J-K,I+K)
            DL = DL - E(-K,1)*E(K-J,I+J)
 20       CONTINUE
          E( J, I) = DU
          E(-J,J+I) = DL
          DD = DD - (E(J,N)*DL + E(-J,1)*DU)
 30     CONTINUE
        E(0,I) = 5D-1 * DD
 40   CONTINUE
C
C***  Assess TRACE [ B * E**-1 ] and clear working storage
C
      DD = ZERO
      DO 60 I=1,N
        MN = -MIN0(M,I-1)
        MP =  MIN0(M,N-I)
        DO 50 K=MN,MP
          DD = DD + B(K,I)*E(-K,K+I)
 50     CONTINUE
 60   CONTINUE
      TRINV = DD
      DO 70 K=1,M
        E( K,N) = ZERO
        E(-K,1) = ZERO
 70   CONTINUE
C
C***  Ready
```

```
C
      RETURN
      END
```

```
C SPLDER.FOR, 1985-06-11
C
C***********************************************************************
*
C
C FUNCTION SPLDER (REAL*8)
C
C Purpose:
C *******
C
C     To produce the value of the function (IDER.eq.0) or of the
C     IDERth derivative (IDER.gt.0) of a 2M-th order B-spline at
C     the point T. The spline is described in terms of the half
C     order M, the knot sequence X(N), N.ge.2*M, and the spline
C     coefficients C(N).
C
C Calling convention:
C ******************
C
C     SVIDER = SPLDER ( IDER, M, N, T, X, C, L, Q )
C
C Meaning of parameters:
C *********************
C
C     SPLDER  ( O )  Function or derivative value.
C     IDER    ( I )  Derivative order required, with 0.le.IDER
C                    and IDER.le.2*M. If IDER.eq.0, the function
C                    value is returned; otherwise, the IDER-th
C                    derivative of the spline is returned.
C     M       ( I )  Half order of the spline, with M.gt.0.
C     N       ( I )  Number of knots and spline coefficients,
C                    with N.ge.2*M.
C     T       ( I )  Argument at which the spline or its deri-
C                    vative is to be evaluated, with X(1).le.T
C                    and T.le.X(N).
C     X(N)    ( I )  Strictly increasing knot sequence array,
C                    X(I-1).lt.X(I), I=2,...,N.
C     C(N)    ( I )  Spline coefficients, as evaluated by
C                    subroutine GCVSPL.
C     L       (I/O)  L contains an integer such that:
C                    X(L).le.T and T.lt.X(L+1) if T is within
C                    the range X(1).le.T and T.lt.X(N). If
```

```
C               T.lt.X(1), L is set to 0, and if T.ge.X(N),
C               L is set to N. The search for L is facili-
C               tated if L has approximately the right
C               value on entry.
C     Q(2*M)  ( W )  Internal work array.
C
C Remark:
C ******
C
C     This subroutine is an adaptation of subroutine SPLDER of
C     the paper by Lyche et al. (1983). No checking is performed
C     on the validity of the input parameters.
C
C Reference:
C *********
C
C     T. Lyche, L.L. Schumaker, & K. Sepehrnoori, Fortran subroutines
C     for computing smoothing and interpolating natural splines.
C     Advances in Engineering Software 5(1983)1, pp. 2-5.
C
C***********************************************************************
*
C
      REAL*8 FUNCTION SPLDER ( IDER, M, N, T, X, C, L, Q )
C
      IMPLICIT REAL*8 (A-H,O-Z)
      PARAMETER ( ZERO=0D0, ONE=1D0 )
      DIMENSION X(N), C(N), Q(2*M)
C
C***  Derivatives of IDER.ge.2*M are alway zero
C
      M2 =  2 * M
      K  = M2 - IDER
      IF (K.LT.1) THEN
        SPLDER = ZERO
        RETURN
      ENDIF
C
C***  Search for the interval value L
C
      CALL SEARCH ( N, X, T, L )
C
C***  Initialize parameters and the 1st row of the B-spline
C***  coefficients tableau
C
      TT  = T
      MP1  = M + 1
      NPM  = N + M
      M2M1 = M2 - 1
      K1   = K - 1
      NK   = N - K
```

```
      LK  = L - K
      LK1 = LK + 1
      LM  = L - M
      JL  = L + 1
      JU  = L + M2
      II  = N - M2
      ML  = -L
      DO 2 J=JL,JU
        IF ((J.GE.MP1).AND.(J.LE.NPM)) THEN
          Q(J+ML) = C(J-M)
        ELSE
          Q(J+ML) = ZERO
        ENDIF
    2 CONTINUE
C
C***  The following loop computes differences of the B-spline
C***  coefficients. If the value of the spline is required,
C***  differencing is not necessary.
C
      IF (IDER.GT.0) THEN
        JL = JL - M2
        ML = ML + M2
        DO 6 I=1,IDER
          JL = JL + 1
          II = II + 1
          J1 = MAX0(1,JL)
          J2 = MIN0(L,II)
          MI = M2 - I
          J  = J2 + 1
          IF (J1.LE.J2) THEN
            DO 3 JIN=J1,J2
              J  = J - 1
              JM = ML + J
              Q(JM) = (Q(JM) - Q(JM-1)) / (X(J+MI) - X(J))
    3       CONTINUE
          ENDIF
          IF (JL.GE.1) GO TO 6
            I1 = I + 1
            J  = ML + 1
            IF (I1.LE.ML) THEN
              DO 5 JIN=I1,ML
                J  = J - 1
                Q(J) = -Q(J-1)
    5         CONTINUE
            ENDIF
    6   CONTINUE
        DO 7 J=1,K
          Q(J) = Q(J+IDER)
    7   CONTINUE
      ENDIF
C
```

```
C***  Compute lower half of the evaluation tableau
C
      IF (K1.GE.1) THEN       !Tableau ready if IDER.eq.2*M-1
        DO 14 I=1,K1
          NKI  = NK + I
          IR  =  K
          JJ  =  L
          KI  =  K - I
          NKI1 = NKI + 1
C***        Right-hand B-splines
          IF (L.GE.NKI1) THEN
            DO 9 J=NKI1,L
              Q(IR) = Q(IR-1) + (TT-X(JJ))*Q(IR)
              JJ   = JJ - 1
              IR   = IR - 1
  9         CONTINUE
          ENDIF
C***        Middle B-splines
          LK1I = LK1 + I
          J1 = MAX0(1,LK1I)
          J2 = MIN0(L, NKI)
          IF (J1.LE.J2) THEN
            DO 11 J=J1,J2
              XJKI  = X(JJ+KI)
              Z    = Q(IR)
              Q(IR) = Z + (XJKI-TT)*(Q(IR-1)-Z)/(XJKI-X(JJ))
              IR   = IR - 1
              JJ   = JJ - 1
 11         CONTINUE
          ENDIF
C***        Left-hand B-splines
          IF (LK1I.LE.0) THEN
            JJ   = KI
            LK1I1 =  1 - LK1I
            DO 13 J=1,LK1I1
              Q(IR) = Q(IR) + (X(JJ)-TT)*Q(IR-1)
              JJ   = JJ - 1
              IR   = IR - 1
 13         CONTINUE
          ENDIF
 14     CONTINUE
      ENDIF
C
C***  Compute the return value
C
      Z = Q(K)
C***  Multiply with factorial if IDER.gt.0
      IF (IDER.GT.0) THEN
        DO 16 J=K,M2M1
          Z = Z * J
 16     CONTINUE
```

```
      ENDIF
      SPLDER = Z
C
C***   Ready
C
      RETURN
      END




C SEARCH.FOR, 1985-06-03
C
C***********************************************************************
*
C
C SUBROUTINE SEARCH (REAL*8)
C
C Purpose:
C *******
C
C     Given a strictly increasing knot sequence X(1) < ... < X(N),
C     where N >= 1, and a real number T, this subroutine finds the
C     value L such that X(L) <= T < X(L+1).  If T < X(1), L = 0;
C     if X(N) <= T, L = N.
C
C Calling convention:
C ******************
C
C     CALL SEARCH ( N, X, T, L )
C
C Meaning of parameters:
C *********************
C
C     N      ( I )  Knot array dimensioning parameter.
C     X(N)   ( I )  Stricly increasing knot array.
C     T      ( I )  Input argument whose knot interval is to
C                   be found.
C     L      (I/O)  Knot interval parameter. The search procedure
C                   is facilitated if L has approximately the
C                   right value on entry.
C
C Remark:
C ******
C
C     This subroutine is an adaptation of subroutine SEARCH from
C     the paper by Lyche et al. (1983). No checking is performed
C     on the input parameters and data; the algorithm may fail if
C     the input sequence is not strictly increasing.
C
```

```
C Reference:
C *********
C
C     T. Lyche, L.L. Schumaker, & K. Sepehrnoori, Fortran subroutines
C     for computing smoothing and interpolating natural splines.
C     Advances in Engineering Software 5(1983)1, pp. 2-5.
C
C***********************************************************************
*
C
      SUBROUTINE SEARCH ( N, X, T, L )
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(N)
C
      IF (T.LT.X(1)) THEN
C***     Out of range to the left
        L = 0
        RETURN
      ENDIF
      IF (T.GE.X(N)) THEN
C***     Out of range to the right
        L = N
        RETURN
      ENDIF
C***   Validate input value of L
      L = MAX0(L,1)
      IF (L.GE.N) L = N-1
C
C***   Often L will be in an interval adjoining the interval found
C***   in a previous call to search
C
      IF (T.GE.X(L)) GO TO 5
      L = L - 1
      IF (T.GE.X(L)) RETURN
C
C***   Perform bisection
C
      IL = 1
    3 IU = L
    4 L = (IL+IU) / 2
      IF (IU-IL.LE.1) RETURN
      IF (T.LT.X(L)) GO TO 3
      IL = L
      GO TO 4
    5 IF (T.LT.X(L+1)) RETURN
      L = L + 1
      IF (T.LT.X(L+1)) RETURN
      IL = L + 1
      IU = N
      GO TO 4
```

```
C
      END


      subroutine calcan(ax,ay,bx,by,ang)
c
c  subroutine calcan calcultes the segment angles to horizontal for
c     any given segment.  both endpoints of the segment are required.
c        ang is the output of the routine.
c
      IMPLICIT REAL*8 (A-H,O-Z)
      PARAMETER ( zz=0D0, pii = 3.1415927 )

      ax = ax - bx
      ay = ay - by
      bx = bx - bx
      by = by - by
c
c  first check for zero values
c
      if ((ax .ne. zz) .and. (ay .ne. zz)) then
        if (ax .gt. zz) then
          if (ay .gt. zz) then
c
c must be in quad I
c
            vv = ay / ax
            ang = datan(vv)
          else
c
c must be in quad IV
c
            vv = ay / ax
            ang = (2.0 * pii) + datan(vv)
          endif
        else
c
c must be in quad II or III [same equation either way]
c
          vv = ay / ax
          ang = pii + datan(vv)
        endif
      else
c
c check where zero values are and assign angle value
c
        if (ay .eq. zz) then
          if (ax .eq. zz) then
            ang = zz
          else
```

```
        ang = pii
      endif
    else
      if (ay .gt. zz) then
        ang = pii / 2.00
      else
        ang = 3.0 * pii / 2.0
      endif
    endif
  endif
end
```

Appendix D: Variable Calculation Program

```
      program numcalc
c
c  this program is designed to calculate or find the values for power
c  squat force data.  the data should be smoothed and the appropriate
c  calculations made for acceleration, velocity, and displacement.  this
c  program contains no tests for the validity of the data.  it reads
c  multiple files of smoothed, derived data.
c
c  output from this program is directed to several files.
c
c  generally, a 'c' in the variable name represents countermovement data
c  and a 'p' represents static [purely concentric] data.
c
c

      IMPLICIT REAL*8 (A-H,O-Z), LOGICAL (L)
      PARAMETER ( K=1, NN=200, MM=10, MM2=MM*2,
NWK=NN+6*(NN*MM+1))
      parameter ( z = 0.0, iz = 0, g = 9.7976, pi = 3.1415927 )
      DIMENSION WX(NN), WY(K), C(NN), WK(NWK), V(MM2)

      dimension ssyc(MM,NN), ssyp(MM,NN), ssxc(MM,NN), ssxp(MM,NN)
      dimension svyc(MM,NN), sayc(MM,NN), svxc(MM,NN), saxc(MM,NN)
      dimension svyp(MM,NN), sayp(MM,NN), svxp(MM,NN), saxp(MM,NN)
      dimension tc(NN), tp(NN), smi(4), sm(4)
      dimension shangc(4,NN), omegac(4,NN), alphac(4,NN)
      dimension shangp(4,NN), omegap(4,NN), alphap(4,NN)
      dimension ec(4,NN), ep(4,NN), ebodc(NN), ebodp(NN)
      dimension workc(NN), pcwork(NN), powc(NN), pcpow(NN)
      dimension penrg(NN), vkenrg(NN), hkenrg(NN), rkenrg(NN)
      dimension cwseg(NN), pwseg(NN), cpseg(NN), ppseg(NN)
c
c  read in data
c     5= (i) countermovent data
c     6= (i) purely concentric data
c     8= (o) calculated variables: avg work, power, elastic energy, etc.
c     3= (o) arrays of instantaneous work, power
c
      open(unit=5, file='cdata', status='old')
      open(unit=6, file='pdata', status='old')
      open(unit=8, file='calcs', status='new')
      open(unit=3, file='calcout', status='new')
c
c  for the segment model the array assignment is as follows:
c        1 = trunk
c        2 = thigh
c        3 = shank
c        4 = bar center
c        5 = head
c        6 = arm
```

```
c       7 = forearm/hand
c       8 = foot
c       9 = COM (system)
c
c
c  angles and moments of inertia are set up in a 4 segment system as follows:
c       1 = trunk
c       2 = thigh
c       3 = shank
c       4 = bar
c   NOTE: angles are segmental to vertical, with bx and by representing
c          the vertex in the equations
c
c   calculate absolute time between frames (frame rate)
c
    AT = 1.0 / 60.0
c
c  first read parameters from file
c
c  then read data for countermovement and purely-concentric lifts
c
    read(5,*) nfc
    read(5,*) ht, bodm, barm, perc
    read(5,*) sm(1),sm(2),sm(3),sm(4)
    read(5,*) smi(1),smi(2),smi(3),smi(4)
    do 10 n = 1,nfc
     time = AT * n
     tc(n) = time
     do 5 i = 1,10
       read(unit=5,FMT=*) rd,rd,ssxc(i,n),ssyc(i,n),
   !          svxc(i,n),svyc(i,n),saxc(i,n),sayc(i,n),rd,rd
  5   continue
 10 continue

    do 20 n = 1,nfc
     do 15 i = 1,4
       read(unit=5,FMT=*) rd,shangc(i,n),omegac(i,n),
   #          alphac(i,n),rd
 15   continue
 20 continue

    read(6,*) nfp
    read(6,*) rd,rd,rd,rd
    read(6,*) rd,rd,rd,rd
    read(6,*) rd,rd,rd,rd
    do 30 n = 1,nfp
     time = AT * n
     tp(n) = time
     do 25 i = 1,10
       read(unit=6,FMT=*) rd,rd,ssxp(i,n),ssyp(i,n),
   !          svxp(i,n),svyp(i,n),saxp(i,n),sayp(i,n),rd,rd
```

```
 25    continue
 30  continue

     do 40 n = 1,nfp
      do 35 i = 1,4
       read(unit=6,FMT=*) rd,shangp(i,n),omegap(i,n),
   #         alphap(i,n),rd
 35   continue
 40   continue

     tmass = barm + bodm
c
c   calculate variables such as work, energy, power, elastic energy
c
c
c  find the time and value for minimum & maximum velocity (conc)
c
     velmin = z
     velmax = z
     j = 1

 500  if (svyc(9,j) .le. velmin) then
        velmin = svyc(9,j)
        istecc = j
      write(8,*) istecc,velmin
      endif
     if (svyc(9,j) .ge. velmax) then
        velmax = svyc(9,j)
        iendcnc = j
      endif

     j = j + 1
     if (j .lt. (nfc-2)) go to 500
c
c   find start of concentric / end of eccentric
c
     j = istecc
 510  if ((svyc(9,j) .lt. z) .and. (svyc(9,j+1) .ge. z)) then
        istcnc = j + 1
        iendecc = j
       else
        j = j + 1
        go to 510
      endif

     timecc = tc(iendecc) - tc(istecc)
     ctimcnc = tc(iendcnc) - tc(istcnc)

     write(8,938) perc, nfc, nfp
     write(8,940)
     write(8,942) velmin, velmax
```

```
      write(8,943) istecc, iendecc, istcnc, iendcnc
      write(8,944) timecc, ctimcnc

  938 format( ' % of 1 RM =',f4.1,' #fr. cms =',i4,' #fr. pcs =',i4)
  940 format( ' info on CMS:')
  942 format( ' minimum velocity= ',f7.4,' m/s     ',
     @ ' maximum velocity=',f7.4,' m/s')
  943 format( ' frame #: st. ecc= ',i3,' end ecc= ',i3,
     % ' st. conc= ',i3,' end conc= ',i3)
  944 format( ' ecc time= ',f7.4,' sec',' conc time= ',f7.4,' sec')

      write(8,998)
  998 format(/' ____________________________________________________')
c
c same for pcs
c
      n = 10
      pvmax = svyp(9,n)

  520 n = n + 1
      if (svyp(9,n) .gt. pvmax) then
        iendpcs = n
        pvmax = svyp(9,n)
        go to 520
       else
        if ( n .lt. (nfp-2)) go to 520
      endif
c
c now work backwards to find start of movement
c
      n = iendpcs
  525 if ((svyp(9,n) .gt. z) .and. (n .gt. 2))then
        n = n - 1
        istpcs = n
        go to 525
      endif

      stimcnc = tp(iendpcs) - tp(istpcs)

      write(8,945)
      write(8,947) pvmax
      write(8,948) istpcs, iendpcs
      write(8,949) stimcnc
      write(8,998)

  945 format( ' info for PCS:')
  947 format( ' maximum velocity=',f9.5,' m/s')
  948 format( ' frame #: st. pcs= ',i3,' end pcs= ',i3)
  949 format( ' purely conc time=',f7.4, ' sec')
c
c   calc work....ecc, conc, inst, & avg
```

```
c
c  CMS work..
c
c       work done on bar and on COM, represented by change
c         in mechanical energy of bar/COM  (Garhammer, 1979)
c
c  for bar
c
      wkebar = (sm(4) * g * (ssyc(4,iendecc) - ssyc(4,istecc))) +
   &        (0.5 * sm(4) * (svyc(4,istecc)**2)) +
   H        (0.5 * sm(4) * (svxc(4,istecc)**2))

      wkcbar = (sm(4) * g * (ssyc(4,iendcnc) - ssyc(4,istcnc))) +
   &        (0.5 * sm(4) * (svyc(4,iendcnc)**2)) +
   H        (0.5 * sm(4) * (svxc(4,iendcnc)**2))
c
c  non-dimensionalized eccentric work
c
      wkebnd = wkebar / (ht * bodm)
c
c net work done by COM
c
      wkmin = (tmass * g * (ssyc(9,iendecc) - ssyc(9,istecc))) +
   &        (0.5 * tmass * (svyc(9,istecc)**2)) +
   H        (0.5 * tmass * (svxc(9,istecc)**2))

      wkmax = (tmass * g * (ssyc(9,iendcnc) - ssyc(9,istcnc))) +
   &        (0.5 * tmass * (svyc(9,iendcnc)**2)) +
   H        (0.5 * tmass * (svxc(9,iendcnc)**2))
c
c instantaneous work (also represented as change in mechanical energy)
c      [for COM only]
c
     do 530 i = 1,(nfc-1)
      workc(i) = (tmass * g * (ssyc(9,i + 1) - ssyc(9,i))) +
   %      (((svyc(9,i+1)**2) - (svyc(9,i)**2)) * tmass / 2.0) +
   H      (0.5 * tmass * ((svxc(9,i+1)**2) - (svxc(9,i)**2)))
  530 continue

     eccsum = z
     m = iz
     do 535 i = istecc,iendecc
      eccsum = eccsum + workc(i)
      m = m + 1
  535  continue

     avgwecc = eccsum / float(m)
c
c  for COM
c
     write(8,950)
```

```
      write(8,952) avgwecc
      write(8,953) wkebar, wkmin
      write(8,954) wkebnd

  950 format( ' eccentric work....')
  952 format( ' average eccentric work (COM)= ',f12.5,' Nm')
  953 format( ' eccentric work done on bar = ',f12.5,' Nm',
     +       ' eccentric work done by COM = ',f12.5,' Nm')
  954 format( ' non-dimensionalized ecc work done on bar= ',f12.6/)
c
c  concentric work
c
c   work done on bar and by COM, represented as change in mechanical
c    energy of bar/COM (Garhammer, 1979)
c
c   non-dimensionalized work on bar
c
      wkcbnd = wkcbar / (ht * bodm)
c
      sumconc = z
      m = iz
      do 540 i = istcnc,iendcnc
        sumconc = sumconc + workc(i)
        m = m + 1
  540 continue

      avgwcnc = sumconc / float(m)

      write(8,955)
      write(8,957) avgwcnc
      write(8,958) wkcbar,wkmax
      write(8,959) wkcbnd

  955 format( ' concentric work for CMS...')
  957 format( ' average concentric work (COM)=',f13.4,' Nm')
  958 format( ' concentric work done on bar = ',f12.5,' Nm',
     +       ' concentric work done by COM = ',f12.5,' Nm')
  959 format( ' non-dimensionalized conc work done on bar= ',f12.6)
c
c work for purely concentric squat
c
c  work done on bar
c
        wkpbar = (sm(4) * g * (ssyp(4,iendpcs) - ssyp(4,istpcs))) +
     &        (0.5 * sm(4) * (svyp(4,iendpcs)**2)) +
     H        (0.5 * sm(4) * (svxp(4,iendpcs)**2))
c
c  non-dimensionalized work on bar
c
      wkpbnd = wkpbar / (ht * bodm)
c
```

```
c  work done by COM
c
c   net work
c
     wkpmax = (tmass * g * (ssyp(9,iendpcs) - ssyp(9,istpcs))) +
    &        (0.5 * tmass * (svyp(9,iendpcs)**2)) +
    H        (0.5 * tmass * (svxp(9,iendpcs)**2))
c
c  instantaneous work
c
     do 550 i = 1,(nfp-1)
       pcwork(i) = (tmass * g * (ssyp(9,i + 1) - ssyp(9,i))) +
    %       (((svyp(9,i+1)**2) - (svyp(9,i)**2)) * tmass / 2.0) +
    H       (0.5 * tmass * ((svxp(9,i+1)**2) - (svxp(9,i)**2)))
  550 continue

     sumpcnc = z
     m = iz
     do 551 i = istpcs,iendpcs
       sumpcnc = sumpcnc + pcwork(i)
       m = m + 1
  551  continue

     avgpcwk = sumpcnc / iendpcs

     write(8,960)
     write(8,957) avgpcwk
     write(8,958) wkpbar, wkpmax
     write(8,959) wkpbnd
     write(8,998)

  960 format( ' work for PCS...')
c
c find work for 4 segment model (cms & pcs)
c
     do 545 i = 1,(nfc-1)
       s1w = (sm(1) * g * (ssyc(10,i + 1) - ssyc(10,i))) +
    %       (((svyc(10,i+1)**2) - (svyc(10,i)**2)) * sm(1) / 2.0) +
    H       (0.5 * sm(1) * ((svxc(10,i+1)**2) - (svxc(10,i)**2)))
       s2w = (sm(2) * g * (ssyc(2,i + 1) - ssyc(2,i))) +
    %       (((svyc(2,i+1)**2) - (svyc(2,i)**2)) * sm(2) / 2.0) +
    H       (0.5 * sm(2) * ((svxc(2,i+1)**2) - (svxc(2,i)**2)))
       s3w = (sm(3) * g * (ssyc(3,i + 1) - ssyc(3,i))) +
    %       (((svyc(3,i+1)**2) - (svyc(3,i)**2)) * sm(3) / 2.0) +
    H       (0.5 * sm(3) * ((svxc(3,i+1)**2) - (svxc(3,i)**2)))
       s4w = (sm(4) * g * (ssyc(4,i + 1) - ssyc(4,i))) +
    %       (((svyc(4,i+1)**2) - (svyc(4,i)**2)) * sm(4) / 2.0) +
    H       (0.5 * sm(4) * ((svxc(4,i+1)**2) - (svxc(4,i)**2)))

       cwseg(i) = s1w + s2w + s3w + s4w
  545 continue
```

```
c
c for eccentric net work done
c
   s1w = (sm(1) * g * (ssyc(10,iendecc) - ssyc(10,istecc))) +
  &        (0.5 * sm(1) * (svyc(10,istecc)**2)) +
  H        (0.5 * sm(1) * (svxc(10,istecc)**2))
   s2w = (sm(2) * g * (ssyc(2,iendecc) - ssyc(2,istecc))) +
  &        (0.5 * sm(2) * (svyc(2,istecc)**2)) +
  H        (0.5 * sm(2) * (svxc(2,istecc)**2))
   s3w = (sm(3) * g * (ssyc(3,iendecc) - ssyc(3,istecc))) +
  &        (0.5 * sm(3) * (svyc(3,istecc)**2)) +
  H        (0.5 * sm(3) * (svxc(3,istecc)**2))
   s4w = (sm(4) * g * (ssyc(4,iendecc) - ssyc(4,istecc))) +
  &        (0.5 * sm(4) * (svyc(4,istecc)**2)) +
  H        (0.5 * sm(4) * (svxc(4,istecc)**2))

   cw4e = s1w + s2w + s3w + s4w
c
c for concentric net work done
c
   s1w = (sm(1) * g * (ssyc(10,iendcnc) - ssyc(10,istcnc))) +
  &        (0.5 * sm(1) * (svyc(10,iendcnc)**2)) +
  H        (0.5 * sm(1) * (svxc(10,iendcnc)**2))
   s2w = (sm(2) * g * (ssyc(2,iendcnc) - ssyc(2,istcnc))) +
  &        (0.5 * sm(2) * (svyc(2,iendcnc)**2)) +
  H        (0.5 * sm(2) * (svxc(2,iendcnc)**2))
   s3w = (sm(3) * g * (ssyc(3,iendcnc) - ssyc(3,istcnc))) +
  &        (0.5 * sm(3) * (svyc(3,iendcnc)**2)) +
  H        (0.5 * sm(3) * (svxc(3,iendcnc)**2))
   s4w = (sm(4) * g * (ssyc(4,iendcnc) - ssyc(4,istcnc))) +
  &        (0.5 * sm(4) * (svyc(4,iendcnc)**2)) +
  H        (0.5 * sm(4) * (svxc(4,iendcnc)**2))

   cw4c = s1w + s2w + s3w + s4w
c
c 4 seg model work for pcs
c
   do 555 i = 1,(nfp-1)
     s1w = (sm(1) * g * (ssyp(10,i + 1) - ssyp(10,i))) +
  %       (((svyp(10,i+1)**2) - (svyp(10,i)**2)) * sm(1) / 2.0) +
  H       (0.5 * sm(1) * ((svxp(10,i+1)**2) - (svxp(10,i)**2)))
     s2w = (sm(2) * g * (ssyp(2,i + 1) - ssyp(2,i))) +
  %       (((svyp(2,i+1)**2) - (svyp(2,i)**2)) * sm(2) / 2.0) +
  H       (0.5 * sm(2) * ((svxp(2,i+1)**2) - (svxp(2,i)**2)))
     s3w = (sm(3) * g * (ssyp(3,i + 1) - ssyp(3,i))) +
  %       (((svyp(3,i+1)**2) - (svyp(3,i)**2)) * sm(3) / 2.0) +
  H       (0.5 * sm(3) * ((svxp(3,i+1)**2) - (svxp(3,i)**2)))
     s4w = (sm(4) * g * (ssyp(4,i + 1) - ssyp(4,i))) +
  %       (((svyp(4,i+1)**2) - (svyp(4,i)**2)) * sm(4) / 2.0) +
  H       (0.5 * sm(4) * ((svxp(4,i+1)**2) - (svxp(4,i)**2)))
```

```
      pwseg(i) = s1w + s2w + s3w + s4w
 555 continue
c
c 4 segment model net work (concentric)
c
     s1w = (sm(1) * g * (ssyp(10,iendpcs) - ssyp(10,istpcs))) +
   &        (0.5 * sm(1) * (svyp(10,iendpcs)**2)) +
   H        (0.5 * sm(1) * (svxp(10,iendpcs)**2))
     s2w = (sm(2) * g * (ssyp(2,iendpcs) - ssyp(2,istpcs))) +
   &        (0.5 * sm(2) * (svyp(2,iendpcs)**2)) +
   H        (0.5 * sm(2) * (svxp(2,iendpcs)**2))
     s3w = (sm(3) * g * (ssyp(3,iendpcs) - ssyp(3,istpcs))) +
   &        (0.5 * sm(3) * (svyp(3,iendpcs)**2)) +
   H        (0.5 * sm(3) * (svxp(3,iendpcs)**2))
     s4w = (sm(4) * g * (ssyp(4,iendpcs) - ssyp(4,istpcs))) +
   &        (0.5 * sm(4) * (svyp(4,iendpcs)**2)) +
   H        (0.5 * sm(4) * (svxp(4,iendpcs)**2))

     pw4c = s1w + s2w + s3w + s4w
c
c write 4 seg model work ouput
c
     write(8,910)
     write(8,915) cw4e, cw4c
     write(8,920) pw4c
     write(8,998)
 910 format(1h , ' work done by 4 segment model****')
 915 format(1h , ' CMS eccentric work = ',f12.4,' Nm',/
   ^          ' CMS concentric work = ',f12.4,' Nm')
 920 format(1h , ' PCS concentric work = ',f12.4,' Nm')
c
c calculate power for ecc, conc, inst, & avg
c
     pwebar = wkebar / timecc
     pwccom = z
     pwecom = z
     do 560 i = 1,(nfc-1)
       powc(i) = workc(i) / AT
       if (powc(i) .le. pwecom) then
         minpfre = i
         pwecom = powc(i)
       endif
       if (powc(i) .ge. pwccom) then
         maxpfrc = i
         pwccom = powc(i)
       endif
 560 continue
     sumeccp = z
     m = iz
     do 565 i = istecc,iendecc
       sumeccp = sumeccp + powc(i)
```

```
      m = m + 1
565  continue

     avgpowe = sumeccp / float(m)

     pwcbar = wkcbar / ctimcnc
     sumcmp = z
     m = iz
     do 570 i = istcnc, iendcnc
       sumcmp = sumcmp + powc(i)
       m = m + 1
570  continue

     avgpowc = sumcmp / float(m)
     pwpbar = wkpbar / stimcnc
     pwpcom = z

     do 580 i = 1,(nfp-1)
       pcpow(i) = pcwork(i) / AT
       if (pcpow(i) .ge. pwpcom) then
          maxpfrp = i
          pwpcom = pcpow(i)
        endif
580  continue

     sumpcp = z
     m = iz
     do 585 i = istpcs, iendpcs
       sumpcp = sumpcp + pcpow(i)
       m = m + 1
585  continue

     avgpcpow = sumpcp / float(m)

     write(8,961)
     write(8,962) avgpowe
     write(8,964) avgpowc
     write(8,965) avgpcpow
     write(8,966)
     write(8,967) pwebar, pwecom, minpfre
     write(8,968) pwcbar, pwccom, maxpfrc
     write(8,969) pwpbar, pwpcom, maxpfrp

961  format( ' power averages:')
962  format( ' ecc power (COM)= ',f12.4, ' Nm/s')
964  format( ' conc power(COM)= ',f12.4, ' Nm/s')
965  format( ' PCS power (COM)= ',f12.4, ' Nm/s')
966  format( ' power calculations:')
967  format( ' bar eccentric power = ',f12.4, ' Nm/s',5x,
  &        ' COM eccentric power = ',f12.4, ' Nm/s',10x,
  #        ' frame of min ecc power = ',i4)
```

```
 968 format( ' bar concentric power = ',f12.4, ' Nm/s     ',
   &       ' COM concentric power = ',f12.4, ' Nm/s',10x,
   #      ' frame of max conc power = ',i4)
 969 format( ' bar PCS power = ',f12.4, ' Nm/s     ',
   &       ' COM PCS power = ',f12.4, ' Nm/s', 10x,
   #      ' frame of max pcs power = ',i4)
c
c  power for 4 segment model
c
      cp4e = z
      cp4c = z
      pp4c = z
      do 587 i = 1,(nfc-1)
       cpseg(i) = cwseg(i) / AT
       if (cpseg(i) .le. cp4e) then
          minpfre = i
          cp4e = cpseg(i)
       endif
       if (cpseg(i) .ge. cp4c) then
          maxpfrc = i
          cp4c = cpseg(i)
        endif
 587 continue
      do 590 i = 1,(nfp-1)
       ppseg(i) = pwseg(i) / AT
       if (ppseg(i) .ge. pp4c) then
            maxpfrp = i
            pp4c = ppseg(i)
        endif
 590 continue
c
c  write power for 4 segment model to output
c
      write(8,925)
      write(8,930) cp4e, minpfre, cp4c, maxpfrc
      write(8,935) pp4c, maxpfrp
      write(8,998)
 925 format(1h , ' peak power for 4 segment model****')
 930 format(1h , ' CMS peak eccentric power = ',f13.5,' Nm/s',
   &        ' frame of min ecc power = ',i4/
   ^       ' CMS peak concentric power = ',f13.5,' Nm/s',10x
   #       ' frame of max conc power = ',i4)
 935 format(1h , ' PCS peak concentric power = ',f13.5,' Nm/s',
   #        10x, ' frame of max pcs power = ',i4)
c
c  calculate energies
c    first separately for ecc, conc, & PCS
c
      sumnrg = z
      prevnrg =tmass*(((svyc(9,istecc)**2)/2.0) +(g*ssyc(9,istecc)))
```

```
      do 600 i = (istecc+1), iendecc
        enrg = tmass * (((svyc(9,i)**2) / 2.0) + (g * ssyc(9,i)))
        diffenrg = abs(enrg - prevnrg)
        prevnrg = enrg
        sumnrg = sumnrg + diffenrg
  600 continue

      eccnrg = sumnrg
      sumnrg = z
      prevnrg =tmass*(((svyc(9,istcnc)**2)/2.0) +(g*ssyc(9,istcnc)))

      do 610 i = (istcnc+1), iendcnc
        enrg = tmass * (((svyc(9,i)**2) / 2.0) + (g * ssyc(9,i)))
        diffenrg = abs(enrg - prevnrg)
        prevnrg = enrg
        sumnrg = sumnrg + diffenrg
  610 continue

      cncnrg = sumnrg
      sumnrg = z
      prevnrg =tmass*(((svyp(9,istpcs)**2)/2.0) +(g*ssyp(9,istpcs)))

      do 620 i = (istpcs+1),iendpcs
        enrg = tmass * (((svyp(9,i)**2) / 2.0) + (g * ssyp(9,i)))
        diffenrg = abs(enrg - prevnrg)
        prevnrg = enrg
        sumnrg = sumnrg + diffenrg
  620 continue

      pcsnrg = sumnrg

c
c  calculate stored elastic energy [see] for point mass
c
      see = ((cncnrg - pcsnrg) / eccnrg) * 100.0

      write(8,979)
      write(8,980)
      write(8,982) eccnrg, cncnrg, pcsnrg
      write(8,985) see

  979 format( 10x,' %%% for point mass (COM)...')
  980 format( 5x,' ecc energy    conc energy     PCS energy')
  982 format( 3f16.3,'        (kg*m*m/s/s)')
  985 format( ' stored elastic energy = ',f9.5, ' %')

c
c  elastic energy for 4 segment model
c
      emince = z
      emaxcc = z
```

```
      emaxpc = z
      do 635 j = 1,nfc
        ebodc(j) = z
        do 630 i = 1,4
          ec(i,j) = (sm(i) * g * ssyc(i,j)) +
     !              (0.5 * sm(i) * (svyc(i,j)**2 + svxc(i,j)**2)) +
     *              (0.5 * smi(i) * omegac(i,j)**2)

          if ( i .eq. 4) then
            penrg(j) = sm(i) * g * ssyc(i,j)
            vkenrg(j) = 0.5 * sm(i) * svyc(i,j)**2
            hkenrg(j) = 0.5 * sm(i) * svxc(i,j)**2
            rkenrg(j) = 0.5 * smi(i) * omegac(i,j)**2
          endif
          ebodc(j) = ebodc(j) + ec(i,j)
          if (((ebodc(j) .le. emince).and.(j .ge. istecc)).or.
     !       (j .eq. 2)) then
            emince = ebodc(j)
            minfrec = j
          endif
          if ((ebodc(j) .ge. emaxcc).and.(j .ge. istcnc)) then
            emaxcc = ebodc(j)
            maxfrcc = j
          endif
 630    continue
 635  continue

      do 645 j = 1,nfp
        ebodp(j) = z
        do 640 i = 1,4
          ep(i,j) = (sm(i) * g * ssyp(i,j)) +
     !              (0.5 * sm(i) * (svyp(i,j)**2 + svxp(i,j)**2)) +
     #              (0.5 * smi(i) * omegap(i,j)**2)

          ebodp(j) = ebodp(j) + ep(i,j)
          if ((ebodp(j) .ge. emaxpc).and.(j .ge. istpcs)) then
            emaxpc = ebodp(j)
            maxfrpc = j
          endif
 640    continue
        write(8,903)j,ebodp(j)
 903  format(1h ,' frame ',i3,' energy= ',f12.3)
 645  continue

      enbe = z
      enbc = z
      enbp = z

      do 650 j = istecc,(iendecc-1)
        enbe = enbe + abs(ebodc(j+1) - ebodc(j))
 650  continue
```

```
      do 660 j = istcnc,(iendcnc-1)
        enbc = enbc + abs(ebodc(j+1) - ebodc(j))
  660   continue

      do 670 j = istpcs,(iendpcs-1)
        enbp = enbp + abs(ebodp(j+1) - ebodp(j))
  670   continue

c
c  calculate see with 4 segment model [storen]
c
      storen = ((enbc - enbp) / enbe) * 100.0

      write(8,990)
      write(8,981) emince, minfrec, emaxcc, maxfrcc
      write(8,984) emaxpc, maxfrpc
      write(8,980)
      write(8,982) enbe, enbc, enbp
      write(8,985) storen

  981 format(1h ,' min energy = ',f13.3,5x,'frame of min = ',i3,8x,
     &       ' max conc energy = ',f13.3,5x,'frame of max = ',i4)
  984 format(1h ,' max pcs energy = ',f13.3,5x,'frame of max = ',i4)
  990 format( 10x,' %%% for 4 segment model...')
c
c calc SEE with methods from Asmussen & Bonde-Peterson (using
c   energy levels of points within the movement
c   -do for both COM and 4 segment model
c
c  using COM
c
      eneg = 0.5*tmass*(svyc(9,istecc)**2) + tmass*g*ssyc(9,istecc)
     eposc = 0.5*tmass*(svyc(9,iendcnc)**2) + tmass*g*ssyc(9,iendcnc)
     eposp = 0.5*tmass*(svyp(9,iendpcs)**2) + tmass*g*ssyp(9,iendpcs)

      dltkpm = ((eposc - eposp) / eneg) * 100.0

      write(8,991)
      write(8,992) eneg, eposc, eposp, dltkpm

  991 format( ' energy values using asmussen/bonde-peterson: ')
  992 format( ' for COM: neg e=',f9.4,' cms pos e=',f9.4,
     ^  ' pcs pos e=',f9.4,' elastic energy contrib=',f9.4)
c
c using 4 segment model
c
      eneg4 = z
      eposc4 = z
      eposp4 = z
```

```
      do 685 i = 1,4
        eneg4 = eneg4 + (0.5 * sm(i) * (svyc(i,istecc)**2) +
     &         sm(i) * g * ssyc(i,istecc) +
     $        0.5 * smi(i) * (omegac(i,istecc)**2))
        eposc4 = eposc4 + (0.5 * sm(i) * (svyc(i,iendcnc)**2) +
     &         sm(i) * g * ssyc(i,iendcnc) +
     $        0.5 * smi(i) * (omegac(i,iendcnc)**2))
        eposp4 = eposp4 + (0.5 * sm(i) * (svyp(i,iendpcs)**2) +
     &         sm(i) * g * ssyp(i,iendpcs) +
     $        0.5 * smi(i) * (omegap(i,iendpcs)**2))
  685 continue

      dltkpm4 = ((eposc4 - eposp4) / eneg4) * 100.0

      write(8,993) eneg4, eposc4, eposp4, dltkpm4

  993 format( ' 4 SEG: neg e=',f9.4,' cms pos e=',f9.4,' pcs pos e=',
     ^ f9.4,' elastic energy contrib=',f9.4)
c
c  write to other output files
c
       if (nfc .gt. nfp) then
        do 675 i = 1,(nfc-1)
          write(3,700) (AT*i),workc(i),powc(i),pcwork(i),pcpow(i)
  675   continue
       else
        do 680 i = 1,(nfp-1)
          write(3,700) (AT*i),workc(i),powc(i),pcwork(i),pcpow(i)
  680   continue
      endif

  700 format(1h ,f6.4,4f12.5)
  710 format(1h ,f6.4,4(2x,f12.6))
  711 format(1h ,f6.4,5(2x,f12.6))

      close(unit=5)
      close(unit=6)
      close(unit=8)
      close(unit=3)

  999 stop
      END
```

Appendix E: SAS Program

```
data lifting;
   infile 'squatn.dat';
   input barwt1-barwt4 prcmax1-prcmax4 tecc1-tecc4 tconc1-tconc4 /
      tpcs1-tpcs4 ctvtp1-ctvtp4 ctvte1-ctvte4 / ptvtp1-ptvtp4
      ptvte1-ptvte4 vecc1-vecc4 / vconc1-vconc4 vpcs1-vpcs4
      weccavg1-weccavg4 / wcncavg1-wcncavg4 wpcsavg1-wpcsavg4
      wecc1-wecc4 / wconc1-wconc4 wpcs1-wpcs4 wndebar1-wndebar4 /
      wndcbar1-wndcbar4 wndpbar1-wndpbar4 peccavg1-peccavg4 /
      pcncavg1-pcncavg4 ppcsavg1-ppcsavg4 pecc1-pecc4 /
      pconc1-pconc4 ppcs1-ppcs4 eeccmin1-eeccmin4 /
      ecncmax1-ecncmax4 epcsmax1-epcsmax4 eecc1-eecc4 /
      econc1-econc4 epcs1-epcs4 elasnrg1-elasnrg4 /
      brbyprc1-brbyprc4 brbywt1-brbywt4;
   zero = 0.00000;

proc means;

proc glm;
   model ctvtp1-ctvtp4 ctvte1-ctvte4 = zero/nouni;
   repeated ctdiff 2;
proc glm;
   model barwt1-barwt4 =/nouni;
   repeated barmass 4;
proc glm;
   model tecc1-tecc4 =/nouni;
   repeated ecctime 4;
proc glm;
   model tconc1-tconc4 tpcs1-tpcs4 =/nouni;
   repeated cptime 2, alltime 4;
proc glm;
   model vecc1-vecc4 =/nouni;
   repeated eccvel 4;
proc glm;
   model vconc1-vconc4 vpcs1-vpcs4 =/nouni;
   repeated cpvel 2, allvel 4;
proc glm;
   model weccavg1-weccavg4 =/nouni;
   repeated avgeccw 4;
proc glm;
   model wcncavg1-wcncavg4 wpcsavg1-wpcsavg4 =/nouni;
   repeated cpavgw 2, allavgw 4;
proc glm;
   model wecc1-wecc4 =/nouni;
   repeated eccwork 4;
proc glm;
   model wconc1-wconc4 wpcs1-wpcs4 =/nouni;
   repeated cpwork 2, allwork 4;
proc glm;
   model wndebar1-wndebar4 =/nouni;
   repeated ndeccwrk 4;
```

```
proc glm;
   model wndcbar1-wndcbar4 wndpbar1-wndpbar4 =/nouni;
   repeated ndcpw 2, ndallw 4;
proc glm;
   model peccavg1-peccavg4 =/nouni;
   repeated avgeccp 4;
proc glm;
   model pcncavg1-pcncavg4 ppcsavg1-ppcsavg4 =/nouni;
   repeated cpavgp 2, allavgp 4;
proc glm;
   model pecc1-pecc4 =/nouni;
   repeated eccpwr 4;
proc glm;
   model pconc1-pconc4 ppcs1-ppcs4 =/nouni;
   repeated cppower 2, allpower 4;
proc glm;
   model eeccmin1-eeccmin4 =/nouni;
   repeated enrgmin 4;
proc glm;
   model ecncmax1-ecncmax4 epcsmax1-epcsmax4 =/nouni;
   repeated cpemax 2, allemax 4;
proc glm;
   model eecc1-eecc4 =/nouni;
   repeated eccenrg 4;
proc glm;
   model econc1-econc4 epcs1-epcs4 =/nouni;
   repeated cpenrg 2, allenrg 4;
proc glm;
   model elasnrg1-elasnrg4 =/nouni;
   repeated elastic 4;
proc glm;
   model brbyprc1-brbyprc4 =/nouni;
   repeated barbyprc 4;
proc glm;
   model brbywt1-brbywt4 =/nouni;
   repeated barbywt 4;
```

Appendix F: Tabled Data

Relative Vertical Displacement of the COM by Load and Condition (%)

Load Percentage	CMS Mean (SD)	PCS Mean (SD)
40%	60.2 (6.4)	60.8 (10.9)
55%	67.2 (3.6)	70.6 (7.4)
70%	72.4 (5.8)	70.1 (2.9)
85%	70.7 (13.0)	72.0 (4.9)

Peak Concentric Velocity by Load and Condition (m/sec)

Load Percentage	CMS Mean (SD)	PCS Mean (SD)
40%	1.08 (0.21)	1.07 (0.19)
55%	1.05 (0.17)	1.04 (0.21)
70%	0.97 (0.24)	0.88 (0.12)
85%	0.75 (0.16)	0.69 (0.12)

Net Concentric Work of the System by Load and Condition (Nm)

Load Percentage	CMS Mean (SD)	PCS Mean (SD)
40%	476 (160)	479 (156)
55%	631 (187)	644 (195)
70%	739 (159)	682 (162)
85%	771 (286)	743 (220)

Non-dimensional Concentric Work Performed on the Bar by Load and Condition

Load Percentage	CMS Mean (SD)	PCS Mean (SD)
40%	1.88 (0.72)	1.86 (0.64)
55%	2.86 (1.00)	2.89 (1.03)
70%	3.57 (0.77)	3.22 (0.79)
85%	3.84 (1.42)	3.70 (1.13)

Average Concentric Work by Load and Condition (Nm)

Load Percentage	CMS Mean (SD)	PCS Mean (SD)
40%	20.2 (6.3)	13.1 (4.2)
55%	23.3 (6.2)	16.0 (4.8)
70%	21.2 (5.8)	14.6 (4.4)
85%	17.2 (4.9)	9.9 (2.9)

Average Concentric Power by Load and Condition (Nm/s)

Load Percentage	CMS Mean (SD)	PCS Mean (SD)
40%	1215 (378)	968 (257)
55%	1399 (372)	1070 (285)
70%	1275 (348)	898 (268)
85%	1030 (296)	641 (194)

Peak Concentric Energy by Load and Condition (Nm)

Load Percentage	CMS Mean (SD)	PCS Mean (SD)
40%	1981 (357)	1970 (328)
55%	2442 (567)	2511 (454)
70%	2788 (527)	2730 (506)
85%	3113 (633)	3184 (781)