# A Distributed Approach to Accounting for Carbon in Wood Products

By: **Gregg Marland**, Eric Marland & Kirk Stellar

Abstract

With an evolving political environment of commitments to limit emissions of greenhouse gases, and of markets to trade in emissions permits, there is growing scientific, political, and economic need to accurately evaluate carbon (C) stocks and flows—especially those related to human activities. One component of the global carbon cycle that has been contentious is the stock of carbon that is physically held in harvested wood products. The carbon stored in wood products has been sometimes overlooked, but the amount of carbon contained in wood products is not trivial, it is increasing with time, and it is significant to some Parties. This paper is concerned with accurate treatment of harvested wood products in inventories of CO2 emissions to the atmosphere. The methodologies outlined demonstrate a flexible way to expand current methods beyond the assumption of a simple, first-order decay to include the use of more accurate and detailed data while retaining the simplicity of simple formulas. The paper demonstrates that a more accurate representation of decay time can have significant economic implications in a system where emissions are taxed or emissions permits are traded. The method can be easily applied using only data on annual production of wood products and two parameters to characterize their expected lifetime. These methods are not specific to wood products but can be applied to long-lived, carbon-containing products from sources other than wood, e.g. long-lived petrochemical products. A single unifying approach that is both simple and flexible has the potential to be both more accurate in its results, more efficient in its implementation, and economically important to some Parties.

**Gregg Marland**, Eric Marland & Kirk Stellar (2010) " A Distributed Approach to Accounting to Carbon in Wood Products" *Mitigation and Adaptation Strategies for Global Change* Volume 15 issue 1 pp. 71-91 Version of Record Available From (www.springer.com)

## 1 Introduction

Concern about global climate change has heightened interest in the global cycling of carbon (C). A complete and accurate description of the global carbon cycle is needed to understand the processes controlling observed increases in the atmospheric concentration of carbon dioxide ($CO_2$) and to mitigate additional increases. With an evolving political environment of commitments to limit emissions of greenhouse gases, and of markets to trade in emissions permits, there is a growing need to accurately evaluate carbon stocks and flows—especially those related to human activities. We would like to understand all of the significant stocks and flows of carbon.

One component of the global carbon cycle that has been particularly contentious is the stock of carbon that is physically held in harvested wood products. The carbon stored in wood products has been sometimes ignored because it is not a large reservoir of carbon, it has been sometimes avoided because it is difficult to evaluate the change in stocks, and it has been sometimes the source of disagreement because there is still no consensus on who should take credit for any change in the amount of stored carbon. And yet, the amount of carbon contained in wood products is not trivial and it is surely significant to some Parties. Ignoring the fact that carbon is stored in wood products generally leads to an overestimate of carbon emissions to the atmosphere. The common procedure to date has been to assume either that all of the carbon in harvested trees is released promptly to the atmosphere as $CO_2$, or that the stock of wood products is not changing with time. These two approaches are mathematically identical. They are also contrary to observations that the stock of wood products is increasing with time (see, for example, Pingoud et al. 2003).

Skog and Nicholson (1998) estimated that in 1990, in the USA, the amount of carbon in wood and paper products, in use and in landfills, was 2.7 Pg C (20% of the amount of carbon in forest trees in the USA) and that this was increasing by 0.06 Pg C per year. The amount of carbon in wood products produced globally in 2000 was 0.71 Pg C, of which 0.37 Pg was in fuel wood and 0.34 Pg was in industrial roundwood (Pingoud et al. 2003). For comparison, the annual increase in the atmospheric content of carbon is now (1998–2007)averaging about 4.1 Pg C/year, a value that is reported with an uncertainty (two sigma) of about +/− 0.3 to 0.4 Pg C (Tans 2007). Pingoud et al. (2003) showed that not only is the stock of carbon in durable wood products increasing with time, but that the national greenhouse gas emissions inventory of some countries can be significantly impacted depending on the details of the rules for accounting for these stocks of wood products. The national, annual inventory of $CO_2$ emissions for major wood producing countries (such as Canada, Finland, New Zealand, and Sweden) can change by as much as 30% depending on how harvested wood products are treated in the inventory (Pingoud et al. 2003). Countries that import or export significant quantities of wood products are impacted differently depending on how the accounts are treated. Accounting for the carbon in wood products has scientific, political and economic implications.

Our concern in this paper is with properly accounting for *how much* carbon is actually stored in wood products. This paper is not focused on the question of *which* Party accounts for carbon stored in wood products. The first question is amenable to technical discussion and our purpose is to show that there are simple mathematical methods that provide a more accurate and more appropriate description of carbon stocks than the prevailing methods in use now. The difference can be significant in an environment where emissions of carbon have regulatory restraint or economic value. The second question has become a political question to be resolved by international negotiators and is discussed only briefly here to provide context.

This paper reviews the approaches that have been suggested, or are currently being used, to deal with the carbon stored in wood products (Section 2). It then describes the mathematical implications of these approaches and suggests an alternative approach (Section 3). Section 4 compares these approaches and Section 5 illustrates the importance of these differences in a system where emissions or emissions permits are valued over time. Section 6 shows the implication of product recycling or re-use, Section 7 deals with the use of discrete data rather than mathematical functions, and Section 8 presents a summary and our conclusions. Throughout this paper the discussion is focused on durable wood products, because it is here that there has been the most discussion and contention, but the observations and conclusions are equally relevant for all long-lived products that contain carbon, including, for example, lubricants, plastics, and fabrics produced from petroleum or natural gas.

## 2 Accounting for stored carbon, past and present

### 2.1 Which Party gets credit for a change in carbon stored in wood products?

The discussion of methods for estimating emissions of $CO_2$ related to harvesting forests and producing wood products has often focused primarily on which Party (producer or consumer) should account for any carbon stored in durable products. This is not our primary concern here but it is an important prerequisite for the physical accounting and we summarize the basic issues and the current status of discussion.

As noted above, the IPCCs 1996 Guidelines for National Greenhouse Gas Inventories (IPCC 1997) made this a moot point by suggesting the default assumption that in a given year there is no net of carbon stored in harvested wood products (HWP). The logic was that the oxidation of wood products during any year is approximately equivalent to the production of wood products in that same year and that there is thus no annual increase in the stock of wood products. This is mathematically identical to assuming that all of the carbon in a forest harvest is released immediately to the atmosphere. As stated in the IPCC Guidelines: “For the purposes of the basic calculations, the recommended default assumption is that all carbon removed in wood and other biomass from forests is oxidized in the year of removal. This is clearly not strictly accurate in the case of some forest products, but is considered a legitimate, conservative assumption for initial calculations.” In concluding, the IPCC guidance “recommends that storage of carbon in forest products be included in a national inventory only in the case where a country can document that existing stocks of long term products are in fact increasing.” (IPCC 1997)

Recognizing that the global stock of wood products is in fact increasing and that the amount of carbon involved might be important to some Parties to the Kyoto Protocol, the IPCC/OECD/IEA convened a workshop in Dakar, Senegal, in 1998, to evaluate approaches for estimating the emissions of $CO_2$ (see Brown et al. 1999) The workshop identified three "approaches" for dealing with harvested wood products but was unable to achieve consensus on which approach to recommend, and the discussion has been stalled at this point ever since.

The three approaches characterized in Dakar were the "atmospheric flow approach", the "stock change approach", and the "production approach". As suggested by the name, "the atmospheric flow approach" would account for carbon flows where and when they occur. Accounts would show carbon uptake where and when forests grow, and carbon releases where and when trees are burned or forest harvests are oxidized to $CO_2$. By contrast, the "stock change approach" would keep account of carbon stocks in forests and harvested forest products. Emissions would be implied if stocks shrunk and sinks would be implied if stocks grew. Accounts would show the increasing stock of carbon in, for example, wooden homes, regardless of where the wood happened to be grown and harvested. An unsustainable harvest would be reflected in a decrease in the stock of carbon in standing forests. The "production approach" would likewise keep account of the stock of carbon in forests and forest products, with the difference that the stock of forest products would be forever tied to the cycle of carbon in the forests from which they were derived. A Party that produced wood products for export would continue to account for the stock of harvested carbon even as the stock changed locations and owners. The three approaches differ in their treatment of stocks and flows and in their definitions of system boundaries. Consequently, they differ in which Party accounts for carbon sequestration in wood products. A fourth approach, labeled the "simple decay approach" (Ford-Robertson 2003), remains in the discussion in spite of recognition that it is fundamentally a simplified version of the production approach (see, for example, UNFCCC 2004, p. 15; and Hashimoto 2008).

There is an extensive literature on the three primary approaches for accounting. The details have been carefully documented in a technical paper prepared by the UNFCCC (2003) and in papers by Brown et al. (1999), Lim et al. (1999), Ford-Robertson (2003), Pingoud et al. (2003), Pingoud (2003), UNFCCC (2004), Cowie et al. (2006) and Hashimoto (2008). Emphasis has been on the political and economic implications of the alternatives for how the sinks and emissions are attributed, and the differences can be quite significant for some countries. The "atmospheric-flow approach" has remained under discussion despite the observation that it may be in conflict with the wording of the Kyoto Protocol: "The net changes in greenhouse gas emissions by sources and removals by sinks resulting from direct human-induced land-use change and forestry activities...measured as verifiable changes in carbon stocks..." (Kyoto Protocol, article 3.3, 1997). Tonn and Marland (2007) have even suggested that carbon storage in wood products requires collaboration of producer and consumer and that the stalemate might be resolved by distributing the "credits" for carbon sequestration between the two Parties.

Discussion of the accounting alternatives has persisted since the Dakar meeting to the extent that the diverging effects of the suggested accounting methods on national greenhouse gas inventories have postponed the inclusion of wood products in the first commitment period of the Kyoto Protocol (UNECE 2008). Accords reached by the UNFCCC at Marrakesh in 2001 (UNFCCC 2002) essentially resulted in the agreement that carbon stored in wood products would not be included in accounting for meeting emissions commitments during the first commitment period (2008–2012) of the Kyoto Protocol (UNFCCC 2003, p. 16) but the Bali Action Plan

(UNFCCC 2008a) left open the way for their inclusion during a second commitment period, after 2012. As of early 2009, draft decisions of the Ad Hoc Working Group on Further Commitments for Annex I Parties under the Kyoto Protocol (UNFCCC 2008b) suggest that for a second commitment period under the Kyoto Protocol there are three alternatives for dealing with harvested wood products: 1.) don't include them at all, 2.) include them only as delayed emissions of harvests from areas specifically included under Articles 3.3, 3.4, or 6 of the Kyoto Protocol, or 3.) "create provisions for including harvested wood products". This UNFCCC document continues to suggest that there are four possible approaches for addressing wood products: "stock changes, production, simple decay and atmospheric flow." Once the possibility for alternatives gets into the political discussion it is very difficult to reach consensus for resolution. A decision is now targeted for the fifth session of the Conference of the Parties serving as the meeting of the Parties to the Kyoto Protocol, meeting in Copenhagen in December, 2009.

If retention of carbon in harvested wood products is ultimately included in accounting of commitments to mitigate $CO_2$ emissions (as seems likely), it makes a significant difference how it is treated numerically, and this is the focus of our paper.

## 2.2 How much is added over time to the store of carbon in wood products?

The net amount of carbon added to the pool of wood products over a given period of time can be evaluated in two basic ways. We can measure the amount of carbon in the pool at two points in time and subtract to find the difference, or we can measure the amount of carbon that has entered the pool and the amount that has left the pool over the time interval of interest. Pingoud et al. (2003) note that there are few cases where the data exist to support the first of these methods. By contrast, there are readily accessible, international data on the rates of production and trade of major categories of primary wood products such as paper and paperboard, sawnwood, and wood-based panels (FAO 2009). Although easy to obtain, the quality of HWP data in the FAO database is variable e.g. +/− 10–15% for OECD countries and as high as +/− 50% for non-OECD countries (Pingoud et al. 2003). Given data on the rate that carbon enters the pool of wood products, the second piece of information needed to estimate the change in stocks is the rate at which carbon leaves the pool of wood products and is released to the atmosphere as $CO_2$, and this is the focus of the current paper.

Pingoud et al. (2003, p. 16) noted that the decay pattern of harvested wood products can be "described by a linear or exponential function or it can follow the logistic equation etc. In real life the decay patterns depend on many socio-economic factors, and the true lifetime of HWP can be much shorter than their technical lifetime". Regarding lifetime models and decay parameters, Pingoud et al. (2003, p. 49) wrote "Increased complexity does not necessarily make such models more reliable or their parameter estimation easier. The main problem is the lack of reliable data." Only a limited fraction of harvested wood products end up in long-term final products due to material losses and residues at every stage in the refining chain, and lifetime estimates of wood products are very uncertain.

The recently developed IPCC 2006 Guidelines for National Greenhouse Gas inventories (IPCC 2007a, b) adopt a simple first order decay assumption for wood products, while acknowledging that this is not the only assumption possible. They suggest that that “Different possibilities include linear decay and more detailed approaches based on studies of the real use of these materials.” They express no preference among the possible decay alternatives. Importantly, the IPCC Guidelines begin by recognizing: “Given that inputs do not in general equal outputs and that carbon can remain stored in HWP for extended periods of time.” (IPCC 2007b, p. 12.6)

The IPCC Tier 1 approach is thus to assume a first order decay of wood products: “This means the annual loss from the stock of products is estimated as a constant fraction of the amount of the stock” (IPCC 2007b, p. 12.9). The IPCC Tier 2 method suggests a similar method but with more country-specific data and with continued use of a first order decay assumption. For Tier 3 methods (country-specific), the IPCC invites more complex, detailed methods and acknowledges that “models could use decay functions other than first order decay” (IPCC 2007b, p. 12.15). The IPCC does not provide support for other approaches to decay but does provide default values for the half life of paper and solidwood products. Pingoud et al. (2003) have compiled a lengthy list of published values for the half life of wood products.

## 2.3 The first-order decay of wood products

Although first-order decay of wood products is not the only approach that has been used, it is the dominant approach and is the approach documented in, for example, the 2006 IPCC Guidelines (IPCC 2007a, b). The first order decay model is framed as a differential equation tracking the rate of change of the stock ($\frac{dS}{dt}$). Production, $J(t)$, is assumed to be a continuous process; decay, or removal from the stock, is assumed to be proportional to the total size of the stock, $\lambda\ S(t)$; and the change in stock is the difference between the rates of production and removal.

$$\frac{dS}{dt} = J(t) - \lambda S(t)$$

where the half life is $\ln(2)/\lambda$ and solutions decay from an initial quantity $S_0$ as $S(t) = S_0 e^{-\lambda t}$.

Marland and Marland (2003) have described the mathematical implications of using first order decay for wood products, most particularly the attractive simplification that is appropriate for the situation of exponential growth in production combined with first order decay of the products. That is, if production is increasing exponentially and the product is subject to first order decay, the annual increase in stocks can be simply approximated as a fraction of the annual rate of production. The coefficient (fraction) to go from rate of production to rate of stock increase will be a simple function of the product decay rate or half life. The IPCC methods capitalize on this approximation by suggesting that for long-lived products, whether of wood or petrochemicals, the annual increase in stocks can be estimated as a simple fraction of the current rate of production.

This approach allows for a range of production functions, including the possibility that $J(t)$ is a sum of more than one component - implying that the stock may be generated from more than one source or process. The removal of stock does not use information regarding the age of the product. This means that a product produced this year is just as likely to be removed from service as a product produced many years previous. We call this the single pool assumption. One implication of this assumption is that the largest rates of decay for a particular year's production occur in the first year after production. This is because that is the time when there is the most stock.

In short, this approach is useful for products that can be treated as a single pool and in which decay occurs most rapidly just after production. Products used for fuel such as gasoline, natural gas, and other short lived products would be likely candidates for this method.

In addition, the approximation used in the IPCC methods assumes that the system has achieved a steady state (equilibrium). The time it takes to reach this steady state varies depending on both the rate of growth of the production and on the decay rate. The longer lived the product, the longer it takes to settle down to a constant fraction (see Marland and Marland 2003).

Other current accounting methods make use of variations of this model and account for carbon release based on this equation reaching a steady state where the decay balances the production in such a way that the decay can be reported as a fraction of the current year's production level. The time it takes to reach such a steady state varies (when the rate of change is below some threshhold) primarily according the rate of decay.

The simple approximation that the change in stocks is a function of the production rate needs to be re-evaluated if the collection of products is not seen as a single, homogeneous pool or if the production cannot reasonably be characterized as exponentially increasing. The circumstances will be different if a single pool is replaced with what we will characterize as a distributed pool. In a distributed pool each years products have an expected life time defined in terms of the year of production and it is best to distinguish this years production from production in prior years. This requires a distributed approach. This approach also removes the need to make assumptions on the nature of increases in production.

The simple pool method is valid, assuming the appropriate use of the simple decay, only if the production term is exponentially increasing. As long as production rates remain exponential this is a reasonable approximation, but many of these rates are projected to plateau or even begin decreasing. For production growth rates that are slower than exponential, the fraction of the current year's production that describes the increase in stocks would be zero (Marland and Marland 2003). In addition, data for most products is irregular and can hardly be considered exponential, or even belonging to any other simple functional form. The constraint on the nature of production, particularly given the tortuous trends in the data, is very limiting and needs to be generalized.

## 3 A distributed approach

The basic premise of first-order decay is that all products in a class exist in a single pool and that the rate of withdrawal from the pool is proportional to the quantity in the pool. In the same manner, the individual units of water in a reservoir are indistinguishable and the rate of leakage at the bottom will depend only on the depth (mass) of water in the reservoir. The highest rate of loss will occur in the first time increment when the mass in the reservoir is greatest. This approach is certainly reasonable for a great many carbon containing products. However, this approach hardly seems appropriate for the oxidation of wood structures, for example, where successive structures are distinguishable and have some finite life expectancy. The longer the expected service life of a class of products, the less likely that first order decay will be appropriate, and the greater the excess retention of carbon stocks with respect to that described by a first order decay representation.

The essential question here is whether we can describe the decay of wood products in a way that is more accurate than simple first-order decay, that is still sufficiently simple that it can be understood and implemented routinely in processing data by international emissions-inventory experts, and that actually makes a difference in national commitments under the Kyoto Protocol (and its successor) and/or in trading of emissions permits.

In addition, many long-lived products do not fit the single pool model very well. A long-lived product is not most likely to decay (or be taken out of service) in the first year after production as is assumed in the first order decay model. Instead, there is an expected lifetime for the product beyond that first year. A particular product will decay with some probability at various times but with higher probability near the expected lifetime. With a large quantity of a particular product, there will then be a distribution of times over which the various portions of the product decays. If we can characterize the distribution, we can more accurately reflect the decay of the product and quantify the changes in the stock in the model.

As an example (see Fig. 1), a product such as a utility pole is not likely to decay fastest just after production. Instead, it has an expected lifetime and a distribution of

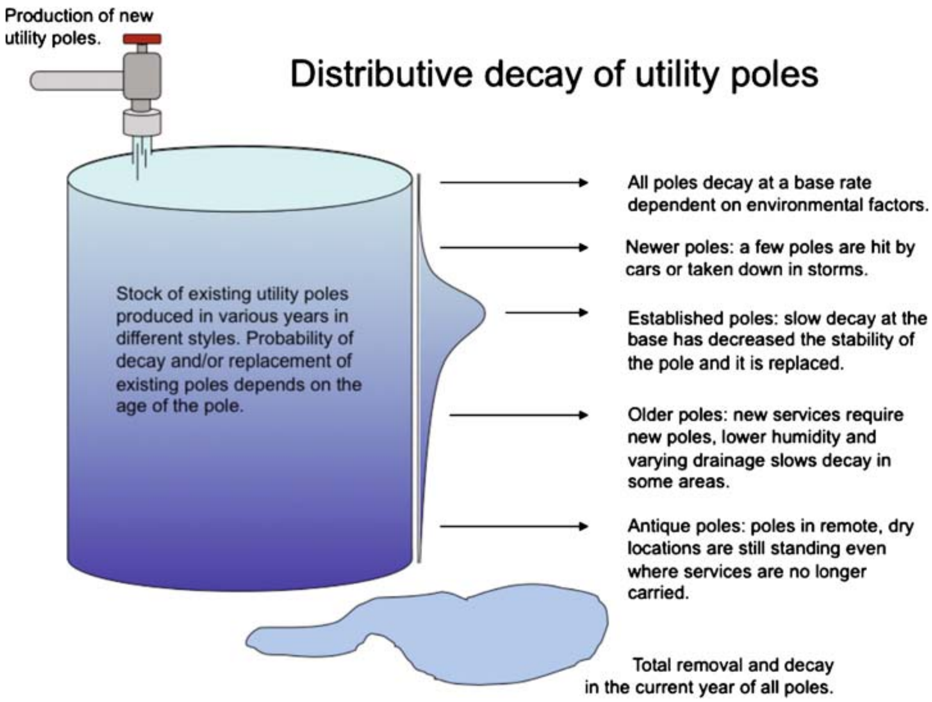


**Fig. 1** A cartoon diagram of distributed decay

times over which different utility poles might be taken out of service. Some utility poles might only last a few years, but most will last several decades if they are properly installed. Only a tiny fraction will be taken down in the first year after installation. Even wood used for fuel does not decay most rapidly in the first year. Most wood fuel is "seasoned" over the course of several months to a year to remove excess water for more efficient and cleaner burning.

There have been earlier efforts that recognized the appropriateness of a distributed approach to product oxidation. Row and Phelps (1996) applied a piecewise distribution to the decay of wood products in a model called HarvCarb aimed at dealing with wood products. The piecewise structure of the distribution was conceptually simple to implement, but computationally very expensive. It did however recognize the importance that the peak decay rate does not necessarily occur at the inital time. Currently used successors to the HarvCarb model, including ForCarb2 and WoodCarb2 (see Skog 2008), do not use the piecewise distributional decay. These models are built on a model originally created by Kim Pingoud, described in Pingoud et al. (2006), and uses an exponential decay term. The exponential decay is framed as a distribution, but is equivalent to the first order decay described above (the equivalence is shown below). The basic structure of the model would easily allow incorporation of the ideas outlined in this paper and is briefly mentioned as an option in IPCC (2007b). The US 1605(b) program for voluntary reporting of greenhouse gas emissions offers the possibility of separate annual tracking and reporting of the changes in wood product pools. The guidelines for the program are based on extensive data and analysis and offer detailed data plus illustrations of the calculations for a distributed approach as described herein, but assume first-order decay for each annual batch of a particular product (US Department of Energy 2007). In this 1605(b) analysis a distributed approach is particularly useful because of the details of the changing product mix.

### 3.1 The distributed decay model

In the distributed decay model we recognize that products decay according to the time since production rather than in proportion to the size of the stock. We also assume that most of the decay might not occur in the first year after production. The decay is based on using a probability distribution for the decay of the product.

$$\frac{dS}{dt} = J(t) - \int_0^t J(t-\tau)P(\tau)d\tau$$

where $J(t)$ is the production term and $P(\tau)$ is the probability distribution for the product's decay likelihood. Here $\tau$ is the integration variable for the time since production. The integral adds up the removal from the stock of all previous years' productions according to the proportion expected to decay in each year following production. This method treats each years' production separately, recognizing that a newly-made product, particularly in the case of wood products, does not have the same probability of removal from the stock as an older or younger product.

In this paper, we use the probability distribution function for the Gamma distribution to demonstrate the model, but there may be other distributions that are equally or more applicable for particular stocks. The choice of the Gamma distribution is based on its extensive use for expected lifetimes and its simplicity without losing much flexibility. The Gamma distribution has two parameters that are used to adjust the characteristics of the distribution, $k$ and $\theta$. From data that relate to the year of peak decay and the year by which 95% of the product has decayed, the two parameters of the distribution, $k$ and $\theta$, are calculated. Any two data points could be used, depending on what data are most readily available for a particular product (half-life and mean life are two such possibilities). With the Gamma distribution, the model becomes,

$$\frac{dS}{dt} = J(t) - \int_0^t J(t-\tau) Gamma(\tau) d\tau$$

or more specifically,

$$\frac{dS}{dt} = J(t) - \int_0^t J(t-\tau) \frac{\tau^{(k-1)}}{\Gamma(k)\theta^k} e^{-\frac{\tau}{\theta}} d\tau$$

where $\Gamma(x)$ is the Gamma function (as opposed to the Gamma distribution) defined by $\Gamma(x) = \int_0^\infty s^{x-1} e^{-s} ds$.

Here, the necessary data to inform the model are the production function $J(t)$ and the two parameters for the distribution of decay. As we show later the decay function can be given in discrete form as well as in the continuous form shown here.

## 3.2 A single pool distribution model

Since the original single pool model is useful in many cases, it might be argued that both models ought to be used, depending on the product. It would be most useful if we could just adjust a few parameters and use the same model structure for different carbon-containing products. Fortunately, we can show that it is not necessary to use multiple models, at least in the situation of the single pool case. The single pool model is in fact a special case of the distributed model using the Gamma distribution. We show the connection here, beginning with the single pool model,

$$\frac{dS}{dt} = J(t) - \lambda S(t). \tag{1}$$

Since this equation is a linear first order differential equation, it can be solved using an integrating factor by standard methods. Whether or not an explicit answer is available depends on the form of the production term. In this case, however, we only want to show the equivalence of the two forms of the model, so we are able to leave the production term as the arbitrary function $J(t)$.

Moving the $\lambda S(t)$ term to the other side and multiplying by the integration factor, $e^{\lambda t}$, we get

$$e^{\lambda t} \frac{dS}{dt} + e^{\lambda t} \lambda S(t) = e^{\lambda t} J(t)$$

Here, by the standard procedure for linear equations, the left hand side can be rewritten as $\frac{d}{dt}(e^{\lambda t} S(t))$ and both sides are integrated from 0 to $t$.

$$e^{\lambda t} S(t) = \int_0^t e^{\lambda \hat{t}} J(\hat{t}) d\hat{t} \tag{2}$$

Again we use an integration variable, $\hat{t}$. We reserve $\tau$ for the final integration variable in order to match notation with the distributed model above.

The standard solution then depends on the integral of the production term multiplied by an exponential. If the production follows a "nice" functional form, the solution can be solved analytically. Otherwise a numerical solution is formed.

$$S(t) = e^{-\lambda t} \int_0^t e^{\lambda \hat{t}} J(\hat{t}) d\hat{t} \tag{3}$$

Now, we will work backward one step to show the relationship to the distributional form of the equation. First we take the derivative of both sides creating an integro-differential equation,

$$\frac{dS}{dt} = e^{-\lambda t} e^{\lambda t} J(t) - \lambda e^{-\lambda t} \int_0^t e^{\lambda \hat{t}} J(\hat{t}) d\hat{t}$$

and then simplify to bring everything inside the integral,

$$\frac{dS}{dt} = J(t) - \int_0^t \lambda e^{-\lambda (t-\hat{t})} J(\hat{t}) d\hat{t} \tag{4}$$

Finally, we use a change of variables with $\tau = t - \hat{t}$ to get

$$\frac{dS}{dt} = J(t) - \int_0^t \lambda e^{-\lambda \tau} J(t-\tau) d\tau. \tag{5}$$

This form of the equation is, in fact, the distributive model with $k = 1$ and $\theta = 1/\lambda$. This shows that the simple single pool model is a subset of the distributive model and can be included when the choice of parameters is appropriate.

The question then comes down to two basic issues, the usability of the distributive model and the benefits of being able to adjust the parameters to fit data where the single pool model is not appropriate. We can also, as we demonstrate below, show that for some products that might previously have been assumed to fit into the single pool model, the fit might be modified slightly to better reflect the data. In other words, our parameter $k$ might not be exactly one, but perhaps only slightly larger. This provides a more accurate fit to the data and improves the accuracy of our calculations.

### 3.3 The distribution of decay through time

The problem is that, in many analyses based on first order decay, the assumption is made that a product decays in simple proportion to the size of the stock at any time. The result is that the highest decay rates occur in the first years after production, and all products of the same type are assumed to decay according to the same timecourse regardless of when they were produced. In Table 1 below, it is illustrated that the peak rate of decay is not occurring in the first year for most products. Also, for the one product where the maximum rate of decay does occur in the first year, it is not clear when during that first year the peak occurs. It has been shown (Pingoud and Wagner 2006) in the case of first order decay the effects of allocating the production to different portions of the year in which they are reported. The same could be done with the data here since it is unclear when during the year the maximum decay

**Table 1** Data on the decay rate of some forest products

| Product (from oak) | Year of maximum decay | 95% decay period (years) | Gamma parameters | |
|---|---|---|---|---|
| | | | $k$ | $\theta$ |
| Waste, bark, fuel | 2 | 18 | 1.305 | 4.918 |
| Pulpwood | 1 | 5 | 1.418 | 1.196 |
| Particleboard | 15 | 40 | 3.676 | 5.419 |
| Pallet, packaging | 2 | 5 | 3.196 | 0.683 |
| Fencing | 40 | 80 | 6.662 | 6.976 |
| Construction | 150 | 300 | 6.740 | 26.045 |
| Mining | 40 | 1000 | 1.128 | 308.594 |

Standard data (years of maximum decay and 95% decay) courtesy of Robert Matthews, Forest Research UK — are converted to the parameters needed for the Gamma distribution model. These parameters were computed numerically based on the decay data from Forest Research. Parameters were calculated using the middle of the year listed as the peak time of decay and the time of 95% decay. Resulting values would be different for $k$ (slightly) and $\theta$ if the beginning or end of the year were used. In particular, "Pulpwood" would have $k = 1$ and $\theta = 1.235$ if the beginning of the year were used

occurred. The parameters of the Gamma distribution are more sensitive for products with short lifetimes.

The graphs in Fig. 2 below show the resulting Gamma distribution and fraction remaining curves (hazard functions) for the parameter values listed in Table 1. It is clearly not the case that most products, particularly long-lived products should be treated as a single pool.

In a distributed decay model, products are not treated as a single pool, but as a series of distinct products, while the rate of product loss is dependent on the time since production.

## 4 Results and comparison

The easiest way to recognize the difference between the current simple, single-pool representation and the distributive decay representation is to see how they differ when modeling data using parameters derived from known characteristics of currently used wood products. In Figs. 3, 4, and 5 we use the data from Table 2 to compare the alternate treatments of pulpwood, pallets and packaging, and fencing. For each figure we show the current simple method where $k = 1$ and compare it to the more accurate representation using the Gamma distribution with $k$ derived from known information about the characteristics of each of these wood products.

Figure 3 shows the decay of pulpwood, which has a maximum rate of decay occurring during the first year and has reached 95% decay during the fifth year. (For this illustration we do not consider that much of the carbon in pulpwood will be carried through to a derivative product.) The dotted line shows the distributed decay model for the decay of the product, simulating the characteristics that have been reported in Table 1. Comparing this to the simple model, where $k = 1$, we find that the two representations are very similar at the temporal scale of years but differ significantly at the scale of months. The single pool model assumes that the maximum rate of decay is at time zero and during the first five months the decay is significantly overestimated with respect to that of the distributed decay model.

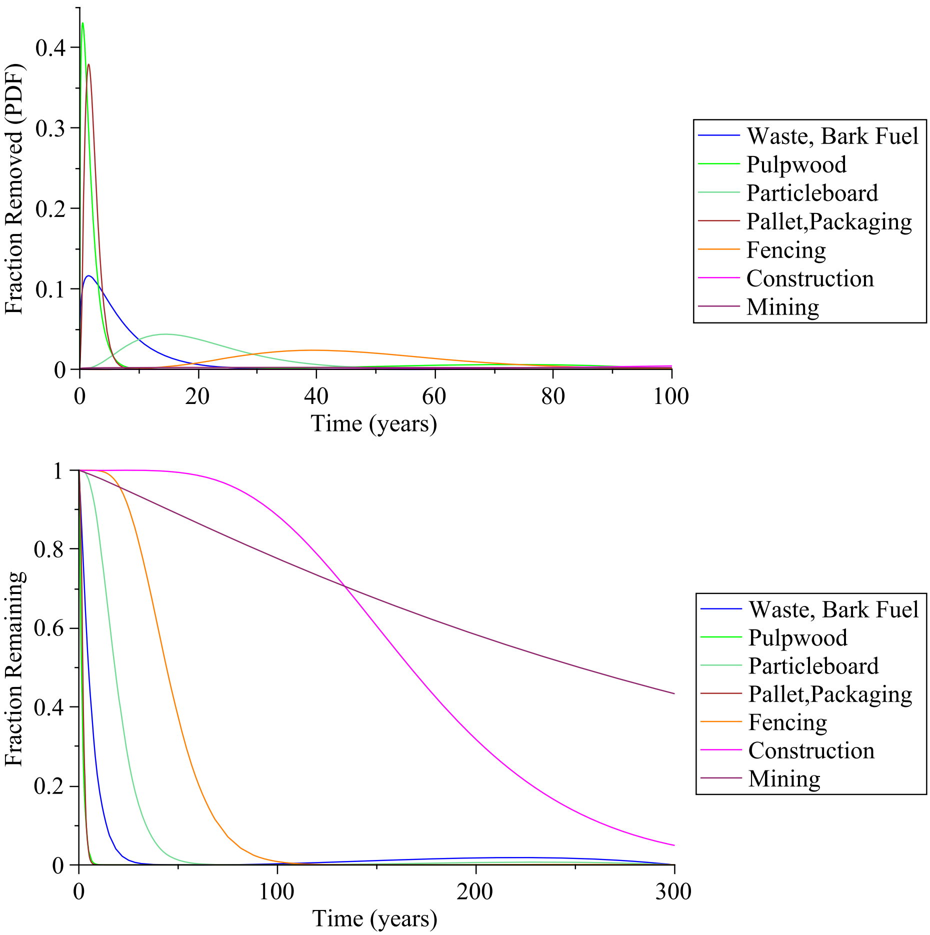


**Fig. 2** Distributive decay where the top plot shows the decay explicitly as a function of time. The bottom plot shows the remaining fraction of the stock left through time. All curves asymptotically approach zero as time increases. Data are from Table 1

Figure 4 models the decay of pallets and packaging products. With a slower decay, or longer life, the difference between the two models is greater, although the pattern is similar. In Fig. 5, the early overestimates of decay last for 20 years.

For some products the difference in the models may be small, but for others, particularly those with long life expectancy, the difference could be very important to some Parties. With the existence of tradable emissions permits, countries or companies could be spending more than required on emissions permits.

## 5 Discounting

In order to mitigate climate change and reduce the release of $CO_2$ into the atmosphere a system is being implemented to monitor, record, and restrict the amount

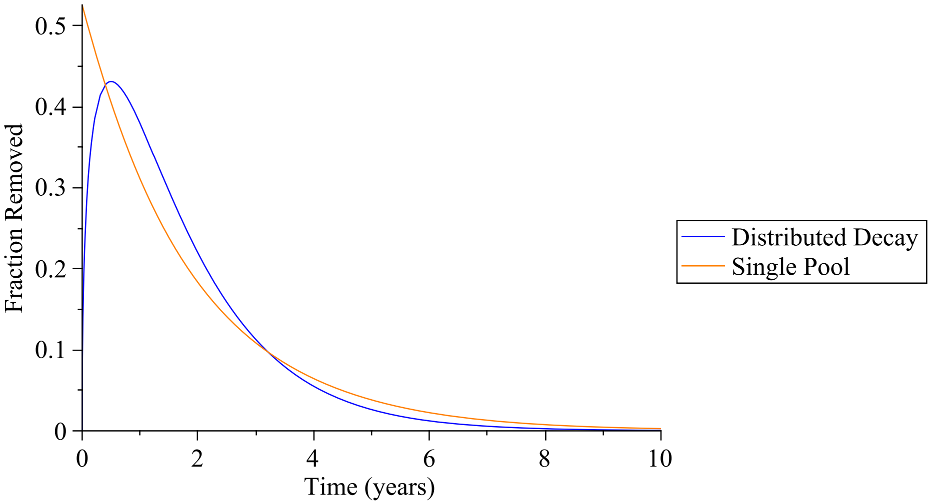


**Fig. 3** Comparison of Gamma distributions showing the decay of pulpwood where $k = 1$ (representative of the simple model) and where $k = 1.418$. Both models are shown with equal half-lives of 1.319

of carbon that is released. An evolving portion of this is a tradable permit system where countries and/or companies are issued permits based on some criteria and then are limited to only releasing $CO_2$ for which they have permits. If a country or company wishes to release additional quantities of carbon they need to purchase the corresponding permits. Countries or companies that release less than their quota can sell their excess permits in secondary markets. This system allows a collective

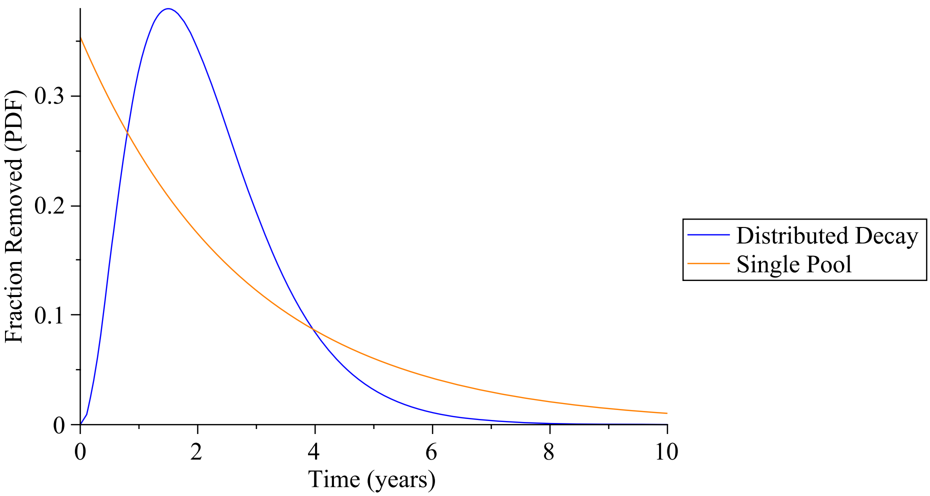


**Fig. 4** Comparison of Gamma distributions showing the decay of pallet and packaging products where $k = 1$ (representative of the simple model) and where $k = 3.196$. Both models are shown with equal half-lives of 1.960

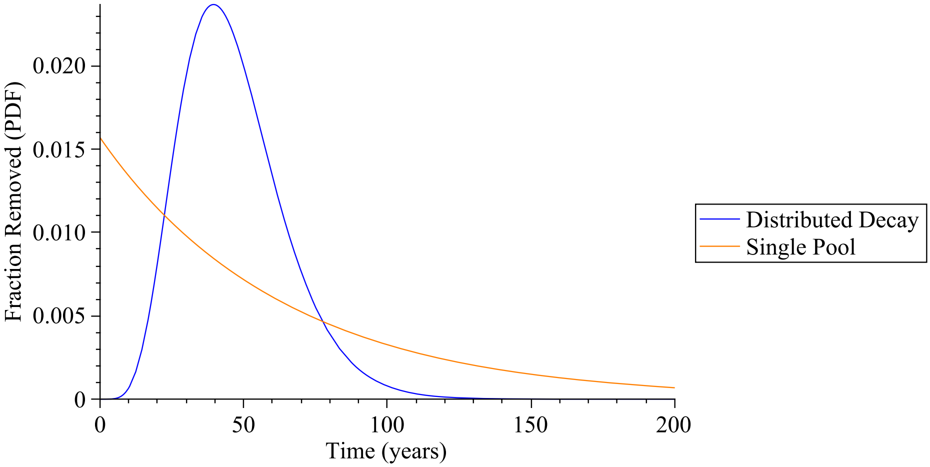


**Fig. 5** Comparison of Gamma distributions showing the decay of fencing where $k = 1$ (representative of the simple model) and where $k = 6.662$. Both models are shown with equal half-lives of 44.171

decision on the total amount of carbon that will be allowed to be released during a given time span without setting absolute restrictions on each individual country or company. This system is proposed to not only restrict the total amount of $CO_2$ released but to promote innovation and research into non-emitting technologies and activities. Alternatively, systems to restrict emissions could impose a tax on emissions or establish emissions targets with penalties on excess emissions.

Considering emissions permits to be tradable assets creates the necessity of investigating the financial implications of $CO_2$ emissions. Given the different paths of emissions illustrated in Figs. 3, 4, and 5, there may be significant financial implications in accurately recognizing the time at which emissions occur. In alternative systems

**Table 2** Cost savings for properly representing the lifetime of wood products when the cost of emissions permits or emissions taxes are discounted at increasing discount rates

| | Percent cost savings from discounting | | |
|---|---|---|---|
| | 0% | 2% | 7% |
| Pulpwood | | | |
| Single pool model | 100% | 97.6639% | 92.2748% |
| Distributed model | 100% | 96.7036% | 89.2252% |
| Difference | 0 | 0.9603% | 3.0495% |
| Pallet and packaging | | | |
| Single pool model | 100% | 98.6524% | 95.4371% |
| Distributed model | 100% | 95.7565% | 86.1345% |
| Difference | 0 | 2.8959% | 9.3027% |
| Fencing | | | |
| Single pool model | 100% | 87.7562% | 67.1899% |
| Distributed model | 100% | 41.8908% | 7.0713% |
| Difference | 0 | 45.8655% | 60.1186% |

Representative products have been chosen to show the effect on products of varying expected lifetime. Values shown are the relative cost of emissions as described by the single pool and distributed models as compared to those that would occur from immediate oxidation at the time of harvest without tradable permits e.g. in a system with taxes on emissions or a penalty on excess emissions there is a similar economic value in recognizing the correct time at which emissions occur. When considering the cost of permits or taxes we consider the current value of $CO_2$ releases that are not immediate but are delayed according to our description of the life of the product. This is the present value of savings if emissions occur in future years rather than in the current year. Looking at the discounted rates of current and future payments will show the difference between the single pool model and the distributed model.

We illustrate by looking at the three oak products shown in Figs. 3, 4, and 5: pulpwood, pallets and packaging, and fencing. We use discount rates of 2% and 7% for each product simply to illustrate the impact of discounting future costs and changing the pattern of reported emissions over time. Table 2 shows the relative cost of emissions permits or taxes as compared to those that would occur under the IPCC default method where harvested materials are assumed to be immediately oxidized. The relevant equation is:

$$\int_0^\infty Gamma(t)\, e^{-\lambda t}\, dt \tag{6}$$

Table 2 shows that the net present value, with a 7%/year discount rate (for the average expected lifetime of fencing), of treating the decay as a Gamma distribution rather than as a first order decay is 60% of the current value of emissions permits. At a 7% discount rate even a single pool model results in a 7% savings with respect to immediate release of carbon. In every case, use of the model based on the Gamma distribution results in a savings in the net present value in taxes or emissions permits, and the savings increase with the expected lifetime of the product and the discount rate on the investment.

## 6 Recycling and use changes

Since not all carbon containing products are oxidized at the same rate, we recognize the need to treat them differently. At the same time, we need to acknowledge that not all products are removed from a stock because of oxidation. Products can be recycled or reclaimed for alternative uses, or sent to a landfill. The distributed model does not present any more difficulties in accounting than previous methods in providing a straightforward method of removing products from a stock and allocating them to different end uses. That is, we expand the notion of removal of the stock to include both decay, landfill storage, and removal to other uses, and we expand the notion of production to include recycled materials. Incorporating these ideas becomes crucial when you realize that in the US much of paper and other wood products end up surviving for long times in landfills and less than half of the carbon in wood and paper is ever converted to CO2 (estimated in Skog and Nicholson 1998).

To incorporate this idea analytically into the model by combining distributions gets complicated very quickly. Fortunately, the removal can easily be simulated using a numerical simulation or by estimating the fraction that was removed from a stock that was moved to another stock rather than oxidized. These resulting fractions of the removal from a particular stock are then treated as the production term in another stock as shown in Fig. 6.

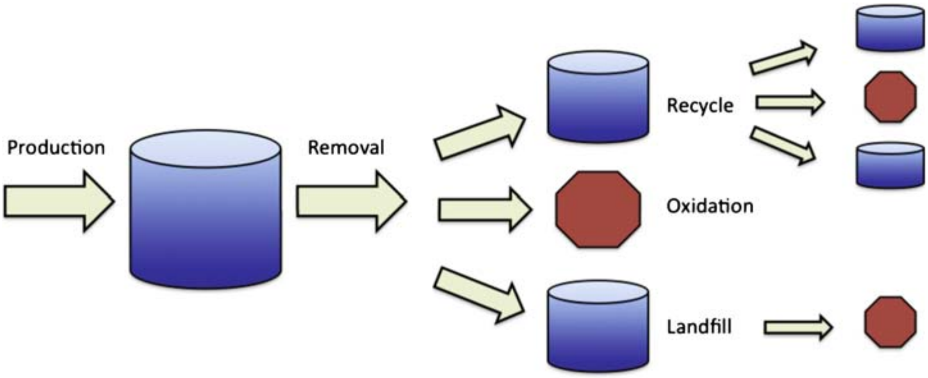


**Fig. 6** Flow of carbon from one stock to another, eventually ending in permanent storage or oxidation

## 7 Handling discrete data

To this point this paper has been based on the functional form of data showing production over time and the pattern of product decay, but, inevitably, data are reported as discrete numbers which represent production values and use changes over the course of an entire year. Since many products are not produced, consumed, or oxidized uniformly over the course of year, it is important to note the potential differences that this non-uniformity might inflict upon stock calculations. Pingoud and Wagner (2006) have discussed different methods of allocating production data into the simple decay model. Their treatment is comprehensive and applies to the distributed method as well. Here we focus instead on the decay portion of the equations.

Decay in the current models is assumed to be a continuous process. Yet, since the production data are tallied in yearly discrete quantities, the decay or removal must also be considered in the same discrete fashion. For the distributed model we can separate the model initially into discrete pieces.

We can begin by breaking the integral into yearly pieces,

$$\int_0^t J(t-\tau)Gamma(\tau)d\tau = \int_0^1 J(t-\tau)Gamma(\tau)d\tau + \int_1^2 J(t-\tau)Gamma(\tau)d\tau + \ldots \tag{7}$$

In summation notation, the expression will include a tail-end piece which reflects a partial year at the end,

$$\int_0^t J(t-\tau)Gamma(\tau)d\tau = \sum_{n=0}^{T-1}\left(\int_n^{n+1} J(t-\tau)Gamma(\tau)d\tau\right) + \int_T^t J(t-\tau)Gamma(\tau)d\tau \tag{8}$$

where $T$ is the time of the last full year. If we also assume that we only take an accounting at the close of each year (that $t$ is an integer) the expression simplifies,

$$\int_0^t J(t-\tau)Gamma(\tau)d\tau = \sum_{n=0}^{t-1}\left(\int_n^{n+1} J(t-\tau)Gamma(\tau)d\tau\right) \tag{9}$$

Now we run into the same issues that Pingoud and Wagner (2006) approached concerning the allocation of discretely (yearly) reported data from continuous processes. While it is possible to include calculations for each of the possibilities outlined in that paper, the mathematics and ensuing calculations are simplified greatly if we assume that production is constant over the course of a year. Assuming the production is constant, the production term $J(t)$ in each of the integrals becomes a constant and can be pulled out of the integral and $J_n$ is then the total production for year $n$.

$$\int_0^t J(t-\tau)Gamma(\tau)d\tau = \sum_{n=0}^{t-1} J_{(t-n)}\left(\int_n^{n+1} Gamma(\tau)d\tau\right) \tag{10}$$

Using this discretization of the integral, and the matching assumption for the production term (piecewise constant) we derive a discrete equation for the change in stock over the course of a year.

$$S_{n+1} - S_n = J_n - \sum_{n=0}^{t-1} J_{(t-n)}\left(\int_n^{n+1} Gamma(\tau)d\tau\right) \tag{11}$$

The interesting result of this approach is that for a particular product, the Gamma integrals can be stored in a spreadsheet and linked to the yearly production data for different products. The change in stocks is then calculated by spreadsheet. No assumptions are needed for the functional form of the production. As we noted in Marland and Marland (2003), the validity of the single pool model depends not only on the decay attributes of a product, but also on the functional form of the production and the time since that functional form was valid. This current result shows a need for only the discrete production data and the Gamma parameters for each stock.

In Table 3, the decay fractions are given for $k = 1.305$ and $\theta = 4.918$. The values are calculated according to the partial integrals above for each of the first 10 years.

Using these yearly decay fractions, we can then form a table to calculate the decay from all years' production (Table 4).

This table then becomes very easy to work with and could easily be calculated automatically in a database with look-up tables for the gamma distributions for the various products. The only data that would need to be entered would be the parameter values for the decay distribution ($k$ and $\theta$) and the yearly production value. For parties with more detailed accounting of products, a discrete decay distribution could be entered with separate tables for subcategories as needed. For less sophisticated users, predefined values for $k$ and $\theta$ can be specified for various products or product groups (such as "Oak, pallets and packaging"). The necessary data needed are then the production values for different products and the name of the product.

**Table 3** The percent of the original years' production that decays in each subsequent year for waste, bark, and fuel in Table 1 ($k = 1.305$, $\theta = 4.918$)

| Oak — waste, bark and fuel | |
|---|---|
| Gamma integrals for $k = 1.305$, $\theta = 4.918$ | |
| Year | Fraction decay |
| 0 | 0.095416 |
| 1 | 0.115683 |
| 2 | 0.110760 |
| 3 | 0.100279 |
| 4 | 0.088402 |
| 5 | 0.076717 |
| 6 | 0.065889 |
| 7 | 0.056173 |
| 8 | 0.047627 |
| 9 | 0.040208 |
| ⋮ | ⋮ |
| Total | 1.000000 |

## 8 Discussion

This paper is concerned with accurate treatment of harvest wood products in inventories of $CO_2$ emissions to the atmosphere. The methodologies outlined in the paper demonstrate a flexible way to expand current methods to include the use of more accurate and detailed data while retaining the simplicity of simple formulas. The methodology also allows the use of the same method, but with different levels

**Table 4** In this example, each years' production (column 2) is multiplied by the fraction that decays in a given year (in a look-up table, see Table 3 as an example) to give the quantity that decays in a given year

| Example table for calculating stock decay | | | | | | | |
|---|---|---|---|---|---|---|---|
| Year | Production (units) | Decay in year | | | | | |
| | | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
| 2010 | 12 | 1.145 | 1.388 | 1.329 | 1.061 | 0.921 | 0.791 |
| 2011 | 15 | 0 | 1.431 | 1.735 | 1.661 | 1.326 | 1.151 |
| 2012 | 19 | 0 | 0 | 1.813 | 2.198 | 2.104 | 1.680 |
| 2013 | 22 | 0 | 0 | 0 | 2.099 | 2.545 | 2.437 |
| 2014 | 28 | 0 | 0 | 0 | 0 | 2.672 | 3.239 |
| 2015 | 29 | 0 | 0 | 0 | 0 | 0 | 2.767 |
| 2016 | 37 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2017 | 46 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2018 | 45 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2019 | 50 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2020 | … | | | | | | |
| Total decay | | 1.145 | 2.819 | 4.877 | 7.019 | 9.568 | 12.065 |
| Change in stock | | 10.855 | 12.181 | 14.123 | 14.981 | 18.432 | 16.935 |
| Total stock | | 10.855 | 23.036 | 37.159 | 52.140 | 70.572 | 87.507 |

The decay quantities are then added in the vertical columns to calculate total decay for a particular calendar year. Values here are calculated for $k = 1.305$, $\theta = 4.918$ (see Table 3). The change in stock is the current year's production minus the total decay in that year and the total stock is the cumulative sum of all previous values for change in stock of detail, for Parties that have different levels of detail in their data collection. The paper demonstrates that the currently most widely used method is a special case of what we propose here and that the more accurate representation of decay time can have significant economic implications in a system where emissions are taxed or emissions permits are traded. The method can be easily applied using only data on annual production of wood products and look-up tables to characterize the two parameters of a gamma distribution.

It should also be noted that these methods are by no means specific to wood products. As we have seen in the graphs and tables, the methods become more useful as the stock deviates from a single pool, simply decaying, product. In other words, long-lived carbon containing products from sources other than wood might benefit from this approach, e.g. long-lived petrochemical products. A single unifying approach that is both simple and flexible has the potential to be more accurate in its results, more efficient in its implementation, and economically important to some Parties.

## References


Brown S, Lim B, Schlamadinger B (1999) Evaluating approaches for estimating net emissions of carbon dioxide from forest harvesting and wood products; meeting report, Dakar, Senegal, 5–7 May 1998; IPCC/OECD/IEA Programme on National Greenhouse Gas Inventories, Intergovernmental Panel on Climate Change, 51 pp

Cowie A, Pingoud K, Schlamadinger B (2006) Stock changes or fluxes? Resolving terminological confusion in the debate on land-use change and forestry. Climate Policy 6:161–171

FAO (2009) Food and Agriculture Organization of the United Nations. http://faostat.fao.org

Ford-Robertson JB (2003) Implications of harvested wood products accounting analysis of issues raised by parties to the UNFCCC and development of a simple decay approach. MAF technical paper no. 2003/5, Ministry of Agriculture and Forestry, Wellington, 30 pp. http://www.maf.gov.nz/forestry/publications/index.htm

Hashimoto S (2008) Different accounting approaches to harvested wood products in national greenhouse gas inventories: their incentives to achievement of major policy goals. Environ Sci Policy 11:756–771

IPCC (1997) Revised 1996 IPCC guidelines for national greenhouse gas inventories. In: Houghton JT, Meira Filho LG, Lim B, Treanton K, Mamaty I, Bonduki Y, Griggs DJ, Callandar BA (eds). Intergovernmental Panel on Climate Change

IPCC (2007a) 2006 IPCC guidelines for national greenhouse gas inventories. Intergovernmental Panel on Climate Change

IPCC (2007b) 2006 IPCC guidelines for national greenhouse gas inventories, volume 4, chapter 12 (Pingoud K, Skog K, Martino DL, Tonosaki M, Xiaoquan Z, Ford-Robertson J (2006) Harvested wood products. 33 pp). Intergovernmental Panel on Climate Change

Kyoto Protocol to the United National Framework Convention on Climate Change (1997)

Lim B, Brown S, Schlamadinger B (1999) Carbon accounting for forest harvesting and wood products: review and evaluation of different approaches. Environ Sci Policy 2(2):207–216

Marland E, Marland G (2003) The treatment of long-lived, carbon containing products in inventories of carbon dioxide emissions to the atmosphere. Environ Sci Policy (6):139–152

Pingoud K (2003) Harvested wood products: considerations on issues related to estimation, reporting and accounting of greenhouse gases. Final report delivered to the UNFCCC secretariat, January 2003

Pingoud K, Wagner F (2006) Methane emissions from landfills and carbon dynamics of harvested wood products: the first-order decay revisited. Mitig Adapt Strategies Glob Chang 11:961–978

Pingoud K, Peräiä A-L, Soimakallio S, Pussinen A (2003) Greenhouse gas impacts of harvested wood products: evaluation and development of methods. In: VTT research notes 2189. VTT, Espoo, Finland

Pingoud K, Skog K, Martino DL, Tonosaki M, Xiaoquan Z, Ford-Robertson J (2006) Chapter 12. Harvested wood products. 33 pp + HWP worksheet MS Excel. In: 2006 IPCC guidelines for national greenhouse gas inventories, vol 4. Agriculture, forestry and other land use. http://www.ipcc-nggip.iges.or.jp/public/2006gl/vol4.htm

Row C, Phelps RB (1996) Wood carbon ows and storage after timber harvest. In: Sampson RN, Hair D (eds) Forests and global change, vol. 2: forest management opportunities for mitigating carbon emissions. American Forests, Washington, DC, pp 27–58

Skog K (2008) Sequestration of carbon in harvested wood products for the United States. For Prod J 58(6):56–72

Skog K, Nicholson GA (1998) Carbon cycling through wood products: the role of wood and paper products in carbon sequestration. For Prod J 48:75–83

Tans P (2007) Trends in atmospheric carbon dioxide global. NOAA/ESRL National Oceanic and Atmospheric Administration/Earth System Research Laboratory (www.esrl.noaa.gov/gmd/ccg/trends). Downloaded 11 December 2007

Tonn B, Marland G (2007) Carbon sequestration in wood products: a method for attribution to multiple parties. Environ Sci Policy 10(2):162–168

UNECE (2008) Harvested wood products in the context of climate change policies: Workshop announcement for a workshop to be held in Geneva, Switzerland, 9–10 September 2008 (accessed 26 August 2008). http://www.unece.org/trade/timber/workshops/2008/hwp/. United Nations Economic Commission for Europe, Timber Committee

UNFCCC (2002) Report of the conference of the parties on its seventh session, held at Marrakesh from 29 October to 10 November 2001, FCCC/CP/2001/13/Add.1, pp 54–62, United Nations Framework Convention on Climate Change, 21 January 2002

UNFCCC (2003) Estimation, reporting, and accounting of harvested wood products: technical paper, FCCC/TP/2003/7. United Nations Framework Convention on Climate Change, 27 October 2003

UNFCCC (2004) Issues relating to harvested wood products; submissions from parties, FCCC/SBSTA/2004/MISC.9. United Nations Framework Convention on Climate Change, Subsidiary Body for Scientific and Technological Advice, 10 May 2004

UNFCCC (2008a) Report of the conference of the parties on its thirteenth session, held in Bali from 3 to 15 December 2007, addendum, part two: action taken by the conference of the parties at its thirteenth session. FCCC/CP/2007/6/Add.1, 14 March 2008, United Nations Framework Convention on Climate Change

UNFCCC (2008b) Land use, land-use change and forestry: draft conclusions proposed by the chair, FCCC/KP/AWG/2008/L.11, 27 August 2008. United Nations Framework Convention on Climate Change, Ad Hoc Working Group on Further Commitments for Annex I Parties under the Kyoto Protocol

US Department of Energy (2007) Technical guidelines: voluntary reporting of greenhouse gases 1605(b) program. Office of Policy and International Affairs, pp 232–233 and part I appendix, at http://www.eia.doe.gov/oiaf/1605/gdlins.html. Downloaded 14 August 2009