8409000

Maynard, Rcbert Lee, Jr.

A STUDY OF THE EFFECTS OF REQUIRED MASTERY STRATEGIES AND THE USE OF CONCRETE MANIPULATIVES ON COLLEGE-AGE REMEDIAL STUDENTS

The University of North Carolina at Greensboro Ed.D. 1983

University
Microfilms
International 300 N. Zeeb Road, Ann Arbor, MI 48106

A STUDY OF THE EFFECTS OF REQUIRED MASTERY STRATEGIES AND THE USE OF CONCRETE MANIPULATIVES ON COLLEGE-AGE REMEDIAL STUDENTS

by

Robert L. Maynard, Jr.

A Dissertation submitted to
the Faculty of the Graduate School at
The University of North Carolina at Greensboro
in Partial Fulfillment
of the Requirements for the Degree
Doctor of Education

Greensboro
1983

Approved by

Dissertation Advisor

APPROVAL PAGE

This Dissertation has been approved by the following committee of the Faculty of the Graduate School at the University of North Carolina at Greensboro.

Dissertation Advisor ______________________

Committee Members ______________________

____

____

____

October 4 1983
Date of Acceptance by Committee

October 4, 1983
Date of Final Oral Examination

MAYNARD, ROBERT L. JR. Ed.D. A Study of the Effects of Required Mastery Strategies and the Use of Concrete Manipulatives on College-Age Remedial Arithmetic Students. (1983)
Directed by Dr. D. Michelle Irwin. Pp. 148.

The purpose of this study was to investigate the effects of required mastery and the use of concrete manipulative materials on achievement, enjoyment of mathematics, and rate of completion in remedial arithmetic classes at a community college. Four classes were used in the study with the treatments as follows: required mastery testing with the use of manipulatives to develop concepts, required mastery testing with the traditional development of concepts, traditional testing with the use of manipulatives, and traditional testing with traditional development. In all four classes lecture-discussion was the primary method of presenting information, and all classes were supplemented by a teacher-directed math lab, audiovisual materials, and study guides which included instuctional objectives.

Eighty-seven of the 133 students (65%), who began the course actually completed it. The Chi-Square test of independence showed that the rate of completion was independent of the method of instruction.

Multivariate analysis of covariance was used to test the effects of required mastery and the use of manipulatives on achievement and enjoyment of mathematics. Two covariates and three dependent variables were used. The covariates

were pretests on achievement and math enjoyment; and the dependent variables were a posttest on math achievement, a posttest on math enjoyment, and the final average on five teacher-made unit tests.

After adjusting for the covariates and the use of manipulatives, required mastery produced a significant multivariate difference based on the two posttests and the unit-tests average. Univariate analyses showed that required mastery produced gains on both the achievement posttest and the math enjoyment posttest but not on the unit-tests average. After adjusting for the covariates and required mastery, the use of manipulatives did not produce a significant multivariate difference. Since the P-value (P = 0.148) was relatively small, univariate analyses were performed. The further analyses showed that the use of manipulatives produced a significant gain on the unit-tests average but not on either posttest. After adjusting for the two covariates and the main effects, the mastery-manipulatives interaction did not produce a significant multivariate difference.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

The Need for Remedial Mathematics

Many high school graduates are entering college without the academic skills needed to do college level work. The recent report released by The National Commission on Excellence in Education [NCEE] (1983) concluded that "the average graduate of our schools and colleges today is not as well educated as the average graduate of 25 or 35 years ago" (p. 12).

Even though they plan to attend college, students are not taking the more demanding college preparatory courses. Between 1964 and 1979 the proportion of students taking the general-track high school program increased from 12% to 42%. Less than one third of recent high school graduates have completed intermediate algebra (NCEE, 1983). There are some indications that the two-decade-old decline in Scholastic Aptitude Test scores has been halted ("SAT Scores Hold Steady," 1982); however, the proportion of students demonstrating superior achievement continues to decline.

Business and industry leaders, as well as college teachers, are complaining about students' lack of skills in reading, writing, and basic mathematics. Up to 13% of all

17-year-olds are functionally illiterate (NCEE, 1983). Many of these unprepared students are actually entering college as demonstrated by the following facts: In 1975 Maeroff reported that 26% of the entering freshmen at Ohio State University had not mastered high school mathematics, and in 1976 Levine reported that 90% of the students entering the General College at the University of Minnesota were incapable of studying college algebra (cited in The Carnegie Foundation for the Advancement of Teaching, 1979). The Southern Regional Education Board (1983) reported that during the seventies about 40% of Louisiana's college-bound high school graduates lacked essential skills required for college-level work. Brawer (1982), in discussing the problem of functional illiteracy among entering community college students wrote: "Indeed, the single thorniest problem for community colleges today is the guiding and teaching of students unprepared for traditional college-level studies" (p.12).

The Colleges' Response

In order to work with increasing numbers of students with inadequate math skills, the colleges and universities are developing remedial (high school level) courses designed to eliminate deficiencies in mathematics and to allow the students to enter the traditional college math sequence. Between 1975 and 1980 institutions' enrollments in remedial mathematics courses increased by 72% and presently account for 16% of all mathematics enrollments. The situation is even more pronounced in two-year colleges where remedial courses now account for 42% of the mathematics enrollment (Conference Board of the Mathematical Sciences [CBMS], 1982). In 1979 remedial mathematics courses were offered in over 95% of all two-year colleges.

Elementary algebra and arithmetic, which were offered by over 80% of the colleges, were the most widely offered courses. Geometry and trigonometry were offered by over 30% of the colleges (Friedlander, 1979).

Rockingham Community College, the site of the proposed study, offers four remedial courses: Arithmetic, Elementary Algebra, Intermediate Algebra, and Geometry. During fall quarter 1981 the four courses accounted for 68% of the total mathematics enrollment.

Many instructional formats have been used in the remedial mathematics classes; however, the traditional lecture format still dominates with over 80% of the two-year colleges having the lecture option available for their remedial mathematics students. The use of math labs and related self-paced arrangements is gaining in popularity (Friedlander, 1979).

Approximately 70% of the two-year colleges have math labs to present or supplement their remedial mathematics courses. Typically, the math lab includes programmed texts which either present the material or supplement the lecture; a variety of audiovisual materials including videotapes, audio cassettes, slides, and filmstrips; worksheets which present additional drill on the concepts covered in class; and either student or professional math tutors (CBMS, 1982).

Classroom organizations range from the traditional teacher-controlled classroom to contractual situations, where the teacher and student together establish performance goals, to completely self-paced organizations. The traditional organization, which is used by approximately 60% of the schools, is the most popular. This is followed by the self-paced format, which is used by almost 25% of the schools. Even though the traditional grading system is dominant, mastery learning approaches are gaining acceptance with over one-third of the two-year colleges having some form of required mastery (Friedlander, 1979).

An examination of the more popular remedial arithmetic textbooks (McKeague, 1981; Bello, 1978) shows that most of the concepts are presented at an abstract level. This presentation encourages students to use logically developed rules and algorithms to solve problems. Even when concrete pictures and diagrams are used to explain the concepts, the problems are solved by applying the rules. The math labs are not labs in the science sense; rather, with the emphasis on programmed textbooks and audiovisual packages, they are more like drill sessions.

In summary, the common characteristic of most remedial courses, regardless of method of presentation or organization, is that they fail to utilize concrete experiences to explain the concepts of mathematics. The courses fail to connect the rules of mathematics to the concrete reality of the student's environment.

The Success of the Response

In a sample of selected two and four-year colleges, Baldwin and others (1975) found that only 14% of the institutions reported that they had evaluated their remedial mathematics program. In the same survey approximately 42% of the institutions indicated satisfaction with their program, and an additional 41% said their program was good but needed improvements. The success of the remedial courses is questionable and relates to the criterion used to measure success.

Of the programs that have been evaluated, the most frequent criterion was rate of completion or the accompanying rate of attrition (Friedlander, 1979). According to Stein (1973) the attrition rate in all community college math courses is often between 40 and 60%. More recent research on remedial mathematics does not suggest a radical departure from Stein's figures. McCoy and Hassett (1980) placed the fall semester attrition rate for remedial courses at a major university at 40% and further stated that the rate typically increases 10 to 15% from fall to spring semester. Spangler and Stevens (1979) reported that for a particular community college, lecture classes in remedial math have normally had an attrition rate of 50 to 60%. By using an individualized math lab approach, the attrition rate was reduced to between 30 and 40%. In the lab sections students who had completed as little as two-thirds of the work received incompletes rather than failing grades; therefore, it is possible that the completion rate was not actually improved.

Archer (1978) did an ex post facto study on the success of a community college remedial mathematics program. He reported that only 47% of the students who began a remedial arithmetic course successfully completed it. Many of the unsuccessful students in a remedial algebra course tried again; however, 78% of the repeaters failed again. Archer reported that 41% of the students who began a beginning algebra course reached a college level math course within two years. In discussing an open-ended independent study math lab, Fast (1980) reported that 46% of the lab students failed to complete any course work. Lecture classes at the same community college fared better but they had an attrition rate of 42%. Fast further reported that developmental classes sometimes lose as many as 80% of their enrolled students. As many as 80% of all incoming remedial mathematics students expressed a severe dislike of math and consequently avoided it whenever possible. Barcus and Kleinstein (1981) reported similar but slightly better results. Fifty-two percent of the students enrolled in a computational skills course at the community college successfully completed the course.

The second method used to evaluate remedial courses was to measure the extent to which remedial courses prepared students for further studies in mathematics. Nowlan (1978) compared the college math performances of students who had completed a developmental program with students of similar ability who had completed only part of the program and with students who had chosen not to take the program. She found that those who had completed the developmental program performed better than either of the other groups; however, all three groups had a cumulative grade point average of less than 2.00.

In evaluating the remedial program at a southern community college, Moore (1974) reported that the program did an adequate job of preparing students for liberal arts math courses but it was not successful in preparing students for the calculus sequence. Archer (1978) reported that 80% of the remedial students who enrolled in college level work were successful; however, as reported earlier only 41% of the beginning remedial algebra students actually made it to the college level courses. In the same study Archer reported that only 44% of the arithmetic graduates were successful in business math. Barcus and Kleinstein (1981) reported that only 64% of the students who passed computational skills chose to enroll in the next level mathematics course and that 47% of those passed the next level course.

Before condemning remedial mathematics, it should be noted that only 57% of the students enrolled in any particular mathematics course passed that course. In comparing various sequences of enrollment, Smith (1982) reported that those students who needed and took remediation stayed enrolled as long as those who did not need remediation. Students who needed but chose not to take remediation were enrolled for significantly fewer semesters.

In general, current remediation programs are successful in preparing students for low level mathematics courses; however, they are not successful in preparing students for higher level precalculus courses (Ajose, 1978). Even with the limited success of remediation, remedial courses have become firmly entrenched in the curriculum of most colleges and universities. Only 1% of the institutions with remedial courses feel that the courses should be discontinued (Baldwin et al., 1975).

Accountability

Remedial mathematics has not escaped the wrath of the accountability movement. Critics question whether the programs actually teach students to do basic math (Cohen, 1982) and an examination of the raw data indicates that the critics have a point. For example, in discussing an ex post facto study on a remedial mathematics program at a regional university, Eisenberg (1981) reported that only 35% of the students who successfully completed the lowest level remedial course took additional math and that 45% of those students failed. In order to receive funding for remedial programs, educators are going to have to address issues such as "How many times should the public pay the schools to try to teach the same competencies to the same people?" (Brawer, 1982, p. 12).

This writer feels that educators need to discuss the problem from two directions. First, educators need to determine why students fail to complete remedial courses, why they fail to register for higher courses, and why they fail the higher courses. One of the purposes of this study is to address the issue of why students fail to complete remedial courses. Second, educators need to focus on successes rather than failures. As Eisenburg (1981) pointed out, by helping those nineteen percent who were successful, the remedial program raised the general educational level of the population and affected many more people than those nineteen percent. In the long run, closing the door to higher education may cost far more than the cost of remediation; however, educators cannot be oblivious to the cost. They must continue to search for more effective and efficient ways to teach the courses, and they must evaluate the effectiveness of their offerings. The fact that so few institutions have evaluated their remedial programs is appalling.

<u>Summary</u>

In order to accommodate increasing numbers of high school graduates who are not prepared for college level work, the colleges and universities are offering a proliferation of remedial courses. Even though most of the colleges indicate that they are satisfied with their remedial offerings, few have done formal evaluations. Studies on the effectiveness of remedial courses show that, as a rule, remedial courses have low completion rates and provide questionable preparation for college level mathematics courses. The critics of remedial education question whether the benefits justify the cost and the burden of proof is being shifted to teachers. Efforts must be made to find ways to improve achievement, improve students' attitudes toward math, and improve rates of completion.

<u>Purpose of Present Study</u>

The purpose of this study is to investigate the effects of required mastery and the use of concrete manipulative materials, with respect to achievement, enjoyment of mathematics, and rate of completion, on remedial arithmetic classes (MAT 101) at Rockingham Community College in North Carolina.

CHAPTER 2

REVIEW OF RESEARCH

Theoretical Basis: Required Mastery

Beginning with E. L. Thorndike and his book *The Psychology of Arithmetic* (1922) there have been numerous educators who have felt that performance and retention could be improved through required mastery or related techniques. Thorndike (1913) published the Law of Effect which said that reinforced behavior is likely to be repeated while nonreinforced behavior tends to become extinct. The problem faced by the educators was to define those behaviors deemed beneficial, elicit those behaviors, and reinforce the behaviors until the student had completely mastered the desired skill. Thorndike's *The Psychology of Arithmetic* was an attempt to define the behaviors by translating the subject content of arithmetic into psychologically formulated-stimulus response bonds (Thorndike, 1922).

Another behaviorist, B. F. Skinner, designed a teaching machine which he felt would solve the technological problems encountered by Thorndike. The machine was designed to give instant reinforcement and allow a student to progress at his or her own pace (Skinner, 1958). Many of the programmed textbooks used in remedial math labs today are a result of the work done by Skinner and his collegues (Resnick & Ford, 1981).

While Skinner and his collegues were working on instructional techniques, another group of psychologists, best exemplified by Benjamin Bloom, were working on more understandable ways to describe the desired behaviors. Bloom (1956) published a Taxonomy of Educational Objectives which was designed to help teachers determine the educational levels to which they were teaching. Writers such as Robert Mager (1962) published programmed books to help teachers write and classify their objectives. The writing of educational objectives has become a major component of an emerging psychology of instruction.

Glaser (1976) presented the components of a psychology of instruction which have become accepted as the framework for required mastery strategies:

1) the writing of instructional objectives
2) description of initial state
3) presenting the instructional sequence
4) evaluation.

In a required mastery strategy if step 4 shows that the student has not mastered the objective or objectives to the desired proficiency, then steps 3 and 4 will be repeated until the proficiency has been obtained. Gagné (1977) further refined steps 1 and 2 with his concept of learning hierarchies; however, the basic model remains the same:

1) tell the learner what he or she is to learn
2) test to see if the learner has the prerequisite

skills and if not use the model to teach those skills

3) present the instructional experience

4) test to determine whether the student has mastered the objectives

5) repeat the instruction and evaluation as needed.

In cases such as remedial instruction, in which the instructors know the specific objectives which need to be taught, there is little doubt that required mastery strategies produce greater achievement gains among the students who complete the course. Block and Burns (1976), in reviewing several studies involving required mastery, reached the following conclusions: Mastery learning approaches result in more overall learning (as measured with achievement tests), less variability in learning, increased learning of higher order skills, and greater retention over time of knowledge-level learning. They were not able to conclude that the observed gains in higher-order skills persisted for more than a short period after the instructional sequence. Spaced review appeared to be needed to maintain the higher-order gains; however, the gain in knowledge-level learning was evident up to one and a half years after the learning experience. There is some doubt as to whether the gains were caused by the required mastery per se or by one of the other factors in the strategy such as informing the student of the objective. One of the purposes of the present study is to determine whether the gains shown by the required mastery strategy can be replicated when the control class as well as the experimental class is informed of the specific objectives to be mastered.

In discussing a modified version of required mastery testing, Thompson (1983) pointed out that required mastery has some difficulties. In some cases required mastery leads to low completion rates. Instead of receiving a low but passing mark, students who are unable to achieve mastery within the allotted time period receive some type of incomplete grade or a withdrawal. Since all students are required to perform at a specified level, required mastery strategies also lead to a limited grade spectrum. By combining high attrition with a limited grade spectrum, one can see that the higher achievement rates of those who complete the course might be the result of the weaker students having dropped the course. Another problem is that required mastery may lead to excessive testing. The extra testing may be a drain on the student's study time as well as the instructor's time. Thompson (1983) suggested competency-unit testing as an alternative. In competency-unit testing, the passing scores are lowered and the number of retests are limited. Once the minimum level is reached, students have the option, but are not required, to attempt mastery.

Theoretical Basis: Use of Manipulatives

An opposing group of theorists feel that learning is not the result of an orderly trip up a hierarchy of behavioral objectives. Rather, learning is the result of an insight gained after having been exposed to a particular learning environment. Jean Piaget (1972), perhaps the best known of the developmental psychologists, claimed that learning is the result of maturation, experience, social transmission, and equilibration. Futhermore, genuine learning can occur only when the learner has the necessary mental equipment to assimilate the new experiences.

Even though many of Piaget's ideas have been incorporated into elementary education (Dunlap & Brennan, 1979), he has had little influence on postsecondary remedial education. For one thing Piaget felt that most youths reached their highest level of development, formal operations, well before they entered college (Piaget, 1972). If most college-age students have reached formal operations, then they have the mental structures to handle the abstract learning experiences provided by college-level courses. However, if they have not reached formal operations, the abstract experiences will have no real meaning for them. Either the students will learn something much different than what was intended, or they will memorize a response which has little reliability or validity (Ginsburg & Opper, 1979).

Research by Renner et al. (1976) shows that a majority of college freshmen have not reached formal operations;

therefore, educators in remedial mathematics cannot continue to ignore the implications of Piaget's stages. This writer has observed that the illogical behavior which Renner predicted will occur if students are taught at a level of cognitive development above their own developmental level. Many conscientious students who complete the course make high scores on their unit tests but soon forget the material and make low scores on the final exam. In addition, the students are unable to apply the rules and concepts outside the context of the classroom. For example, students can solve percentage increase problems in the math classroom; however, they cannot solve similar problems in their biology lab.

To survive in a system that requires them to work at a level over their heads, students have only two options--memorize or cheat. Unfortunately, a traditional system of instruction and evaluation rewards those students who memorize and leads teachers into a false impression that their students are actually learning. This false sense of success prevents the teachers from designing meaningful learning situations that would help the students achieve the formal operational level of development. Teachers continue to present material at the abstract level; however, the remedial mathematics students are not prepared to deal with this abstract presentation of concepts. They need additional concrete experiences and opportunities for developing concepts through actual manipulation of materials (Renner et al., 1976).

Piaget's research has no direct implications for education (Ginsburg & Opper, 1979); however, many theorists have drawn indirect inferences. Adler (1966), Bruner (1960), and Ginsburg and Opper (1979) all agree that when initially introduced to a new concept, the learner should be physically involved with concrete manipulatives. Ginsburg and Opper suggested that even for students working at the formal operational level, it would be helpful to drop back to the concrete operational level when introducing a new concept.

Dunlap and Brennan (1979) reported that research on teaching young children suggests that manipulative aids will help children understand the principles of mathematics. In general, they concluded that math instruction should begin with concrete experiences, move to semi-concrete pictures or diagrams, and conclude with abstract symbols. They cautioned that manipulatives must be carefully selected and carefully matched to the mathematical concept to be taught. When possible, more than one device should be used to introduce a concept and each child must be taught to use the aid. Watching the teacher perform the manipulation is useless; each child must actually perform the manipulation. Learning does not occur from the manipulatives themselves but from the child's physical action upon the manipulatives.

The above comments are directed toward children studying arithmetic in elementary school. One of the purposes of the present study is to determine whether the above comments are applicable to college-age remedial arithmetic students.

<u>Method of Instruction: Effect on Achievement</u>

Research shows that something other than instructional format has the greatest effect on achievement and attrition in remedial mathematics. Ajose (1978) reviewed approximately 30 studies which compared the traditional lecture method with various individualized instructional methods such as programmed learning, tutorial, contract learning, and televised instruction. None of the alternative methods provided consistent evidence of improved achievement or lower attrition rates. In fact Ajose reported that students who received remedial instruction through the traditional lecture approach did better in succeeding college-level mathematics courses than students who had been taught through an individualized lab approach. More recent research tends to support the conclusion drawn by Ajose. Williams (1980) found no significant difference in achievement for remedial mathematics students taught through a small-group discussion approach versus an individual approach, and Beal (1978) found similar results when comparing individualized and traditional instructional methods. In one study even a voluntary math lab designed to supplement traditional instruction failed to produce greater achievement than the traditional instruction alone (Blount, 1980).

Several researchers have found evidence of increased achievement when various strategies have been used in conjunction with traditional lecture. Reese (1977) found that the lecture approach with a mastery learning strategy was more effective than lecture with traditional testing procedures. The students in the experimental group were given study guides which included instructional objectives. During the class period the teacher lectured, worked problems at the board, and assisted the students with the use of various audiovisual aids. The textbook for the experimental section was programmed. The mastery level was 80% and those who failed were given tutorial assistance and extra work. When the students felt that they had mastered the material, they took the retest. The control group used a standard textbook, the instructors gave traditional lectures, and no instructional objectives were given to the students. In a similar study Schwartz (1980) found that a mastery learning strategy, which included short introductory lectures, the use of carefully sequenced examples and exercises, frequent formative testing, immediate feedback, and a follow-up of extra problems when needed, produced greater achievement than the traditional approach.

McCoy and Hassett (1980) reported: "Improvements in student performance in both self-study and group based courses have been produced through mastery testing" (p. 22). In a study of remedial students in a large urban community college, Akst (1976) found that self-paced classes had achievement gains significantly greater than did group-paced classes and that required-mastery sections out-performed single-testing sections; however, the completion rate was lower in self-paced retesting sections.

Other researchers have found that the use of behavorial objectives increased students' achievement on departmental final exams (Houston, 1977; Drennen, 1971). Unlike Reese, Houston did not use required mastery testing in either case. Except for the fact that experimental sections received a copy of the objectives, both experimental and control sections were taught by the traditional methods. Drennen suggested that the improvement may be due to the fact that instructors who gave objectives were better organized and more task oriented. Students who were taught using instructional objectives and given a copy of the objectives did not score higher than those taught by objectives but not given a copy of the objectives. In other words, students' awareness of the objectives did not increase achievement; however, instructors' use of the objectives did increase achievement.

Poage (1973) found that students who have their regular classroom instruction supplemented by an individualized, teacher-directed math lab do significantly better than students who have their classroom instruction supplemented by a student-directed lab. In another study, required homework was found to have no effect on the students' achievement; however, the use of required problem sessions did improve achievement with one session per week superior to no sessions and two sessions superior to one session (Bickford, 1979). The use of such a seminar session was also supported in a study conducted by Slate (1975). He found that when all four sections were supplemented by an audiovisual lab with paid student tutors, a seminar approach was more effective than either self-paced instruction, a-one-day-per-week lecture arrangement, or small-group discussion.

In reviewing the research this writer noticed that experiments in which the teacher volunteered for the experimental section and the control section was assigned as part of another teacher's normal teaching load were usually successful. Studies in which both groups were taught by teachers who volunteered freqently produced no significant difference.

Method of Instruction: Effect on Rate of Completion

Several investigators have looked for a relationship between method of instruction and rate of completion;

however, they found no statistically significant results (McCoy & Hassett, 1980; Muha, 1974; Bluman, 1971; Drennen, 1971). Akst (1976) reported lower completion rates for students in self-paced retesting sections compared to those in single-testing sections. This suggests that the rate of completion is not in and of itself sufficient to evaluate remedial programs. The evidence suggests a fairly constant across-the-board attrition rate. After reporting a 45% attrition rate for an experimental laboratory section, Williams (1973) stated: "In the more difficult subject areas such as mathematics, I believe we must learn to live with the dropout problem" (p.45).

Method of Instruction: Effect on Attitude Toward Math

Even though the recent innovative approaches have had little effect on achievement in remedial mathematics classes, many have improved the students' attitudes toward math significantly more than has the traditional approach. Approaches designed to present the instructor as a helpful, supportive mentor have significantly improved the student attitude toward math (Muha, 1974; Slate, 1975).

Blount (1980) found that the availability of an individualized math lab to supplement traditional instruction improved the students' attitude significantly more than the improvement gained by the traditional method alone. Required mastery strategies produced higher gains in students' attitude toward math (Schwartz, 1980) and the use of behavioral objectives resulted in students' giving the teacher higher ratings (Drennen, 1971).

It appears that individual study methods alone do not produce a significantly greater gain in students' attitudes toward math. Williams (1980) found that students taught through the small-group approach had better attitudes toward math than students taught through an individual work approach.

At present there appears to be no consistent evidence that improved attitudes resulted in either greater achievement or higher rates of completion. Bickford (1979) reported that the students' attitude toward math had no observable influence on the students' achievement. In discussing the backgrounds and attitudes of college students whose lower American College Test (ACT) scores were in math, Bellile (1980) suggested that their attitudes may have contributed to their present state. She concluded that the students demonstrated an unwillingness to apply themselves to learn content and techniques that they perceived as useless and boring.

Developmental Level

Some researchers feel that the lack of achievement gains is due to the fact that all of the methods teach at the wrong level of cognitive development (Renner et al., 1976). According to studies by Jean Piaget (1972), individuals go through four stages of cognitive development as they mature from infancy to adulthood. The four stages and the approximate ages are as follows: sensory motor, 0-2 years; pre-operational, 2-7 years; concrete operational, 7-11 years; and formal operational, 12 years through adulthood. Theoretically most, if not all, college students should be at the formal operational stage of cognitive development; therefore, colleges classes, remedial and regular, have traditionally been taught at the formal operational level (Plymale & Jarrell, 1982). In order to correct several years of math deficiencies in a very short period of time, remedial mathematics instructors have used the supposed superiority of adults in the area of hypothetical-deductive and abstract reasoning to justify presenting the material at an extremely rapid pace. The remedial arithmetic textbooks rely heavily on the students being able to handle the "all-other-things-being-equal" and the "if-then-therefore" constructs. According to Renner et al. (1976), both constructs are characteristics of the formal operational learner.

Recent research indicates that the majority of the college remedial mathematics students are not at the formal operational level of cognitive development (Robicheaux, 1981). Available evidence indicates that approximately one-half of all students entering college cannot cope with abstract propositions and that figure is fairly constant across colleges. Since college teachers expect most students to be at the formal operational level, they often create an educational situation with which the students cannot cope. This disparity between where the students are and where the teacher perceives them to be may contribute to the high attrition rate in college courses (Renner et al., 1976). Renner et al. stated, "College students are generally not given the learning opportunities they need to develop logical thoughts with abstract propositions" (p. 111). The statement is supported by a study conducted by Plymale and Jarrell (1982). They studied a sample of sophomore students enrolled in a state-supported university and compared the cognitive development levels of students enrolled in two divisions of the university--the college of education and the community college. The community college had an open-door admissions policy while the school of education had a more traditional admissions policy. The study showed that 48% of all college students tested were not capable of performing at the cognitive development level necessary for success in college classes. Somewhat surprisingly they found no significant difference between the developmental level of the community college students, and the students in the school of education. In any case, the results of their study do not show that a significant number of students reached the formal operational stage during their freshman year of college. The findings of

Renner et al. were also supported by Parete (1979), who tested beginning freshmen at a branch campus of a major university. Of 231 students tested, approximately one-half were at the concrete operational stage, one-tenth were transitional, and the remainder had reached formal operations. Since research shows a high correlation between ACT test scores and scores on Piaget's tasks (Plymale & Jarrell, 1982), and since remedial arithmetic students generally have very low ACT scores, one would expect an even greater percentage of students in remedial arithmetic who are below the formal operational level. Robicheaux (1981) in studying the relationship between course performance and Piagetian functioning level found only 5% of the developmental mathematics students functioning at the formal operational level.

Manipulatives

Research on the use of manipulatives in college remedial mathematics classes is limited; however, that limited research combined with research on math classes for elementary teachers gives some indication of the effect of the use of manipulatives on mathematics achievement, rate of completion, and students' attitudes toward mathematics in college remedial mathematics classes.

Statements by Kenney (1965) convinced this writer that elementary education majors are not much different in math ability than remedial mathematics students, and that, as a consequence, research on math classes for elementary education majors is relevant to the present study. Kenney reported that 55% of the elementary education majors scored below the median of eighth and ninth grade pupils on a contemporary mathematics test and further stated that in mathematics, elementary teachers lack understanding in language, vocabulary, concepts, relationships, and generalizations.

Research in elementary education mathematics classes indicates that the use of manipulative labs in place of lecture has a detrimental effect on achievement (Warkentin, 1975; Kulm, 1977); however, Warkentin found that students in the lab section had a better attitude toward math than did the students in the lecture section. When a manipulative lab was used to supplement the traditional lecture, the results slightly favored the use of the manipulatives lab over the unsupplemented lecture. In a study by Fitzgerald (1968) the manipulative materials for the lab were selected to complement and parallel the concepts covered in lecture. Even though the mathematical competencies of the students were unchanged by the lab experience, the students felt highly positive about their lab experiences. Summarizing the results of a similar study, Fuson (1975) stated

> Although there is no precise measure of the size of increase, trainees both thought they increased and actually seemed to increase their understanding of elementary mathematical concepts (p. 59).

Weissglass (1977) compared a small-group discussion and laboratory class with a traditional lecture class. The lab section used manipulatives such as attribute blocks, Cuisenaire rods, geoboards, tanagrams, geoblocks, and dice to investigate mathematical concepts. There was no significant difference in achievement gains between the two groups; however, the lab class had a lower attrition rate--36% to 51%.

Barnett and Eastman (1978) did a study to see if students who were taught using manipulatives would be able to use the manipulatives to teach elementary school children more effectively than students who were taught using pictures and diagrams only. They found that their students did not learn to teach better by actually using manipulatives; however, the students did a superior job of learning the related math concepts. The researchers suggested that more time should be spent using manipulatives to teach prospective elementary teachers the actual mathematics that they will be expected to teach in the future.

Results of research in remedial mathematics are generally comparable to the results cited for mathematics for elementary teachers. Harris (1979) compared a class of remedial students taught ratio and proportions by the traditional example method with a class of remedial students taught the same unit through the use of manipulatives. She found no significant difference in immediate learning; however, over time there was a significant loss of learning for students taught by example only, while there was not a significant loss for students taught using manipulatives.

Wepner (1980) reported on a study in which Piagetian techniques were used to teach a unit on percentages. The techniques used included a sequential development beginning with ideas defined through concrete examples, the use of initial problems which required students to raise questions and predict outcomes, and explanations by the teacher. The experimental group had a significantly greater posttest score than did the control group which was taught by the traditional method. Wepner concluded, "It appears that Piaget's theory can be successfully applied to the mathematics instruction of adult remedial students" (p. 13). Drapac (1981) conducted a study in which math tiles were used to teach operations on integers, combining like terms, operations on polynomials, and factoring to a group of college students in remedial algebra. The control group was taught using examples only. After the treatment the experimental group had significantly greater achievement test scores than did the control group, a significantly more positive overall attitude toward math, and significantly more confidence in their ability to do math. However, during the treatment, there was a decline in the students' attitude toward the usefulness of math. About one-half of the students reported that they enjoyed working with the manipulatives.

Wepner (1982) reported exceptionally high completion rates for a developmental program which was taught using a Piagetian instructional approach. The program consists of one remedial arithmetic course, two developmental algebra courses, and two intermediate algebra proficiency courses. Students who were exceptionally weak in computations took, in sequence, the remedial arithmetic course, one of the developmental algebra courses, and one of the proficiency courses. The developmental course which the students took was determined by their performance on a placement test and the proficiency course was determined by their major. Instruction in all of the courses proceeded along a continuum from the concrete to the abstract and emphasis was on process rather than on specific rules. Wepner (1982) reported that 90% of the students in both arithmetic and algebra achieved proficiency in their respective courses. She attributed the high success rate to

> the use of a Piagetian instructional approach; the use of peer tutors; a stratified placement procedure whereby students are grouped more homogeneously according to mathematical ability; and teacher commitment to the success of the remedial process (p. 1).

Another factor which may have accounted for the success rate is an extremely strict attendance policy. The seventh absence, excused or unexcused, earned an automatic F for the course. Since there was no control group, it is impossible to determine which of the factors contributed significantly to success.

Characteristics of an Effective Class

Several writers have attempted to identify key elements of effective remedial programs and effective instruction in general. In successful remedial programs the instructors typically decide what is to be learned, the method of instruction, and what is expected of the students. Students must be actively involved in the learning process for substantial and frequent periods of time. Programs with retention rates greater than 50% shared three characteristics: Full-time faculty taught remedial courses, tutorial assistance was provided, and expenditures per student were high (Southern Region Education Board, 1983). Cronbach and Snow (cited in Resnich & Ford, 1981) suggested highly structured teaching for remedial students of low mathematical ability. Good and Grouws (1979) listed the following characteristics of successful teachers: They present information actively and clearly, they are task focused, they are nonevaluative and create a relatively relaxed learning atmosphere, and they express high achievement expectations. Even though Good and Grouws were referring to elementary teachers, the writer feels that the same characteristics apply to postsecondary remedial mathematics teachers.

Summary

Research indicates that attempts to individualize instruction have had little effect on mathematics achievement or rate of completion; however, traditional instruction when used in conjunction with behavioral objectives and required mastery techniques seems to improve instruction. There is some doubt whether the gains in achievement are a result of required mastery testing or the instructional techniques that accompany the strategy. For example, achievement gains were recorded when objectives were used even though required mastery testing was not used. Since research indicates that required mastery may reduce rates of completion, it leaves the possibility that achievement gains observed in required mastery classes could be the the result of greater attrition among weaker students. The present study will examine both of the above possibilities.

Other researchers have indicated that the cognitive development of students in remedial courses is at a different level than that needed for the traditional abstract presentation generally used in college remedial math courses. The highly abstract follow-the-definition-or-rule format assumes that students are able to work at the formal operational level of cognitive development; however, research suggests that sixty percent or more of beginning college students have not reached the formal operational stage of development. There is evidence that instructional methods employing a linear progression from the concrete or intuitive to the abstract produces achievement gains; however, findings are not consistent. If the instructor is well organized and aware of the personal needs of students, remedial students profit from a highly-structured, teacher-controlled environment. Teachers' feelings about teaching remedial courses are important indicators of the success of remedial programs.

CHAPTER 3

METHODOLOGY

Overview

The purpose of this study was to investigate the effects of required mastery and the use of concrete manipulative materials on achievement, enjoyment of mathematics, and the completion rate in remedial arithmetic classes (MAT 101) at Rockingham Community College (RCC). Four MAT 101 classes were used in the study with treatments as follows: required mastery with the use of manipulative materials to develop concepts and procedures, required mastery with the traditional abstract development of concepts and procedures, traditional testing with the use of manipulative materials, and traditional testing with the abstract development of concepts and procedures. Except for the two main variables, all four classes used methodologies that previous research has indicated to be most successful. All four classes utilized teacher-directed math labs in which the students got individual tutoring and used audiovisual materials related to class instruction. Study guides which included behavioral objectives, assignments, and back-up audiovisual materials were distributed to all classes. Lecture was used in all classes; however, the lectures were different. In the two classes which used manipulatives, the instructors attempted to explain rules and procedures through the use of concrete materials such as Cuisenaire rods, colored disks, geoboards, chips, and cubes. The other two classes had traditional lectures which basically followed the textbook. In all classes after each concept had been presented, the students practiced the concept by working in small groups. Certain features such as required assignments and required attendance, which are not necessarily supported by research but have proven to be successful at RCC, were used in all four classes (see Appendix A for the four syllabi).

Subjects

The subjects in the study were 133 college transfer and technical students at Rockingham Community College who registered for one of the four-day sections of MAT 101 during the Fall Quarter of 1982. On the basis of a placement test, the students were encouraged, but not required, to take the remedial course before registering for any higher-level math courses. All of the students demonstrated a lack of understanding of fractions, decimals, and percentages; most possessed a history of math avoidance.

Student Assignment to Classes

Sections of the course were offered daily at 10:00 A.M. and at 11:00 A.M. Originally, it was planned to force the sections to fill evenly and use an even-odd technique to split each section in half; however, unknown schedule pressures caused about three times as many students to register for the class at 11:00 as registered for the class at 10:00. At the end of registration, 95 students had registered for the 11:00 A.M. section and only 38 had registered for the 10:00 section. The decision was made to divide the 11:00 section into three equal classes rather than force students into the 10:00 A.M. time slot. In order to divide the 11:00 section, the names of all students were written in alphabetical order and numbered one through 95. Students whose numbers were equivalent modulo three were grouped together to create 2 classes of 32 students each and one class of 31 students. Therefore, the study, which included all daytime MAT 101 students, started with four classes having 38, 31, 32, and 32 students, respectively.

Teacher Assignment to Classes

Three teachers, all of whom requested assignment to remedial courses, were involved in the study. The author taught two of the sections, Mrs. Susan Clark taught one section, and Mrs. Norma Maynard taught one section. All three teachers had previously taught MAT 101 and all three had extensive experience in teaching remedial mathematics. The educational backgrounds of the three were also similar. All three had a Master of Education degree with a major in mathematics from the University of North Carolina at Greensboro. The teacher assignments to sections were based on convenience and availability and were made prior to the student assignments. The author taught the 10:00 class (Section 1) and one of the 11:00 classes (Section 4) while Mrs. Clark and Mrs. Maynard taught the other two 11:00 classes (Section 3 and Section 2, respectively).

<u>Assignment of Treatments to the Classes</u>

Once the students and teachers were assigned, the treatments were randomly assigned to the classes. Section one was assigned to have required mastery with manipulatives, section two was assigned traditional testing with manipulatives, section three was assigned required mastery without manipulatives, and section four was assigned traditional testing without manipulatives. Table 1 summarizes teacher and treatment assignments.

<u>Procedures Used in All Four Classes</u>

With the exception of required mastery testing and the use of manipulative materials, all four classes used common procedures which previous research had indicated would increase achievement scores or improve students' attitudes toward mathematics. For example, all students were given study guides (see Appendix B for sample) which included instructional objectives. Other common procedures are explained in the following paragraphs.

Table 1

<u>Teacher and Treatment Assignments</u>

Time	Section	Instructor	Treatment
10:00	01	B. Maynard	Required Mastery With Manipulatives
11:00	02	N. Maynard	Traditional Testing With Manipulatives
11:00	03	S. Clark	Required Mastery Without Manipulatives
11:00	04	B. Maynard	Traditional Testing Without Manipulatives

A math lab was available to all students. The math lab was staffed by the math faculty from 9:00 A.M. to 1:00 P.M. daily. During the lab students could obtain answers to their questions, help with their assignments, and back-up audiovisual packages. Most of the objectives had filmstrips with accompanying audio tapes as their back-up. Students who missed class were required to do the audiovisual back-ups for the objectives they missed, and all students having trouble with a particular objective were encouraged to do the back-ups. Students were encouraged to get individual tutoring whenever they did not understand the concepts presented in class.

All classes were instructor controlled with lecture as the primary means of presenting the concepts. All of the objectives were presented through lecture and the lecture pace controlled the pace of the class. Attendance was required and all students were required to take unit tests on designated test days. Care was taken to make sure that during each day all four classes were presented exactly the same objectives. Students were expected to complete their assignments daily; however, the assignments for a unit were not collected until the day of the unit test. Hindsight shows that in all four sections, many of the students were delinquent in doing assignments. In a typical class period the instructor would lecture for approximately 30 minutes and give students approximately 20 minutes to work individually or in small groups. The purpose of work periods was to allow students to get immediate practice with the concepts presented during the lectures.

The same textbook, Introductory Mathematics by Charles McKeague, was used in all four classes and the assignments from the textbook were the same. With the exception of manipulatives aids, all four classes used the same materials. The same pretest, unit tests, final exam, and enjoyment scale were used in all four classes and all classes used the same grade-assignment scale.

Procedures Unique to Required Mastery

In the two required mastery sections, students who scored below 80% on a unit test were required to retake the test until they achieved the 80% mastery level. Students were required to take the first retake within one week of the unit test. No additional assignments were required and the students made the decision as to when they were prepared to take the retest. Approximately 90% of the students demonstrated mastery on each unit test or on the first retake. If students failed to score 80% or better on a retake, then they were required to meet with the instructor and develop a comprehensive study plan that would allow them to master the old material and keep up with the new material. Usually, the plan involved individual tutoring and additional assignments. When the instructor felt that a student had mastered the old material, the student was allowed to take a second retest. If mastery was not demonstrated, the study plan was adjusted and the procedure was repeated. Since the study plans had the potential of being extremely time consuming for students, instructors were afraid that the weaker students would withdraw from the class rather than make a study plan; however, as will be reported later, such was not the case. If a difference is found due to required mastery, it will be the result of retesting and the accompanying extra study rather than other factors frequently associated with the required mastery strategy.

Procedures Unique to Manipulatives Sections

In the two sections which used manipulative aids, mathematical concepts were first presented through use of the aids and then extended to the abstract. Recommendations concerning the use of manipulative aids given by Dunlap and Brennan (1979) were used throughout. For example, more than one device was used to introduce each new concept. Cuisenaire rods were used to introduce the concept of equivalent fractions, the concept was reinforced through the use of rulers graduated to sixteenths, and the concept was extended to the semi-concrete through the use of the number line. Finally the concept was extended to the abstract by relating the observed concrete attribute to the Fundamental Theorem of Fractions. (If a/b represents a fraction and c is a non-zero integer, then a/b = ac/bc.) In other words concrete experiences were related to the familar abstract rule for reducing or expanding fractions: If one multiplies or divides both the numerator and denominator of a fraction by the same non-zero integer, then the new fraction will be equivalent to the original. By having seen that a segment 5/8 inches long is the same length as a segment 10/16 inches long, the student should realize that 5/8 and 10/16 name the same quantity.

During lectures, the instructor introduced each new concept through the use of a concrete manipulative, picture, diagram, or an example related to the students' environment, and then showed students how to use the materials. During the open labs students were given lab assignments in which they actually performed the manipulations and drew intuitive conclusions (see Appendix C for sample labs). During individual or small-group sessions, the students were encouraged to extend concepts to the abstract level. Since rules were listed in their textbook, many students simply looked up rules and presented them to their group. In such cases the instructor encouraged students to discuss why the rule worked and when it could be applied.

Students were encouraged to use the insight gained through the use of manipulatives to estimate answers before doing the calculations. For example, when adding 2 1/4 and 3 1/2 the students were encouraged to draw a mental picture of a number line graduated to fourths and to think of starting at 2 1/4 and moving 3 1/2 units to the right. The purpose of such procedures was to help students catch their own "careless errors" and to give them an intuitive feel for mathematics.

Procedures Unique to the Nonmanipulatives Sections

In the sections that did not use manipulatives, concrete experiences were replaced by drill. During lectures, the instructors presented a rule or procedure, gave a logical mathematical explanation of why the rule worked, gave a real example showing the usefulness of the rule, and worked examples using the rule. After the lecture, the instructors wrote several problems on the board. As the students were working the problems individually or within a small group, the instructors circulated among the students checking their work, answering their questions, and correcting their mistakes. Any problems not completed during the class period were completed during the open lab periods. Students were expected to complete the assignments from their textbook in addition to the drill given in class.

Research Design to Measure the Effects of Mastery and the Use of Manipulatives on Math Achievement and Enjoyment of Mathematics

A multivariate analysis of covariance (Ray, 1982) was used to determine the effect of required mastery and the use of manipulatives on math achievement and enjoyment of mathematics. The dependent variables in the study were defined as follows:

Y_1 = Score on the final exam,

Y_2 = Final average on five instructor-prepared unit tests,

Y_3 = Final score on Aiken's Math Enjoyment Scale.

Form B of the Arithmetic Skills test published by the College Board, Princeton, New Jersey, was used as the final exam. Norms for the test were developed by administering the test to a nationwide sample of college freshmen who had one year or less of high school algebra. The mean number of correct answers for the norming sample was 24.46 with a standard deviation of 6.48. The KR-20 reliability estimate for the test was 0.87 and the standard error of measurement was 2.1. Topics covered by the test include operations with whole numbers, operations with fractions, operations with decimals and percents, and applications involving computations (Guide to the Use of the Descriptive Tests of Mathematics Skills, 1979). The author considered the final exam to be a measure of complete term achievement.

Students were given teacher-made unit tests on each of the five units. Scores on the five tests were averaged to create Y_2. Each unit test consisted of 20 questions which covered the objectives presented in the unit study guides (see Appendix D for a copy of the unit tests). All students were required to take unit tests on specified test days. The students in the required mastery sections were required to retake alternate forms of the unit tests until they scored 80% or better; however, only their first score on each unit was used for the study. The unit test average was considered to be a measure of short-term achievement.

Aiken's Math Enjoyment Scale is an eleven-item opinionnaire arranged in a Likert-type format. Students respond to each item by indicating whether they strongly agree, agree, have no opinion, disagree, or strongly disagree. Seven of the items are worded such that "strongly agree" indicates high enjoyment, and the remaining four items are worded such that "strongly disagree" indicates high enjoyment. Responses to each item were coded as -2, -1, 0, 1, or 2, with higher scores indicating greater enjoyment of mathematics.

Norms for the scale were developed by administering the 11 items along with additional filler items to 185 freshmen students (98 women and 87 men) at a southeastern college. The filler items were designed to measure another aspect of students' attitudes toward mathematics. Students' total scores were computed by adding their 11 item scores. The norming sample had a mean score of -0.06 with a standard deviation of 11.06. Correlation coefficients were calculated between each of the 11 item scores and the total score. All item score/total score correlation coefficients were greater than 0.75. Based on the norming sample, the scale had an internal consistency alpha coefficient of 0.95 (Aiken, 1974).

Aiken (1974) also presented evidence of acceptable content and discriminant validity. Item 2 which reads, "Mathematics is enjoyable and stimulating to me", had an item score/total score correlation coefficient of 0.91. Such a high correlation coefficient and the high alpha coefficient suggest that, for the norming sample, the scale reliably measured a construct called "enjoyment of mathematics". Item score/total score correlation coefficients were calculated for each of the filler items and none of the coefficients was as large as 0.75. The correlation between the total score on the 11 items and the total score on filler items was 0.64; therefore, while there is considerable overlap between the two sets of items, they do not measure identical constructs.

Following the procedures used by Aiken, the enjoyment scale along with filler items was administered to the subjects as a posttest on math enjoyment. The filler items were not scored; however, the students had no way of knowing which items were to be scored and which were to be ignored.

Two covariates were used in the study. Form A of the Arithmetic Skills test was used to correct for initial differences in achievement and Aiken's Math Enjoyment Scale was used to correct for initial differences in math enjoyment. The 11 items used on the pretest for math enjoyment were the same items used on the posttest; however, the filler items were different. The students should not have been able to recognize that the actual scales were the same. The covariates were assigned as follows:

X_1 = Raw score on Form A of Arithmetic Skills

X_2 = Raw score on Aiken's Math Enjoyment Scale.

The two categorical variables in the study were the use of manipulatives and the use of required mastery testing. The values were assigned as follows:

$$X_3 = \begin{cases} 0 \text{ if traditional testing} \\ 1 \text{ if required mastery} \end{cases}$$

$$X_4 = \begin{cases} 0 \text{ if manipulatives were not used} \\ 1 \text{ if manipulatives were used} \end{cases}$$

The two categorical variables were crossed to create a 2 X 2 factorial design with two covariates. Table 2 shows the full factorial variable-treatment assignment.

Table 2

Full Factorial Variable Assignments

X_4 \ X_3	0	1
0	Section 4	Section 3
1	Section 2	Section 1

In addition to the mastery-manipulative interaction term created by the design, a mastery-pretest interaction term was also examined. The author had reason to believe that mastery, if it had an effect at all, might affect the students with low pretest scores more than it would affect students with high pretest scores. Theoretically, the students with the higher scores would be less likely to retest; therefore, they would not be affected by the retesting strategy. Even though there was no theoretical justification for additional interaction terms, all possible combinations of covariate-by-treatment interaction terms were added to the model and analyzed by the General Linear Models subprogram of the Statistical Analysis System (SAS) (Goodnight, Stall, & Sarle, 1982). The unique contribution of each of the two, three, and four-way interaction terms was observed. The model was reduced by eliminating all covariate-treatment interaction terms which had a $\underline{P}$ value greater than 0.2. Only the mastery-pretest interaction term survived the reduction. The multivariate model used for the study is:

$$(Y_1, Y_2, Y_3)' =$$

$$\underline{B_0} + \underline{B_1}X_1 + \underline{B_2}X_2 + \underline{B_3}X_3 + \underline{B_4}X_4 + \underline{B_5}X_3X_4 + \underline{B_6}X_1X_3 + \epsilon$$

where

$$\underline{B_0} = (B_{01}, B_{02}, B_{03})'$$

$$\underline{B_1} = (B_{11}, B_{12}, B_{13})'$$

$$\vdots$$

$$\underline{B_6} = (B_{61}, B_{62}, B_{63})'.$$

Research Design to Measure the Effects of Required Mastery and the Use of Manipulatives on the Rate of Completion

The Chi-Square test of independence was used to determine whether the rate of completion was independent of the method of instruction. In addition an attempt was made to contact every student who missed as many as five consecutive classes and determine the reason that the student had decided to withdraw from the class. A descriptive analysis was completed to see if there were ways that the attrition rate could be lowered. Finally, an analysis of variance was performed on the pretest scores of the students who dropped. The purpose was to determine whether there was a difference, among the four classes, in the quality of students who dropped.

Internal Validity

Due to the uneven fill rate for the 10:00 A.M. and 11:00 A.M. sections, the author was concerned there might be an uncontrolled force that would affect the internal validity of the study. He was concerned there might be an initial difference between the two groups of students and that this difference might influence the final results. In order to test the reality of this concern, the students who registered for the class at 10:00 were compared to the students who registered for the class at 11:00 to see if there were observable differences in prior math achievement, enjoyment of math, ratio of females to males, or ratio of college transfer students to technical students. The two covariates were used as measures of prior math achievement and enjoyment of math, respectively. The 10:00 students had a mean achievement test score of 20.4 and a mean math enjoyment score of 4.3; and the 11:00 students had a mean achievement test score of 20.9 and a mean math enjoyment score of 3.1. Neither of the differences was significant at the 0.05 level of significance (see Table 3). The two groups were also compared to see if there were initial differences in the ratios of females to males or college transfer to technical students. The 10:00 time slot had 26.3% males and 73.7% females, and 26.3% college transfer and 73.7% technical students. The 11:00 class had 27.8% males and 72.2% females, and 28.4% college transfer and 71.6% technical students (see Table 4). None of the differences was significant. It was concluded that there were no initial differences in these variables which were caused by the uneven fill rates.

Table 3

Comparison of Initial Achievement and Attitude Differences

	Mean Achievement Pretest Score	Mean Attitude Pretest Score	t-test for Differences Between Means	Critical t
10:00	20.4	5.3	0.53	1.96
11:00	20.9	3.1	1.74	1.96

Table 4

Comparison of Initial Sex and Classification Differences

	% male	% female	% College Transfer	% Technical
10:00	26.3%	73.7%	26.3%	73.7%
11:00	27.8%	72.2%	28.4%	71.6%

Another potential threat to internal validity involved mortality. Whenever there are subjects who drop out during the experiment, there is a possibility that differences in the quality of losses will induce nonequivalence. In order to see whether there were differences among the classes in the initial achievement level of the students who withdrew, an analysis of variance was performed. The calculated *F* was 0.91 and the critical *F* was 2.92; therefore, it was concluded that there were no differences in the quality of students who dropped the courses (see Table 5).

Table 5

Analysis of Withdrawals Across Classes

Section	Number of Withdrawals	Mean Achievement Pretest Scores	Mean Math Enjoyment Pretest Scores
01	10	18.6	3.2
02	9	20.9	5.1
03	9	18.4	3.6
04	6	22	5.8

Critical $F(0.05, 3, 30) = 2.94$

Calculated $F(\text{achievement}) = 0.91$

Calculated $F(\text{enjoyment}) = 0.21$

The classes were also compared using the enjoyment pretest score of those who dropped as the dependent variable. Again, no significant differences were found (see Table 5). It was concluded that no differences among the groups were introduced through mortality.

Care was taken to insure that no differential treatments were introduced by accident. The author provided lesson plans for all classes and the three instructors followed the plans. Since all three instructors had previous experience in teaching the course, they were able to anticipate most of the students' questions and prepare consistent answers. All four classes were paced identically with the same objectives being presented in each class each day.

In summary, any posttest differences observed should be the result of the treatments and not the result of any pre-existing or accidentally induced differences.

Assumptions

In order to use multivariate analysis of covariance certain assumptions must be made. It is assumed that the errors, ϵ_i, are independent ($\mathrm{cov}(\epsilon_i, \epsilon_j) = 0$ for $i \neq j$) and normally distributed with mean of 0 and a constant variance of σ^2. It is assumed that the error matrices are equal across the treatment groups and at each level of the covariates, and that the set of dependent variables is multi-normally distributed. It is further assumed that each of the dependent variables can be written as a linear combination of the independent variables, that the covariates are independent of the treatments, and that the slope is the same for all treatment groups (Hair, Anderson, Tatham, & Grablowsky, 1979). Monte Carlo type experiments show that the F test is robust with regard to violations of the assumptions provided that deviations are not great and the sample sizes are equal (Harris, cited in Hair et al., 1979). Since the Wilks' Lambda, which will be used to test for multivariate significance, can be converted to an *F* statistic, it is also robust when sample sizes are equal. The following discussion will be directed toward the three univariate analyses of covariance; however, the same arguments apply to the multivariate case.

It can be argued that neither the independence nor normality assumptions are violated; however, according to research by Block and Burns (1976), the homogeneity of variance assumption is probably violated. Since the student is the unit of analysis and since all students within a class received the same treatment, there are potential independence problems. For example, an unplanned response by a student or teacher has the potential of affecting the entire class. The author has no evidence that the potential problems actually developed. Each of the teachers was asked to note and report all observed abnormalities; however, none were reported. Also, during exams, students worked independently of each other. Since the students taking the course at 10:00 were shown to be experimentally equivalent to those taking the course at 11:00, and since there were no observed contaminating episodes, the subject and treatment assignments should ensure the independence assumption. The subprocedure Normal of the Procedure Univariate of SAS (Delong, 1982) was used to check the normality of the residuals of each of the dependent variables. There was no evidence to reject the overall normality of the residuals or the normality of the residuals within any of the classes. Stem and leaf plots were also examined. None indicated a serious deviation from normality.

Since Block and Burns (1976) concluded that required mastery strategies should reduce the variability of achievement test scores, the univariate procedure was used to calculate the variances of each dependent variable within each of the four classes. Table 6 shows the classes and the variances of the dependent variables within the classes.

Table 6

<u>Variances of Dependent Variables Within Each Class</u>

		Variables		
		Y_1	Y_2	Y_3
Classes	No Man. No Mastery	43.5	470.8	85.5
	No Man. Mastery	15.2	182.5	32.6
	Man. No Mastery	26.3	273.2	45.7
	Man. Mastery	12.0	135.9	52.4

An examination of Table 6 shows that the variances of Y_1

and Y_2, the two measures of achievement, tend to be smaller in the mastery sections. An analysis of the residuals within each class yields similar results; therefore, there is evidence that the homogenity of variance assumption is violated. Elashoff (1969) pointed out that the homogeneity of variance assumption can be violated in two ways. It has already been pointed out that the variances are unequal across treatments. Now the variance of each Y that depends on the value of the covariates will be discussed. The procedure Plot of SAS (Goodnight, 1982) was used to plot the residuals against each of the covariates. No patterns could be detected; therefore, the variances of the dependent variables within a treatment are the same for each covariate but the variances are unequal across treatments. Following the advice of Hair et al. (1979), the author decided to equalize the class sizes, so as to reduce the effects of heterogeneity.

The normal procedure for equalizing sample size is to randomly drop subjects from the larger samples until one achieves equality. The procedure is valid whenever there is no systematic force causing the samples to be unequal. In order to search for such a force the author compared the covariate scores of those who withdrew with those who did not withdraw. The students who withdrew had a mean achievement score of 19.8 and those who completed the course had a mean score of 21.6. The calculated _t_ for the difference between means was 1.34 which was not significant. The math enjoyment pretest score of those who withdrew was 4.3 and the score for those who did not drop was 3.6. The calculated *t* (*t* = -0.47) was not significant. It has already been shown that there were no observable between-class differences in the quality of students who withdrew; therefore, it was concluded that the pattern of withdrawals was random. The normal procedure for equalizing sample size is valid for this study. Subjects were randomly dropped from Sections 01, 03, and 04 in order to create 4 classes of 22 each.

In order to test the linearity assumption, the residuals were plotted against the predicted values. Norusis (1982) says that if there is no observable pattern, then the linearity assumption is satisfied. An examination of the three plots (see Figures 1-3) shows no noticeable patterns.

The homogeneity of regression assumption is violated. The need for a required mastery-pretest score interaction term shows that the slopes are not the same over treatment groups. Elashoff (1969) indicated that violations of the homogeneity of regression assumptions tends to make the *F* test more conservative; therefore, any significant results should be valid. As a check, the author analyzed the treatment effects without including the pretest score-required mastery interaction term in the model. The conclusions were the same as those drawn using the complete model.

Summary

In summary, the homogeneity of variance assumption is probably violated; however, the results will not be biased as long as the sample sizes are equal. The equality of slopes assumption is violated since a covariate-treatment interaction term was added to the model; however, deleting the term does not change the conclusions. There is no evidence that any of the other assumptions were violated. The lack of a pattern between residuals and predicted values indicates that the chosen model is appropriate to describe the relationship between the dependent and independent variables. There is no combination of the given independent variables that would produce a more linear relationship. In other words, no quadratic terms or additional interaction terms are needed.

Figure 1. Plot of residuals versus predicted values for the achievement posttest.

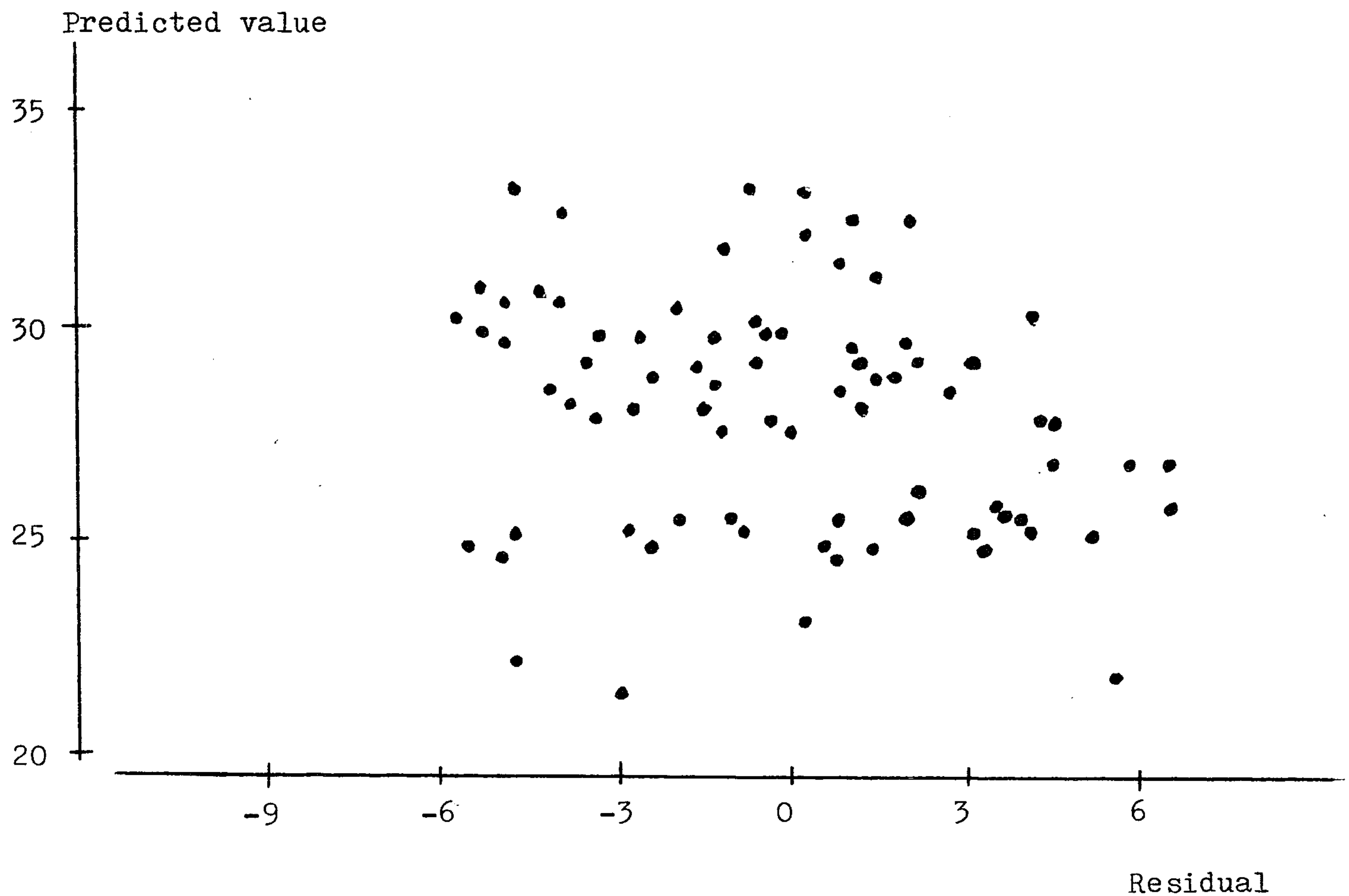


Figure 2. Plot of residuals versus predicted values for the unit-tests average.

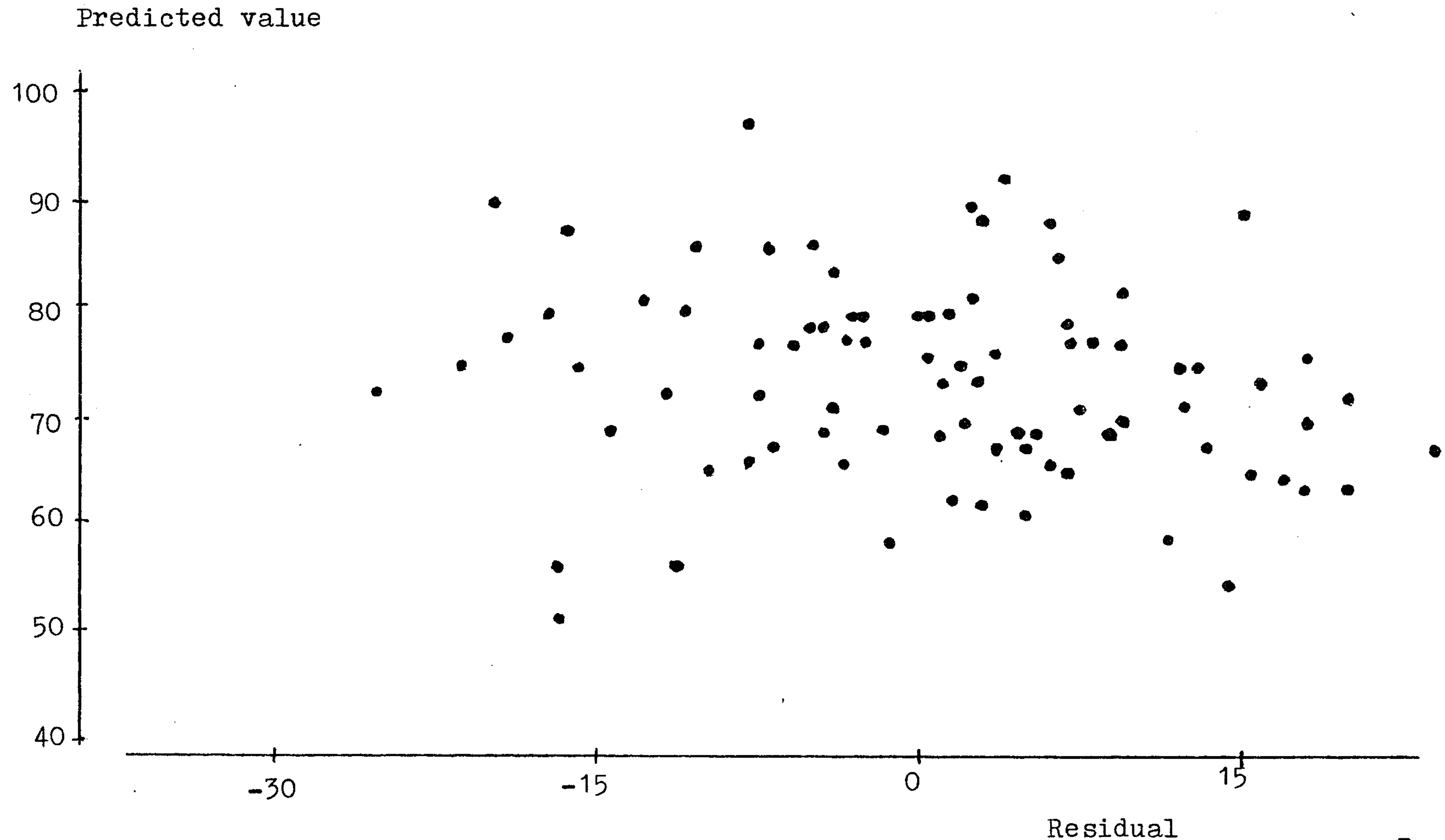
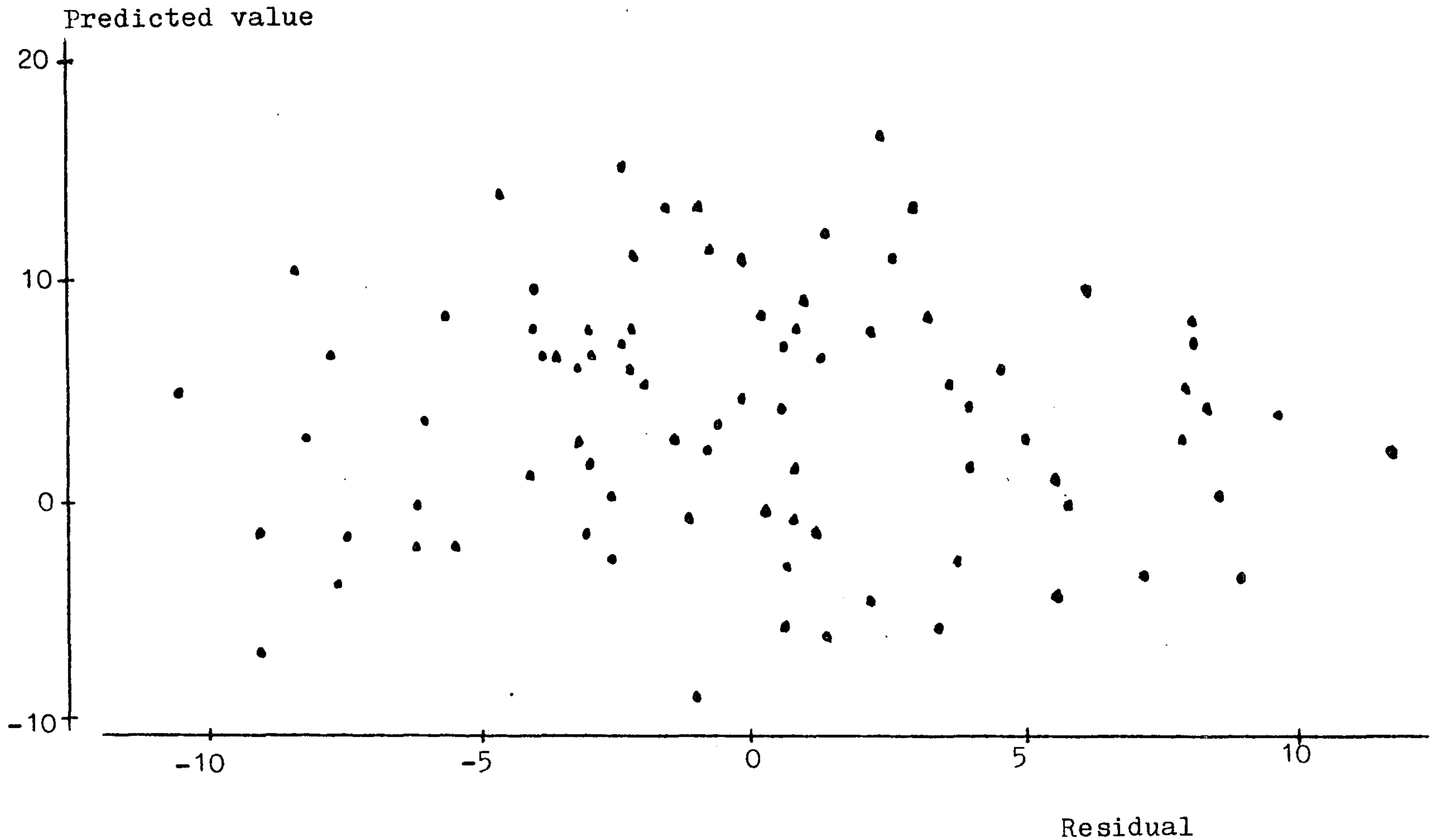


Figure 3. Plot of residuals versus predicted values for the enjoyment posttest.



CHAPTER 4

ANALYSES

This chapter reports the results of statistical analyses which were made in comparing the four classes. A total of 14 analyses were performed to determine whether any of the categorical variables or the interaction terms had an effect on completion rate, complete-term achievement, short-term achievement, or math enjoyment. As used in this study, attrition and rate of completion are not complementary terms. The rate of attrition was determined by comparing the numbers of students who withdrew before the end of the term to the number who enrolled. The rate of completion was determined by comparing the number of students who earned a grade of C or better with the number who enrolled.

Chi Square Analysis

Analysis I: Rate of Completion

Null Hypothesis: The number of students who completed the course is independent of the method of instruction.

Research Hypothesis: The number of students who completed the course is dependent on the method of instruction.

<u>Conclusion</u>: A Chi-Square test of independence was performed (see Table 7). Rates of completion were not significantly different at the 0.05 level of significance.

Table 7

<u>Contingency Table for Rate of Completion</u>

Section

	1	2	3	4	Total
No. who completed	28 [24.28]	19 [20.28]	20 [20.93]	20 [20.93]	87
No. who failed to complete	10 [13.14]	12 [10.72]	12 [11.07]	12 [11.07]	46
Total	38	31	32	32	133

<u>Note</u>: Numbers in boxes are the expected cell values.

$$\chi^2 = \sum \frac{(O - E)^2}{E} = 1.62; \quad \chi^2_{(.05,3)} = 7.82$$

<u>Multivariate Analyses</u>

In each of Analyses II-V Multivariate Analysis of Covariance (Ray, 1982) was performed using achievement and math enjoyment pretest scores as covariates; final exam scores, unit-tests averages, and posttest scores on math enjoyment as dependent variables; and required mastery and the use of manipulatives as categorical independent variables. The analyses were performed using the MANOVA subprocedure of the procedure GLM of SAS (Goodnight et al., 1982).

Analysis II: Multivariate Effect of the Required Mastery-Manipulatives Interaction

<u>Null Hypothesis</u>: There was no significant multivariate difference among the classes due to a required mastery-manipulatives interaction, having adjusted for the main effects and the covariates.

<u>Research Hypothesis</u>: There was a significant multivariate difference among the classes due to a required mastery-manipulatives interaction, having adjusted for the main effects and the covariates.

<u>Conclusion</u>: The data failed to yield sufficient evidence to reject the null hypothesis. There was no significant multivariate difference due to a required mastery-manipulatives interaction. The Wilks' Lambda criterion yielded $\underline{F}(3,79) = 0.46$ ($\underline{P} = 0.7154$). Since the multivariate test produced no evidence of a significant difference, no univariate analyses of the mastery-manipulative interaction were conducted.

Analysis III: Multivariate Effect of Required Mastery

<u>Null Hypothesis</u>: There was no significant multivariate difference among the classes due to

required mastery, having adjusted for the covariates and the use of manipulatives.

Research Hypothesis: There was a significant multivariate difference among the classes due to required mastery, having adjusted for the covariates and the use of manipulatives.

Conclusion: The data yielded evidence to reject the null hypothesis. After adjusting for the achievement pretest score, the math enjoyment pretest score, and the use of manipulatives, required mastery did produce a significant multivariate difference among the classes. The multivariate difference was based on final exam scores, unit-tests averages, and posttest scores on math enjoyment. The Wilks' Lambda criterion yielded $F(3, 79) = 4.52$ ($P = 0.0057$). Since the multivariate test produced evidence of a significant difference, univariate analyses were performed.

Analysis IV: Multivariate Effect of the Use of Manipulatives

Null Hypothesis: There was no significant multivariate difference among the classes due to the use of manipulatives, having adjusted for the covariates and required mastery.

Research Hypothesis: There was a significant multivariate difference among the classes due to the use of manipulatives, having adjusted for the covariates and required mastery.

Conclusion: The data failed to yield sufficient evidence to reject the null hypothesis. After adjusting for the achievement pretest score, the math enjoyment pretest score, and required mastery, the use of manipulatives did not produce a significant multivariate difference among the classes. The Wilks' Lambda criterion yielded $F(3,79) = 1.82$ ($P = 0.1480$). Even though the multivariate difference was not significant at the 0.05 level of significance, the P-value of 0.1480 means there is an 85% probability of some non-zero differences among the means. In an attempt to report all of the facts, the univariate analyses will be reported; however, the reader is warned that the multivariate difference was only significant at the 0.148 level of significance.

Analysis V: Multivariate Effect of the Achievement Pretest-Required Mastery Interaction

Null Hypothesis: There was no significant multivariate difference among the classes due to an achievement pretest-required mastery interaction, having adjusted for the achievement pretest score, the math enjoyment pretest score, required mastery, and the use of manipulatives.

Research Hypothesis: There was a significant multivariate difference among the classes due to an achievement pretest-required mastery interaction, having adjusted for the achievement pretest score, the math enjoyment pretest score, required mastery, and the use of manipulatives.

Conclusion: The data yielded sufficient evidence to reject the null hypothesis. After adjusting for the achievement pretest score, the math enjoyment pretest score, the use of manipulatives, and required mastery, the achievement pretest score-required mastery interaction did produce a significant multivariate difference among the classes. The multivariate difference was based on final exam scores, unit-tests scores, and posttest scores on math enjoyment. The Wilks' Lambda criterion yielded $F(3,79) = 2.93$ ($P = 0.0383$). Since the multivariate test produced evidence of a significant difference, univariate analyses were performed.

Univariate Analyses

In Analyses VI-VIII univariate analyses were performed to test the effects of required mastery on each of the three dependent variables (see Table 8).

Table 8

Univariate Effects of Required Mastery (X_3) Adjusting for the Achievement Pretest Score (X_1), the Math Enjoyment Pretest Score (X_2), and the Use of Manipulatives (X_4)

Complete Linear Model:

$Y(i) = B_{0i} + B_{1i}X_1 + B_{2i}X_2 + B_{3i}X_3 + B_{4i}X_4 + B_{5i}X_3X_4 + B_{6i}X_1X_3 + \epsilon$ for $i = 1, 2, 3$.

Effect on the Final Exam Scores (Y_1)

Source	SS	DF	MS	F	P Value
Required Mastery	84.48*	1	84.84	6.14	0.0153
Error	1114.84	81	13.76		

Effect on the Unit-Test Average (Y_2)

Source	SS	DF	MS	F	P Value
Required Mastery	64.97*	1	64.97	0.42	0.5185
Error	12512.75	81	154.48		

Effect on the Posttest Score for Math Enjoyment (Y_3)

Source	SS	DF	MS	F	P Value
Required Mastery	231.45*	1	231.45	9.44	0.0029
Error	1986.98	81	24.53		

* reduction of SS error due to using:

$Y_i = B_{0i} + B_{1i}X_1 + B_{2i}X_2 + B_{3i}X_4 + B_{4i}X_3 + \epsilon$ rather than

$Y_i = B_{0i} + B_{1i}X_1 + B_{2i}X_2 + B_{3i}X_4 + \epsilon$ where $i = 1, 2, 3$.

In each case an analysis of covariance was performed using the achievement pretest score and the pretest score on math enjoyment as covariates, and required mastery and the use of manipulatives as independent variables. The General Linear Models Procedure of SAS (Goodnight et al., 1982) was used to perform the analyses. All univariate analyses were performed using the Type II Sum of Squares which Ray (1982) defined as follows: "The Type II _SS_ are the reduction in error _SS_ due to adding the term after all other terms have been added to the model except terms that contain the effect being tested " (p. 164).

Analysis VI: Univariate Effect of Required Mastery on the Final Exam

Null Hypothesis: There was no significant difference in the final exam scores among the classes, having adjusted for the covariates and the use of manipulatives.

Research Hypothesis: There was a significant difference in the final exam scores among the classes, having adjusted for the covariates and the use of manipulatives.

Conclusion: The data yielded evidence to reject the null hypothesis. After adjusting for the covariates and the use of manipulatives, required mastery did produce a significant difference in final exam scores. An analysis of the Type II _SS_ yielded $F(1,81) = 6.14$ ($P = 0.0153$).

Analysis VII: Univariate Effect of Required Mastery on the Unit-Tests Average

Null Hypothesis: There was no significant difference in the unit-tests averages among the classes, having adjusted for the covariates and the use of manipulatives.

Research Hypothesis: There was a significant difference in the unit-tests averages among the classes, having adjusted for the covariates and the use of manipulatives.

Conclusion: The data failed to yield evidence to reject the null hypothesis. After adjusting for the covariates and the use of manipulatives, required mastery did not produce a significant difference in unit-tests averages. An analysis of the Type II _SS_ yielded $F(1,81) = 0.42$ ($P = 0.5185$).

Analysis VIII: Univariate Effect of Required Mastery on the Math Enjoyment Posttest Score.

Null Hypothesis: There was no significant difference in the math enjoyment posttest scores among the classes, having adjusted for the covariates and the use of manipulatives.

<u>Research Hypothesis</u>: There was a significant difference in the math enjoyment posttest scores among the classes, having adjusted for the covariates and the use of manipulatives.

<u>Conclusion</u>: The data yielded evidence to reject the null hypothesis. After adjusting for both covariates and the use of manipulatives, required mastery did produce a significant difference in math enjoyment posttest scores. An analysis of the Type II <u>SS</u> yielded $\underline{F}(1,81) = 9.44$ ($\underline{P} = 0.0029$).

In Analyses IX-XI univariate analyses were performed to test the effects of the use of manipulatives on each of the three dependent variables (see Table 9). In each case an analysis of covariance was performed using the achievement pretest score and the math enjoyment pretest score as covariates, and required mastery and the use of manipulatives as independent variables. The General Linear Models procedure of SAS (Goodnight et al., 1982) was used to perform the analyses. The reader is reminded that the multivariate test had a <u>P</u>-value of 0.148 and that care should be used in applying the results of the analyses.

Table 9

Univariate Effects of the Use of Manipulatives (X_4) Adjusted for the Achievement Pretest Score (X_1), the Pretest Score on Math Enjoyment (X_2), and the Use of Manipulatives (X_4)

Complete Linear Model:

$Y(i) = B_{0i} + B_{1i}X_1X_3 + B_{2i}X_2 + B_{3i}X_3 + B_{4i}X_4 + B_{5i}X_3X_4 + \epsilon$

for i = 1, 2, 3.

Effect on the Final Exam Scores (Y_1)

Source	SS	DF	MS	F	P Value
Manipulatives	47.36*	1	47.36	3.44	0.0672
Error	1114.84	81	13.76		

Effect on the Unit-Tests Average (Y_2)

Source	SS	DF	MS	F	P Value
Manipulatives	718.33*	1	718.33	4.65	0.034
Error	12512.75	81	154.48		

Effect on the Math Enjoyment Posttest Score (Y_3)

Source	SS	DF	MS	F	P Value
Manipulatives	51.01*	1	51.01	2.08	0.15
Error	1986.98	81	24.53		

* reduction of SS error due to using:

$Y_i = B_{0i} + B_{1i}X_1X_3 + B_{2i}X_2 + B_{3i}X_3 + B_{4i}X_4 + \epsilon$ rather than

$Y_i = B_{0i} + B_{1i}X_1X_3 + B_{2i}X_2 + B_{4i}X_3 + \epsilon$ where i = 1, 2, 3.

Analysis IX: Univariate Effect of the Use of Manipulatives on the Final Exam

<u>Null Hypothesis</u>: There was no significant difference in the final exam scores among the classes, having adjusted for the covariates and required mastery.

<u>Research Hypothesis</u>: There was a significant difference in the final exam scores among the classes, having adjusted for the covariates and required mastery.

<u>Conclusion</u>: The data failed to yield sufficient evidence to reject the null hypothesis at the 0.05 level of significance. After adjusting for both covariates and required mastery, the use of manipulatives did not produce a significant difference in final exam scores. An analysis of the Type II <u>SS</u> yielded $\underline{F}(1, 81) = 3.44$ ($\underline{P} = 0.0672$).

Analysis X: Univariate Effect of the Use of Manipulatives on the Unit-Tests average

<u>Null Hypothesis</u>: There was no significant difference in the unit-tests averages among the classes, having adjusted for the covariates and required mastery.

<u>Research Hypothesis</u>: There was a significant difference in the unit-tests averages among the

classes, having adjusted for the covariates and required mastery.

Conclusion: The data yielded sufficient evidence to reject the null hypothesis. After adjusting for both covariates and required mastery, the use of manipulatives did produce a significant difference among the unit-tests averages. An analysis of the Type II SS yielded $F(1, 81) = 4.65$ ($P = 0.034$).

Analysis XI: Univariate Effect of the Use of Manipulatives on the Math Enjoyment Posttest Score

Null Hypothesis: There was no significant difference in the math enjoyment posttest scores among the classes, having adjusted for the covariates and required mastery.

Research Hypothesis: There was a significant difference in the math enjoyment posttest scores among the classes, having adjusted for the covariates and required mastery.

Conclusion: The data failed to yield evidence to reject the null hypothesis. After adjusting for both covariates and required mastery, the use of manipulatives did not produce a significant difference in math enjoyment posttest scores. An analysis of the Type II SS yielded $F(1, 81) = 2.08$ ($P = 0.1532$)

In Analyses XII-XIV univariate analyses were performed to test the effect of the achievement pretest-required mastery interaction on each of the three dependent variables (see Table 10). In each case a multiple regression analysis was performed using

$Y_i = B_{0i} + B_{1i}X_1 + B_{2i}X_2 + B_{3i}X_3 + B_{4i}X_4 + B_{5i}X_3X_4 + B_{6i}X_1X_3 + \epsilon$, where $i = 1, 2, 3$, as the complete model and

$Y_i = B_{0i} + B_{1i}X_1 + B_{2i}X_2 + B_{3i}X_3 + B_{4i}X_4 + B_{5i}X_3X_4 + \epsilon$, where $i = 1, 2, 3$, as the reduced model. The General Linear Models procedure of SAS (Goodnight et al., 1982) was used to perform the analyses. The sum of squares of the residuals was used as SS error and the reduction of the sum of squares SS error due to adding $B_{6i}X_1X_3$ was used as the SS hypothesis.

Analysis XII: Univariate Effect of the Entering Achievement Level-Required Mastery Interaction on the Final Exam Score

Null Hypothesis: $B_{61} = 0$.

Research Hypothesis: $B_{61} \neq 0$.

Conclusion: The data failed to yield sufficient evidence to reject the null hypothesis at the 0.05 level of significance. After adjusting for the main effect due to the achievement pretest, the math enjoyment pretest score, required mastery, the use of manipulatives, and the mastery-manipulatives interaction, the required mastery-entering achievement

Table 10

Univariate Effect of the Entering Achievement Level-Required Mastery Interaction

Complete Model:

$Y_i = B_{0i} + B_{1i}X_1 + B_{2i}X_2 + B_{3i}X_3 + B_{4i}X_4 + B_{5i}X_3X_4 + B_{6i}X_1X_3 + \epsilon$, for i = 1, 2, 3.

Reduced Model:

$Y_i = B_{0i} + B_{1i}X_1 + B_{2i}X_2 + B_{3i}X_3 + B_{4i}X_4 + B_{5i}X_3X_4 + \epsilon$, for i = 1, 2, 3.

Effect on the Final Exam (Y_1)

Source	SS	DF	MS	F	P Value
Achievement/Mastery	49.11	1	49.11	3.57	0.0625
Error	1114.84	81	13.76		

Effect on the Unit-Tests Average (Y_2)

Source	SS	DF	MS	F	P Value
Achievement/Mastery	16.65	1	16.65	0.11	0.7435
Error	12512.75	81	154.48		

Effect on the Math Enjoyment Posttest Score (Y_3)

Source	SS	DF	MS	F	P Value
Achievement/Mastery	145.46	1	145.46	5.93	0.0171
Error	1986.98	81	24.53		

level interaction failed to produce a significant difference among the final exam scores. An analysis of the Type II *SS* yielded $F(1, 81) = 3.57$ ($P = 0.0625$).

Analysis XIII: Univariate Effect of the Entering Achievement Level-Required Mastery Interaction on the Unit-Tests Average

Null Hypothesis: $B_{62} = 0$.

Research Hypothesis: $B_{62} \neq 0$.

Conclusion: The data failed to yield evidence to reject the null hypothesis. After adjusting for the main effect due to the achievement pretest, the math enjoyment pretest score, required mastery, the use of manipulatives, and the mastery-manipulatives interaction, the required mastery-entering achievement level interaction failed to produce a significant difference among the unit-tests averages. An analysis of the Type II *SS* yielded $F(1, 81) = 0.11$ ($P = 0.7435$).

Analysis XIV: Univariate Effect of the Entering Achievement Level-Required Mastery Interaction on the Math Enjoyment Posttest Score

Null Hypothesis: $B_{63} = 0$.

Research Hypothesis: $B_{63} \neq 0$.

Conclusion: The data yielded evidence to reject the null hypothesis. After adjusting for the main effects due to the achievement pretest, the math enjoyment pretest score, required mastery, the use of manipulatives, and the required mastery-manipulatives interaction, the achievement pretest-required mastery interaction produced a significant difference among the math enjoyment posttest scores. An analysis of the Type II SS yielded $F(1, 81) = 5.93$ ($p = 0.0171$).

Summary

1. Neither required mastery nor the use of manipulatives had an effect on attrition.
2. Required mastery produced a significant multivariate difference among the treatment groups, using final exam scores, unit-tests averages, and math enjoyment posttest scores as dependent variables. Univariate analyses showed that required mastery produced significant differences on the final exam and the math enjoyment posttest scores. In all cases, the effects due to mastery were adjusted for the two covariates and the use of manipulatives.
3. The use of manipulatives failed to produce a significant multivariate difference among the treatment groups using final exam scores, unit-tests averages, and math enjoyment posttest scores as dependent variables. The relatively low, but not significant, p value (0.148) suggested that information might be gained by performing

univariate analysis. The use of manipulatives produced a significant difference based on unit-tests averages.

4. The required mastery-manipulatives interaction failed to produce a significant multivariate difference among the treatment groups using final exam scores, unit-tests averages, and math enjoyment posttest scores as dependent variables. The _P_ value was high; therefore, no univariate analyses were performed.

5. The achievement pretest score-required mastery interaction produced multivariate differences among the treatment groups, using final exam scores, unit-tests averages, and math enjoyment posttest scores as dependent variables. Univariate analyses showed that the interaction produced a significant difference on math enjoyment posttest scores. In all cases effects due to the achievement pretest-required mastery interaction were adjusted for all main effects and for the mastery-manipulatives interaction.

CHAPTER 5

CONCLUSIONS AND IMPLICATIONS

The primary purpose of this chapter is to consider the results of the investigation and to discuss possible reasons for findings of significance or lack of significance. The implications that the study holds for teaching remedial mathematics and for future research will also be discussed. Basic areas to be addressed are rate of completion, complete-term achievement, short-term achievement, and math enjoyment. Finally, the achievement and enjoyment variables will be considered as a single factor called "the overall success of the instruction".

Rate of Completion

Analysis I showed that rate of completion is independent of method of instruction. Neither required mastery nor the use of manipulatives nor an interaction between the two treatments affected the rate of completion. This finding contradicts earlier research (Akst, 1976) which implied that completion rate was lowered by required mastery strategies. One possible reason for the conflicting results is that, in the present study, efforts were taken to reduce attrition in all classes. Appointments were scheduled with those students who began to accumulate excessive absences and an attempt was made to call all students who missed three consecutive class periods. During the appointment or call, the instructor attempted to determine why the student had excessive absences, discussed possible ways to remedy the situation, offered to help the student make up missed work, and offered to help the student officially withdraw if that was the only viable choice. Unfortunately, no record was kept on the number of calls that resulted in a student's returning to class; however, the instructors were able to contact 30 of the 34 students who withdrew during the quarter. If care is taken to control attrition, the remedial math instructor can use required mastery without adversly affecting the completion rate. Additional research is needed to determine whether the practice of calling students, which was done in all classes, significantly affects attrition.

In doing the present study and in attempting to increase internal validity, the author was interested in determining the factors which caused students to drop out of the study. The instructors were able to discuss the reasons for withdrawing with 30 of the 34 students who dropped the course during the study. Since the students' grades were determined in a completely objective manner and were not affected by what the students reported, there is no reason to suspect that the reasons given were not true. The reasons for withdrawal were classified and placed in one of eight categories (see Table 11).

The majority of the withdrawals were for job-related reasons or personal problems. The job-related reasons given included changing shifts which created a time conflict, getting a job, and working too many hours to continue with the present load. Personal problems included prolonged illness of a child, emotionally draining divorce proceedings, being sentenced to an active jail term, and moving out of the state in order to handle family affairs. Three of the withdrawals were because of illness which forced the student to miss an excess of five consecutive days. For the above cases the students and instructors agreed that the students would benefit by withdrawing and starting over the following quarter.

Other non-school-related reasons included transportation problems and financial problems. It was concluded there was nothing the math instructors could or should have done to prevent those withdrawals. Three of the students withdrew for reasons they attributed to the school but not directly to the math class. Two of the three felt that their advisors allowed them to register for too many hours, and the third left school because the school did not offer the program he wanted. Again there was little the math instructors could have done.

Three of the students withdrew for reasons directly connected with the math class. The actual reasons were as follows: "too much homework", "need to develop a study plan but cannot attend math lab", and "not making the grades that I want to make." After examining the data the author tends to agree with Williams (1973). Perhaps we should accept a fairly high rate of attrition and concentrate on creating the best possible learning experience for those who remain. The instructors should be more concerned about the twelve students who remained enrolled but failed to satisfactorily complete the course.

Table 11

Reasons Given for Withdrawing

Reason	Number
School Related, Math	3
School Related, Non-Math	3
Job Related	8
Financial	2
Transportation	2
Personal Problems	9
Illness	3
Unable to contact and/or classify	4
Total	34

Complete-Term Achievement

For the purposes of this study, complete-term achievement is defined as the achievement gains students made during the complete quarter. Form A of the Arithmetic

Skills test was used to measure students' entering achievement levels and Form B of the same test was used to measure students' overall arithmetic achievement level at the end of the quarter. The difference between the two scores is a measure of achievement gain due to experiences encountered during the quarter. All four classes demonstrated significant gains in achievement (see Table 12). None of the 95% confidence interval estimates of the true mean gain in achievement contained 0.

Table 12

Complete-Term Achievement Gains

Section	Mean Gain	Standard Deviation	95% Confidence Interval Estimate
01	8.68	3.68	[7.90, 9.46]
02	6.91	4.87	[5.87, 7.95]
03	6.64	4.59	[5.66, 7.62]
04	4.36	3.47	[3.62, 5.10]

Analysis VI showed that required mastery had an overall effect on the mean final exam score after correcting for entering achievement level, entering math enjoyment level, and the use of manipulatives. Comparing that result with the above data shows that required mastery has a positive effect on complete-term achievement. Except for the retesting feature, all of the classes had the usual aspects of the required mastery strategy; therefore, the observed gains are due to factors directly related to retesting. Since there were no observed across-class differences in the entering achievement levels among those who withdrew, the achievement gains cannot be attributed to attrition patterns favorable to required mastery. It must be concluded that required mastery, as used in this study, will improve the measured complete-term achievement of remedial arithmetic students; and this gain in achievement is not at the expense of increased attrition.

After correcting for the effects due to required mastery, the use of manipulatives did not have a significant effect on complete-term achievement ($\underline{P}$ = 0.0672); however, Section 02, which used manipulatives, but not mastery, had an observed gain that was greater than that of Section 03, which used mastery but not manipulatives (see Table 12). While one cannot conclude that the use of manipulatives will improve complete-term achievement, one can certainly claim that a teacher could use the manipulatives and concrete examples to replace the usual abstract follow-the-rules approach without harming achievement. Using class and lab time to provide the students with concrete experiences is at least as effective as the drill they replaced.

One limitation of the present study is that some of the students in the manipulatives sections failed to participate fully in the concrete experiences. Some chose not to do the labs; therefore, they benefited only from the class experiences. (Some of the students in the other sections chose not to do the drill.) Since the $\underline{P}$ value is relatively low, additional research is needed in which the researcher produces a greater incentive for all students to complete the labs. In the present study, as in many classrooms, the real incentive was to score well on the exams. With the lab grade contributing so little to the final grade (see Appendix I) the student could afford, point-wise, not to complete the labs.

Even though not significant at the 0.05 level of significance, required mastery seemed more beneficial for students with lower entering achievement levels. The achievement pretest-required mastery interaction produced a significant multivariate difference and a univariate difference ($\underline{P}$ = 0.0625) on the achievement posttest scores. An examination of the plots of achievement gain versus entering achievement level shows a slight negative correlational pattern for the sections that did not use required mastery and a moderate negative correlational pattern for the required-mastery sections (see Figures 4-5). The slight negative correlational pattern is expected due to the tendency for a regression toward the mean and due to the

Figure 4. Plot of achievement gains versus achievement pretest scores for required mastery sections.

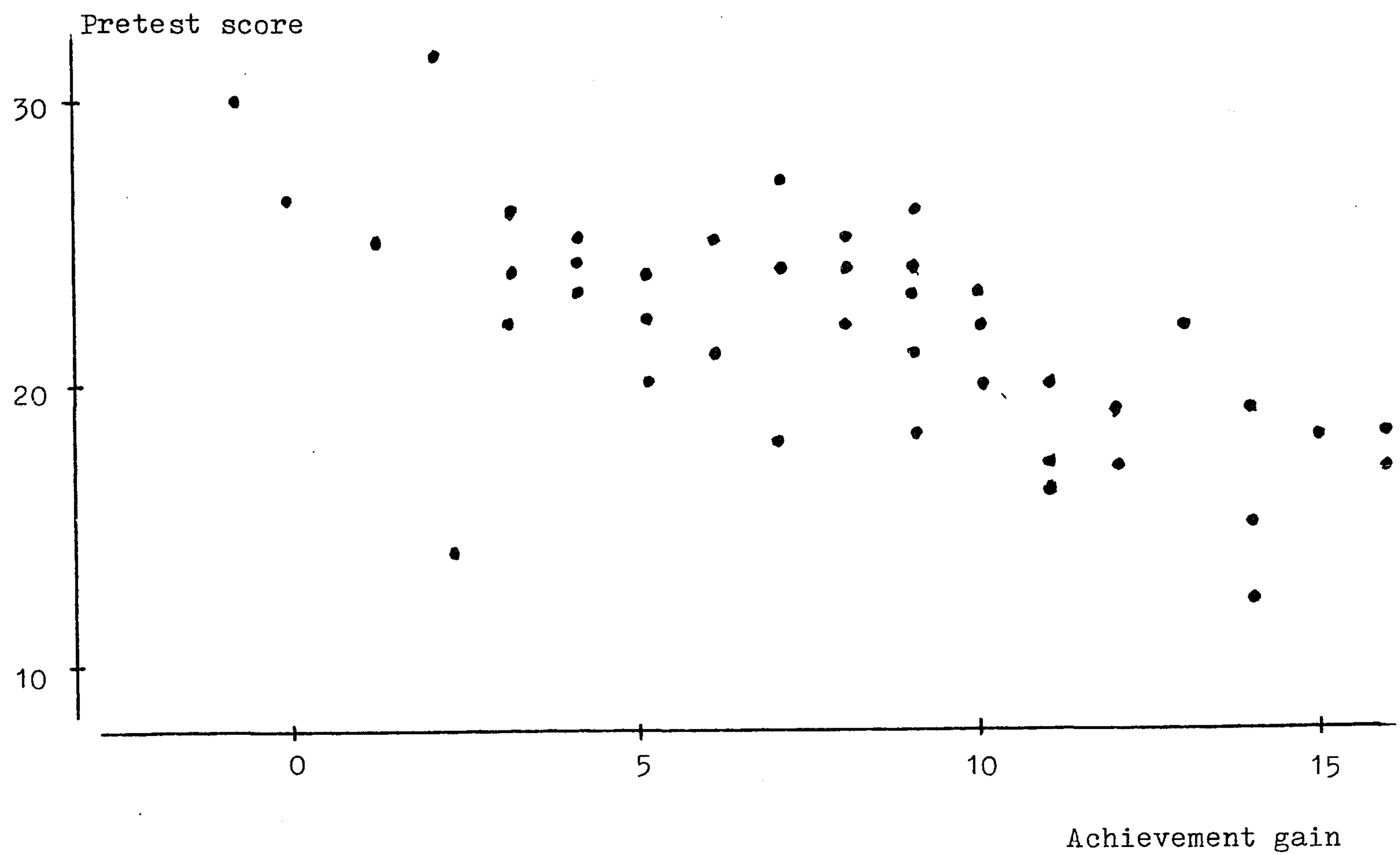
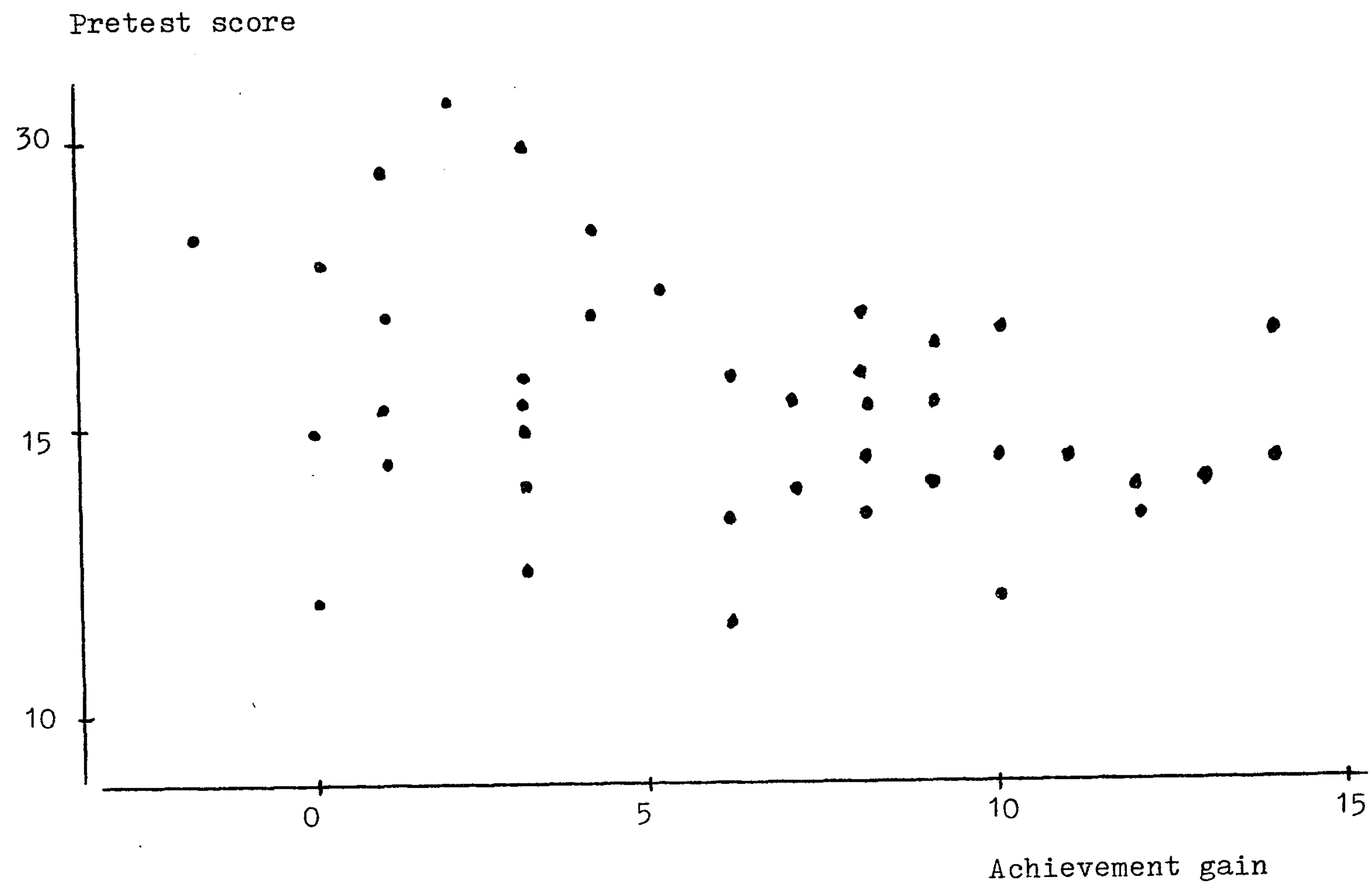


Figure 5. Plot of achievement gains versus achievement pretest scores for the traditional testing sections.



fact that the pretest score is included in the calculation of achievement gain (Bereiter, 1963). The moderate negative correlational pattern for the required-mastery sections means that in those sections, students who had lower achievement pretest scores had achieved greater gains than did those with higher pretest scores. The observed results were expected since theoretically students with higher entering achievement levels should be less likely to participate in retesting.

Short-Term Achievement

Short-term achievement was measured by unit-tests averages. In contrast to the final exam which was comprehensive, unit tests were given approximately every two weeks and covered a relatively small amount of content. To a certain extent, the unit tests and the final exam measured the same thing, math achievement; however, since there was not a perfect correlation between the two variables (r = 0.74), they were measuring a slightly different type of achievement. The unit tests required less broad integration, but they required a deeper, more intuitive understanding of each specific concept (see Appendix D for the unit tests). They tested to a much greater depth and each problem required more steps than did the problems on the final exam. Mean unit-tests averages adjusted for entering achievement are given in Table 13.

Table 13

Final Average on the Five Unit Tests Adjusted for Entering Achievement Level

Section	Treatment	Adjusted Mean Score
01	Both Mastery and Manipulatives	77.4
02	Manipulatives but not Mastery	75.9
03	Mastery but not Manipulatives	70.2
04	Neither Mastery nor Manipulatives	69.8

Analysis X showed that the use of manipulatives significantly affected unit-tests averages and the above data show that it affected the averages positively. Many of the lab experiences required the students to consider the underlying reasons for a rule or procedure, rather than simply applying the rule. All of the classes had the reasons explained to them but at different cognitive levels. Perhaps the students in the traditional sections did not understand the abstract explanations as well as the students in the manipulative sections understood the concrete explanations. The extra drill and repeated practice allowed the traditional students to develop rote procedures to work simple problems and they were able to remember the procedures for the complete term; however, they did not develop the understanding to solve problems which were more difficult and required several steps. The use of manipulatives and the emphasis on estimation allowed the students to perform lengthy calculations with less chance of error. It is concluded that the use of manipulatives along with concrete explanations will improve performance on the unit tests in remedial arithmetic classes.

The fact that required mastery did not significantly affect unit-tests averages futher supports the contention that it was the actual studying for the retest and the resulting ability to understand new concepts that improved complete-term achievement. If the students had been working harder to avoid having to take a retest, then their unit-tests averages would have been significantly higher.

Math Enjoyment

Aiken's Math Enjoyment Scale was administered to the classes as a pretest and again as a posttest. Class means for the differences in math enjoyment scores along with 95% confidence interval estimates of the true mean gains in enjoyment scores are presented in Table 14. Three of the classes had confidence interval estimates which included 0; therefore, one cannot conclude that these classes produced a gain in math enjoyment scores. Only section one, which had both mastery and the use of manipulatives, showed a significant increase in math enjoyment.

Table 14

Complete-Term Changes in Math Enjoyment

Section	Mean Change	Standard Deviation	95% Confidence Interval Estimate
01	3.18	4.56	[2.21, 4.15]
02	0.82	6.83	[-0.64, 2.28]
03	0.23	5.61	[-0.97, 1.43]
04	-0.50	5.29	[-1.63, 0.63]

In the planning stages of the study, the author felt that required mastery might lower students' math enjoyment scores. He felt that forcing the students to make study plans and attend a lab for extra help would create dislike for the subject; however, the results did not support the contention. Analysis XI, which showed that the use of manipulatives had no effect on math enjoyment, and Analysis VIII, which showed that required mastery did affect math enjoyment, indicate that the observed difference is due to required mastery. Futhermore, Analysis XIV and an examination of initial achievement, math enjoyment plots (see Figures 6-7) shows that students with an initial low achievement score benefited more from required mastery than those with higher pretest scores. Students with an initial low achievement score probably had a history of math failure. The required mastery strategy, which required retesting and the developing of study plans, allowed these

Figure 6. Plot of math enjoyment gains versus achievement pretest scores for required mastery sections.

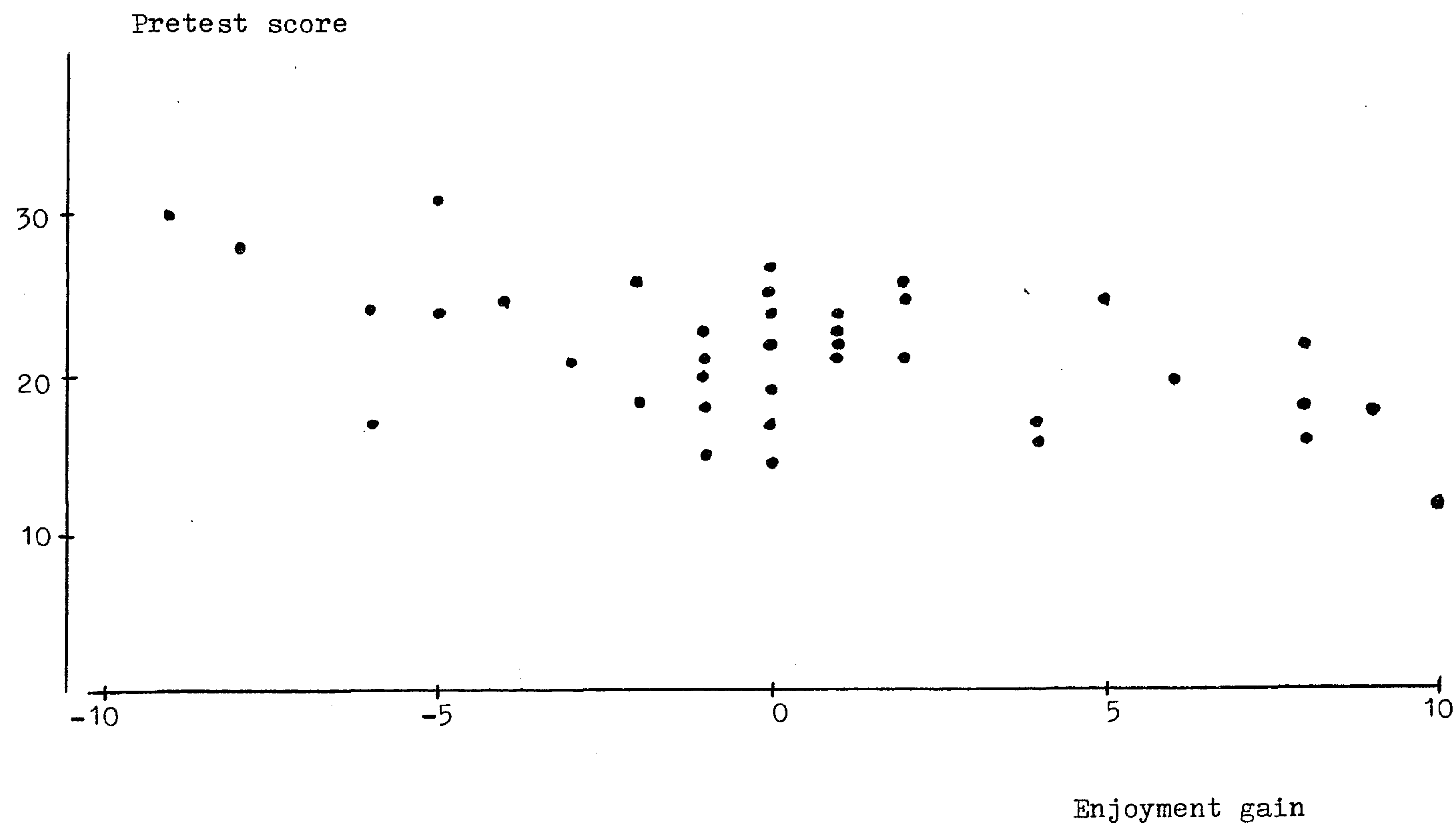
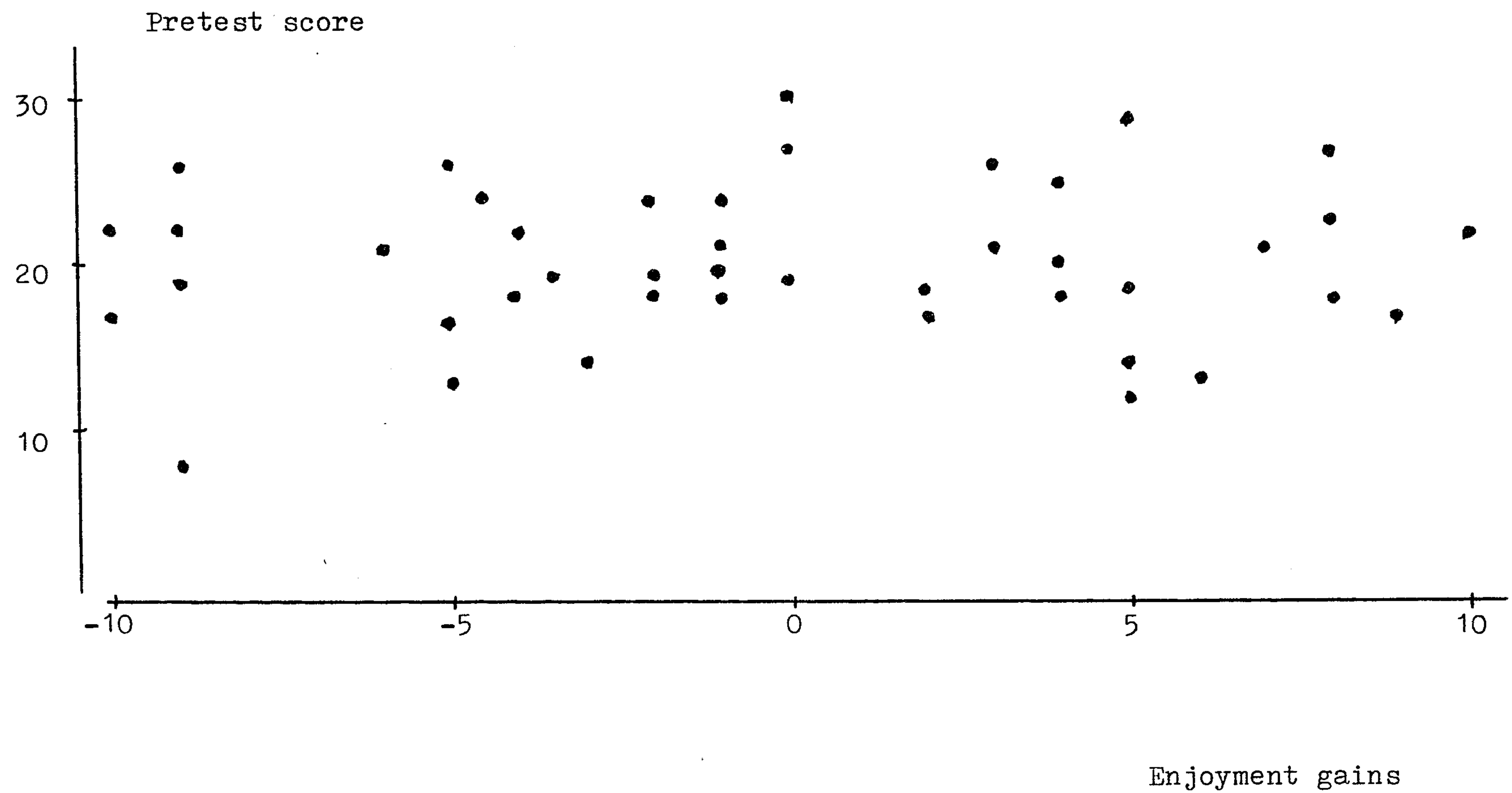


Figure 7. Plot of math enjoyment gains versus achievement pretest scores for traditional testing sections.



students to experience success. The successful experiences evidently led to increased enjoyment of mathematics. The fact that the use of manipulatives did not significantly affect the enjoyment-of-math posttest scores indicates that teaching at students' cognitive levels of development did not contribute to their enjoyment of the subject. Since most of the students in required-mastery sections scored well on their retests, it is concluded that success on the tests contributed more to the enjoyment of mathematics than did intuitive understanding. It is concluded that required mastery significantly increased the students' enjoyment of remedial arithmetic and that the remedial arithmetic instructors can use manipulatives and concrete explanations without fear of reducing enjoyment.

Overall Success

For the purpose of the present study, overall success was taken to be the vector score composed of final exam scores, unit-tests averages, and math enjoyment posttest scores. The significant multivariate results of the study showed that required mastery contributed positively to the overall success of the remedial arithmetic course. In addition, required mastery was more beneficial for students with initially low achievement scores than for students with higher initial achievement. While there was some indication that the use of manipulatives contributed to greater overall success ($\underline{P}$ = 0.148) the contribution was not significant.

External Validity

The author feels that the results of the present study can be generalized to any population of remedial mathematics students for which the following conditions are true: The instructors know the objectives which should be taught, and a majority of the students are below the formal operational level of cognitive development. The results should not be generalized to college-level courses or to remedial courses in other disciplines.

Summary

Based on a review of the research and on the conclusions reached through the present study, the author recommends that instructors of remedial mathematics implement both required mastery strategies and the use of manipulatives. There is cumulative evidence that required mastery strategies result in improved complete-term achievement and in the enjoyment of mathematics, and the improvement does not come at the expense of the completion rate. The findings in the present study imply that the use of manipulatives will improve short-term achievement, will probably improve complete-term achievement($\underline{P}$ = 0.0672), and will not adversely affect either math enjoyment or the rate of completion. There was no interaction between the two treatments; therefore, the instructor could get the main effect advantages of either treatment without implementing the other.

More research is needed on matching the students' cognitive level with the level of presentation. Researchers need to develop studies in which the incentives are present for all subjects to participate fully in the lab exercises. Research is needed to measure the effectiveness of both required mastery and the use of manipulatives over time spans greater than one quarter. Studies which measure the success of the students in their next math courses are needed and attempts must be made to determine why such large percentages of students never take additional math courses.

Finally, all existing remedial courses need to be evaluated in terms of success within the course, completion rates, and success at the next level; and the results of the evaluation should be published. Due to the nature of remediation, remedial instructors cannot expect the same success rates as their peers in college-level courses; however, they do need some indications of what is acceptable. The author wonders how many successful remedial programs have been revised into something less effective because the instructor had unrealistic expectations. Remedial education needs its own realistic definitions of success and those definitions must extend beyond the remedial programs. For example, a remedial course which has a near perfect completion rate is useless if the students fail the next level course, whereas a course with a completion rate of 30% may be quite valuable if nearly all of the completing students take and pass their college-level courses.

BIBLIOGRAPHY

Adler, I. (1966). Mental growth and the art of teaching. Arithmetic Teacher, 13, 576-584.

Aiken, L. R. (1974). Two scales of attitude toward mathematics. Journal for Research in Mathematics Education, 5(2), 67-71.

Ajose, S. A. (1978). A review of research on the effectiveness of remedial mathematics programs in two-year colleges. Paper presented at the annual meeting of the American Mathematical Association of Two-Year Colleges, Houston, TX. (ERIC Document Reproduction Service No. ED 164017)

Akst, G. R. (1976). A study of the effect on learning of pacing and testing procedures in a two-year college remedial mathematics course. Dissertation Abstracts International, 37, 2035A. (University Microfilms No. 76-21, 768)

Archer, J. A. (1978). Developmental mathematics: An ex post facto study. Community/Junior College Research Quarterly, 3(1), 75-86.

Baldwin, James and others. (1975). Survey of developmental mathematics courses at colleges in the United States. Garden City, New York: American Mathematical Association of Two-Year Colleges. (ERIC Document Reproduction Service No. ED 125688)

Barcus, H., & Kleinstein, J. (1980-1981). Description and assessment of basic mathematics skills course, computational skills, at Westchester Community College. Journal of the CUNY Mathematics Discussion Group, 8, 36-66.

Barnett, J. C., & Eastman, P. M. (1978). The use of manipulative materials and student performance in the enactive and iconic modes. Journal for Research in Mathematics Education, 9, 94-101.

Beal, K. M. (1978). A study of attitude and achievement in basic mathematics utilizing individualized and traditional instruction at Haskell Indian Junior College. Dissertation Abstracts International, 39, 6561A. (University Microfilms No. 7910611)

Bellile, E. G. (1980). Background, attitude, and problem solving characteristics of college students whose lowest ACT scores are in math. Dissertation Abstracts International, 41, 1450A. (University Microfilms No. 80-22, 001)

Bello, I. (1978). Contemporary basic mathematical skills. New York: Harper and Row.

Bereiter, C. (1963). Some persisting dilemmas in the measurement of change. In C. W. Harris (Ed.), Problems in Measuring Change. Madison, WI: University of Wisconsin Press.

Bickford, E. T. (1979). Identification of important factors contributing to the academic achievement of the student in the remedial mathematics course in the junior college (Doctoral dissertation, Georgia State University, 1979). Dissertation Abstracts International, 40, 3169A.

Block, J. H., & Burns, R. B. (1976). Mastery learning. In L. S. Shulman (Ed.), Review of Research in Education (Vol 4, pp. 3-50). Itasca, IL: American Educational Research Association.

Bloom, B. S. (1956). Taxonomy of educational objectives. New York: Longmans Green.

Blount, M. A., Sr. (1980). Effects of a recycling laboratory on attitude toward and achievement in mathematics among college freshmen (Doctoral dissertation, University of Connecticut, 1980). Dissertation Abstracts International, 41, 1990A.

Bluman, A. G. (1971). Development of a laboratory method of instruction in mathematics at the community college (Doctoral dissertation, University of Pittsburgh, 1971). Dissertation Abstracts International, 32, 1970A-1971A.

Brawer, F. B. (1982, Spring). Literacy, transfer, and effectiveness: The community colleges' accelerating mission. ERIC: Junior College Resource Review. pp. 11-15.

Bruner, J. S. (1960). On learning mathematics. The Mathematics Teacher, 53, 610-619.

The Carnegie Foundation for the Advancement of Teaching. (1979). Missions of the college curriculum: A contemporary review with suggestions. San Francisco: Jossey-Bass.

Cohen, A. M. (1982, Spring). Ten criticisms of developmental education. ERIC: Junior College Resource Review, pp.17-19.

Conference Board of the Mathematical Sciences. (1982. January-February). Survey reveals dramatic changes in collegiate mathematics. Focus: The Newsletter of the Mathematical Association of America, pp. 1-6.

Delong, D. M. (1982). Univariate [Computer program]. Cary, NC: Statistical Analysis System.

Drapac, G. L. (1981). Development of manipulative materials for elementary algebra for college students and evaluation of their effect on achievement, attitude toward mathematics, and math anxiety. Dissertation Abstracts International, 42, 120A-121A. (University Microfilms No. 8114251)

Drennen, R. L. (1971). The effect of specific performance objectives on student achievement, attitude, and attrition in remedial mathematics at Jefferson State Junior college (Doctoral dissertation, Auburn University, 1971). Dissertation Abstracts International, 32, 4325A.

Dunlap, W. P., & Brennan, A. H. (1979). Developing mental images of mathematical processes. Learning Disability Quarterly, 2(2), 89-96.

Eisenburg, T. (1981). Remedial mathematics and open admissions. School Science and Mathematics, 81(4), 341-346.

Elashoff, J. D. (1969). Analysis of covariance: A delicate instrucment. American Educational Research Journal, 6, 383-400.

Fast, C. (1980, May). Making the best better: The LCC individualized math story. Eugene, OR: Oregon Develmental Educational Association. (ERIC Document Reproduction service No. ED 187 371)

Fitzgerald, W. (1968). A math lab for prospective elementary school teachers. Arithmetic Teacher 15, 547-549.

Friedlander, J. (1979, August). Developmental mathematics. ERIC: Junior College Resource Review, pp. 1-3.

Fuson, K. (1975). The effect on preservice elementary teachers of learning mathematics and means of teaching mathematics through the actual manipulation of material. Journal for Research in Mathematics Education, 6, 51-63.

Fuys, D. J. & Tischer, R. W. (1979). Teaching mathematics in the elementary school. Boston: Little, Brown.

Gagné, R. M. (1977). The conditions of learning (3rd ed.). New York: Holt, Rinehart, & Winston.

Ginsburg, H., & Opper, S. (1979). Piaget's theory of intellectual development (2nd ed.). Englewood Cliffs, NJ: Prentice-Hall.

Glaser, R. (1976). Components of a psychology of instruction: Toward a science of design. Review of Educational Research, 46, 1-24.

Good, T. L. & Grouws, D. A. (1979). Teaching and mathematics learning. Educational Leadership, 37(1), 39-45.

Goodnight, J. H. (1982). Plot [Computer program]. Cary, NC: Statistical Analysis System.

Goodnight, J. H., Stall, J. P., & Sarle, W. S. (1982). General linear models [Computer program]. Cary, NC: Statistical Analysis System.

Guide to the use of the descriptive test of mathematics skills. (1979). Princeton, NJ: Educational Testing Service.

Hair, J. F. Jr., Anderson, R. E., Tatham, R. L., & Grablowsky, B. J. (1979). Multivariate data analysis with readings. Tulsa: Petroleum Publishing Company.

Harris, C. H. (1979). A study of the effects of using manipulatives in teaching a unit on ratio and proportion to freshment in a compensatory course. Dissertation Abstract International, 40, 1932A-1933A. (University Microfilm No. 7923595)

Houston, C. (1977). The improving of academic performance using instructional objectives. MATYC Journal, 11(2), 23-31.

Kenny, R. A. (1965). Mathematical understanding of elementary school teachers. Arithmetic Teacher, 12, 431-442.

Kulm, G. (1977). Laboratory and lecture approaches to mathematics instruction for concrete and formal college students. Paper presented at the annual meeting of the American Educational Research Association, New York. (ERIC Document Reproduction Service No. ED 139 657)

Mager, R. F. (1962). Preparing instructional objectives. Palo Alto, CA: Fearon Publishers.

McCoy, K. M. & Hassett, M. S. (1980). Improving mastery learning courses through post-quiz drill. The AMATYC Review, 2(1), 22-24.

McFadden, S., Anderson, K., & Schaff, O. (1975). Mathlab-junior high. Eugene, OR: Action Math Associates.

McKeague, C. P. (1981). Introductory mathematics. Belmont, CA: Wadsworth Publishing Company.

Moore, C. (1974). A study of the effectiveness of the developmental mathematics at Danville Community College. Danville, VA: Danville Community College. (ERIC Document Reproduction Service No. ED 097 049)

Muha, J. C. (1974). A study comparing the traditional approach versus an experimental approach to teaching remedial math in the community college. A practicum presented to Nova University. (ERIC Document Reproduction Service No. ED 104 488)

The National Commission on Excellence in Education. (1983, May 4). A nation at risk: The imperative for educational reform. The Chronicle of Higher Education, 26(10), 11-16.

Norusis, M. J. (1982). Basic statistics and operations: SPSS introductory guide. New York: McGraw Hill.

Nowlan, H. C. (1978). Successful completion of a developmental mathematics course at an open door institution as a criterion for success in a college level mathematics course (Doctoral dissertation, University of Mississippi, 1978) Dissertation Abstracts International, 39, 3432A-3433A.

Parete, Jr. J. D. (1979). Formal reasoning abilities of college age students: An investigation of the concrete and formal reasoning stages formulated by Jean Piaget. Dissertation Abstracts International, 39, 6006A. (University Microfilms No. 7908195)

Piaget, J. (1972). Intellectual evolution from adolescence to adulthood. Human Development, 15, 1-12.

Plymale, S. H., & Jarrell, B. J. (1982). A comparison of community college and college education sophomores using Piaget's cognitive development model. Community College Review, 9(3), 19-21.

Poage, M. L. (1973). The use of a teacher-directed laboratory to individualize instruction for remedial mathematics in college courses. Dissertation Abstracts International, 33, 4692-A. (University Microfilms No. 73-5467)

Ray, A. A. (Ed.). (1982). SAS users guide: Statistics. Cary, NC: SAS Institute Inc.

Reese, R. L. (1977). A comparative study of the lecture method of instruction with the lecture method used in conjunction with mastery learning in teaching intermediate algebra at a Florida junior college (Doctoral dissertation, University of Mississippi, 1976). Dissertations Abstracts International, 37, 4904A.

Renner, J. W., Stafford, D. G., Lawson, A. E., McKinnon, J. W., Friot, F. E., & Kellog, D. H. (1976). Research, teaching, and learning with the Piaget model. Norman: University of Oklahoma Press.

Resnick, L. B., & Ford, W. W. (1981). The psychology of mathematics for instruction. Hillsdale, NJ: Lawrence Erlbaum Associates.

Robicheaux, R. T. Sr. (1981). Developmental mathematics course performance of college freshmen as related to Piagetian cognitive functioning level. Dissertation Abstracts International,43, 388-A. (University Microfilms No. 82-11, 203)

SAT scores hold steady for first time in 14 years. (1982, March-April). Focus: The Newsletter of the Mathematical Association of America, p. 2.

Schwartz, H. I. (1980). A mastery learning study in remedial mathematics in an urban community college (Doctoral dissertation, Columbia University, 1980). Dissertation Abstracts International, 41, 577A-578A.

Skinner, B. F. (1958). Teaching machines. Science, 128, 969-977.

Slade, A. R. (1975). The comparative effectiveness of four techniques of instruction of arithmetic in a community college (Doctoral dissertation, George Peabody College for Teachers, 1975). Dissertation Abstracts International, 36, 2078A-2079A.

Smith, J. L. (1982). A study of the relationships of sequences of enrollment in college remedial mathematics to grades in a subsequent college algebra course and to persistence in college (Doctoral dissertation, University of Alabama, 1982). Dissertation Abstracts International, 43, 1457-A.

Southern Regional Education Board. (1983). Remedial education in college: The problem of underprepared students. Issues in Higher Education, 20, 1-5.

Stein, S. K. (1973). Mathematics for the captured student. Two-Year College Mathematics Journal, 4(3), 62-71.

Spangler, R. C., & Stevens, L. P. (1979). Mathematics K-14. Community and Junior College Journal, 49(7), 28-30.

Survey reveals dramatic changes in collegiate mathematics. (1982, January-February). Focus: The Newsletter of the Mathematical Association of America, pp. 1-6.

Thompson, R. B. (1983, Winter). Competency unit testing with limited repetitions. Mathematics in College: A Journal of the Instructional Resource Center and the CUNY Mathematics Discussion Group, pp. 6-15.

Thorndike, E. L. (1913). Educational psychology (Vol. 2). New York: Teachers College, Columbia University.

Thorndike, E. L. (1922). _The psychology of arithmetic_. New York: Macmillan.

Warkentin, G. (1975). The effect of mathematics instruction using manipulative models on attitude and achievement of prospective teachers. _Journal for Research in Mathematics Education_, _6_, 88-94.

Weissglass, J. (1977). Mathematics for elementary teaching: A small-group laboratory approach. _American Mathematical Monthly_, _84_, 377-382.

Wepner, G. (1980). _Improving one characteristic of formal operations in adult remedial students through a Piagetian oriented instructional unit on percent_. Paper presented to the annual meeting of the Interdisciplinary International Conference, Los Angeles. (ERIC Document Reproduction Service No. ED 184 881)

Wepner, G. (1982). _Remedial/developmental mathematics program of the division of basic studies_. Unpublished manuscript, Ramapo College of New Jersey

Williams, P. D. (1980). Discovery learning: The differential effects of small-group work and individual work on mathematics achievement and attitudes of college students in remedial mathematics. _Dissertation Abstract International_, _41_, 578A. (University Microfilms No. 8018332)

Williams, R. C. (1973). The mathematics laboratory and the single student. _The Two-Year College Mathematics Journal_, _4_(1), 40-47.

APPENDIX A

COURSE SYLLABI

SYLLABUS FOR MAT 101-01

TITLE: Arithmetic

COURSE DESCRIPTION: A remedial course designed for students who need to develop basic arithmetic skills. Topics to be covered include: operations on whole numbers, fractions, decimals, ratio and proportion, percent, and measurement.

CREDIT: 3-4-5

TARGET GROUP: Math 101 is designed for college students who need help with basic arithmetic skills.

PAY-OFF: Math 101 will do the following:

(1) Count toward the math requirement for the A.A.S. degree
(2) Count as an elective toward the A.G.E. degree
(3) Prepare students for higher math courses.

TEXTBOOK: Introductory Mathematics by Charles R. McKeague

TIME: Successful students report spending 1-2 hours each night reviewing their notes and doing the assignments, 2-4 hours studying for each unit test, and 4-6 hours studying for the final.

CLASS ATTENDANCE: Students are expected to attend class every class period; however, it is understood that they may be forced to miss because of illness, death in their families, or similar emergencies. When students must miss class, they should contact their instructor to explain the situation and get assignments. Any student who is absent five consecutive days without contacting the instructor will be considered as having abandoned the course and will be dropped. If students need to withdraw, they should tell their instructor at once. Students who abandon the course will get a grade of WF.

Each student will have an attendance, classwork grade that will count as a unit test grade. The student will be awarded two points for each day that he or she is present, on time, and participates in class activities. Students tardy by no more than 15 minutes will be awarded one point. The attendance grade will be the ratio of points earned to possible points.

MATH LAB: Four hours of lab time are required per week--two are scheduled within the class hours and two are open lab hours which will be scheduled at the first class meeting. The math lab is designed to be a place where the student can come for quiet study, individual tutoring, group work, taking tests, reviewing tests, and doing lab assignments. Study carrels are available for quiet study and tables are available for group work. Students needing individual tutoring should report to the lab instructor and students needing tests should report to their class instructor. Materials for the week's lab assignment will be on the activities table. The lab assignments will relate to the topics discussed in class and will give the students concrete, manipulative activities to perform. The lab assignment, which will be collected and graded each Friday, will count as a unit test grade.

COURSE CONTENT: Math 101 consists of the following units: whole numbers, fractions, decimals, percents and proportions, and measurement. See the unit study guides for a detailed listing of objectives and learning activities.

MATERIALS: Each student should have the textbook and a loose-leaf notebook for their assignments. The instructor will provide each student with study guides which include objectives, assignments, and suggested learning activities. Lab assignment sheets will be distributed weekly.

CLASS INSTRUCTION: All objectives will be covered through lecture and most objectives will be covered through backup audio tapes and filmstrips or frames (see the unit study guides). Typically the instructor will lecture approximately 30 minutes each class period. During the lecture the student should take notes and make sure that he or she understands the concepts covered. During the remaining 20 minutes the students will work in small groups of 4-5 on activities reinforcing the concepts covered in lecture. The instructor will move from group to group answering questions and giving hints. After each class meeting, students are expected to complete the assignments for the objectives covered during class. If any student is unable to complete the assignments, he or she should attend the math lab and get individual tutoring or do the backup activities. Students who miss class are required to do the backup activities.

ASSIGNMENTS: Students should complete all assignments on loose-leaf notepaper. The assignments, properly labeled and in correct order, must be turned in before students take their tests. The instructor will randomly choose and grade problems from the assignments for each topic. The final assignment average will count as a unit test grade.

<u>COURSE COMPLETION</u>: In order to complete the course, the student must score 80% or better on each unit test and score 70% or better on the final exam.

<u>TESTING</u>: The following statements cover the testing policy:

1. Students are required to take unit tests during the designated class period. The tests will be returned and reviewed the following day.

2. If a student scores below 80%, then he or she must retake the test. Retakes, which may be taken during the instructors office or lab hours, should be taken on the day after the test was returned and reviewed. In any case the retake must be taken within one week of the original test.

3. If a student scores below 80% on the retest, then he or she must immediately make an appointment to develop a comprehensive study plan that will allow the student to learn the old material, make up the test, and keep up with the new material. The study plan will typically involve an additional hour or more per day in the math lab with most of the time spent in individual tutoring. Failure to make and comply with the study plan will result in the student being withdrawn from class with a grade of WF.

4. For grading purposes each retake will carry a 5 point penalty. For example, a score of 80% on the second retake would earn the student credit for the unit; however, a 70 would be recorded as the unit grade.

5. All tests and make-ups are 50-minute tests. Students may not leave the testing station from the time they start the test until they complete it or the time expires. While taking a test the student should have two sharpened pencils. The student should not have notes, books, calculators, or extra paper. The test paper and all worksheets, including scratch work, are to be turned in.

6. A student may take the final exam two times. Students who fail to reach the minimum score of 70 will not receive credit for the course.

GRADING: For grading purposes each unit test will count as one score, the assignment average will count as one score, the lab grade will count as one score, the attendance grade will count as one score, and the final exam will count as two scores. The ten scores will be averaged to determine the students final numerical average. The following symbols will be used.

A-The student completed the course with an average of 90 or better.
B-The student completed the course with an average from 80 to 89.
C-The student completed the course with an average from 70 to 79.
D-The student completed the course with an average below 70.
S-The student completed the course on the S-U option.
F-The student did not complete the course.
U-The student took but did not complete the course on the S-U option.
W-The student withdrew within the first four weeks of the quarter.
WP-The student withdrew after 4 weeks; at the time of withdrawal the student had passed all unit tests given to date.
WF-The student withdrew after 4 weeks; at the time of withdrawal the student had not passed all unit tests given to date.

SYLLABUS FOR MAT 101-02

TITLE: Arithmetic

COURSE DESCRIPTION: A remedial course designed for students who need to develop basic arithmetic skills. Topics to be covered include: operations on whole numbers, fractions, decimals, ratio and proportion, percent, and measurement.

CREDIT: 3-4-5

TARGET GROUP: Math 101 is designed for college students who need help with basic arithmetic skills.

PAY-OFF: Math 101 will do the following:

(1) Count toward the math requirement for the A.A.S. degree
(2) Count as an elective toward the A.G.E. degree
(3) Prepare students for higher math courses.

TEXTBOOK: Introductory Mathematics by Charles R. McKeague

TIME: Successful students report spending 1-2 hours each night reviewing their notes and doing the assignments, 2-4 hours studying for each unit test, and 4-6 hours studying for the final.

CLASS ATTENDANCE: Students are expected to attend class every class period; however, it is understood that they may be forced to miss because of illness, death in their families, or similar emergencies. When students must miss class, they should contact their instructor to explain the situation and get assignments. Any student who is absent five consecutive days without contacting the instructor will be considered as having abandoned the course and will be dropped. If students need to withdraw, they should tell their instructor at once. Students who abandon the course will get a grade of WF.

Each student will have an attendance, classwork grade that will count as a unit test grade. The student will be awarded two points for each day that he or she is present, on time, and participates in class activities. Students tardy by no more than 15 minutes will be awarded one point. The attendance grade will be the ratio of points earned to possible points.

MATH LAB: Four hours of lab time are required per week--two are scheduled within the class hours and two are open lab hours which will be scheduled at the first class meeting. The math lab is designed to be a place where the student can come for quiet study, individual tutoring, group work, completing the backup exercises and doing lab assignments. Study carrels are available for quiet study and tables are available for group work. Students needing individual tutoring should report to the lab instructor. Materials for the week's lab assignment will be on the activities table. The lab assignments will relate to the topics discussed in class and will give the students concrete, manipulative activities to perform. The lab assignment, which will be collected and graded each Friday, will count as a unit test grade.

COURSE CONTENT: Math 101 consists of the following units: whole numbers, fractions, decimals, percents and proportions, and measurement. See the unit study guides for a detailed listing of objectives and learning activities.

MATERIALS: Each student should have the textbook and a loose-leaf notebook for their assignments. The instructor will provide each student with study guides which include objectives, assignments, and suggested learning activities. Lab assignment sheets will be distributed weekly.

CLASS INSTRUCTION: All objectives will be covered through lecture and most objectives will be covered through backup audio tapes and filmstrips or frames (see the unit study guides). Typically the instructor will lecture approximately 30 minutes each class period. During the lecture the student should take notes and make sure that he or she understands the concepts covered. During the remaining 20 minutes the students will work in small groups of 4-5 on activities reinforcing the concepts covered in lecture. The instructor will move from group to group answering questions and giving hints. After each class meeting, students are expected to complete the assignments for the objectives covered during class. If any student is unable to complete the assignments, he or she should attend the math lab and get individual tutoring or do the backup activities. Students who miss class are required to do the backup activities.

ASSIGNMENTS: Students should complete all assignments on loose-leaf notepaper. The assignments, properly labeled and in correct order, must be turned in before students take their tests. The instructor will randomly choose and grade problems from the assignments for each topic. The final assignment average will count as a unit test grade.

COURSE COMPLETION: In order to complete the course, the student must have a final average of 60% or better and score 70% or better on the final exam.

TESTING: The following statements cover the testing policy:

1. Students are required to take unit tests during the designated class period. The tests will be returned and reviewed the following day.

2. All tests are 50-minute tests. Students may not leave the testing station from the time they start the test until they complete it or the time expires. While taking a test, the student should have two sharpened pencils. The student should not have notes, books, calculators, or extra paper. The test paper and all worksheets, including scratch work, are to be turned in.

3. A student may take the final exam two times. Students who fail to reach the minimum score of 70 will not receive credit for the course.

GRADING: For grading purposes each unit test will count as one score, the assignment average will count as one score, the lab grade will count as one score, the attendance grade will count as one score, and the final exam will count as two scores. The ten scores will be averaged to determine the students final numerical average. The following symbols will be used.

A-The student completed the course with an average of 90 or better.
B-The student completed the course with an average from 80 to 89.
C-The student completed the course with an average from 70 to 79.
D-The student completed the course with an average from 60 to 69.
S-The student completed the course on the S-U option.
F-The student did not complete the course.
U-The student took but did not complete the course on the S-U option.
W-The student withdrew within the first four weeks of the quarter.
WP-The student withdrew after 4 weeks; at the time of withdrawal the student had a unit test average of 70% or better.
WF-The student withdrew after 4 weeks; at the time of withdrawal the student had a unit test average below 70%.

SYLLABUS FOR MAT 101-03

TITLE: Arithmetic

COURSE DESCRIPTION: A remedial course designed for students who need to develop basic arithmetic skills. Topics to be covered include: operations on whole numbers, fractions, decimals, ratio and proportion, percent, and measurement.

CREDIT: 3-4-5

TARGET GROUP: Math 101 is designed for college students who need help with basic arithmetic skills.

PAY-OFF: Math 101 will do the following:

(1) Count toward the math requirement for the A.A.S. degree
(2) Count as an elective toward the A.G.E. degree
(3) Prepare students for higher math courses.

TEXTBOOK: Introductory Mathematics by Charles R. McKeague

TIME: Successful students report spending 1-2 hours each night reviewing their notes and doing the assignments, 2-4 hours studying for each unit test, and 4-6 hours studying for the final.

CLASS ATTENDANCE: Students are expected to attend class every class period; however, it is understood that they may be forced to miss because of illness, death in their families, or similar emergencies. When students must miss class, they should contact their instructor to explain the situation and get assignments. Any student who is absent five consecutive days without contacting the instructor will be considered as having abandoned the course and will be dropped. If students need to withdraw, they should tell their instructor at once. Students who abandon the course will get a grade of WF.

Each student will have an attendance, classwork grade that will count as a unit test grade. The student will be awarded two points for each day that he or she is present, on time, and participates in class activities. Students tardy by no more than 15 minutes will be awarded one point. The attendance grade will be the ratio of points earned to possible points.

MATH LAB: Four hours of lab time are required per week--two are scheduled within the class hours and two are open lab hours which will be scheduled at the first class meeting. The math lab is designed to be a place where the student can come for quiet study, individual tutoring, group work, taking tests, and reviewing tests. Study carrels are available for quiet study and tables are available for group work. Students needing individual tutoring should report to the lab instructor, and students needing tests should report to their class instructor.

COURSE CONTENT: Math 101 consists of the following units: whole numbers, fractions, decimals, percents and proportions, and measurement. See the unit study guides for a detailed listing of objectives and learning activities.

MATERIALS: Each student should have the textbook and a loose-leaf notebook for their assignments. The instructor will provide each student with study guides which include the objectives, assignments, and suggested learning activities.

CLASS INSTRUCTION: All objectives will be covered through lecture and most objectives will be covered through backup audio tapes and filmstrips or frames (see the unit study guides). Typically the instructor will lecture approximately 30 minutes each class period. During the lecture the student should take notes and make sure that he or she understands the concepts covered. During the remaining 20 minutes the students will work in small groups of 4-5 on activities reinforcing the concepts covered in lecture. The instructor will move from group to group answering questions and giving hints. After each class meeting, students are expected to complete the assignments for the objectives covered during class. If any student is unable to complete the assignments, he or she should attend the math lab and get individual tutoring or do the backup activities. Students who miss class are required to do the backup activities.

ASSIGNMENTS: Students should complete all assignments on loose-leaf notepaper. The assignments, properly labeled and in correct order, must be turned in before students take their tests. The instructor will randomly choose and grade problems from the assignments for each topic. The final assignment average will count as a unit test grade.

COURSE COMPLETION: In order to complete the course, the student must score 80% or better on each unit test and score 70% or better on the final exam.

TESTING: The following statements cover the testing policy:

1. Students are required to take unit tests during the designated class period. The tests will be returned and reviewed the following day.

2. If a student scores below 80%, then he or she must retake the test. Retakes, which may be taken during the instructors office or lab hours, should be taken on the day after the test was returned and reviewed. In any case the retake must be taken within one week of the original test.

3. If a student scores below 80% on the retest, then he or she must immediately make an appointment to develop a comprehensive study plan that will allow the student to learn the old material, make up the test, and keep up with the new material. The study plan will typically involve an additional hour or more per day in the math lab with most of the time spent in individual tutoring. Failure to make and comply with the study plan will result in the student being withdrawn from class with a grade of WF.

4. For grading purposes each retake will carry a 5 point penalty. For example, a score of 80% on the second retake would earn the student credit for the unit; however, a 70 would be recorded as the unit grade.

5. All tests and make-ups are 50-minute tests. Students may not leave the testing station from the time they start the test until they complete it or the time expires. While taking a test the student should have two sharpened pencils. The student should not have notes, books, calculators, or extra paper. The test paper and all worksheets, including scratch work, are to be turned in.

6. A student may take the final exam two times. Students who fail to reach the minimum score of 70 will not receive credit for the course.

<u>GRADING</u>: For grading purposes each unit test will count as one score, the assignment average will count as two scores, the attendance grade will count as one score, and the final exam will count as two scores. The ten scores will be averaged to determine the students final numerical average. The following symbols will be used.

A-The student completed the course with an average of 90 or better.
B-The student completed the course with an average from 80 to 89.
C-The student completed the course with an average from 70 to 79.
D-The student completed the course with an average below 70.
S-The student completed the course on the S-U option.
F-The student did not complete the course.
U-The student took but did not complete the course on the S-U option.
W-The student withdrew within the first four weeks of the quarter.
WP-The student withdrew after 4 weeks; at the time of withdrawal the student had passed all unit tests given to date.
WF-The student withdrew after 4 weeks; at the time of withdrawal the student had not passed all unit tests given to date.

SYLLABUS FOR MAT 101-04

TITLE: Arithmetic

COURSE DESCRIPTION: A remedial course designed for students who need to develop basic arithmetic skills. Topics to be covered include: operations on whole numbers, fractions, decimals, ratio and proportion, percent, and measurement.

CREDIT: 3-4-5

TARGET GROUP: Math 101 is designed for college students who need help with basic arithmetic skills.

PAY-OFF: Math 101 will do the following:

(1) Count toward the math requirement for the A.A.S. degree
(2) Count as an elective toward the A.G.E. degree
(3) Prepare students for higher math courses.

TEXTBOOK: Introductory Mathematics by Charles R. McKeague

TIME: Successful students report spending 1-2 hours each night reviewing their notes and doing the assignments, 2-4 hours studying for each unit test, and 4-6 hours studying for the final.

CLASS ATTENDANCE: Students are expected to attend class every class period; however, it is understood that they may be forced to miss because of illness, death in their families, or similar emergencies. When students must miss class, they should contact their instructor to explain the situation and get assignments. Any student who is absent five consecutive days without contacting the instructor will be considered as having abandoned the course and will be dropped. If students need to withdraw, they should tell their instructor at once. Students who abandon the course will get a grade of WF.

Each student will have an attendance, classwork grade that will count as a unit test grade. The student will be awarded two points for each day that he or she is present, on time, and participates in class activities. Students tardy by no more than 15 minutes will be awarded one point. The attendance grade will be the ratio of points earned to possible points.

MATH LAB: Four hours of lab time are required per week--two are scheduled within the class hours and two are open lab hours which will be scheduled at the first class meeting. The math lab is designed to be a place where the student can come for quiet study, individual tutoring, group work, and completing backup exercises. Study carrels are available for quiet study and tables are available for group work. Students needing individual tutoring should report to the lab instructor.

COURSE CONTENT: Math 101 consists of the following units: whole numbers, fractions, decimals, percents and proportions, and measurement. See the unit study guides for a detailed listing of objectives and learning activities.

MATERIALS: Each student should have the textbook and a loose-leaf notebook for their assignments. The instructor will provide each student with study guides which include objectives, assignments, and suggested learning activities.

CLASS INSTRUCTION: All objectives will be covered through lecture and most objectives will be covered through backup audio tapes and filmstrips or frames (see the unit study guides). Typically the instructor will lecture approximately 30 minutes each class period. During the lecture the student should take notes and make sure that he or she understands the concepts covered. During the remaining 20 minutes the students will work in small groups of 4-5 on activities reinforcing the concepts covered in lecture. The instructor will move from group to group answering questions and giving hints. After each class meeting, students are expected to complete the assignments for the objectives covered during class. If any student is unable to complete the assignments, he or she should attend the math lab and get individual tutoring or do the backup activities. Students who miss class are required to do the backup activities.

ASSIGNMENTS: Students should complete all assignments on loose-leaf notepaper. The assignments, properly labeled and in correct order, must be turned in before students take their tests. The instructor will randomly choose and grade problems from the assignments for each topic. The final assignment average will count as a unit test grade.

COURSE COMPLETION: In order to complete the course, the student must have a final average of 60% or better and score 70% or better on the final exam.

<u>TESTING</u>: The following statements cover the testing policy:

1. Students are required to take unit tests during the designated class period. The tests will be returned and reviewed the following day.

2. All tests are 50-minute tests. Students may not leave the testing station from the time they start the test until they complete it or the time expires. While taking a test, the student should have two sharpened pencils. The student should not have notes, books, calculators, or extra paper. The test paper and all worksheets, including scratch work, are to be turned in.

3. A student may take the final exam two times. Students who fail to reach the minimum score of 70 will not receive credit for the course.

<u>GRADING</u>: For grading purposes each unit test will count as one score, the assignment average will count as two scores, the attendance grade will count as one score, and the final exam will count as two scores. The ten scores will be averaged to determine the students final numerical average. The following symbols will be used.

A-The student completed the course with an average of 90 or better.
B-The student completed the course with an average from 80 to 89.
C-The student completed the course with an average from 70 to 79.
D-The student completed the course with an average from 60 to 69.
S-The student completed the course on the S-U option.
F-The student did not complete the course.
U-The student took but did not complete the course on the S-U option.
W-The student withdrew within the first four weeks of the quarter.
WP-The student withdrew after 4 weeks; at the time of withdrawal the student had a unit test average of 70% or better.
WF-The student withdrew after 4 weeks; at the time of withdrawal the student had a unit test average below 70%.

APPENDIX B

SAMPLE STUDY GUIDES

STUDY GUIDE
WHOLE NUMBERS

OBJECTIVE 1

Give the place value for specified digits in a given whole number, write whole numbers in expanded notation, write the word name for numerals given in digit form, and give the digit form for numerals written in words.

Examples:
a) Give the place value of the 7 in 97 281.
b) Write 102 321 in expanded notation.
c) Write 6 998 454 in words.
d) Write four billion, twenty thousand, four hundred thirty-two with digits instead of words.

Reference:
Textbook, section 1.1, pages 1-6.

Assignments:
Set 1.1, problems 5, 11, 13, 15, 17, 19, 21, 29, 33, 37, 39, 41, 43, 45, 47, 51, 53, 55, 57, 59, 61, 63, 65.

Backup:
Competency Skills in Arithmetic, Module 1, Frames 1-10. Do all of the problems given in the frames and do problem 1 on the Module 1 Practice Sheet.

OBJECTIVE 2

Place given whole numbers on the number line and give the correct order relation (<, >, =) betweem two whole numbers.

Examples:
a) Place 7 on the given number line

0 10

b) Give the correct order relation between 19 and 27.

Reference:
The set of whole numbers denoted by W is defined as $W = \{0, 1, 2, 3, 4, \ldots\}$. The three dots indicate that the whole numbers continue infinitely in the pattern established. The whole numbers can be placed on a number line.

0 1 2 3 4 5 6 7 8 9 10 ...

Definition:
If a is to the left of b on the number line, then a is less than b ($a < b$).

Examples:
a) 10 is less than 12
b) 2 < 7

Definition:
If a is to the right of b on the number line, then a is greater than b (a > b).

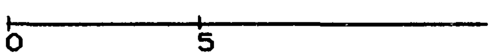
Examples:
a) 12 is greater than 10
b) 7 > 2

Note:
When using the symbol, the arrow always points to the smaller number. The symbol "=" is read "is equal to "

Example:
a) 6 = 6

Assignment:
1. Copy the number line below and place the numbers on the line.

a) 8	c) 1
b) 6	d) 3



2. Give the correct relation using words (less than, greater than, or equal to).

a) 20; 33	d) Nine thousand and two; 9,002
b) 627; 470	e) Three hundred thirty three; 320
c) 29; 64	f) 10,000; 9,990

3. Give the correct relation using symbols (>, <, =)
a) 3; 0
b) 602; 700
c) 1,020,000; 1,019,842
d) 987; 234
e) 3,000,197; 4,000,000
f) 1981; 2001

Answers:
2. a) 20 is less than 33
   b) 627 is greater than 470
   c) 29 is less than 64
   d) Nine thousand and two is equal to 9,002
   e) Three hundred thirty three is greater than 320
   f) 10,000 is greater than 9,990
3. a) 3 > 0
   b) 602 < 700

c) 1,020,000 > 1,019,842
d) 987 > 234
e) 3,000,197 < 4,000,000
f) 1981 < 2001

Backup:
Objective 2 has no backup. If you do not understand the objective, see the lab instructor for tutoring and additional exercises.

OBJECTIVE 3

Round given whole numbers to any specified position.

Example:
Round 1267 to the nearest one hundred.

Reference:
Testbook, section 2.2, pp.43-45.

Assignment:
Set 2.2, problems 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 37, 50, 51, 53.

Backup;
Math House Proficiency Review Tapes, Unit B, Tape 12.
Do worksheet 12A, Sections 1, 2, 3.

OBJECTIVE 4

Given an addition problem in symbols or words, solve the problem and identify the addends and the sum.

Examples:
a) 98 + 2 146 + 981 = ___________.
b) Bill had 41 strokes on the front nine and 44 strokes on the back nine. What was Bill's score for 18 holes?
c) Given 9 + 5 = 14, the addends are ____ and ____.
The sum is ____.

Reference:
Textbook, section 1.2, pp.6-12, section 2.1, pp. 35-42.

Assignment:
Set 1.2, problems 1, 3, 5, 7, 9, 11, 17, 19, 65, 67, 69, 71, 73, 75, 77, 81, 83, 85, 87; set 2.1, problems 11, 17, 21, 23, 25, 27, 35, 39, 41, 43, 45, 47, 55, 57, 62.

Backup:
Competency Skills in Arithmetic, Module 1, Frames 10-30.
Do all of the exercises in the frames and do problem 2 on the Module 1 Practice Sheet.

OBJECTIVE 5

Given a subtraction problem in symbols or works, solve the problem; identify the minuend, subtrahend, and the difference; and check the difference by addition

Examples:
a) Solve and check: 7801 - 4929 = ________.
b) Given 29 - 14 = 15; the minuend is ________, the subtrahend is _________, and the difference is ______.
To check the problem one should add ________ and ________ . to get ________.
c) In 1981 RCC had 1594 students and in 1982 RCC had 1704 students. Determine the amount of increase.

Reference:
Textbook, section 1.3, pp. 13-17; section 2.3, pp. 46-51.

Assignment:
Set 1.3, problems 1, 5, 9, 13, 15, 17, 19, 21, 25, 29, 31, 35, 39, 43, 47, 51, 60, 64, 68, 83, 87,; Set 2.3, problems 1, 5, 9, 13, 17, 19, 21, 23, 25, 27, 31, 33, 35, 39, 43, 45, 46, 47, 59.

Backup:
Competency Skills in Arithmetic, Module 1, Frames 31-51. Do all of the exercises in the frames and do problem 3 on the Module 1 Practice Sheet.

OBJECTIVE 6

Given a multiplication problem in symbols or words, solve the problem and identify the factors and the product.

Examples:

a) (2 841)(189) = ________.
b) Given (41)(20)= 820, _____ and _____ are called factors and ______ is the product.
c) A car can travel 22 miles on 1 gallon of gas. At the same rate, how far can the car travel on 95 gallons of gas?

Reference:
Teatbook, aection 1.4, pp.17-22; aection 2.5, pp. 56-61.

Assignment:
Set 1.4, problems 1, 2, 3, 4, 5, 7, 9, 11, 13, 14, 15, 17, 19, 21, 25, 27, 67, 69, 73,; Set 2.5, problems 11, 15, 19, 23, 27, 31, 35, 39, 41, 44, 45, 47, 48.

Backup:
Competency Skills in Arithmetic, Module 2, Frames 1-40. Do all of the exercises in the frames and do problems 1, 2, and 3 on the Module 2 Practice Sheet.

OBJECTIVE 7

Given a division problem in symbols or words, solve the problem; identify the dividend, divisor, and quotient; and check the quotient by multiplication.

Example:
a) 2 844 ÷ 43 = _____.
b) Given 27/9 = 3, _____ is the dividend, _____ is the divisor, and _____ is the quotient.
c) Given $41\overline{)82}$ with quotient 2, _____ is the dividend, _____ is the divisor, and _____ is the quotient.
d) How many 15 feet pieces of string can one cut from a ball of string 5000 feet long?

Reference:
Textbook, section 1.5, pp. 23-27; section 2.6, pp.61-68.

Assignment:
Set 1.5, problems 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 23, 27, 29, 31, 33, 35, 39, 47, 51, 59, 63, 64, 65, 67, 79, 80, 81; Set 2.6, problems 1, 5, 9, 13, 17, 19, 21, 24, 25, 31, 33, 35, 37, 40, 42.

Backup:
Competency Skills in Arithmetic, Module 3, Frames 1-57. Do all of the exercises in the frames and do problems 1, 2, 3, and 4 on the Module 3 Practice Sheet.

OBJECTIVE 8

Evaluate powers and identify the base and the exponent.

Examples:
a) Evaluate $(2)^3$
b) In 2^4, ______ is the exponent and ______ is the base.

Reference:
Textbook, section 1.6, pp. 28-31; section 2.4, pp. 51-55.

Assignment:
Set 1.6, problems 1, 5, 9, 13, 17, 21, 25, 29, 33, 36, 42, 45, 47, 50, 51, 52, 53, 54, 56; Set 2.4, problems 1, 5, 9, 13, 17, 19, 21, 25, 29, 39, 41, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82.

Backup:
Math House Proficiency Review Tapes. Unit A, Tape 1.
Do worksheets 1A and 1B, and do practice sheet 1C.

OBJECTIVE 9

State the rule for the order of operations for evaluating whole number expression, evaluate whole number expressions given in words or symbols, and solve word problems requiring the use of two or more operations.

Examples:
a) Evaluate 2(3) + 4[18 - 5(7 - 4)]
b) Evaluate 3 times the difference of 6 and 1.
c) Jim earns $948 a month in take home pay. Jim pays $180 rent, a $140 car payment, and a $100 payment on his charge card bill. How much will Jim have left?

Reference:
Textbook, Section 2.7, pp. 68-73.

Assignment:
Set 2.7, problems 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 25, 27, 29, 31, 33, 37, 39, 41, 43, 44, 51, 53, 55, 57.

Backup:
Video tape series to accompany Elementary Algebra, Chapter 1, section 1. Do problems 31-50 on page 5 of the book Elementary Algebra.

OBJECTIVE 10

Evaluate algebraic expressions for given whole number values for the variables.

Example:
Evaluate 2a - 3b for a = 10 and b = 3.

Reference:
Algebraic expressions have symbols (letters) which may stand for whole numbers. In any given problem a symbol can stand for only one whole number; however, the same symbol may have a different value in the next problem. To evaluate algebraic expressions, replace the symbols by their values and follow the order of operations given in Objective 9.

Example 1:
Evaluate $2l + 2w$ for $l = 18$ and $w = 15$

$$
\begin{aligned}
2l + 2w &= 2(18) + 2(15) \\
&= 36 + 30 \\
&= 66
\end{aligned}
$$

Example 2:
Evaluate $d = rt$ for $r = 55$ and $t = 3$

$$
\begin{aligned}
d &= rt \\
&= 55(3) \\
d &= 165
\end{aligned}
$$

Example 3:
Evaluate $b^2 - 4ac$ for $b = 8$, $a = 4$, and $c = 1$

$$
\begin{aligned}
b^2 - 4ac &= 8^2 - 4(4)(1) \\
&= 64 - 16 \\
&= 48
\end{aligned}
$$

Assignment:

Evaluate the following:

1. $2a + 3b$ for $a = 5$, $b = 7$
2. $2l + 2w$ for $l = 31$, $w = 15$
3. lw for $l = 9$, $w = 2$
4. a^2 for $a = 9$, $b = 4$
5. a^2b for $a = 5$, $b = 2$
6. ab^2 for $a = 5$, $b = 2$
7. $l - a(b - 2)$ for $l = 1$, $a = 50$, $b = 2$
8. $\dfrac{m(x - y)}{4}$ for $m = 12$, $x = 15$, $y = 10$
9. $4 + 3a^2 - a^3$ for $a = 2$
10. $\dfrac{c - d}{11}$ for $c = d = 5$

Answers:

1.	31	6.	20
2.	92	7.	1
3.	18	8.	15
4.	81	9.	8
5.	50	10.	0

Backup:
Objective 10 has no backup. If you do not understand the objective, see the lab instructor for tutoring and additional exercises.

STUDY GUIDE
DECIMALS

OBJECTIVE 1

Give the place value for specified digits in a given decimal, write the word name for a decimal given in digit form, and give the digit form for decimals written in words.

Examples:
a) Give the place value for the 8 in 27.1083.
b) Write 2.361 in words.
c) Write four and fifty-two hundredths in digit form.

Reference:
Textbook, section 5.1, pp. 149-151.

Assignment:
Set 5.1, problems 1,3, 7, 9, 11, 15, 17, 19, 26, 27, 29, 31, 33, 35.

Backup:
Math House Proficiency Review Tapes; Tape 11. Do worksheets 11A and 11B, and do practice sheet 11C.

OBJECTIVE 2

Approximate decimals to any given positions and give the correct order relationship between given pairs of decimals.

Examples:
a) Approximate 3.2781 to the nearest one hundredth.
b) Give the correct symbol (<, >, =) to describe the relationship between 7.238 and 7.24.

Reference:
Textbook, section 5.1, p.152.

Assignment:
Set 5.1, problems 37-46, 48, 49, 59, 63, 67.

Backup:
Math House Proficiency Review Tapes; Tape 12. Do worksheets 12A and 12B, and do practice sheet 12C.

OBJECTIVE 3

Add and subtract decimals.

Examples:
a) 7 + 8.23 + 0.005 = ______.
b) Determine the sum of 9.1 and 17.632.
c) Subtract 19.13 from 25.

Reference:
Textbook, section 5.2, pp. 154-156.

Assignment: Set 5.2, problems 1, 3, 5, 7, 9, 11, 15, 21, 23, 25, 29, 31, 33, 35, 37, 39, 42, 44, 45-50, 51, 55, 59.

Backup:
Math House Proficiency Review Tapes; Tapes 13 and 14. Do worksheets 13A, 13B, 14A, and 14B; and do practice sheets 13C and 14C.

OBJECTIVE 4

Multiply decimals.

Examples:
a) Multiply 4.71 by 3.62.
b) (4.1)(.0023) = ______.

Reference:
Textbook, section 5.3, pp. 158-160.

Assignment:
Set 5.3, problems 1, 3, 5, 7, 9, 11, 13, 15, 19, 21, 23, 25, 27, 49, 51, 53-58, 61, 67.

Backup:
Math House Proficiency Review Tapes; Tape 15. Do worksheets 15A and 15B, and do practice sheet 15C.

OBJECTIVE 5

Divide decimals and approximate the quotient to any given position.

Examples:
a) Divide 28.73 by 4.1 and round the answer to tenths.
b) 17.005 ÷ 4.32 = ______ to the nearest one thousandth.

Reference:
Testbook, section 5.4, pp. 162-166.

Assignment:
Set 5.4, problems 1, 7, 13, 17, 19, 25, 27, 29, 31, 33, 35, 39, 45, 51, 53, 55, 57, 59, 61.

Backup:
Math House Proficiency Review Tapes; Tape 16. Do worksheets 16A and 16B; and do practice sheet 16C, problems 1, 2, 3, 4, 7, 8, 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 24, 25.

OBJECTIVE 6

Convert terminating decimals to fractions and convert fractions to decimals correct to any given decimal position.

Examples:
a) Convert 0.125 to a fraction and reduce the fraction to lowest terms.
b) Convert 3/16 to a decimal.
c) Convert 4/11 to a decimal correct to the nearest one thousandth.

Reference:
Textbook, section 5.5, pp.167-170.

Assignment:
Set 5.5, problems 1, 3, 11, 13, 15, 17, 21, 23, 25, 29, 33, 35, 57, 59, 63, 64, 65, 69.

Backup:
Competency Skills in Arithmetic, Module 8, frames 1-9. Do all of the exercises on the frames and do problems 1 and 2 on the Module 8 practice sheet.

OBJECTIVE 7

Determine the square root of a perfect square, use a calculator to approximate the square root of a number which is not a perfect square, and evaluate expressions involving square roots.

Examples:
a) Approximate $\sqrt{55}$ to the nearest one hundredth.
b) $\sqrt{1296}$ = ______.
c) $15\sqrt{9} - 9\sqrt{16}$ = ______.

Reference:
Textbook, section 5.6, pp. 173-176.

Assignment:
Set 5.6, problems 1-8, 9, 11, 13, 17, 19, 23, 25, 27, 29-32, 33, 39, 41, 43, 45, 47, 49, 53, 55, 57, 59.

Backup:
Basic Arithmetic by Moon, Konrad, Klentos, and Newmyer; Unit 25, frames 1-9. Do study exercise 1, p. 264.

OBJECTIVE 8

Follow the order of operations to evaluate expressions involving decimals and expressions involving both decimals and fractions.

Examples:
a) Evaluate $(4.2)(30.1) - 91.4 \div 0.2$.
b) Evaluate $19/50\ (1.32 + 0.48)$.

Reference:
Textbook, section 5.3, p.160; section 5.5, p.171.

Assignment:
Set 5.3, problems 29, 33, 35, 39, 41, 43; set 5.5, problems 37, 39, 41, 43, 45, 47.

Backup:
There is no backup for this objective. If you need help or extra problems, see the lab instructor.

OBJECTIVE 9

Evaluate algebraic expressions involving decimals and fractions.

Examples:
a) Evaluate $3a - 4(b - a)$ for $a = 1.2$ and $b = 1.4$.
b) Evaluate $5/9(F - 32)$ for $F = 98.6$.

Reference:
Textbook, section 8.5, p. 277; section 10.1, pp. 318-319.

Assignment:
Set 10.1, problems 73, 74, 75; Set 8.5, problems 23, 25, 26, 46.

Backup:
This objective has no backup. See your lab instructor for help or additional problems.

OBJECTIVE 10

Solve word problems requiring the use of decimals and one or more of the basic operations.

Example:
A checking account had a beginning balance of $576.72. Checks were written for $57.06, $128.24, and $23.09. A deposit of $322 was made. What is the current balance?

Reference:
Textbook, sections 5.2, 5.3, and 5.4.

Assignment:
Chapter 5 Diagnostic Test, problems 35-40.

Backup:
<u>Competency Skills in Arithmetic</u>, Module 8, frames 10-51. Do all of the exercises on the frames and do problems 3, 4, 5, and 6 from the Module 8 practice sheet.

APPENDIX C

SAMPLE LABS

LAB 3

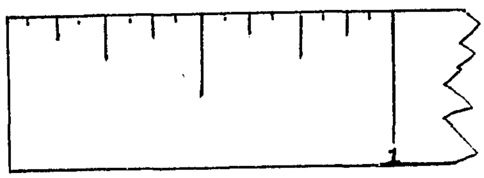
MATERIALS: Cuisenaire Rods

1. Let the orange rod be 1 unit.
   a) What color represents 1/2 of the orange rod? 1/10? 2/10? 7/10?
   b) Show 1/2 + 1/5 by placing the 1/2 rod and the 1/5 rod end to end. What color rod is the same length as 1/2 + 1/5?
      Therefore 1/2 + 1/5 = 7/10.
   c) Use rods to solve the following problems:
      1/2 +1/5
      1/5 + 7/10
      1/2 + 1/5 + 3/10
      3/5 + 1/10
      9/10 + 1/2
      3/5 - 1/2
      1/2 - 1/10
      1 - 3/5
2. Let the brown rod be 1 unit.
   a) What color represents 1/2 of the brown rod? 1/4? 1/8? 3/8? 1 1/4?
   b) Use the rods to solve the following problems:
      5/8 + 1/4
      1/4 + 3/8
      1/2 + 7/8
      1/2 + 1/8 + 3/4
      3/4 + 5/8
      1 1/4 - 7/8
   c) 1/2 X 3/4 means 1/2 of 3/4; therefore 1/2 X 3/4 is represented by the rod that is 1/2 of the dark green rod. 1/2 X 3/4 = 3/8 since the light green rod is 3/8.
      Use the rods to solve the following problems:
      1/2 X 1/4
      1/2 X 1 1/4
      1/2 X 1/2
   d) 1/2 ÷ 1/4 is the same as asking how many 1/4's does it take to equal 1/2. Use the rods to answer the question and use the rods to solve the following problems:
      3/4 ÷ 1/4
      3/4 ÷ 3/8
      1/2 ÷ 1/8
      1 1/8 ÷ 3/8
      1 1/4 ÷ 1/2
3. Choose your unit rod in order to allow you to do the following operations and then complete the problems:
   a) 2/3 + 4/9
   b) 1/2 X 4/9
   c) 1 1/3 ÷ 2/9

<u>LAB 4</u>

<u>MATERIALS</u>: Rulers

1. Locate the inch edge on your ruler.
   a) How many marks do you have between the end of the ruler and 1 inch inclusive?
   This means that your ruler allows you to measure to the nearest 1/16 of an inch.
2. Carefully study the markings on your ruler between 0 and 1 inch and compare them to the enlarged drawing below.



a) $\frac{1}{2} = \frac{2}{4} = \frac{?}{8} = \frac{?}{16}$

b) $\frac{12}{16} = \frac{?}{8} = \frac{?}{4}$

c) $\frac{6}{16} = \frac{?}{8}$

d) $\frac{7}{8} = \frac{?}{16}$

3. Measure the following segments to the nearest 1/16 inch.

   Express your answers in reduced form.

   a)

   b)

   c)

   d)

4. Draw segments the following lengths:

   a) 1 3/8"

   b) 7 3/4"

   c) 4 3/16"

   d) 5 7/8"

5. By drawing segments end to end and measuring the total length, add the following fractions:

   a) 3 1/4 + 2 1/2

   b) 5 3/8 + 1 3/4

   c) 2 9/16 + 3 7/8

   d) 7/16 + 1/2

6. Use your ruler to help you subtract the following:

   a) 7 3/4 - 6 1/2

   b) 5 3/16 - 3 7/8

   c) 4 1/2 - 3/4

   d) 5 - 1 1/4

7. Draw a segment 7 1/2 inches long. Divide the segment into 5 nearly equal parts. Use your drawing to estimate 2/5 of 7 1/2 or 2/5 X 7 1/2. In a like manner estimate the following products:

   a) 2/3 X 7 1/2

   b) 3/4 X 5 1/4

   c) 1/4 X 6

   d) 2/7 X 4 1/2

8. Draw the necessary segments to estimate the following products:

   a) 2 1/2 X 1 3/4

   b) 1 1/4 X 3 1/2

9. Consider 8 1/2 ÷ 1 1/4. One way of approaching the problem is to ask how many 1 1/4 inch segments are in a segment 8 1/2 inches long. One cannot use a ruler to get the exact answer; however, one can get a good estimate. Estimate the following quotients:

a) 3 1/2 ÷ 1/4

b) 4 1/2 ÷ 1 1/2

c) 5 1/4 ÷ 1 1/4

d) 6 ÷ 1 7/8

e) 8 1/2 ÷ 1 1/4

LAB 9

MATERIALS: Geoboard, rubber bands, and dot paper.

1. Get the geoboard, a rubber band, and dot paper.

a) Make 8 different figures, each with an area of 4 square units.

Record the results on dot paper.

b) Make triangles which have the following areas:

1/2 sq. unit

1 sq. unit

1 1/2 sq. units

2 sq. units

3 sq. units

Record each on dot paper.

c) Make each of the following figures on your geoboard and give the area of each:

i) A rectangle with length of 4 units and width of 2 units

ii) Three different triangles with a base of 3 units and a height of 2 units

iii) A trapezoid with bases of 4 units and 2 units and a height of 2 units

iv) A parallelogram with a base of 3 units and a height of 2 units

Record each of your figures on dot paper

APPENDIX D

UNIT TESTS

MATH 101 WHOLE NUMBERS

1. Give the place value of the 2 in 7,126,345.

2. Write 5,190,021,400 in words.

3. Use the correct order symbol (<, >, or =) in the blank between the two numbers that follow: 2____14.

4. Round 1,289 to the nearest hundred.

5. Round 361,345 to the nearest ten thousand.

6. In the problem 420 ÷ 35 = 12, identify the a) divisor and b) quotient.

Perform the indicated operations.

7. 137 + 1682 +17 +4

8. 6004 - 135

9. 116(39)

10. 77,824 ÷ 256

11. 6^3

12. State the rule for the order of operations for evaluating whole number expressions.

13. Evaluate 6 + 4(3).

14. Evaluate 118 - 3(5 - 2).

15. Evaluate (a + b)/c if a = 4, b = 6, and c = 2.

16. Evaluate $2 + 4a^2 - 5b$ if a = 3 and b = 6.

17. A man had $789 in his checking account. He wrote checks of $95, $200, and $135. What was the balance in his account?

18. A secretary can type 74 words per minute. How long will it take her to type 24,050 words.

19. An automobile salesman sells 36 cars at $7,589 each. What is the total amount of his sales?

20. A student is saving money to buy a car. He has now saved $4,100. If he saves $450 more he can buy the car. How much does the car cost?

MATH 101 FRACTIONS

1. Place 1 7/8 on the number line below.

2. Reduce 28/35 to lowest terms.

3. Express 8 7/10 as an improper fraction.

4. Give the correct order relationship (<, >, =) between 1 6/7 and 15/8.

5. (7/8)(4/5)(15/28) =______.

6. Determine the product of 1 2/3 and 1 1/2.

7. 5 - 3 2/3 = _______.

8. 4 2/3 divided by 7 1/2 = ______.

9. Determine the sum of 18 1/4, 24, and 30 7/8.

10. 7/12 + 8/15 = ______.

11. What fraction is 2 3/4 less than 8 1/2?

12. Subtract 9 7/8 from 12.

13. Simplify 1/2 + 3/4 - 5/8 - 3/10.

14. 1 2/3 + 3/4[3 2/3 - 18(7/2 - 3 1/2)].

15. $\frac{7/8 + 3/4}{2\ 3/4 - 1\ 1/2}$ = _____.

16. Evaluate 5/9(F - 32) for F = 86.

17. Ms Parttimer worked 2 1/4 hours on Thursday, 4 hours on Friday and 6 1/2 hours on Saturday. How many hours did she work during the three day period?

18. Mr. Hobby needs a piece of plywood 8 feet long and 2 3/4 feet wide. He has a new sheet of plywood 8 feet long and 4 feet wide. How much should he cut off of the new sheet in order to get the desired piece?

19. Mrs. Fixit needs short braces that are 15 3/4 inches long. How many braces can she cut from a board that is 10 ft. long?

20. Round 8,147 to tens.

MATH 101 DECIMALS

1. Give the place value of the 2 in 314.126.

2. Approximate 49.231 to the nearest tenth.

3. Give the correct symbol (<, >, =) to describe the relationship between 9.196 and 9.2.

4. 7.813 + 9 + 2.1617 =_____.

5. Subtract 17.13 from 20.2.

6. Determine the product of 3.12 and 0.124.

7. (9.8)(2.2) = ______.

8. Divide 92.3 by 2.4 and round the quotient to the nearest hundredth.

9. 14.2 - 0.002 = _____.

10. Convert 0.625 to a fraction and reduce to lowest terms.

11. Convert 5/6 to a decimal correct to the nearest thousandth.

12. $\sqrt{144}$ = _____.

13. $3\sqrt{4} + 5\sqrt{16}$ = _____.

14. Evaluate 9.1 + 6.2 ÷ 3.1 - 5.

15. Evaluate 1/4(20.24) - 1/5(15.7).

16. Evaluate 3.1 - 2.4[3.20 - 2(1 + 0.6)].

17. Evaluate 5/9(F - 32) for F = 99.5.

18. Evaluate 2l + 2w for l = 17.6 and w = 9.31.

19. Carol has $12. She wants to buy records that cost $1.69 each. How many records can she buy? (Assume that there is no sales tax.)

20. Bill bought 2 shirts for $14.50 each and a pair of pants for $24.95. The sales tax is $2.16. How much change should he get from a $100 bill?

MATH 101 PROPORTION AND PERCENT

1. Express the ratio of 3 quarters to 5 dimes in simpliest form.

2. Carolina won 32 games and lost 2 games. Give the ratio of games won to games played.

3. Solve for a: $\frac{3}{a} = \frac{12}{32}$.

4. Solve for x: $\frac{x}{1/4} = \frac{8}{1/2}$.

5. A recipe for 4 servings of pudding calls for 0.4 liters of milk. How much milk is needed to make 15 servings?

6. It takes a machine 6 minutes to process 9000 cards. At the same rate how many cards can the machine process in 8 minutes.

7. Convert 12% to a fraction.

8. Convert 3.25 to a percent.

9. Convert 3/25 to a percent.

10. Convert 23% to a decimal.

11. Convert 1/4% to a fraction.

12. Convert 37.5% to a decimal.

13. 48 is what percent of 300?

14. 96 is 25% of what number?

15. What is 80% of 240?

16. The sales tax rate is 4%. How much sales tax must be paid on a coat which is priced at $89.

17. During the summer 80% of RCC's nursing graduates passed their state boards. If 24 graduates passed their boards, give the total number of graduates.

18. Determine the simple interest earned on $1600 invested for 2 years at 14% simple interest.

19. Ms Needsmoney borrowed $200 at 18% simple interest. She agreed to repay the loan plus interest at the end of 6 months. How much will she have to pay in all?

20. A blazer with a list price of $100 has been marked down to $60. Give the percentage of discount to the nearest percentage.

MATH 101 MEASUREMENT

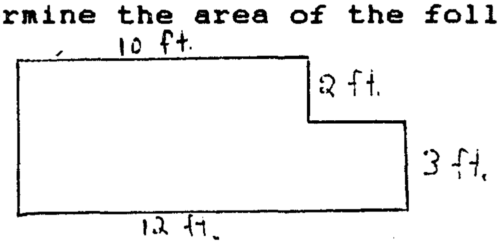
1. Measure the given segment to the nearest 1/8 inch.

2. Convert 48 inches to yards.

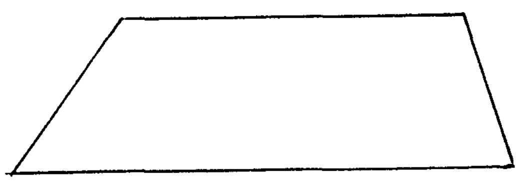
3. Give the appropriate metric unit of length to use for expressing the distance between Reidsville and Eden.

4. Convert 2 874 m to kilometers.

5. Determine the area of the following figure:



6. Determine the area of the given figure.



Make all measurements to the nearest 0.1 cm.

7. What U.S. unit of area is normally used to measure the area of a sheet of notepaper?

8. Convert 225 ft^2 to square yards.

9. Name four metric units used to measure area.

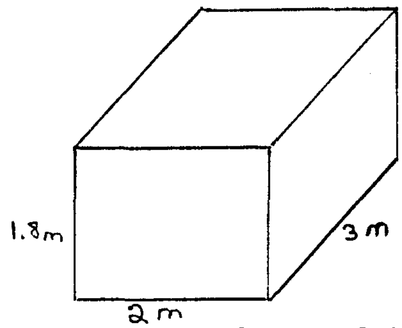
10. Convert 1220 cm^2 to square meters.

11. Give the U.S. unit of volume used to express the amount of cola that a person would drink at one time.

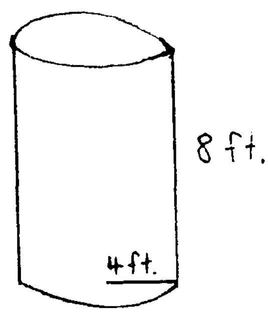
12. Convert 54 in^3 to cubic feet.

13. Give the metric unit of volume normally used to measure liquid medicines.

14. Determine the volume of the given figure.



15. Determine the volume of the given figure.



16. Convert 8,942 cm^3 to liters.

17. Name 3 U.S. units used to measure weight.

18. Convert 176 ounces to pounds.

19. Give the appropriate metric unit to express the mass of a large box of cheese.

20. Convert 4.2 g to milligrams.