Exploring Long Division Through Division Quilts

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Abstract:

As a collective, all of the authors agree that at some point in our teaching careers we recognized that there were minimal ways to demonstrate division when teaching the algorithm in isolation; furthermore, there are rare opportunities to adhere to the expectations in mathematics education (Common Core Standards Initiative, 2011; NCTM, 2000) that students should be able to identify and use relationships between operations. Imaging is an important activity (Richardson, Pratt & Kurtts, 2010; Wheatley, 1998) that allows for such opportunities. In this article, we outline a series of activities that provide students different representations of division to help them achieve understanding and proficiency of the algorithm due to their understandings of the visual aide. It is important to state here that we believe students should be able to use the division algorithm, but only as it is attached to meaning making and images. We argue that by using Division Quilts, students are better able to demonstrate their comprehension of division, through student work as well as standardized assessments.

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*****Note: Full text of article below**

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About the Cover
The image on the cover depicts the centroid of a triangle.

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TABLE OF CONTENTS

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Thomasenia Lott Adams: *Mathematics Proficiency: What is it? How do we help students get it?*

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Exploring Long Division through Division Quilts

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Kerri Richardson, UNC at Greensboro, Greensboro, NC

New teachers in elementary schools often follow the same strategy for teaching division as they experienced in school, memorizing the algorithm. In this article Tina Lupton shares her experiences in the transition from helping students "understand" division – by reciting "divide, multiply, subtract, bring down" again and again until they could perform proficiently – to affording opportunities for students to "imagine" division.

As a collective, all of the authors agree that at some point in our teaching careers we recognized that there were minimal ways to demonstrate division when teaching the algorithm in isolation; furthermore, there are rare opportunities to adhere to the expectations in mathematics education (Common Core Standards Initiative, 2011; NCTM, 2000) that students should be able to identify and use relationships between operations. Imaging is an important activity (Richardson, Pratt & Kurtts, 2010; Wheatley, 1998) that allows for such opportunities. In this article, we outline a series of activities that provide students different representations of division to help them achieve understanding and proficiency of the algorithm due to their understandings of the visual aide. It is important to state here that we believe students should be able to use the division algorithm, but only as it is attached to meaning making and images. We argue that by using Division Quilts, students are better able to demonstrate their comprehension of division, through student work as well as standardized assessments.

Multiplication and division are inverse operations, and both can be understood through models of grouping/fair sharing, arrays and area model (Ma, 1999; Reynolds & Wheatley, 2010). Progressing through understandings of these three models strengthens connections between multiplication and division, as well as expanding understandings of algebraic structures and geometric structures. Starting students with Base-10 blocks to represent division as fair sharing allows connections between personal experiences and mathematical concepts. Moving to division as an array and an area model provides more images with which students can work to achieve deeper understandings and connections among mathematical concepts. We provide Division Quilts as one way to foster opportunities for conceptual development, promoting exploration of patterns, shapes, measurements, spatial relationships, and number sense (Dark, 2007). Specifically, there is a cognitive leap that occurs between grouping/fair sharing and area models to represent division. Division Quilts provide an opportunity for individual students to transition to the more sophisticated model in their own time.

The data collected in this study comes from multiple 4th grade classrooms in a Title 1 School (2008-2012) where over 80% of the students are considered living at or below the poverty level[1]. The demographics of the classrooms over the years have consisted of approximately of 13-15 boys, 10-12 girls, 5-8 English as a Second Language Learners, 2-4 EC students (1 B.E.D., 2 ADHD, 1 OHI), and 2-4 Academically Gifted students. Division Quilts are used as a strategy that meets the needs of these diverse learners.

[1] In 2010 NC adopted the Common Core State Standards creating a lapse in comparable data.

Based on the data collected from the 2008 and 2009 Benchmark Assessments, the content standard for three digit by two digit division increased 12% after the first year of implementing Division Quilts. The 12% increase meant that students were now performing at 61% proficiency with division. Division Quilts made a significant impact on the comprehension of number sense. The greatest impact that Division Quilts had was on standard for strategies for multiplying and dividing numbers, which increased 22% from 2008 to 2009.

Division Quilts

Division Quilts are created on grid paper by outlining the dividend then shading the divisor with different colors to reflect the quotient. The outlined polygon is referred to as the dividend, and once shaded by the pattern of the divisors, it is now a "quilt." To engage students in thinking about division, the teacher begins the lesson by asking students to represent 25 ÷ 7 using Base-10 Blocks. The representations created by the students exhibit a fair sharing model, which is an easy model for students to understand. From there, the teacher will transition to the Division Quilts, using the same division problem of 25 ÷ 7. The problem is selected because it includes remainders, skip-counting by 5, and it is a small enough dividend that it is not numerically challenging. The first quilt should be done the same by all students. The teacher can suggest to students to find the dividends by counting by a factor of 25, which are 1, 5, and 25. In this problem counting by 5 is the most efficient choice. Starting with the first unit/block/square at the top left of the grid paper, everyone counts to the right 5, then from that unit count down rows by multiples of 5 until the dividend of 25 is outlined (Figure 1a). Once the dividend is verified (Figure 1b), the outline is then colored with a heavy marker to enclose the dividend for the quilt (Figure 1c). For future Division Quilts, students choose the polygon to be outlined, based on the level of the student. For some students, it may be best to have a pre-drawn outline and have them record the dividend. For other students it may be enriching to tell them the dividend and let them decide how to draw the outline. As long as the outline encloses the dividend as a polygon (closed, straight, non-overlapping figure) then the quotient can still be found. It is not necessary that the outline is an array, square or rectangle; some quilts may need to have "loose strings" where a few units are needed that exceeds a quadrilateral (Figure 2). The less complex the outline, the easier the next steps will be, and as students become more proficient, they will move toward creating rectangles. To differentiate, the teacher can guide students to group the units by a factor of the number and skip count until they get to the dividend. If students find it difficult to skip count by the divisor, then it may be easiest to count by 5's or 10's until they are close to or on the number.

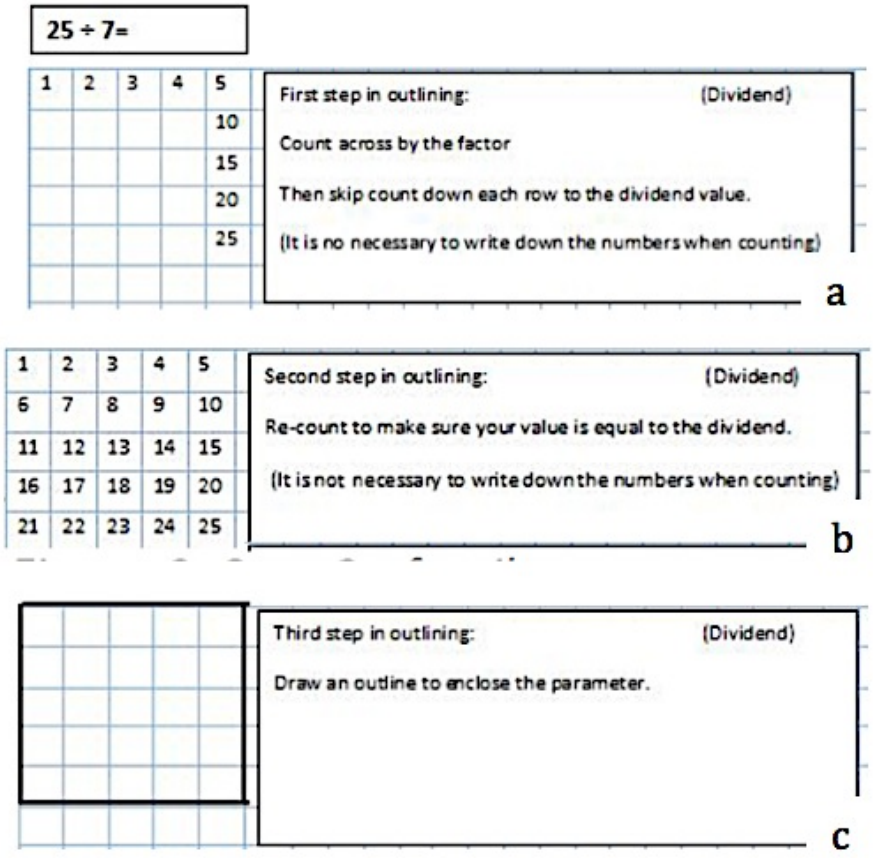

Figure 1: Building the Quilt in Steps

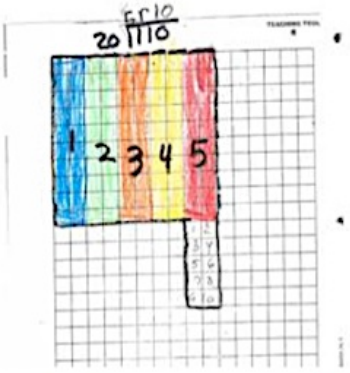

Figure 2: A non-rectangular quilt

Now that the outlined dividend creates the parameter and is verified, the next step is to color in the divisor. In this case the divisor is 7, so each student picks a color and shades in 7 blocks (Figure 3). Count each unit one at a time 1-7. Show the students that they can count vertically, horizontally, or in a pattern that makes sense, as long as the 7 units are adjacent. Students should be discouraged from doing "checkerboard" style coloring. Remind them that the divisor is a collection, so the group is colored together.

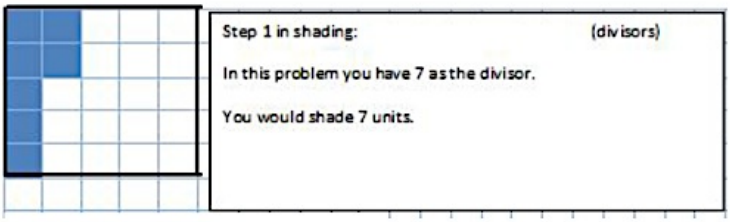

Figure 3: Step 4 of the Quilt

Next have the students count 7 new units and color them a different color (Figure 4). Students will continue a pattern of changing colors and coloring groups, until they cannot make any more groups of the divisor. In this example, there are 4 white spaces left, representing the remainder. Number each white unit that is remaining. Now that the quilt is colorful and remainders are labeled, students number each of the groups of 7 with a heavy marker. The labels allow students see the image of the quotient.

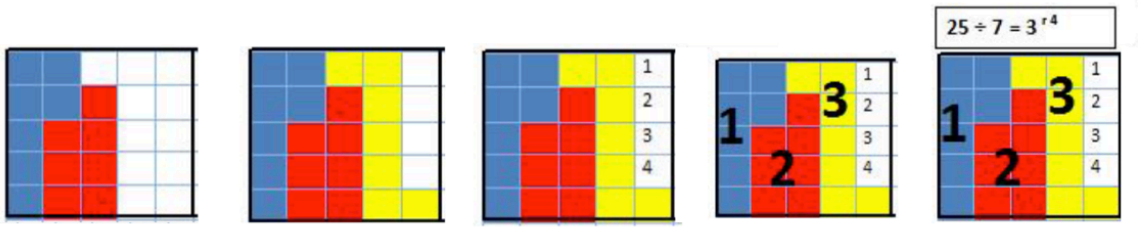

Figure 4: Steps 5 through 9

Once the quilt has been made, the teacher should review the process. Reviewing the steps allows the teacher to answer questions, clear up misconceptions, review the vocabulary, label the vocabulary terms on the quilt, or go over alternative options in making the quilt. The next step is to connect the process of creating the Division Quilt to the division algorithm. The process of creating the quilt can relate to the "divide, multiply, subtract, bring down" strategy. Each time a group is colored, a piece of the dividend is being removed, which is subtraction. With the first colored group (25 - 7), 18 blocks of the dividend remain. As long as the number of blocks remaining is greater than the divisor, then the process is to continue dividing the quilt into groups, explaining this can help clarify to students why we "Bring down" in the algorithm. The repeated grouping of 7 shows the meaning of division. Each additional group increases the number of groups of size 7 that are colored. Once all sets of the divisor (7) are colored (3 groups of 7), the number of groups times the divisor gives the dividend with the remainder of the uncolored blocks (3 times 7 is 21 units with 4 units left). Through the process the teacher can ask students to pause and count how many groups have been made, if they are to multiply the divisor by the number of groups, what part of the quilt would that answer represent? (The answer is the dividend.) It is important to not only have students go through the process, but to also have conversations and engage mathematical thinking about the process.

Advancing to the next step of interpreting the remainder is an easy progression. Ask students what would happen with the 4 units that are left over. Prompt two different scenarios to suggest when the remainder affects the quotient from being simply interpreted as a number versus a logical answer. For example, if there are 25 students and 7 can fit in a canoe, how many canoes are needed? Students are required to think logically and take into consideration what a remainder represents. In this scenario it means that 3 canoes are not sufficient because 4 students would not be able to partake in the trip, so a fourth canoe would be required in order to include those that did not make a complete group. Another scenario is a wedding photographer creating a 30-page album with 236 photos, and the

bride and groom want the same number of pictures on each page. What is the maximum number of photos that can be placed on each page? With this scenario the interpretation of the remainder provides the number of pictures that will not be included in the album, but it will not affect how many pictures will be placed on each page. The photographer could place as many as 7 pictures on a page, using 210 pictures, and so 26 pictures would be left out of the album. Students are required by state standards to analyze remainders in real life word problems. They are expected to consider situations where the remainder indicates that the value of the quotient is not enough to accommodate, to consider that the remainder is the desired answer, or to consider that the value of the quotient is sufficient as is.

Once students have completed their first Division Quilt along with the teacher's modeling and have had time to process the strategy, they can proceed with guided modeling. With guided modeling the teacher will present another problem, such as 56 ÷ 8, and pose questions to guide the students through the steps. The teacher would start by asking, "If I am dividing 56 by 8 then what is my dividend?" 56. "If my dividend is 56 then what is the first thing I need to do to set up the problem?" Find a factor of 56. "What are the factors of 56?" 1, 2, 4, 7, 8, 14, 28, 56. "So which factor should we use?" Appropriate factors would be 4, 7, or 8. This is where the students decide which factor they want to count by or if they want to count by 5's or 10's, depending on their understanding of factors and multiples. If they count by 5, then they would be able to get to 55 and then would need to include one extra unit underneath to have the whole dividend.

Once students have decided which factor works best for them, prompt students to consider the different shapes and ask if it will change the quotient. The answer is that it will not because all of the shapes have the same dividend, even if they look differently. While students are verifying their outlined dividend, check students' understandings. The next step is for students to color a group of size 8 each time to show the divisor. The teacher transitions between modeling for all to see and monitoring students. When students have finished, ask "How was this quilt different from the first quilt?" One answer is that the second quilt has no remainder so all units are colored in with nothing left over. More questions can be used as needed, depending on the struggles and success of the students.

Gauge each student's ability to determine which students need pre-outlined dividends and which are able to outline the dividends on their own. After the first two quilts have been modeled, it would be acceptable to have students create their own problems or give them several to try on their own alternating between problems that have remainders and those that do not. Some students may be able to do triple digit dividends with single and double-digit divisors. (Note: For these advanced problems, use 1 cm grid paper so there are enough units. Figure 6 shows an example of this.)

Mathematical Purposes of Division Quilts

Division Quilts address the mathematical ideas of quotient, divisor, dividend, remainder, the meaning of division, and how multiplication and division are inversely related. Modeling can help promote problem solving and provide rich learning experiences (English, Fox, & Walters, 2005). Initially, Division Quilts should be introduced by pre-drawing a quilt outline for students for the purpose of primarily teaching them about the groups, or parts (divisor). While doing this activity, it is important to prompt students' thinking by asking questions such as: "Why does each group have x number of blocks?" "Why would I want to use a new color when I build a new group?" "How can I draw a group?" When focusing on the dividend they will see the "whole" that is being divided in the problem and help them understand what they are dividing. During this phase, questions can be asked, such as: "How many total blocks did you need?" "How did you find your outline?" "Could there have been an easier way to find your quilt?" "Was there a pattern?" The result then becomes a natural focal point, leading to

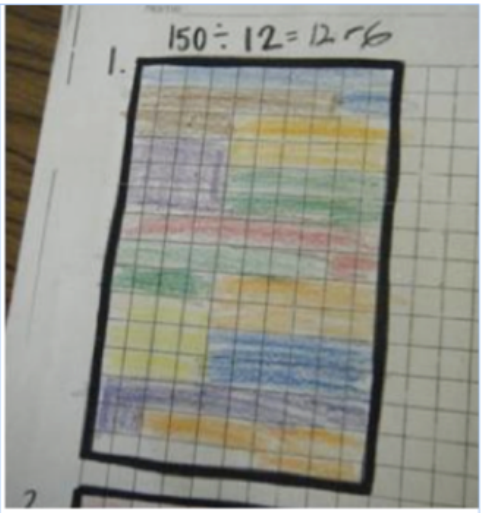

Figure 5: Modeling the Division Quilt

discussions about the quotient. Finally, problems can be introduced in which remainders becomes a part of the result, from the image of an excess of white blocks/squares/units.

Once students have all possible groups colored, prompt student thinking with questions such as: "How many blocks are left?" "Why should we not color them?" "How many more blocks do you need to make a group?" "Can you have more white blocks left then the value of your divisor"?" What if you did not have any white blocks left?"
It is best for students to use an array of colors so they can see each group individually. Students will determine what they want their groups to look like based on the amount of the divisor and the length of the Division Quilt. Some students will count their groups horizontally across the quilt; others will make a pattern/tessellation. After one group is formed, it is important for students to keep their patterns similar and interchange colors so they can determine where one group ends and another begins (Figure 5).

Concepts in Division Quilts
With the use of Division Quilts, students are exposed to the terminology of the long division algorithm; understanding the terms in division helps students in problem solving with division. Division Quilts create an image for the dividend, the inside of the quilt. A common misconception for students is being able to understand what they are dividing, so if students can start division by physically seeing what they are dividing, there is an increase in their success to interpret division problems because what they understand is more concrete.

Another common misconception for students is the importance of remainders. Division Quilts are a great way for students to visually see what a remainder is, as well as how the value of the remainder may or may not change the interpretation of the quotient depending on the situation and whether to exclude or include the remaining pieces. Some students believe that the remainder is how many groups are left, instead of thinking how many individual items are left. Some students' misconceptions of remainders stem from inexperience of manipulating a problem. Many times students will go through the mathematical process of division and know that there is a remainder but do not know what it means. By seeing the white space in the quilt, students can recognize that it is part of the dividend, but not part of the divisor or the quotient. In addition, Division Quilts can help students make connections to multiplication. Prompting students by asking questions that relate the image to multiplication as well as division will generate opportunities for them to recognize this relationship and will help them build their understanding between operations.

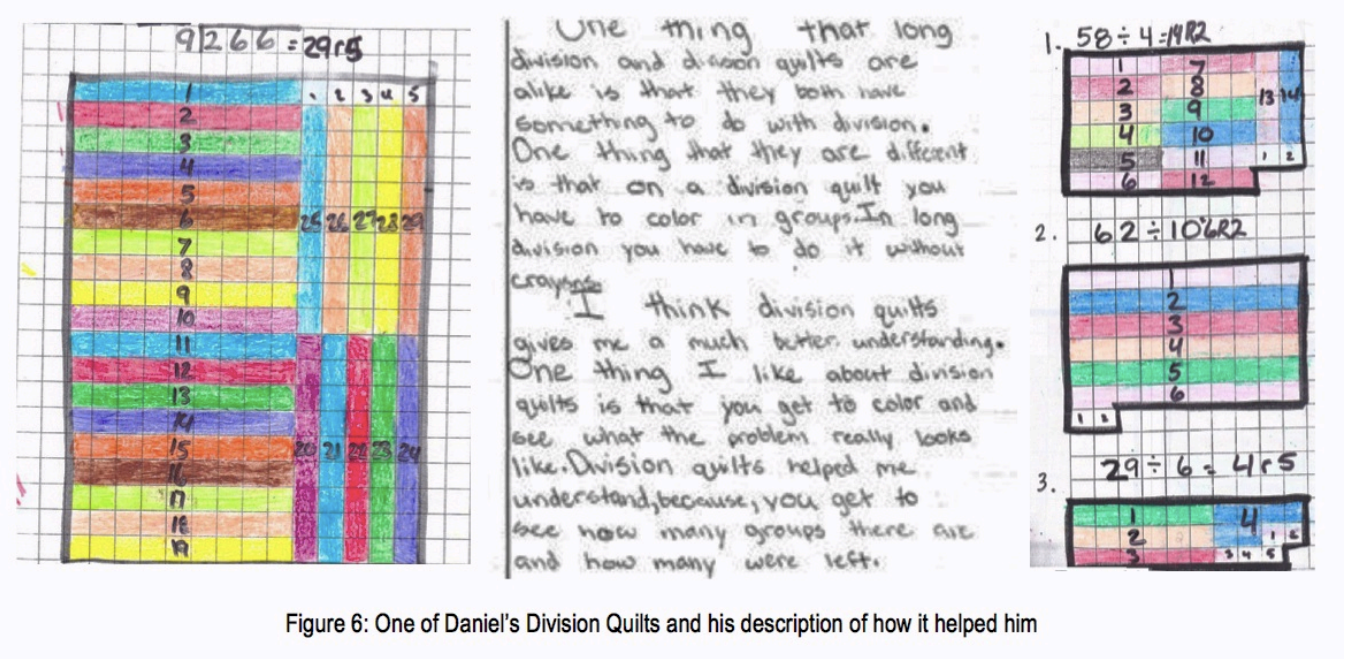

Figure 6: One of Daniel's Division Quilts and his description of how it helped him

Examples of quilts made by Mrs. Lupton's students and their comments on Division Quilts are shown in Figure 6. Richardson and Stein (2008) state that when students make connections with prior knowledge, they are able to

speak more “mathematically.” The use of Division Quilts supports this statement, as exhibited in the students’ comments and models in Figure 6.

Final Thoughts

Division Quilts have a significant impact on a student’s understanding of division and multiplication. Buschman (2003) states that students enjoy problem solving more when the way they solve the problem makes sense to them. Based on students’ perspectives, Division Quilts provide a clear explanation of how to solve division problems, whereas the long division algorithm allows for many possibilities to make mistakes, especially if the students are not confident and knowledgeable of the algorithm. Modeling of division affords opportunities for meeting the needs of students on any learning level. Division Quilts build students’ interest in division and provides a way for them to engage meaningfully when exploring division problems.

References

Buschman, L. (2003). Children who enjoy problem solving. *Teaching Children Mathematics, 9* (9), 539-544.

Common Core State Standards Initiative. (2011). *Mathematics grade 5 introduction. Common Core Standards*. Washington, D.C.: NGA Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO). Retrieved from http://www.corestandards.org/the-standards/mathematics/grade-5/introduction/.

Dark, D. (2007). Making "cute" count. *Teaching Children Mathematics, 14* (3), 153-159.

English, L., Fox, J., & Walters, J. (2005). Problem posing and solving with mathematical modeling. *Teaching Children Mathematics, 12* (3), 156-162.

Ma, L. (1999). *Knowing and teaching mathematics: Teachers’ understanding of fundamental mathematics in China and the United States*. Mahweh, NJ: Lawrence Erlbaum.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

North Carolina Department of Public Instruction. (1999). *North Carolina standard course of study: Mathematics*. Retrieved from http://www.ncpublicschools.org/curriculum/mathematics

Richardson, K., Pratt, S., & Kurtts, S. (2010). Imagery and utilization of an area model as a way of teaching long division: Meeting diverse student needs. *Oklahoma Journal of School Mathematics, 2* (1), 14-24.

Richardson, K., & Stein, C. (2008). Developing spatial sense and communication skills. *Mathematics Teaching in the Middle School, 14* (2), 101-107.

Russell, D. (2009). *Learning long division: Start with the basics*. Retrieved from http://math.about.com/od/arithmetic/ss/Base10Division_2.htm.

Wheatley, G. (1998). Imagery and mathematics learning. *Focus on Learning Problems in Mathematics, 20* (2/3), 65-77.

Wheatley, G., & Reynolds, A. (2010). *Coming to know number: A mathematics activity resource for elementary school teachers, 2nd Ed*. Bethany Beach, DE: Mathematics Learning.