JESSUP, NAOMI ALLEN, Ph.D. Understanding Teachers' Noticing of Children's Mathematical Thinking in Written Work from Different Sources. (2018)
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Expertise in teacher noticing of children's thinking is central to a vision of responsive teaching in which teachers regularly elicit and build on children's thinking during instruction (Richards & Robertson, 2016). In mathematics classrooms, this core instructional practice of noticing children's mathematical thinking repeatedly occurs during instruction and involves attending to and making sense of children's mathematical thinking (Sherin, Jacobs, & Philipp, 2011). Teachers daily have opportunities to notice children's mathematical thinking during their conversations with students and in students' written work. However, expertise in noticing children's mathematical thinking does not develop automatically or through years of teaching, and teachers need support developing noticing expertise. To help teachers develop noticing expertise, professional developers often employ artifacts of practice (e.g., video clips and student written work) from teachers' own classrooms as well as strategically selected artifacts from classrooms taught by teachers unfamiliar to the PD participants. This study explored the potential differences in teachers' noticing with written work from these two sources—teachers' own classrooms and classrooms unfamiliar to the teachers. Drawing on the construct of framing (Goffman, 1974), particular attention was paid to the various frames (or lenses) teachers used during noticing.

Using a context of professional development focused on children's mathematical thinking in the domain of fractions, this three-phase study explored teachers' noticing and their use of frames by investigating the relationship between teachers' noticing of children's mathematical thinking in written work from their own classrooms versus unfamiliar classrooms. In the first phase, this study identified the frames individual teachers used when noticing children’s thinking in written work from their own classrooms. The second phase explored the frames that small groups of teachers used when collectively noticing children’s thinking in written work from unfamiliar classrooms during professional development. The third phase used in-depth interviews to investigate the relationship between the quality of teacher noticing and the use of frames of six teachers who were asked to notice children’s thinking in written work on the same problem from their own classrooms and from unfamiliar classrooms.

Findings identified six frames teachers used while noticing children's mathematical thinking in written work from the two sources, and they fell into three broad categories: (a) noticing focused on the child’s current mathematical performance, (b) noticing focused on the child’s non-mathematical performance, and (c) noticing that compared the child’s performance to the expected performance based on the child’s past performance, the performance of the rest of the class, or curricular or testing guidelines. Confirmation of these frames in three data sets highlighted the variety of ways teachers reason during noticing, suggesting that frames are a useful construct for understanding the complexity of teachers’ noticing because frames capture the multiple and sometimes competing ideas that teachers need to coordinate.

When comparing teachers’ noticing of children’s thinking in written work from their own classrooms versus unfamiliar classrooms, a lack of substantial evidence was found to distinguish the sources in terms of the use of particular frames, the prevalence of particular frames, or the quality of teachers' noticing of children's thinking. Further, there was evidence that teachers "imagined" insider knowledge of children from unfamiliar classrooms to assist with their noticing, which might explain why engaging with written work from either source did not seem to change the quality of teachers' noticing. On the other hand, comparative analyses identified a distinction between teachers' use of frames when they were considering one child's strategy versus several children's strategies regardless of whether the written work came from the teachers' classrooms or unfamiliar classrooms. Specifically, when teachers' noticing focused on more than one child, more frames and a greater variety of frames were invoked. Implications for professional development focus on the need to appreciate and address teachers' coordination of multiple frames and the idea that the use of these frames depends less on the source of the written work and more on the number of children involved in the task.

UNDERSTANDING TEACHERS' NOTICING OF CHILDREN'S MATHEMATICAL THINKING IN WRITTEN WORK FROM DIFFERENT SOURCES

by

Naomi Allen Jessup

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Approved by

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Committee Chair

This dissertation is dedicated in loving memory to my mother and to my husband and children who have sacrificed, supported, and encouraged me to pursue my dreams.

APPROVAL PAGE

This dissertation written by NAOMI ALLEN JESSUP has been approved by the following committee of the Faculty of The Graduate School at The University of North Carolina at Greensboro.

Committee Chair ______________________________

Committee Members ______________________________

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Date of Acceptance by Committee

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Date of Final Oral Examination

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TABLE OF CONTENTS

LIST OF TABLES

Page

LIST OF FIGURES

CHAPTER I

INTRODUCTION

There is a current vision of mathematics instruction articulated throughout research and policy documents that calls for teachers to attend to children's thinking in productive ways. The importance of mathematics teaching that foregrounds children's thinking to promote learning for all children derives from a robust research base (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Jackson & Cobb, 2010; Jacobs & Empson, 2016; Munter, 2014; NRC, 2001). Similarly, policy documents such as the *Common Core State Standards for Mathematics* (National Governors Association, 2010) and *Principles to Action* (National Council of Teachers of Mathematics, 2014) reiterate the importance of eliciting and building on children's thinking as meaningful practices of mathematics teaching. In short, this vision of mathematics instruction highlights teachers' use of evidence of children's mathematical thinking as a basis for making continual adjustments to instruction that support and extend children's learning.

This dissertation study focused on this vision of instruction, which has been referred to as *responsive teaching* because responding to children's mathematical thinking as an approach to support student learning outcomes is foregrounded (Robertson, Scherr, & Hammer, 2016). In choosing this emphasis, I also acknowledge that there are other ways for teaching to be responsive in the classroom. For example, culturally responsive teaching is another vision of instruction, which foregrounds the importance of eliciting and utilizing children's cultural identities in all aspects of learning (Gay, 2002; Ladson-Billings, 1995). I believe the two types of responsiveness to be mutually reinforcing and my focus on children's thinking is based on the premise that children have a wealth of knowledge and experiences that they bring to the classroom and that are reflected in their mathematical thinking. In turn, it is the teachers' responsibility to facilitate instruction from children's individual knowledge and skills by watching and listening and responding. Thus, in responsive teaching, children are provided opportunities to develop in their thinking, and teachers use their knowledge of how particular children, and children in general, make sense of mathematical ideas to support and extend children's thinking (Jacobs & Ambrose, 2008; Jacobs & Empson, 2016).

Responsive teaching, like all teaching, is complex and composed of a collection of practices to help support student learning (Grossman & McDonald, 2008; Jacobs & Spangler, 2017; Lampert, 2010). Many current efforts focus on identifying and promoting core instructional practices that are research-based, support student and teacher learning, and can be accessed and learned in a variety of settings (Grossman, Hammerness, & McDonald, 2009; Jacobs & Spangler, 2017). While the field has not developed a consensus regarding core practices that are responsive to children's thinking, I join others in arguing that teacher noticing is a core practice of responsive teaching (Jacobs & Spangler, 2017).

Teacher Noticing—Core Practice of Responsive Teaching

Noticing refers to the general everyday process of making observations in which many things are competing for our attention and sense making. Teacher noticing is a more intentional type of noticing (Mason, 2002) in a complex classroom environment, in which so much occurs that it is hard to attend to everything with an equal amount of consideration. Teacher noticing is a construct that has the potential to uncover what teachers find important in a teaching episode, specifically regarding students and learning. In this study, I focus on a specialized type of teacher noticing, *professional noticing of children's mathematical thinking,* that is closely linked to my vision of responsive teaching, which emphasizes building on children's mathematical thinking. Professional noticing of children's mathematical thinking includes the three interrelated skills of attending to children's strategies, interpreting children's mathematical understandings, and deciding how to respond on the basis of children's understandings (Jacobs, Lamb, & Philipp, 2010). Professional noticing expertise is necessary, but not sufficient, for responsive teaching and honing in on children's thinking for use in instructional decision making is an acquired expertise (Jacobs, Lamb, & Philipp, 2010; Louie, 2016; Sherin, Jacobs, & Philipp, 2011).

Expertise in Noticing Children's Mathematical Thinking

Teacher noticing of children's mathematical thinking is challenging. Classrooms are complex environments composed of different interactions that occur throughout the instructional setting. Teachers must determine which aspects of classroom instruction are important while making in-the-moment decisions. Further, there is a range of factors that could shape teachers' noticing, such as teaching environments, preferences, biases, and specialized content knowledge (Sherin et al., 2011). Despite the challenges in developing expertise in teacher noticing, research has shown that it is a learnable practice.

Generally, teachers do not develop noticing expertise automatically, even after years of teaching experience (Jacobs et al., 2010), but there is evidence that with support, noticing expertise can improve for both prospective teachers (Callejo & Zapatera, 2017; Fernández, Llinares, & Valls, 2012; Schack et al., 2013) and practicing teachers (Floro & Bostic, 2017; Jacobs et al., 2010; van Es & Sherin, 2008).

The development of teachers' noticing expertise often occurs in professional development settings in which the practices of teaching are decomposed into manageable parts (Jacobs & Spangler, 2017; Grossman et al., 2009). Teachers then work with these parts through face-to-face interactions with students or engagement with artifacts of practice (e.g., student written work and classroom video). When artifacts of practice are used to promote growth in noticing expertise, these artifacts can come from teachers' own classrooms or can be strategically selected by facilitators from classrooms unfamiliar to the teachers. The inclusion of artifacts from the two sources—teachers' own classrooms and unfamiliar classrooms—has shown promise in supporting the development of teacher noticing expertise during PD, but additional research is needed to understand the potential differences of teacher noticing prompted by each source. Teachers may draw upon the use of insider knowledge of their students when noticing children's mathematical thinking in artifacts from their own classrooms which is not possible in artifacts from unfamiliar classrooms. This insider knowledge potentially influences how closely teachers' noticing is reflective of the mathematical thinking represented in current artifacts.

Study Origins

The idea for this study developed from literature on PD and teacher noticing as well as my observations when working on the *Responsive Teaching in Elementary Mathematics* (RTEM) study. RTEM was a 4-year professional development design study interested in characterizing teachers' development of responsiveness to children's mathematical thinking in the domain of fractions. I observed the same teachers in PD and their classrooms and saw differences in those teachers' noticing of children's mathematical thinking in their own classrooms versus in PD when the written work was mostly strategically selected by the facilitator from unfamiliar classrooms. In the PD, teachers generally seemed to notice children's thinking in written work more effectively than in their own classrooms—they were more likely to attend closely to the details of the students' thinking represented in the strategies, interpret the students' understanding based on evidence found within the strategy, and decide how to respond based on the students' understanding. In contrast, when reflecting on their own lessons, they generally used less specificity when discussing strategy details, and their interpretations of students' understandings and decisions about next instructional steps sometimes used evidence from the strategies in the written work but other times relied more on previous interactions with the students and sometimes were not even mathematically focused. Although prior interactions and non-mathematical foci maybe be useful at times, they often seemed to overwhelm the teachers' noticing in a way that minimized the mathematical work the child had actually done. These types of differences in how teachers noticed children's thinking in the written work in the two settings caused me to wonder about the extent to which teachers foregrounded children's mathematical thinking when noticing children's thinking in written work from their own classrooms versus unfamiliar classrooms, and what implications these differences might have for the use of both types of artifacts in professional development. To better understand the additional, and potentially competing lenses teachers may use when noticing children's thinking in written work from their own classrooms, I drew on the construct of framing.

Teacher Noticing and Framing

The construct of framing is a potential tool to understand the complexity of teachers' noticing of children's mathematical thinking. *Frames* are the lenses used as individuals structure information for the sense-making process of filtering and discarding irrelevant information (Goffman, 1974). Frames provide structures that help people classify, organize, and interpret their experiences, and thus the use of frames refers to the "active sense-making that teachers engage in" (Sherin & Russ, 2014, p. 6). In settings that support the development of expertise in noticing children's mathematical thinking, a children's thinking lens is foregrounded to help teachers attend to and make sense of salient mathematical details within children's strategies. In this study, I chose framing to explore the use of a children's mathematical thinking lens and other lenses that may enhance or impede the use of this lens in teacher noticing. In particular, I am interested in understanding the relationship between teachers' use of frames and their quality of noticing of children's mathematical thinking in written work from their own classrooms and those from unfamiliar classrooms. In this way, I can consider how the context of schooling and experiences with students from the teachers' own classes influence the quality of their noticing expertise. Teaching is not context-free and past and current contextual factors often shape pedagogical decisions, and thus the frames teachers use.

Overview of Dissertation

The purpose of this study was to characterize teacher noticing of children's mathematical thinking to understand differences in noticing expertise and the use of frames employed in noticing as teachers engaged with different sources of student written work. The study involved three phases (See Figure 1.1).

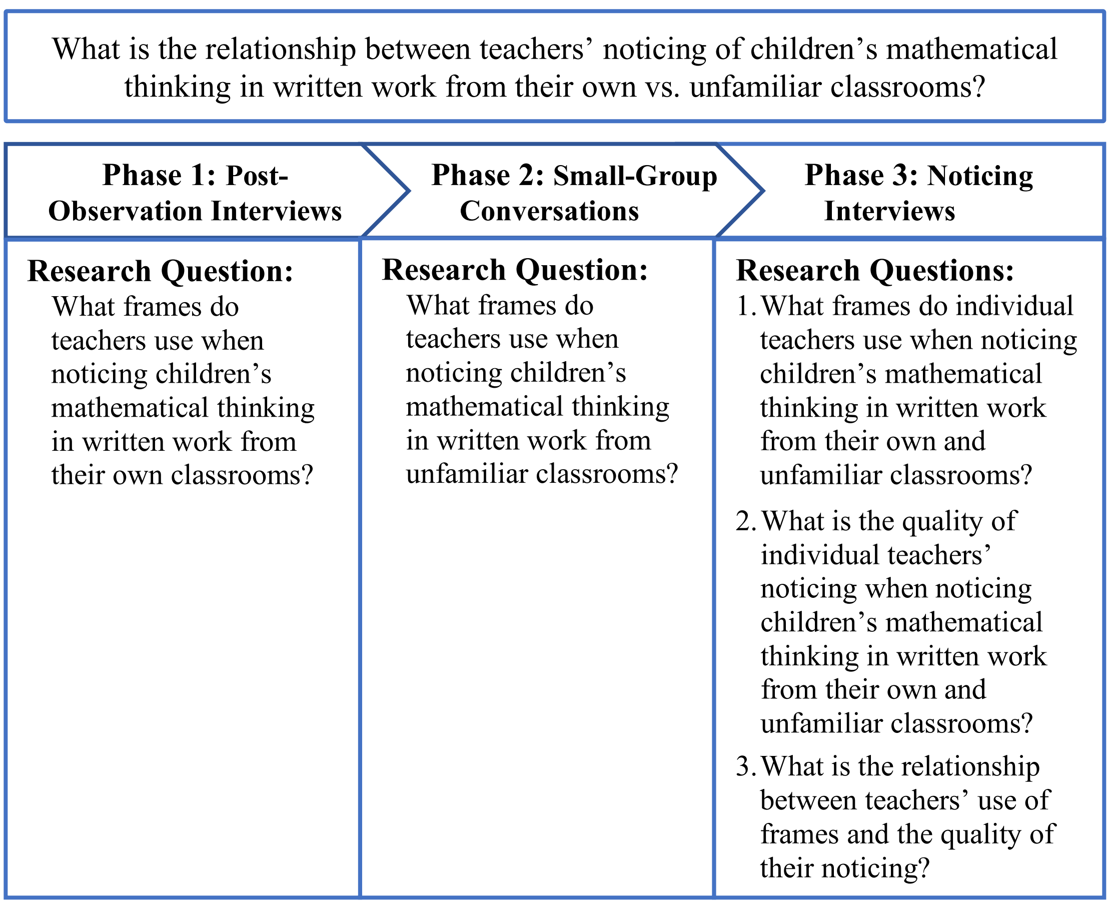

Figure 1.1. Overview of Dissertation Phases.

In the first two phases, I started with existing data from the larger RTEM project to understand the range of frames teachers used when noticing children's mathematical thinking in written work from their own and unfamiliar classrooms. During the first phase, I focused on identifying a list of frames in teachers' noticing of written work from their own classrooms by analyzing post-observation interviews. During the second phase, I extended my understanding of frames used when teachers noticed children's thinking in written work from their own classrooms to their noticing in written work from classrooms that were unfamiliar. Specifically, I analyzed the frames used in conversations of small groups of teachers as they participated in noticing activities during the RTEM PD. I began with frames identified from the first phase while leaving room for the emergence of additional frames. By the end of the second phase, a comprehensive list of frames had been identified by looking at teachers' noticing of children's mathematical thinking in written work from the two sources. However, the teachers in Phase 1 and Phase 2, while overlapping, were not identical and their noticing was only captured in a general fashion. In other words, teachers broadly engaged with the three interrelated skills of noticing children's mathematical thinking—attending to strategy details, interpreting children's understandings based on strategy details, and deciding how to respond on the basis of those understandings—but specific prompts linked to each skill were not asked consistently.

In the final phase of my study, I collected new data, using what was learned in the first two phases to develop an extensive noticing interview that did address the three noticing skills explicitly. I worked with a set of six teachers and investigated how the same teacher noticed children’s mathematical thinking in written work from both sources. I interviewed each teacher twice using written work linked to the same story problem—once with written work from her own classroom and once with a common set of written work from an unfamiliar classroom. I looked specifically at (a) teachers’ use of frames, (b) the quality of teachers’ noticing expertise, and (c) the relationship between teachers’ use of frames and the quality of their noticing.

Study Contributions

Research on teacher noticing continues to build in popularity, and thus the knowledge base continues to expand. My study contributes to this knowledge base in several ways. First, few noticing studies include the use of frames to uncover the multiple influences on teachers’ reasoning during noticing (for exceptions, see Louie, 2016 and Sherin & Russ, 2014). My study was designed to identify the variety of frames teachers use when noticing children’s thinking in written work from two sources.

Second, my study investigated potential differences in frames used when teachers notice children’s mathematical thinking in written work from their own classrooms and unfamiliar classrooms. Unlike most noticing studies, my study incorporated the use of artifacts from teachers’ own classrooms and unfamiliar classrooms *in the same study* to understand differences in the quality of teachers’ noticing and the role that frames may play in those differences.

Third, common methodological approaches to capturing teacher noticing involve written responses to prompts or interviews with minimal follow-up to teachers’ ideas.

Phase 3 of my study incorporated the use of interviews, involving think-aloud prompts and follow-up questions, to more extensively capture teachers' noticing.

Finally, my study has practical implications for those who use written work to support the development of noticing expertise. The potential influence of frames or the source of the written work on teachers' noticing could not only affect facilitators' use of particular written-work artifacts in PD but also identify a need to help teachers learn to coordinate multiple frames during noticing.

Outline of Dissertation

This dissertation is organized into six chapters. In this chapter, I introduced the problem and provided a rationale for my study. Chapter 2 reviews the literature about teaching that foregrounds children's mathematical thinking, teacher noticing of children's mathematical thinking, the construct of framing, and the use of framing while noticing. Chapters 3–5 provide the methods and findings for the three phases of my study. In addition, Chapter 3 begins with a discussion of the broader RTEM project in which my study resides. In Chapter 6, I synthesize the findings from all three phases of the study, and discuss implications and limitations of the work. I conclude Chapter 6 by outlining future areas of research related to this topic.

CHAPTER II

REVIEW OF THE LITERATURE

During the past decade, mathematics education research has developed a robust knowledge base for understanding and characterizing a vision of teaching that centralizes taking up children's mathematical ideas during instruction. This vision has been conceptualized in various forms and described by different terms such as *high-quality mathematics instruction* (Munter, 2014) *ambitious teaching* (Jackson & Cobb, 2010) and more recently *responsive teaching* (Jacobs & Empson, 2015; Richards & Robertson, 2016), but all share the idea that children's thinking is foregrounded. In this study, I adopt the conceptualization of responsive teaching characterized by Roberston, Scherr, and Hammer (2016) as including three features: (a) teachers foreground attention to children's ideas; (b) teachers recognize ways the disciplinary content, in this case mathematics, connects with children's ideas; and (c) teachers take up and pursue children's ideas.

This vision of instruction that is responsive to children's mathematical thinking connects to the large and growing body of research on children's thinking that has documented benefits for both children and teachers (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema et al., 1996; Jacobs, Franke, Carpenter, Levi, & Battey, 2007). Children have shown gains in student achievement (Carpenter et al.,1989;

Fennema et al., 1996; Jacobs et al., 2007), and also other benefits from participating in rich classroom environments that provide opportunities for children to not only share their reasoning but also engage in the reasoning of their peers (Cobb et al.,1991; Wilson & Berne, 1999). Teachers benefit from engaging with children's mathematical thinking because they gain access to the children's thinking which can sometimes highlight mathematical ideas that may differ from the teachers' ideas and can help to guide future instruction. My study is situated in this vision of responsive teaching and focuses specifically on one of the core instruction practices of this type of instruction, noticing of children's mathematical thinking. In the follow sections, I describe the construct of teacher noticing, its importance, and the landscape of research on teacher noticing. I then turn to my specific focus, teacher noticing of children's mathematical thinking and connect it with the construct of framing as a way to better understand the reasoning underlying teacher noticing, especially in relation to teachers' familiarity with what is being noticed. The chapter concludes with a discussion of methodological issues and the design of my study.

Construct of Teacher Noticing

Teacher noticing is distinct from the broader construct of noticing, which refers to general observations that occur in everyday life. Teacher noticing is a more intentional type of noticing (Mason, 2002) with roots in the concept of professional vision. Goodwin (1994) defined professional vision as "socially organized ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group'' (p. 606). In any profession, members of that community become sensitized to notice certain things in their professional settings, and expertise in attention to salient details increases the ability to process information more productively (Taylan, 2015). Teacher noticing is no exception. The classroom is a complex environment, and there is so much occurring that it is impossible, and perhaps not even desirable, for teachers to attend to everything with an equal amount of importance. Teacher noticing is a construct that describes what teachers find important in a teaching episode, specifically regarding students and their learning. I join others in arguing that research on teacher noticing is worthwhile for understanding how teachers focus on salient aspects of instruction that is responsive to children's thinking (Jacobs & Spangler, 2017).

Importance of Teacher Noticing

Jacobs and Spangler (2017) argued that teacher noticing is a core practice of responsive teaching. Teachers must consider a range of students' ideas as they arise while making in-the-moment decisions, and what they choose to attend to or miss while noticing can impact what students learn. Further, expertise in teacher noticing has been positively linked to productive in-the-moment decision making. Choppin (2011) studied how teachers' noticing of student thinking supported secondary mathematics teachers' adaptation of challenging tasks during instruction. Results showed a relationship between what teachers noticed and whether the adjustment of tasks maintained high levels of the cognitive demand. Additionally, findings suggested that a consistent focus on student thinking provided teachers with opportunities to develop a deep understanding of how their students engaged with the mathematical ideas within the tasks.

Understanding the practice of teacher noticing has also been useful in helping teachers gain expertise in other disciplines. For example, Barnhart and van Es (2015) highlighted the benefits of supporting secondary science prospective teachers in a video-based noticing course designed to assist teachers in their licensure assessment. The primary goal of the course supported prospective teachers in using evidence of student thinking to (a) attend to student thinking in teacher-student interactions, (b) analyze student understandings from those interactions, and (c) decide next steps based on those analyses. Results indicated prospective teachers enrolled in the course demonstrated higher levels of expertise in their overall attention, interpretation, and response to student thinking on their licensure assessment. In contrast, teachers not registered in the class demonstrated little to no attention to student thinking despite specific student-thinking prompts in the assessment.

In summary, teacher noticing is a core instructional practice that has been shown to support both practicing and prospective teachers in being responsive to children's ideas in complex teaching environments. However, the construct of teacher noticing has been operationalized in multiple ways, with different emphases being foregrounded depending on the researcher, and I now turn to characterizing this landscape of research.

Landscape of Research on Teacher Noticing

Research on teacher noticing continues to grow in popularity deepening the field's knowledge base of this core practice and includes two books (see Schack, Fisher, & Wilhelm, 2017; Sherin, Jacobs, & Philipp, 2011), a compendium chapter (see Jacobs & Spangler, 2017), and a plethora of research articles. Jacobs and Spangler (2017) noted that although some research has focused on documenting the range of what teachers have found noteworthy, most studies have focused on how or the extent to which teachers notice something of interest to researchers. In the following sections, I briefly share three of these areas of interest that have captured the attention of noticing researchers: teacher noticing of children's mathematical thinking, teacher noticing of equity indicators of in mathematics instruction, and curricular noticing. I then turn to the conceptualization of teacher noticing of children's thinking used in my study.

Teacher noticing of children's mathematical thinking. The most researched type of teacher noticing has been teacher noticing of children's mathematical thinking. Some researchers have focused on teacher noticing of children's thinking and the connections to research-based learning trajectories or frameworks that derive from long-standing research programs (see, e.g., Jacobs et al., 2010; Schack et al., 2013). Others have focused on teacher noticing of children's use of key mathematical concepts in particular mathematical domains. Mathematical concepts targeted in these studies have included multiplicative reasoning in proportional problems (Fernández, Llinares, & Valls, 2013), algebraic thinking (Walkoe, 2015), and generalizations about additive-growth patterns (Zapatera & Callejo, 2013).

Teacher noticing of equity indicators in mathematics instruction. A growing area of research in teacher noticing has been how and the extent to which teachers notice matters related to equity in mathematics instruction. This research has identified a variety of equity indicators resulting in research that has focused on teacher noticing of student participation during discussions (Kalinec-Craig, 2017;Wager, 2014), equitable practices (Hand, 2012; van Es, Hand, & Mercado, 2017), or culture and power dynamics (Louie, 2016). Some of this worked targeted specific populations of teachers (Kalinec-Craig, 2017) or particular populations of students (Fernandes, 2012).

Curricular noticing. An emerging area of research, curricular noticing, combines research on noticing and teachers' use of curriculum materials to understand how teachers interact with curriculum materials to support their curricular reasoning and decision-making. Amador and colleagues (2017) defined *curricular noticing* as the ways teachers make sense of the complexity of pedagogical opportunities in written or digital curricular materials. Curricular noticing, composed of three components, includes teachers' attending to specific aspects of curricular materials, interpreting what was attended to, and resulting curricular decisions. Curricular noticing is different than the above two categories of teacher noticing because it does not take place in the moment, which extends the boundaries of the current conceptualization of teacher noticing as an in-the-moment practice (Sherin, 2017).

Teacher Noticing of Children's Mathematical Thinking in This Dissertation Study

In my study, I focus on *professional noticing of children's mathematical thinking* as described by Jacobs and colleagues (2010). They identified three interrelated component skills that include attending to the details in children's strategies, interpreting children's mathematical understandings reflected in those strategy details, and deciding how to respond on the basis of children's mathematical understandings. In the sections below, I will describe each of the three component skills. Note that this set of component skills distinguishes this conceptualization of noticing from other conceptualizations that focus solely on teachers' attention in complex instructional environments (Star, Lynch, & Perova, 2011) or only on teachers' attention to and interpretation of what was seen (Goldsmith & Seago, 2011; Sherin & van Es, 2008).

Skill 1: Attending to children's strategies. For the component skill of attending to children's strategies, Jacobs and colleagues (2010) focused on the extent teachers with different professional development experiences attended to the mathematical details in individual children's strategies by describing the mathematical details, patterns, and nuances in the individual children's strategies.

Skill 2: Interpreting children's mathematical understandings. Research has shown that children's strategy details often reflect a nuanced picture of what the children understand (Carpenter, Fennema, Franke, Levi, & Empson, 2015). Jacobs and colleagues' (2010) description of interpreting children's mathematical understanding focused on teachers' reasoning and the extent to which it was "consistent with both the details of specific child's strategies and the research on children's mathematical understanding" (p. 4). The researchers also recognized the impossibility of providing a complete description of children's understandings on the basis of children's work on a single problem, but they argued that some components of children's understandings can be reflected in strategy details.

Skill 3: Deciding how to respond on the basis of children's mathematical understandings. Research has identified a variety of in-the-moment teacher moves that support or extend children's understandings of mathematical ideas (Jacobs & Ambrose, 2008; Jacobs & Empson, 2016). Jacobs et al. (2010) described this third skill of noticing as teachers' intended response to build on children's understanding. They recognized that no perfect next move exists and instead their focus was on the reasoning teachers used when they learned about the children's understandings and whether the teachers' reasoning was consistent with that child's understandings and research on the development of children's mathematical thinking.

In summary, the three component skills of Jacobs et al.'s (2010) professional noticing of children's mathematical thinking can be taken to be integral to teaching that is responsive to children's thinking. These skills are worthy of study because teachers do not automatically have this expertise, but Jacobs et al's (2010) study showed that they can be learned, with sustained support. Professional development designed to help teachers gain this support often uses artifacts of practice, such as video clips and student written work, and these artifacts can be drawn from multiple sources.

Teacher Noticing with Artifacts from Multiple Sources

Teacher noticing is a complex practice and supporting teachers' noticing expertise is additionally complex because of the contextual factors that shape teachers' decision making. In PD, facilitators often use artifacts of practice from teachers' own classrooms as well as those that they strategically select from classrooms unfamiliar to the teachers in the PD. Artifacts from different sources have different affordances. For instance, when teachers are noticing children's thinking in artifacts from their own classrooms, they can use insider knowledge of those children whereas this knowledge is unavailable with artifacts from unfamiliar classrooms. This difference raises the question of whether teachers' noticing (or the quality of their noticing) may be different depending on their familiarity with the artifacts used. Few researchers have directly studied this idea, but I will share two studies which have compared teacher noticing using video artifacts from both sources.

In the first study focused on science teacher noticing, Seidel et al., (2011) analyzed written comments of physics teachers' responses to videos of lessons of the teachers' own teaching and then of another teacher on the same physics topic. The researchers were interested in the teachers' overall reaction to the video, so teachers were asked to watch each video and make a comment for each 10 minute segment. In the second study focused on mathematics teacher noticing, Kleinknecht & Schneider (2013) analyzed written responses of eighth-grade mathematics teachers' responses to videos presented by a web-based tool of lessons of the teachers' own teaching and then of another teacher. Teachers were asked specific questions about what they noticed in each video, but were also provided opportunities to comment on any scene of interest to them. In both studies, teachers had recently completed video-based professional development in which both types of video clips were used to support the development of professional vision (noticing and knowledge-based reasoning) of individual teachers.

Across both studies, results showed differences in teachers' engagement with video from their own teaching versus facilitator-selected videos based on (a) overall instances of noteworthy interactions identified, (b) resonance with their own practice, (c) evaluation of classroom events, and (d) reflection on possible alternatives. First, teachers noticed more instances of noteworthy interactions when watching video clips of their own teaching in contrast to the teaching of others which meant teachers viewed clips from their own classrooms more positively than clips from other teacher's classrooms. Second, teachers who were noticing videos of their own teaching commented more about how the representation of teaching resonated with them in comparison to noticing in video from facilitator-selected videos. This finding suggests that teachers may have found it challenging to engage with video clips of teaching from other teachers because what was represented in the video was not closely linked their own experiences teaching the same topic. Third, teachers were more willing to evaluate instruction critically when the video was from others' classrooms meaning that teachers' personal connection to their own videos may have interfered with their ability to think critically about their instruction. Fourth, teachers minimally engaged with classroom instances that were identified as negative in their own videos, specifying less possible alternatives. When teachers were asked to push for specific ways to provide alternatives for those negative events, teachers engaging with facilitator-selected videos supplied explanations of ways to improve those events. In contrast, teachers did not provide an alternative or provided minimal responses when noticing videos from their own classrooms

In summary, the lenses teachers used in their noticing in classroom video clips from their own classrooms and those from other classrooms were different. Teachers engaged more with video clips from their own classrooms, but found difficulty in reflecting on any aspect of their instruction that was not as positive. The differences in teachers' engagement with artifacts from teachers' own and unfamiliar classrooms supports the need to investigate teachers' noticing in artifacts from both sources further. Similar to these comparisons of teachers' noticing with video clips from the two sources, my study explored the potential differences in teacher noticing of children's thinking in written work from the two sources. I drew upon the construct of framing to help explore these differences because framing allowed me to consider the various frames (or lenses) teachers used while noticing.

Construct of Framing

Framing or frame analysis was theorized by sociologist Goffman (1974) to explain how individuals organize their experiences and perception of those experiences. Goffman argued that people frame things every day to organize their understanding of something—individuals actively classify, organize, and interpret their life experiences to make sense of them. Additionally, people filter important information and discard what is not needed depending on the situation. In essence, framing provides a lens for engaging with complex environments and a person's framing of an event establishes meaning for the individual whether or not he or she is aware of the lens. For instance, consider the different purposes teachers use when examining written work and how each purpose has guiding principles and values that shape how teachers engage with and structure their analysis of the work. One purpose a teacher might have is looking for the correct answer, which can prompt teachers to separate that specific feature of the strategy from other strategy features as the basis for student understanding and may even cause teachers to minimize their appreciation for student understanding reflected in other strategy features. Thus, the frame of "looking for the correct answer" provides a structure for teachers to make sense of the student thinking represented in the written work.

I argue that framing is a broad concept that captures the knowledge, beliefs, and experiences that can play a role in teacher noticing. However, teachers often have multiple frames that they are invoking while noticing and the coordination of these frames can be complicated because they can be complementary or competing. I use Jessup, Hewitt, Jacobs and Empson's (2015) work on teachers' perspectives on children's fraction strategies to illustrate how the idea of complementary or competing frames can play out in teachers' noticing of children's mathematical thinking. Jessup and colleagues (2015) investigated the ways teachers, at the start of PD, made sense of children's fraction strategies before being formally introduced to a research-based framework of children's thinking to support teachers' perspectives on those strategies. Groups of elementary school teachers were asked to analyze a set of 12 pieces of student written work on the same fraction story problem and order the strategies in terms of sophistication. Teachers' rationales for why a particular strategy was more sophisticated than another identified sets of strategy features they preferred. These identified strategy features were sometimes similar to and sometimes different than those leveraged in the research-based children's thinking frameworks. As teachers engaged in the PD, one could imagine how these two sets of frames—teachers' own set of preferences for strategy features based on their prior experiences and the set of research-based strategy features privileged in the PD—would need to be coordinated in teachers' noticing of children's thinking in fraction strategies. This act of coordination would be complicated because some ideas are complementary and others are conflicting. The construct of framing could shed light on this coordination as teachers work to gain noticing expertise. Thus, combining the study of framing with the study of teacher noticing has potential.

Connection of Framing to Teacher Noticing

The inclusion of the use of framing to investigate further the complexity of teacher noticing in mathematics education can be attributed to the work of Sherin and Russ (2014) who examined teachers' use of framing in their reasoning about classroom events in video. In interviews, a group of secondary mathematics teachers were asked to comment on what they noticed in four short video clips from unfamiliar teachers' classrooms. Analyses led to the identification of 13 frames that shaped the ways teachers made sense of what they noticed in classroom events depicted in video. The authors argued that while their identification of frames occurred in an interview context, teachers' use of frames would continue to shape teachers' noticing during instruction.

Other research in mathematics education has used framing, especially the coordination of multiple frames, to understand the complexity of teaching. Louie (2016) observed high school mathematics teachers engaged in an equity-oriented PD in different settings (i.e., classrooms, mathematics department meetings, and PD) and used framing to capture teachers' explanations of what it meant to be mathematically capable and who is or can become mathematically capable. Findings indicated tensions in the use of multiple frames that were sometimes in competition. For example, William, a teacher, interacted with a small group of students working on a hypotenuse problem in a geometry class. During his interaction, he sometimes framed the students as capable and other times as incapable. Specifically, upon his arrival at the table, William noticed that two students had solved a problem using similar representations but arrived at different answers. William's initial engagement with the students framed them as *mathematically capable* by asking each student a series of questions to elicit their thinking. However, as the students (whose native languages were not English) struggled to articulate their reasoning, William began to ask a series of closed questions that prompted students to answer giving one- to three- word responses. In the subsequent interaction, William started to explain the mathematically important elements of the Pythagorean theorem, framing the students as dependent on the teacher to provide reasoning. This study highlights the multiple frames used by a teacher as he noticed children's mathematical thinking but also the tension in what gets foregrounded during certain instances. Both studies pushed me to consider how frames in teacher noticing were not mutually exclusive and the use of multiple frames within different contexts further points to the complexity of teaching and teacher noticing. These studies brought about the need to integrate the study of framing into the study of teacher noticing and next, I consider the typical measurement approaches and methodological challenges associated with studying teacher noticing.

Measurement Approaches to Studying Teacher Noticing

Because teacher noticing occurs in-the-moment and is not visible, it is difficult to know everything teachers are attending to and making sense of during instruction. Researchers have used a variety of approaches to capture teacher noticing by making it visible, but these approaches are not without their methodological challenges. Measuring teacher noticing is hard due to its hidden nature, and there are benefits and drawbacks to the various approaches used to understand this complex practice.

Approaches to capturing teacher noticing using artifacts of practice. A standard approach for capturing teacher noticing involves providing teachers with artifacts from classrooms and asking them to describe their noticing. Teaching has been portrayed through different types of artifacts, such as classroom photographs of instruction (Oslund & Crespo, 2014), video of individual students solving a range of problems (Jacobs, Lamb, Philipp, & Schappelle, 2011), video vignettes of classroom events (Blomberg, Stürmer, & Seidel, 2011; Santagata & Yeh, 2013), or student written work (Callejo & Zapatera, 2016; Fernández, Llinares, & Valls, 2013; Jacobs et al., 2010).

Researchers have assessed teacher noticing expertise with artifacts from both unfamiliar classrooms and teachers' own classrooms, and there are advantages and disadvantages to each approach. When using artifacts from unfamiliar classrooms, researchers can easily compare noticing across teachers because everyone is noticing with a common artifact. However, researchers miss how teachers may use contextual information when noticing with familiar students. In contrast, when using artifacts from teachers' own classrooms, researchers can learn about teachers' perspectives on their students and classroom events. However, comparing noticing expertise across teachers is challenging because different artifacts have different strategy details thus providing different opportunities to notice.

In both approaches to capturing teacher noticing, teachers are generally asked to discuss what they notice in those artifacts through written responses, group discussions, or interviews, but there are often few follow-up questions so some of teachers' noticing is not accessed. In written responses, teachers are asked to respond to prompts related to the researchers' conceptualization of noticing, and the opportunity to ask follow-up questions is not available. In group discussions with their peers and interviews that capture teacher noticing, teachers respond to a series of questions related to noticing, but the use of clarifying questions to probe teachers' responses have typically been limited (Ainley & Luntley, 2007; Sherin & van Es, 2009; Walkoe, 2015).

Analysis of teacher noticing. In the analyses of the data used to measure teacher noticing, researchers have often coded teacher responses according to levels of teacher noticing expertise. These levels provide some insight into the quality of teachers' noticing, but can often mask the underlying reasoning used in teacher noticing (Sherin & Russ, 2014; Sherin & Star, 2011). Additional research is needed to reveal further teachers' underlying reasoning—frames used during teacher noticing—that could explain differences in teachers' noticing in artifacts from teachers' own classrooms versus classrooms unfamiliar to teachers.

My dissertation study is designed to explicitly address some of the methodological challenges identified above, including using both types of artifacts (common artifacts from unfamiliar classrooms and individualized artifacts from teachers' own classrooms), designing interviews to include think-aloud protocols and numerous follow-up questions, and analyzing the data for not only teacher noticing quality but also the frames invoked.

Dissertation Study Design

My dissertation study, which has three phases, explored teachers' noticing and their use of frames by examining the relationship between teachers' noticing of children's mathematical thinking in written work from their own classrooms versus unfamiliar classrooms. In the first two phases, I investigated what frames teachers used when noticing children's mathematical thinking in written work from each source separately—one in Phase 1 and one in Phase 2. The third phase of the study investigated the use of frames and the quality of teachers' noticing through the use of individual interviews to maximize the potential for characterizing teachers' noticing expertise by probing teachers' reasoning related to certain aspects of noticing and the use of frames. The third phase of the study also provided the chance to compare the same teacher's noticing of children's thinking on the same problem in written work from both sources. In the next chapter, I focus on Phase 1 in which I explored teachers' use of frames when noticing children's mathematical thinking in written work from their own classrooms.

CHAPTER III

PHASE 1: POST-OBSERVATION INTERVIEWS

In three phases, this dissertation study explored teachers' noticing and their use of frames to understand the relationship between teachers' noticing of children's mathematical thinking in written work from their own classrooms versus unfamiliar classrooms. In this chapter, I focus on Phase 1 in which I used post-observation interviews to identify the frames teachers used when noticing children's mathematical thinking in written work from their own classrooms. However, because my dissertation study was situated in the larger RTEM project, I begin the chapter with descriptions of relevant methodological information—professional development and participants—for this larger project.

Methods of the RTEM Study

The RTEM study engaged upper elementary school teachers in multi-year PD to study and support the development of responsive-teaching expertise in the domain of fractions. Because RTEM provided the backdrop for my dissertation study and the data for Phases 1 and 2, I use the next two sections to provide descriptions of the PD and participants for the larger RTEM study.

RTEM professional development. The RTEM professional development was guided by a vision of responsive teaching in which teachers' decisions about what to pursue and how to pursue it are constantly adjusted in-the-moment of instruction in response to the details of children's mathematical thinking (Jacobs & Empson, 2016). RTEM PD supported this vision of teaching by foregrounding the importance of eliciting and building on children's mathematical thinking through engagement with multiple frameworks linked to research on children's mathematical thinking and instructional practices. The children's thinking frameworks drawn from Empson and Levi's (2011) work introduced teachers to children's thinking about fractions in grades 3–5 through problem-type and strategy frameworks. The problem-type frameworks included distinctions of fraction story problems based on how children distinguish between the structure of those problems. Each problem type in the framework was linked to a strategy framework that included a range of typical strategies children use, ordered to reflect increasing levels of understanding of fractions. During the PD, teachers were supported in learning how to use these children's thinking frameworks when interacting with children.

The RTEM PD also focused on two instructional-practice frameworks that are central to teaching in a manner that is responsive to children's mathematical thinking. The first framework, noticing children's thinking (Jacobs, Lamb, & Philipp, 2010), engaged teachers in focusing on and making sense of children's mathematical thinking in their comments, questions, and written work. This framework consisted of the three interrelated component skills of attending to the details in children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings. The second instructional framework focused on questioning to support and extend children's mathematical thinking (Jacobs & Ambrose, 2008; Jacobs & Empson, 2016). During the PD, teachers engaged with these frameworks in relation to classroom artifacts (video and written work) and work with children. The goal of these PD activities was to help teachers develop an appreciation for the idea that building on children's mathematical thinking of whole number concepts and fractions can advance children's mathematical ideas. It is important to note that when written work artifacts were used in this PD, they were sometimes strategically selected in advance from the classrooms of teachers outside of the PD and other times brought by participating teachers from their own classrooms.

The frameworks used in PD had strong ties to the research and professional development project Cognitively Guided Instruction (CGI) (Carpenter, Fennema, Franke, Levi, & Empson, 2015) that takes a strength-based approach to examine how children think about mathematics. CGI has documented benefits in learning for both teachers and children (Carpenter, Fennema, Franke, Levi, & Empson, 2015; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). Like CGI, the RTEM PD was unscripted and aimed at helping teachers understand how children think about mathematics and how to use knowledge of children's thinking to make instructional decisions.

Teachers in the RTEM project participated in sustained professional development over three years. Each year, the PD was comprised of 8.5 days of workshops that spanned the summer (4.5 days) and school year (2 days in the fall and 2 days in the spring), totaling more than 150 hours across the 3 years. During the workshops, teachers engaged in numerous activities to support their responsiveness to children's fraction thinking. These activities engaged teachers in: (a) learning about research on children's thinking, (b) analyzing children's strategies in video and written work, (c) questioning and planning for instruction for individual and groups of students, (d) working with children's thinking in their classrooms, and (e) adapting existing curricular resources. Additionally, each year teachers worked with colleagues at their school sites to participate in four self-guided discussions through the Collaborative Inquiry Tool (CIT). The CIT is a web-based tool developed to support teachers' face-to-face conversations with one another at school sites. In small groups, teachers examined their own students' written work and that of their colleagues' written work to further their noticing of children's mathematical thinking. The CIT helped to increase the amount of time teachers engaged with the PD ideas between workshop sessions.

RTEM participants. The RTEM project worked with three cohorts of teachers in grades 3–5 and a few elementary mathematics instructional specialists or coaches. Each cohort had a staggered start and the first two cohorts completed three years of the PD whereas the last cohort only completed one year of PD as part of the project but had other opportunities to continue their learning with locally-offered PD. The RTEM project worked with 92 participants (82 females and 10 males): 35 third-grade teachers, 29 fourth-grade teachers, 21 fifth-grade teachers, 1 second-third grade teacher, 3 fourth-fifth grade teachers and 3 teachers who were instructional specialists or coaches. At the start of the PD, these participants ranged in years of teaching experience (0–34 years, $M = 10$ years) and about one third had participated in previous CGI PD on children's mathematical thinking with whole numbers.

RTEM teachers were drawn from three school districts that were close in proximity, involving 11–15 schools per district, in the southern region of the United States (see Figure 2.1). All three districts supported the PD and instruction that was responsive to children's mathematical thinking, and they were purposefully selected because of their varying instructional contexts. Two of the districts had a long tradition of supporting their teachers in learning about children's mathematical thinking to inform instruction, and multiple district-created resources were available. The third district had only recently started to shift towards instruction that was responsive to children's mathematical thinking and hence resources were still emerging.

		District A	District B	District C
Students classified as Limited English Proficiency		33%	47%	9%
Students who qualified for free or reduced-cost lunch		61%	71%	40%
Student race and ethnicity classifications	White	48%	36%	68%
	Hispanic	45%	46%	12%
	Black	2%	2%	10%
	Other	5%	15%	10%

Figure 2.1. RTEM PD District Demographic Data.

Note. These district demographic data reflect the school year in which the highest number of teachers were involved in the study. Data and demographic classifications were drawn from a state-level database.

Methods of Phase 1: Post-Observation Interviews

In Phase 1 of my study, I examined post-observation interviews of 42 teachers in the RTEM project who were observed teaching a fraction lesson and then interviewed after the lesson to share their thoughts on the lesson. My goal was to understand the frames teachers used during noticing of children's mathematical thinking in written work from their own students.

Participants. This first phase of my study involved criterion sampling of a subset of teachers from the RTEM project. Criterion sampling is a form of qualitative sampling that involves selecting cases that meet some criteria of importance to construct a comprehensive understanding of a phenomenon (Creswell, 2013; Patton, 2002).

My criteria for inclusion were (a) the teacher's use of equal-sharing problems in the lesson we observed and (b) the lesson had a complete set of written work. Equal sharing problems are a type of fraction story problem in which a total number of items is distributed to a certain number of groups and, in this dissertation, I focus on equal sharing problems in which the answer is a fractional amount. An example of an equal sharing problem is: *Four children want to share 10 brownies so that everyone gets exactly the same amount. How much brownie can each child have?* By holding constant the mathematics in the observations, I could more easily compare teachers' noticing, and a focus on equal sharing provided several advantages. Specifically, much of the PD focused on this type of problem, which is appropriate for grades 3–5 and particularly powerful for helping children understand fraction concepts. Thus, teachers were familiar with these types of problems and had already had an opportunity to try them with their students. In addition, the majority of the classroom observations conducted involved lessons with equal sharing problems.

These 40 teachers (35 females and 5 males) were drawn from all three PD cohorts and were spread across the grade levels targeted by the PD: 16 third-grade teachers, 13 fourth-grade teachers, 9 fifth-grade teachers, and 2 fourth-fifth grade teachers. These teachers were also distributed across the three school districts and reflected a range in overall years of teaching experience (1–34 years, *M* = 11 years) and years of teaching experience specific to grades 3–5 (0–19 years, *M* = 6 years). Further, about one third had participated in previous CGI PD on children's mathematical thinking with whole numbers.

Data source. Data in the first phase were drawn from existing data collected as part of the RTEM data-collection process. Data included 40 audio-recorded, semi-structured interviews (5–15 minutes) that took place following an observation of each teacher's classroom instruction. Teachers were asked to pose an equal sharing problem during their instruction that they considered appropriate for their class, including problem context and number choices. After the lesson, teachers were immediately interviewed about many aspects related to the observation. This study focused on the portion of the interview in which teachers were asked to identify a child's piece of written work from the lesson that was interesting to them and then discuss what stood out about the child's thinking. Additional questions were sometimes posed to ask teachers to describe the details of the child's strategy, his or her understanding reflected in the strategy, and instructional next steps for the child based on that understanding, but not all of these additional questions were posed in all interviews.

Data analysis. A grounded theory approach (Charmaz 2006; Corbin & Strauss, 2008) was selected to analyze teachers' conversations in their post-observation interviews due to a limited knowledge base regarding the specific frames used while engaging with children's mathematical thinking about written work. Grounded theory analysis entails creating codes through the iterative study of the data, thus allowing codes to emerge through multiple rounds of coding. In Phase 1 of this study, I used Charmaz's (2006) approach to grounded theory analysis that consisted of developing initial sets of codes, applying identified codes, and then looking across the coded data to determine broader categories. My analysis of the post-observation interviews occurred in multiple stages and was guided by the question: "What frames do teachers typically draw upon when making sense of their own students' written work?" The intention of this guiding question was to help me look for both the *children's thinking* frame that was often emphasized in the PD, and any other frames teachers utilized. First, I transcribed and divided the post-observation interviews into idea units that indicated when a single topic was discussed (Jacobs & Morita, 2002). Second, I determined the focus of each idea unit—the child, the child within class, or beyond the child and class. Third, I used an iterative process of looking holistically across idea units to generate six codes that reflected the six frames used in teachers' interviews about the written work from their own classrooms. Fourth, I used these six frames to code the entire data set. Fifth, I noted the prevalence of the six frames, identifying whether or not each teacher had used each frame at least once.

Phase 1: Results

In the first phase of my study, I found that teachers used multiple frames when noticing children's mathematical thinking in written work from their own classrooms. Thus, the main result of the analysis of teachers' interviews was the identification of the six frames used. These six frames fell within three broad categories: current mathematical performance, non-mathematical performance, and mathematical performance comparisons. Table 1.1 at the end of the results section provides a summary of the three broad categories and their corresponding frames, including their prevalence across teachers. The next sections describe the categories and their corresponding frames in detail using examples from the data, followed by an expanded example showing the use of multiple frames within a single teacher's interview.

Current mathematical performance category: Children's thinking frame. This first category consisted of one frame, the *children's thinking* frame, in which teachers highlighted the current mathematical details within a child's strategy in their response. For example, during her interview, a teacher described the details of a child's strategy for the problem: *Bryan has 12 friends that shared 8 cakes equally. How much cake will each friend get?* Her comments focused on the strategy depicted in Figure 2.2 in which the child drew 8 rectangles to represent the cakes, split each cake into twelfths, and numbered them 1–12 to indicate giving each friend one twelfth from each cake. The child circled the "12" in each cake and then combined the one twelfth from each cake by repeatedly adding one twelfth eight times to get eight twelfths. When asked about this strategy, the teacher shared these details by stating,

> He started with 12 friends up at the top and then drew 8 cakes. He knew to divide those cakes into 12 and numbered them 1, 2, 3, 4, 5 6, 7, 8, etc., and divided them into twelfths. He circled the twelfth piece on each [cake] and used repeated addition to get eight twelfths.

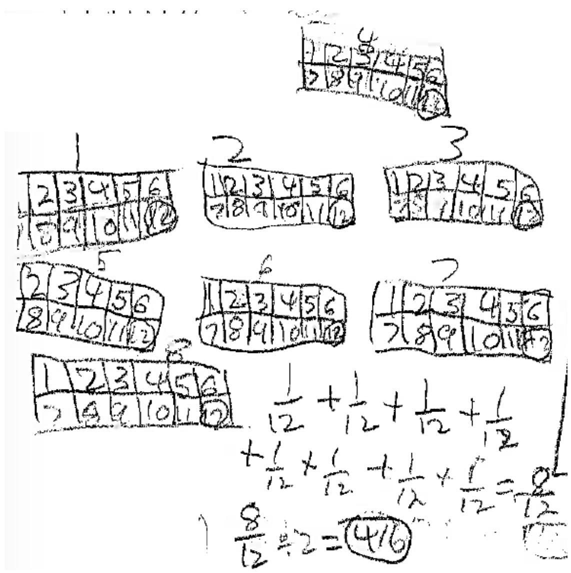

Figure 2.2. Children's Thinking Frame Example: Student Response to the Problem of 12 Children Sharing 8 Cakes.

In this example, the teacher attended to the mathematically significant details of partitioning each item into the number of sharers (12), distributing the twelfths to each friend, and combining the unit fractions from different wholes to generate the final answer. A *children's thinking* frame was central to the RTEM PD, so it was not surprising this frame was used by itself by 100% of the teachers.

Non-mathematical performance category: Confidence and behavior frames. The second category, non-mathematical performance frames, included two frames in which teachers attended to the mathematical details of the child's strategy but highlighted something other than the mathematical aspects of the child's problem solving. Non-mathematical frames were sometimes used to emphasize the progress the child made in solving the problem despite certain issues or as a way of making excuses for what the child was unable to accomplish.

In the *confidence* frame, teachers attended to the details in a child's strategy but highlighted the child's mathematical confidence in relation to his or her strategy. For example, a teacher was asked to describe the details of a child's strategy for the problem in which 10 children were sharing 3 cakes (see Figure 2.3). The child drew 3 rectangles and split each rectangle into tenths. Then the child shaded one tenth from each rectangle and combined each one tenth to get three tenths. This child used a strategy similar to the strategy described for the *children's thinking* frame—the child partitioned the items into the number of sharers (in this case, 10) and then combined the unit fractions for the final answer. However, this teacher's observations had a different focus, such as when she noted, "At first they did nothing. I had to walk them through the problem. They have the knowledge to solve the problem, but [did] not have the confidence to do it on their own without assistance." In this excerpt, the teacher did not attend to the mathematical details of the child's ideas but excused the child's initial inability to solve the problem independently due to a lack of confidence. The *confidence* frame was used by 25% of the teachers.

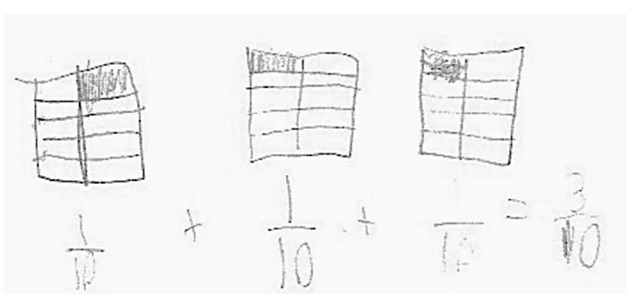

Figure 2.3. Confidence Frame Example: Student Response to the Problem of 10 Children Sharing 3 Cakes.

The *behavior* frame was another non-mathematical frame in which teachers attended to the details in a child's strategy but highlighted specific behaviors of the child in relation to his or her problem solving. In these instances, teachers' statements often suggested that a child's behavior had a causal relationship to what was represented in the written work. For example, for the problem in which 5 children were sharing 2 sticks of licorice, a child drew two rectangles for the sticks of licorice and 5 lines for the children (see Figure 2.4). Then the child split each stick of licorice into fifths, and numbered each fifth 1–10, but no final answer was given. A closer look into the child's strategy indicated the lines and arrows were used to purposefully denote passing out to 2 one fifths to each child. When asked to respond to the question, "What stood out to you in the child's work?" the child's teacher stated,

> I couldn't understand why [pause]. For the problem, … he had the drawing and he had it divided but he struggles a lot with attention and gets side tracked. He had it all right there [referring to the drawing] and he had the pieces at the top. He didn't realize he missed the piece at the bottom.

The teacher attributed the incomplete set of two fifths (four instead of five) in the child's equation at the bottom of the page to his difficulty focusing on the problem. Only two teachers used the *behavior* frame, but this frame was included because it speaks to ways others have found that teachers sometimes categorize students (Horn, 2007).

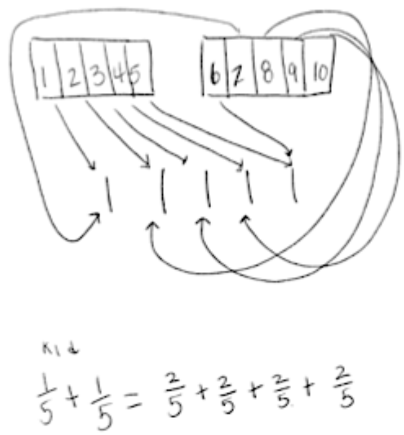

Figure 2.4. Behavior Frame Example: Student Response to the Problem of 10 Children Sharing 2 Pieces of Licorice.

Mathematical performance comparison category: past performance, class performance, and broader scope frames. This third category included three frames that underscored the comparison of the child's current mathematical performance to the child's performance in previous problems, the performance of others in the class on this problem or previous problems, or curricular or testing goals for that grade level. Teachers often explicitly expressed these frames in terms of comparison by discussing the child's work in terms of its consistency with the teacher's knowledge and experiences from previous involvement with that specific child, other children in the classroom, or curriculum or testing goals.

In the *past performance* frame, teachers attended to the details in a child's strategy but highlighted how the child's performance on the problem compared with prior work from that child and often mentioned typical strategies used by the child or his or her progress over time. For example, for the problem involving 6 campers who wanted to share 2 pizzas equally, a teacher was asked to describe the details of a child's strategy (see Figure 2.5). The child solved the problem using a valid strategy in which he drew two rectangles and split each rectangle into sixths, but this strategy was incomplete and this child did not have an answer written. The teacher said, "He drew two rectangles and split each into 6 pieces, but no answer was written. I was surprised because yesterday, he had an invalid strategy. He might know more than I thought." These statements indicated the teacher originally anticipated the child would solve the problem using an invalid strategy based on their work from the previous day. The teacher was surprised the child used a valid strategy and acknowledged a potential underestimation of the child's understandings. The *past performance* frame was used by 63% of the teachers.

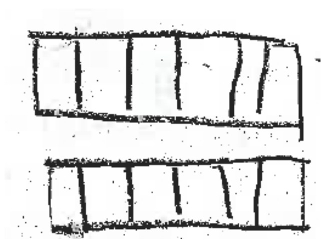

Figure 2.5. Past Performance Frame Example: Student Response to the Problem of 6 Campers Sharing 2 Pizzas.

Related to the *past performance* frame was the *class performance* frame in which teachers attended to the mathematical details in a child's strategy but highlighted how this child's performance on the problem compared with the performance of the rest of the class on this problem or previous similar problems. There were two versions of this frame, and each is illustrated below: (a) comparing the child's performance to the performance of rest of the class or (b) comparing the child's performance to the performance of individual students in the class by thinking about ways to create strategic pairings of this child with another child in the class so that they could learn from each other.

In the first version of the *class performance* frame, teachers compared the child's performance with the performance of rest of the class on that problem or previous similar problems. For example, a child solved a problem about 20 cookies being shared by 8 friends by drawing 20 tallies for each cake and 8 circles for the friends (see Figure 2.6). His answer was 2 4/8 which he indicated by circling two sets of 8 tallies (and putting 2 tallies in each child's circle) and drawing four circles for the remaining cookies, partitioning each into eighths, and numbering the eighths 1–8 to indicate distribution. When probed about the child's understandings, the teacher stated, "He understands that if there are 20 [items], you have 2 groups of 8 with a remainder of 4. He did the division problem in his head, while everyone else [in the class] first passed out wholes individually [one-by-one]. In some ways, he is more advanced than the others." She surmised that the child mentally knew that 20 cookies divided by 8 children would mean 2 whole cookies per child with 4 cookies remaining without having to pass out cookies one by one. Her assumption was likely based on the child's circling of the two sets of 8 tallies, and the possibility that the child's partitioning of the 4 leftover cookies at the top of the page might have been his first written step when problem solving. With this interpretation of the child's strategy, the teacher used the details of the child's strategy to compare his level of understanding to the majority of the class.

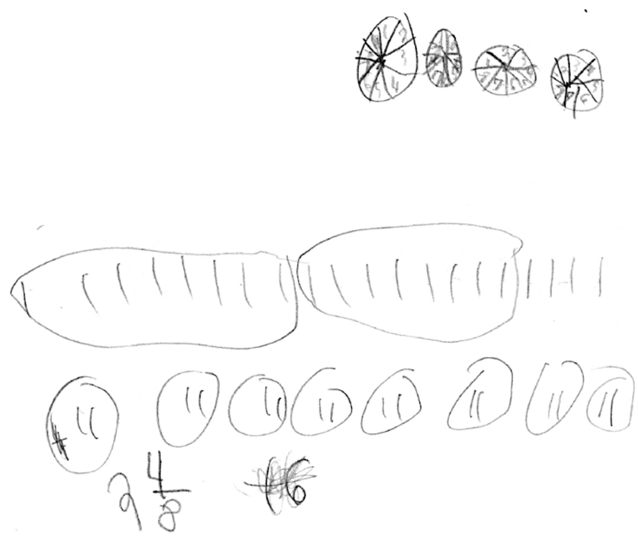

Figure 2.6. Class Performance Frame Example: Student Response to the Problem of 8 Friends Sharing 20 Cookies.

In the second version of the *class performance* frame, teachers not only distinguished the child's performance from others in the class but also considered the benefits of connecting children with different strategies. Specifically, they considered how a child with a small group of their peers could either offer support to their peers or vice versa when sharing diverse strategies. For example, in the problem in which 7 cookies were shared by 3 friends a child drew 7 rectangles and split each one into halves (see Figure 2.7). Then, the child numbered the halves 1–3 to distribute the halves, which left two remaining halves. When asked about instructional next steps for the child, the teacher stated,

> I want her to explain her thinking to another student and compare it [her partitions] to another student to see how [the other student] partitioned differently [not using halves]. By pairing her up with another student, that could get her to see how to partition to be able to combine all the pieces.

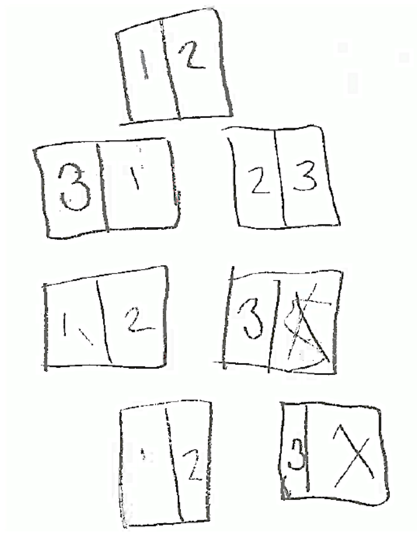

Figure 2.7. Class Performance Frame Example: Student Response to the Problem of 3 Friends Sharing 7 Cookies.

In this example, the teacher wanted to group this child with another child in the class who had solved the problem correctly by partitioning in a way that did not leave any cookies unused. The teacher considered the child's partitioning into halves and how she would benefit from comparing her strategy with the strategy of another child who used a different strategy, with the goal of getting her to partition differently so that she could solve the problem correctly. The *class performance frame* was used by 70% of the teachers.

The third frame in the mathematical performance comparison category, the *broader scope* frame, drew on teachers' expectations from goals that were beyond the performance of the specific children in the teachers' class. In this frame, teachers attended to the mathematical details in a child's strategy but highlighted how a child's performance on this problem compared with the broader curricular or testing goals for the grade level. For instance, in the problem of 10 people sharing 85 pounds to determine how many pounds each person would get, a child solved by writing a division problem (backwards) 10 ÷ 85 = 8 remainder 5. The child stated in their explanation, they knew 8/8 made a whole and that the remaining 5 was really 5/8 (see Figure 2.8).

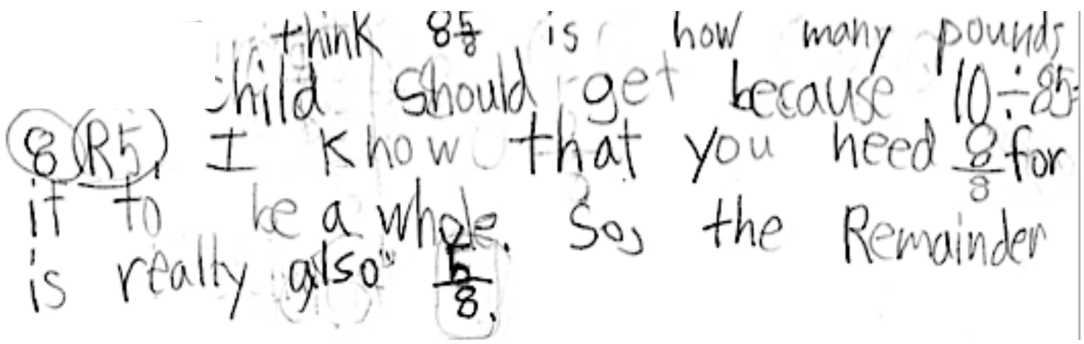

Figure 2.8. Class Performance Frame Example: Student Response to the Problem of 10 People Sharing 85 Pounds.

The teacher used a *broader scope* frame when asked to share next steps for this child. She responded, “I want [the class] to start notating their thinking more since on their [state standardized test] they have to write out everything. They cannot draw to show their work.” Here, the teacher framed her analysis to focus on the desired use of more notation for the child and the entire class based on her knowledge of testing expectations. The *broader scope* frame was used by 30% of the teachers.

Extended example of frames in use. When teachers talked about their students’ written work, they often used multiple frames or the same frames multiple times throughout the interview. The following example shows the use of various frames in the interview with a teacher, Ms. Young, and written work from her student, Jordan. Ms. Young posed the following problem to the class: *10 friends want to share 19 brownies* *equally. How much brownie will each friend get?* Jordan solved the problem by drawing 19 rectangles to represent the brownies (See Figure 2.9). She numbered the first ten brownies 1–10 to give each friend one brownie. Jordan then split the next five brownies into halves and gave one half to each friend, as indicated by the numbers 1–10 in each half. Finally, Jordan split the remaining four brownies into fifths and gave each person one fifth of a brownie from every two brownies, as indicated by the numbers 1–10 in the fifths of the first two brownies and again in the second two brownies. Thus, each friend should have received 1 whole brownie, 1/2 of a brownie, and 2/5 of a brownie, totaling 1 9/10 brownies. However, Jordan incorrectly combined her pieces to answer 1 7/10. It is important to note that Jordan solved this problem using a valid strategy but had an incorrect answer given her error on the final step of combining the three different-sized pieces.

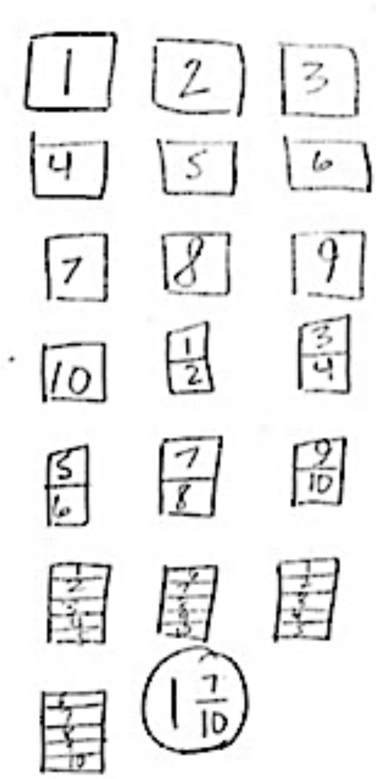

Figure 2.9. Jordan's Strategy for the Problem of 10 Children Sharing 19 Brownies.

Below is an excerpt from an interview about Jordan's strategy in which multiple frames were used:

> **Interviewer:** What stood out to you about Jordan's work?
>
> **Ms. Young**: Jordan's answer did not match her work. It's okay, but she is just not confident in her work. That is why she added them altogether. However, the rest of the class did what I expected them to do. Most of the class broke up the brownies into tenths, like I expected. Jordan did not do what I expected her to do.

First, Ms. Young observed the mathematically important detail that Jordan's answer of 1 7/10 did not match how she had assigned each child 1 whole brownie, 1/2 of a brownie, and 2 one fifths of a brownie. Ms. Young was surprised that Jordan responded to the problem incorrectly despite having a valid strategy. She used the *confidence frame* as a rationale for Jordan's mistake of adding the fraction pieces incorrectly. According to Ms. Young, Jordan's overall lack of confidence in her work was the reason for adding incorrectly. She also compared Jordan's work to the work of other students in the class, using the *class performance* frame, because unlike Jordan, most of the class had solved the problem in the way Ms. Young anticipated they would. Ms. Young had an overall expectation that the class would solve the problem by partitioning all the brownies into the number of sharers (10) and Jordan did not partition as anticipated, which was a reason her work stood out.

Later in the interview, Ms. Young also used the *past performance* frame to discuss Jordan's understandings:

> **Interviewer**: Based on Jordan's work, what do you think she understands?
>
> **Ms. Young:** I think she can be flexible with her fractions. It's just interesting that she broke it up into halves first and then fifths. Usually, she does division and writes out the answer (without drawing a picture), so I don't know why she ended up drawing a picture today.

Ms. Young started to describe the mathematically important details in Jordan's strategy—her process of partitioning the brownies into halves and fifths. Then the conversation shifted and Ms. Young utilized the *past performance* frame to express how she expected Jordan to solve the problem with a more advanced strategy than using a picture because drawing a picture was not common for Jordan. According to Ms. Young, Jordan typically solved these problems by writing a division equation and then the answer, but for this problem she drew a picture and used halves and fifths. In sum, Ms. Young's interview reflected a use of three frames as she noticed Jordan's mathematical thinking in her written work. This extended example illustrates how teachers sometimes used multiple frames but it is important to mention that not all frames were used in every teacher's interviews.

Phase 1: Conclusion

In the first phase of my study, I explored the frames teachers used when noticing children's mathematical thinking in written work from their own classrooms. I hypothesized that since these 40 teachers participated in PD that emphasized a *children's thinking* frame, they would use that particular frame in their interviews. I was correct in my conjecture in that 100% of the teachers used the *children's thinking* frame. However, this phase uncovered the use of multiple frames in teachers' noticing in addition to the *children's thinking* frame. Teachers not only highlighted the current mathematical details within the child's strategy, but also used frames that focused on non-mathematical aspects of the child's performance or drew comparisons between the child's performance and the expected performance based on the child's past performance, the performance of the rest of the class, or curricular or testing guidelines. Thus, framing served as a useful construct for understanding the complexity of teachers' noticing of children's thinking in written work from their own classrooms.

In this first phase of my study, six frames emerged from teachers' noticing of children's mathematical thinking in one source of written work—teachers' own classrooms—providing insight into my overall investigation of teacher noticing in written work from multiple sources. The second phase of my study extended my understanding of frames by considering teachers' noticing of children's thinking in written work from unfamiliar classrooms.

Table 1.1

Frames Used by Teachers While Engaging With Children's Mathematical Thinking

Category of Frames	Frames	Definition	Example	No. (%) of teachers using frame N = 40
Current Mathematical Performance	Children's Thinking	Teacher highlights the mathematical details within a child's strategy.	"He started with 12 friends up at the top and then drew 8 cakes. He knew to divide those cakes into 12 and numbered them 1, 2, 3, 4, 5 6, 7, 8, etc., and divided them into twelfths. He circled the twelfth piece on each [cake] and used repeated addition to get eight twelfths." (Figure 2.2)	40 (100%)
Non-Mathematical Performance	Confidence	Teacher attends to the mathematical details in a child's strategy but highlights a child's confidence related to his or her problem-solving performance.	"At first they did nothing. I had to walk them through the problem. They have the knowledge to solve the problem, but [did] not have the confidence to do it on their own without assistance." (Figure 2.3)	10 (25%)

Category of Frames	Frames	Definition	Example	No. (%) of teachers using frame N = 40
	Behavior	Teacher attends to the mathematical details in a child's strategy but highlights a child's behavior related to his or her problem-solving performance.	"I couldn't understand why [pause]. For the problem, … he had the drawing and he had it divided but he struggles a lot with attention and gets side tracked. He had it all right there [referring to the drawing] and he had the pieces at the top. He didn't realize he missed the piece at the bottom." (Figure 2.4)	2 (5%)
Mathematical Performance Comparisons	Past Performance	Teacher attends to the mathematical details in a child' s strategy but highlights how the child's performance on this problem compares with his or her prior work.	"He drew two rectangles and split each into 6 pieces, but no answer was written. I was surprised because yesterday, he had an invalid strategy. He might know more than I thought." (Figure 2.5)	25 (63%)
	Class Performance	Teacher attends to the mathematical details in a child's strategy but highlights how the child's performance compares with the performance of the rest of the class on this problem or previous similar problems.	"He understands that if there are 20 [items], you have 2 groups of 8 with a remainder of 4. He did the division problem in his head, while everyone else [in the class] first passed out wholes individually [one-by-one]. In some ways, he is more advanced than the others." (Figure 2.6) "I want her to explain her thinking to another	28 (70%)

Category of Frames	Frames	Definition	Example	No. (%) of teachers using frame N = 40
			student and compare it [her partitions] to another student to see how [the other student] partitioned differently [not using halves]. By pairing her up with another student, that could get her to see how to partition to be able to combine all the pieces." (Figure 2.7)	
	Broader Scope	Teacher attends to the mathematical details in a child's strategy but highlights how the child's performance compares with the broader curriculum or testing goals for the grade level.	"I want [the class] to start notating their thinking more since on their [state standardized test] they have to write out everything. They cannot draw to show their work." (Figure 2.8)	12 (30%)

CHAPTER IV

PHASE 2: PD CONVERSATIONS

In Phase 2 of the study, I extended my understanding of individual teachers' use of frames as they noticed children's mathematical thinking in written work from their own classrooms to explore the range of frames used when small groups of teachers worked together in PD to notice children's mathematical thinking in written work from unfamiliar classrooms. In this phase, I specifically addressed the following question: What frames do teachers use when noticing children's mathematical thinking in written work from unfamiliar classrooms?

Methods of Phase 2: PD Conversations

In this phase, I again drew on existing data from the RTEM project, and to facilitate connections with Phase 1, I kept the focus on equal sharing problems. I chose to investigate video-recorded PD conversations of small groups of teachers who were asked to examine equal sharing strategies in written work that was strategically selected by the facilitator and drawn from classrooms unfamiliar to teachers. I chose to include only the conversations for which I had a video because video was needed to understand how teachers were interacting with the specific features of the written work (e.g., representations, partitions, distributions, etc.).

Participants. Phase 2 of the study involved criterion sampling (Patton, 2002) of a subset of teachers from the RTEM study. The criterion used for selection was any teacher who participated in the video-recorded PD conversations selected for analysis. Participants in Phase 2 consisted of 37 teachers (34 females and 3 males) including 17 third grade teachers, 9 fourth-grade teachers, 9 fifth grade teacher, 1 second-third grades teacher, and 1 fourth-fifth grades teacher. These teachers were distributed across the 3 school districts and reflected a range in overall years of teaching experience (1–32 years, *M* = 7 years) and years of teaching experience specific to grades 3–5 (0–17 years, *M* = 4 years). Participants were drawn from all three cohorts, and because all the PD conversations selected occurred during the first year of the PD, participants were early in their learning about children's fraction thinking. However, about one third had previously participated in CGI PD on children's mathematical thinking with whole numbers. Note that there was some overlap of participants with Phase 1 in that 18 teachers participated in both phases.

Data source. Data in the second phase were drawn from existing data collected as part of the RTEM study data-collection process and included 5 PD activities totaling 17 video-recorded conversations of small groups of 2–4 teachers engaging with written work from unfamiliar classrooms (See Table 2.1 for an overview of the 5 activities). These PD conversations ranged in duration (11–35 minutes, *M* = 21 minutes) and in the activity directions. Sometimes teachers were asked to look through sets of written work to sort strategies based on similar features and then either order them regarding levels of sophistication or generate follow-up questions to pose to the authors of the written work.

Other times, prompts called for teachers to categorize strategies based on a specific framework of children's fraction thinking used throughout the PD (Empson & Levi, 2011).

Similar to the Phase 1 interviews, the PD conversations in Phase 2 provided a general sense of teachers' noticing of children's mathematical thinking in equal sharing problems. However, during Phase 2, teachers noticed children's mathematical thinking in small groups using a set of written work from unfamiliar classrooms in contrast to noticing an individual child’s thinking in one piece of written work from their own classes in Phase 1.

Table 2.1

Activity Overview of PD Conversations

PD Activity	Number of PD Activities (*N*=17)	Activity Instruction	Problem	Cohort
1	2	Sort written work and pose questions	6 children sharing 14 brownies	1
2	3	Categorize strategies based on the equal sharing framework	6 children sharing 10 sandwiches	2
3	4	Sort written work and order in terms of levels of sophistication	6 children sharing 16 brownies	2
4	4	Categorize strategies based on the equal sharing framework	4 children sharing 15 sandwiches	2
5	4	Sort written work and order in terms of levels of sophistication	6 children sharing 16 brownies	3

Data analysis. Analysis of PD conversations occurred in three stages. In the first stage, conversations for each episode were divided into idea units that captured when a single topic was discussed by a group of teachers (Jacobs & Morita, 2002). During the second stage, the teachers' focus for each idea unit was determined for coding—the child, the child within the class, or beyond the child and the class. In this stage, the six frames from Phase 1—*children's thinking*, *confidence*, *behavior*, *past performance*, *class performance*, and *broader scope*—were applied in the coding of each idea unit. Although those frames served as a basis for the analysis, opportunities were available for additional frames to surface from the data. In the final stage, I noted the prevalence of the six frames, identifying whether or not each small group had used the frame at least once.

Phase 2: Results

The primary goal of the analysis of the PD conversations was to confirm the presence of the existing frames identified in Phase 1, enhance my understanding of these frames, and provide a chance for the emergence of new frames. No additional frames were identified, and I found that some but not all of the six frames used in individual teachers' noticing of children's mathematical thinking in written work from their own classrooms applied during small group conversations of teachers' noticing in written work from unfamiliar classrooms. Overall, there was generally less use of frames in the PD conversations which were relatively short and did not include any follow-up questions. Further, of the six previously identified frames, three persisted (*children's thinking*, *class performance*, and *broader scope*) and three were absent (*confidence*, *behavior*, and *past* *performance*). See Table 2.2 for a summary of the prevalence of frames across PD conversations.

Table 2.2

Prevalence of Frames in PD Conversations

Frames	No. (%) PD Conversations (N = 17)
Current Mathematical Performance	
Children's Thinking	17 (100%)
Non-Mathematical Performance	
Confidence	0%
Behavior	0%
Mathematical Performance Comparison	
Past Performance	0%
Class Performance	3 (18%)
Broader Scope	3 (18%)

This pattern also helped me understand important distinctions between the ways teachers engaged with the different frames. Specifically, the *children's thinking* frame drew upon teachers' engagement with the written work being discussed. The *class performance* and *broader scope* frames drew upon teachers' prior experiences as upper elementary school teachers and, in particular, their work with students on equal sharing problems and mathematics standards. The *confidence*, *behavior*, and *past performance* frames require teachers' ability to apply insider knowledge based on their experiences with a specific child—therefore, it is not surprising that these three frames were evident in Phase 1 data when the written work was from teachers' classrooms and were not evident in Phase 2 when the written work was from children unfamiliar to the teachers.

In the sections that follow, I provide examples of each of the frames that were used in Phase 2, highlighting ways in which my understanding of the frames was elaborated

Children's thinking frame. Teachers' use of the *children's thinking* frame in 100% of the conversations was expected because within the PD, teachers were given multiple opportunities to notice children's mathematical thinking in equal sharing problems, often using the equal sharing framework as a tool for making sense of children's equal sharing strategies. Further, because teachers did not know the child or children authoring the written work, a focus on things related to the PD content was likely.

In the following example, I provide a glimpse of what group conversations looked like when focused on children's thinking. Teachers were asked to categorize strategies according to the equal sharing framework for the problem: *Subway provided 10 sandwiches for a child's birthday party. If there were 6 guests at the party, how much sandwich would each guest get?* Below is an excerpt from a transcript of a group's discussion about Max's strategy (see Figure 3.1) in which the children's thinking frame is used.

T1: So, the first six squares are the six--.

T2: --these are the guests. So, he knew that everyone gets one whole and then knew—

T1: And then he cut those four sandwiches left over there.

T2: but he has this last one circled (pause)

T1: Yeah because you can have six halves to give to each guest and then you have that one whole left over. He didn't even use the last one. So he is non-anticipatory because he halved the remaining [sandwiches] and just went to halving.

T2: Yeah and he did nothing with that last sandwich.

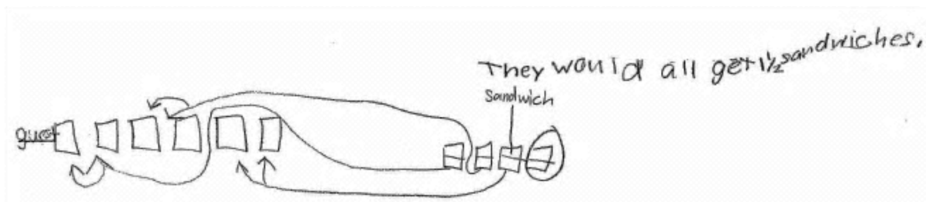

Figure 3.1. Max's Strategy for the Subway Problem of 6 Guests Sharing 10 Sandwiches.

In this discussion of Max's strategy, teachers used the *children's thinking* frame by focusing on the details of his strategy. Teachers noted that the six rectangles on the left of the paper were the guests, and the four rectangles on the right of the paper represented the sandwiches. Likewise, they recognized Max knew that each person would get a whole sandwich, even though this distribution was not indicated in his picture. Teachers further noted that Max partitioned the remaining rectangles into halves, passed out a half to each guest, and had a whole left over. Teachers said Max's strategy was *non-anticipatory*—a categorization linked to the equal sharing strategy framework used in the PD (Empson & Levi, 2011)—because his drawing indicated there was no coordination in the initial partitions between the amount being shared and the number of sharers (in this case, 6). Overall, the teachers' conversation about Max's strategy was very focused on the details within his strategy.

Class performance frame. Similar to teachers' use of the *class performance* frame in the Phase 1, teachers attended to the mathematical details in a child's strategy but highlighted the connections to their own classrooms. Specifically, even though the written work came from unfamiliar classrooms, the teachers envisioned themselves as each child's classroom teacher and compared the unfamiliar child's thinking represented in the written work to children's thinking in their own classrooms and even considered the ways that they would engage with children in their classrooms.

In the following example of a PD conversation, teachers were asked to look through a set of written work for the following problem: *Subway provided 15 sandwiches for a child's birthday party. If there were 4 guests at the party, how much sandwich would each guest get?* The teachers were focused on Eric's strategy (Figure 3.2) in which he drew 15 circles to represent the sandwiches. He knew that since there were 4 guests each person could get 3 sandwiches [denoted by 4 x 3 = 12], so he circled 3 sandwiches and distributed each set of 3 to a guest. There were 3 sandwiches remaining, and he split two sandwiches into halves and one into fourths. Eric passed out each fractional amount (one half and one fourth) to each guest, but his final answer was "5 sandwiches."

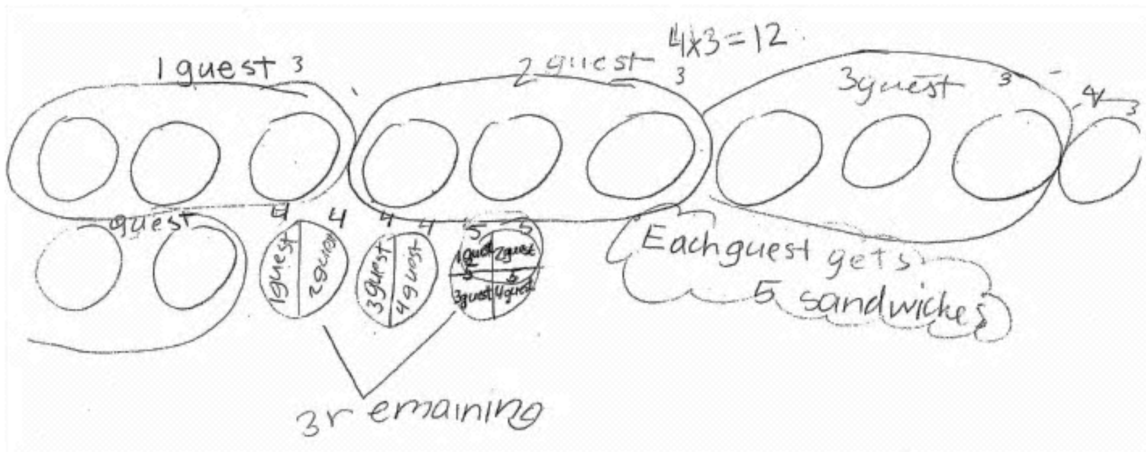

Figure 3.2. Eric’s Strategy for the Subway Problem of 4 Guests Sharing 15 Sandwiches.

During the conversation around Eric's strategy, teachers discussed that Eric shared the sandwiches equally with the 4 guests but they were concerned about how he partitioned the remaining 3 sandwiches into different fractional quantities and that each guest received 5 sandwiches. Next, the conversation shifted to focus on ways the teachers have dealt with similar strategies in their own classes by encouraging students to consider using different representations, such as changing the representation from circles to rectangles. Applying these previous teaching experiences to Eric’s strategy, teachers discussed how using rectangles for sandwiches, would have potentially helped Eric partition the remaining sandwiches based on the number of sharers (fourths instead of halves and fourths) thereby eliminating a need to add fractions with unlike denominators. This example illustrates the use of the *class performance* frame because when teachers attended to Eric’s partitioning of the three remaining brownies into different fractional amounts, not only did the teachers compare Eric's strategy to strategies used by other students in their classrooms, but also they chose to support Eric similarly to how they would help their students.

Broader scope frame. The *broader scope* frame was used similarly to Phase 1, in that teachers attended to the mathematical details in a child's strategy but highlighted how the child's performance compared to teachers' experiences with curricular or testing goals of the assumed grade level of the student. Throughout the PD, teachers engaged with written work from grades 3–5 and sometimes the grade level was noted on the written work and other times it was not mentioned. Sometimes when the grade level was not mentioned, teachers would assume the grade of the child was their grade level. In the following example, the teachers discussed Julia's strategy (see Figure 3.3) for the following problem: *Subway provided 10 sandwiches for a child's birthday party. If there were 6 guests at the party, how much sandwich would each guest get?* Julia knew that each guest would get 1 sandwich, which would use 6 sandwiches and leave 4 remaining. Next Julia divided the 4 sandwiches among the 6 guests, mentally, by dividing 4 by 6 thereby giving each person 4/6 for an answer of 1 4/6 sandwiches per guest.

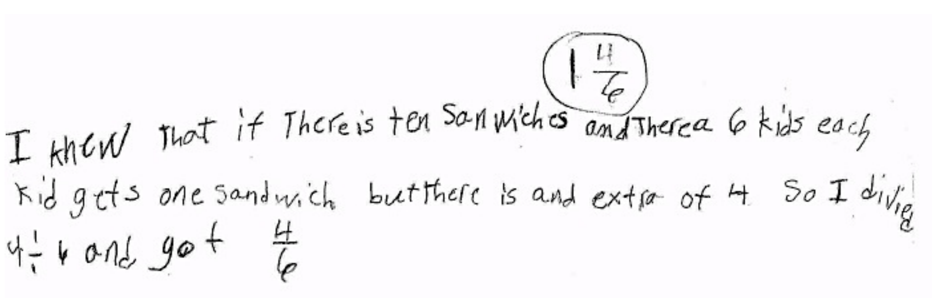

Figure 3.3. Julia's Strategy for the Subway Problem of 6 Guests Sharing 10 Sandwiches.

At the time of the conversation, teachers were using the *Common Core State Standards for Mathematics* (National Governors Association, 2010) to guide instruction, and one of the standards for fractions in fifth grade called for students to interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). For example, children can interpret 3/4 as the result of dividing 3 by 4 when 3 wholes are shared equally among 4 people. One group of teachers referenced these standards in their discussion of the details of Julia's strategy. For example, a fourth grade teacher focused on Julia's partitioning of the 4 remaining sandwiches and the equation $4 \div 6 = 4/6$. This teacher mentioned Julia's equation could easily be a learned strategy and that Julia may not know what the quantity 4/6 meant. In disagreement with this teacher's conjecture, a fifth-grade teacher compared Julia's strategy to the above standard for her fifth-graders and shared that Julia could understand the quantity based on this teacher's experiences addressing this mathematics standard with equal sharing problems in her own classroom. She specifically argued that it was possible that after Julia passed out one whole sandwich to 6 kids leaving 4 leftovers, she knew that she could divide the remaining sandwiches by the number of kids to get 4/6. This example was considered a use of the *broader scope* frame because the fifth-grade teacher used her previous experiences with a particular standard to support the claim that Julia may have an understanding of the quantity 4/6 instead of it being a learned strategy.

Phase 2: Conclusion

In Phase 2, I extended my understanding of individual teachers' use of frames when noticing children's mathematical thinking in written work from their own classrooms to written work from unfamiliar classrooms. Of the six frames previously identified, half were never used (i.e., *confidence, behavior,* and *past performance)*, and this absence was understandable given that these frames are more closely linked to teachers' insider knowledge of their own students. Teachers do not have this type of insider knowledge for students unfamiliar to them. Of the three frames that were used, the children's *thinking frame* was used similarly and just as extensively as by teachers in Phase 1. The teachers' use of the *class performance* and *broader scope* frames showed similarities as well, but their use of the *class performance* frame with unfamiliar work highlighted the role that teachers' prior experiences play in their noticing. Similar to teachers' use of the *class performance* frame in the previous phase, teachers compared the child's performance to the performance of the rest of the class. The difference was that in some uses of the *class performance* frame, teachers explicitly envisioned themselves as the classroom teacher of the student even though the written work was from an unfamiliar classroom, and they responded in the same ways they would if the written work were from their own classrooms. The teachers' way of envisioning themselves as the classroom teacher and engaging with the written work as such, caused me to wonder whether an "imagined" insider knowledge in the three frames that teachers did not use in this phase could also play a role in the teachers' noticing of children's mathematical thinking in written work from unfamiliar classrooms.

Phase 1 and Phase 2 investigated the frames teachers used when noticing children's mathematical thinking in written work. In Phase 1, individual teachers were asked to notice children's mathematical thinking in written work for a range of equal sharing problems from teachers' own classrooms. In Phase 2, small groups of teachers were asked to notice children's mathematical thinking in written work for a variety of equal sharing problems from unfamiliar classrooms. In the next phase of my study, I explored the relationship between the quality of six teachers' noticing and their use of frames during in-depth interviews in which they were asked to notice children's mathematical thinking in written work for the same equal sharing problem from their own classrooms and unfamiliar classrooms.

CHAPTER V

PHASE 3: NOTICING INTERVIEWS

In Phase 1 and Phase 2, I used existing data from the RTEM project to explore the frames teachers used when noticing children's mathematical thinking in written work from two sources—teachers' own classrooms and unfamiliar classrooms. In both phases, teachers broadly engaged with the three interrelated skills of noticing children's mathematical thinking across a variety of equal sharing problems, but follow-up questions to explore these three noticing skills and teachers' use of frames were minimal or non-existent. In Phase 3, I built on what was learned in the earlier phases to collect and analyze a new set of data to investigate how the same teacher noticed children's mathematical thinking in written work from the two sources—her own classroom and an unfamiliar classroom. Phase 3 kept the focus on equal sharing problems but explicitly addressed the three component skills of noticing in a setting that allowed for follow-up questions. This additional probing allowed me to track the frames each teacher used and the quality of each component skill of noticing with the two sources. Overall, the goal of Phase 3 was to begin to identify the relationship between teachers' use of frames and their noticing quality, with special attention to the role that the source of the written work might play. In the following sections, I will share the methods and findings of Phase 3 in which I specifically explored the following three research questions:

(1) What frames do individual teachers use when noticing children's mathematical thinking in written work from their own and unfamiliar classrooms?

(2) What is the quality of individual teachers' noticing when noticing children's mathematical thinking in written work from their own and unfamiliar classrooms?

(3) What is the relationship between teachers' use of frames and the quality of their noticing?

Methods of Phase 3: Noticing Interviews

In Phase 3, I interviewed six teachers to investigate how the same teacher noticed children's mathematical thinking in written work from different sources but involving the same story problem. Thus, teachers were interviewed twice using written work linked to the following fraction story problem:

The baker has 10 small cakes to share equally among 6 children. How much cake does each child get?

One interview focused on written work from teachers' own classrooms and the other on a common set of written work from a classroom unfamiliar to the teachers.

Participants. I worked with 6 teachers from District C in a single RTEM cohort at the end of their third (final) year of the PD (See Table 3.1). I used homogenous sampling—a type of purposeful sampling used to describe a particular subgroup in depth and reduce variation, thus resulting in participants that have a shared set of characteristics (Creswell, 2013; Teddlie, 2007). Specifically, I held the teachers' district, cohort, and number of years of PD constant. I wanted to hold these factors constant to facilitate comparison across the six teachers given my expectation that contextual factors (e.g., instructional policies, adopted textbooks, curriculum pacing, PD support, etc.) could shape the frames teachers used when noticing children's mathematical thinking and potentially the quality of their noticing.

Table 3.1

Phase 3 Participant Teaching Experience

	T1	T2	T3	T4	T5	T6
Current Grade Level	3rd	4th	3rd	4th	4th	4th
Years of teaching						
Total	9	6	5	15	8	20
K–2	0	0	0	5	0	12
3–5	9	6	5	10	8	8
Current school	9	6	5	9	8	5
District C	9	6	5	10	8	5

I selected teachers from District C because this district was in the early stages of embracing a vision of mathematics instruction that is responsive to children's thinking, and thus district philosophies and resources were still emerging, providing fertile ground for frames that could compete with a children's thinking frame (See Figure 2.1 for the demographic data of District C.). All teachers (N=8) from District C in Cohort 3 were invited to participate, but 2 teachers were unavailable due to scheduling conflicts (e.g., district-wide science professional development at grade 5). The 6 participating teachers were female and included 2 third grade and 4 fourth grade teachers. Participating teachers ranged in years of teaching experience (5–20 years, M = 11 years) with the majority of those years in grades 3–5. Most of their teaching experiences in grades 3–5 were at their current schools in District C.

Data sources. Data in Phase 3 included two video-recorded semi-structured interviews of each teacher engaged in noticing children's mathematical thinking in written work from teachers' own classrooms and unfamiliar classrooms, totaling 12 interviews. The first six interviews involved a common set of written work (researcher-selected) from students unfamiliar to the teachers, and interviews lasted from 29–53 minutes. The second six interviews involved written work from teachers' own classrooms, and interviews lasted from 32–43 minutes. Interviews took place after school or on the weekend, and each teacher's two interviews took place within a week of each other. I purposefully sequenced the interviews in this order because I conjectured that the quality of teachers' noticing of children's mathematical thinking would be stronger in written work from unfamiliar classrooms. When teachers do not know the child, they may be more likely to focus solely on the current mathematical details within the child's strategy rather than use their insider knowledge of that child, which can be faulty and compete with the mathematical understanding reflected in the child's current strategy. The next sections describe the protocol for both noticing interviews followed by the process for selecting written work for each interview.

Protocol for both noticing interviews. Teachers were shown four pieces of written work and asked to respond orally to prompts linked to the three component skills of noticing: attending, interpreting and deciding how to respond to children's mathematical thinking. I used two prompts for the component skill of deciding how to respond: (a) deciding how to respond with a follow-up problem and (b) deciding how to respond in a one-on-one conversation with one of the students (see Appendix A). In addition to providing noticing prompts, I listened to the teachers' responses and posed follow-up questions as general questions, probes for frames, and probes for rationales linked to specific noticing skills. Note that I never explicitly mentioned the frames by names and instead chose to follow-up on teachers' comments that spontaneously invoked the use of frames.

Noticing interview 1: Selection of written work from unfamiliar classrooms. I selected four pieces of written work from unfamiliar classrooms for the cake problem (see Appendix B, and all strategies were given female names to reduce any gender biases related to children's levels of understanding associated by gender (Leyva, 2017). In a review of research on gender in mathematics education, Leyva noted findings from studies that reported teachers' use of a deficit view of female students' understandings related to mathematics tasks in comparison to males in their classrooms. All of the strategies selected were valid, in that the correct answer could be derived from the strategy, and each used some form of fraction notation. In addition, my selection criteria ensured that the set of strategies: (a) reflected a range of children's understandings (Empson & Levi, 2011), (b) demonstrated complexity in the overall representation, and (c) included a range of strategy features for which teachers' perspectives on the desirability of these features have been shown to be inconsistent with the research on children's thinking (Jessup, Hewitt, Jacobs, & Empson, 2015). Each of these three criteria are elaborated in the following sections.

Range of children's understandings. I wanted to ensure that the set of strategies reflected the range of levels of understanding depicted in the equal sharing framework used in the PD (Empson and Levi, 2011). This framework described three categories of strategies based, in part, on the ways children did or did not coordinate their partitions with the number of sharers, and these categories were ordered to reflect increasing levels of understanding of fractions: *non-anticipatory direct modeling*, *emergent anticipatory direct modeling*, and *anticipatory*. The four pieces of written work reflected each of the three categories, with two examples of the category reflecting the least amount of understanding, one example in the middle category, and one example reflecting the most amount of understanding. In the following sections, I describe the four pieces of written work and use them to define and illustrate the three strategy categories.

First, Alicia's strategy (see Figure 4.1) shows the least amount of understanding of fractions and is considered a *non-anticipatory direct modeling strategy* because she represented every quantity and every share in the problem and did not coordinate the amount being shared and the number of sharers at the start of the strategy. Specifically, Alicia drew 10 big circles to represent the 10 cakes and then 6 smaller circles to represent the 6 children. The 10 cakes showed evidence of erasures, suggesting that Alicia may have initially partitioned everything in thirds, erased her work (probably because that partitioning did not yield an even number of pieces for the 6 children), and then partitioned everything in fourths. After Alicia distributed 6 fourths to each child (by numbering each fourth 1–6), she erased the fourth partitions in the last circle, and instead partitioned the circle based on the number of sharers (6). She also indicated that she gave each person one sixth by numbering each sixth 1–6. For her final answer, Alicia stated each person received 6/4 and 1/8 of the cakes. She likely counted the number of one fourths that each person received to arrive at the answer of 6/4, but her answer of 1/8 is more puzzling. The 1/8 likely came from the last circle that was cut into sixths instead of eighths, perhaps indicating that she was unsure how to label the one sixth pieces.

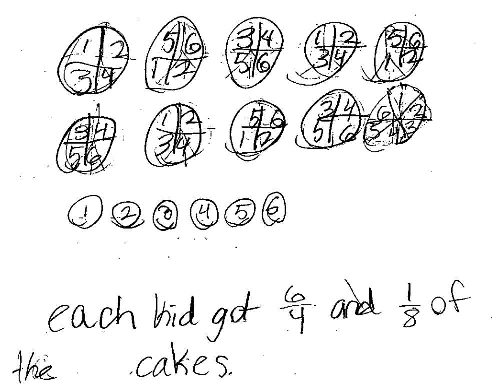

Figure 4.1. Alicia's Non-Anticipatory Direct Modeling Strategy for the Problem of 6 Children Sharing 10 Small Cakes.

Second, Emily's strategy (see Figure 4.2) is also considered *non-anticipatory direct modeling*. Specifically, Emily represented her 10 cakes as circles and her 6 people as stick figures and distributed a whole cake to each child by drawing lines from each cake to a child, and then marked out that cake. At the bottom of the page, she also drew three circles, each with a person inside, and kept a running total of the quantities distributed to that person in the circle. At this point, she included a 1 (for 1 whole cake) in each circle. Emily then split the 4 remaining cakes into halves and re-drew these quantities as eight separate halves, labeling that portion of the drawing as "8 halves." Next, Emily distributed a half to each child and indicated this distribution by drawing a line from each half to one child's circle and writing "1/2" in that circle to represent how much that child received. She crossed out the six halves that she had used in that distribution and re-drew the remaining two halves as 6 sixths, essentially splitting each half into three pieces. Each sixth was again distributed with the distribution indicated by lines from each piece to the child's circle and the amount (this time, "1/6) written in that child's circle. Emily did not provide a final answer, but based on the amounts indicated in each circle, each child received 1 whole, 1/2 and 1/6.

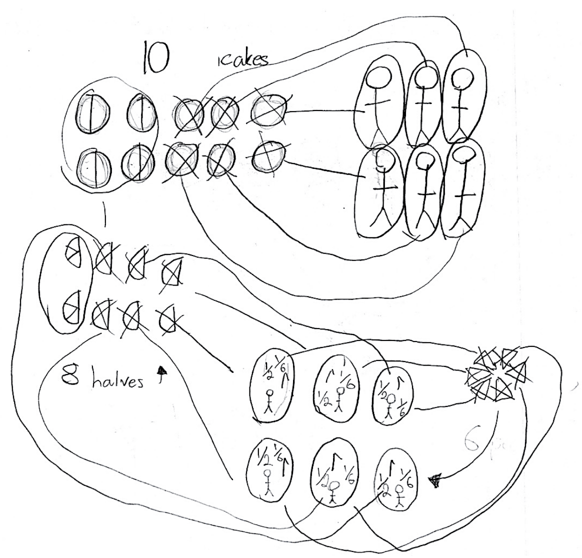

Figure 4.2. Emily's Non-Anticipatory Direct Modeling Strategy for the Problem of 6 Children Sharing 10 Small Cakes.

Third, Katie's strategy (see Figure 4.3) shows the next highest level of understanding of fractions and is considered an *emergent anticipatory direct modeling strategy* because she again represented every quantity and every share in the problem, but this time, her partitions showed that she coordinated the shared items and number of sharers from the beginning of her strategy. Specifically, Katie represented her cakes by drawing ten rectangles and children using six stick figures. She coordinated the number of sharers by grouping two cakes to share among six children. Therefore, she partitioned each cake into thirds, allowing each child to receive one third of a cake for every two cakes. Katie distributed 5 one thirds to each child by repeatedly numbering the pieces 1–6, which indicated which piece went to each of the 6 children. Katie's final answer indicated that each kid received 1 2/3 cakes, but it is unclear how Katie grouped her 5 one thirds to get 1 whole and 2 thirds.

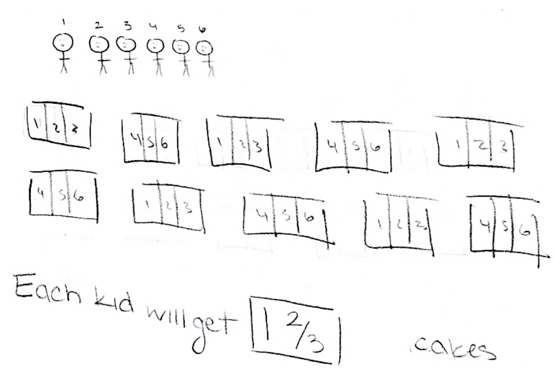

Figure 4.3. Katie's Emergent Anticipatory Direct Modeling Strategy for the Problem of 6 Children Sharing 10 Small Cakes.

Fourth, Monica's strategy (see Figure 4.4) shows the most amount of understanding of fractions and is considered an *anticipatory strategy* because she mentally coordinated the amount being shared and the number of sharers. Specifically, Monica was able to mentally coordinate 10 cakes being shared by 6 children and determine that each child would receive 10/6 cakes. She worked exclusively with symbols, without needing to represent the cakes and children. It is unclear whether Monica began with the 10/6 or the equation, but either way, she knew that this problem could be solved by division. Monica wrote the equation 10 ÷ 6 to represent the 10 cakes shared among 6 children, and she demonstrated her understanding of the quantities by writing "cakes" under 10, "divided among" under the division symbol, and "children" under the 6. Monica knew that 10 ÷ 6 was 10/6 and that 10/6 was the same as 1 4/6, but it is unclear how she knew the equivalence relationship between 10/6 and 1 4/6.

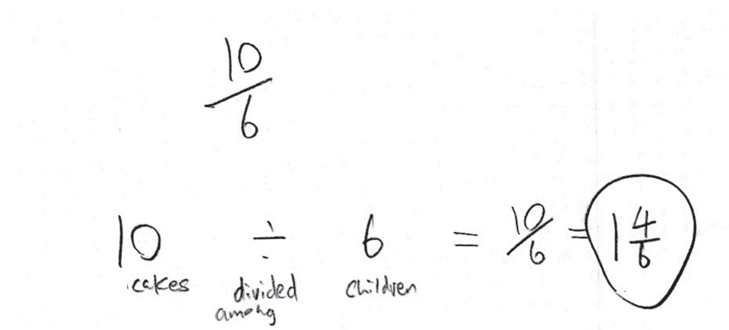

Figure 4.4. Monica's Anticipatory Strategy for the Problem of 6 Children Sharing 10 Small Cakes.

Complexity in the overall representation. I selected written work that demonstrated complexity in the overall representation—giving teachers much to notice. I looked for representations that were messy, such as the use of several lines or arrows showing distributions or had erasures showing multiple attempts (e.g., Alicia's strategy). Additionally, I considered representations that included multiple parts or multiple types of partitions (e.g., Emily's strategy). Lastly, I selected written work that included nonconventional fraction notation or pictures (e.g., Emily's strategy).

Range of controversial strategy features. I selected written work that included a range of strategy features for which teachers' perspectives on the desirability of these features have been shown to be inconsistent with the research on children's thinking.

Jessup and colleagues (2015) studied teachers in their first year of the RTEM PD and captured their initial perspectives on desirable strategy features for children's work on equal sharing problems prior to learning about the corresponding research. Findings identified five strategy features teachers preferred and often used as their rationale for indicating why a particular strategy was more sophisticated than another: (a) leftover items were partitioned; (b) fraction notation was predominantly used (vs. drawing); (c) whole items were distributed prior to any partitioning; (d) the largest possible partitions were used; and (e) the answer was in the form of a mixed number (vs. an improper fraction or non-conventional notation). The first two perspectives were consistent with research on children's fraction thinking, whereas the last three perspectives were not. I was particularly interested in selecting written work that provided an opportunity for teachers to use these inconsistent perspectives. Specifically, I chose a problem in which there were more items than the number of sharers so that teachers had an opportunity to consider whether children had distributed whole items. Second, so teachers had an opportunity to consider whether children had used the largest partition, I chose a set of strategies in which multiple sized partitions were visible—across the four strategies, there was a range of initial partitions (e.g., halves, thirds, and fourths) and some strategies that used multiple partitions. Third, I selected strategies that were all valid but had different forms of the answer. There was one incorrect answer and three correct answers in different forms (i.e., mixed number, improper fraction, and non-conventional [Emily's specification of the pieces 1, 1/2, 1/6 rather than a combined amount]) so that teachers had an opportunity to compare them.

Noticing interview 2: Selection of written work from teachers' own classrooms. Immediately following the first noticing interview, teachers were given copies of the same cake problem to pose to their classes. Teachers were asked to pose the problem to their class on the same day as their second noticing interview, which occurred within a week of their first interview. Teachers were encouraged to integrate the problem into their mathematics class in a way that was typical for their problem-solving lessons, and no additional preparation was needed before posing the problem. I attempted to reduce chances for teachers to review the written work before the noticing interview by collecting the work at the end of the teachers' designated mathematics lesson in which they posed the problem.

In preparation for each teacher's noticing interview involving written work from their own classrooms, I needed to select four pieces to discuss from her class set of written work. Therefore, between the mathematics lesson and the interview, I analyzed the class set and selected four pieces that matched as closely as possible the criteria I used for written work from unfamiliar classrooms. First, I tried to select strategies that captured a range of understandings based on the equal sharing framework. When possible, I included one strategy from each equal sharing strategy category to reflect a similar range of understandings reflected in written work from unfamiliar classrooms. Second, I attempted to mirror similar complexities found in the first set of written work in terms of the overall complexity of the representations. I tried to select representations that were messy, showed potential erasures, included multiple parts or multiple types of partitions, or used nonconventional fraction notation or pictures. Third, I tried to select strategies that included features for which teachers' perspectives on the desirability of these features have been shown to be inconsistent with the research on children's thinking. In addition to the criteria used when selecting work from unfamiliar classrooms, I also considered the ratio of male and female students in the overall selection of strategies to make sure both genders were included. I tried to pay close attention so that within a set of written work there was not a dominance of one gender showing the most understanding or the least understanding, when possible.

In my selection of the written work, I followed a process which began by looking through a teacher's entire set of written work and sorting strategies based on the use of pictures. Strategies were then organized into two main stacks; (1) strategies that used pictures or (2) strategies in which the child partitioned mentally or symbolically without representing items. Within the stack of strategies using pictures, I sorted strategies based on how they partitioned (e.g., halves, thirds, fourths, and others). Across the set of strategies using pictures, I kept track of strategies that used a picture only, fraction words, fraction notation only, or a mix of pictures, fraction words, and fraction notation. In the stack of strategies that reflected mental partitioning without representing items, I sorted strategies according to the use of equations. Strategies were grouped according to no equation used (answer only), use of addition equation only, or use of an equation that combined addition and multiplication. Across both main stacks of strategies, I noted if there was an error in the strategy reasoning, an incorrect answer with a valid strategy, or if the strategy was unique in relation to the rest of class. Once the set of written work was sorted, I used the three main criteria described above to select the four strategies and then tried to include a variety of the other features I tracked. The set of written work selected from each teacher's class is found in Appendix C.

***Data analysis*.** The goal of the analyses of the noticing interviews was to understand the frames teachers used during noticing, the quality of teachers' noticing, and the relationship between the two. Analyses in this phase were divided into three stages. In the first stage, I analyzed frames used by teachers as they noticed children's mathematical thinking in written work from their own classrooms and from unfamiliar classrooms. In the second stage of the analysis, I explored the quality of teachers' noticing in written work from different sources. In the third stage, I looked across both teachers' use of frames during noticing and the quality of their noticing of children's thinking to consider a possible relationship.

*Frames***.** The analysis of teachers' use of frames in the noticing interviews occurred similarly to the three stages used in Phase 2, but in this case I tracked the number of times that teachers used each frame rather than simply attending to whether or not a teacher used a particular frame. In the first stage, each interview transcript was divided into idea units that captured when a single topic was discussed (Jacobs & Morita, 2002). The total number of ideas units across all interviews was 307 (168 in interviews with written work from unfamiliar classrooms and 139 in interviews with written work from teachers' own classrooms), and individual teachers generated 17–35 idea units in a single interview (17–35 when the written work was from unfamiliar classrooms and 19–28 when it was from their own classrooms). The second stage determined the focus of each idea unit for coding—the child, the child within class, or beyond the child and class.

The six previously identified frames (i.e., *children's thinking*, *confidence*, *behavior*, *past performance*, *class performance*, and *broader scope)* from Phases 1 and 2 were used as the primary set of codes, while leaving opportunities for the emergence of new codes from the data, but none occurred. In the third and final stage, the prevalence of frames used in the noticing interviews was analyzed, and in this case I tracked both whether or not each teacher used a frame and how often. Interrater reliability was determined by having a second individual code both of the interviews for two teachers (for a total of 4 interviews). These teachers were one third of my participants and their use of frames reflected approximately 30% of the total number of frames used. In selecting the two teachers for coding by myself and another individual, I tried to ensure that all of my frames were represented at least once. Interrater reliability was calculated at 80% or higher, and any discrepancies were resolved through discussion.

*Noticing***.** The analyses of the quality of teachers' noticing of children's mathematical thinking in written work from both sources began by dividing the interview transcripts into the three component skills of noticing—attending, interpreting, and deciding how to respond—for coding. I developed a holistic rubric that characterized teachers' use of children's mathematical thinking as the basis of their evidence by starting with the noticing interviews in which the written work was the same. Coding each component skill of noticing was influenced by previous research on noticing children's mathematical thinking (see Jacobs, Lamb, Philipp, & Schappelle, 2011; Jacobs, Lamb, & Philipp, 2010).

First, I determined a score for each teacher's response for each component skill on a 3-point continuum: robust, limited, or lack of evidence of use of children's mathematical thinking. I looked for evidence of use of children's thinking after the specific prompt related to that component skill and throughout the entire interview in case there was related information elsewhere. For the attending component skill, I looked for how the teachers attended to the mathematically important details within the children's strategies. Discussion of mathematically important details for equal sharing strategies included details such as whether the children represented every share, partitioned based on the number of sharers, distributed equal shares, used fraction notation and words, or combined unit fractions for a final answer. For the interpreting component skill, I considered how teachers made sense of the details within the children's strategies linked to research on children's strategies for equal sharing problems (Empson & Levi, 2011). Specifically, I paid attention to whether the teacher focused on what the children understood versus did not understand and perhaps what the teacher wished the children had done. There were two prompts for the component skill of deciding how to respond. First, in deciding how to respond with a follow-up problem, I looked for how each teacher made sense of the children's strategies, developed a problem that was based on what was learned about the children's understandings, and the consistency with research on children's thinking. Second, in deciding how to respond in a one-on-one conversation, I considered whether the teacher referenced the child's thinking, left opportunities for the child's future thinking, and the consistency with research on children's thinking.

I calculated each teacher's overall noticing score by holistically looking across the teacher's noticing scores for each component skill. I wanted to weight each component skill—attending, interpreting, and deciding how to respond—the same so I thought of the teacher's two deciding-how-to-respond scores together when determining the overall noticing score. The following sections provide a sense of each level of the overall noticing scores, which varied on these three dimensions related to the component skills of noticing: (a) attention to the mathematically important details of the strategy, (b) link between the teachers' reasoning about the children's understandings and strategies, and (c) appreciation of the children's mathematical thinking in determining next instructional steps.

Robust evidence of using children's mathematical thinking. Robust responses provided substantial evidence of attention to the mathematically important details in the children's strategies, and possible interpretations of the children's reasoning were based on evidence within the strategies. Robust responses also showed an appreciation for the children's sense-making in the teachers' proposed next steps.

Limited evidence of using children's mathematical thinking. Limited responses provided some evidence of attention to the mathematically important details in the children's strategies, and possible interpretations of the children's reasoning were based on evidence within the strategies. Responses that were considered limited demonstrated some level of appreciation for children's thinking but were vague. Often limited responses were general in their plans for next instructional steps.

Lack of evidence of using children's mathematical thinking. Responses that were coded as lack of evidence provided no to minimal evidence of attention to the mathematically important details in the children's strategies. Interpretations of the children's reasoning discussed were often not linked to the strategies and instead focused on possible ways the teachers would have wanted the children to reason. Lack of evidence responses often did not show appreciation for the children's sense-making, but instead, the focus for the next instructional steps was more on the teacher's goals, which were typically outside of the understandings the children had shown in the strategies.

Interrater reliability was determined by having a second individual code both of the interviews for two teachers (for a total of 4 interviews). These teachers were one third of my participants and were selected to ensure that all levels of noticing quality were represented at least once and multiple levels were represented for each component skill of noticing. Interrater reliability was calculated at 80% or higher, and any discrepancies were resolved through discussion.

*Relationship between use of frames and quality of teachers' noticing***.** The analysis of the relationship between teachers' use of frames and the quality of their noticing included looking for patterns that emerged in both analyses across teachers and each noticing component skill. First, I looked across teachers to compare their use of frames in both interviews and when each frame was used within each noticing component skill. Next, I looked across each noticing component skill for all teachers to determine the frequency of frames used, paying special attention to the number of frames used in addition to the *children's thinking* frame. Finally, I compared the frequency and range of frames used with each noticing score of robust, limited, and lack of evidence of use of children's thinking. I noted any prevalent and interesting patterns that emerged to determine the relationship between teachers' use of frames and the quality of their noticing.

Phase 3: Results

In Phase 3 of my study, I confirmed the use of all six frames identified in the previous phases thereby highlighting the variety of ways teachers reasoned during noticing and suggesting that frames are a useful construct for understanding the complexity of teachers' noticing. In addition, through my six cases, I was able to not only compare teachers' use of frames with written work from teachers' own classrooms and unfamiliar classrooms but also begin to explore the relationship between teachers' use of frames and the quality of their noticing. My results are organized according to the three research questions addressed in Phase 3:

(1) What frames do individual teachers use when noticing children's mathematical thinking in written work from their own and unfamiliar classrooms?

(2) What is the quality of individual teachers' noticing when noticing children's mathematical thinking in written work from their own and unfamiliar classrooms?

(3) What is the relationship between teachers' use of frames and the quality of their noticing?

Research Question 1. The first research question was answered by comparing teachers' use of frames in Phase 3 with the results from Phases 1 and 2 as well as comparing teachers' use of frames with written work from the two sources. (See Tables 3.2 and 3.3 for a summary of teachers' use of frames in Phase 3.)

Table 3.2

Teachers' Use of Frames With Written Work From Their Own Classrooms

	Number of Instances (%)						Total Number of Instances
	T1 *N* = 23	T2 *N* = 19	T3 *N* = 21	T4 *N* = 28	T5 *N* = 22	T6 *N* = 26	*N* = 139 (%)
Current Mathematical Performance	**17 (74%)**	**14 (74%)**	**20 (95%)**	**21 (75%)**	**18 (82%)**	**23 (88%)**	**113 (81%)**
Children's Thinking	17 (74%)	14 (74%)	20 (95%)	21 (75%)	18 (82%)	23 (88%)	113 (81%)
Non-Mathematical Performance	**0**	**0**	**0**	**1 (4%)**	**2 (9%)**	**0**	**3 (2%)**
Confidence	0	0	0	0	2 (9%)	0	2 (1%)
*Behavio*r	0	0	0	1 (4%)	0	0	1 (<1%)
Mathematical Performance Comparisons	**6 (26%)**	**5 (26%)**	**1 (5%)**	**6 (21%)**	**2 (9%)**	**3 (12%)**	**23 (17%)**
Past Performance	2 (9%)	0	1 (5%)	4 (14%)	0	2 (8%)	9 (7%)
Class Performance	4 (17%)	5 (26%)	0	2 (7%)	1 (5%)	1 (4%)	13 (9%)
Broader Scope	0	0	0	0	1 (5%)	0	1 (<1%)

Table 3.3

Teachers' Use of Frames With Written Work From Unfamiliar Classrooms

	Number of Instances (%)						Total Number of Instances
	T1 N = 31	T2 N = 31	T3 N= 17	T4 N = 25	T5 N = 29	T6 N = 35	*N* = 168 (%)

Current Mathematical Performance	**27 (81%)**	**25 (81%)**	**16 (94%)**	**21 (84%)**	**26 (90%)**	**28 (80%)**	**143 (85%)**
Children's Thinking	27 (81%)	25 (81%)	16 (94%)	21 (84%)	26 (90%)	28 (80%)	143 (85%)
Non-Mathematical Performance	**0**	**0**	**0**	**0**	**0**	**1 (3%)**	**1 (<1%)**
Confidence	0	0	0	0	0	1 (3%)	1 (<1%)
Behavior	0	0	0	0	0	0	0
Mathematical Performance Comparisons	**4 (13%)**	**6 (19%)**	**1 (6%)**	**4 (16%)**	**3 (10%)**	**6 (17%)**	**24 (14%)**
Past Performance	1 (3%)	1 (3%)	1 (6%)	0	1 (3%)	1 (3%)	5 (3%)
Class Performance	2 (6%)	4 (13%)	0	4 (16%)	1 (3%)	5 (14%)	16 (10%)
Broader Scope	1 (3%)	1 (3%)	0	0	1 (3%)	0	3 (2%)

Comparison of teachers' use of frames to phases 1 and 2 results. Across the 12 noticing interviews, I confirmed the use of all six frames identified in the previous phases and no new frames emerged. I also noted that their relative prevalence was similar, with teachers demonstrating extensive use of the current mathematical performance frame (*children's thinking*), some use of the mathematical performance comparison frames (*past performance*, *class performance*, and *broader scope*) and rare use of the non-mathematical frames (*confidence*, and *behavior*). Also noteworthy was that all teachers used multiple frames across their two interviews; all teachers used the *children's thinking* frame and at least one other frame, and 8 of the 12 interviews used the *children's thinking frame* and at least 2 other frames.

Link between teachers' use of frames and the source of written work. There was lack of substantial evidence to distinguish the sources in terms of the use of particular frames or the prevalence of frames. Specifically, all 6 of the frames were used with written work from teachers' own classrooms and all but the *behavior* frame were used with written work from unfamiliar classrooms. Further, the prevalence of the three categories of frames was similar across the two sources: more than 80% for the current mathematical performance (*children's thinking*) frame, between 10–20% for the mathematical performance comparison frames, and less than 5% for the non-mathematical frames. Finally, all 6 teachers used the *children's thinking* frame and at least 1 additional frame when noticing with either source.

Compared to the results from previous phases, one important difference did occur in the teachers' use of frames with unfamiliar written work. In earlier phases, the *past performance* and *confidence* frames were only used with written work from teachers' own classrooms in that teachers drew on their previous experiences with the child's problem solving—insider knowledge of the child—to make comparisons or describe the child's confidence. In Phase 3, teachers used the *past performance* or *confidence* frames when noticing children's thinking in written work from their own classrooms and unfamiliar classrooms in similar ways. In other words, teachers used the *past performance* and *confidence* frames to compare the child's current performance to the child's "imagined" previous problem-solving experiences, such as their use of representations or overall confidence in using particular partitions. The example below illustrates Teacher 1's use of *the past performance* frame involving "imagined" insider knowledge to discuss Emily's understandings (see Figure 4.2 or Appendix B for Emily's strategy).

> I'm thinking Emily is very, very dependent on her direct modeling, but I think that she—Let's see. Hang on; I've got to look at this. She's just very dependent on her direct modeling because—I am very impressed that she was able to turn those three [2 halves cut into thirds], that three into a sixth. She knew that was six parts.

In this example, Teacher 1 noted that Emily was able to reason that the two halves that were split into thirds were actually one sixth of the cake. Teacher 1 was impressed with Emily's ability to partition a half into three parts and label them as sixths. In addition to this attention to the details of Emily's strategy, Teacher 1 repeatedly mentioned that Emily was dependent on her direct modeling. Even though Emily was not her student, Teacher 1 seemed to generalize Emily's problem solving far beyond this particular strategy by claiming that she was "very very dependent on her direct modeling." In other words, Teacher 1 "imagined" Emily's history—a history she would have known for her own students—in her interpretation of Emily's strategy. In summary, analysis of teachers' use of frames in these noticing interviews expanded my understanding of the use of insider knowledge when teachers notice children's mathematical thinking in written work from unfamiliar classrooms. "Imagined" insider knowledge was used in the *past performance* frame by 5 of the 6 teachers and by one teacher with the *confidence* frame.

Research Question 2. The second research question investigated the quality of teachers' noticing children's mathematical thinking in written work from the two sources. Results indicated a small range in teachers' overall noticing scores and a lack of substantial evidence was found to distinguish the quality of teacher's overall noticing when working with written work from the two sources. (See Figure 4.5 for a summary of teachers' noticing scores in the two interviews.)

Teacher	**Source of Written Work**	**Noticing Scores for the Component Skills**				*Overall Noticing Score*
		Attending	*Interpreting*	*Deciding How to Respond (Problem)*	*Deciding How to Respond (Conversation)*	
T1	Own	Robust	Limited	Limited	Limited	Limited
	Unfamiliar	Robust	Limited	Limited	Lack	Limited
T2	Own	Robust	Lack	Robust	Limited	Limited
	Unfamiliar	Limited	Limited	Robust	Limited	Limited
T3	Own	Limited	Limited	Limited	Lack	Limited
	Unfamiliar	Limited	Lack	Lack	Limited	Lack
T4	Own	Limited	Limited	Limited	Limited	Limited
	Unfamiliar	Robust	Lack	Limited	Lack	Limited
T5	Own	Limited	Limited	Limited	Limited	Limited
	Unfamiliar	Robust	Limited	Lack	Robust	Limited
T6	Own	Robust	Robust	Robust	Limited	Robust
	Unfamiliar	Robust	Robust	Robust	Limited	Robust

Figure 4.5. Teachers' Noticing Scores from Both Sources.

Note. Cells that are shaded represent pairs of scores in which teachers' noticing scores differed with different sources of written work.

Overall quality of teacher noticing. These data revealed a small range for the overall quality of noticing children's mathematical thinking. Nine of the interviews demonstrated overall limited evidence of use of children's mathematical thinking, 2 demonstrated robust evidence, and 1 demonstrated lack of evidence. These results are expected given that teachers were at the end of their third year in the RTEM PD and it is challenging to develop overall noticing expertise—teachers had made substantial progress in using children's thinking but still had room to grow. To give a general sense of teachers' engagement with children's mathematical thinking at each level (i.e., robust, limited, and lack of evidence), I provide examples of responses at each level for the component skill of attending to the strategy details with Monica's strategy (see Figure 4.4 for the responses and Appendix B for Monica's strategy).

Teacher 5's response was considered robust evidence of using children's thinking because she attended to most of the mathematically important details in the child's strategy and captured the overall essence of the strategy. Specifically, she discussed how Monica was thinking about the 10/6 and linked this answer to the story context of 10 things shared amongst six children. The teacher also conjectured how Monica arrived at 1 4/6 by taking 6 sixths from the 10/6 to make a whole leaving 4/6 left over. Teacher 3's response was considered limited evidence of using children's thinking because her response provided only some evidence of attention to the mathematically important details in Monica's strategy. Her description generally connected Monica's equation to the story context regarding the process of dividing 10 cakes by 6 children. She also shared Monica's answer, but did not conjecture how Monica arrived at the 10/6 or 1 4/6. Teacher 2's response was coded as lack of evidence of using children's thinking because there was minimal attention to the details in Monica's strategy. Teacher 2 focused mainly on Monica's answer, and did not consider ways she could have arrived at her answer.

***Prompt:* Describe in detail what Monica did to solve the problem.**		
Robust Evidence of Using Children's Mathematical Thinking	*Limited Evidence of Using Children's Mathematical Thinking*	*Lack of Evidence of Using Children's Mathematical Thinking*
Teacher 5: "Monica said 10 sixths. She said 10 cakes divided among six children equals 10 sixths, equals 1 and 4 sixths. She knew that she had 10-cakes and she had to divide that among six children. That would be 10 sixths and if it weren't for the 1 and 4 sixths, I would think that maybe I would need to ask her some questions. I felt like I would assume that that was a procedure. But with her saying it equals 1 and 4-sixths, I feel like she's able to take the 10 sixths and know that there's 6 [6 sixths] to make a whole and 4 sixths leftover."	Teacher 3: "We have Monica, which like I said, she just went to notating it. She knew that she was dividing the ten cakes by the six children. She ended up with the fraction 10 sixths and then she was able to turn that into an improper [sic] fraction."	Teacher 2: "[My class] solved this two weeks ago…Another student demonstrated this similar skill, and they took the total number, and they put it over how many children. I mean like made a division problem out of it, at which, yay, I mean it was really neat to see. Then I see that [Monica] knows that she has one left over. She has one per person with four left over. And she took the 4 sixths and made it to a fraction, which is what we [the class] started out talking about. So, to me, I wonder if she is using division—she did, she used division. Like, how does she know that?"

Figure 4.6. Sample Responses for Attending to the Details of Monica's Strategy for the Problem of 6 Children Sharing 10 Small Cakes.

Link between teachers' quality of noticing and the source of written work. In a comparison of the quality of each teacher's noticing of children's mathematical thinking in written work from their own classrooms versus unfamiliar classrooms, there was a lack of substantial evidence found to distinguish teachers' overall scores and within the three component skills (see Figure 4.5). Teachers were relatively consistent in that their noticing scores with their own and unfamiliar written work were either identical or only one level apart (and differences were not always in the same direction). Specifically, 5 teachers' overall noticing scores were the same with both sources of written work, and 1 teacher's overall score was 1 level off. Of the twenty-four possible pairs of scores within the component skills, 12 pairs were the same, and 7 were one level higher with written work from teachers' own classrooms and 5 were one level higher with written work from unfamiliar classrooms.

Research Question 3. The third research question asked**:** What is the relationship between teachers' use of frames and the quality of their noticing? As the results for the previous two research questions indicated, there was a lack of substantial evidence distinguishing the use of frames and the quality of teachers' noticing of children's mathematical thinking in written work from teachers' own classrooms versus unfamiliar classrooms. Therefore, in this section, I generally discuss the relationship between the teachers' use of frames and the quality of their noticing in terms of the set of 12 interviews as a whole. However, Figures 4.7 and 4.8 do provide a summary of teachers' noticing scores and their use of frames for each component skill with each source. Overall, I did not find a direct link between teachers' use of frames and the quality of their noticing, but I did identify two findings that merit further research: the possibility that some frames may compete with a focus on children's thinking and a possible link between teachers' use of frames and the number of strategies under consideration.

		Teacher 1	Teacher 2	Teacher 3	Teacher 4	Teacher 5	Teacher 6
Attending	Score	*Robust*	*Robust*	*Limited*	*Limited*	*Limited*	*Robust*
	Frames	CMT, CP	CMT	CMT	CMT, PP	CMT, C	CMT
Interpreting	Score	*Limited*	*Lack*	*Limited*	*Limited*	*Limited*	*Robust*
	Frames	CMT, CP, CP, PP, PP	CMT, CP, CP, CP	CMT	CMT, B, PP, PP	CMT, C, BS	CMT, PP, PP
Deciding How To Respond (Problem)	Score	*Limited*	*Robust*	*Limited*	*Limited*	*Limited*	*Robust*
	Frames	CMT, CP	CMT, CP, CP	CMT, PP	CMT, CP, CP	CMT, CP	CMT, CP
Deciding How to Respond (Questions)	Score	*Limited*	*Limited*	*Lack*	*Limited*	*Limited*	*Limited*
	Frames	CMT	CMT	CMT	CMT, PP	CMT	CMT
Overall Score		**Limited**	**Limited**	**Limited**	**Limited**	**Limited**	**Robust**

Figure 4.7. Teachers' Noticing Scores and Use of Frames With Their Own Written Work.

Note. Teachers' noticing scores are in italics, and their use of frames is captured with the following abbreviations. The *children's thinking* frame is signified by CMT, *confidence* frame as C, *behavior* frame as B, *past performance* frame as PP, *class performance* frame as CP, and *broader scope* frame as BS.

		Teacher 1	Teacher 2	Teacher 3	Teacher 4	Teacher 5	Teacher 6
Attending	Score	*Robust*	*Limited*	*Limited*	*Robust*	*Robust*	*Robust*
	Frames	CMT	CMT, CP, PP	CMT, PP	CMT, CP	CMT	CMT
Interpreting	Score	*Limited*	*Limited*	*Lack*	*Lack*	*Limited*	*Robust*
	Frames	CMT, BS, PP	CMT, BS, CP	CMT	CMT, CP	CMT, PP	CMT, C
Deciding How To Respond (Problem)	Score	*Limited*	*Robust*	*Lack*	*Limited*	*Lack*	*Robust*
	Frames	CMT, CP, CP	CMT, CP, CP	CMT	CMT	CMT, BS, CP	CMT, CP, CP, CP, PP
Deciding How to Respond (Questions)	Score	*Lack*	*Limited*	*Limited*	*Lack*	*Robust*	*Limited*
	Frames	CMT	CMT	CMT	CMT, CP, CP	CMT	CMT, CP, CP
Overall Score		**Limited**	**Limited**	**Lack**	**Limited**	**Limited**	**Robust**

Figure 4.8. Teachers' Noticing Scores and Use of Frames with Unfamiliar Written Work.

Note. Teachers' noticing scores are in italics, and their use of frames is captured with the following abbreviations. The *children's thinking* frame is signified by CMT, *confidence* frame as C, *behavior* frame as B, *past performance* frame as PP, *class performance* frame as CP, and *broader scope* frame as BS.

Lack of a direct link between teachers' use of frames and the quality of their noticing. The data did not show a direct link between teachers' use of frames and the quality of their noticing. There were examples of all levels of noticing using only a *children's thinking* frame and all levels of noticing using multiple frames, which indicate that the use of frames alone does not dictate the quality of teachers' noticing of children's mathematical thinking. The following teacher responses illustrate this idea.

When Teacher 1 and Teacher 4 were asked to generate the problem (or problems) they would pose next in relation to the set of unfamiliar written work, both of their responses were considered limited evidence of using children's mathematical thinking. However, they differed in terms of their use of frames—Teacher 4 only used the *children's thinking* frame in comparison to Teacher 1 one who used multiple frames. Consider Teacher 4's decisions about appropriate next problem(s) for the unfamiliar students: Alicia, Emily, Katie, and Monica:

> [For Alicia, Emily, and Katie] So maybe some more practice with similar problems with thoughts to how many shares they need to make. And Monica used the procedure. She used division. [pause] Whether or not she understands the amounts that she's working with, I don't know, but she does know it's 10 cakes divided among six children. So with Monica, I guess I would have her explain it to me if her understanding was there, then I would want to move her into a more difficult problem.

Teacher 4 showed limited evidence of using children's mathematical thinking in her response because her description of the next problem was general for all students. For Alicia, Emily, and Katie, she wanted the children to practice similar problems and it is not clear what the teacher means. Perhaps Teacher 4 wanted the children to solve equal sharing problems in which the number of items to be shared was greater than the number of sharers, or maybe a problem that provided multiple ways of partitioning. Similarly, she wanted Monica to work on more difficult problems but again that statement is ambiguous. Note that Teacher 4's response used only the *children's thinking* frame because she highlighted the details in Monica's strategy to consider which problem to pose. In contrast, Teacher 1 also showed limited evidence of using children's mathematical thinking but used multiple frames in her response (the *children's thinking* frame and the *class performance* frame). In Teacher 1's response, I highlight her use of both frames to provide a sense for how the use of multiple frames does not always enhance or hinder teachers' ability to decide how to respond on the basis of children's understandings. When asked what problem she would pose next, Teacher 1 responded:

> She [Emily] has all these arrows drawn to where each person goes. Let's see. But I don't see that she's written down an actual answer. I'm trying to look in her circles and see what she has on each circle. She has one, and then one half and then one sixth in each circle so maybe that's what she was thinking there.
>
> Then this kid [Alicia] just put them all in fourths, drew out her 10, gave them all fourths and then did the last one with eighths and she says, each kid got six fourths and one eighth of the cakes.
>
> These two [Emily and Alicia], I think are just trying to divide it up in any possible way. I'm just thinking if I could do maybe—I'm not sure, I'm just trying to think of a different number set that may make it a little easier for them to see, to comprehend with what they are already trying to do. I feel that with Monica, I might want to try to push her further a little bit, but the other two I feel like I might want to go back and see what they really understand before we go on.
>
> For one thing, what I would do is, I would have another kiddo see if they could explain what they were thinking because I always have them [my class] put it all on the board and I'll ask them if they can explain what they

> were thinking and then I ask them, "Well, where do you see this in her work or his work? Is it the same?" or those kinds of things. We [my class] talk through those types of things like that anyway. But as far as another problem, let's see.

Teacher 1's response is similar to Teacher 4's response in that she considered creating problems for Emily and Alicia that were different from the ones she would create for Monica based on how each child solved the problem. Teacher 1's suggestions for another problem for Emily, Alicia, and Monica were also vague. She wanted an easier number set for Emily and Alicia and a way to push Monica further in her thinking. Teacher 1's overall goal for her next problems was for her to gain a better understanding of the children's strategies. However, she did not provide specifics for next problems and talked generally, thus her response was considered limited evidence of using children's mathematical thinking. Teacher 1 used a *children's thinking* frame in the first two paragraphs of her response, highlighting the different partitions and ways Emily and Alicia solved the problem. Later in the conversation, she used the *class performance* frame in discussing ways she engaged her class in understanding each other's mathematical thinking. Rather than creating a next problem, Teacher 1 talked as if Emily, Alicia, and Monica were students in her class and explained that she would have the children share their strategies with each other and think across the strategies. Teacher 1's combination of using the *children's thinking* frame and *class performance* frame did not enhance or hinder the quality of her noticing. In contrast, there were instances in which the use of frames other than the *children's thinking* frame did seem to hinder the quality of teachers' noticing—it seemed as if these frames competed with a focus on children's thinking.

Competing frames. Results indicated the use of multiple frames during teacher noticing had the potential to compete with teachers' ability to notice children's thinking, although, as in the above example, that was not always the case. Nonetheless, it is important to understand how these competing frames sometimes had a negative effect. Consider the following examples of two teachers using the *class performance* frame in their description of details of Monica's strategy (see Appendix B for her strategy). In one example, the *class performance* frame competes with a focus on children's thinking and in the other example it does not.

In the first example of the use of the *class performance* frame (see Figure 4.9), Teacher 1's response was considered robust evidence of using children's mathematical thinking. She attended to some of the mathematically important details in Monica's strategy, such as linking Monica's equation to the 10 cakes divided by the 6 children. She did not explicitly consider possible ways Monica knew 10/6 was 1 4/6, but in her use of the *class performance* frame, she compared her experiences with similar strategies in her own class to discuss how she encourages her class to notate their thinking about how 10/6 is equivalent to 1 4/6. Here, Teacher 1 included some attention to children's thinking in her use of the *class performance* frame in her discussion of Monica's strategy, and when she coordinated these ideas, she foregrounded children's thinking.

Robust Evidence of Using Children’s Mathematical Thinking
Teacher 1: “[Monica], who when I first looked at it, I thought to myself, well, this is very procedural because she has 10 over six but then out here, she has 10 cakes divided among six children. She literally wrote out the verbiage underneath to explain her thinking… Monica used an equation, and she said 10 divided by six, but underneath she explained her thinking. 10 cakes divided among six children equals 10 over six. Then after she had the 10 over six she put that's one whole and 4 sixths. I probably would have had my students show me how did [they] know that 10 sixths were one and 4 sixths—kind of show me that.”

Figure 4.9. Non-Competing Use of the Class Performance Frame in a Teacher Noticing Example.

Lack of Evidence Using Children’s Mathematical Thinking
Teacher 2: “[My class] solved this two weeks ago…Another student demonstrated this similar skill, and they took the total number, and they put it over how many children. I mean like made a division problem out of it, at which, yay, I mean it was really neat to see. Then I see that [Monica] knows that she has one left over, she has one per person with four left over. And she took the 4 sixths and made it to a fraction.”

Figure 4.10. Competing Use of the Class Performance Frame in a Teacher Noticing Example.

In contrast, in the second example (see Figure 4.10), Teacher 2’s response was considered lack of evidence for using children’s thinking and, in this case, the *class performance* frame was foregrounded thereby competing with her ability to notice children’s mathematical thinking. In her description of Monica’s strategy, Teacher 2 began by comparing Monica’s strategy to that of similar strategies used by her class two weeks prior. Then Teacher 2 shifted back to Monica’s strategy and focused on the answer of 1 and 4/6. She talked broadly about Monica's answer and how from 10/6 Monica knew each person would get one whole and 4 sixths. Teacher 2's explanation of Monica's strategy included some attention to children's thinking and some discussion of her own class, but in the coordination of the two ideas, Monica's thinking was minimized. In Teacher 2's description of Monica's strategy, it is very difficult to get the gist of the strategy, and thus the use of the *class performance* frame hindered her ability to notice children's mathematical thinking. Teachers' use of frames that compete with a focus on children's thinking requires further research but the current findings suggest that they are worthy of study. The next section identifies an unexpected finding that I believe is also worthy of further study.

Link between teachers' use of frames and the number of strategies under consideration. A pattern emerged in teachers' use of frames when they were considering one child's strategy versus several children's strategies (regardless of the source of the written work). Teachers were specifically asked to focus on an individual child's strategy in the prompts for the component skills of attending and deciding how to respond in a one-on-one conversation. (Note that for the attending prompt, teachers were asked to describe multiple children's strategies, but one at a time, thus forcing a focus on individual children.) In contrast, for the component skills of interpreting and deciding how to respond with a follow-up problem, teachers could choose to focus on only an individual child, multiple children by considering their understandings or instructional needs individually, or multiple children as a group. There was a striking pattern in the differential use of frames between the two sets of component skills (see Figure 4.11).

The component skills with the open prompt—allowing for a focus on individual students or students as a group—involved more use of frames whether looking (a) overall at the total number of frames used (not including the c*hildren's thinking* frame) or (b) by interview, identifying the number of interviews that used a *children's thinking* frame and more than one additional frame. When teachers are noticing the thinking of more than one child, there are many decisions and areas of focus that could impact the quality of teachers' noticing, and these findings suggest that frames may play an even greater role in teachers' noticing as the number of children are increased, and thus may require attention in PD.

Prompts related to the component skills of noticing		Total number of frames used in the 12 interviews (not including the *children's thinking* frame)	Number of interviews using a *children's thinking* frame and more than 1 additional frame (*N*=12)
Required focus on individual students	*Attending*	7	1
	Deciding how to respond (conversation)	5	2
Open focus (individual students or students as a group)	*Interpreting*	21	7
	Deciding how to respond (problem)	17	6

Figure 4.11. Comparing the Use of Frames When the Noticing Task Focused on Individual or Groups of Students.

Conclusion

In Phase 3, I explored the relationship between teachers' use of frames and the quality of their noticing in written work from their own classrooms and unfamiliar classrooms. All frames from the previous phases were confirmed and the prevalence of frames used by teachers suggests that framing is a useful construct for understanding the complexity of teachers' noticing. In a comparison of teachers' noticing of children's thinking in written work from their own classrooms versus unfamiliar classrooms, there was a lack of substantial evidence found to distinguish the two sources in terms of the use of particular frames, the prevalence of particular frames, or the quality of teachers' noticing of children's thinking. Drawing written work from either source did not change teachers' noticing and, in fact, there was evidence that teachers used "imagined" insider knowledge of children from unfamiliar classrooms to assist with their noticing. In addition, there was evidence that some teachers struggled with one or more frames competing with a focus on children's thinking and this idea needs further exploration. Finally, a distinction between teachers' use of frames was identified when they were considering one child's strategy versus several children's strategies regardless of the source. Specifically, when teachers' noticing focused on more than one child, more frames were invoked. Implications for those who study and support teacher noticing in professional developed are addressed in the next chapter.

CHAPTER VI

CONCLUSION

This study examined the noticing of teachers who participated in a professional development project that supported teachers' responsiveness to children's mathematical thinking through the engagement with research-based frameworks of children's thinking and frameworks of instructional practices. In three phases, I investigated teachers' use of frames and the quality of their noticing in written work from their own classrooms and classrooms unfamiliar to them to understand the relationship between framing and noticing.

In the first phase, I identified frames individual teachers used when noticing children's thinking in written work from their own classrooms. Findings identified that teachers used six frames that fell into three broad categories: (a) noticing focused on the child's current mathematical performance, (b) noticing focused on the child's non-mathematical performance, and (c) noticing that compared the child's performance to the child's previous performance, the performance of others in the class on this problem or previous problems, or curricular or testing goals for that grade level.

In the second phase, I explored the frames that small groups of teachers used when collectively noticing children's thinking in written work from unfamiliar classrooms during professional development. Results confirmed the use of half of the frames and the frames that were absent made sense because they required "imagined" insider knowledge of the child which teachers did not have because the written work was from unfamiliar classrooms. The findings from this phase also enhanced understanding of teachers' use of the *class performance* frame in terms of the ways teachers envisioned themselves as the classroom teachers when working with written work from unfamiliar classrooms.

In the third phase, I used in-depth interviews to investigate the relationship between the quality of teacher noticing and the use of frames of six teachers who were asked to notice children's thinking in written work on the same problem from their own classrooms and from unfamiliar classrooms. Key findings included confirmation of the existence of the six frames and their prevalence as well as the lack of direct links between the source of written work and the use of frames or the quality of teachers' noticing.

The rest of this chapter is organized to highlight my study's contribution to our understanding of the construct of noticing, the methodologies for capturing noticing, and the design of PD intended to support the development of noticing expertise. I conclude the chapter with a discussion of study limitations and possible directions for future research.

Contributions to the Construct of Noticing

The literature on teacher noticing continues to grow as the field seeks to "define, describe, and capture what is essentially invisible" (Schack et. al., 2017, pp. 6) in efforts to support teacher learning. My study contributes to the research base by confirming the use of frames during noticing and some potential areas for future study.

In three different data sets, my study identified teachers' use of frames that included the *children's thinking* frame supported in the professional development and other frames. The overwhelming use of the *children's thinking* frame with written work from both sources suggests that this frame, which is essential for responsive teaching, can be readily adopted. However, my studied also identified five other frames that teachers used, which suggests that they needed to coordinate multiple frames in their noticing of children's thinking in written work. Consideration of teachers' use and coordination of frames helps us to better understand the complexity of teacher noticing and what is occurring as teachers notice.

My identification of six frames also connects with the study of Sherin and Russ (2014) in which they identified 13 frames in their exploration of teachers' noticing in video. Rather than identify one-to-one mappings between their frames and my frames, I note similarities in the ways teachers engaged in noticing. Both studies highlighted teachers' abilities to engage in noticing in ways that move their reasoning beyond what is currently represented in the artifact presented. Specifically, the use of frames allowed teachers to envision themselves as the classroom teacher when noticing with artifacts unfamiliar to them. Additionally, teachers used frames to make comparisons related to their previous experiences. While both studies explored teachers' use of frames during noticing in the context of interviews, I join others in arguing that during instruction the use of frames could continue to influence the quality of teachers' noticing (Sherin & Russ, 2014).

Finally, although I did not find a direct link between teachers' use of frames and the quality of their noticing, more study is warranted because there was some evidence that in some situations, teachers occasionally struggled to stay focused on children's thinking. Further, my initial conjecture about the quality of teachers' noticing being linked to the source of written work did not hold. It is possible that the source did not matter because, when engaging with unfamiliar written work, teachers "imagined" insider knowledge of children and drew upon their own grade level experiences to fill in the gaps. Therefore, future studies are needed to examine teachers' ability to use this type of reasoning during noticing.

Contributions to the Methodologies for Capturing Noticing

The methodological challenges of capturing noticing have been raised throughout the literature as the field of noticing continues to grow and evolve (see Jacobs & Spangler, 2017; Schack, Fisher, & Wilhelm, 2017; Sherin, Jacobs, & Philipp, 2011). In a commentary on these methodological challenges, Jacobs (2017) noted the difficulty with collecting teacher noticing data in written form without opportunities to ask follow-up questions. My study expanded the methodology for capturing teacher noticing by using individual interviews that included follow-up questions that probed teachers' ideas related to each component skill of noticing. These types of follow-up questions provide more insight into teachers' reasoning, making this hidden practice more visible.

In addition, my study built on Sherin and colleagues' (2011) recommendation to use think alouds (Ericsson & Simon, 1993) in the study of teacher noticing. In my study, teachers were asked to think aloud during their interviews which made their thinking more visible while noticing children's mathematical thinking. The inclusion of think alouds in noticing interviews allowed me to capture teachers' reasoning while it was occurring as teachers engaged in each component skill of noticing.

Contributions to the Design of PD Intended to Support Noticing Expertise

In a recent review of research on mathematics teachers' professional development, Sztajn and colleagues (2017) identified a prominence of studies that involved the use of research-based frameworks of children's thinking to foreground teachers' attention to children's thinking. In those studies, frameworks of children's thinking guided the overall design of PD to support teachers' development of a *children's thinking* frame in particular content areas. The RTEM PD is representative of these studies and my dissertation findings suggest that professional developers need to be aware that teachers may adopt the *children's thinking* frame promoted while also using other frames. My data revealed that multiple frames were used by multiple teachers with written work from teachers' own and unfamiliar classrooms. Therefore, facilitators need to listen for and address the use of additional frames, particularly when those frames do not foreground children's thinking.

Because the data revealed that sometimes (but not always) teachers struggled to keep a focus on children's mathematical thinking in their coordination of multiple frames, PD facilitators may want to discuss with teachers this coordination. Consideration of the specific prompts used in PD activities may help facilitators in supporting teachers to think about coordinating multiple frames. Data revealed that there was a consistent use of multiple frames, but more often when teachers were asked to focus on more than one child. Thus, while this type of prompt may help teachers to invoke multiple frames, PD facilitators may, at other times, choose to use prompts linked to a single child to limit the number of frames invoked.

Findings also indicated teachers' abilities to notice children's mathematical thinking were similar with written work from teachers' own classrooms and unfamiliar classrooms. In other words, the source of written work did not seem to matter in terms of teachers' use of frames or their quality of noticing, meaning that teachers' noticing of children's thinking in written work from unfamiliar classrooms can serve as proxy for their noticing in written work from their own classrooms. This finding is helpful for facilitators who should be able to productively use written work from both sources to support the development of teachers' noticing of children's mathematical thinking.

Although there was a lack of substantial evidence found in my comparison of the quality of teachers' noticing in written work from the two sources, some of the teachers self-reported a perceived difficulty and a perceived advantage in noticing children's thinking in written work from other classrooms. Teachers mentioned that they felt they were making assumptions about the child's overall thinking based on a lack of insider knowledge. For example, Teacher 3 from Phase 3 stated, "To me if you don't know the children, it makes it more difficult because there's a lot of assumptions I can make. However, that's where the better questions come from." In this quote, Teacher 3 indicated that because she did not know the children, she was concerned about making assumptions regarding their mathematical thinking. However, the teacher also recognized that not knowing the child caused her to really focus on asking what she considered "better" questions. It is therefore possible that asking teachers to notice with unfamiliar written work may be effective in limiting teachers' assumptions; the fear of making assumptions when noticing children's mathematical thinking in written work from unfamiliar classrooms may have pushed teachers to more closely attend to the mathematical thinking present in the children's strategies.

Limitations

As with any study, there were limitations in this study, and I have chosen to highlight three. First, all three phases of my study focused on teachers' noticing of children's mathematical thinking in written work related to equal-sharing problems. This focus allowed me to explore teachers' noticing with this critical problem type in depth and to easily make comparisons across the three phases of my study, but it left open the question of teachers' noticing with other types of fraction problems. Second, I studied only six teachers in Phase 3 and thus the findings provide only an initial foray into the research questions, and generalizations must be made with caution. Third, the teachers in Phase 3 were purposefully selected so that they all taught in the same district and had just completed three years of PD focused on children's fraction thinking. This homogeneity allowed me to explore a particular context in depth and more easily make comparisons, but it did not allow me to investigate the contextual influences of other districts or other types of PD support.

Future Research

The existence and confirmation of frames used by teachers while noticing children's mathematical thinking provided insight into the complexity of the construct of teacher noticing and requires further investigation. Specifically, future research is needed to investigate the same relationship with a larger sample of teachers drawn from multiple districts and with a range of experience with various PD opportunities.

This study also sparked other interests in terms of teachers' use of frames during noticing while teaching in classrooms and with other types of noticing. First, my study explored the relationship between teachers' use of frames and their noticing of children's mathematical thinking in written work outside of classrooms. Future research is needed to expand our understanding of teachers' use of frames during noticing in classrooms as teachers circulate when students are solving problems and as teachers select and sequence written work for classroom discussions.

Second, my study combined the construct of framing with one of the most researched types of teacher noticing, noticing of children's mathematical thinking. As the field continues to grow to consider other types of teacher noticing, so could research on the incorporation of teachers' use of frames. In particular, future research could explore whether the same or different frames occur when the focus is on teachers' noticing of equity indicators in mathematics instruction.

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APPENDIX A

NOTICING INTERVIEWS

The interview protocols for the noticing interviews with written work from teachers' own classrooms and unfamiliar classrooms were almost identical. Both interviews began by reminding the teacher of the problem:

You have been given 4 students' written work for solving this problem:

> *The baker has 10 small cakes to share equally among 6 children. How much cake does each child get?*

The interview involving unfamiliar written work also included the following introduction:

This work was completed in a third-fourth-grade combination class in October. The teacher's directions were simply "Here is a problem for you to try," and students were able to solve the problem any way they wanted. When students finished this problem, other problems were available for them to try.

Both interviews then continued by addressing each of the component skills of noticing in the following order: (a) deciding how to respond with a follow-up problem, (b) attending, (c) interpreting, and (d) deciding how to respond with a one-on-one conversation. When needed general follow-up questions, such as the following, were posed:

- *I heard you say that.....*
- *What did you mean when you said....?*
- *What do you mean by that?*
- *You keep mentioning ________. Why is that important to you?*

Deciding How to Respond (Follow-Up Problem)
Given the 4 students' written work for solving this problem, what problem or problems might you pose next? I am interested in how you think about selecting next problems, but I do not believe that there is ever a single best problem. (Read the following for the interview involving unfamiliar work–*I also recognize that if you were the teacher of these students, you would have additional information to inform your selection*)

What was your rationale for selecting the problem or problems?

Potential Follow-Up Questions
**Push to make sure they give a specific problem.*

- *What do you mean when you said a harder problem?*
- *Why did you choose this problem? Problem type?*
- *Can you give an example of what you mean by…?*
- *Can you tell me why you choose those number choices? Or why would that be a good number choice?*

Attending
Please describe in detail what you think each child did in response to this problem. (Ask the teacher to describe each strategy individually.)

Student 1

Student 2

Student 3

Student 4

Potential Follow-Up Questions

- *For each piece of student written work, where did you start to make sense of how the student solved the problem?*

Interpreting
Please explain what you learned about these children's understandings. (Look for the use of evidence in determining these understandings.)

Can you put the four strategies in order from least to most understanding or group them if they are at the same level?

Deciding How to Respond (One-on-One Conversation)
Imagine that you were able to have a one-on-one conversation with one of the students. Which student would you choose?

Describe some ways you might respond to his or her work on this problem, and explain why you chose those responses.

Potential Follow-Up Questions

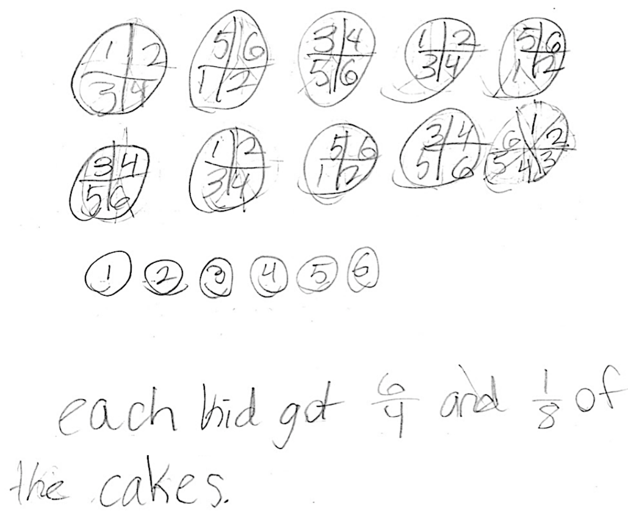
- *Why did you choose _______ to have a one-on-one conversation?*

APPENDIX B

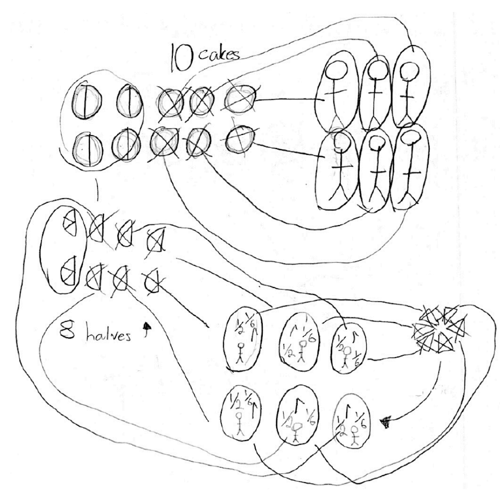
WRITTEN WORK FROM UNFAMILIAR CLASSROOMS

Problem: The baker has 10 small cakes to share equally among 6 children. How much cake does each child get?

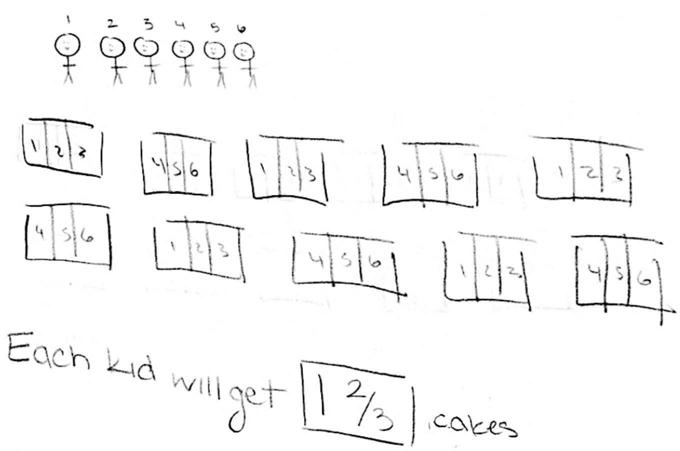
Alicia's Strategy



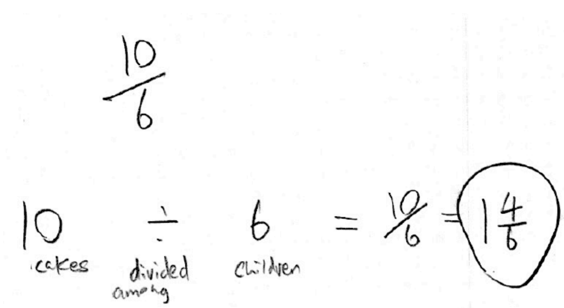
Emily's Strategy



Katie's Strategy



Monica's Strategy



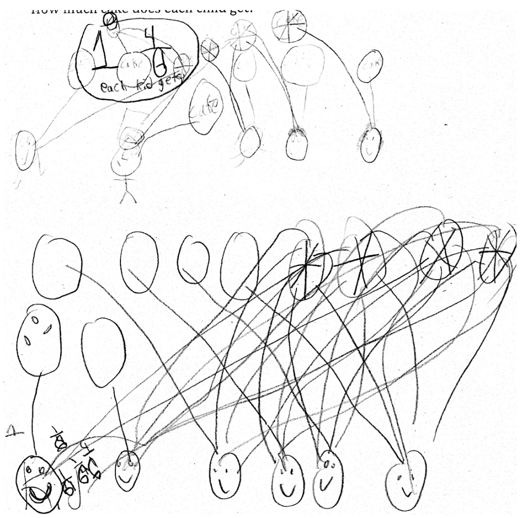
APPENDIX C

WRITTEN WORK FROM TEACHERS' OWN CLASSROOMS

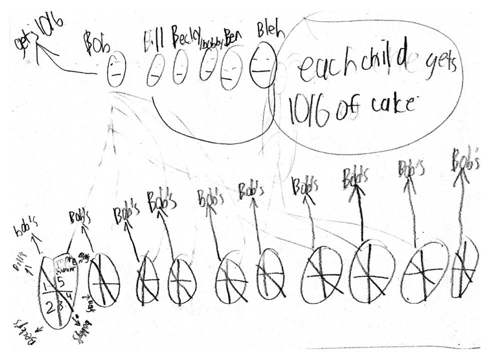
Teacher 1

Problem: The baker has 10 small cakes to share equally among 6 children. How much cake does each child get?

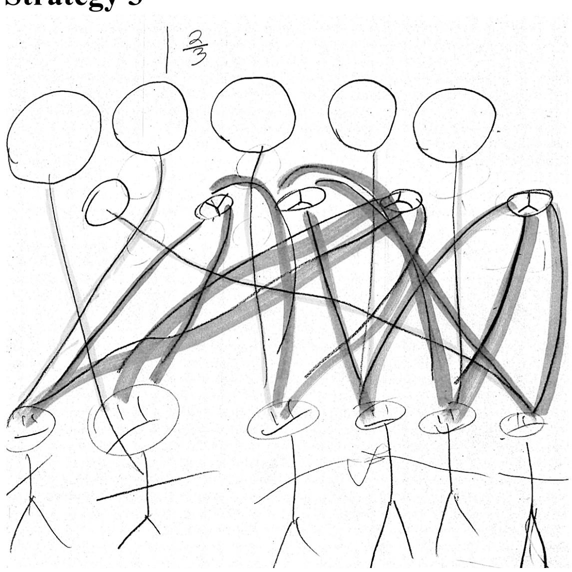
Strategy 1



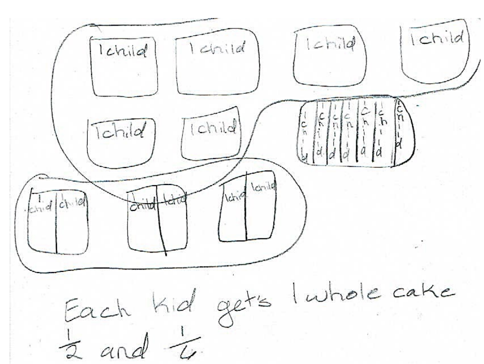
Strategy 2



Strategy 3



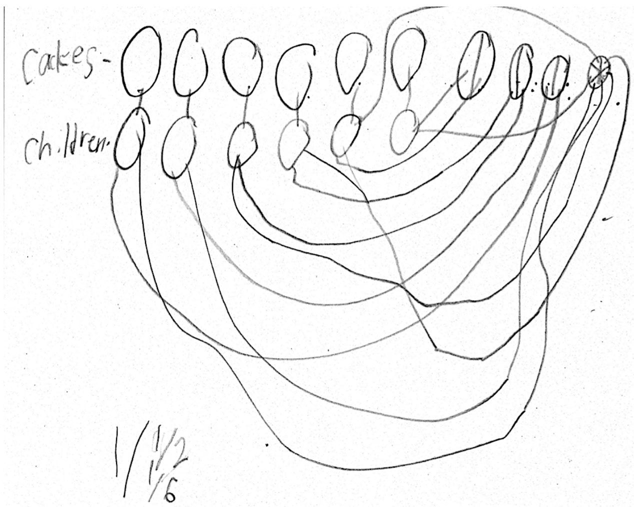
Strategy 4



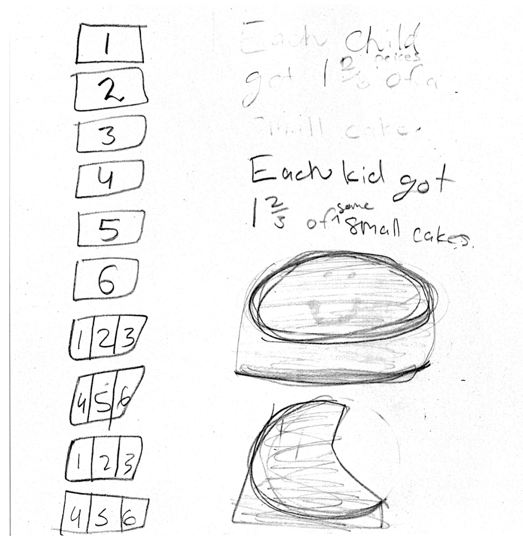
Teacher 2

Problem: The baker has 10 small cakes to share equally among 6 children. How much cake does each child get?

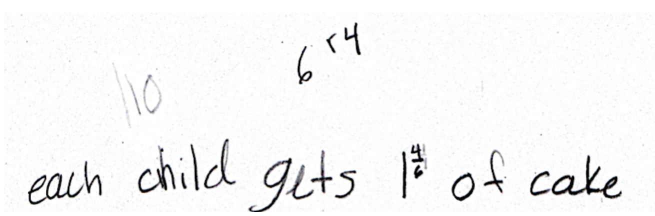
Strategy 1



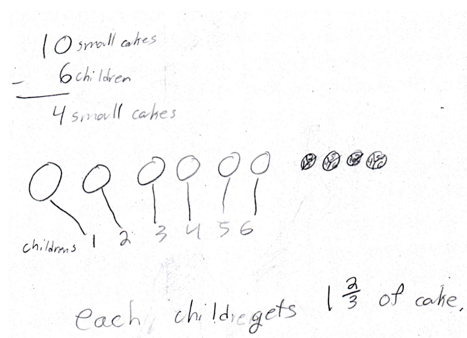
Strategy 2



Strategy 3



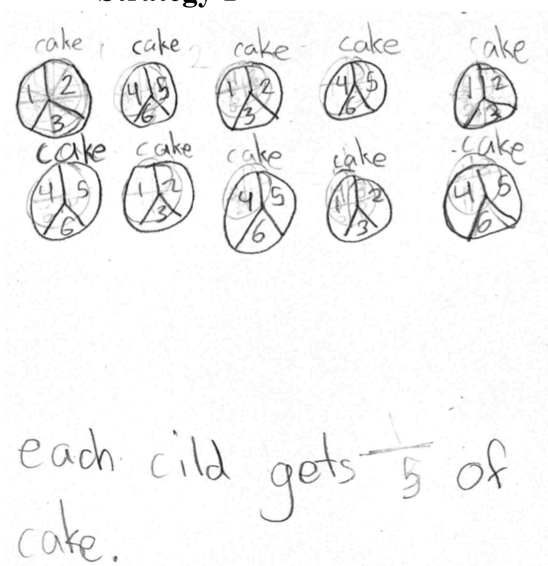
Strategy 4



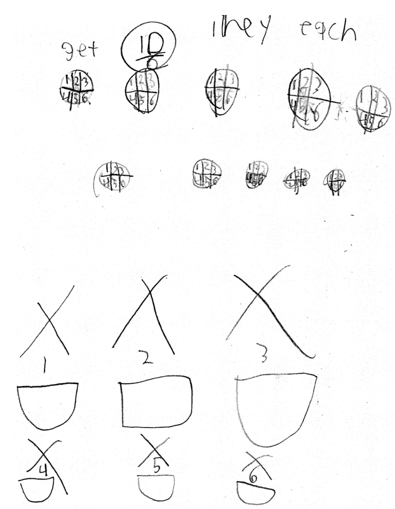
Teacher 3

Problem: The baker has 10 small cakes to share equally among 6 children. How much cake does each child get?

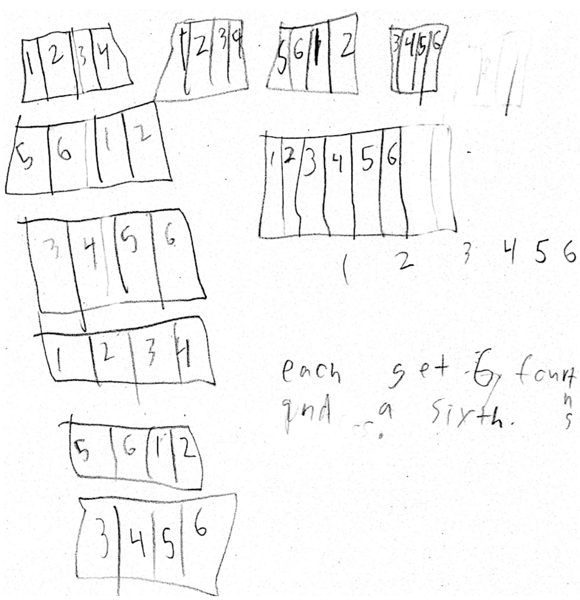
Strategy 1



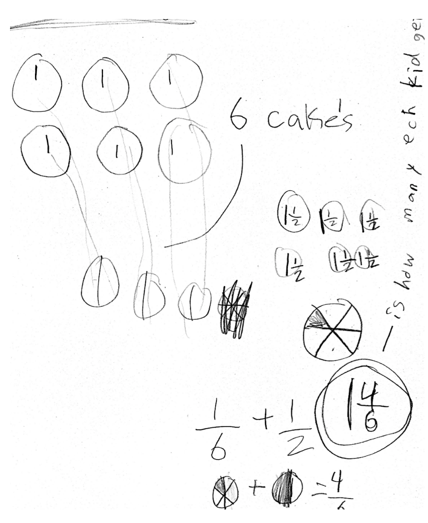
Strategy 2



Strategy 3



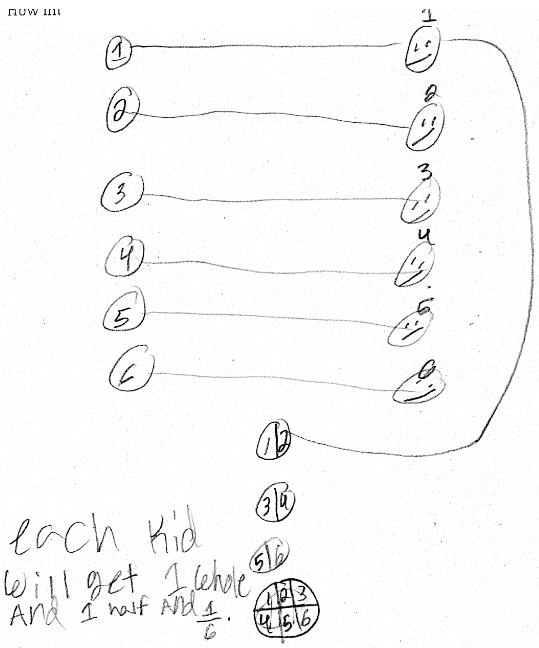
Strategy 4



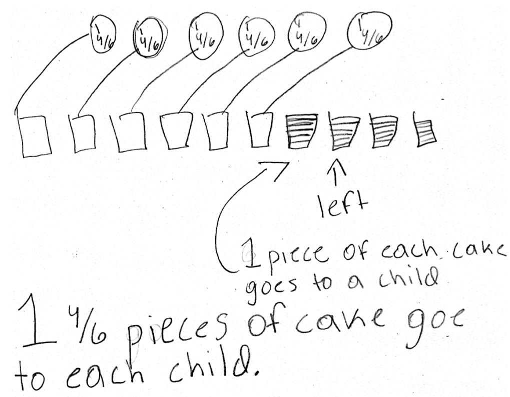
Teacher 4

Problem: The baker has 10 small cakes to share equally among 6 children. How much cake does each child get?

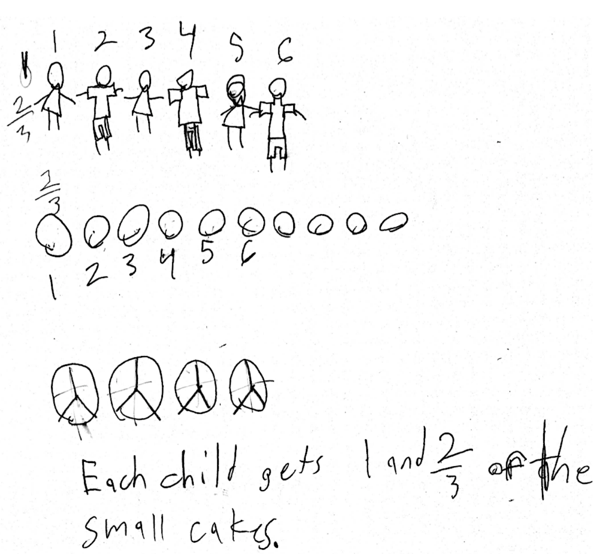
Strategy 1



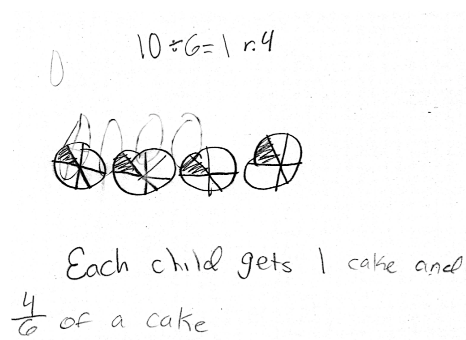
Strategy 2



Strategy 3



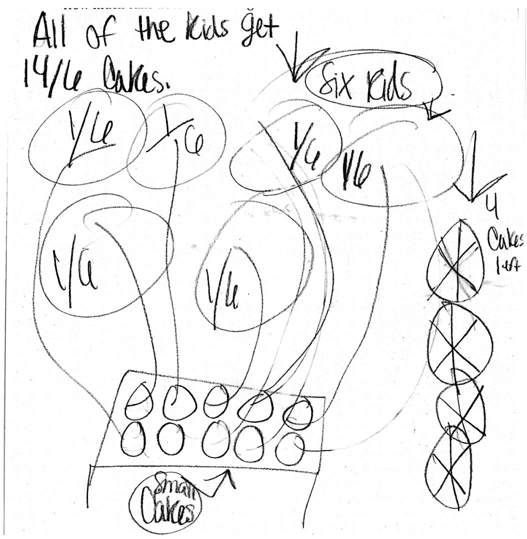
Strategy 4



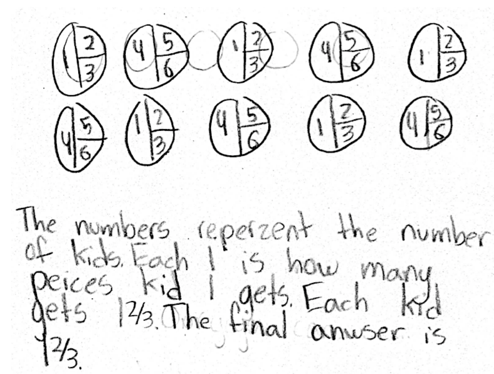
Teacher 5

Problem: The baker has 10 small cakes to share equally among 6 children. How much cake does each child get?

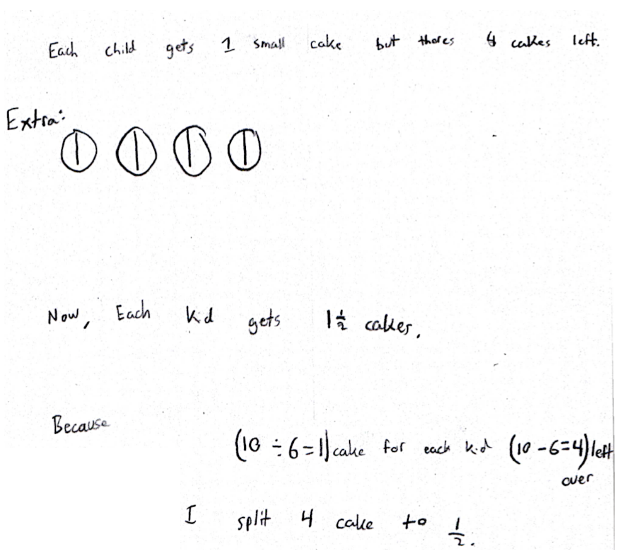
Strategy 1



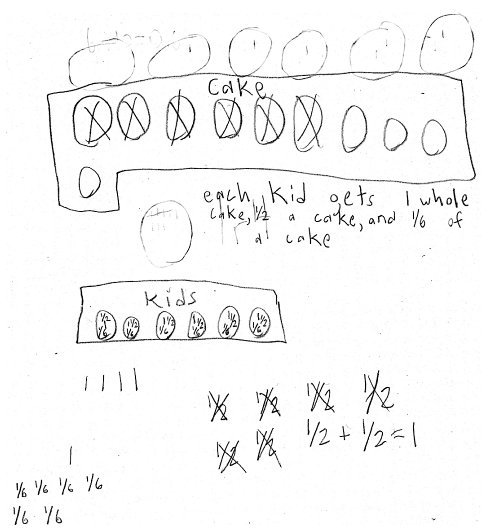
Strategy 2



Strategy 3



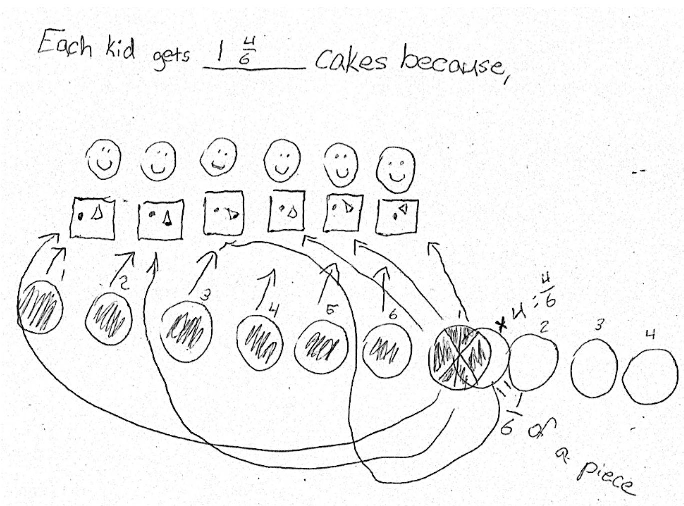
Strategy 4



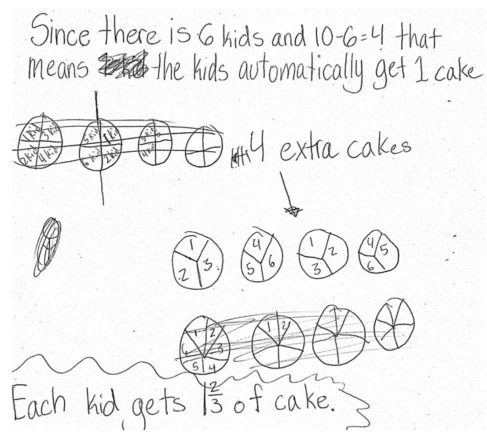
Teacher 6

Problem: The baker has 10 small cakes to share equally among 6 children. How much cake does each child get?

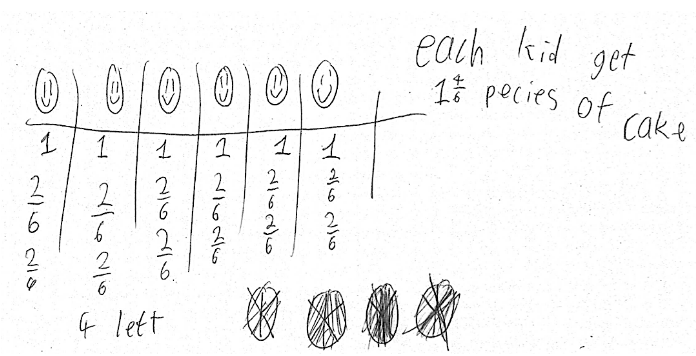
Strategy 1



Strategy 2



Strategy 3



Strategy 4